LINEAR PROGRAMMING

CHAPTER – 12

LINEAR PROGRAMMING

LINEAR INEQUATION

If a, b, c $\in$ R, then the equation ax + by = c is called a **linear equation** in two variables x, y whereas inequalities of the form $ax + by \leq c$, $ax + by \geq c$, $ax + by < c$ & $ax + by > c$ are called **Linear Inequations** in two variables x & y. We know that the graph of the equation ax + by = c is a straight line which divides the xy-plane into two parts

(i) $ax + by \leq c$ (ii) $ax + by \geq c$. These two are known as the **half spaces.**

In set form {(x, y) : ax + by = c} is the straight line whereas, sets $\{(x, y) : ax + by \leq c\}$ and $\{(x, y) : ax + by \geq c\}$ are **closed half spaces** and the sets $\{(x, y) : ax + by < c\}$ and $\{(x, y) : ax + by > c\}$ are **open half spaces**. These half spaces are also known as the solution sets of the corresponding inequation.

GRAPHS OF LINEAR IN EQUATIONS

Consider a linear inequation $ax + by \leq c$. Drawing the graph of a linear inequation means finding its solution set.

Steps to draw the graph:

To draw the graph of an equation, following procedures are to be made-

(i) Write the inequation $ax + by \leq c$ into an equation ax + by = c which represent a straight line in xy-plane.

(ii) Put y = 0 in ax + by = c to get point where the line meets x- axis. Similarly, put x = 0 to obtain a point where the line meets y- axis. Join these two points to obtain the graph of the line.

(iii) If the inequation is > or <, then the points lie on this line does not consider and line is drawn dotted or discontinuous.

(iv) If the inequation is $\geq$ or $\leq$, then the point lie on the line consider and line is drawn black (bold) or continuous.

(v) This line divides the plane XOY in two region.

To find the region that satisfies the inequation, *we apply the following rules-*

(a) Choose a point [If possible (0, 0)] not lying on this line.

(b) Substitute its coordinates in the inequation. If the inequation is satisfied, then shade the portion of the plane which contains the chosen point, otherwise shade the portion which does not contain this point. The shaded portion represents the solution set.

Note

In case of inequations $ax + by \leq c$ and $ax + by \geq c$ points on the line are also a part of the shaded region while in case of $ax + by < c$ and $ax + by > c$ points on the line ax + by = c are not included in the shaded region.

SIMULTANEOUS LINEAR INEQUATION IN TWO VARIABLES

Since the solution set of a system of simultaneous linear inequation is the set of all points in two dimensional space which satisfy all the inequations simultaneously. Therefore to find the solution set we find the region of the plane common to all the portions comprising the solution sets of given inequations. In case there is no region common to all the solution of the given inequations, we say that the solution set is void or empty.

FEASIBLE REGION

The limited (bounded) region of the graph made by two inequations is called **Feasible Region**. All the coordinates of the points in feasible region constitutes the solutions of system of inequations.

LINEAR PROGRAMMING PROBLEMS

Linear Programming is a device to optimize the results which occurs in business under some restrictions. A general Linear Programming problem can be stated as follows:

Given a set of m linear inequalities or equations in n variables, we wish to find non- negative values of these variables which will satisfy these inequalities or equations and maximize or minimize some linear functions of the variables.

The general form of Linear Programming Problems (L.P.P.) is-

Maximize (Minimize) $z = c_1x_1 + c_2x_2 + \dots + c_nx_n$ subjected to

$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \{\leq, =, \geq\} b_1$

$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \{\leq, =, \geq\} b_2$

....

$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \{\leq, =, \geq\} b_n$

and $x_1, x_2, x_3, \dots, x_n \geq 0$

where $x_1, x_2, x_3, \dots x_n$ are the variables whose values are to be determined and are called the **decision variables**. The inequation are called **constraints** and the function to be maximized or minimized is called the **objective function**.

SOME DEFINITIONS

(i) Solution : A set of values of the decision variables which satisfy the constraints of a Linear Programming Problem (L.P.P.) is called a solution of the L.P.P.

(ii) Feasible Solution : A solution of L.P.P. which also satisfy the non- negative restrictions of the problem is called the feasible solution.

(iii) Optimal Solution : A feasible solution which maximize or minimize i.e. which optimize the objective function of L.P.P. called an optimal solution.

Note

A Linear Programming Problem may have many optimal solution. If a L.P.P. has two optimal solution, then there are an infinite number of optimal solutions.

(iv) Iso-Profit Line: The line is drawn in geometrical area of feasible region of L.P.P. for which the objective function remains constant at all the points lie on this line, is called iso-profit line.

GRAPHICAL METHOD OF SOLUTION OF LINEAR PROGRAMMING PROBLEMS

The graphical method for solving linear programming problems is applicable to those problems which involve only two variables. This method is based upon a theorem, called **extreme point theorem**, which is stated as follows-

Extreme Point Theorem: If a L.P.P. admits an optimal solution, then at least one of the extreme (or corner) points of the feasible region gives the optimal solution.

Working Rule:

(i) Find the solution set of the system of simultaneous linear inequations given by constraints and non- negativity restrictions.

(ii) Find the coordinates of each of corner points of the feasible region.

(iii) Find the values of the objective function at each of the corner points of the feasible region.

(ii) Find the coordinates of each of corner points of the feasible region.

(iii) Find the values of the objective function at each of the corner points of the feasible region. By extreme point theorem one of the corner points will provide the optimal value of the objective function. The coordinates of that corner point determine the optimal solution of the L.P.P.

(i) If it is not possible to determine the point at which the suitable solution found, then the solution of problem is unbounded.

(ii) If feasible region is empty, then there is no solution for the problem.

(iii) Nearer to the origin, the objective function is minimum and that of further from the origin the objective function is maximum.

CONVEX SETS

In linear programming problem mostly feasible solution is a polygon in first quadrant this polygon is a convex. It means that if two points of polygon are connecting by a line then the line must be inside to polygon.

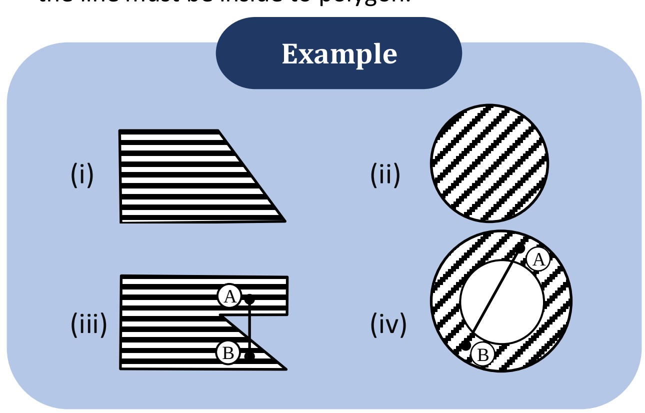


Figure (i) and (ii) are convex set while (iii) & (iv) are not convex set. It can be easily seen that the intersection of two convex sets is a convex set and the set of all feasible solutions of a LPP is also a convex set..

QUESTIONS

MCQ

Q1. Which of the following is true for the LPP
$5x + 2y \leq 10, x \geq 0, y \geq 0$:
(a) The region is bounded
(b) The region is unbounded
(c) Can't be determined
(d) None

Q2. Solve the LPP:
Minimize $Z = 200x + 500y$ subjected to
$x + 2y \geq 10$
$3x + 4y \leq 24$
$x \geq 0, y \geq 0$
(a) 2000 (b) 2500
(c) 2300 (d) 3000

Q3. Old hens can be bought for Rs 2.00 each and young ones at Rs 5.00 each. The old hens lay
3 eggs per week and the young hens lay 5 eggs per week, each egg being worth 30 paise. A hen costs ₹1.00 per week to feed. A man has only ₹ 80" to spend for hens. Formulate the problem for maximum profit per week, assuming that he cannot house more than 20 hens.
(a) Maximize $Z = \frac{5y-x}{10}$ s t: $x \geq 0, y \geq 0, x + y \leq 20$ and $2x + 5y \leq 80$
(b) Maximize $Z = \frac{5y+x}{10}$ s t: $x + y \leq 20$ and $2x + 5y \leq 80$
(c) Maximize $Z = \frac{5y-x}{10}$ s t: $x \geq 0, y \geq 0, x - y \leq 20$ and $2x + 5y \leq 80$
(d) Maximize $Z = \frac{5y-x}{10}$ s t: $x + y \leq 20$ and $2x + 5y \leq 80$

Q4. Feasible region in the set of points which satisfy
(a) The objective functions
(b) Some the given constraints
(c) All of the given constraints
(d) None of these

Q5. Of all the points of the feasible region for maximum or minimum of objective function the points
(a) Inside the feasible region
(b) At the boundary line of the feasible region
(c) Vertex point of the boundary of the feasible region
(d) None of these

Q6. Objective function of a linear programming problem is
(a) a constraint
(b) function to be optimized
(c) A relation between the variables
(d) None of these

Q7. A set of values of decision variables which satisfies the linear constraints and non-negativity conditions of a L.P.P. is called its
(a) Unbounded solution (b) Optimum solution
(c) Feasible solution, (d) None of these

Q8. A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16.If the profit on a necklace is ₹100 and that on a bracelet is ₹300. Formulate an LPP for finding how many of each should be produced daily to maximize the profit? It is being given that at least one of each must be produced.
(a) $Max\ Z = 100x + 300y, s.t\ x + y \leq 24, 2x + y \leq 32, x + 1 \geq 0, y + 1 \geq 0$
(b) $Max\ Z = 100x + 300y, s.t\ x + y \leq 24, 2x + y \geq 32, x - 1 \geq 0, y - 1 \geq 0$
(c) $Max\ Z = 100x + 300y, s.t\ x + y \leq 24, 2x + y \leq 32, x \geq 0, y \geq 0$
(d) $Max\ Z = 100x + 300y, s.t\ x + y \leq 24\ \ 2x + y \leq 32, x - 1 \geq 0, y - 1 \geq 0$

Q9. Minimize Z=3x+2y subject to the constraints:
$x + y \geq 8, 3x + 5y \leq 15, x \geq 0, y \geq 0$
The point at which the optimal solution will exist is:
(a) (0,8) (b) (0,3)
(c) (0,0) (d) None

Q10. Minimize and Maximize $z = 5x + 2y$ subject to :
$x - 2y \leq 2,\ 3x + 2y < 12,\ -3x + 2y \leq 3, x \geq 0, y \geq 0$
The optimal solutions for the above LPP is:
(a) 0 , 10 respectively (b) 3, 15 respectively
(c) 0 , 19 respectively (d) 3 , 19 respectively

Q11. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹ 35 per package of nuts and ₹ 14 per package of bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates each machine for at most 12 hours a day? Convert it into an LPP and solve graphically. How many nuts and bolts should be produced daily to maximize the profit?
(a) 4 nuts and 4 Bolts (b) 3 nuts and 3 Bolts
(c) 5 nuts and 5 Bolts (d) 2 nuts and 2 Bolts

Q12. Two tailors A and B earn ₹ 150 and ₹ 200 per day respectively. A can stitch 6 shirts and 4 pants per day, while B can stitch 10 shirts and 4 pants per day. Form a L.P.P. to minimize the labour cost.

