CHAPTER 10

ALGEBRAIC EXPRESSIONS AND IDENTITIES

Exercise 10.1

1. Identify the terms, their numerical as well as literal coefficients in each of the following expressions:

(i) $12x^2yz - 4xy^2$

(ii) $8 + mn + nl - lm$

(iii) $\frac{x^2}{3} + \frac{y}{6} - xy^2$

(iv) $-4p + 2.3q + 1.7r$

Solution:

	Terms	Numerical coefficient	Literal coefficient
i.	$12x^2yz$ $-4xy^2$	12 -4	x^2yz xy^2
ii.	8 mn nl $-lm$	8 1 1 -1	- mn nl ln
iii.	$\frac{x^2}{3}$ $\frac{y}{6}$ $-xy^2$	$\frac{1}{3}$ $\frac{1}{6}$ -1	x^2 y xy^2
iv.	$-4p$ $2.3q$ $1.7r$	-4 2.3 1.7	p q r

2. Identify monomials, binomials, and trinomials from the following algebraic expressions:

(i) $5p \times q \times r^2$

(ii) $3x^2 + y \div 2z$
(iii) $-3 + 7x^2$
(iv) $\frac{(5a^2 - 3b^2 + c)}{2}$
(v) $7x^5 - \frac{3x}{y}$
(vi) $5p \div 3q - 3p^2 \times q^2$
Solution:
(i) $5p \times q \times r^2 = 5pqr^2$
As this algebraic expression has only one term, its therefore a monomial.

(ii) $3x^2 + y \div 2z = \frac{3x^2}{2z} + \frac{y}{2z}$
As this algebraic expression has two terms, its therefore a binomial.

(iii) $-3 + 7x^2$
As this algebraic expression has two terms, its therefore a binomial.

(iv) $\frac{(5a^2 - 3b^2 + c)}{2} = \frac{5a^2}{2} - \frac{3b^2}{2} + \frac{c}{2}$
As this algebraic expression has three terms, its therefore a trinomial.

(v) $7x^5 - \frac{3x}{y}$
As this algebraic expression has two terms, its therefore a binomial.

(vi) $5p \div 3q - 3p^2 \times q^2 = \frac{5p}{3q} - 3p^2q^2$
As this algebraic expression has two terms, its therefore a binomial.

3. Identify which of the following expressions are polynomials. If so, write their degrees.

(i) $\frac{2}{5x^4} - \sqrt{3x^2} + 5x - 1$

(ii) $7x^3 - \frac{3}{x^2} + \sqrt{5}$

(iii) $4a^3b^2 - 3ab^4 + 5ab + \frac{2}{3}$

(iv) $2x^2y - \frac{3}{xy} + 5y^3 + \sqrt{3}$

Solution:

(i) It is a polynomial and the degree of this expression is 4.

(ii) It is not a polynomial.

(iii) It is a polynomial and the degree of this expression is 5.

(iv) It is not a polynomial.

4. Add the following expressions:

(i) ab – bc, bc – ca, ca – ab

(ii) $5p^2q^2 + 4pq + 7$, $3 + 9pq - 2p^2q$

(iii) $l^2 + m^2 + n^2$, $lm + mn$, $mn + nl$, $nl + lm$

(iv) $4x^3 - 7x^2 + 9$, $3x^2 - 5x + 4$, $7x^3 - 11x + 1$, $6x^2 - 13x$

Solution:

(i) ab – bc, bc – ca, ca – ab

On adding the expressions, we have

$\Rightarrow ab - bc + bc - ca + ca - ab = 0$

(ii) $5p^2q^2 + 4pq + 7$, $3 + 9pq - 2p^2q^2$

On adding the expressions, we have

$= 5p^2q^2 + 4pq + 7 + 3 + 9pq – 2p^2q^2$

$= 5p^2q^2 – 2p^2q^2 + 4pq + 9pq + 7 + 3$

$= 3p^2q^2 + 13pq + 10$

(iii) $l^2 + m^2 + n^2$, $lm + mn$, $mn + nl$, $nl + lm$

On adding the expressions, we have

$= l^2 + m^2 + n^2 + lm + mn + mn + nl + nl + lm$

$= l^2 + m^2 + n^2 + 2lm + 2mn + 2nl$

(iv) $4x^3 – 7x^2 + 9$, $3x^2 – 5x + 4$, $7x^3 – 11x + 1$, $6x^2 – 13x$

On adding the expressions, we have

$= 4x^3 – 7x^2 + 9 + 3x^2 – 5x + 4 + 7x^3 – 11^2 + 1 + 6x^2 – 13x$

$= 4x^2 + 7x^3 – 7x^2 + 3x^2 + 6x^2 – 5x – 11x – 13x + 9 + 4 + 1$

$= 11x^3 – 2x^2 – 29x + 14$

5. Subtract:

(i) $8a + 3ab – 2b + 7$ from $14a – 5ab + 7b – 5$

(ii) $8xy + 4yz + 5zx$ from $12xy – 3yz – 4zx + 5xyz$

(iii) $4p^2q – 3pq + 5pq^2 – 8p + 7q -10$ from $18 – 3p – 11q + 5pq –$

$2pq^2 + 5p^2q$
Solution:

(i) Subtracting $8a + 3ab – 2b + 7$ from $14a – 5ab + 7b – 5$, we have

$= (14a – 5ab + 7b – 5) – (8a + 3ab – 2b + 7)$

$= 14a – 5ab + 7b – 5 – 8a – 3ab + 2b – 7$

$= 6a – 8ab + 9ab – 12$

(ii) Subtracting $8xy + 4yz + 5zx$ from $12xy – 3yz – 4zx + 5xyz$, we have

$= (12xy – 3yz – 4zx + 5xyz) – (8xy + 4yz + 5zx)$

$= 12xy – 3yz – 4zx + 5xyz – 8xy – 4yz – 5zx$

$= 4xy – 7yz – 9zx + 5xyz$

(iii) Subtracting $4p^2q – 3pq + 5pq^2 – 8p + 7q – 10$ from $18 – 3p – 11q + 5pq – 2pq^2 + 5p^2q$, we have

$= (18 – 3p – 11q + 5pq – 2pq^2 + 5p^2q) – (4p^2q – 3pq + 5pq^2 – 8p + 7q – 10)$

$= 18 – 3p – 11q + 5pq – 2pq^2 + 5p^2q – 7p^2q + 3pq – 5pq^2 + 8p – 7q + 10$

$= 28 + 5p – 78q + 8pq – 7pq^2 + p^2q$

6. Subtract the sum of $3x^2 + 5xy + 7y^2 + 3$ and $2x^2 – 4xy – 3y^2 + 7$ from $9x^2 – 8xy + 11y^2$
Solution:

First, adding $3x^2 + 5xy + 7y^2 + 3$ and $2x^2 – 4xy – 3y^2 + 7$, we have

$= 3x^2 + 5xy + 7y^2 + 3 + 2x^2 – 4xy – 3y^2 + 7$

$= 5x^2 + xy + 4y^2 + 10$

Now,

Subtracting $5x^2 + xy + 4y^2 + 10$ from $9x^2 – 8xy + 11y^2$

$= (9x^2 – 8xy + 11y^2) – (5x^2 + xy + 4y^2 + 10)$

$= 9x^2 – 8xy + 11y^2 – 5x^2 – xy – 4y^2 – 10$

$= 4x^2 – 9xy + 7y^2 – 10$

7. What must be subtracted from $3a^2 – 5ab – 2b^2 – 3$ to get $5a^2 – 7ab – 3b^2 + 3a$?
Solution:

From the question, its understood that we have to subtract $5a^2 – 7ab – 3b^2 + 3a$ from $3a^2 – 5ab – 2b^2 – 3$

$= 3a^2 – 5ab – 2b^2 – 3 – (5a^2 – 7ab – 3b^2 + 3a)$

$= 3a^2 – 5ab – 2b^2 – 3 – 5a^2 + 7ab + 3b^2 – 3a$

$= -2a^2 + 2ab + b^2 – 3a – 3$

8. The perimeter of a triangle is $7p^2 – 5p + 11$ and two of its sides are $p^2 + 2p – 1$ and $3p^2 – 6p + 3$. Find the third side of the triangle.
Solution:

Given,

Perimeter of a triangle $= 7p^2 – 5p + 11$

And, two of its sides are $p^2 + 2p - 1$ and $3p^2 - 6p + 3$

We know that,

Perimeter of a triangle = Sum of three sides of triangle

$\Rightarrow 7p^2 - 5p + 11 = (p^2 + 2p - 1) + (3p^2 - 6p + 3) +$ (Third side of triangle)

$7p^2 - 5p + 11 = (4p^2 - 4p + 2) +$ (Third side of triangle)

$\Rightarrow$ Third side of triangle $= (7p^2 - 5p + 11) - (4p^2 - 4p + 2)$

$= (7p^2 - 4p^2) + (- 5p + 4p) + (11 - 2)$

$= 3p^2 - p + 9$

Thus, the third side of the triangle is $3p^2 - p + 9$.

