7

Assignment Problem and Sequencing

Let's Study

- Definition of Assignment Problem
- Assignment model
- Hungarian method of solving Assignment Problem
- Special cases of Assignment Problem
- Sequencing Problem
- Types of Sequencing Problem
- Finding an optimal sequence

Let's Recall

Linear Programming Problem

Let's :Learn

Introduction to Assignment Problem:

We often come across situations in which we have to assign n jobs to n workers. All n workers are capable of doing all jobs, but with a varying cost. Hence our task is to find the best possible assignment that gives maximum efficiency and minimum cost e.g. assigning activities to students, subjects to teachers, different routes of pizza delivery boys, salesmen to different regions, jobs to machines, products to factories, research problems to teams, vehicles and drivers to different routes etc. A problem of this nature is called an assignment problem.

7.1 Definition of Assignment Problem:

Assignment problem is a special type of problem which deals with allocation of various resources to various activities on one to one basis. It is done in such a way that the total cost or time involved in the process is minimum or the total profit is maximum.

Conditions:

i) Number of jobs is equal to number of machines or workers.

ii) Each worker or machine is assigned to only one job.

iii) Each worker or machine is independently capable of handling any job.

iv) Objective of the assignment is clearly specified (minimizing cost or maximizing profit)

Assignment Model:

Given n workers and n jobs with the cost of every worker for every job, the problem is to assign each worker to one and only one job so as to optimize the total cost.

Let C_{ij} be the cost of assigning i^{th} worker to j^{th} job, x_{ij} be the assignment of i^{th} worker to j^{th} job

and $x_{ij} = 1$, if i^{th} worker is assigned to j^{th} job

$= 0$, otherwise

Following table represents the cost of assigning n workers to n jobs.

Worker	Jobs					
	1	2	3	...	...	n
1	C_{11}	C_{12}	C_{13}	...	...	C_{1n}
2	C_{21}	C_{22}	C_{23}	...	...	C_{2n}
3	C_{31}	C_{32}	C_{33}	...	...	C_{3n}
. . .	. . .	. . .	. . .	. . .	. . .	. . .
n	C_{nl}	C_{n2}	...	...	...	C_{nn}

The objective is to make assignments that minimize the total cost.

Thus, an assignment problem can be represented by n x n matrix which covers all the n! possible ways of making assignments.

Assignment Problem is a special case of Linear Programming Problem.

Assignment problem can be expressed symbolically as follows:

$$\text{Minimize } Z = \sum_{i=1}^{n}\sum_{i=1}^{n} C_{ij}x_{ij}$$

Subject to constrains

$$\sum_{j=1}^{n} x_{ij} = 1;\ i = 1, 2, 3, n$$

(exactly one job is assigned to i^{th}worker)

$$\sum_{i=1}^{n} x_{ij} = 1;\ i = 1, 2, 3, n$$

(exactly one worker is assigned to j^{th} job

where x_{ij} takes a value 0 or 1.

Let's Discuss ...

Let us consider the following problem.

Due to neglect, your home is in serious need of repair. You approach 3 contractors for remodeling and repairing. Suppose you get quotations as shown below.

Contractors	Home Repairs (Cost)
	Price (in Rs)
Amar	27,980
Akabar	31,640
Anthony	29,330

Naturally, we may think of Amar as he is giving the best overall price. However, if we think of individual items and get the prices per repair item from our contractors, this would be more beneficial in two ways:

1. We may save time since all contractors will be working on different repair items simultaneously.
2. We may get a better price by hiring contractors based on their lowest item cost.

Take a look at the following table containing prices of individual items:

Contractors	**Home Repairs (Cost)**			**Total**
	Price (in Rs)			
	Flooring	Painting	Aluminum Sliding Window	
Amar	14440	8500	5040	Rs. 27,980
Akbar	13840	13300	4500	Rs. 31,640
Anthony	14080	11200	4050	Rs. 29,330

We wish to hire one contractor for one job to minimize both the time and cost. For example, we may choose the following:

Contractors	**Home Repairs (Cost)**			**Total**
	Price (in Rs)			
	Flooring	Painting	Aluminum Sliding Window	
Amar	14440	8500	5040	Rs. 26,390
Akbar	13840	13300	4500	
Anthony	14080	11200	4050	

By this strategy, we can minimize the total cost and also the total time required to complete the job. Though this looks simple, the problem is difficult to solve for larger number of contractors and many more repairs.

An assignment problem can be represented by n × n matrix which constitutes n! possible ways of making assignments. Finding an optimal solution by writing all the n! possible arrangements is time consuming. Hence there is a need of an efficient computational technique for solving such problems.

There is an interesting and easy method to solve this type of problems called **Hungarian Method.**

The Hungarian Method is an optimization algorithm that solves an Assignment Problem.

7.2 Hungarian Method:

Hungarian method is based on the following properties:

1) If a constant (positive or negative) is added to every element of any row or column in the given cost matrix, an assignment that minimizes the total cost in the original

matrix also minimizes the total cost in the revised matrix.

2) In an assignment problem, a solution having zero total cost of assignment is an optimal solution.

The Hungarian algorithm can be explained with the help of the following example.

Consider an example where 4 jobs need to be performed by 4 workers, one job per worker. The matrix below shows the cost of assigning a certain worker to a certain job. The objective is to minimize the total cost of assignment.

Workers	Jobs			
	J_1	J_2	J_3	J_4
W_1	62	63	50	72
W_2	57	35	49	60
W_3	21	49	15	56
W_4	18	19	78	23

Let us solve this problem by Hungarian method.

Step 1: Subtract the smallest element of each row from every element of that row.

Workers	Jobs			
	J_1	J_2	J_3	J_4
W_1	12	13	0	22
W_2	22	0	14	25
W_3	6	34	0	41
W_4	0	1	60	5

Step 2: Subtract the smallest element of each column from every element of that column.

Workers	Jobs			
	J_1	J_2	J_3	J_4
W_1	12	13	0	17
W_2	22	0	14	20
W_3	6	34	0	36
W_4	0	1	60	0

Step 3: Assign through zeros.

Workers	Jobs				
			√		
W_1	12	13	$\boxed{0}$	17	√
W_2	22	$\boxed{0}$	14	20	
W_3	6	34	~~0~~	36	√
W_4	$\boxed{0}$	1	60	~~0~~	

Observe that third row does not contain an assignment.

Step 4 :

1. Mark (√) the row (R_3).
2. Mark (√) the columns (C_3) having zeros in the marked rows.
3. Mark (√) the row (R_1) which contains assignment in marked column.
4. Draw straight lines through **marked columns** and **unmarked rows**.

Workers	Jobs				
			√		
W1	12	13	$\boxed{0}$	17	√
W2	22	$\boxed{0}$	14	20	
W3	6	34	~~0~~	36	√
W4	$\boxed{0}$	1	60	~~0~~	

All zeros can be covered using 3 lines.

Therefore, number of lines required = 3 and order of matrix = 4

Hence, the number of lines required $\neq$ order of matrix.

Therefore we continue with the next step to create additional zeros.

Step 4:

(i) Find the smallest uncovered element (6)

(ii) Subtract this number from all uncovered elements and add it to all elements which lie at the intersection of two lines and other elements on the lines remain unchanged.

Workers	Jobs			
	J_1	J_2	J_3	J_4
W_1	6	7	0	11
W_2	22	0	20	20
W_3	0	28	0	30
W_4	0	1	66	0

Step 5: Assigning through zeros we get.

