UNIT 8

Arithmetic Progression (A.P.)

1. CONCEPT OF SEQUENCE, SERIES AND PROGRESSION

(i) Sequence

(a) Collection of numbers arranged in some definite order and formed according to some rule is called a sequence. A sequence is called to be finite or infinite if the number of terms in it is finite or infinite. A sequence having all real terms is called a real sequence.

(b) *Terms of a sequence*

The different numbers in a sequence are called its terms. These are denoted by $t_1, t_2, t_3, ..., t_n$ or $a_1, a_2, a_3, ..., a_n$. For example, in a sequence 1, 3, 5, 7, ..., 25, the number 5 is its t_3 i.e., third term. The nth term is called the general term of the sequence and is written as t_n or a_n.

A sequence can also be written by writing the nth term like $t_n = 2n + 5$.

The sequence corresponding to tn $= 2n + 5$ is obtained by putting $n = 1, 2, 3,$ ......

$\therefore$ The above sequence is 7, 9, 11, ...

Similarly, if $tn = n^2 + 3$, then the sequence is 4, 7, 12, 19, ...

ILLUSTRATIVE EXAMPLES

Example 1. If $t_n = n(n+3)$, find the difference of its 5th term and 2nd term i.e., $t_5 - t_2$

Sol: $t_n = n(n+3)$

$\therefore \quad t_5 = n(n+3)$

$\therefore \quad t_5 - t_2 = 5 \times 8 - 2 \times 5 = 40 - 10 = 30.$

Ans. 30

Example 2. If $a_n = \dfrac{n^2}{3n+2}$, find a_5.

Sol: $a_1\ a_5 = \dfrac{1}{5} \times \dfrac{25}{17} = \dfrac{5}{17}$

Ans. $\dfrac{5}{17}$

Example 3. If $t_n = \dfrac{1+(-2)^n}{n-1}$, find $t_6 - t_5$

Sol: $t_6 = \dfrac{1+(-2)^6}{6-1} = \dfrac{1+64}{5} = \dfrac{65}{5} = 13$

$t_5 = \dfrac{1+(-2)^5}{5-1} = \dfrac{1+32}{4} = \dfrac{31}{4}$

$\therefore \quad t_6 = t_5 = 13 - \left(-\dfrac{31}{4}\right) = 13 + \dfrac{31}{4} = \dfrac{83}{4} = 20\dfrac{3}{4}$

Ans. $20\dfrac{3}{4}$

(ii) Progression

The sequence that follows a certain pattern is called a progression

(iii) Arithmetic Progression (A.P)

A sequence in which the difference of two terms (any term and its preceding term) is the same (constant) throughout is called an Arithmetic Progression or Arithmetic sequence. The constant difference is called the common difference and is usually denoted by *d*.

(iv) Given below are the sequences in Arithmetic Progressions.

1. $1, 7, 13, 19, ...$ The common difference $d = 6$
2. $13, 18, 23, 28, ...$ Here $d = 5$
3. $a, a+d, a+2d, a+3d, ...$
4. $50, 43, 36, 29, ...$ Here $d = -7$.

2. FIRST TERM AND COMMON DIFFERENCE OF AN A.P.

The most general form of an A.P. is

$a, a+d, a+2d, a+3d, ...$

where 'a' is the first term and d the common difference of an A.P. Clearly, $d = t_n - t_{n-1}$, where n is a natural number.

Also the nth term of an A.P. is $a + (n-1)d$ and hence $t_n = a + (n-1)d$

This formula imolves four quantities a, d, n and t_n. If any three of these are known, fourth can be determined.

Example 4. The first term of an A.P. is 5 and its common difference is –3.

Find the 11th term of an A.P.

Sol: $t_n = a + (n-1)d$ where $n = 11, a = 5, d = -3$

$\therefore \quad t_{11} = 5 + (10) \times (-3) = -25$

Ans. –25

Example 5. Find the 20th term of an A.P. whose 5th term is 15 and the sum of its 3rd and 8th terms is 34.

