DPP – 10

CLASS –12th

TOPIC – COMPOSITION OF FUNCTIONS

Q.1 Find gof and fog when f: R → R and g : R → R is defined by

(i) $f(x) = 2x + 3$ and $g(x) = x^2 + 5$.

(ii) $f(x) = 2x + x^2$ and $g(x) = x^3$

(iii) $f(x) = x^2 + 8$ and $g(x) = 3x^3 + 1$

(iv) $f(x) = x$ and $g(x) = |x|$

(v) $f(x) = x^2 + 2x - 3$ and $g(x) = 3x - 4$

(vi) $f(x) = 8x^3$ and $g(x) = x^{1/3}$

Q.2 Let f = {(3, 1), (9, 3), (12, 4)} and g = {(1, 3), (3, 3) (4, 9) (5, 9)}. Show that gof and fog are both defined. Also, find fog and gof.

Q.3 Let f = {(1, −1), (4, −2), (9, −3), (16, 4)} and g = {(−1, −2), (−2, −4), (−3, −6), (4, 8)}. Show that gof is defined while fog is not defined. Also, find gof.

Q.4 Let A = {a, b, c}, B = {u, v, w} and let f and g be two functions from A to B and from B to A, respectively, defined as: f = {(a, v), (b, u), (c, w)}, g = {(u, b), (v, a), (w, c)}.

Show that f and g both are bijections and find fog and gof.

Q.5 Find fog (2) and gof (1) when f: R → R; $f(x) = x^2 + 8$ and g: R → R; $g(x) = 3x^3 + 1$.

Q.6 Let R+ be the set of all non-negative real numbers. If f: $R^+ \to R^+$ and g : $R^+ \to R^+$ are defined as $f(x)=x^2$ and $g(x)=+\sqrt{x}$, find fog and gof. Are they equal functions.

Q.7 Let f: R → R and g: R → R be defined by $f(x) = x^2$ and $g(x) = x + 1$. Show that fog ≠ gof.

SOLUTION

(MATHS) **RELATIONS & FUNCTIONS**

DPP – 10

CLASS – 12^{th}

TOPIC – COMPOSITION OF FUNCTIONS

Sol.1 **(i)** Given, $f: R \to R$ and $g: R \to R$

So, $gof: R \to R$ and $fog: R \to R$

Also given that $f(x) = 2x + 3$ and $g(x) = x^2 + 5$

Now, $(gof)(x) = g(f(x))$

$= g(2x + 3)$

$= (2x + 3)^2 + 5$

$= 4x^2 + 9 + 12x + 5$

$= 4x^2 + 12x + 14$

Now, $(fog)(x) = f(g(x))$

$= f(x^2 + 5)$

$= 2(x^2 + 5) + 3$

$= 2x^2 + 10 + 3$

$= 2x^2 + 13$

(ii) Given, $f: R \to R$ and $g: R \to R$

so, $gof: R \to R$ and $fog: R \to R$

$f(x) = 2x + x^2$ and $g(x) = x^3$

$(gof)(x) = g(f(x))$

$= g(2x + x^2)$

$= (2x + x^2)^3$

Now, $(fog)(x) = f(g(x))$

$= f(x^3)$

$= 2(x^3) + (x^3)^2$

$= 2x^3 + x^6$

(iii) Given, $f: R \to R$ and $g: R \to R$

So, $gof: R \to R$ and $fog: R \to R$

$f(x) = x^2 + 8$ and $g(x) = 3x^3 + 1$

$(gof)(x) = g(f(x))$

$= g(x^2 + 8)$

$= 3(x^2 + 8)^3 + 1$

Now, $(fog)(x) = f(g(x))$

$= f(3x^3 + 1)$

$= (3x^3+1)^2 + 8$

$= 9x^6 + 6x^3 + 1 + 8$

$= 9x^6 + 6x^3 + 9$

(iv) Given, f: R → R and g: R → R

So, gof: R → R and fog: R → R

$f(x) = x$ and $g(x) = |x|$

$(gof)(x) = g(f(x))$

$= g(x)$

$= |x|$

Now $(fog)(x) = f(g(x))$

$= f(|x|)$

$= |x|$

(v) Given, f: R → R and g: R → R

So, gof: R → R and fog: R → R

$f(x) = x^2 + 2x - 3$ and $g(x) = 3x - 4$

$(gof)(x) = g(f(x))$

$= g(x^2 + 2x - 3)$

$= 3(x^2 + 2x - 3) - 4$

$= 3x^2 + 6x - 9 - 4$

$= 3x^2 + 6x - 13$

Now, $(fog)(x) = f(g(x))$

$= f(3x - 4)$

$= (3x - 4)^2 + 2(3x - 4) - 3$

$= 9x^2 + 16 - 24x + 6x - 8 - 3$

$= 9x^2 - 18x + 5$

(vi) Given, f: R → R and g: R → R

So, gof: R → R and fog: R → R

$f(x) = 8x^3$ and $g(x) = x^{1/3}$

$(gof)(x) = g(f(x))$

$= g(8x^3)$

$= (8x^3)^{1/3}$

$= [(2x)^3]^{1/3}$

$= 2x$

Now, $(fog)(x) = f(g(x))$

$= f(x^{1/3})$

$= 8(x^{1/3})^3$

$= 8x$

Sol.2 Given f = {(3, 1), (9, 3), (12, 4)} and g = {(1, 3), (3, 3) (4, 9) (5, 9)}

f : {3, 9, 12} → {1, 3, 4} and g : {1, 3, 4, 5} → {3, 9}

Co-domain of f is a subset of the domain of g.