Workers	Jobs			
	J_1	J_2	J_3	J_4
W_1	6	7	$\boxed{0}$	11
W_2	22	$\boxed{0}$	20	20
W_3	$\boxed{0}$	28	$\cancel{0}$	30
W_4	$\cancel{0}$	1	66	$\boxed{0}$

Now, each row and each column contains an assignment.

Hence, optimal solution is obtained and the optimal assignment is as follows.

Worker 1 should perform job 3, worker 2 job 2, worker 3 job 1 worker 4 job 4 i.e. $W_1 \rightarrow J_3$, $W_2 \rightarrow J_2$, $W_3 \rightarrow J_1$, $W_4 \rightarrow J_4$

Total Minimum Cost = 50+35+21+23 = Rs. 129

Steps of the Hungarian Method :

Following steps describe the Hungarian Method.

Step1. Subtract the minimum cost in each row of the cost matrix from all the elements in the respective row.

Step2. Subtract the minimum cost in each column of the cost matrix from all the elements in the respective column

Step3. Starting with the first row, examine the rows one by one until a row containing exactly single zero is found. Make an assignment by marking (☐) that zero. Then cross (×) all other zeros in the column in which the assignment was made. This eliminates the possibility of making further assignments in that column.

Step 4. After examining all the rows, repeat the same procedure for columns. i.e. examine the columns one by one until a column containing exactly one zero is found. Make an assignment by marking (☐) that zero. Then cross (×) all other zeros in the row in which the assignment was made.

Step 5. Continue these successive operations on rows and columns until all the zeros have been either assigned or crossed out and there is exactly one assignment in each row and in each column. In such case optimal solution is obtained.

Step 6. There may be some rows (or columns) without assignments i.e. the total number of marked zeros is less than the order of the cost matrix. In such case, proceed to step 7.

Step 7. Draw the least possible number of horizontal and vertical lines to cover all zeros. This can be done as follows:

i) Mark (✓) the rows in which no assignment has been made.

ii) Mark (✓) the column having zeros in the marked rows.

iii) Mark (✓) rows which contain assignments in marked columns.

iv) Repeat 2 and 3 until the chain of marking is completed.

v) Draw straight lines through marked columns.

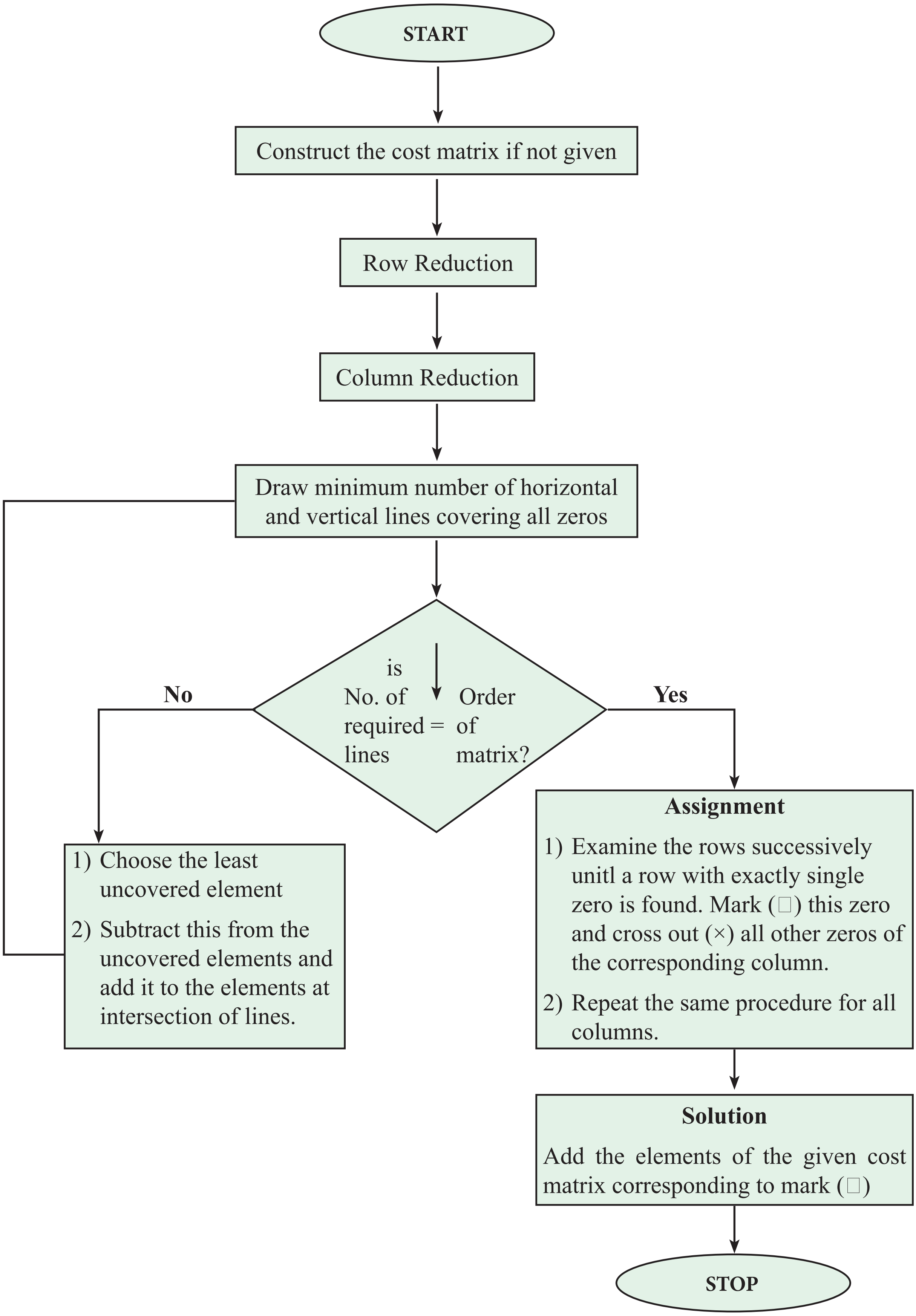
vi) Draw straight lines through unmarked rows.

By this way we draw the minimum number of horizontal and vertical lines required to cover all zeros. If the number of lines is less than the order of matrix, then there is no solution. And if the minimum number of lines is equal to the order of matrix, then there is a solution and it is optimal.

Step 8. If minimum number of lines < order of matrix, then

a) Select the smallest element not covered by any of the lines of the table.

Flow Chart of Hungarian Metod:



Draw minimum number of horizontal and vertical lines covering all zeros

1) Choose the least uncovered element

2) Subtract this from the uncovered elements and add it to the elements at intersection of lines.

Assignment

1) Examine the rows successively unitl a row with exactly single zero is found. Mark (□) this zero and cross out (×) all other zeros of the corresponding column.

2) Repeat the same procedure for all columns.

Solution

Add the elements of the given cost matrix corresponding to mark (□)

b) Subtract this value from all the uncovered elements in the matrix and add it to all those elements which lie at the intersection of horizontal and vertical lines.

Step 9. Repeat steps 4, 5 and 6 until we get the number of lines covering all zeros equal to the order of matrix. In this case, optimal solution can be obtained.

Step 10. We now have exactly one marked (☐) zero in each row and each column of the cost matrix. The assignment schedule corresponding to these zeros is the optimal assignment.

Let's Note :-

The Hungarian Method was developed and published in 1955 by Harold Kuhn, who gave the name 'Hungarian Method' as the algorithm was largely based on the earlier works of two Hungarian mathematicians : Dènes Kőnig and Jenő Egervàry.

SOLVED EXAMPLES

Ex.1.

A departmental store has four workers to pack their goods. The times (in minutes) required for each worker to complete the packings per item sold is given below. How should the manager of the store assign the jobs to the workers, so as to minimize the total time of packing.