Sol. $t_5 = t_5$ and $t_3 + t_8 = 34$

$\Rightarrow \quad a + 4d = 15 \quad ...(i)$

and $\quad (a+2d) + (a+7d) = 34$

$\Rightarrow \quad 2a + 9d = 34 \quad ...(ii)$

(i) and (ii) gives d = 4 and a = –1

$\therefore \quad t_{20} = a + 19d = -1 + 19 \times 4\ 75$

Ans. 75

Example 6. The angles of a quadrilateral are in A.P. The greatest angle is thrice the least angle. Find the greatest angle.

Sol: Let the angles be $a, a+d, a+2d, a+3d$

$a + 3d = 3a \quad \Rightarrow \quad 3d = 2a \quad \text{or} \quad d = \dfrac{2a}{3}.$

Now $a + a + d + a + 2a + a + 3d = 360°$

$\Rightarrow \quad 4a + 6d = 360°$

so $\quad 4a + 4a = 360°$

or $\quad a = 45°$

$\therefore \quad$ Greatest angle $= 3 \times 45°\ 135°$.

Ans. 135°

3. SUM TO FIRST N TERMS OF AN A.P.

Let a be the first term and d, the common difference. Let sum to first n terms of an A.P. be S_n. Let l be the last term i.e., t_n of the series.

$\therefore \quad l = a + (n-1)d$

$S_n = a + (a+d) + (a+2d) + ... + (l-2d) + (l-d) + l ... \quad ...(i)$

Also,writing the above terms in the reverse order, we get

$S_n = l + (l-d) + (l-2d) + ... + (a+2d) + (a+d) + a ...$ ...(ii)

Adding (i) and (ii) we get

$2S_n = (a-l) + (a-l) + (a-l) + ... + (a-l) + (a-l) + (a-l)$

$= n(a+l)$

$\Rightarrow \quad S_n = \frac{n}{2}(a+l)$ ... (iii)

Since $l = t_n = a + (n-1)d$

$\therefore \quad S_n \frac{n}{2}[2a + (n-1)d]$ ... (iv)

(*iii*) and (*iv*) are the rules of finding the sum of first n terms of an A.P.

Note: In an A.P, $t_n = S_n - S_{n-1}$

Example 7. If $t_n = (3+4n)$ of an $a-p$, then the sum of the its 15 terms is:

Sol: First terms $t_1 = 3 + 4 = 7$

15th term $= 3 + 4 \times 15 = 63$

$\therefore \quad S_{15} = \frac{15}{2}[t_1 + t_{15}]$

$\frac{15}{2}(7+63) = 15 \times 35 = 525$

Ans. 525.

Example 8. If $t_3 = 15, S_{10} = 120$, then the tenth term of the series i:

Sol: $t_3 = a + 2d = 15$ ...(i)

$S_{10} = \frac{10}{2}[2a + 9d] = 120$

$S_{10} \Rightarrow 2a + 9d = 24$ ...(ii)

(*i*) and (*ii*) gives $a = \frac{87}{5}$ and $d = -\frac{6}{5}$

$\therefore \quad t_{10} = a + 9d = \frac{87}{5} + 9 \times \left(-\frac{6}{5}\right)$

$= 6\frac{3}{5}$

Ans. $6\frac{3}{5}$

MULTIPLE CHOICE QUESTIONS

Tick (✓) the correct choice amongst the following-

1. If tn = $\begin{cases} n^2, \text{when } n \text{ is even} \\ n^2 + 1, \text{when } n \text{ is odd} \end{cases}$

find $t_{15} - t_{10}$

(a) 116 (b) 126
(c) 106 (d) 226

2. The 5th terms of the sequence defined by $t_1 = 2, t_2 = 3$ and $t_n = t_{n-1} + t_{n-2}$ for $n \geq 3$

(a) 13 (b) 15
(c) 16 (d) 18

3. The sum of the 4th and 8th term of an A.P is 24 and the sum of its 6th and 10th terms is 44. Find the first term.

(a) 13 (b) 12
(c) –13 (d) –14

4. The nth terms of an A.P. $\frac{1}{m}, \frac{m+1}{m}, \frac{2m+1}{m}, ...$ is:

(a) $\frac{m+1-mn}{m}$ (b) $\frac{mn-m+1}{m}$
(c) $\frac{mn-m-n}{m}$ (d) $\frac{mn+m-n}{m}$