(a) $Max\ z = 150x + 200y$ s.t $x \geq 0, y \geq 0, 3x + 5y \geq 30, x + y \geq 8$

(b) $Min\ z = 150x + 200y$ s.t $x \geq 0, y \geq 0, 3x + 5y \geq 30, x + y \geq 8$

(c) $Max\ z = 150x + 200y$ s.t $3x + 5y \geq 30, x + y \geq 8$

(d) $Max\ z = 150x + 200y$ s.t $x \leq 0, y \leq 0, 3x + 5y \leq 30, x + y \leq 8$

Q13. Solve the following problem graphically: Maximize $Z = 5x + 2y$ subject to the constraints $3x + 5y \leq 15, 5x + 2y \leq 10, x, y \geq 0$

(a) $Z = 0\ at\ (0,0)$ (b) $Z = 10\ at\ (2,0)$

(c) $Z = 10\ at\ \left(\frac{20}{19}, \frac{45}{19}\right)$ (d) None

Q14. Solve the following problem graphically: Minimize $Z = 3x + 5y$ subject to the constraints $x + y = 6, x \leq 4, y \leq 5, x \geq 0, y \geq 0$

(a) 20 (b) 21

(c) 28 (d) 25

Q15. Solve the following problem graphically: Maximize $Z = 2x + 3y$ subject to the constraints $x + y \leq 1, 2x + 2y \geq 6, x \geq 0, y \geq 0$

(a) Unbounded region (b) No feasible region

(c) 9 (d) 6

Q16. Solve the following problem graphically: Maximize $Z = x + 2y$ subject to the constraints $x - y \geq 0, 2y \leq x + 2, x, y \geq 0$

(a) Unbounded region (b) No feasible region

(c) 0 (d) 6

Q17. A manufacturer produces two types of steel trunks. He has two machines A and B. The first type of trunk requires 3 ours on machine A and 3 hours on machine B. The second type of trunk requires 3 hours on machine A and 2 hours on machine B. Machines A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of ₹30 and ₹25 per trunk of the first type and second type respectively. Form an LPP for how many trunks of each type must he make each day to make maximum profit?

(a) Max Z=30x+25y s.t x +y$\leq 6, 3x + 2y \leq 15, x \geq 0, y \geq 0$

(b) M in Z=30x+25y s.t x +y$\leq 6, 3x + 2y \leq 15, x \geq 0, y \geq 0$

(c) Max Z=30x+25y s.t x +y$\geq 6, 3x + 2y \geq 15, x \geq 0, y \geq 0$

(d) Min Z=30x+25y s.t x +y$\geq 6, 3x + 2y \geq 15, x \leq 0, y \leq 0$

Q18. Find the maximum value of Z = 7X + 7Y,subject to the constraints. x ≥ 0, y ≥ 0, x + y ≥ 2 and 2x + 3y ≤ 6.

(a) 21 (b) 20

(c) 19 (d) 25

Q19. Maximize Z = 4x + 9y, subject to the constraints $x \geq 0, y \geq 0, x + 5y \leq 200, 2x + 3y \leq 134.$

(a) 360 (b) 0

(c) 268 (d) 382

Q20. Find the minimum value of Z = 3x + 5y, subject to the constraints $-2x + y \leq 4, x + y \geq 3, x - 2y \leq 2, x \geq 0, y \geq 0$

(a) 20 (b) 15

(c) $\frac{29}{3}$ (d) None

Q21. Maximize Z = 3x + 5y, subject to the constraints $x + 2y \leq 2000, x + y \leq 1500, y \geq 0, y \leq 600, x \geq 0$

(a) 0 (b) 3000

(c) 5400 (d) 5500

Q22. Mr. Das wants to invest ₹12000 in public provident fund (PPF) and in national bonds. He has to invest at least ₹1000 in PPF and at least ₹2000in bonds. If the rate of interest on PPF is 12% per annum and that on bonds is 15% per annum, how should he invest the money to earn maximum annual income?

(a) Rs.1000 in PPF and Rs.11000 in national bonds

(b) Rs.1000 in PPF and Rs.2000 in national bonds

(c) Rs.10000 in PPF and Rs.2000 in national bonds

(d) None

Q23. A small firm manufactures necklace and bracelets. The total number of necklace and bracelet that it can handle per day is at most 24. It takes 1 hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is ₹100 and that on a bracelet is ₹300, how many of each should be produced daily to maximize the profit? It is being given that at least one of each must be produced.

(a) 2 necklaces and 2 bracelets

(b) 2 necklaces and 15 bracelets

(c) 16 necklaces and 8 bracelets

(d) 23 necklaces and 1 bracelet

Q24. A firm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. They need certain nutrients, named as X,Y,Z. the pigs are fed on two products, A and B. One unit of product A contain 36 unit of X, 3 units of Y and 20 units of Z, while one unit of product B contain 6 units of X, 12 units of Y and 10 units of Z. the minimum requirement of X, Y, Z are 108 units, 36 units and 100 units respectively. Product A costs ₹20 per unit and product B costs ₹40 per unit. How many units of each product must be taken to minimize the cost?

(a) 0 units of fertilizer A and 18 units of fertilizer B

(b) 4 units of fertilizer A and 2 units of fertilizer B

(c) 2 units of fertilizer A and 6 units of fertilizer B

(d) 12 units of fertilizer A and 0 units of fertilizer B

Q25. A dietician wishes to mix two types of food, X and Y, in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food X contains 2 units/kg of vitamin A and 1 unit /kg of vitamin C, while food Y contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹5 per kg to purchase the food X and ₹7 per kg to purchase the food Y. Determine the minimum cost of such a mixture.

(a) 56 (b) 28

(c) 58 (d) 38

Q26. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of mineral and 1400 calories. Two food, A and B, are available at a cost of ₹4 and ₹3 per unit respectively. If one unit of A contains 200 units of vitamins, 1 unit of mineral and 40 calories, and 1 unit of B contains 100 units of vitamins, 2 units of mineral and 40 calories, find what combination of foods should be used to have the least cost.

(a) 0 units of food A and 140 units of food B

(b) 20 units of food A and 30 units of food B

(c) 5 units of food A and 30 units of food B

(d) None

Q27. A housewife wishes to mix together two kinds of food, X and Y, in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of 1 kg of each food are given below.

	Vitamin A	Vitamin B	Vitamin C
Food X	1	2	3
Food Y	2	2	1

If 1 kg of food X cost ₹6 and 1 kg of food Y costs ₹10, find the minimum cost of the mixture which will produce the diet.

(a) 80 (b) 56

(c) 60 (d) 52

Q28. A manufacture produces two types of steel trunks. He has two machines, A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type required 3 hours on machine A and 2 hours on Machine A and 2 hours on machine B. Machine A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of ₹30 and ₹25 per trunk of the first type and second type respectively. How may trunks of each type must he make each day to make the maximum profit?

(a) 3 trunks of each type (b) 5 trunks of each type

(c) 6 trunks of each type (d) 2 trunks of each type

Q29. Kellogg is a new cereal formed of a mixture of bran and rice, that contains at least 88 grams of protein and at least 36 milligrams of iron. Knowing that bran contains 80 grams of protein and 40 milligrams of iron per kilograms, and that rice contains 100 grams of protein and 30 milligrams of iron per kilogram, find the minimum cost producing this new cereal if bran costs ₹5 per kilogram and rice costs ₹4 per kilogram.

(a) 4.8 (b) 5.1

(c) 4.6 (d) 5.5

Q30. Maximize = 60x + 15y, subject to the constraints $x + y \leq 50, 3x + y \leq 90, x, y \geq 0$

(a) 1500 (b) 750

(c) 0 (d) 1800

SUBJECTIVE QUESTIONS

Q1. Graph the solution set of the following inequations:

$x + y \leq 40$

$x + 2y \leq 60$

and $x, y \geq 0$.

Q2. A vertex of the linear inequalities $2x + 3y \leq 6$, $x + 4y \leq 4$ and $x, y \leq 0$, is-

Q3. Find the minimum value of $z = 2x_1 - 10x_2$ subjected to constraints $x_1 - x_2 \geq 0$, $x_1 - 5x_2 \leq -5$ & $x_1, x_2 \geq 0$

Q4. Find the maximum value of Max. $z = 6x_1 + 10x_2$

subject to $3x_1 + 5x_2 \leq 13$

$5x_1 + 3x_2 \leq 15$ And $x_1, x_2 \geq 0$

Q5. Find the maximum value of $z = 3x_1 + 2x_2$, Subject to $x_1 + x_2 \geq 1$ $x_2 - 5x_1 \leq 0$

$5x_2 - x_1 \geq 0$ $x_1 - x_2 \geq -1$

$x_1 + x_2 \leq 6$ $x_1 \leq 3$

and $x_1 \geq 0, x_2 \geq 0$

NUMERICAL TYPE QUESTIONS

Q1. The maximum value of Max. $z = 5x_1 + 7x_2$ subjected to $x_1 + x_2 < 4$, $3x_1 + 8x_2 < 24$, $10x_1 + 7x_2 < 35$ and $x_1 > 0$, $x_2 > 0$ ______.

Q2. The maximum value of max. $z = 3x_1 + 4x_2$ subject to $5x_1 + 4x_2 \leq 200$, $3x_1 + 5x_2 \leq 150$, $5x_1 + 4x_2 \geq 100$, $8x_1 + 5x_2 \geq 80$ and $x_1, x_2 \geq 0$ __________.

Q3. The minimum value of P = 6x + 16 y subject to constraints $x \leq 40$, $y \geq 20$ and $x, y \geq 0$ is _______.

Q4. If $3x_1 + 5x_2 \leq 15$, $5x_1 + 2x_2 \leq 10$, $x_1, x_2 \geq 0$, then the maximum value of $5x_1 + 3x_2$ _________-.

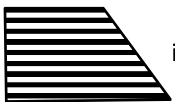
Q5. To maximize the objective function z = 2x + 3y under the constraints $x + y \leq 30$, $x - y \geq 0$, $y \leq 12$, $x \leq 20$, $y \geq 3$ and $x, y \geq 0$, is at (x , y) then x + y = ______.

TRUE AND FALSE

Q1. A solution of L.P.P. which also satisfy the non- negative restrictions of the problem is called the non feasible solution.

Q2. A feasible solution which maximize or minimize i.e. which optimize the objective function of L.P.P. called an optimal solution.

Q3. The line is drawn in geometrical area of feasible region of L.P.P. for which the objective function remains constant at all the points lie on this line, is called iso-profit line.

Q4.  is convex set

Q5. The linear inequations or inequalities or restriction on the variables of a linear programming problem are constraints**.**

ASSERTION AND REASONING

Directions: (Q1 -5) In the following questions , A statement of Assertion (A) is followed by a statement of Reason (R).

(a) Both A and R are true but R is the correct explanation of A

(b) Both A and R are true but R is Not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

Q1. **Assertion (A) :** The graph of inequations $3x + 2y \le 6$ and $6x + 4y \ge 20$, are disjoint.

Reason(R): The equations, corresponding to inequalities $3x + 2y \le 6$ and $6x + 4y \ge 20$, are $3x + 2y = 6$ and $6x + 4y = 20$. So the lines represented by these equations are parallel.

Q2. **Assertion (A) :** Inequation $y - x \le 0$ represents the half plane that contains the negative x-axis

Reason (R): The position of points O (0,0) and P (2, –2) in the region of graph of inequations $2x - 3y < 5$, will be inside and outside respectively.

Q3. **Assertion (A) :** The solution set of the inequation $2x + y > 5$ is open half plane not containing the origin.

Reason(R): The vertex of common graph of inequalities $2x + y \ge 2$ and $x - y \le 3$, is $\left(\frac{5}{3}, -\frac{4}{3}\right)$

Q4. **Assertion (A) :** The region represented by the inequation system $x, y \ge 0$, $y \le 6$, $x + y \le 3$, is unbounded in first quadrant

Reason (R): In equations $3x - y \geq 3$ and $4x - y > 4$ is true for positive value of x and negative value of y .