Workers	Packing of			
	Books	Toys	Crockery	Cutlery
A	0	11	10	8
B	13	2	12	2
C	3	4	6	1
D	4	15	4	9

Solution:

Let us solve this problem by Hungarian method.

Step1: Subtract the smallest element of each row from every element of that row.

Workers	Packing of			
	Books	Toys	Crockery	Cutlery
A	0	8	7	5
B	11	0	10	⊗
C	2	3	5	0
D	⊗	11	0	5

Step 2: Since all column minimums are zero, no need to subtract anything from columns.

Step 3: Assigning through zeros we get,

Workers	Packing of			
	Books	Toys	Crockery	Cutlery
A	$\boxed{0}$	8	7	5
B	11	$\boxed{0}$	10	⊗
C	2	3	5	$\boxed{0}$
D	⊗	11	$\boxed{0}$	5

$\therefore$ Optimal assignment schedule is :

A →Books, B→ Toys, C→ Cutlery, D→Crockery. Total Minimum Time = 3 + 2 + 4 + 1 = 10 minutes.

Ex.2.

Solve the following assignment problem for minimization.

Operator	Machine				
	I	**II**	**III**	**IV**	**V**
1	18	24	19	20	23
2	19	21	20	18	22
3	22	23	20	21	23
4	20	18	21	19	19
5	18	22	23	22	21

Solution:

Let us solve this problem by Hungarian method.

Step1: Subtract the smallest element of each row from every element of that row.

	I	**II**	**III**	**IV**	**V**
1	0	6	1	2	4
2	1	3	2	0	3
3	2	3	0	1	3
4	2	0	3	1	1
5	0	4	5	4	3

Step2: Subtract the smallest element of each column from every element of that column.

	I	II	III	IV	V
1	0	6	1	2	4
2	1	3	2	0	3
3	2	3	0	1	2
4	2	0	3	1	0
5	0	4	5	4	2

Step3: Draw minimum number of lines (horizontal and vertical) that are required to cover all zeros in the matrix.

	I	II	III	IV	V	
1	$\boxed{0}$	6	1	2	4	√
2	1	3	2	$\boxed{0}$	3	
3	2	3	$\boxed{0}$	1	2	
4	2	$\boxed{0}$	3	1	$\otimes$	
5	$\otimes$	4	5	4	2	√

Here, minimum number of lines (4) < order of matrix (5). Therefore we continue with the next step to create additional zeros.

Step 4:

(i) Find the smallest uncovered element (1)

(ii) Subtract this number from all uncovered elements and add it to all elements which lie at the intersection of two lines.

	I	II	III	IV	V	
1	$\otimes$	5	$\otimes$	1	3	√
2	2	3	2	$\boxed{0}$	3	
3	3	2	$\boxed{0}$	1	2	√
4	3	$\boxed{0}$	3	1	$\otimes$	
5	$\boxed{0}$	3	4	3	1	√

Here, minimum number of lines (4) < order of matrix (5). Therefore we continue with the next step to create additional zeros.

Step5:

(i) Find the smallest uncovered element (1)

(ii) Subtract this number from all uncovered elements and add it to all elements which lie at the intersection of two lines.

(iii) Then assign through zeros.

	I	II	III	IV	V
1	$\boxed{0}$	4	$\otimes$	$\otimes$	2
2	3	3	3	$\boxed{0}$	3
3	3	1	$\boxed{0}$	$\otimes$	1
4	4	$\boxed{0}$	4	$\otimes$	$\otimes$
5	$\otimes$	2	4	2	$\boxed{0}$

Optimal Solution: 1→I, 2→IV, 3→III,

4→II, 5→V

Minimum Value = 18 + 18 + 20 +18 + 21 = 95

7.3 Special Cases of Assignment Problem:

The assignment problem is generally defined as a problem of minimization. In practice, some situations are like Assignment Problem but with some variations. The following four variations are more common and can be solved using the Hungarian method.

I. Unbalanced assignment problem:

An unbalanced assignment problem is one in which the number of resources is not equal to the number of activities i.e. the cost matrix of an assignment problem is not a square matrix (no. of rows ≠ no. of columns).

An unbalanced assignment problems can be balanced by adding dummy resources/tasks (row/column) with zero costs.

II. Maximization Problem:

Sometimes the assignment problem may deal with maximization of the objective function. To solve such a problem, we need to convert it to minimization so that we can solve it using Hungarian Method. This conversion to minimization problem can be done in either of the following ways:

(i) by subtracting all the elements from the largest element of the matrix

(ii) by multiplying all the elements of the matrix by '-1'

Then this equivalent minimization problem can be solved using Hungarian method.

III. Restricted assignment problem:

An assignment problem involving restrictions on allocation due to personal, technical, legal or other reasons is called a restricted assignment problem. A restricted assignment problem does not allow some worker(s) to be assigned to some job(s). It can be solved by assigning a very high cost (or infinite cost) to the restricted cells where assignment cannot be made.

IV. Alternative optimal solutions:

An alternate (multiple) solution exists for an assignment problem when the final assignment matrix contains more than the required number of zeros. In this case, assignments can be made through zeros arbitrarily, keeping in mind that each row and each column can contain only one assignment.

SOLVED EXAMPLES

Ex.1. [Unbalanceed assignment problem]

A departmental head has four subordinates, and three tasks to be performed. The subordinates differ in efficiency. Estimated time for that task would take to perform each given in the matrix below How should the tasks be allotted so as to minimize the total man hours?

Job	Man			
	M_1	M_2	M_3	M_4
A	7	2	6	3
B	3	7	5	4
C	5	4	3	7

Solution:

Step1: Observe that the number of rows is not equal to number of columns in the above matrix. Therefore it is an unbalanced assignment problem. It can be balanced by introducing a dummy job D with zero cost.

Job	Man			
	M_1	M_2	M_3	M_4
A	7	2	6	3
B	3	7	5	4
C	5	4	3	7
D	0	0	0	0

Step2: Subtract the smallest element of each row from every elements of that row.

Job	Man			
	M_1	M_2	M_3	M_4
A	5	0	4	1
B	0	4	5	1
C	2	1	0	4
D	0	0	0	0

Step3: Since all the column minimums are zeros, no need to subtract anything from columns.

Step4: Assigning through zeros we get,

Job	Man			
	M_1	M_2	M_3	M_4
A	5	$\boxed{0}$	4	1
B	$\boxed{0}$	4	2	1
C	2	1	$\boxed{0}$	4
D	⊗	⊗	⊗	$\boxed{0}$

Optimal Solution:

Job	Man	Man hours
A	M_2	2
B	M_1	3
C	M_3	3
	Total	8

Ex.2. [Maximization Case and Alternative Optimal Solutions]

A marketing manager has list of salesmen and towns. Considering the capabilities of the salesmen and the nature of towns, the marketing manager estimates amounts of sales per month (in thousand rupees) for each salesman in each town. Suppose these amounts are as follows:

Salesman	Town				
	T_1	T_2	T_3	T_4	T_5
S_1	37	43	45	33	45
S_2	45	29	33	26	41
S_3	46	32	38	35	42
S_4	27	43	46	41	41
S_5	34	38	45	40	44

Find the assignment of salesmen to towns that will result in maximum sale.

Solution: The above maximization problem can be converted into the equivalent minimization problem by subtracting all the matrix elements from the largest element which is 46. Then the resulting matrix is.