Exercise 10.2

1. Find the product of:

(i) $4x^3$ and $-3xy$

(ii) 2xyz and 0

(iii) $-\left(\frac{2}{3}\right)p^2q$, $\left(\frac{3}{4}\right)pq^2$ and 5pqr

(iv) -7ab, $-3a^3$ and $-\left(\frac{2}{7}\right)ab^2$

(v) $-\frac{1}{2}x^2 - \left(\frac{3}{5}\right)xy$, $\left(\frac{2}{3}\right)yz$ and $\left(\frac{5}{7}\right)xyz$

Solution:

Product of:

(i) $4x^3$ and $-3xy = 4x^3 \times (-3xy) = -12x^{3+1}\, y = -12x^4y$

(ii) 2xyz and 0 = 2xyz × 0 = 0

(iii) $\left(-\frac{2}{3}p^2q\right) \times \left(\frac{3}{4}pq^2\right) \times (5pqr)$

$= -\frac{2}{3} \times \frac{3}{4} \times 5 \times p^2q \times pq^2 \times pqr$

$= -\frac{5}{2}p^4q^4r$

(iv) $(-7ab) \times (-3a^3) \times \left(-\frac{2}{7}ab^2\right)$

$= (-7) \times (-3) \times \left(-\frac{2}{7}\right) \times ab \times a^3 \times ab^2$

$= -6a^5b^3$

(v) $\left(-\frac{1}{2}x^2\right) \times \left(-\frac{3}{5}xy\right) \times \left(\frac{2}{3}yz\right) \times \left(\frac{5}{7}xyz\right)$

$= \left(-\frac{1}{2}\right) \times \left(-\frac{3}{5}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{5}{7}\right) \times x^2 \times xy \times yz + xyz$

$= \frac{1}{7}x^4y^3z^2$

2. Multiply:

(i) (3x – 5y + 7z) by – 3xyz

(ii) ($2p^2$ – 3pq + $5q^2$ + 5) by – 2pq

(iii) $\left(\frac{2}{3a^2b} - \frac{4}{5ab^2} + \frac{2}{7ab} + 3\right)$ by 35ab

(iv) ($4x^2$ – 10xy + $7y^2$ – 8x + 4y + 3) by 3xy

Solution:

(i) $-3xyz \times (3x - 5y + 7z)$

$= (-3xyz) \times 3x + (-3xyz) \times (-5y) + (-3xyz) \times (7z)$

$= -9x^2yz + 15xyz^2 - 21xyz^2$

(ii) $-2pq \times (2p^2 - 3pq + 5q^2 + 5)$

$= (-2pq) \times 2p^2 + (-2pq) \times (-3pq) + (-2pq) \times (5q^2) + (-2pq) \times 5$

$= -4p^3q + 6p^2q^2 - 10pq^3 - 10pq$

(iii) $\left(\frac{2}{3a^2b} - \frac{4}{5ab^2} + \frac{2}{7ab} + 3\right)$ by 35ab

$= \left(\frac{2}{3}\right) a^2b \times 35ab - \left(\frac{4}{5}\right) ab^2 \times 35ab + \left(\frac{2}{7}\right) ab \times 35ab + 3 \times 35ab$

$= \left(\frac{70}{3}\right) a^3b^2 - 28a^2b^3 + 10a^2b^2 + 105ab$

(iv) $(4x^2 - 10xy + 7y^2 - 8x + 4y + 3)$ by 3xy

$= 4x^2 \times 3xy - 10xy \times 3xy + 7y^2 \times 3xy - 8x \times 3xy + 4y \times 3xy + 3 \times 3xy$

$= 12x^3y - 30x^2y^2 + 21xy^3 - 24x^2y + 12xy^2 + 9xy$

3. Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively:

(i) (p^2q, pq^2)
(ii) $(5xy, 7xy^2)$
Solution:

(i) Given, sides of a rectangle are p^2q and pq^2

hence,

Area = $p^2q \times pq^2 = p^{2+1} \times q^{2+1} = p^3q^3$

(ii) Given, sides are $5xy$ and $7xy^2$

Hence,

Area = $5xy \times 7xy^2 = 35x^{1+1} \times y^{1+2} = 35x^2y^3$

4. Find the volume of rectangular boxes with the following length, breadth and height respectively:
(i) $5ab, 3a^2b, 7a^4b^2$
(ii) $2pq, 4q^2, 8rp$
Solution:

Given are the length, breadth and height of a rectangular box:

(i) $5ab, 3a^2b, 7a^4b^2$

$\therefore$ Volume = Length × breadth × height

$= 5ab \times 3a^2b \times 7a^4b^2$

$= 5 \times 3 \times 7 \times a^{1+2+4} \times b^{1+1+2}$

$= 105a^7b^4$

(ii) $2pq, 4q^2, 8rp$

∴ Volume = Length × breadth × height

$= 2pq \times 4q^2 \times 8rp$

$= 2 \times 4 \times 8 \times p^{1+1} \times q^{1+2} \times r$

$= 64p^2q^3r$

5. Simplify the following expressions and evaluate them as directed:
(i) $x^2(3 - 2x + x^2)$ for $x = 1$; $x = -1$; $x = \frac{2}{3}$ and $x = -\frac{1}{2}$
(ii) $5xy(3x + 4y - 7) - 3y(xy - x^2 + 9) - 8$ for $x = 2, y = -1$
Solution:

(i) $x^2(3 - 2x + x^2)$

For $x = 1$; $x = -1$; $x = \frac{2}{3}$ and $x = -\frac{1}{2}$

$x^2(3 - 2x + x^2) = 3x^2 - 2x^3 + x^4$

(a) For $x = 1$

$3x^2 - 2x^3 + x^4 = 3(1)^2 - 2(1\ )^3 + (1)^4$

$= 3 \times 1 - 2 \times 1 + 1$

$= 3 - 2 + 1 = 2$

(b) For $x = -1$

$3x^2 - 2x^3 + x^4 = 3(-1)^2 - 2(-1)^3 + (-1)^4$

$= 3 \times 1 - 2 \times (-1) + 1$

$= 3 + 2 + 1 = 6$

(c) For $x = \frac{2}{3}$

$3x^2 - 2x^3 + x^4 = 3\left(\frac{2}{3}\right)^2 - 2\left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^4$

$= 3 \times \left(\frac{4}{9}\right) - 2 \times \left(\frac{8}{27}\right) + \left(\frac{16}{81}\right)$

$= \left(\frac{4}{3}\right) - \left(\frac{16}{27}\right) + \left(\frac{16}{81}\right)$

$= \frac{108 - 48 + 16}{81}$

$= \frac{124 - 48}{81}$

$= 76/81$

(d) For $x = -\frac{1}{2}$

$3x^2 - 2x^3 + x^4 = 3\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^4$

$= 3 \times \left(\frac{1}{4}\right) - 2 \times \left(-\frac{1}{8}\right) + \left(\frac{1}{16}\right)$

$= \left(\frac{3}{4}\right) + \frac{1}{4} + \left(\frac{1}{16}\right)$

$= \frac{12 + 4 + 1}{16} = \frac{17}{16}$

(ii) $5xy(3x + 4y - 7) - 3y(xy - x^2 + 9) - 8$

$= 15x^2y + 20xy^2 - 35xy - 3xy^2 + 3\,x^2y - 21y - 8$

$= 18x^2y + 17xy^2 – 35xy – 27y – 8$

When x = 2, y = -1, we have

$= 18(2)^2 \times (-1) + 17(2)(-1)^2 – 35(2)(-1) – 27(-1) – 8$

$= 18 \times 4 \times (-1) + 17 \times 2 \times 1 – 35 \times 2 \times (-1) – 27 \times (-1) – 8$

$= -74 + 34 + 70 + 27 – 8$

$= 131 – 80 = 51$

6. Add the following:
(i) $4p(2 – p^2)$ and $8p^3 – 3p$
(ii) $7xy(8x + 2y – 3)$ and $4xy^2(3y – 7x + 8)$
Solution:

Adding,

(i) $4p(2 – p^2)$ and $8p^3 – 3p$

$= 8p – 4p^3 + 8p^3 – 3p$

$= 5p + 4p^3$

$= 4p^3 + 5p$

(ii) $7xy(8x + 2y – 3)$ and $4xy^2(3y – 7x + 8)$

$= 56x^2y + 14xy^2 – 21xy + 12xy^3 – 28x^2y^2 + 32xy^2$

$= 12xy^3 – 28x^2y^2 + 56x^2y + 46xy^2 – 21xy$

7. Subtract: (i) $6x(x - y + z) - 3y(x + y - z)$ from $2z(-x + y + z)$ (ii) $7xy(x^2 - 2xy + 3y^2) - 8x(x^2y - 4xy + 7xy^2)$ from $3y(4x^2y - 5xy + 8xy^2)$

Solution:

Subtracting,

(i) $6x(x - y + z) - 3y(x + y - z)$ from $2z(-x + y + z)$

$\Rightarrow 6x^2 - 6xy + 6xz - 3xy - 3y^2 + 3yz$ from $- 2xz + 2yz + 2z^2$

$= (-2xz + 2yz + 2z^2) - (6x^2 - 6xy + 6xz - 3xy - 3y^2 + 3yz)$

$= - 2xz + 2yz + 2z^2 - 6x^2 + 6xy - 6xz + 3xy + 3y^2 - 3yz$

$= 9xy - yz - 8zx - 6x^2 + 3y^2 + 2z^2$

(ii) $7xy(x^2 - 2xy + 3y^2) - 8x(x^2y - 4xy + 7xy^2)$ from $3y(4x^2y - 5xy + 8xy^2)$

$\Rightarrow 7x^3y - 14x^2y^2 + 21xy^3 - 8x^3y + 32x^2y - 56x^2y^2$ from $12x^2y^2 - 15xy^2 + 24xy^3$

$= (12x^2y^2 - 15xy^2 + 24xy^3) - (7x^3y - 14x^2y^2 + 21xy^3 - 8x^3y + 32x^2y - 56x^2y^2$

$= 12x^2y^2 - 15xy^2 + 24xy^3 - 7x^3y + 14x^2y^2 - 12xy^3 + 8x^3y - 32x^2y + 56x^2y^2$

$= 82x^2y^2 + 3xy^3 + x^3y - 15xy^2 - 32x^2y$

Exercise 10.3

1. Multiply:
(i) $(5x - 2)$ by $(3x + 4)$
(ii) $(ax + b)$ by $(cx + d)$
(iii) $(4p - 7)$ by $(2 - 3p)$
(iv) $(2x^2 + 3)$ by $(3x - 5)$
(v) $(1.5a - 2.5b)$ by $(1.5a + 2.56)$
(vi) $\left(\frac{3}{7}p^2 + 4q^2\right)$ by $\left(p^2 - \frac{3}{4}q^2\right)$
Solution:

(i) $(5x - 2)$ by $(3x + 4)$

$= (5x - 2) \times (3x + 4)$

$= 5x\,(3x + 4) - 2\,(3x + 4)$

$= 15x^2 + 20x - 6x - 8$

$= 15x^2 + 14x - 8$

(ii) $(ax + b)$ by $(cx + d)$

$= (ax + b) \times (cx + d)$

$= ax\,(cx + d) + b\,(cx + d)$

$= acx^2 + adx + bcx + bd$

(iii) $(4p - 7)$ by $(2 - 3p)$

$= (4p - 7) \times (2 - 3p)$

$= 4p(2 – 3p) -7(2 – 3p)$

$= 8p – 12p^2 – 14 + 21p$

$= 29p – 12p^2 – 14$

(iv) $(2x^2 + 3)$ by $(3x – 5)$

$= (2x^2 + 3) (3x – 5)$

$= 2x^2(3x – 5) + 3(3x – 5)$

$= 6x^3 – 10x^2 + 9x – 15$

(v) $(1.5a – 2.5b)$ by $(1.5a + 2.5b)$

$= (1.5a – 2.5b) (1.5a + 2.5b)$

$= 1.5a(1.5 + 2.5b) – 2.5b(1.5a + 2.5b)$

$= 2.25a^2 + 3.75ab – 3.75a6 – 6.25b^2$

$= 2.25a^2 – 6.25b^2$

(vi) $\left(\frac{3}{7}p^2 + 4q^2\right)$ By $7\left(p^2 - \frac{3}{4}q^2\right)$

$= \left(\frac{3}{7}p^2 + 4q^2\right) \times 7\left(p^2 - \frac{3}{4}q^2\right)$

$= 7\left(\frac{3}{7}p^2 + 4q^2\right)\left(p^2 - \frac{3}{4}q^2\right)$

$= 7\left[\frac{3}{7}p^2\left(p^2 - \frac{3}{4}q^2\right) + 4q^2\left(p^2 - \frac{3}{4}q^2\right)\right]$

$= 7\left[\frac{3}{7}p^4 - \frac{9}{28}p^2q^2 + 4p^2q^2 - 3q^4\right]$

$= 3p^4 - \frac{9}{4}p^2q^2 + 28p^2q^2 - 21q^4$

$= 3p^4 - \frac{9p^2q^2+112p^2q^2}{4} - 21q^4$

$= 3p^4 - \frac{103}{4}p^2q^2 - 21q^4$

2. Multiply:
(i) (x – 2y + 3) by (x + 2y)
(ii) (3 – 5x + 2x²) by (4x – 5)

Solution:

(i) (x – 2y + 3) by (x + 2y)

= (x – 2y + 3) × (x + 2y)

= x (x + 2y) – 2y(x + 2y) + 3 (x + 2y)

= x^2 + 2xy – 2xy – $4y^2$ + 3x + 6y

= x^2 – $4y^2$ + 3x + 6y

(ii) (3 – 5x + $2x^2$) by (4x – 5)

= (4x – 5) (3 – 5x + $2x^2$)

= 4x(3 – 5x + $2x^2$) – 5(3 – 5x + $2x^2$)

= 12x – $20x^2$ + $8x^3$ – 15 + 25x – $10x^2$

$= 8x^3 – 30x^2 + 37x – 15$

3. Multiply:
(i) $(3x^2 – 2x – 1)$ by $(2x^2 + x – 5)$
(ii) $(2 – 3y – 5y^2)$ by $(2y – 1 + 3y^2)$
Solution:

(i) $(3x^2 – 2x – 1)$ by $(2x^2 + x – 5)$

$= (3x^2 – 2x – 1) (2x^2 + x – 5)$

$= 3x^2(2x^2 + x – 5) – 2x(2x^2 + x – 5) -1(2x^2 + x – 5)$

$= 6x^4 + 3x^3 – 15x^2 – 4x^3 – 2x^2 + 10x – 2x^2 – x + 5$

$= 6x^4 – x^3 – 19x^2 + 9x + 5$

(ii) $(2 – 3y – 5y^2)$ by $(2y- 1 + 3y^2)$

$= (2 – 3y – 5y^2) \times (2y- 1 + 3y^2)$

$= 2(2y – 1 + 3y^2) – 3y (2y – 1 + 3y^2) -5y^2(2y – 1 + 3y^2)$

$= 4y – 2 + 6y^2 – 6y^2 + 3y – 9y^3 – 10y^3 + 5y^2 – 15y^4$

$= -15y^4 – 19y^3 + 5y^2 + 7y – 2$

4. Simplify:
(i) $(x^2 + 3) (x – 3) + 9$
(ii) $(x + 3) (x – 3) (x + 4) (x – 4)$
(iii) $(x + 5) (x + 6) (x + 7)$

(iv) $(p + q - 2r)(2p - q + r) - 4qr$
(v) $(p + q)(r + s) + (p - q)(r - s) - 2(pr + qs)$
(vi) $(x + y + z)(x - y + z) + (x + y - z)(-x + y + z) - 4zx$
Solution:

(i) $(x^2 + 3)(x - 3) + 9$

$= x^2 (x - 3) + 3(x - 3) + 9$

$= x^2 - 3x^2 + 3x - 9 + 9$

$= x^3 - 3x^2 + 3x$

(ii) $(x + 3)(x - 3)(x + 4)(x - 4)$

$= \{(x + 3)(x - 3)\} \times \{(x + 4)(x - 4)\}$

$= \{x (x - 3) + 3 (x - 3)\} \{x (x - 4) + 4 (x - 4)\}$

$= (x^2 - 3x + 3x - 9) \{x^2 - 4x + 4x - 16\}$

$= (x^2 - 9)(x^2 - 16)$

$= x^2 (x^2 - 16) - 9 (x^2 - 16)$

$= x^4 - 16x^2 - 9x^2 + 144$

$= x^4 - 25x^2 + 144$

(iii) $(x + 5)(x + 6)(x + 7)$

$= \{(x + 5) \times (x + 6)\} (x + 7)$

$= (x^2 + 6x + 5x + 30)(x + 7)$

$= (x^2 + 11x + 30)(x + 7)$

$= x(x^2 + 11x + 30) + 7(x^2 + 11x + 30)$

$= x^3 + 11x^2 + 30x + 7x^2 + 77x + 210$

$= x^3 + 18x^2 + 107x + 210$

(iv) $(p + q - 2r)(2p - q + r) - 4qr$

$= p(2p - q + r) + q(2p - q + r) - 2r(2p - q + r) - 4qr$

$= 2p^2 - pq + pr + 2pq - q^2 + qr - 4pr + 2qr - 2r^2 - 4qr$

$= 2p^2 - q^2 - 2r^2 + pq - 3pr - 2qr$

(v) $(p + q)(r + s) + (p - q)(r - s) - 2(pr + qs)$

$= (pr + ps + qr + qs) + (pr - ps - qr + qs) - 2pr - 2qs$

$= 0$

(vi) $(x + y + z)(x - y + z) + (x + y - z)(-x + y + z) - 4zx$

$= x^2 - xy + xz + xy - y^2 + yz + xz - yz + z^2 - x^2 + xy + xz - xy + x^2 + yx + xz - yz - z^2 - 4zx$

$= 0$

5. If two adjacent sides of a rectangle are $5x^2 + 25xy + 4y^2$ and $2x^2 - 2xy + 3y^2$, find its area.

Solution:

Given,

The adjacent sides of a rectangle are $5x^2 + 25xy + 4y^2$ and $2x^2 – 2xy + 3y^2$

So,

Area of rectangle = Product of two adjacent sides

$= (5x^2 + 25xy + 4y^2) (2x^2 – 2xy + 3y^2)$

$= 10x^4 – 10x^3y + 15x^2y^2 + 50x^3y – 50x^2y^2 + 75xy^3 + 8x^2y^2 – 8xy^3 + 12y^4$

$= 10x^4 + 40x^3y – 27x^2y^2 + 67xy^3 + 12y^4$

Thus,

The area of the rectangle is $10x^4 + 40x^3y – 27x^2y^2 + 67xy^3 + 12y^4$.

Exercise 10.4

1. Divide:

(i) – $39pq^2r^5$ by – $24p^3q^3r$

(ii) –a^2b^3 by a^3b^2

Solution:

(i) – $39pq^2r^5$ (÷) – $24p^3q^3r$

$= \frac{-39pq^2r^5}{-24p^3q^3r}$

$= \left(\frac{-39}{-24}\right) \times \left(\frac{pq^2r^5}{p^3q^3r}\right)$

$= \frac{13}{8} \times \frac{r^4}{p^2q} = \frac{13r^4}{4p^2q}$

(ii) $-\frac{3}{4}a^2b^3 \div \frac{6}{7}a^3b^2$

$= \frac{-\frac{3}{4}a^2b^3}{\frac{6}{7}a^3b^2}$

$= \left(\frac{\frac{-3}{4}}{\frac{6}{7}}\right) \times \left(\frac{a^2b^3}{a^3b^2}\right)$

$= \left(\frac{-3}{4} \times \frac{7}{6}\right) \times \left(\frac{b}{a}\right)$

$= \frac{-7}{8} \times \frac{b}{a} = \frac{-7b}{8a}$

2. Divide:

(i) $9x^4 - 8x^3 - 12x + 3$ by $3x$

(ii) $14p^2q^3 - 32p^3q^2 + 15pq^2 - 22p + 18q$ by $- 2p^2q$.