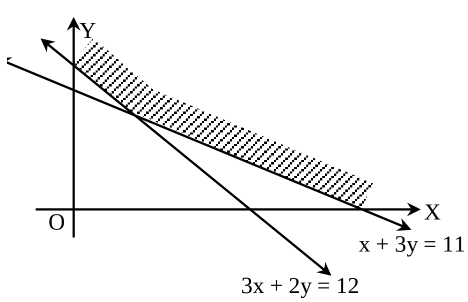
Q5. **Assertion (A) :** The bounded region for inequations $x + y \le 1$ and $x - y \le 1$ is situated all the four quadrants

Reason(R): The graph of inequations $x \le y$ and $y > x + 3$ located in II, III quadrants

HOMEWORK

MCQ

Q1. For the following feasible region, the linear constraints are-



(a) $x \ge 0, y \ge 0, 3x + 2y \ge 12, x + 3y \ge 11$

(b) $x \ge 0, y \ge 0, 3x + 2y \le 12, x + 3y \ge 11$

(c) $x \ge 0, y \ge 0, 3x + 2y \le 12, x + 3y \le 11$

(d) None of these

Q2. The region represented by $2x + 3y - 5 \le 0$ and $4x - 3y + 2 \le 0$, is-

(a) not in first quadrant

(b) bounded in first quadrant

(c) unbounded in first quadrant

(d) None of these

Q3. A wholesale merchant wants to start the business of cereal with Rs.24,000. Wheat is Rs.400 per quintal and rice is Rs.600 per quintal. He has capacity for store 200 quintal cereal. He earns the profit Rs.25 per quintal on wheat and Rs.40 per quintal on rice. If he store x quintal rice and y quintal wheat, then for maximum profit the objective function is -

(a) $25x + 40y$ (b) $40x + 25y$

(c) $400x + 600y$ (d) $\frac{400}{40}x + \frac{600}{25}y$

Q4. For the constraint of a linear optimizing function $z = x_1 + x_2$, given by $x_1 + x_2 \le 1$, $3x_1 + x_2 \ge 3$ and $x_1, x_2 \ge 0$-

(a) There are two feasible regions

(b) There are infinite feasible regions

(c) There is no feasible region

(d) None of these

Q5. On maximizing $z = 4x + 9y$ subject to $x + 5y \leq 200$, $2x + 3y \leq 134$ and $x, y \geq 0$, then z =

(a) 380 (b) 382

(c) 384 (d) None of these

Q6. For the L.P. problem Min. $z = -x_1 + 2x_2$ such that $-x_1 + 3x_2 \leq 0$, $x_1 + x_2 \leq 6$, $x_1 - x_2 \leq 2$ and $x_1, x_2 \geq 0$, then x_1 =

(a) 2 (b) 8

(c) 10 (d) 12

Q7. The intermediate solutions of constraints must be checked by substituting them back into-

(a) object function (b) constraint equations

(c) not required (d) None of these

Q8. The maximum value of $Z = 4x + 2y$ subjected to the constraints $2x + 3y \leq 18$, $x + y \geq 10$; $x, y \geq 0$, is-

(a) 36 (b) 40

(c) 20 (d) None of these

Q9. For the L.P. problem Min $z = 2x_1 + 3x_2$ such that $-x_1 + 2x_2 \leq 4$, $x_1 + x_2 \leq 6$, $x_1 + 3x_2 > 9$ and $x_1, x_2 \geq 0$

(a) $x_1 = 1.2$ (b) $x_2 = 2.6$

(c) $z = 10.2$ (d) All the above

Q10. For the L.P. problem Min $z = x_1 + x_2$ such that $5x_1 + 10x_2 \leq 0$, $x_1 + x_2 \geq 1$, $x_2 \leq 4$ and $x_1, x_2 \geq 0$

(a) there is a bounded solution

(b) there is no solution

©there are infinite solutions

(d) None of these

SUBJECTIVE QUESTIONS

Q1. For the constraints of a L.P. problem given by $x_1 + 2x_2 \leq 2000$, $x_1 + x_2 \leq 1500$, $x_2 \leq 600$ and $x_1, x_2 \geq 0$, which one of the following points does not lie in the positive bounded region-

Q2. A firm makes pants and shirts. A shirt takes 2 hours on machine and 3 hours of man labour while a pent takes 3 hours on machine and 2 hours of man labour. In a week there are 70 hours machine and 75 hours of man labour available. If the firm determine to make x shirts and y pants per week, then for this the linear constraints are-

Q3. Mohan wants to invest the total amount of Rs. 15,000 in saving certificates and national saving bonds. According to rules, he has to invest at least Rs. 2000 in saving certificates and Rs. 2500 in national saving bonds. The interest rate is 8% on saving certificate and 10% on national saving bonds per annum. He invest Rs. x in saving certificates and Rs. y in national saving bonds. Then the objective function for this problem is

Q4. The point at which the maximum value of x + y subject to the constraints $2x + 5y \leq 100$, $\frac{x}{25} + \frac{y}{40} \leq 1$, $x, y \geq 0$ is obtained is-

Q5. The solution of a problem to maximize the objective function $z = x + 2y$ under the constraints $x - y \leq 2$, $x + y \leq 4$ and $x, y \geq 0$, is-

NUMERICAL TYPE QUESTIONS

Q1. Maximize $Z = 3x + 4y$ subject to $2x + 2y \leq 80$, $2x + 4y \leq 120$ ______.

Q2. Minimize $Z = 2x + 4y$ subject to $x + y \geq 8$, $x + 4y \geq 12$, $x \geq 3$, $y \geq 2$ _________-

Q3. Minimize $Z = 30x + 20y$ subject to $x + y \leq 8$, $x + 4y \geq 12$, $5x + 8y = 20$, $x, y \geq 0$.______.

Q4. Maximize $Z = 3x + 3y$, if possible, subject to the constraints $x - y \leq 1$, $x + y \geq 3$, $x, y \geq 0$________-.

Q5. The minimum value of $3x + 5y$ subject to the constraints $-2x + y \leq 4$, $x + y \geq 3$, $x - 2y \leq 2$, $x, y \geq 0$ _______.

TRUE AND FALSE

Q1. A set of values of the decision variables which satisfy the constraints of a linear programming problem (LPP) is called a solution of the LPP.

Q2. There is a method to solved a linear programming problem graphically i.e., Non corner – point method.

Q3. The linear function Z which is to be maximized or minimized is called the objective function.

Q4. A solution of a LPP is an infeasible solution, if it does not satisfy the non- negativity restriction.

Q5. An optimal solution of a LPP, if it exists, occurs at one of the extreme (corner) points of the convex polygon of the set of all feasible solution.

ASSERTION AND REASONING

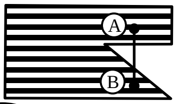
Directions: (Q1 -5) In the following questions, A statement of Assertion (a) is followed by a statement of Reason (R).

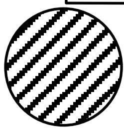
(a) Both A and R are true but R is the correct explanation of A

(b) Both A and R are true but R is Not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

Q1. **Assertion (a):**  is not a convex set

Reason(R):  is not a convex set

Q2. **Assertion (a):** A set of values of the decision variables which satisfy the constraints of a Linear Programming Problem (L.P.P.) is called a solution of the L.P.P.
Reason (R): A Linear Programming Problem have only two optimal solution.

Q3. **Assertion (a):** The graph of the equation ax + by = c is a straight line which divides the xy-plane into two parts known as the half spaces.
Reason(R): In set form {(x, y) : ax + by = c} is the straight line whereas, sets {(x, y) : ax + by ≤ c} and {(x, y) : ax + by ≥ c} are closed half spaces and the sets {(x, y) : ax + by < c} and {(x, y) : ax + by > c} are open half spaces

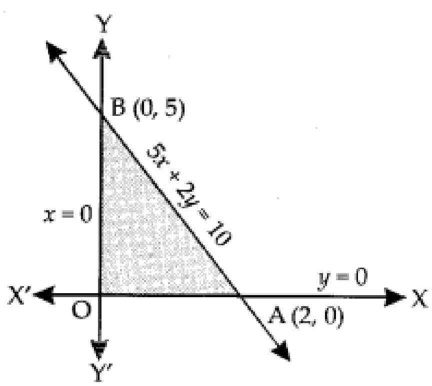
Q4. **Assertion (a):** there is no region common to all the solution of the given inequations, we say that the solution set is non empty
Reason (R): The limited (bounded) region of the graph made by two inequations is called Feasible Region.

Q5. **Assertion (a):** The general form of Linear Programming Problems (L.P.P.) is-
Maximize (Minimize) $z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$
subjected to
$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \{\le, =, \ge\} b_1$
$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \{\le, =, \ge\} b_2$
....
$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \{\le, =, \ge\} b_n$
and $x_1, x_2, x_3, \ldots, x_n \ge 0$
Reason(R): Where $x_1, x_2, x_3, \ldots x_n$ are the variables whose values are to be determined and are called the **decision variables**. The inequation are called **constraints** and the function to be maximized or minimized is called the **objective function**.

SOLUTIONS

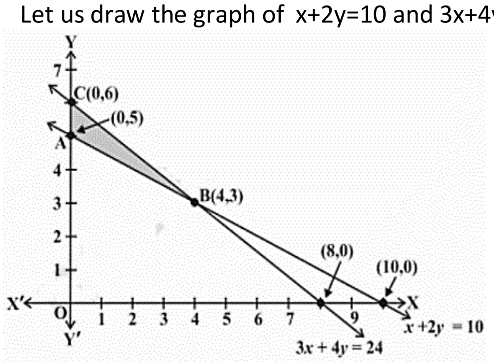
MCQ

S1. **(a)**



The above region is bounded.

S2. **(c)** Let us draw the graph of x+2y=10 and 3x+4y=24



Corner Point	Value of Z
(0,5)	2500
(4,3)	2300(Minimum)
(0,6)	3000

Hence, the minimum value of Z is 2300" is at the point (4,3).

S3. **(a)** Let ‘ x" ‘ be the number of old hens and ‘ y ‘ the number of young hens.
Profit =(3x+5y)$\frac{30}{100} - (x + y) \ldots (1)$
$= \frac{9x}{10} + \frac{3}{2}y x - y$
$= \frac{y}{2} - \frac{x}{10} = \frac{5y-x}{10}$
∴LPP is:
Maximize $Z = \frac{5y-x}{10}$ subject to:
$x \ge 0, y \ge 0, x + y \le 20$ and $2x + 5y \le 80$

S4. **(c)** (c) All of the given constraints

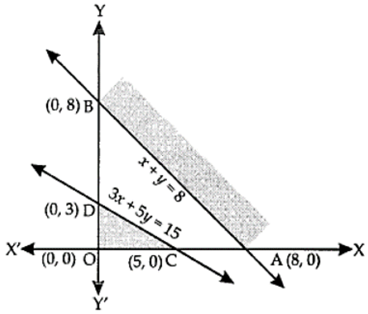
S5. **(c)** (c) Vertex point of the boundary of the feasible region

S6. **(b)** (b) function to be optimized

S7. **(c)** Feasible solution

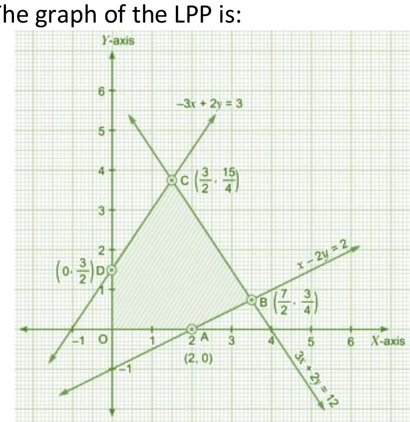
S8. **(d)** Let ‘x’ necklaces and ‘y’ bracelets be manufactured per day.
Then LPP problem is:
Maximize Z = 100x+300y
Subject to the constraints: $x + y \le 24$,
$x + \left(\frac{1}{2}\right)y \le 32 \Rightarrow 2x + y \le 32$
And $x \ge 1, y \ge 1 \Rightarrow x - 1 \ge 0, y - 1 \ge 0$

S9. **(d)** The graph for the LPP:
Minimize Z=3x+2y subject to the constraints:
$x + y \ge 8, 3x + 5y \le 15, x \ge 0, y \ge 0$



Clearly, there is no feasible region thus there is no solution.