So, gof exists and gof: {3, 9, 12} → {3, 9}

(gof) (3) = g (f (3)) = g (1) = 3

(gof) (9) = g (f (9)) = g (3) = 3

(gof) (12) = g (f (12)) = g (4) = 9

⇒ gof = {(3, 3), (9, 3), (12, 9)}

Co-domain of g is a subset of the domain of f.

So, fog exists and fog: {1, 3, 4, 5} → {3, 9, 12}

(fog) (1) = f (g (1)) = f (3) = 1

(fog) (3) = f (g (3)) = f (3) = 1

(fog) (4) = f (g (4)) = f (9) = 3

(fog) (5) = f (g (5)) = f (9) = 3

⇒ fog = {(1, 1), (3, 1), (4, 3), (5, 3)}

Sol.3 Given f = {(1, −1), (4, −2), (9, −3), (16, 4)} and g = {(−1, −2), (−2, −4), (−3, −6), (4, 8)}

f: {1, 4, 9, 16} → {-1, -2, -3, 4} and g: {-1, -2, -3, 4} → {-2, -4, -6, 8}

Co-domain of f = domain of g

So, gof exists and gof: {1, 4, 9, 16} → {-2, -4, -6, 8}

(gof) (1) = g (f (1)) = g (−1) = −2

(gof) (4) = g (f (4)) = g (−2) = −4

(gof) (9) = g (f (9)) = g (−3) = −6

(gof) (16) = g (f (16)) = g (4) = 8

So, gof = {(1, −2), (4, −4), (9, −6), (16, 8)}

But the co-domain of g is not the same as the domain of f.

So, fog does not exist.

Sol.4 Given f = {(a, v), (b, u), (c, w)}, g = {(u, b), (v, a), (w, c)}.

Also given that A = {a, b, c}, B = {u, v, w}

Now we have to show f and g both are bijective.

Consider f = {(a, v), (b, u), (c, w)} and f: A → B

Injectivity of f: No two elements of A have the same image in B.

So, f is one-one.

Surjectivity of f: Co-domain of f = {u, v, w}

Range of f = {u, v, w}

Both are the same.

So, f is onto.

Hence, f is a bijection.

Now consider g = {(u, b), (v, a), (w, c)} and g: B → A

Injectivity of g: No two elements of B have the same image in A.

So, g is one-one.

Surjectivity of g: Co-domain of g = {a, b, c}

Range of g = {a, b, c}

Both are the same.

So, g is onto.

Hence, g is a bijection.

Now we have to find fog,

we know that the co-domain of g is the same as the domain of f.

So, fog exists and fog: {u v, w} → {u, v, w}

(fog) (u) = f (g (u)) = f (b) = u

(fog) (v) = f (g (v)) = f (a) = v

(fog) (w) = f (g (w)) = f (c) = w

So, fog = {(u, u), (v, v), (w, w)}

Now we have to find gof,

Co-domain of f is the same as the domain of g.

So, fog exists and gof: {a, b, c} → {a, b, c}

(gof) (a) = g (f (a)) = g (v) = a

(gof) (b) = g (f (b)) = g (u) = b

(gof) (c) = g (f (c)) = g (w) = c

So, gof = {(a, a), (b, b), (c, c)}

Sol.5 Given f: R → R; $f(x) = x^2 + 8$ and g: R → R; $g(x) = 3x^3 + 1$.

Consider (fog) (2) = f (g (2))

$= f(3 \times 2^3 + 1)$

$= f(3 \times 8 + 1)$

= f (25)

$= 25^2 + 8$

= 633

(gof) (1) = g (f (1))

= g (1^2 + 8)

= g (9)

= 3 × 9^3 + 1

= 2188

Sol.6 Given f: $R^+ \to R^+$ and g: $R^+ \to R^+$

So, fog: $R^+ \to R^+$ and gof: $R^+ \to R^+$

Domains of fog and gof are the same.

Now we have to find fog and gof also we have to check whether they are equal or not,

Consider (fog) (x) = f (g (x))

= f ($\sqrt{x}$)

= $\sqrt{x^2}$

= x

Now consider (gof) (x) = g (f (x))

= g (x^2)

= $\sqrt{x^2}$

= x

So, (fog) (x) = (gof) (x), $\forall x \in R+$

Hence, fog = gof

Sol.7 Given f: R → R and g: R → R.

So, the domains of f and g are the same.

Consider (fog) (x) = f (g (x))

= f (x + 1) = $(x + 1)^2$

= $x^2 + 1 + 2x$

Again consider (gof) (x) = g (f (x))

= g (x^2) = $x^2 + 1$

So, fog ≠ gof