5. If the numbers $3k+4$, $7k+1$ and $12k-5$ are in A.P, then the value of k is

(a) 2 (b) 3
(c) 4 (d) 5

6. An A – P consists of 50 terms of which 3rd term is 12 and the last term is 106. Its 29th term is

(a) 58 (b) 60
(c) 61 (d) 64

7. The 4th term of A.P. is equal to 3 times the first term and 7th term excess which the third term by 1. Find its nth term.

(a) $n+2$ (b) $3n+1$
(c) $(2n+1)$ (d) $3n+2$

8. If 5 times the 5th term of an A.P. is the same as 7 times the 7th term, then find its 12th terms.

(a) 0 (b) 11
(c) 14 (d) 18

9. For what value of n, the nth terms of an A.P. is

(i) 63, 65, 67, ... and

(ii) 3, 10, 17, ... are equal?

(a) 10 (b) 11
(c) 12 (d) 13

10. Which term of an A.P. 3, 15, 27, 39, ... will be 132 more than its 54th term?

(a) 1st (b) 63rd
(c) 65th (d) None of these

11. Find the sum of first 31 terms of an A.P. whose nth term is $\left(3+\frac{2n}{3}\right)$.

(a) $423\frac{2}{3}$ (b) $413\frac{1}{3}$

(c) $417\frac{2}{3}$ (d) $419\frac{2}{3}$

12. If the sum of first n terms of an A.P. is $3x^2 - 2n$, then its 19th. term is

(a) 123 (b) 118
(c) 109 (d) 107

13. If the third and 11th terms of an A.P. are 8 and 20 respectively, find the sum of first ten terms.

(a) $105\frac{1}{2}$ (b) 108

(c) $117\frac{1}{2}$ (d) $203\frac{1}{2}$

14. How many terms of the A.P. 9, 17, 25, ... must be taken to give the sum 636?

(a) 15 (b) 14
(c) 13 (d) 12

15. A man saves Rs. 320 during the first month, Rs. 360 in the second month, Rs. 400 in the third month. If he continues his savings in this sequence, in how many months will he save Rs. 20,000?

(a) 28 (b) 25
(c) 22 (d) 20

ANSWERS

1. (b)	2. (a)	3. (c)	4. (b)	5. (b)
6. (d)	7. (c)	8. (a)	9. (d)	10. (c)
11. (a)	12. (c)	13. (c)	14. (d)	15. (b)

SOLUTIONS

[For Some Selected Problems]