S10. **(c)** The graph of the LPP is:



Corner Point	Value of Z
O (0,0)	0(Minimum)
A (2,0)	10
H$\left(\frac{7}{2},\frac{3}{4}\right)$	19(Maximum)
G$\left(\frac{3}{2},\frac{15}{4}\right)$	15
F$\left(0,\frac{3}{2}\right)$	3

Hence, $Z_{max} = 19$ at $\left(\frac{7}{2},\frac{3}{4}\right)$ and $Z_{Min} = 0$ at $(0,0)$

S11. **(b)** Let 'x' and 'y' be the number of packages of nuts and bolts respectively.

We have the following constraints:

$x \geq 0 \ldots (1)$

$y \geq 0 \ldots (2)$

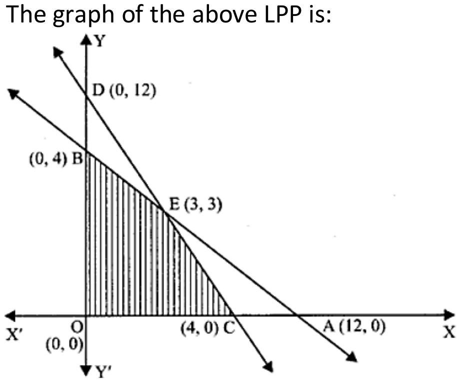
$x + 3y \leq 12 \ldots (3)$

$3x + y \leq 12 \ldots (4)$

Now the profit= 35x+ 14y …. (5)

We are to maximize P subject to constraints (1) -(4).

The graph of the above LPP is:



Corner Point	Value of Z
(0,0)	0
(4,0)	140
(3,3)	147(Maximum)
(0,4)	56

Hence, the minimum value of Z is 147 is at the point (3,3)i.e 3 nuts and 3 Bolts

S12. **(a)** Let the tailor A work for 'x' days and B for 'y' days.

We have:	Shirts	Pants
Tailor (A)	6	4
Tailor (B)	10	4

We have the following constraints:

$x \geq 0 \ldots (1)$

$y \geq 0 \ldots (2)$

$6x + 10y \geq 60$

i.e. $3x + 5y \geq 30 \ldots (3)$

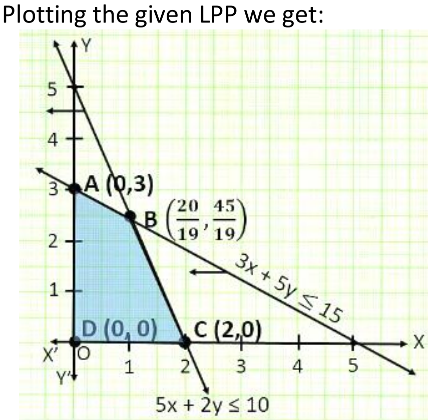
$4x + 4y \geq 32$

i.e. $x + y \geq 8 \ldots (4)$

The objective function, or the cost Z is:

Z = 150x + 200 y s.t constraint (1)-(4) follow.

S13. **(d)** Plotting the given LPP we get:

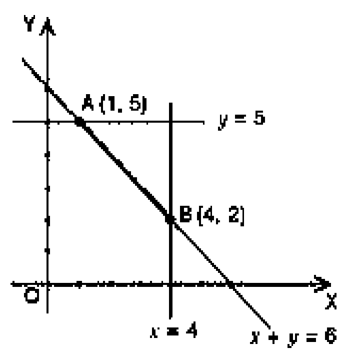


Calculating Z at different corner points we get:

$Z = 0\ at\ (0,0)\ , Z = 10\ at\ (2,0)\ , Z = 10\ at\ \left(\frac{20}{19},\frac{45}{19}\right), Z = 6\ at\ (0,3)$

Thus, we have multiple optimal solutions.

S14. **(c)** The graph of the LPP is:

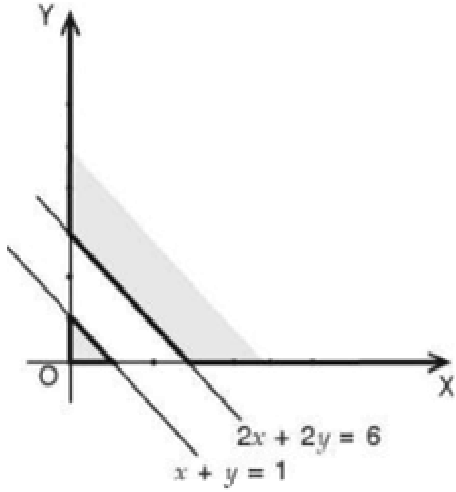


Here the feasible region in the line segment AB, with corner points A (1,5) and B (4,2).

At these corner points the value of Z is 28 and 22 respectively.

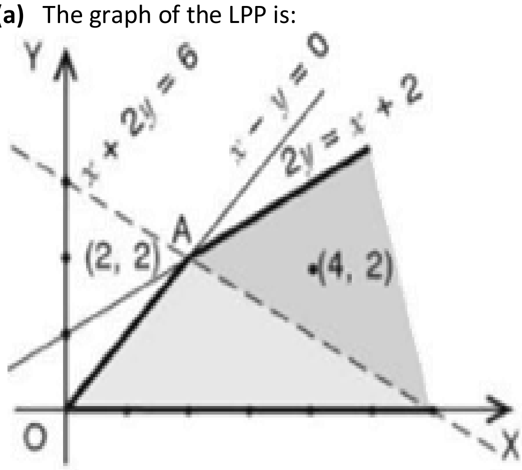
Hence the optimal solution is 28 at x=4, y=2.

S15. **(b)** The graph for the LPP:



As there is no point satisfying the given conditions, the problem is infeasible.

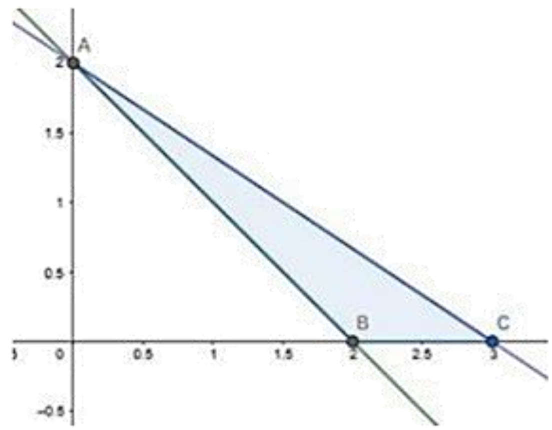
S16. **(a)** The graph of the LPP is:



We note that the feasible region is unbounded.
At the corner points
(0,0) Z=0
(2,2) Z=6
But as the feasible region is unbounded, we cannot say whether the largest value 6 is maximum or not.

S17. **(a)** Let x and y be the number of steel trunks of first type and second type produced by the manufacturer. As the profit on the first type of trunk is ₹30 and on the second type of trunk is ₹25, so the total profit " Z=30x+25y(₹).
Hence, the problem can be formulated as an L.P.P. as follows :
Maximize $Z\& = 30x + 25y$
$3x + 3y\& \leq 18$ i.e. $x + y \leq 6$
$3x + 2y\& \leq 15$
$X \geq 0, \geq 0$

S18. **(a)** The feasible region determined by the constraints x ≥ 0, y ≥ 0, x + y ≥ 2 and 2x + 3y ≤ 6 is given by

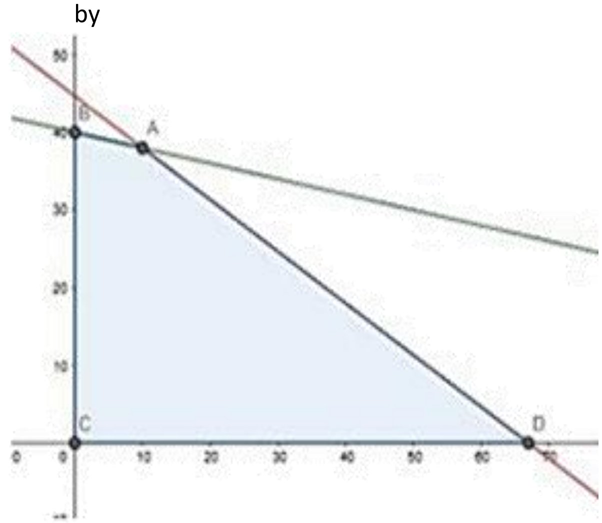


The corner points of the feasible region is A (0,2), B (2,0), C (3,0). The values of Z at the following points are

Corner point	Z = 7x + 7y	
A(0 , 2)	14	
B(2 , 0)	14	
C(3,0)	21	Maximum

The maximum value of Z is 21 at point C (3,0).

S19. **(d)** The feasible region determined by the constraints $x \geq 0, y \geq 0, x + 5y \leq 200, 2x + 3y \leq 134$ is given by

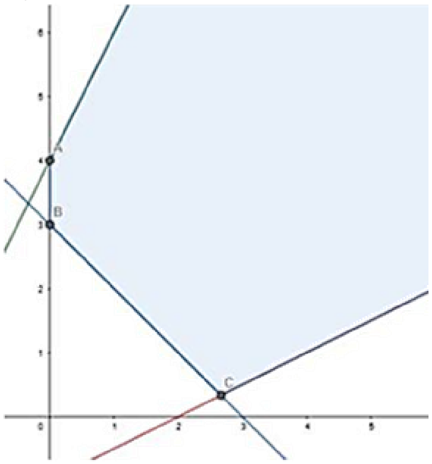


The corner points of feasible region are A(10,38), B(0,40) ,C(0,0), D(67,0) . The values of Z at the following points are

Corner point	Z = 4x + 9y	
A(10 , 38)	382	Maximum
B(0 ,40)	360	
C(0.0)	0	
D(67 , 0)	268	

The maximum value of Z is 382 at point A(10,38) .

S20. **(c)** The feasible region determined by the $-2x + y \le 4, x + y \ge 3, x - 2y \le 2, x \ge 0, y \ge 0$ is given by



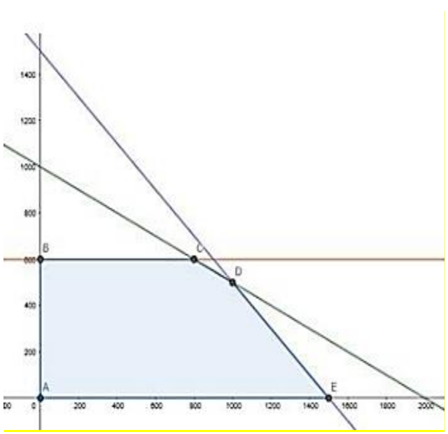
Here the feasible region is unbounded. The vertices of the region are A(0,4), B(0,3), C$\left(\frac{8}{3}, \frac{1}{3}\right)$.

The values of Z at the following points are

Corner Point	Z = 3x + 5y	
A(0 , 4)	20	
B(0 , 3)	15	
C$\left(\frac{8}{3}, \frac{1}{3}\right)$	$\frac{29}{3}$	Minimum

The minimum value of Z is $\frac{29}{3}$ at point $C\left(\frac{9}{3}, \frac{1}{3}\right)$

S21. **(d)** The feasible region determined by $x + 2y \le 2000, x + y \le 1500, y \ge 0, y \le 600, x \ge 0$ is given by



The corner points of the feasible region are A(0,0), B(0,600), C(800,600), D(1000,500), E(1500,0). The value of Z at the corner points are

Corner point	Z = 3x + 5y	
A(0,0)	0	
B(0,600)	3000	
C(800 , 600)	5400	
D(1000 , 500)	5500	Maximum
E(1500 , 0)	4500	

The maximum value of Z is 5500 at point D(1000,500).