Solution:

(i) $\frac{9x^4-8x^3-12x+3}{3x}$

$= \frac{9x^4}{3x} - \frac{8x^3}{3x} - \frac{12x}{3x} + \frac{3}{3x}$

$= 3x^3 - \frac{8}{3}x^2 - 4 + \frac{1}{x}$

(ii) $\frac{14p^2q^3-32p^3q^2+15pq^2-22p+18q}{-2p^2q}$

$= \frac{14p^2q^3}{-2p^2q} - \frac{32p^3q^2}{-2p^2q} + \frac{15pq^2}{-2p^2q} - \frac{22p}{-2p^2q} + \frac{18q}{-2p^2q}$

$= -7q^2 + 16pq - \frac{15q}{2p} + \frac{11}{pq} - \frac{9}{p^2}$

3. Divide:
(i) $6x^2 + 13x + 5$ by $2x + 1$
(ii) $1 + y^3$ by $1 + y$
(iii) $5 + x - 2x^2$ by $x + 1$
(iv) $x^3 - 6x^2 + 12x - 8$ by $x - 2$
Solution:

(i) $6x^2 + 13x + 5 \div 2x + 1$

$$\begin{array}{r|l|l}
2x+1 & 6x^2 + 13x + 5 & 3x+5 \\
 & 6x^2 + 3x & \\
 & \ \ - \quad\ \ - & \\
\hline
 & \quad 10x + 5 & \\
 & \quad 10x + 5 & \\
 & \quad\ - \quad\ - & \\
\hline
 & \qquad 0 & \\
\hline
\end{array}$$

∴ Quotient = $3x + 5$ and remainder = 0

(ii) $1 + y^3 \div 1 + y$

$$
\begin{array}{r|l|l}
y+1 & y^3 + 1 & y^2 - y + 1 \\
 & y^3 + y^2 & \\
 & - \quad - & \\ \hline
 & \quad -y^2 + 1 & \\
 & \quad -y^2 - y & \\
 & \quad + \quad + & \\ \hline
 & \qquad\quad y + 1 & \\
 & \qquad\quad y + 1 & \\
 & \qquad\quad - \quad - & \\ \hline
 & \qquad\qquad 0 & \\ \hline
\end{array}
$$

∴ Quotient = $y^2 - y + 1$ and remainder = 0

(iii) On arranging the terms of dividend in descending order of powers of x and then dividing, we get

$-2x^2 + x + 5 \div x + 1$

$$
\begin{array}{r|l|l}
x+1 & -2x^2 + x + 5 & -2x + 3 \\
 & -2x^2 - 3x & \\
 & + \qquad + & \\ \hline
 & \qquad 3x + 5 & \\
 & \qquad 3x + 3 & \\
 & \qquad - \quad - & \\ \hline
 & \qquad\quad 2 & \\ \hline
\end{array}
$$

∴ Quotient = $-2x + 3$ and remainder = 2

(iv) $x^3 - 6x^2 + 12x - 8 \div x - 2$

$$
\begin{array}{r|l|l}
x-2 & x^3 - 6x^2 + 12x - 8 & x^2 - 4x + 4 \\
 & x^3 - 2x^2 & \\
 & - \quad + & \\
\hline
 & \quad -4x^2 + 12x & \\
 & \quad -4x^2 + 8x & \\
 & \quad + \qquad - & \\
\hline
 & \qquad\qquad 4x - 8 & \\
 & \qquad\qquad 4x - 8 & \\
 & \qquad\qquad - \quad + & \\
\hline
 & \qquad\qquad\quad 0 & \\
\hline
\end{array}
$$

∴ Quotient = $x^2 - 4x + 4$ and remainder = 0

4. Divide:
(i) $6x^3 + x^2 - 26x - 25$ by $3x - 7$
(ii) $m^3 - 6m^2 + 7$ by $m - 1$
Solution:

(i) $6x^3 + x^2 - 26x - 25 \div 3x - 7$

$$
\begin{array}{r|l|l}
3x-7 & 6x^3 + x^2 - 26x - 25 & 2x^2 + 5x + 3 \\
 & 6x^3 - 14x^2 & \\
 & - \quad + & \\
\hline
 & \quad 15x^2 - 26x - 25 & \\
 & \quad 15x^2 - 35x & \\
 & \quad - \qquad + & \\
\hline
 & \qquad\qquad 9x - 25 & \\
 & \qquad\qquad 9x - 21 & \\
 & \qquad\qquad - \quad + & \\
\hline
 & \qquad\qquad\qquad -4 & \\
\hline
\end{array}
$$

∴ Quotient = $2x^2 + 5x + 3$ and remainder = – 4

(ii) $m^3 - 6m^2 + 7 \div m - 1$

$$
\begin{array}{r|l|l}
m-1 & m^3 - 6m^2 + 7 & m^2 - 5m - 5 \\
 & m^3 - m^2 & \\
 & - \quad + & \\
\hline
 & \quad -5m^2 \qquad + 7 & \\
 & \quad -5m^2 + 5m & \\
 & \quad + \qquad - & \\
\hline
 & \qquad -5m + 7 & \\
 & \qquad -5m + 5 & \\
 & \qquad + \qquad - & \\
\hline
 & \qquad\qquad 2 & \\
\hline
\end{array}
$$

∴ Quotient = $m^2 - 5m - 5$ and remainder = 2.

5. Divide:
(i) $a^3 + 2a^2 + 2a + 1$ by $a^2 + a + 1$
(ii) $12x^3 - 17x^2 + 26x - 18$ by $3x^2 - 2x + 5$
Solution:

(i) $a^3 + 2a^2 + 2a + 1 \div a^2 + a + 1$

$$
\begin{array}{r|l|l}
a^2 + a + 1 & a^3 + 2a^2 + 2a + 1 & a + 1 \\
 & a^3 + a^2 + a & \\
 & - \quad - \quad - & \\
\hline
 & \qquad a^2 + a + 1 & \\
 & \qquad a^2 + a + 1 & \\
\hline
 & \qquad\quad 0 & \\
\hline
\end{array}
$$

∴ Quotient = $a + 1$ and remainder = 0.

(ii) $12x^3 – 17x^2 + 26x – 18 \div 3x^2 – 2x + 5$

$$
\begin{array}{r|l|l}
3x^2 - 2x + 5 & 12x^3 - 17x^2 + 26x - 18 & 4x - 3 \\
 & 12x^3 - \ \ 8x^2 \ + 20x & \\
 & - \qquad + \qquad\quad - & \\
\hline
 & \qquad\quad -9x^2 + 6x - 18 & \\
 & \qquad\quad -9x^2 + 6x - 15 & \\
 & \qquad\quad + \qquad - \qquad + & \\
\hline
 & \qquad\qquad\qquad\qquad\quad -3 & \\
\hline
\end{array}
$$

∴ Quotient = 4x – 3 and remainder = -3

6. If the area of a rectangle is $8x^2 – 45y^2 + 18xy$ and one of its sides is $4x + 15y$, find the length of adjacent side.

Solution:

Given,

Area of rectangle = $8x^2 – 45y^2 + 18xy$ and, one side = $4x + 15y$

∴ Second (adjacent) side = $\frac{\text{Area of rectangle}}{\text{One side}}$

= $8x^2 – 45y^2 + 18xy \div 4x + 15y$

$$
\begin{array}{r|l|l}
4x + 15y & 8x^2 + 18xy - 45y^2 & 2x - 3y \\
 & 8x^2 + 30x & \\
 & - \quad\ - & \\
\hline
 & \qquad\quad -12xy - 45y^2 & \\
 & \qquad\quad -12xy - 45y^2 & \\
 & \qquad\quad + \qquad\quad + & \\
\hline
 & \qquad\qquad\qquad\quad 0 & \\
\hline
\end{array}
$$

Thus, length of the adjacent side is 2x – 3y.

Exercise 10.5

1. Using suitable identities, find the following products:

(i) (3x + 5) (3x + 5)

(ii) (9y – 5) (9y – 5)

(iii) (4x + 11y) (4x – 11y)

(iv) $\left(\frac{3m}{2}+\frac{2n}{3}\right)\left(\frac{3m}{2}-\frac{2n}{3}\right)$

(v) $\left(\frac{2}{a}+\frac{5}{b}\right)\left(2a+\frac{5}{b}\right)$

(vi) $\left(\frac{p^2}{2}+\frac{2}{q^2}\right)\left(\frac{p^2}{2}-\frac{2}{q^2}\right)$

Solution:

(i) $(3x + 5)(3x + 5)$

$= (3x + 5)^2$

$= (3x)^2 + 2 \times 3x \times 5 + (5)^2$ [Using, $(a + b)^2 = a^2 + 2ab + b^2$]

$= 9x^2 + 30x + 25$

(ii) $(9y - 5)(9y - 5)$

$= (9y - 5)^2$

$= (9y)^2 - 2 \times 9y \times 5 + (5)^2$ [Using, $(a - b)^2 = a^2 - 2ab + b^2$]

$= 81y^2 - 90y + 25$

(iii) $(4x + 11y)(4x - 11y)$

$= (4x)^2 - (11y)^2$

$= 16x^2 - 121y^2$ [Using, $(a + b)(a - b) = a^2 - b^2$]

(iv) $\left(\frac{3m}{2}+\frac{2n}{3}\right)\left(\frac{3m}{2}-\frac{2n}{3}\right)$

$= \left(\frac{3m}{2}\right)^2 - \left(\frac{2n}{3}\right)^2$

$= \frac{9m^2}{4} - \frac{4n^2}{9}$ [Using, $(a + b)(a – b) = a^2 – b^2$]

(v) $\left(\frac{2}{a} + \frac{5}{b}\right)\left(2a + \frac{5}{b}\right)$

$= \left(\frac{2}{a} + \frac{5}{b}\right)^2$

$= \left(\frac{2}{a}\right)^2 + 2\left(\frac{2}{a}\right)\left(\frac{5}{b}\right) + \left(\frac{5}{b}\right)^2$ [Using, $(a + b)^2 = a^2 + 2ab + b^2$]

$= \frac{4}{a^2} + \frac{20a}{b} + \frac{25}{b^2}$

(vi) $\left(\frac{p^2}{2} + \frac{2}{q^2}\right)\left(\frac{p^2}{2} - \frac{2}{q^2}\right)$

$= \left(\frac{p^2}{2}\right)^2 - \left(\frac{2}{q^2}\right)^2$ [Using, $(a + b)(a – b) = a^2 – b^2$]

$= \frac{p^4}{4} - \frac{4}{q^4}$

2. Using the identities, evaluate the following:
(i) 81^2
(ii) 97^2
(iii) 105^2
(iv) 997^2
(v) 6.1^2
(vi) 496 × 504
(vii) 20.5 × 19.5
(viii) 9.62
Solution:
(i) $(81)^2 = (80 + 1)^2$

$= (80)^2 + 2 \times 80 \times 1 + (1)^2$ [Using, $(a + b)^2 = a^2 + 2ab + b^2$]

$= 6400 + 160 + 1$

$= 6561$

(ii) $(97)^2 = (100 - 3)^2$

$= (100)^2 - 2 \times 100 \times 3 + (3)^2$ [Using, $(a - b)^2 = a^2 - 2ab + b^2$]

$= 10000 - 600 + 9$

$= 10009 - 600$

$= 9409$

(iii) $(105)^2 = (100 + 5)^2$

$= (100)^2 + 2 \times 100 \times 5 + (5)^2$ [Using, $(a + b)^2 = a^2 + 2ab + b^2$]