1. $t_{10} = 10^2 = 100$

$t_{15} = (15)^2 + 1 = 226$

$\therefore \quad t_{15} - t_{10} = 226 - 100 = 126.$

2. $t_1 = 2, t_2 = 3$ and $t_n = t_{n-1} + t_{n-2}$

$t_3 = t_2 + t_1 = 3 + 2 = 5$

$t_4 = t_3 + t_2 = 5 + 3 = 8$

$t_5 = t_4 + t_3 = 8 + 5 = 13$

3. $t_4 + t_8 = a + 3d + a + 7d = 2a + 10d$

$\therefore \quad 2a + 10d = 24 \quad \text{or} \quad a + 5d = 12 \qquad ...(i)$

Also $t_6 + t_{10} = 44 \quad \Rightarrow \quad a + 7d = 22 \qquad ...(ii)$

(i) and (ii) gives $d = 5$ and $a = -13$

4. $$a = \frac{1}{m}$$

$$d = \frac{m+1}{m} - \frac{1}{m} = 1\frac{1}{m} - \frac{1}{m} = 1$$

$$\therefore \quad t_n = + (n-1)d = \frac{1}{m} + (n-1)$$

$$= \frac{mn - m + 1}{m}$$

5. Clearly

$$(7k + 1) - (3k + 4) = (12k - 5) - (7k + 1)$$

$\Rightarrow \qquad k = 3$

6. $t_3 = 12$ and $t_{50} = 106$

$\Rightarrow \qquad a = 2d = 12 \qquad ...(i)$

and $a + 49d = 106 \qquad ...(ii)$

(i) and (ii) gives $a = 8, d = 2$

$\therefore \; t_{29} = a + 28d = 8 + 56 = 64$

7. $t_4 = 3t_1, \;\; \Rightarrow \;\; a + 3d = 3a$

or $\qquad d = \frac{2}{3}a$

$t_7 = 2t_3 + 1$

$\Rightarrow \qquad a + 6d = 2(a + 2d) + 1$

$\Rightarrow \qquad a + 2d - 1 = 2 \times \frac{2}{3} - 1$ or $a = 3$

$d = \frac{2}{3} \times 3 = 2$

$\therefore \qquad t_n = a + (n-1)d = 3\,(n-1) \times 2 = (2n + 1)$

8. $t_5 = a + 4d$ and $t_7 = a + 6d$

Now $\qquad 5t_5 = 7t_7$

$\Rightarrow \qquad 5(a + 4d) = 7(a + 6d)$

$\Rightarrow \qquad a + 11d = 0$

$t_{12} = \mathrm{a} + 11d = 0$

9. t_n of series (i) is $63 + (n-1) \,.\, 12 = 61 + 2n$

t_n of series (ii) is $7n - 4$

$\Rightarrow \; 61 + 2n = 7n - 4 \quad$ so $n = 13$

10. Let t_n be the terms of the series 3, 15, 27, 39,...

$\Rightarrow \; t_n 3 + (n-1) \times 2 = 12n - 9$

$t_{54} = 3 + 53 \times 12 = 639$

$\therefore \; t_{54} + 132 = 639 + 132 = 771$

$\Rightarrow \; 771 = 12n - 9 \quad$ or $\quad 12n = 780 \quad$ or $\quad n = 65$

11. $a_1 = 3 + \frac{2}{3} = \frac{11}{3}$

$a_{31} = 3 + \frac{2}{3} \times 31 = 3 + \frac{62}{3} = \frac{71}{3}$

$\therefore \qquad S_{31} = \frac{31}{2} \times \left(\frac{11}{3} + \frac{71}{3}\right) = \frac{31}{2} \times \frac{82}{3} = \frac{1271}{3}$

$= 423\frac{2}{3}$

12. $t_{19} = S_{19} - S_{18}$

$= (3 \times 19^2 - 2 \times 19) - (3 \times 18^2 - 2 \times 18)$

$= (1083 - 38) - (972 - 36)$

$= 109$

13. $a + 2d = 8 \qquad$ and $\qquad a + 10d = 20$

$\Rightarrow \qquad a = 5, \quad d = \frac{3}{2}$

$\therefore \; S_{10} = \frac{10}{2}\,[2a + (10-1)d]$

$= 5\,[2 \times 5 + 9 \times \frac{3}{2}]$

$= 117\frac{1}{2}.$

14. $a = 9, d = 8, S_n = 636$

$\therefore \; 636 = \frac{n}{2}\,[2 \times 9 + (\mathrm{n} - 1) \times 8]$

$\Rightarrow \; 4n^2 + 5n - 636 = 0$

$\Rightarrow \; (n - 12)\,(4n + 53) = 0$

$\Rightarrow \; n = 12 (\therefore \;\; \neq -\frac{53}{4})$

15. $a = 320, d = 40.$

$S_n = 20{,}000.$

It is required to find n.

$S_n = 20{,}000 = \frac{n}{2}\,[2 \times 320 + (n-1) \times 40]$

$\Rightarrow \; 500 = \frac{n}{2}\,[15 + (n-1)] = \frac{n}{2}\,(n + 15)$

$\Rightarrow \; n^2 + 15n - 1000 = 0$

$\Rightarrow \; n = 25.$