S22. **(a)** Let the invested money in PPF be x and in national bonds be y.

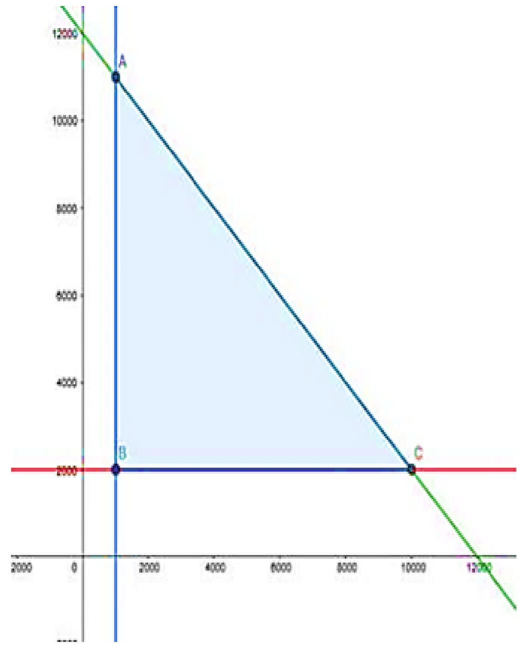
∴According to the question,

$x + y \le 12000$

$x \ge 1000, y \ge 2000$

Maximize Z = 0.12x + 0.15y

The feasible region determined by $x + y \le 12000, x \ge 1000$



The corner points of the feasible region are A(1000,11000) , B(1000,2000) and C(10000,2000) .

The value of Z at the corner point are

Corner point	Z = 0.12x + 0.15y	
A(1000 , 11000)	1770	Maximum
B(1000 , 2000)	420	
C(10000,2000)	1500	

The maximum value of Z is 1770 at point A(1000,11000).

So, he must invest Rs.1000 in PPF and Rs.11000 in national bonds.

The maximum annual income is Rs.1770 .

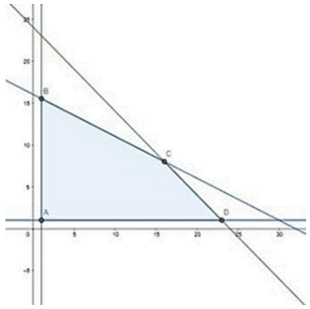
S23. **(b)** Let the firm manufacture x number of necklaces and y number of bracelets a day.

∴According to the question,

$x + y \le 24, 0.5x + y \le 16, x \ge 1, y \ge 1$

Maximize Z = 100x + 300y

The feasible region determined by $x + y \le 24, 0.5x + y \le 16, x \ge 1, y \ge 1$ is given by

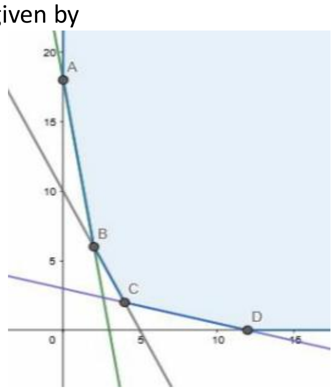


The corner points of the feasible region are A(1,1), B(1,15.5) , C(16,8) , D(23,1).The number of bracelets should be whole number. Therefore, considering point (2,15). The value of Z at corner point is

Corner point	Z = 100x+ 300y	
A(1 ,1)	400	
B(2 , 15)	4700	Maximum
C(16 , 8)	4000	
D(23 , 1)	2600	

The maximum value of Z is 4700 at point B (2,15).
∴ The firm should make 2 necklaces and 15 bracelets.

S24. (b) Let x and y be number of units of products of A and B.
∴According to the question,
$36x + 6y \geq 108, 3x + 12y \geq 36, 20x + 10y \geq 100, x \geq 0, y \geq 0$
Minimize Z = 20x + 40y
The feasible region determined $36x + 6y \geq 108, 3x + 12y \geq 36, 20x + 10y \geq 100, x \geq 0, y \geq 0$ Is given by

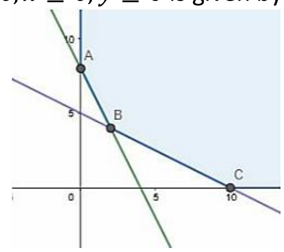


The feasible region is unbounded. The corner points of feasible region are A(0,18) , B(2,6) , C(4,2), D(12,0).
The value of Z at corner points are

Corner point	Z = 20x + 40y	
A(0,18)	720	
B(2 , 6)	280	
C(4 ,2)	160	Maximum
D(12 ,0)	240	

The minimum value of Z is 160 at point (4,2). Hence, the firm should buy 4 units of fertilizer A and 2 units of fertilizer B to achieve minimum expense of Rs.160.

S25. (d) Let x and y be number of units of X and Y.
∴According to the question,
$2x + y \geq 8, x + 2y \geq 10, x \geq 0, y \geq 0$
The feasible region determined by $2x + y \geq 8, x + 2y \geq 10, x \geq 0, y \geq 0$ is given by

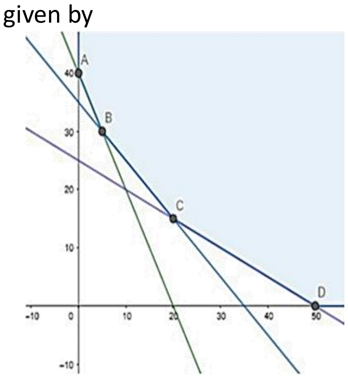


The feasible region is unbounded. The corner points of feasible region are A(0,8) , B(2,4) , C(10,0). The value of Z at corner points are

Corner point	Z = 5x + 7y	
A(0,0)	56	
B(2 ,4)	38	Minimum
C(0,0)	50	

The minimum value of Z is 160 at point (4,2). Hence, the dietician should mix 2 units of X and 4 units of Y to meet the requirements at minimum cost of Rs.38.

S26. (c) Let x and y be number of units of food A and B.
∴ According to the question, $200x + 100y \geq 4000, x + 2y \geq 50, 40x + 40y \geq 1400, x \geq 0, y \geq 0$ Minimize Z = 4x + 3y
The feasible region determined $200x + 100y \geq 4000, x + 2y \geq 50, 40x + 40y \geq 1400, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,40) , B(5,30) , C(20,15), D(50,0).The value of Z at corner points are

Corner point	Z = 4x + 3y	
A(0 , 40)	120	
B(5 , 30)	110	Minimum
C(20,15)	125	
D(50,0)	200	

The minimum value of Z is 110 at point (5,30). Hence, the diet should contain 5 units of food A and 30 units of food B for the least cost.

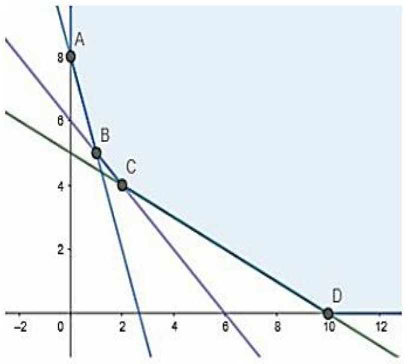
S27. (d) Let x and y be number of kilograms of food x and y.

∴According to the question,

$x + 2y \geq 10, 2x + 2y \geq 12, 3x + y \geq 8, x \geq 0, y \geq 0$

Minimize Z = 6x + 10y

The feasible region determined $x + 2y \geq 10, 2x + 2y \geq 12, 3x + y \geq 8, x \geq 0, y \geq 0$ is given by



The feasible region is unbounded. The corner points of feasible region are A(0,8) , B(1,5) , C(2,4), D(10,0). The value of Z at corner points are

Corner point	Z = 6x + 10y	
A(0 , 8)	80	
B(1 , 5)	56	
C(2 ,4)	52	
D(10 ,0)	60	Minimum

The minimum value of Z is 52 at point (2,4). Hence, the diet should contain 2 kgs of food X and 4 kgs of food Y for the least cost of Rs. 52.

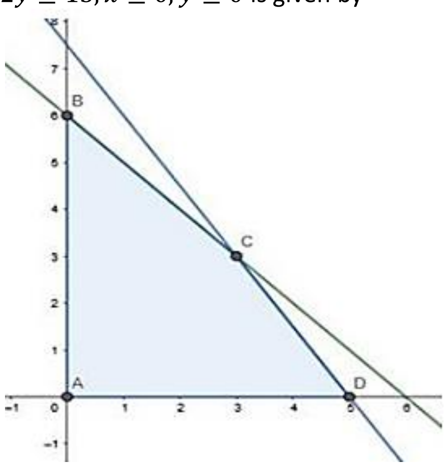
S28. (a) Let the manufacturer manufacture x and y numbers of type 1 and type 2trunks.

∴According to the question,

$3x + 3y \leq 18, 3x + 2y \leq 15, x \geq 0, y \geq 0$

Maximize Z = 30x + 25y

The feasible region determined $3x + 3y \leq 18, 3x + 2y \leq 15, x \geq 0, y \geq 0$ is given by



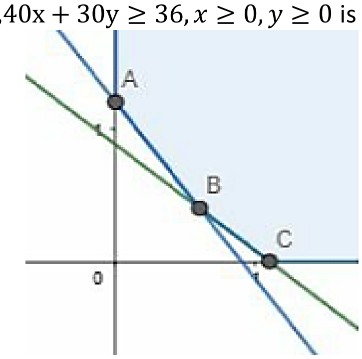
The corner points of feasible region are A(0,0), B(0,6) , C(3,3) , D(5,0).The value of Z at corner point is

Corner point	Z = 30x + 25y	
A(0 ,0)	0	
B(0,6)	150	
C(3,3)	165	Maximum
D(5,0)	150	

The maximum value of Z is 165 and occurs at point (3,3). The manufacturer should manufacture 3 trunks of each type to earn maximum profit of Rs.165.

S29. (c) Let x and y be number of kilograms of bran and rice.

∴According to the question, $80x + 100y \geq 88, 40x + 30y \geq 36, x \geq 0, y \geq 0$ Minimize Z = 5x + 4y The feasible region determined $80x + 100y \geq 88, 40x + 30y \geq 36, x \geq 0, y \geq 0$ is given by



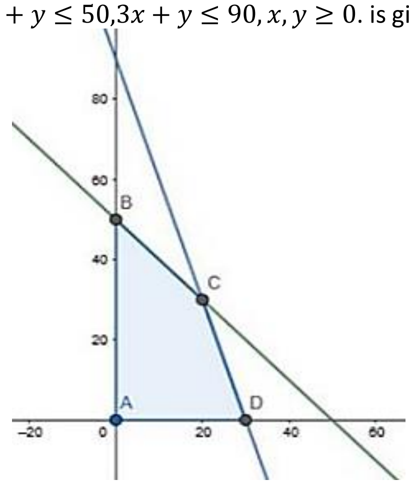
The feasible region is unbounded. The corner points of feasible region are A(0,1.2) , B(0.6,0.4) , C(1.1,0).

The value of Z at corner points are

Corner point	Z = 5x + 4y	
A(0 , 1.2)	4.8	
B(0.6 , 0.4)	4.6	Minimum
C(1.1 , 0)	5.5	

The minimum value of Z is 4.6 at point (0.6,0.4). Hence, the diet should contain 0.6 kgs of bran and 0.4 kgs of rice for achieving minimum cost of Rs.4.6.