$= 10000 + 1000 + 25$

$= 11025$

(iv) $(997)^2 = (1000 - 3)^2$

$= (1000)^2 - 2 \times 1000 \times 3 + (3)^2$ [Using, $(a - b)^2 = a^2 - 2ab + b^2$]

$= 1000000 - 6000 + 9$

$= 1000009 - 6000$

$= 994009$

(v) $(6.1)^2 = (6 + 0.1)^2$

$= (6)^2 + 2 \times 6 \times 0.1 + (0.1)^2$ [Using, $(a + b)^2 = a^2 + 2ab + b^2$]

$= 36 + 1.2 + 0.01$

$= 37.21$

(vi) 496×504

$= (500 - 4)(500 + 4)$ [Using, $(a + b)(a - b) = a^2 - b^2$]
$= (500)^2 - (4)^2$

$= 250000 - 16$

$= 249984$

(vii) 20.5×19.5

$= (20 + 0.5)(20 - 0.5)$ [Using, $(a + b)(a - b) = a^2 - b^2$]
$= (20)^2 - (0.5)^2$

$= 400 - 0.25$

$= 399.75$

(viii) $(9.6)^2 = (10 - 0.4)^2$

$= (10)^2 - 2 \times 10 \times 0.4 + (0.4)^2$ [Using, $(a - b)^2 = a^2 - 2ab + b^2$]
$= 100 - 8.0 + 0.16$

$= 92.16$

3. Find the following squares, using the identities:

(i) $\mathbf{(pq + 5r)^2}$ **(ii)** $\left(\frac{5a}{2} - \frac{3b}{5}\right)^2$

(iii) $\left(\sqrt{2a} + \sqrt{3b}\right)^2$ **(iv)** $\left(\frac{2x}{3y} - \frac{3y}{2x}\right)^2$

Solution:

(i) $(pq + 5r)^2$

$= (pq)^2 + 2 \times pq \times 5r + (5r)^2$ [Using, $(a + b)^2 = a^2 + 2ab + b^2$]
$= p^2q^2 + 10pqr + 25r^2$

(ii) $\left(\frac{5a}{2}-\frac{3b}{5}\right)^2$

$= \left(\frac{5a}{2}\right)^2 - 2 \times \left(\frac{5a}{2}\right) \times \left(\frac{-3b}{5}\right) + \left(\frac{3b}{5}\right)^2$ [Using, $(a - b)^2 = a^2 - 2ab + b^2$]

$= \frac{25a^2}{4} - 3ab + \frac{9b^2}{25}$

(iii) $\left(\sqrt{2a} + \sqrt{3b}\right)^2$

$= \left(\sqrt{2a}\right)^2 + 2 \times \sqrt{2}a \times \sqrt{3}b + \left(\sqrt{3}b\right)^2$

[Using, $(a + b)^2 = a^2 + 2ab + b^2$]

$= 2a^2 + 2\sqrt{6}ab + 3b^2$

(iv) $\left(\frac{2x}{3y}-\frac{3y}{2x}\right)^2$

$= \left(\frac{2x}{3y}\right)^2 - 2 \times \frac{2x}{2y} \times \frac{3y}{2x} + \left(\frac{3y}{2x}\right)^2$ $\{(a-b)^2 = a^2 - 2ab + b^2\}$

$= \frac{4x^2}{9y^2} - 2 + \frac{9y^2}{4x^2}$

4. Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$, find the following products:
(i) $(x + 7)(x + 3)$
(ii) $(3x + 4)(3x - 5)$
(iii) $(p^2 + 2q)(p^2 - 3q)$
(iv) $(abc + 3)(abc - 5)$
Solution:
(i) $(x + 7)(x + 3)$

$= (x)^2 + (7 + 3)x + 7 \times 3$

$= x^2 + 10x + 21$

(ii) $(3x + 4)(3x - 5)$

$= (3x)^2 + (4 - 5)(3x) + 4 \times (-5)$

$= 9x^2 - 3x - 20$

(iii) $(p^2 + 2q)(p^2 - 3q)$

$= (p^2)^2 + (2q - 3q)p^2 + 2q \times (-3q)$

$= p^4 - p^2q - 6pq$

(iv) $(abc + 3)(abc - 5)$

$= (abc)^2 + (3 - 5)abc + 3 \times (-5)$

$= a^2b^2c^2 - 2abc - 15$

5. Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$, evaluate the following:
(i) 203×204
(ii) 8.2×8.7
(iii) 107×93
Solution:
(i) 203×204

$= (200 + 3)(200 + 4)$

$= (200)^2 + (3 + 4) \times 200 + 3 \times 4$

$= 40000 + 1400 + 12$

$= 41412$

(ii) 8.2×8.7

$= (8 + 0.2)(8 + 0.7)$

$= (8)^2 + (0.2 + 0.7) \times 8 + 0.2 \times 0.7$

$= 64 + 8 \times (0.9) + 0.14$

$= 64 + 7.2 + 0.14$

$= 71.34$

(iii) 107×93

$= (100 + 7)(100 – 7)$

$= (100)^2 + (7 – 7) \times 100 + 7 \times (-7)$

$= 10000 + 0 – 49$

$= 9951$

6. Using the identity $a^2 – b^2 = (a + b)(a – b)$, find
(i) $53^2 – 47^2$
(ii) $(2.05)^2 – (0.95)^2$
(iii) $(14.3)^2 – (5.7)^2$
Solution:
(i) $53^2 – 47^2$

$= (50 + 3)(50 – 3)$

$= (50)^2 – (3)^2$

$= 2500 - 9$

$= 2491$

(ii) $(2.05)^2 - (0.95)^2$

$= (2.05 + 0.95)\ (2.05 - 0.95)$

$= 3 \times 1.10$

$= 3.3$

(iii) $(14.3)^2 - (5.7)^2$

$= (14.3 + 5.7)\ (14.3 - 5.7)$

$= 20 \times 8.6$

$= 172$

7. Simplify the following:

(i) $(2x + 5y)^2 + (2x - 5y)^2$

(ii) $\left(\frac{7}{2}a - \frac{5}{2}b\right)^2 - \left(\frac{5}{2}a - \frac{7}{2}b\right)^2$

(iii) $(p^2 - q^2r)^2 + 2p^2q^2r$

Solution:

(i) $(2x + 5y)^2 + (2x - 5y)^2$ [Using, $(a \pm b)^2 = a^2 \pm 2ab + b^2$]

$= (2x)^2 + 2 \times 2x \times 5y + (5y)^2 + (2x)^2 - 2 \times 2x \times 5y + (5y)^2$

$= 4x^2 + 20xy + 25y^2 + 4x^2 - 20xy + 25y^2$

$= 8x^2 + 50y^2$

(ii) $\left(\frac{7}{2}a - \frac{5}{2}b\right)^2 - \left(\frac{5}{2}a - \frac{7}{2}b\right)^2$

$= \left[\left(\frac{7}{2}a\right)^2 - 2 \times \frac{7}{2}a \times \frac{5}{2}b - \left(\frac{5}{2}b\right)^2\right] - \left[\left(\frac{5}{2}a\right)^2 - 2 \times \frac{5}{2}a \times \frac{7}{2}b - \left(\frac{7}{2}b\right)^2\right]$

$= \left[\frac{49}{4}a^2 - \frac{35}{2}ab + \frac{25}{4}b^2\right] - \left[\frac{25}{4}a^2 - \frac{35}{2}ab + \frac{49}{4}b^2\right]$

$= \frac{49}{4}a^2 - \frac{35}{2}ab + \frac{25}{4}b^2 - \frac{25}{4}a^2 + \frac{35}{2}ab - \frac{49}{4}b^2$

$= \frac{49}{4}a^2 - \frac{25}{4}a^2 + \frac{25}{4}b^2 - \frac{49}{4}a^2$

$= \frac{24}{4}a^2 + \frac{-24}{4}b^2$

$= 6a^2 - 6b^2$

(iii) $(p^2 - q^2r)^2 + 2p^2q^2r$ [Using, $(a - b)^2 = a^2 - 2ab + b^2$]

$= (p^2)^2 - 2 \times p^2 \times q^2r + (q^2r)^2 + 2p^2q^2r$

$= p^4 - 2p^2q + q^4r^2 + 2p^2q^2r$

$= p^4 + q^4r^2$

8. Show that:

(i) $(4x + 7y)^2 - (4x - 7y)^2 = 112xy$

(ii) $\left(\frac{3}{7}p - \frac{7}{6}q\right)^2 + pq = \frac{9}{49}p^2 + \frac{49}{36}q^2$

(iii) $(p - q)(p + q) + (q - r)(q + r) + (r - p)(r + p) = 0$

Solution:

(i) Taking LHS, we have

LHS $= (4x + 7y)^2 - (4x - 7y)^2$ [Using, $(a \pm b)^2 = a^2 \pm 2ab + b^2$]

$= [(4x)^2 + 2 \times 4x \times 7y + (7y)^2] - [(4x)^2 - 2 \times 4x + 7y + (7y)^2]$

$= (16x^2 + 56xy + 49y^2) - (16x^2 - 56xy + 49y^2)$

$= 16x^2 + 56xy + 49y^2 - 16x^2 + 56xy - 49y^2$

$= 112xy = \text{RHS}$

(ii) Taking LHS, we have

LHS = $\left(\frac{3}{7}p - \frac{7}{6}q\right)^2 + pq$

$= \left(\frac{3}{7}p\right)^2 - 2 \times \frac{3}{7}p \times \frac{7}{6}q + \left(\frac{7}{6}q\right)^2 + pq$

$\{(a-b)^2 = a^2 - 2ab + b^2\}$

$= \frac{9}{49}p^2 - pq + \frac{49}{36}q^2 + pq$

$= \frac{9}{49}p^2 + \frac{49}{36}q^2 = \text{RHS}$

(iii) Taking LHS, we have

LHS = $(p - q)(p + q) + (q - r)(q + r) + (r - p)(r + p)$

$= p^2 - q^2 + q^2 - r^2 + r^2 - p^2$ [Using, $(a + b)(a - b) = a^2 - b^2$]