Salesman	**Town**				
	T_1	T_2	T_3	T_4	T_5
S_1	9	3	1	13	1
S_2	1	17	13	20	5
S_3	0	14	8	11	4
S_4	19	3	0	5	5
S_5	12	8	1	6	2

Now, we can solve this problem by Hungarian method.

Step1: Subtract the smallest element of each row from every element of that row.

Salesman	**Town**				
	T_1	T_2	T_3	T_4	T_5
S_1	8	2	0	12	0
S_2	0	16	12	19	4
S_3	0	14	8	11	4
S_4	19	3	0	5	5
S_5	11	7	0	5	1

Step2: Subtract the smallest element of each column from every element of that column.

	T_1	T_2	T_3	T_4	T_5
S_1	8	0	0	7	0
S_2	0	14	12	14	4
S_3	0	12	8	6	4
S_4	19	1	0	0	5
S_5	11	5	0	0	1

Step3: Draw minimum number of lines (horizontal and vertical) that are required to cover all zeros in the matrix.

	T_1	T_2	T_3	T_4	T_5
S_1	8	0	0	7	0
S_2	0	14	12	14	4
S_3	0	12	8	6	4
S_4	19	1	0	0	5
S_5	11	5	0	0	1

Therefore, number of lines required (4) < order of matrix (5)

Therefore we continue with next step to create additional zeros.

Step 4:

(i) Find the smallest uncovered elements (4).

(ii) Subtract this number from all uncovered elements and add it to all elements which lie at the intersection of two lines.

	T_1	T_2	T_3	T_4	T_5
S_1	12	0	0	7	0
S_2	0	10	8	10	0
S_3	0	8	4	2	0
S_4	23	1	0	0	5
S_5	15	5	0	0	1

Step5: We return to step 3 i.e. again we determine the minimum number of lines required to cover all zeros in the matrix.

	T_1	T_2	T_3	T_4	T_5
S_1	12	0	0	7	0
S_2	0	10	8	10	0
S_3	0	8	4	2	0
S_4	23	1	0	0	5
S_5	15	5	0	0	1

Number of lines required (5) = Order of matrix

Therefore optimal assignment can be made.

Optimal assignment: Optimal assignment can be made through zeros.

Note that after assigning $S_1 \rightarrow T_1$, each row and column more than one zeros. Therefore alternate optimal solutions exist. Assigning through zeros in different ways, we get two different assignments:

(i)

	T_1	T_2	T_3	T_4	T_5
S_1	12	$\boxed{0}$	⊗	7	⊗
S_2	⊗	10	8	10	$\boxed{0}$
S_3	$\boxed{0}$	8	4	2	⊗
S_4	23	1	⊗	$\boxed{0}$	5
S_5	15	5	$\boxed{0}$	⊗	1

$S_1 \rightarrow T_2$, $S_2 \rightarrow T_5$, $S_3 \rightarrow T_1$, $S_4 \rightarrow T_4$, $S_5 \rightarrow T_3$

Maximum Sale = 43 + 41 + 46 + 41 + 45

= 216 thousand rupees.

(ii)

	T_1	T_2	T_3	T_4	T_5
S_1	12	$\boxed{0}$	⊗	7	⊗
S_2	$\boxed{0}$	10	8	10	⊗
S_3	⊗	8	4	2	$\boxed{0}$
S_4	23	1	$\boxed{0}$	⊗	5
S_5	15	5	⊗	$\boxed{0}$	1

$S_1 \rightarrow T_2$, $S_2 \rightarrow T_1$, $S_3 \rightarrow T_5$, $S_4 \rightarrow T_3$, $S_5 \rightarrow T_4$

Maximum Sale = 43 + 45 + 42 + 46 + 40

= 216 thousand rupees.

- **Observe that the amount of Maximum Sale is same in both the cases.**

Ex.3. [Restricted assignment problem]

Three new machines M_1, M_2, M_3 are to be installed in a machine shop. There are four vacant places A, B, C, D. Due to limited space, machine M_2 can not be placed at B.

The cost matrix (in hundred rupees) is as follows :

Machines	Places			
	A	B	C	D
M_1	13	10	12	11
M_2	15	-	13	20
M_3	5	7	10	6

Determine the optimum assignment schedule.

Solution:

Step1:

(a) Observe that the number of rows is not equal to number of columns in the above matrix. Therefore it is an unbalanced assigned problem. It can be balanced by introducing a dummy job D with zero cost.

(b) Also, it is a restricted assignment problem. So we assign a very high cost '∞' to the prohibited cell.

Machines	Places			
	A	B	C	D
M_1	13	10	12	11
M_2	15	∞	13	20
M_3	5	7	10	6
M_4	0	0	0	0

Step2: Subtract the smallest element of each row from every element of that row.

Machines	Places			
	A	B	C	D
M_1	3	0	2	1
M_2	2	∞	0	7
M_3	0	2	5	1
M_4	0	0	0	0

Step3: Since all the column minimums are zeros, no need to subtract anything from columns.

Step4: Assigning through zeros we get,

Machines	Places			
	A	B	C	D
M_1	3	$\boxed{0}$	2	1
M_2	2	∞	$\boxed{0}$	7
M_3	$\boxed{0}$	2	5	1
M_4	$\otimes$	$\otimes$	$\otimes$	$\boxed{0}$

Optimal Solution:

Machnine	Place	Man hours
M_1	A	10
M_2	B	13
M_3	C	5
	Total	28

Therefore, Total Minimum Cost = 28 hundred rupees.

Let's Remember

- Assignment Problem is a special case of LPP in which every worker or machine is assigned only one job.
- Objective of the assignment is clearly specified (minimizing cost or maximizing profit).
- Hungarian Method is used to solve a minimization assignment problem.
- Special Cases of Assignment Problem:

1) **Unbalanced assignment problem:**

(No. of rows $\neq$ No of columns)

An unbalanced assignment problem can be balanced by adding dummy row/column with zero costs.

2) **Maximization Problem:**

Such problem is converted to minimization by subtracting all the elements from the largest element of the matrix. Then this can be solved by Hungarian method.

3) **Restricted assignment problem:**

It can be solved by assigning a very high cost (infinite cost) to the restricted cell.

4) **Alternative optimal solutions:**

If the final assignment matrix contains more than the required number of zeros, assign through zeros arbitrarily.

EXERCISE 7.1

1. A job production unit has four jobs A, B, C, D which can be manufactured on each of the four machines P, Q, R and S. The processing cost of each job for each machine is given in the following table:

Job	Machines (Processing Cost in Rs.)			
	I	II	III	IV
P	31	25	33	29
Q	25	24	23	21
R	19	21	23	24
S	38	36	34	40

Find the optimal assignment to minimize the total processing cost.

2. Five wagons are available at stations 1, 2, 3, 4 and 5. These are required at 5 stations I, II, III, and IV and V. The mileage between various stations are given in the table below. How should the wagons be transported so as to minimize the mileage covered?

	I	II	III	IV	V
1	10	5	9	18	11
2	13	9	6	12	14
3	3	2	4	4	5
4	18	9	12	17	15
5	11	6	14	19	10

3. Five different machines can do any of the five required jobs, with different profits resulting from each assignment as shown below:

Job	Machines (Profit in Rs.)				
	A	B	C	D	E
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

Find the optimal assignment schedule.

4. Four new machines M_1,M_2,M_3 and M_4 are to be installed in a machine shop. There are five vacant places A, B. C, D and E available. Because of limited space, machine M_2 cannot be placed at C and M_3 cannot be placed at A. The cost matrix is given below.

Machines	Places				
	A	B	C	D	E
M1	4	6	10	5	6
M2	7	4	-	5	4
M3	-	6	9	6	2
M4	9	3	7	2	3

Find the optimal assimment schedule.