S30. (d) The feasible region determined by the constraints $x + y \leq 50, 3x + y \leq 90, x, y \geq 0.$ is given by



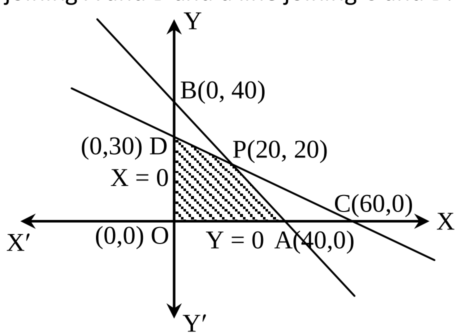
The corner points of feasible region are A(0,0),B(0,50), C(20,30), D(30,0) . The values of Z at the following points is

Corner point	Z = 60 x + 15y	
A(0 , 0)	0	
B(0 , 50)	750	
C(20 , 30)	1650	
D(30 , 0)	1800	Maximum

The maximum value of Z is 1800 at point A(30,0)

SUBJECTIVE QUESTIONS

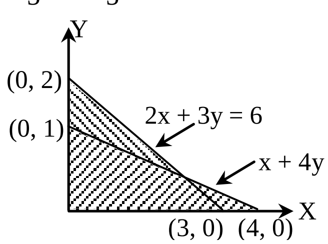
S1. Write the given inequations into equations. We have x + y = 40, x + 2y = 60 and x = 0, y = 0.
The line x + y = 40 meets the x-axis at A(40, 0) and y-axis at B(0, 40). The line x + 2y = 60 meets the x-axis at C (60,0) and y-axis at D (0,30).Draw a line joining A and B and a line joining C and D.



Let AB and CD meet at P. P will be (20, 20). Then the shaded region OAPD, common to all the solutions of the given inequation, will be the solution set of the given inequations.

S2. Write the given inequation into equation we get 2x + 3y = 6 and x + 4y = 4, on solving we get

$x = \frac{12}{5}$, $y = \frac{2}{5}$.



Hence a vertex is $\left(\frac{12}{5}, \frac{2}{5}\right)$

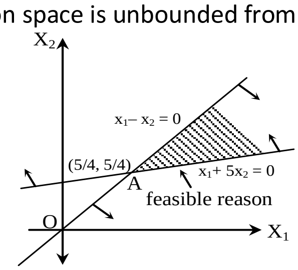
S3. The bounding lines corresponding to the inequalities of the given constraints are :

$x_1 - x_2 = 0$ $\qquad$ $x_1 - 5x_2 = -5$

& $x_1 = 0$, $x_2 = 0$

Draw these lines in a two dimensional space and consider the solution space for each given inequality.

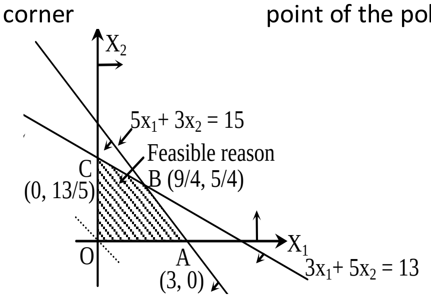
We find that the feasible region i.e., their common solution space is unbounded from one side.



It is clear from the figure that the objective function z attains its minimum value at the point A which is the intersection of the two lines $x_1 - x_2 = 0$ and $-x_1 + 5x_2 = 0$.

Solving them we find that $x_1 = x_2 = \frac{5}{4}$. Hence z is minimum when $x_1 = \frac{5}{4}$, $x_2 = \frac{5}{4}$ and then its minimum value is $2 \times \frac{5}{4} - 10 \times \frac{5}{4} = -10$

S4. Drawing the bounding lines corresponding to the given inequalities and considering their common solution space, we find that the feasible region is given by the shaded area OABC. Every point of this region gives a feasible solution of the problem whereas its optimal solution is attained at some corner point of the polygon OABC.



We find that the coordinates of the vertices A, B, C are (3, 0), $\left(\frac{9}{4}, \frac{5}{4}\right)$, $\left(0, \frac{13}{5}\right)$ respectively. Now calculating the value of the objective function at these points we have.

At A.(3, 0) ; z = 6.3 + 10.0 = 18 At B $\left(\frac{9}{4}, \frac{5}{4}\right)$; $z = 6.\frac{9}{4} + 10.\frac{5}{4} = 26$

At C$\left(0, \frac{13}{5}\right)$; $z = 6.0 + 10.\frac{13}{5} = 26$

It is observed that z is maximum at B$\left(\frac{9}{4}, \frac{5}{4}\right)$ as well as at C$\left(0, \frac{13}{5}\right)$ and its maximum value is 26. As a matter of fact every point on the line segment BC gives this max. value; because in this case the line of objective function lies along one boundary line of the feasible region. In such a case the problem has infinite number of optimal solutions.

S5. Equalities corresponding to the given constraints are

$x_1 + x_2 = 1$ $\qquad$ $x_2 - 5x_1 = 0$

$5x_2 - x_1 = 0$ $\qquad$ $x_1 - x_2 = -1$

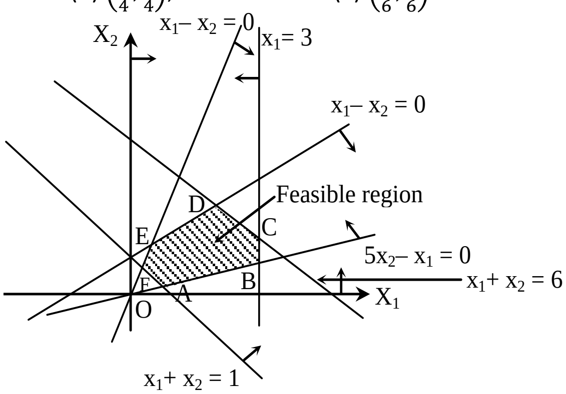
$x_1 + x_2 = 6$

Draw the straight lines represented by these equations in a two dimensional space. These are the bounding lines for the given inequalities. Now considering the solution space for each inequality of the given constraints, we find that the feasible region i.e., their common solution space is given by the shaded region ABCDEF. Every point of this region gives feasible solution of the problem whereas the optimal solution is attained at one of the vertices of the polygon ABCDEF. Coordinates of the six vertices are :

(A) $\left(\frac{5}{6}, \frac{1}{6}\right)$; $\qquad$ (B) $\left(3, \frac{3}{5}\right)$;

(c) (3,3) ; $\qquad$ (D) $\left(\frac{5}{2}, \frac{7}{2}\right)$;

(E) $\left(\frac{1}{4}, \frac{5}{4}\right)$; $\qquad$ (F) $\left(\frac{1}{6}, \frac{5}{6}\right)$.



So we find that

At A $\left(\frac{5}{6}, \frac{1}{6}\right)$; z = $3.\frac{5}{6} + 2.\frac{1}{6} = \frac{17}{6}$

At B $\left(3, \frac{3}{5}\right)$; z = $3.3 + 2.\frac{2}{5} = \frac{51}{5}$

At C (3, 3); z = 3.3 + 2.3 = 15

At D $\left(\frac{5}{2}, \frac{7}{2}\right)$; z = $3.\frac{5}{2} + 2.\frac{7}{2} = \frac{29}{2}$

At E $\left(\frac{1}{4}, \frac{5}{4}\right)$; z = $3.\frac{1}{4} + 2.\frac{5}{4} = \frac{13}{4}$

At F $\left(\frac{1}{6}, \frac{5}{6}\right)$; z = $3.\frac{1}{6} + 2.\frac{5}{6} = \frac{13}{6}$

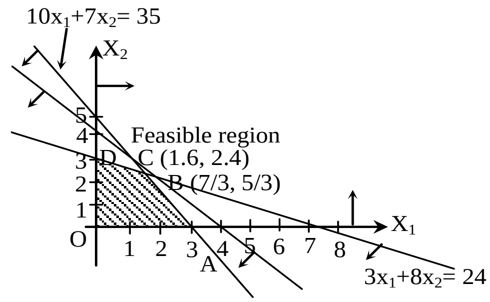
Thus z is maximum at C where $x_1 = 3$, $x_2 = 3$ and max. z = 15.

NUMERICAL TYPE QUESTIONS

S1. (24.8) Let us draw the lines

$x_1 + x_2 = 4$ $\qquad$ $3x_1 + 8x_2 = 24$

$10x_1 + 7x_2 = 35$ $\qquad$ and $x_1 = 0$, $x_2 = 0$



which corresponds to the inequalities of the given constraints. On considering the solution space for each of the given inequality, we find that the common solution space, represented by the shaded area OABCD, is the feasible region. Now to search the maximum value of z which is at one of the corners of the polygon OABCD, we find that At A(3.5, 0) ; z = 5.3.5 + 7.0 = 17.5

At B $\left(\frac{7}{3}, \frac{5}{3}\right)$; z = $5.\frac{7}{3} + 7.\frac{5}{3} = 23.3$

At C (1.6, 2.4) ; z = 5.1.6 + 7.2.4 = 24.8

At D (0, 3) ; z = 5.0 + 7.3 = 21

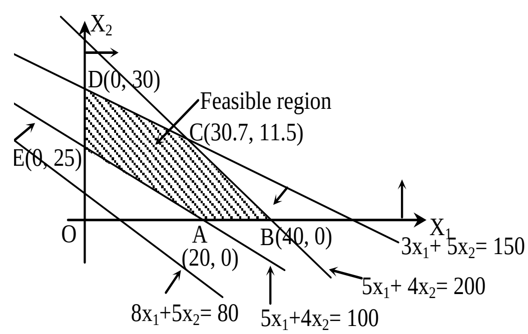
Thus z is maximum at C where x_1= 1.6, x_2 = 2.4 and max. z = 24.8

S2. **(138.1)** The bounding lines for the given inequalities are

$5x_1 + 4x_2 = 200$ $\qquad$ $3x_1 + 5x_2 = 150$

$5x_1 + 4x_2 = 100$ $\qquad$ $8x_1 + 5x_2 = 80$

And $x_1 = 0$, $x_2 = 0$



Draw these lines in a two dimensional space and consider the solution space for each inequality of the given constraints. We find that the shaded area ABCDE is their common solution space (feasible region.)

Every point of this region gives a feasible solution of the problem and the optimal solution is attained at one of the corners of this polygon. We find that the coordinates of the corner points A, B, C, D, E are (20, 0) ; (40,0) ; (30.7 , 11.5) ; (0, 30) (0,25) respectively

At A (20, 0) $\qquad$; z = 3 × 20 + 0 = 60

At B (40, 0) $\qquad$; z = 3 × 40 + 0 = 120

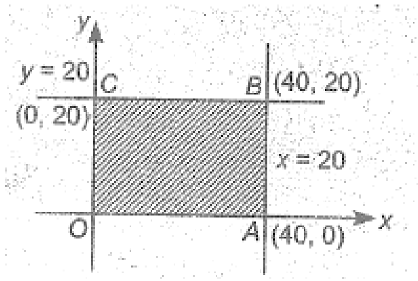
At C (30.7, 11.5) ;z =3× 30.7 + 4×11.5= 138.1

At D (0, 30) $\qquad$; z = 0 + 4 × 30 = 120

At E (0, 25) ; z = 0 + 4 × 25 = 100

The z is maximum at C where x_1 = 30.7 and x_2 = 11.5, maximum z = 138.1

S3. **(0)**



Clearly feasible region is OABCOOABCO

Now value of objective function at points

O(0,0) O (0,0) P=6 (0)+16 (0)=0P=6 (0)+16 (0)=0

A(40,0) A (40,0) P = 6 (40) + 16 (0) =240 P= 6 (40) + 16 (0) = 240

B(40,20) B(40,20) P=6(40)+16 (20)=560P=6 (40)+16 (20)=560

C(0.20) C (0.20), P=6 (0)+16 (20)=320P=6 (0)+16 (20)=320

∴ Minimum value of PP is 0 .