$= 0 = \text{RHS}$

9. If $x + \frac{1}{x} = 2$, evaluate:

(i) $x^2 + \frac{1}{x^2}$ (ii) $x^4 + \frac{1}{x^4}$

Solution:

(i) We have, $x + \frac{1}{x} = 2$

On squaring on both sides, we get

$\left(x + \frac{1}{x}\right)^2 = 2^2$

$x^2 + 2 \times x \times \frac{1}{x} + \frac{1}{x^2} = 4$

$x^2 + 2 + \frac{1}{x^2} = 4$

$x^2 + \frac{1}{x^2} = 4 - 2$

Thus,

$x^2 + \frac{1}{x^2} = 2$

(ii) Again squaring, we get

$\left(x^2 + \frac{1}{x^2}\right)^2 = 2^2$

$x^4 + 2 \times x^2 \times \frac{1}{x^2} + \frac{1}{x^4} = 4$

$x^4 + 2 + \frac{1}{x^4} = 4$

$x^4 + \frac{1}{x^4} = 4 - 2$

Thus,

$x^4 + \frac{1}{x^4} = 2$

10. If $x - \frac{1}{x} = 7$, evaluate:

(i) $x^2 + \frac{1}{x^2}$ **(ii) $x^4 + \frac{1}{x^4}$**

Solution:

We have, $x - \frac{1}{x} = 7$

On squaring on both sides, we get

$\left(x - \frac{1}{x}\right)^2 = 7^2$

$x^2 - 2 \times x^2 \times \frac{1}{x} + \frac{1}{x^2} = 49$

$x^2 - 2 + \frac{1}{x^2} = 49$

$x^2 + \frac{1}{x^2} = 49 + 2$

Thus,

$x^2 + \frac{1}{x^2} = 51$

(ii) Again squaring, we get

$\left(x^2 + \frac{1}{x^2}\right)^2 = 51^2$

$x^4 + \frac{1}{x^4} + 2 \times x^2 \times \frac{1}{x^2} = 2601$

$x^4 + \frac{1}{x^4} + 2 = 2601$

$x^4 + \frac{1}{x^4} = 2601 - 2$

Thus,
$x^4 + \frac{1}{x^4} = 2599$

11. If $x^2 + \frac{1}{x^2} = 23$, evaluate:
(i) $x + \frac{1}{x}$ **(ii) $x - \frac{1}{x}$**
Solution:
We have, $x^2 + \frac{1}{x^2} = 23$

(i) $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$= 23 + 2$

= 25

Taking square root on both sides, we get

$\left(x + \frac{1}{x}\right) = \pm 5$

Thus, $x + \frac{1}{x} = 5$ or -5

(ii) $\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$

= 23 – 2

= 21

Taking square root on both sides, we get

$\left(x + \frac{1}{x}\right) = \pm\sqrt{21}$

Thus, $x + \frac{1}{x} = \sqrt{21}$ or $-\sqrt{21}$

12. If a + b = 9 and ab = 10, find the value of $a^2 + b^2$.

Solution:

Given,

a + b = 9 and ab = 10

Now, squaring a + b = 9 on both sides, we have

$(a + b)^2 = (9)$

$a^2 + b^2 + 2ab = 81$

$a^2 + b^2 + 2 \times 10 = 81$

$a^2 + b^2 + 20 = 81$

$a^2 + b^2 = 81 - 20 = 61$

$\therefore a^2 + b^2 = 61$

13. If a – b = 6 and $a^2 + b^2 = 42$, find the value of
Solution:

Given

$a - b = 6$ and $a^2 + b^2 = 42$

$a - b = 6$

Now, squaring $a - b = 6$ on both sides, we have

$(a - b)^2 = (6)^2$

$a^2 + b^2 - 2ab = 36$

$42 - 2ab = 36$

$2ab = 42 - 36 = 6$

$ab = \frac{6}{2} = 3$

$\therefore ab = 3$

14. If $a^2 + b^2 = 41$ and ab = 4, find the values of
(i) a + b
(ii) a – b
Solution:

Given, $a^2 + b^2 = 41$ and $ab = 4$

(i) $(a + b)^2 = a^2 + b^2 + 2ab$

$= 41 + 2 \times 4$

$= 41 + 8$

$= 49$

$\therefore a + b = \pm 7$

(ii) $(a - b)^2 = a^2 + b^2 - 2ab$

$= 41 - 2 \times 4$

$= 41 - 8$

$= 33$

$\therefore a - b = \pm \sqrt{33}$

Mental Maths

Question 1: Fill in the blanks:

(i) A symbol which has a fixed value is called a ……….

(ii) A symbol which can be given various numerical values is called ……….. or ………

(iii) The various parts of an algebraic expression separated by + or – sign are called ………..

(iv) An algebraic expression having ………… terms is called a binomial.

(v) Each of the quantity (constant or literal) multiplied together to form a product is called a ………… of the product.

(vi) The terms having same literal coefficients are called a ………… otherwise they are called …………

(vii) Degree of the polynomial is the greatest sum of the powers of ………… in each term.

(viii)An identity is an equality which is true for ………… of variables in it.

(ix) $(a + b)^2 =$ …………

(x) $(x + a)(x + b) = x^2 +$ ………… + ab.

(xi) Dividend = ………… + remainder.

Solution:

(i) A symbol which has a fixed value is called a constant.

(ii) A symbol which can be given various numerical values is called variable or literal.

(iii) The various parts of an algebraic expression separated by + or – sign are called terms.

(iv) An algebraic expression having two terms is called a binomial.

(v) Each of the quantity (constant or literal) multiplied together to form a product is called a factor of the product.

(vi) The terms having same literal coefficients are called alike terms otherwise they are called unlike terms.

(vii) Degree of the polynomial is the greatest sum of the

powers of variables in each term.
(viii)An identity is an equality which is true
for all values of variables in it.
(ix) $(a + b)^2 = a^2 + 2ab + b^2$.
(x) $(x + a)(x + b) = x^2 + (a + b)x + ab$.
(xi) Dividend = divisor × quotient + remainder.

Question 2: State whether the following statements are true (T) or false (F):
(i) An algebraic expression having only one term is called a monomial.
(ii) A symbol which has fixed value is called a literal.
(iii) The term of an algebraic expression having no literal factor is called its constant term,
(iv) In any term of an algebraic expression the constant part is called literal coefficient of the term.
(v) An algebraic expression is called polynomial if the powers of the variables involved in it in each term are non-negative integers.
(vi) $5x^2y^2z$, $3x^2zy^2$, $-\frac{4}{5}zx^2y^2$ are unlike terms.
(vii) $5x + \frac{2}{x} + 7$ is a polynomial of degree 1.
(viii) $3x^2y + 7x + 8y + 9$ is a polynomial of degree 3.
(ix) Numerical coefficient of $-7x^3y$ is 7.
(x) An equation is true for all values of variables in it.
(xi) $(a - b)^2 + 2ab = a^2 + b^2$.
Solution:
(i) An algebraic expression having only one term is called a monomial. True.
(ii) A symbol which has fixed value is called a literal. False
Correct:
It is called constant.

(iii) The term of an algebraic expression having no literal factor is called its constant term. True.
(iv) In any term of an algebraic expression, the constant part is called literal coefficient of the term. False
Correct:
It is called the constant coefficient.

(v) An algebraic expression is called polynomial if the powers of the variables involved in it in each term are non-negative integers. True.

(vi) $5x^2y^2z$, $3x^2zy^2$, $-\frac{4}{5}zx^2y^2$ are unlike terms. False
Correct:
They are like terms.
(vii) $5x + \frac{2}{x} + 7$ is a polynomial of degree 1. False
Correct:
$\because \frac{2}{x}$ has negative power of y i.e. x^{-1}.
(viii) $3x^2y + 7x + 8y + 9$ is a polynomial of degree 3. True.
(ix) Numerical coefficient of $-7x^3y$ is 7. False Correct: It is -7.
(x) An equation is true for all values of variables in it. False
Correct:
It is true for some specific values of variables.
(xi) $(a – b)^2 + 2ab = a^2 + b^2$. True

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 18):

Question 3: The literal coefficient of $-9xyz^2$ is
(a) -9
(b) xy
(c) xyz^2

(d) -9xy
Solution:
Literal co-efficient of $-9xyz^2$ is -9 (a)

Question 4: Which of the following algebraic expression is
(a) $3x^2 - 2x + 7$
(b) $\frac{5x^3}{2x} + 3x^2 + 8$
(c) $3x + \frac{2}{x} + 7$
(d) $\sqrt{2}x^2 + \sqrt{3}x + \sqrt{6}$
Solution:
$3x^2 - 2x + 7$ is not an algebraic expression. (c)

Question 5: Which of the following algebraic expressions is not a monomial?
(a) $3x \times y \times z$
(b) -5pq
(c) $8m^2 \times n \div 31$
(d) $7x \div y - z$
Solution:
$7x \div y - z = \frac{7x}{y} - z$
It is not monomial.
(As it has two terms) (d)

Question 6: Degree of the polynomial $7x^2yz^2 + 6x^3y^2z^2 - 5x + 8y$ is
(a) 4
(b) 5
(c) 6
(d) 7

Solution:

Degree of polynomial

$7x^2yz^2 + 6x^3y^2z^2 - 5x + 8y$ is $3 + 2 + 2 = 7$ (d)

Question 7: $a(b - c) + b(c - a) + c(a - b)$ is equal to

(a) $ab + bc + ca$

(b) 0

(c) $2(ab + bc + ca)$

(d) none of these

Solution:

$a(b - c) + b(c - a) + c(a - b)$

$= ab - ac + bc - ab + ac - bc = 0$ (b)

Question 8: $\frac{7}{5}xy - \frac{2}{3}xy + \frac{8}{9}xy$ is equal to

(a) $\frac{73}{45}xy$ **(b) $-\frac{73}{45}xy$**

(c) xy **(d) none of these**

Solution:

$\frac{7}{5}xy - \frac{2}{3}xy + \frac{8}{9}xy$

$= \frac{63xy - 30xy + 40xy}{45} = \frac{73}{45}xy$ (a)