5. A. company has a team of four salesmen and there are four districts where the company wants to start its business. After taking into account the capabilities of salesmen and the nature of districts, the company estimates that the profit per day in rupees for each salesman in each district is as below :

Salesman	District			
	1	2	3	4
A	16	10	12	11
B	12	13	15	15
C	15	15	11	14
D	13	14	14	15

Find. the assignment of salesman to various districts which will yield maximum profit.

6. In the modification of a plant layout of a factory four new machines M_1, M_2, M_3 and M_4 are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M2 can not be placed at C and M3 can not be placed at A the cost of locating a machine at a place (in hundred rupees) is as follows.

Machines	Location				
	A	B	C	D	E
M1	9	11	15	10	11
M2	12	9	-	10	9
M3	-	11	14	11	7
M4	14	8	12	7	8

Find the optimal assignment schedule.

SEQUENCING PROBLEM

Introduction to Sequencing Problem:

Suppose we have two machines - A : Cutting and B: Sewing machine Suppose there are two items I and II to be processed on these machines in the order A-B.

The machines can handle only one job at a time and the time taken in hours by the machines to complete the jobs is given by following table.

Machine	Item	
	I	II
A	6	3
B	3	6

Then there are two ways of completing this task

(i) Processing in the order I-II

(ii) Processing in the order II-I

Case i. Let us start with item I at 0 hours. Then we get

Item	Machine			
	A		B	
	In	Out	In	Out
1	0	6	6	9

Processing of item I starts at 0 hrs and is completed at 6 hrs, Note that during this time, though machine B is idle, it can not process job II, since cutting is required completed before sewing.

Once the processing of item I is completed on machine A at 6 hours, it is shifted to machine B for sewing immediately.

Machine B being idle, item I is immediately taken for processing on machine B without any wastage of time.

Therefore, 'time in' for item I on machine B is 6 and time out is 6 + 3 = 9; where 3 hrs is the time required for the processing of item I on machine B.

While item I is being processed on machine B. Machine A is free and hence, it can take item II for processing.

Thus, item II enters machine A at 6 hrs and since it needs 3 hrs for cutting (refer to table 1) it gets out at 9 hrs from machine A.

At 9 hrs machine B is available, and hence can take, item II for processing at that time.

Item II requires 6 hrs on machine B, and will be out from machine B at 9 + 6 = 15 hrs. as shown below :

Item	Machine			
	A		B	
	In	Out	In	Out
I	0	6	6	9
II	6	9	9	15

Thus, the processing of items I and II in the order I - II takes 15 hours.

Case ii.

Let us now see what happens if we change the order of processing the two items i.e. processing item II first and item I second, (II-I)

Repeating the same process as in **case (i)** we get

Item	Machine			
	A		B	
	In	Out	In	Out
II	0	3	3	9
I	3	9	9	12

Therefore, processing of items in the order II-I takes 12 hours.

Observe that -

Order of processing the items	Time required to complete the task
I-II	15 hrs
II-I	12 hrs.

By a mere change in the order of processing of the two jobs, we could save 3 hrs.

Thus, it is very important to decide the order in which the jobs should be lined up for processing so as to complete the entire schedule in minimum time.

Such type of problem where, one has to determine the order or sequence in which the jobs are to be processed through machines so as to minimize the total processing time is called a **'sequencing problem.'**

Conditions:

1) No machine can process more than one job at a time.
2) Each job, once started, must be processed till its completion.
3) The processing times are independent of the order of processing the jobs.
4) Each machine is of different type.
5) The time required to transfer a job from one machine to another is negligible.

Terminology:

1) Total Elapsed Time:

It is the time required to complete all the jobs i.e. the entire task.

Thus, total elapsed time is the time between the beginning of the first job on the first machine till the completion of the last job on the last machine.

2) Idle Time:

Idle time is the time when a machine is available but not being used, i.e. the machine is available but is waiting for a job to be processed.

General Sequencing Problem:

Let there be `n' jobs, to be performed one at a time, on each of 'm' different machines, where the order of processing on machines and the processing time of jobs on machines is known to us. Then our aim is to find the optional sequence of processing jobs that minimizes the total processing time or cost.

Hence our job is to find that sequence out of $(n!)^m$ sequences, which minimizes the total elapsed time.

NOTATION

A_i, B_i : Processing time required by i^{th} job on machine A and machine B (i =1, 2, 3...n)

T : Total elapsed time

X_A, X_B: Idle times on machines A, B from end of $(i-1)^{th}$ job to the start of i^{th} job

Type of sequencing problems:

I. Sequencing n jobs on two machine.

II. Sequencing n jobs on three machine.

7.4.1 Sequencing n jobs on Two Machine:

Let there be 'n' job each of which is to be processed through two machines say A and B in the order AB. Let the processing time A_1, A_2, A_3, ...A_n, B_1, B_2, B_3...B_n be given.

Algorithm to find Optimal Sequence:

1) Find out Min $\{A_i, B_i\}$

2) (a) If the minimum processing time is A_r, then process r^{th} job first.

   (b) If the minimum processing time is B_s, then process s^{th} job in the last.

3) Case of tie: Tie can be broken arbitrarily.

4) Cross off the jobs already placed in the sequence and repeat steps 1 to 3 till all the jobs are placed in the sequence.

5) Once the sequence is decided, prepare the work table and find total elapsed time.

SOLVED EXAMPLES

Ex.1. We have five jobs each of which has to go through the Machine M_1 and M_2 in the order M_1, M_2. Processing time (in hours) are given as:

Job	I	II	III	IV	V
Machine A	3	3	7	5	2
Machine B	6	4	2	1	5

Determine a sequence of these job that will minimize the total elapsed time T, idle time for machine M_1 and idle time for machine M_2.

Solution:

Observe that Min $\{A_i, B_i\}$ = 1, which corresponds to job IV on machine B.

Therefore, job IV is placed last in the sequence.

				IV

Then the problem reduces to :

Job	I	II	III	V
Machine A	3	3	7	2
Machine B	6	4	2	5

Now,

Min$\{A_i, B_i\}$ = 2, which corresponds to job V on machine A & job III on machine B is placed. Therefore, job V is placed first and job III is placed next to last.

V			III	IV

Then the problem reduces to:

Job	I	II
Machine A	3	3
Machine B	6	4

Now, Min$\{A_i, B_i\}$ = 3, which corresponds to job I and job II on machine A.

Therefore, job I and II can be placed after job V in any order. i.e.

V	I	II	III	IV

or

V	II	I	III	IV

Therefore, the optimal sequence is: V-I-II-III-IV or V-II-I-III-IV

(i) Total elapsed time for sequence V-I-II-III-IV

Job	Machine A		Machine B	
	In	Out	In	Out
V	0	2	2	7
I	2	5	7	13
II	5	8	13	17
III	8	15	17	19
IV	15	20	20	21

(ii) Total elapsed time for sequence V-II-I-III-IV

Job	Machine M_1		Machine M_2	
	In	Out	In	Out
V	0	2	2	7
II	5	8	11	17
I	2	5	7	11
III	8	15	17	19
IV	15	20	20	21

(Observe that, through the optimal sequence are different, total elapsed time is same i.e. 21 hrs)

$\therefore$ Total elapsed time = 21 hrs.

Idle time for machine A = T - (sum of processing times of all jobs on M_1)

= 21- 20

= 1hrs.

Idle time for machine B = T - (sum of processing times of all jobs on M_2)

= 21 - (6 + 4 + 2 + 1 + 5)

= 21 - 18

= 3 hrs.