S4. **($12\frac{7}{19}$)** The shaded region represent the feasible region, hence $p = 5x_1 + 3x_2$ obviously it is maximum at $\left(\frac{20}{19}, \frac{45}{19}\right)$

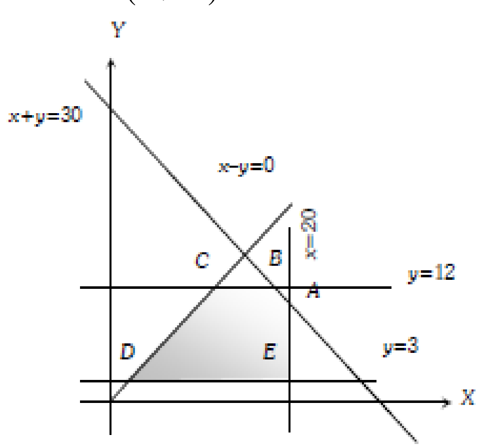
Max $p = 5x_1 + 3x_2 = 5\left(\frac{20}{19}\right) + 3\left(\frac{45}{19}\right)$

$= \frac{100}{19} + \frac{135}{19} = \frac{235}{19} = 12\frac{7}{19}$.

S5. **(30)** The objective function is Max $z = 2x + 3y$.

The vertices are A(20,10), B(18,12), C(12,12), D(3,3) and $E(20,\ 3)$.

Hence the maximum value of the objective function will be at $(18,\ 12)$.



⇒ $x = 18$, $y = 12$, then x + y = 30

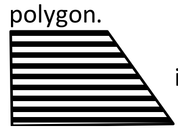
TRUE AND FALSE

S1. **(False)** A solution of L.P.P. which also satisfy the non-negative restrictions of the problem is called the feasible solution.

S2. **(True)** A feasible solution which maximize or minimize i.e. which optimize the objective function of L.P.P. called an optimal solution.

S3. **(True)** The line is drawn in geometrical area of feasible region of L.P.P. for which the objective function remains constant at all the points lie on this line, is called iso-profit line.

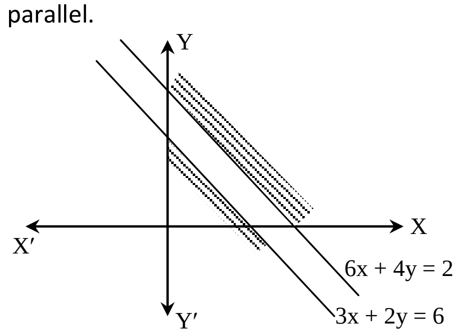
S4. **(True)** In linear programming problem mostly feasible solution is a polygon in first quadrant this polygon is a convex. It means that if two points of polygon are connecting by a line then the line must be inside to polygon.

 is convex set

S5. **(True)** The linear inequations or inequalities or restriction on the variables of a linear programming problem are constraints.

ASSERTION AND REASONING

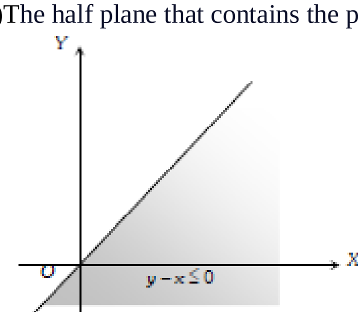
S1. **(a)** The equations, corresponding to inequalities 3x + 2y ≤ 6 and 6x + 4y ≥ 20, are 3x + 2y = 6 and 6x + 4y = 20. So the lines represented by these equations are parallel.



Hence the graphs are disjoint.

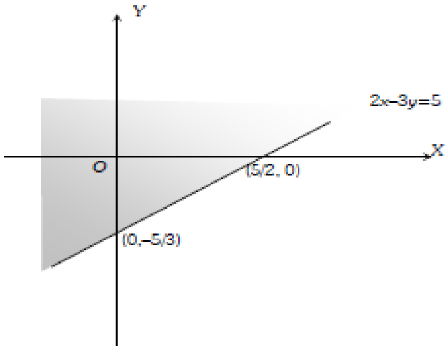
Thus, both A and R are true and R is correct explanation of A.

S2. **(d)** (A)The half plane that contains the positive x-axis



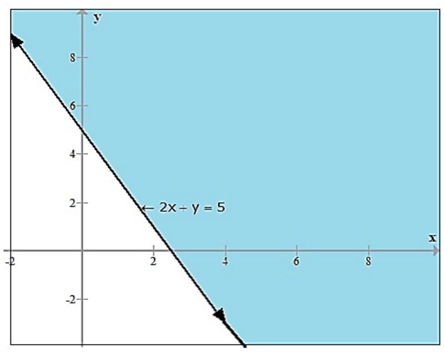
A is not true.

(R) O inside and P outside



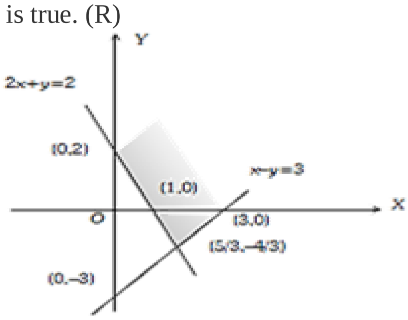
Thus R is true.

S3. **(b)** (A)



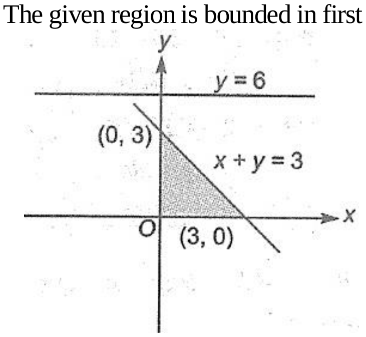
Thus the solution set is the open half plane not containing the origin.

A is true. (R)



Then correct ans is $\left(\frac{5}{3}, -\frac{4}{3}\right)$. R is true.

S4. **(d)** (a) The given region is bounded in first quadrant.



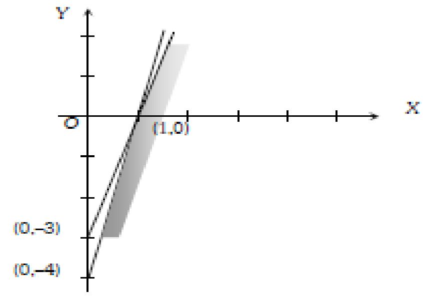
Thus A is not true.

(R) Following figure will be obtained on drawing the graphs of given inequations.

From $3x - y \geq 3, \frac{x}{1} + -\frac{y}{-3} = 1$

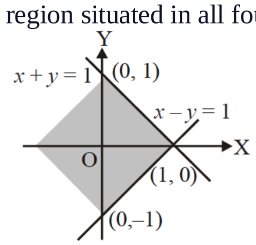
From $4x - y > 4, \frac{x}{1} + \frac{y}{-4} = 1$

Clearly the common region of both is true for positive value of (x, y). It is also true for positive value of x and negative value of y.



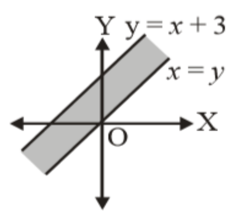
R is true.

S5. **(c)** (A) As shown in graph drawn for $x + y = 1$ and $x - y = 1$ the origin included in the area. Hence the bounded region situated in all four quadrant.



A is true. (R)

The shaded area is the required area given in graph as below.



Hence, it is in I, II and III quadrant.

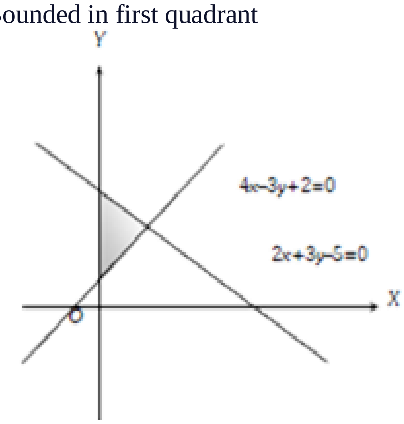
R is false.

HOMEWORK

MCQ

S1. **(a)** For the line $3x+2y=12$; origin is not included in the region hence the constraint corresponding to this line is $3x+2y\geq 12$. similarly for the line $x+3y=11$; origin is not included in the region hence the constraint corresponding to this line is $x+3y\geq 11$.

S2. **(b)** Bounded in first quadrant



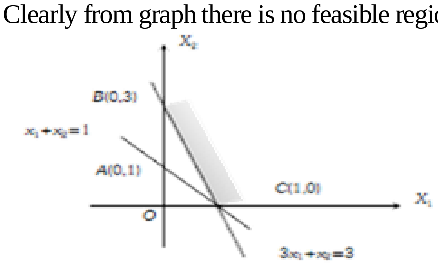
S3. **(b)** Given that: $40x+25y$

Let, the quantity of the rice is x quintal and wheat is y quintal.

According to question profit: rice per quintal is 40 rupees per quintal and for wheat is 25 rupees per quintal.

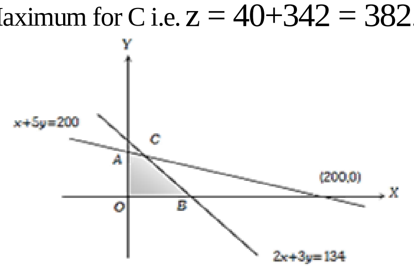
Therefore, Net profit is $40x+25y$.

S4. **(c)** Clearly from graph there is no feasible region.



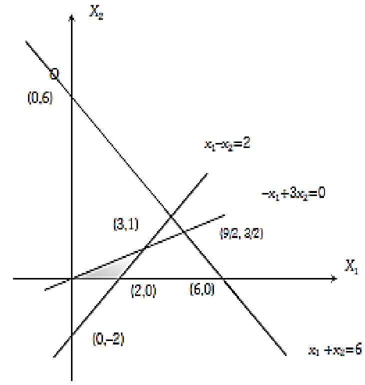
S5. **(b)** Here, A=(0,40),B=(67,0) and C=(10,38).

Maximum for C i.e. z = 40+342 = 382.



S6. **(a)** (3,1),(2,0) are vertices of Min Z for $(2,\ 0)$,

Hence $x_1=2$.



S7. **(b)** The intermediate solutions of constraints must be checked by substituting them back into Constraint equations. Constraint equations means the Constraint Equation is an equation representing any constraints that you are given in the problem.

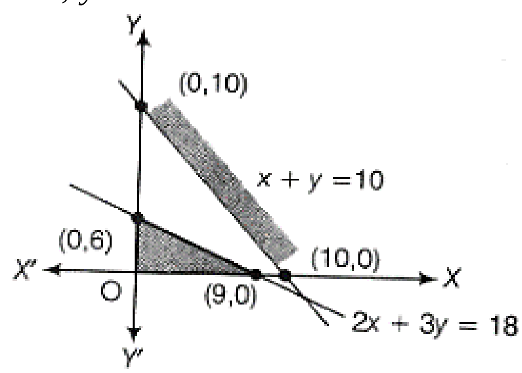
S8. **(d)** $Z = 4x+2y$

Subject to constraints

$2x+3y \leq 18$,

$x+y \geq 10$ and

$x,\ y \geq 0$



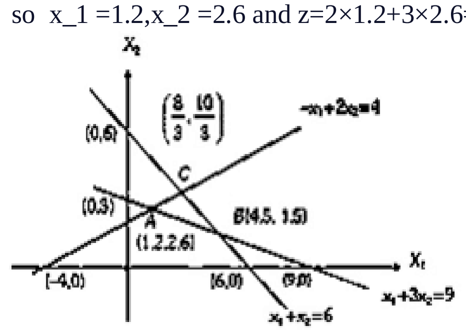
There is no common area in the first quadrant. Hence, the objective function Z cannot be maximized.