Question 9: $(3p^2qr^3) \times (-4p^3q^2r^2)$ x $(7pq^3r)$ is equal to

(a) $84p^6q^6r^6$

(b) $-84p^6q^6r^6$

(c) $84p^6q^5r^6$

(d) $-84p^6q^5r^6$

Solution:

$(3p^2qr^3) \times (-4p^3q^2r^2) \times (7pq^3r)$

$= 3(-4) \times 7p^{2+3+1} \times q^{1+2+3} \times r^{3+2+1}$
$= -84p^6q^6r^6$ (b)

Question 10: $3m \times (2m^2 – 5mn + 4n^2)$ is equal to
(a) $6m^3 + 15m^2n – 12mn^2$
(b) $6m^3 – 15m^2n + 12mn^2$
(c) $6m^3 – 15m^2n – 12mn^2$
(d) $6m^3 + 15m^2n + 12mn^2$
Solution:
$3m \times (2m^2 – 5mn + 4n^2)$

$= 6m^3 – 15m^2n + 12mn^2$ (b)

Question 11: Volume of a rectangular box whose adjacent edges are $3x^2y$, $4y^2z$ and $5z^2x$ respectively is
(a) 60xyz
(b) $60x^2y^2z^2$
(c) $60x^3y^3z^3$
(d) none of these
Solution:
Volume $= 3x^2y \times 4y^2z \times 5z^2x = 60x^3y^{32}z^3$ (c)

Question 12: (x – 1) (x + 2) is equal to
(a) 2x + 3
(b) $x^2 + 2x + 2$
(c) $x^2 + 3x + 2$
(d) $x^2 + 2x + 3$
Solution:
$(x + 1) (x + 2) = x^2 + (1 + 2)x + 1 \times 2$

$= x^2 + 3x + 2$ (c)

Question 13: If x + $\frac{1}{x}$ = 2, then $x^2 + \frac{1}{x^2}$ is equal to

(a) 4
(b) 2
(c) 0
(d) none of these

Solution:

$x + \frac{1}{x} = 2$

Squaring both sides,

$\left(x + \frac{1}{x}\right)^2 = (2)^2$

$\Rightarrow x^2 + \frac{1}{x^2} + 2 \times \text{x} \times \frac{1}{\text{x}} = 4$

$\Rightarrow x^2 + \frac{1}{x^2} = 4 - 2$

$\Rightarrow x^2 + \frac{1}{x^2} = 2$ (b)

Question 14: If $x^2 + y^2 = 9$ and xy = 8, then x + y is equal to

(a) 25
(b) 5
(c) -5
(d) ±5

Solution:

$x^2 + y^2 = 9$, xy = 8

$(x + y)^2 = x^2 + y^2 + 2xy$

$= (9)^2 + 2 \times 8$

$= 9 + 16 = 25 = \pm (5)$ (d)

Question 15: $(102)^2 - (98)^2$ is equal to

(a) 200

(b) 400
(c) 600
(d) 800
Solution:
$(102)^2 - (98)^2$
$= (102 + 98) \times (102 - 98)$
$= 200 \times 4 = 800$ (d)

Question 16: $-50x^3y^2z^2$ divided by $-5xyz$ is equal to
(a) 10xyz
(b) $10x^2yz$
(c) -10xyz
(d) $-10x^2yz$
Solution:
$\frac{-50x^3y^2z^2}{-5xyz} = 10x^2yz$ (b)

Question 17: 96 × 104 is equal to
(a) 9984
(b) 9974
(c) 9964
(d) none of these
Solution:
$96 \times 104 = (100 - 4)(100 + 4)$
$= (100)^2 - (4)^2$
$= 10000 - 16 = 9984$ (a)

Question 18: If the area of a rectangle is $24(x^2yz + xy^2z + xyz^2)$ and its length is 8xyz, then its breadth is:
(a) 3(x + y + z)

(b) 3xyz
(c) 3(x + y – z)
(d) none of these
Solution:
In a rectangle
Area = $24(x^2yz + xy^2z + xyz^2)$
Length = 8xyz
Breadth = $\frac{24xyz(x+y+z)}{8xyz}$ = 3(x + y + z) (a)

Higher Order Thinking Skills (Hots)

Question 1: Using the identity (a + b) = $a^2 + 2ab + b^2$, derive the formula for $(a + b + c)^2$. Hence, find the value of $(2x – 3y + 4z)^2$.
Solution:
$(a + b)^2 = a^2 + 2ab + b^2$
$(a + b + c)^2 = (a + b)^2 + c^2 + 2(a + b)c$
$= a^2 + b^2 + 2ab + c^2 + 2ca + 2bc$
$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
$(2x - 3y + 4z)^2 = (2x)^2 + (3y)^2 + (4z)^2 + 2 \times 2x \times (-3y)$
$+ 2(-3y)(4z) + 2(4z)(2x)$
$= 4x^2 + 9y^2 + 16z^2 - 12xy - 24yz + 16zx$

Question 2: Using the product of algebraic expressions, find the formulas for $(a + b)^2$ and (a – b).

Solution:
$(a + b)^2 = (a + b)^2 (a + b)$
$= (a^2 + 2ab + b^2)(a + b)$
$= a(a^2 + 2ab + b^2) + b(a^2 + 2ab + b^2)$
$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$

$= a^{32} + b^3 + 3a^2b + 3ab^2$
$= a^2 + b2^2 + 3ab(a + b)$

$(a - b)^2 = (a - b)(a - b)^2$
$= (a - b)(a^2 - 2ab + b^2)$
$= a(a^2 - 2ab + b^2) - b(a^2 - 2ab + b^2)$
$= a^3 - 2a^2b + ab^2 - a^2b + 2ab^2 - b^3$
$= a^3 - b^3 - 3a^2b + 3ab^2$
$= a^3 - b^3 - 3ab(a - b)$

Check Your Progress

1. Add the following expressions:
(i) $-5x^2y + 3xy^2 – 7xy + 8, 12x^2y – 5xy^2 + 3xy – 2$
(ii) $9xy + 3yz – 5zx, 4yz + 9zx – 5y, -5xz + 2x – 5xy$
Solution:

(i) $(-5x^2y + 3xy^2 – 7xy + 8) + (12x^2y – 5xy^2 + 3xy – 2)$

$= 7x^2y – 2xy^2 – 4xy + 6$

(ii) $(9xy + 3yz – 5zx) + (4yz + 9zx – 5y, -5xz + 2x – 5xy)$

$= 4xy + 7yz – zx + 2x – 5y$

2. Subtract:
(i) $5a + 3b + 11c – 2$ from $3a + 5b – 9c + 3$
(ii) $10x^2 – 8y^2 + 5y – 3$ from $8x^2 – 5xy + 2y^2 + 5x – 3y$
Solution:

(i) $5a – 3b + 11c – 2$ from $3a + 5b – 9c + 3$

$= (3a + 5b – 9c + 3) – (5a – 3b + 11c – 2)$

$= 3a + 5b – 9c + 3 – 5a + 3b – 11c + 2$

$= -2a + 8b – 20c + 5$

(ii) $10x^2 – 8y^2 + 5y – 3$ from $8x^2 – 5xy + 2y^2 + 5x – 3y$

$= (8x^2 – 5xy + 2y^2 + 5x – 3y) – (10x^2 – 8y^2 + 5y – 3)$

$= 8x^2 – 5xy + 2y^2 + 5x – 3y – 10x^2 + 8y^2 – 5y + 3$

3. What must be added to $5x^2 – 3x + 1$ to get $3x^3 – 7x^2 + 8$?
Solution:

From the question, the required expression is

$= (3x^3 – 7x^2 + 8) – (5x^2 – 3x + 1)$

$= 3x^3 – 7x^2 + 8 – 5x^2 + 3x – 1$

$= 3x^3 – 12x^2 + 3x + 7$

4. Find the product of:
(i) $3x^2y$ and $-4xy^2$
(ii) $-\left(\frac{4}{5}\right) xy,\ \left(\frac{5}{7}\right) yz$ and $-\left(\frac{14}{9}\right) zx$
Solution:

Product of:

(i) $3x^2y$ and $-4xy^2$

$= 3x^2 \times (-4xy^2)$

$= -12x^{2+1}\ y^{1+2}$

$= 12x^3y^3$

(ii) $-\left(\frac{4}{5}\right) xy,\ \left(\frac{5}{7}\right) yz$ and $-\left(\frac{14}{9}\right) zx$

$= -\left(\frac{4}{5}\right) xy \times \left(\frac{5}{7}\right) yz \times -\left(\frac{14}{9}\right) zx$

$= -\left(\frac{4}{5}\right) \times \left(\frac{5}{7}\right) \times -\left(\frac{14}{9}\right) x^2y^2z^2$

$= \left(\frac{8}{9}\right) x^2y^2z^2$

5. Multiply:
(i) $(3pq - 4p^2 + 5q^2 + 7)$ by $-7pq$
(ii) $\left(\frac{3}{4}x^2y - \frac{4}{5}xy + \frac{5}{6}xy^2\right)$ by $-15xyz$
Solution:
(i) $(3pq - 4p^2 + 5q^2 + 7) \times (-7pq)$

$= -7pq \times 3pq - 7pq \times (-4p^2) + (-7pq)(5q^2) - 7pq \times 7$

$= -21p^2q^2 + 28p^3q - 35pq^3 - 49pq$

(ii) $\left(\frac{3}{4}x^2y - \frac{4}{5}xy + \frac{5}{6}xy^2\right) \times (-15xyz)$

$= -15xyz\left(\frac{3}{4}x^2y - \frac{4}{5}xy + \frac{5}{6}xy^2\right)$

$= -15xyz \times \frac{3}{4}x^2y - 15xyz \times \left(\frac{-4}{5}xy\right) - 15xyz\left(\frac{5}{6}xy^2\right)$

$= \frac{-45}{4}x^3y^2z + 12x^2y^2z - \frac{25}{5}x^2y^3z$

6. Multiply:
(i) $(5x^2 + 4x - 2)$ by $(3 - x - 4x^2)$
(ii) $(7x^2 + 12xy - 9y^2)$ by $(3x^2 - 5xy + 3y^2)$
Solution:
(i) $(5x^2 + 4x - 2) \times (3 - x - 4x^2)$

$= 5x^2(3 - x - 4x^2) + 4x(3 - x - 4x^2) - 2(3x - x - 4x^2)$

$= 15x^2 - 5x^3 - 20x^4 + 12x - 4x^2 - 16x^3 - 6x + 2x + 8x^2$

$= -20x^4 – 21x^3 + 19x^2 + 14x – 6$

(ii) $(7x^2 + 12xy – 9y^2)$ x $(3x^2 – 5xy + 3y^2)$

$= 7x^2(3x^2 – 5xy + 3y^2) + 12xy(3x^2 – 5xy + 3y^2) – 9y^2(3x^2 – 5xy + 3y^2)$

$= 21x^4 – 35x^3y + 21x^2y^2 + 36x^3y – 60x^2y^2 + 36xy^3$

$– 27x^2y^2 + 45xy^3 – 27y^4$

$= 21x^4 + x^3y + 81xy^3 – 66x^2y^2 – 27y^4$

7. Simplify the following expressions and evaluate them as directed:

(i) $(3ab – 2a^2 + 5b^2)$ x $(2b^2 – 5ab + 3a^2) + 8a^3b – 7b^4$ for a = 1, b = -1

(ii) $(1.7x – 2.5y) (2y + 3x + 4) – 7.8x^2 – 10y$ for x = 0, y = 1.