Ex. 2. A book has one printing machine, one binding machine and manuscripts of 7 different books. The times required for performing printing and binding operations for different books are shown below:

Book	1	2	3	4	5	6	7
Printing time (hours	20	60	50	30	110	25	55
Binding time (hours)	25	40	43	24	80	35	40

Decide the optimum sequence of processing of books in order to minimize the total time required to bring out all the books.

Solution:

Let A be the printing machine and

B be the binding machine

Observe that Min$\{A_i, B_i\}$ = 20, which corresponds to 1st Book on machine A

Therefore, book 1 is processed first on machine M_1.

1						

Then the problem reduces to:

Book	2	3	4	5	6	7
A	60	50	30	110	25	55
B	40	45	24	80	35	40

Now, Min$\{A_i, B_i\}$ = 24, which corresponds to 4th book on machine B.

1						4

Therefore, book 4 is processed in the last.

Then the problem reduces to:

Book	2	3	5	6	7
A	60	50	110	25	55
B	40	45	80	35	40

Now, Min$\{A_i, B_i\}$ = 25, which corresponds to book 6 on machine A.

1	6					4

Book	2	3	5	7
A	60	50	110	55
B	40	45	80	40

Min$\{A_i, B_i\}$ = 40, which corresponds to book 2 and 7 are processed last but before 4th book in any order i.e.

1	6	3	5	2	7	4

or

1	6	3	5	7	2	4

Then the problem reduces to

Book	3	5
A	50	110
B	45	80

Now, Min $\{A_i, B_i\}$ = 45, which corresponds to book 3 on machine B. Therefor, book 3 is processed after book 6 and at the remaining.

Optional Sequence is :

1	6	3	5	2	7	4

or

1	6	3	5	7	2	4

Total elapsed time.

Job	Machine A		Machine B	
	In	Out	In	Out
1	0	20	20	45
6	20	45	45	80
3	45	95	95	140
5	95	205	205	285
2	205	265	285	325
7	265	320	325	365
4	320	350	365	389

$\therefore$ Total elapsed time = 389 hrs.

Idle time for machine A = 389 - 350 = 39 hrs.

Idle time for machine B = 20 + 15 + 65 = 100 hrs. or = 389 - 289 = 100 hrs.

7.4.2 Sequencing 'n' Jobs on Three Machines:

Let there be 'n' jobs each of which is to be processed through three machines say A, B and C in the order ABC. To solve this problem -

(i) first reduce it to the 'n job 2 machine' problem and determine the optimal sequence.

(ii) once the sequence is determined, go back to the original 3 machines and prepare the work table for 3 machines.

Conditions for reducing a 3 machine problem to a 2 machine problem:

To convert a 3 machine problem into a 2 machine problem, at least one of the following conditions must hold true.

1) The minimum processing time for machine A is greater than or equal to the maximum processing time for machine B.

i.e. Min $A_i \geq$ Max B_i, i = 1, 2, 3, ... n

OR

2) The minimum processing time for machine C is greater than or equal to the maximum processing time for mac-hine B.

i.e. Min C_i, $\geq$ Max B_i, i = 1, 2, 3, ... n

PROCEDURE

Step1. If either one of the above conditions holds, go to step 2. If not, the method fails.

Step 2. Introduce two fictitious machines say G and H such that

$G_i = A_i + B_i$

$H_i = B_i + C_i$, i = 1, 2, 3, ... n

Where G_i and H_i are the processing times of i^{th} job on machines G and H respectively. Now, solve the problem as n jobs 2 machines (G, H) problem as before.

SOLVED EXAMPLES

Ex.1. Determine the optimal sequence of job that minimizes the total elapsed time for the date given below (processing time on machines is given in hours). Also find total elapsed time T and the idle time for three machines.

Job	I	II	III	IV	V	VI	VII
Machine A	3	8	7	4	9	8	7
Machine B	4	3	2	5	1	4	3
Machine C	6	7	5	11	5	6	12

Solution: Here, minA=3, min C=5, and max B=5

Since min C ≥ max B is satisfied, the problem can be converted into a two machine problem.

Let G and H be two fictions machines such that G = A + B and H = B + C

Then the problem can be written as

Job	I	II	III	IV	V	VI	VII
Machine G	7	11	9	9	10	12	10
Machine H	10	10	7	16	6	10	15

Using the optimal sequence algorithm, the following optimal sequence can be obtained.

I	IV	VII	VI	II	III	V

Total elapsed time.

Job	Machine A		Machine B		Machine C	
	In	Out	In	Out	In	Out
I	0	3	3	7	7	13
IV	3	7	7	12	13	24
VII	7	14	14	17	24	36
VI	14	22	22	26	36	42
II	22	30	30	33	42	49
III	30	37	37	39	49	54
V	37	46	46	47	54	59

∴ Total elapsed time = 59 hrs.

Idle time for machine A = 59 - 46 = 13 hrs.

Idle time for machine B = 59 - 22 = 37 hrs.

Idle time for machine C = 59 - 52 = 7 hrs.

Ex. 2. Find the sequence that minimizes the total times required in performing following jobs on three machines in the order ABC. Processing times (in hrs.) are given in the following table.

Job	1	2	3	4	5
Machine A	8	10	6	7	11
Machine B	5	6	2	3	4
Machine C	4	9	8	6	5

Solution: Here, min A = 6, min C = 4 and max B = 6

Since min A ≥ max B is satisfied, the problem can be converted into a two machine problem.

Let G and h be two fictitious machines such that G = A + B and H = B + C

Then the problem can be written as.

Job	1	2	3	4	5
Machine G	13	16	8	10	15
Machine H	9	15	10	9	9

Using the optimal sequence algorithm, the following optimal sequence can be obtained.

3	2	5	4	1

Total elapsed time

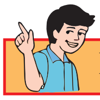
Job	Machine A		Machine B		Machine C	
	In	Out	In	Out	In	Out
3	0	6	6	8	8	16
2	6	16	16	22	22	31
5	16	27	27	31	31	36
4	27	34	34	37	37	43
1	34	42	42	47	47	51

∴ Total elapsed time = 51 hrs.

Idle time for machine A = 51 - 42 = 9 hrs.

Idle time for machine B = 6 + 8 + 5 + 3 + 5
+ (51-47) = 31 hrs.

Idle time for machine C=8 + 6 +1 + 4=19hrs.



Let's Remember

- **Sequencing problem :** In sequencing problems, one has to determine the order or sequence in which the jobs are to be processed through machines so as to minimize the total processing time.
- **Total Elapsed Time:** It is the time required to complete all the jobs i.e. the entire task.
- **Idle Time:** Idle time is the time when a machine is available, but is not being used.

- **Types of sequencing problems :**
- **Sequencing n jobs on Two machines :**
- **Sequencing n jobs on Three machines:**

To convert a 3 machine problem into a 2 machine problem, at least one of the following conditions must hold true.

1) Min $A_i \geq$ Max B_i OR 2) Min $C_i \geq$ Max B_i

$i = 1, 2, 3, \ldots n$

If either one of the above conditions hold, introduce two fictitious machines say

- G and H such that : $G_i = A_i + B_i$

  $H_i = B_i + C_i$, $i = 1, 2, 3, \ldots n$

  If not, the problem cannot be solved.

EXERCISE 7.2

1. A machine operator has to perform two operations, turning and threading on 6 different jobs. The time required to perform these operations (in minutes) for each job is known. Determine the order in which the jobs should be processed in order to minimize the total time required to complete all the jobs. Also find the total processing time and idle times for turning and threading operations.