S9. **(d)** The graph of linear programming problem is as given below,

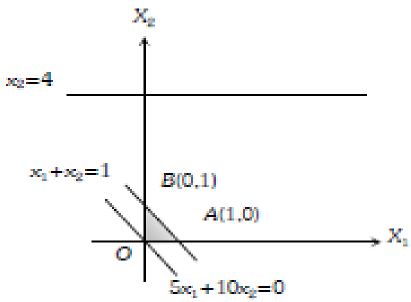
Hence the required feasible region is given by the graph whose vertices are $A(1.2, 2.6), B(4.5,1.5)$ and $C(38,310)$

Thus objective function is minimum at $A(1.2, 2.6)$

so x_1 =1.2,x_2 =2.6 and z=2×1.2+3×2.6=10.2

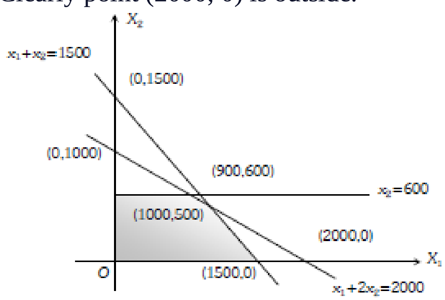


S10. **(c)** As there may be infinite values of x_1 and x_2 on line $x_1 + x_2 = 1$



SUBJECTIVE QUESTIONS

S1. Clearly point (2000, 0) is outside.



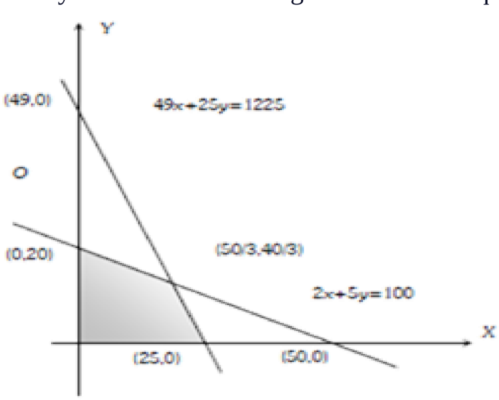
S2.

Type of items	Working time on machine	Man labour
Shirt (x)	2 hours	3 hours
Pant (y)	3 hours	2 hours
Availability	70 hours	75 hours

Linear constraints are $x \geq 0, y \geq 0, 2x + 3y \leq 70, 3x + 2y \leq 75$.

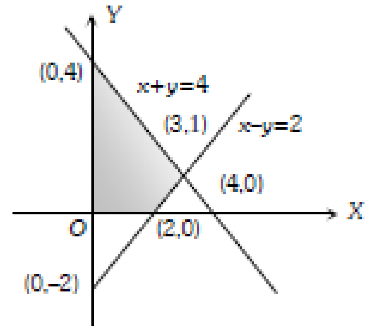
S3. Objective function is given by profit function $-x.\frac{8}{100} + y \times \frac{10}{100} - 0.08x + 0.10y.$

S4. Since the points given in (a), (b) and (c) does not satisfy the given inequalities.



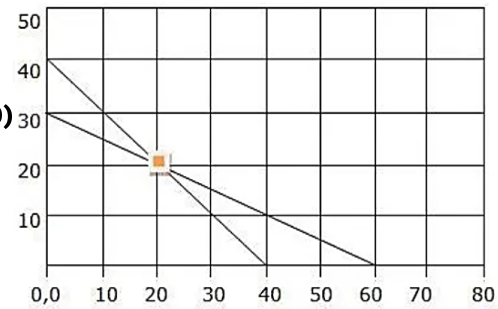
S5. $z = x + 2y$

Max $z = 0 + 4(2) = 8$



NUMERICAL TYPE QUESTIONS

S1. **(140)**



2x +2y ≤ 80; when x = 0 , y = 40 and when y = 0 , x = 40

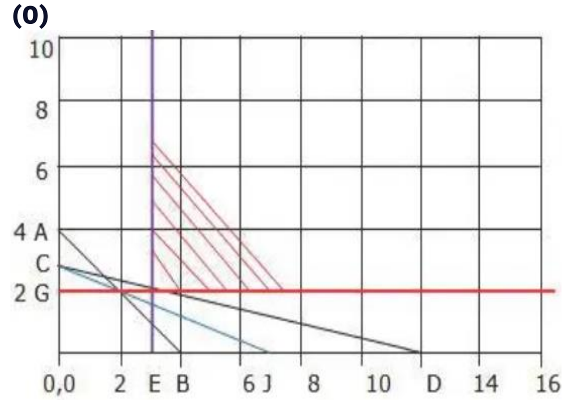
2x +4y ≤ 120; when x = 0 , y = 30 and when y = 0 , x =60

The intersection of the two plotted lines gives (20 , 20) .
Feasible area is 30 – C - 40

Corner point	Value of Z = 3x + 4y
0, 0	0
0, 30	120
20, 20	140
40, 0	120

The maxima is obtained at x = 20 , y = 20 and is 140

S2. **(0)**



2x +2y ≥ 8 ; when x = 0 , y = 4 and when y = 0 , x = 4 line AB

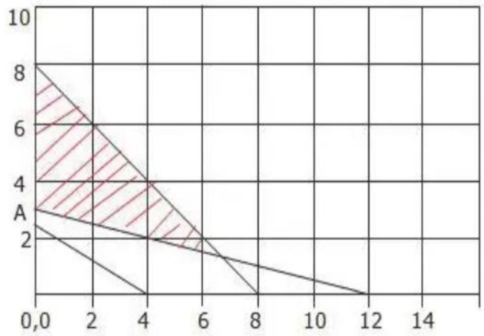
x + 4y ≥ 12 ; when x = 0 , y = 3 and when y = 0 , x = 12 line CD x ≥ 3 , y ≥ 2 are the lines parallel to y -axis and x-axis respectively.

The diverging shaded area in red lines is the area of feasible solution. This area is unbounded .

Z = 2x + 4y at (3 , 2) = 14

Plot 2x + 4y > 14 line CJ to see if there is any common region. There is no common region so there is no optimal solution.

S3. (226.66)



x + y ≤ 8 ; when x = 0 , y = 8 and when y = 0 , x= 8 , line 8 – 8

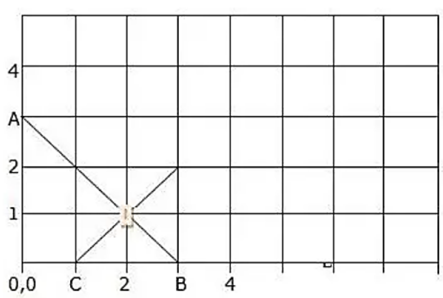
x + 4y ≥ 12 ; when x = 0 , y = 3 and when y = 0 , x = 12 line A – 12

5x + 8 y = 20 ; when x = 0 , y = $\frac{5}{2}$ and when y = 0 , x = 4
The shaded area in red is the area of feasible solution.

Corner point	Value of Z = 30x + 20y
0, 3	60
0, 8	160
6.66, 1.33	226.66

The maxima is obtained at x = 6.66 , y = 1 . 33 and is 226 .66

S4. (0)



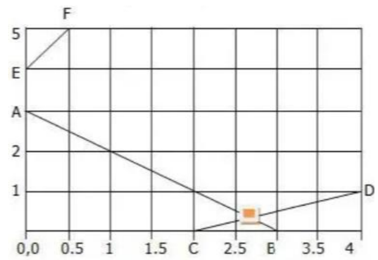
x – y ≤ 1 ; when x = 0 , y = 1 and when y = 0 , x = 2

x + y ≥ 3 ; when x = 0 , y = 3 and when y = 0 , x= 3 , line AB a unbounded region A- C-D is obtained using the constraints.

Corner point	Value of Z = 3x + 3y
0, 3	9
2, 1	9

So, optimal solution does not exist.

S5. (0)



-2x + y ≤ 4 ; or y = 2x + 4 ; when x = 0 , y = 4 and when y = 0 , x = -2 line EF

x+ y ≥ 3 ; or y = -x + 3 ; when x = 0 , y = 3 when y = 0 ,x = 3 ; line AB

x – 2y ≤ 2 ; or y = 0.5 x -1 ; when x = 0 , y = -1 and when y = 0 , x = 2 line CD

The feasible solution is the unbounded area with F – E -A -G -D

Corner point	Value of Z = 3x + 5y
(2.67, 0.33)	Minimum 9.66
(0, 3)	15
(0, 4)	20

To check whether it is the minimal value plot the objective function with a value less than 9 .66 or y = - 0.6x -1.932

It can be seen that the value of x and y are always negative. So , there is no optimal solution.

TRUE AND FALSE

S1. (true) A set of values of the decision variables which satisfy the constraints of a linear programming problem (LPP) is called a solution of the LPP.

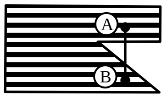
S2. (False) There is a method to solved a linear programming problem graphically i.e., corner – point method.

S3. (True) The linear function Z which is to be maximized or minimized is called the objective function.

S4. (True) A solution of a LPP is an infeasible solution, if it does not satisfy the non-negativity restriction.

S5. (True) An optimal solution of a LPP , if it exists , occurs at one of the extreme (corner) points of the convex polygon of the set of all feasible solution.

ASSERTION AND REASONING

S1. **(c)** 

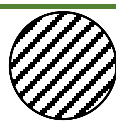
........(i)  ....(ii)

Figure (i) is not convex set while (ii) is convex set. It can be easily seen that the intersection of two convex sets is a convex set and the set of all feasible solutions of a LPP is also a convex set.

A is true but R is false.

S2. **(c)** **Assertion (A):** A set of values of the decision variables which satisfy the constraints of a Linear Programming Problem (L.P.P.) is called a solution of the L.P.P.

A is true

Reason (R): A Linear Programming Problem have only two optimal solution.

R is False , since A Linear Programming Problem may have many optimal solution. If a L.P.P. has two optimal solution, then there are an infinite number of optimal solutions.

S3. **(a)** **Assertion (A)** **:** The graph of the equation $ax + by = c$ is a straight line which divides the xy-plane into two parts known as the half spaces.

Reason(R): In set form $\{(x, y) : ax + by = c\}$ is the straight line whereas, sets $\{(x, y) : ax + by \le c\}$ and $\{(x, y) : ax + by \ge c\}$ are closed half spaces and the sets $\{(x, y) : ax + by < c\}$ and $\{(x, y) : ax + by > c\}$ are open half spaces

Both A and R are true and R is correct explanation of A.

S4. **(d)** (A) is false

There is no region common to all the solution of the given inequations, we say that the solution set is void or empty.

(R) is true

The limited (bounded) region of the graph made by two inequations is called Feasible Region.

S5. **(a)** **Assertion (A) :** The general form of Linear Programming Problems (L.P.P.) is-

Maximize (Minimize) $z = c_1x_1 + c_2x_2 + ...+ c_nx_n$

subjected to

$a_{11} x_1 + a_{12} x_2 +......+ a_{1n} x_n \{\le, =, \ge\} b_1$

$a_{21} x_1 + a_{22} x_2 + ...+ a_{2n} x_n \{\le, =, \ge\} b_2$

....

$a_{m1} x_1 + a_{m2} x_2 +......+ a_{mn} x_n \{\le, =, \ge\} b_n$

and $x_1, x_2, x_3, ..., x_n \ge 0$

Reason(R): Where $x_1, x_2, x_3, ...x_n$ are the variables whose values are to be determined and are called the **decision variables**. The inequation are called **constraints** and the function to be maximized or minimized is called the **objective function**.

Both A and R are true and R is correct explanation of A.