Solution:

(i) $(3ab – 2a^2 + 5b^2) \times (2b^2 – 5ab + 3a^2) + 8a^3b – 7b^4$

$= 3ab (2b^2 – 5ab + 3a^2) – 2a^2(2b^2 – 5ab + 3a^2)$

$+ 5b^2(2b^2 – 5ab + 3a^2) + 8a^3b – 7b^4$

$= 6ab^{32} – 15a^2b^2 + 9a^3b – 4a^2b^2 + 10a^3b$

$– 6a^4 + 10b^4 – 25ab^3 + 15a^2b^2 + 8a^3b – 7b^4$

$= 27a^3b – 4a^2b^2 – 19ab^3 – 6a^4 + 3b^4$

Putting, a = 1 and b = (-1)

$= 27(1\ )^3 (-1) – 4(1)^2 (-1)^2 – 19 (1) (-1)^3 – 6(1)^4 + 3(-1)^4$

$= -27 – 4 + 19 – 6 + 3$

$= -37 + 22$

$= -15$

(ii) $(1.7x – 2.5y) (2y + 3x + 4) – 7.8x^2 – 10y$

$1.7x(2y + 3x + 4) – 2.5y(2y + 3x + 4) – 7.8x^2 – 10y$

$= 3.4xy + 5.1x^2 + 6.8x – 5y^2 – 7.5xy – 10y – 7.8x^2 – 10y$

$= -2.7x^2 – 4.1xy – 5y^2 + 6.8x – 20y$

Putting, $x = 0$ and $y = 1$

$= -2.7 \times 0 – 4.1 \times 0 \times 1 – 5(1)^2 + 6.8 \times 0 – 20 \times 1$

$= 0 + 0 – 5 + 0 – 20$

$= -25$

8. Carry out the following divisions:
(i) $66pq^2r^3 \div 11qr^2$
(ii) $(x^3 + 2x^2 + 3x) \div 2x$
Solution:

(i) $\frac{66pq^2r^3}{11qr^2}$

$= 6pq^{2-1}r^{3-2}$

$= 6pqr$

(ii) $\frac{(x^3+2x^2+3x)}{2x}$

$= \frac{x^3}{2x} + \frac{2x^2}{2x} + \frac{3x}{2x}$

$= \frac{1}{2}x^2 + x + \frac{3}{2}$

9. Divide $10x^4 – 19x^3 + 17x^2 + 15x – 42$ by $2x^2 – 3x + 5$.
Solution:

$(10x^4 – 19x^3 + 17x^2 + 15x – 42) \div (2x^2 – 3x + 5)$

Performing long division, we have

$$
\begin{array}{r|l}
 & 5x^2 - 2x - 7 \\
\hline
2x^2 - 3x + 5 & 10x^4 - 19x^3 + 17x^2 + 15x - 42 \\
 & 10x^4 - 15x^3 + 25x^2 \\
 & - \quad\; + \qquad - \\
\hline
 & \qquad -4x^3 - 8x^2 + 15x \\
 & \qquad -4x^3 + 6x^2 - 10x \\
 & \qquad + \quad\; - \qquad + \\
\hline
 & \qquad\qquad -14x^2 + 25x - 42 \\
 & \qquad\qquad -14x^2 + 21x - 35 \\
 & \qquad\qquad + \qquad - \qquad + \\
\hline
 & \qquad\qquad\qquad\qquad 4x - 7 \\
\hline
\end{array}
$$

Thus, Quotient = $5x^2 – 2x – 7$ and Remainder = $4x – 7$

10. Using identities, find the following products:
(i) (3x + 4y) (3x + 4y)
(ii) $\left(\frac{5a}{2} - b\right)\left(\frac{5a}{2} - b\right)$
(iii) (3.5m – 1.5n) (3.5m + 1.5n)
(iv) (7xy – 2) (7xy + 7)
Solution:

(i) $(3x + 4y)(3x + 4y)$

$= (3x + 4y)^2$

$= (3x)^2 + 2 \times 3x \times 4y + (4y)^2$ [Using, $(a + b)^2 = a^2 + 2ab + b^2$]
$= 9x^2 + 24xy + 16y^2$

(ii) $\left(\frac{5a}{2} - b\right)\left(\frac{5a}{2} - b\right)$

$= \left(\frac{5a}{2} - b\right)^2$

$= \left(\frac{5a}{2}\right)^2 + 2 \times \frac{5a}{2} \times (-b) + (b)^2$ [Using, $(a - b)^2 = a^2 - 2ab + b^2$]

$= \frac{25a^2}{4} - 5ab + b^2$

(iii) $(3.5m - 1.5n)(3.5m + 1.5n)$

$= (3.5m)^2 - (1.5n)^2$ [Using, $(a - b)(a + b) = a^2 - b^2$]
$= 12.25m^2 - 2.25n^2$

(iv) $(7xy - 2)(7xy + 7)$

$= (7xy)^2 + (-2 + 7) \times (7xy) + (-2) \times 7$

[Using, $(x + a)(x + b) = x^2 + (a + b)x + ab$]

$= 49x^2y^2 + 35xy - 14$

11. Using suitable identities, evaluate the following:
(i) 105^2
(ii) 97^2
(iii) 201×199
(iv) $87^2 - 13^2$
(v) 105×107

Solution:

(i) $(105)^2 = (100 + 5)^2$

$= (100)^2 + 2 \times 100 \times 5 + (5)^2$ [Using, $(a + b)^2 = a^2 + 2ab + b^2$]

$= 10000 + 1000 + 25$

$= 11025$

(ii) $(97)^2 = (100 - 3)^2$

$= (100)^2 - 2 \times 100 \times 3 + (3)^2$ [Using, $(a - b)^2 = a^2 - 2ab + b^2$]

$= 10000 - 600 + 9$

$= 10009 - 600$

$= 9409$

(iii) $201 \times 199 = (200 + 1)(200 - 1)$

$= (200)^2 - (1)^2$ [Using, $(a + b)(a - b) = a^2 - b^2$]

$= 40000 - 1$

$= 39999$

(iv) $87^2 - 13^2$

$= (87 + 13)(87 - 13)$ [Using, $a^2 - b^2 = (a + b)(a - b)$]

$= 100 \times 74$

$= 7400$

(v) 105×107

$= (100 + 5)(100 + 7)$

$= (100)^2 + (5 + 7) \times 100 + 5 \times 7$

[Using, $(x + a)(x - b) = x^2 + (a + b)x + ab$]

$= 10000 + 1200 + 35$

$= 11235$

12. Prove that following:
(i) $(a + b)^2 – (a – b)^2 + 4ab$
(ii) $(2a + 3b)^2 + (2a – 3b)^2 = 8a^2 + 18b^2$
Solution:

(i) Taking the RHS, we have

RHS $= (a – b)^2 + 4ab$

$= a^2– 2ab + b^2 + 4ab$

$= a^2 + 2ab + b^2$

$= (a + b)^2 =$ L.H.S.

(ii) Taking the LHS, we have

LHS $= (2a + 3b)^2 + (1a – 3b)^2$

$= (2a)^2 + 2 \times 2a \times 3b + (3b)^2 + (2a)^2 – 2 \times 2a \times 3b + (3b)^2$

$= 4a^2 + 12ab + 9b^2 + 4a^2 – 12ab + 9b^2$

$= 8a^2 + 18b^2 =$ RHS

13. If $x + \frac{1}{x} = 5$, evaluate
(i) $x^2 + \frac{1}{x^2}$ (ii) $x^4 + \frac{1}{x^4}$
Solution:

(i) We have, $x + \frac{1}{x} = 5$

On squaring on both sides, we get

$\left(x + \frac{1}{x}\right)^2 = 5^2$

$x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} = 25$

$x^2 + 2 + \frac{1}{x^2} = 25$

$x^2 + \frac{1}{x^2} = 25 - 2$

Hence, $x^2 + \frac{1}{x^2} = 23$

(ii) Again, squaring $x^2 + \frac{1}{x^2} = 23$ on both sides, we get

$\left(x^2 + \frac{1}{x^2}\right)^2 = 23^2$

$x^4 + \frac{1}{x^4} + 2 \times x^4 \times \frac{1}{x^4} = 529$

$x^4 + \frac{1}{x^4} + 2 = 529$

$x^4 + \frac{1}{x^4} = 529 - 2$

Hence,

$x^4 + \frac{1}{x^4} = 527$

14. If a + b = 5 and $a^2 + b^2 = 13$, find ab.
Solution:

Given,

$a + b = 5$ and $a^2 + b^2 = 13$

On squaring $a + b = 5$ both sides, we get

$(a + b)^2 = (5)^2$

$a^2 + b^2 + 2ab = 25$

$13 + 2ab = 25 \Rightarrow 2ab = 25 - 13 = 12$

$\Rightarrow ab = \frac{12}{2} = 6$

$\therefore ab = 6$