Job	1	2	3	4	5	6
Time for turning	3	12	5	2	9	11
Time for threading	8	10	9	6	3	1

2. A company has three jobs on hand. Each of these must be processed through two departments, in the order AB where

   Department A : Press shop and

   Department B : Finishing

   The table below gives the number of days required by each job in each department

Job	I	II	III
Department A	8	6	5
Department B	8	3	4

Find the sequence in which the three jobs should be processed so as to take minimum time to finish all the three jobs. Also find idle time for both the departments.

3. An insurance company receives three types of policy application bundles daily from its head office for data entry and tiling. The time (in minutes) required for each type for these two operations is given in the following table:

Policy	1	2	3
Data Entry	90	120	180
Filing	140	110	100

Find the sequence that minimizes the total time required to complete the entire task. Also find the total elapsed time and idle times for each operation.

4. There are five jobs, each of which must go through two machines in the order XY. Processing times (in hours) are given below. Determine the sequence for the jobs that will minimize the total elapsed time. Also find the total elapsed time and idle time for each machine.

Job	A	B	C	D	E
Machine X	10	2	18	6	20
Machine Y	4	12	14	16	8

5. Find the sequence that minimizes the total elapsed time to complete the following jobs in the order AB. Find the total elapsed time and idle times for both the machines.

Job	I	II	III	IV	V	VI	VII
Machine A	7	16	19	10	14	15	5
Machine B	12	14	14	10	16	5	7

6. Find the optimal sequence that minimizes total time required to complete the following jobs in the order ABC. The processing times are given in hrs.

(i)

Job	I	II	III	IV	V	VI	VII
Machine A	6	7	5	11	6	7	12
Machine B	4	3	2	5	1	5	3
Machine C	3	8	7	4	9	8	7

(ii)

Job	1	2	3	4	5
Machine A	5	7	6	9	5
Machine B	2	1	4	5	3
Machine C	3	7	5	6	7

7. A publisher produces 5 books on Mathematics. The books have to go through composing, printing and binding done by 3 machines P, Q, R. The time schedule for the entire task in proper unit is as follows.

Book	A	B	C	D	E
Machine P	4	9	8	6	5
Machine Q	5	6	2	3	4
Machine R	8	10	6	7	11

Determine the optimum time required to finish the entire task.

MISCELLANEOUS EXERCISE - 7

I) Choose the correct alternative.

1. In sequencing, an optimal path is one that minimizes ....

   (a) Elapsed time (b) Idle time

   (c) Both (a) and (b) (d) Ready time

2. If job A to D have processing times as 5, 6, 8, 4 on first machine and 4, 7, 9, 10 on second machine then the optimal sequence is :

   (a) CDAB (b) DBCA

   (c) BCDA (d) ABCD

3. The objective of sequencing problem is

   (a) to find the order in which jobs are to be made

   (b) to find the time required for the completing all the job on hand

   (c) to find the sequence in which jobs on hand are to be processed to minimize the total time required for processing the jobs

   (d) to maximize the cost

4. If there are n jobs and m machines, then there will be........... sequences of doing the jobs.

   (a) mn (b) m(n!)

   (c) n^m (d) $(n!)^m$

5. The Assignment Problem is solved by

   (a) Simplex method,

   (b) Hungarian method

   (c) Vector method,

   (d) Graphical method,

6. In solving 2 machine and n jobs sequencing problem, the following assumption is wrong

   (a) No passing is allowed

   (b) Processing times are known

   (c) Handling time is negligible

   (d) The time of passing depends on the order of machining

7. To use the Hungarian method, a profit maximization assignment problem requires

   (a) Converting all profits to opportunity losses

   (b) A dummy person or job

   (c) Matrix expansion

   (d) Finding the maximum number of lines to cover all the zeros in the reduced matrix

8. Using Hungarian method the optimal assignment obtained for the following assignment problem to minimize the total cost is :

Agent	Job			
	A	B	C	D
1	10	12	15	25
2	14	11	19	32
3	18	21	23	29
4	15	20	26	28

   (a) 1 — C, 2 — B , 3 — D, 4 — A

   (b) 1 — B, 2 — C, 3 — A, 4 — D

   (c) 1 — A, 2 — B, 3 — C, 4 — D

   (d) 1 — D, 2 — A, 3 — B, 4 — C

9. The assignment problem is said to be unbalance if
   - (a) Number of rows is greater than number of columns
   - (b) Number of rows is lesser than number of columns
   - (c) Number of rows is equal to number of columns
   - (d) Both (a) and (b)

10. The assignment problem is said to be balanced if
    - (a) Number of rows is greater than number of columns
    - (b) Number of rows is lesser than number of columns
    - (c) Number of rows is equal to number of columns
    - (d) If the entry of row is zero

11. The assignment problem is said to be balanced if it is a
    - (a) Square matrix (b) Rectangular matrix
    - (c) Unit matrix (d) Triangular matrix

12. In an assignment problem if number of rows is greater than number of columns then
    - (a) Dummy column is added
    - (b) Dummy row is added
    - (c) Row with cost 1 is added
    - (d) Column with cost 1 is added

13. In a 3 machine and 5 jobs problem, the least of processing times on machine A, B and C are 5, 1. and 3 hours and the highest processing times are 9, 5, and 7 respectively, then it can be converted to a 2 machine problem if order of the machines is:
    - (a) B-A-C, (b) A-B-C
    - (c) C - B - A (d) Any order

14. The objective of an assignment problem is to assign
    - (a) Number of jobs to equal number of persons at maximum cost
    - (b) Number of jobs to equal number of persons at minimum cost
    - (c) Only the maximize cost
    - (d) Only to minimize cost

II) Fill in the blanks.

1. An assignment problem is said to be unbalanced when ....
2. When the number of rows is equal to the number of columns then the problem is said to be ....assignment problem.
3. For solving an assignment problem the matrix should be a ....matrix.
4. If the given matrix is not a .... matrix, the assignment problem is called an unbalanced problem.
5. A dummy row(s) or column(s) with the cost elements as .... the matrix of an unbalanced assignment problem as a square matrix.
6. The time interval between starting the first job and completing the last. job including the idle time (if any) in a particular order by the given set of machines is called ....
7. The time for which a machine j does not have a job to process to the start of job i is called ....
8. Maximization assignment problem is transformed to minimization problem by subtracting each entry in the table from the.................. value in the table.
9. When an assignment problem has more than one solution, then it is............... optimal solution.
10. The time required for printing of four books A, B, C and D is 5, 8, 10 and 7 hours. While its data entry requires 7, 4, 3 and 6 hrs respectively. The sequence that minimizes total elapsed time is..........................

III) State whether each of the following is True or False.

1. One machine - one job is not an assumption in solving sequencing problems.

2. If there are two least processing times for machine A and machine B, priority is given for the processing time which has lowest time of the adjacent machine.
3. To convert the assignment problem into a maximization problem, the smallest element in the matrix is deducted from all other elements.
4. The Hungarian method operates on the principle of matrix reduction, whereby the cost table is reduced to a set of opportunity costs.
5. In a sequencing problem, the processing times are dependent of order of processing the jobs on machines.
6. Optimal assignments are made in the Hungarian method to cells in the reduced matrix that contain a zero.
7. Using the Hungarian method, the optimal solution to an assignment problem is found when the minimum number of lines required to cover the zero cells in the reduced matrix equals the no of persons.
8. In an assignment problem, if number of column is greater than number of rows, then a dummy column is added,.
9. The purpose of dummy row or column in an assignment problem is to obtain balance between total number of activities and total number of resources.
10. One of the assumptions made while sequencing n jobs on 2 machines is : two jobs must be loaded at a time on any machine.

PART - I

IV) Solve the following problems.

1. A plant manager has four subordinates, and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. This estimate of the times each man would take to perform each task is given in the effectiveness matrix below.

	I	II	III	IV
A	7	25	26	10
B	12	27	3	25
C	37	18	17	14
D	18	25	23	9

How should the tasks be allocated, one to a man, as to minimize the total man hours?

2. A dairy plant has five milk tankers, I, II, III. IV & V. These milk tankers are to be used on five delivery routes A, B, C, D & E. The distances (in kms) between the dairy plant and the delivery routes are given in the following distance matrix.

	I	II	II	IV	V
A	150	120	175	180	200
B	125	110	120	150	165
C	130	100	145	160	175
D	40	40	70	70	100
E	45	25	60	70	95

How should the milk tankers be assigned to the chilling center so as to minimize the distance travelled?

3. Solve the following assignment problem to maximize sales:

Salesmen	Territories				
	I	II	III	IV	V
A	11	16	18	15	15
B	7	19	11	13	17
C	9	6	14	14	7
D	13	12	17	11	13

4. The estimated sales (tons) per month in four different cities by five different managers are given below:

Manager	Cities			
	P	Q	R	S
I	34	36	33	35
II	33	35	31	33
III	37	39	35	35
IV	36	36	34	34
V	35	36	35	33

Find out the assignment of managers to cities in order to maximize sales.

5. Consider the problem of assigning five operators to five machines. The assignment costs are given in following table.

Operator	Machine				
	1	2	3	4	5
A	6	6	-	3	7
B	8	5	3	4	5
C	10	4	6	-	4
D	8	3	7	8	3
E	7	6	8	10	2

Operator A cannot be assigned to machine 3 and operator C cannot be assigned to machine 4. Find the optimal assignment schedule.

6. A chartered accountant's firm has accepted five new cases. The estimated number of days required by each of their five employees for each case are given below, where - means that the particular employee can not be assigned the particular case. Determine the optimal assignment of cases of the employees so that the total number of days required to complete these five cases will be minimum. Also find the minimum number of days.

Employee	Cases				
	I	II	III	IV	V
E_1	6	4	5	7	8
E_s	7	-	8	6	9
E_3	8	6	7	9	10
E_4	5	7	-	4	6
E_5	9	5	3	10	-

PART - II

1. A readymade garments manufacturer has to process 7 items through two stages of production, namely cutting and sewing. The time taken in hours for each of these items in different stages are given below:

Items	1	2	3	4	5	6	7
Time for Cutting	5	7	3	4	6	7	12
Time for Sewing	2	6	7	5	9	5	8

Find the sequence in which these items are to be processed through these stages so as to minimize the total processing time. Also find the idle time of each machine.

2. Five jobs must pass through a lathe and a surface grinder, in that order. The processing times in hours are shown below. Determine the optimal sequence of the jobs. Also find the idle time of each machine.

Job	I	II	III	IV	V
Lathe	4	1	5	2	5
Surface grinder	3	2	4	3	6

3. Find the sequence that minimizes the total elapsed time to complete the following jobs. Each job is processed in order AB.

	Jobs (Processing times in minutes						
	I	II	III	IV	V	VI	VII
Machine A	12	6	5	11	5	7	6
Machine B	7	8	9	4	7	8	3

Determine the sequence for the jobs so as to minimize the processing time. Find the total elapsed time and the idle times for both the machines.

4. A toy manufacturing company produces five types of toys. Each toy has to go through three machines A, B, C in the order ABC. The time required in hours for each process is given in the following table.

Type	1	2	3	4	5
Machine A	16	20	12	14	22
Machine B	10	12	4	6	8
Machine C	8	18	16	12	10

Solve the problem for minimizing the total elapsed time.

5. A foreman wants to process 4 different jobs on three machines: a shaping machine, a drilling machine and a tapping machine, the sequence of operations being shaping-drilling-tapping. Decide the optimal sequence for the four jobs to minimize the total elapsed time. Also find the total elapsed time and the idle time for every machine.

Job	Shaping (minutes)	Drilling (minutes)	Trapping (Minutes)
1	13	3	18
2	18	8	4
3	8	6	13
4	23	6	8

Activities... Assignment Problem

1. Given below the costs of assigning 3 workers to 3 jobs. Find all possible assignments by trial and error method.

	Jobs		
Workers	X	Y	Z
A	11	16	21
B	20	13	17
C	13	15	12

Among these assignments, find the optimal assignment that minimizes the total cost.

2. Show that the optimal solution of an assignment problem is unchanged if we add or subtract the same constant to the entries of any row or column of the cost matrix.

3. Construct a 3 × 3 cost matrix by taking the costs as the first 9 natural numbers and arranging them row wise in ascending order. Find all possible assignments that will minimize the total sum.

4. Given below the costs (in hundred rupees) of assigning 3 operators to 3 different machines. Find the assignment that will minimize the total cost. Also find the minimum cost.

Operators	Machines		
	I	II	III
A	$3i + 4j$	$2j^2 + 5i$	$5j - 3i$
B	$i^3 + 8j$	$7i + j^2$	$4i + j$
C	$2i^2 - 1$	$3i + 5j$	$i^3 - 4j$

Where, i stands for number of row and j stands for number of column.

5. A firm marketing a product has four salesmen S_1, S_2, S_3 and S_4. There are three customers C_1, C_2 and C_3. The probability of making a sale to a customer depends upon the salesman customer support. The Table below represents the probability with which each of the salesmen can sell to each of the customers.

Customers	Salesmen			
	S1	S2	S3	S4
C1	0.7	0.4	0.5	0.8
C2	0.5	0.8	0.6	0.7
C3	0.3	0.9	0.6	0.2

If only one salesman is to be assigned to one customer, what combination of salesmen and customers shall be optimal? Profit obtained by selling one unit to C_1 is Rs. 500, to C_2 is Rs.450 and to C_3 is Rs. 540. What is the total expected profit

Activities

1. Give two different examples of sequencing problems from your daily life.

2. Let there be five jobs I, I1, III, IV & V to be processed on two machines A and B in the order AB. Take the first 5 composite numbers as the processing times on machine A for jobs I. II. III, IV, V respectively and the first five odd numbers as the processing times on machine B for jobs V, IV, III, II, I respectively. Find the sequence that minimizes the total elapsed time. Also find the total elapsed time and idle times on both the machines.

3. Determine the optimal sequence of jobs that minimizes the total elapsed time. Processing times are given in hours. Also find total elapsed time and idle times for the machines.

Job	I	II	III	IV	V	VI
Machine A	$2\frac{1}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	$2\frac{3}{4}$	$3\frac{1}{4}$	$\frac{9}{4}$
Machine B	$3\frac{1}{2}$	$4\frac{1}{2}$	$\frac{7}{4}$	$2\frac{1}{4}$	$\frac{5}{4}$	$1\frac{1}{2}$

4. Consider 4 jobs to be processed on 3 machines A, B and C on the order ABC. Assign processing times to jobs and find the optimal sequence that minimizes the total processing time. Also find the elapsed time and idle times for all the three machines.

5. (a) Determine the optimal sequence of jobs that minimizes the total elapsed time based on the following information. Processing time on machines is given in hours, and passing is not allowed. Find total elapsed time and idle times for the machines.

Job	A	B	C	D	E	F	G
Machine X	13	18	17	14	19	18	17
Machine Y	14	13	12	15	11	14	13
Machine Z	16	17	14	21	14	16	22

(b) What happens if we change the processing times on machine Z corresponding to jobs C and E and take them as 15 instead of 14? .

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