Unit 9

Elements of Symbolic Logic

Origin of symbolic logic:

In the history of western logic, symbolic logic is a relatively recent development. What sets symbolic logic apart from traditional logic is its leaning towards mathematics. Aristotle, the Greek thinker, in the fourth century BC, laid the foundation of logic as a science of sciences. Logic for him is a science which can serve as the backbone of all other scientific queries. He was the first to propose that logic is the indispensable tool which can provide a set of correct rules for correct reasoning, and that these rules will be applicable in all scientific studies and discussions. Aristotle's conception of logic as a formal study of valid inferences was accepted by all logicians for nearly 2000 years.

In the seventeenth century G.W. Von Leibniz, a mathematician turned philosopher, however, could see that Aristotle and his followers' style of reasoning needs modification. But he merely suggested, and never worked or showed the direction which logic possibly could take. It is only in the nineteenth century that the thinkers started actualizing the ideas conceived by Leibniz. For nearly two thousand years since its inception, logic did not progress so much as it had done in last one hundred and fifty years. This is mostly because of rapid development in mathematics and its use in logic. Since Leibniz's discovery of Differential Calculus, mathematics developed rapidly. Soon, it became the 'key-science'.

With the ascending of the mathematical method, George Boole (1815-1864), an English mathematician, rediscovered logic and it's potential. He mathematized logic by highlighting the resemblances between disjunction and conjunction of concepts and addition and multiplication operations of mathematics. Through his efforts Class Calculus emerged in logic. While Boole and others like De Morgan (1806-1871), Charles Sanders Pierce (1839-1914) were mathematizing logic, Gottlob Frege (1824-1925), a German mathematician, attempted something revolutionary. He tried to reduce all mathematics into logic by analyzing the concept of number. This is known as the thesis of logicism. In order to achieve this aim he invented a language; a Begriffsschrift (language of thought) to free logical thinking from the dominion of ordinary language and its grammar. The new language which is also called artificial language makes use of logical notations. These notations can express much larger complex sentences than Aristotle's logic could do. This achievement of Frege ushered in the age of symbolic or mathematical logic for which he is rightly called the father of modern symbolic logic.

Arguments presented in natural language whether in English or in any other language are sometimes difficult to evaluate because of the ambiguous, vague and equivocal nature of words and idioms in which they are expressed. To avoid these impurities of ordinary language the logicians as well as some of the mathematicians have developed specialized technical language and vocabulary which is called artificial language or symbolic language.

Nature of symbolic language

Ideograms and Phonograms:

In sharing the same platform with mathematics, modern logicians made extensive use of symbols. Symbols are ideograms, written words are phonograms. Ideograms stand directly for ideas whereas phonograms stand for sounds that we make while speaking words. Every word of a language is a phonogram. For example, the written words 'question mark' are phonograms because they are the written characters of the sounds we make while uttering these words. But '?' is an ideogram. This does not represent written characters of the sounds, but stands for the idea directly. Similarly, 'minus' is a phonogram, but '-' is an ideogram; 'Square root of 10' is phonogram, but $\sqrt{10}$ is an ideogram. Symbolic logic makes use of ideograms instead of phonograms.

There is no doubt the classical logic used symbols like S, P, M which represent minor term, major term and middle term in a categorical syllogism respectively. A, E, I, O symbols were also used to denote categorical propositions by the traditional thinkers. But the use of symbols was very little and it was restricted to a few symbols only. Modern symbolic logic made extensive use of symbols both as variables and constants.

Types of symbol

Variables:

Two types of symbol are used in logic - variables and constants. A variable symbol keeps on changing its value from argument to argument. These symbols are 'dummies'. They do not have fixed values. For example, 'P' in one argument may stand "It is very pleasant today", 'P' may stand for 'She is Priya" in another argument. Similarly 'P' may represent 'We will go to Palampur' in yet another argument. Here symbol 'P' is variable. Many more types of variables are used in modern logic such as propositional variables (which represent propositions), predicate variables (which represent predicates of proposition), class variables (which represent classes) and many more types. In the traditional logic variables were used for the three terms of categorical syllogism, S, P, M only. This shows that the use of symbols and variables, was known to Aristotle though it was restricted only to a few symbols.

Constants:

A constant symbol should be distinguished from a variable symbol. Constant symbols do not change their values in the domain of logic whereas variables do not have fixed values. Aristotle and other traditional logicians never used symbols for constants. The use of symbols for constants was done for the first time in the modern logic. Some important constant symbols used in elementary symbolic logic are as follows:

1. ' $\sim$ ' for negation; called (curl or tilde)

2. 'V' for the relation of 'either, or' in inclusive sense ; called wedge. (disjunction)

3. 'Λ' for the relation of 'either, or' in exclusive sense called alternation.

4. ' • ' for the relation of 'and'; called dot.

5. '⊃ ' for the relation of 'if, then' and is called horse shoe.

6. '≡' for the relation of 'if and only if' and is called equivalent.

Uses of Symbols:

A question many arise why the modern logicians make extensive use of symbols? What are the advantages of using symbols in logic? There are at least two major advantages of using symbols in logic: First, the logical form of an argument becomes explicit by using symbols. The complexity of language may hide the structure of an argument, and thus can conceal its logical form. Earlier in this chapter it has been emphasized that ordinary language which makes use of phonograms is not fit for evaluating arguments in logic. Symbolic language is better than the ordinary language for the logicians.

The use of language in logic has to be very clear as the slightest variation in the interpretation of the meanings of terms or propositions can change the logical form of the reasoning. By replacing language by symbols, the logical form of an argument becomes explicit. Once right logical form of an argument is captured, its validity can be decided quickly and accurately. A question may arise why are the logicians interested in the logical form an of argument? This is because the validity of an argument is decided by the logical form of the argument.

Second, the use of symbols in logic is seen as an economical device. The long and big arguments become small and handy expressions after symbolization. They can be operated quickly and easily. The chances of committing errors in deciding their validity are thus reduced.

Symbolization

One of the important tasks in modern logic is to to express propositions and arguments in symbols. This process is called symbolization. One letter from alphabet represents one simple proposition. For Example:

1. "Sita is a student" is symbolized as S.
2. "Mohan is citizen of India" is symbolized as C.
3. "Qutab Minar is in Delhi" is symbolized as Q.
4. "Sita is a good student and she is hardworking" is symbolized as: S and H.
5. "If it rains, then we shall go for picnic" is symbolized as :

If R, then G.

6. "Either it rains or it will be very humid" is symbolized as Either R or H.

Negative proposition such as:

"Sachin is not a mathematician" is symbolized as $\sim$ S

Symbolization of Compound Propositions

Conjunctive: When two simple propositions are in conjoined by 'and' then the conjoined form is called conjunctive proposition.

For example :

"Sita is intelligent and she is hardworking student" is symbolized as:

I.H

I, H are called conjuncts

"Sita is intelligent" - I

"Sita is hardworking" - H.

Besides 'and', there are many words such as yet, but, both, although, however, moreover, neither nor, as well as, while etc. for which '.' is used.

For example:

1. "She is poor yet she is honest".

   P. H

2. "He is intelligent but could not clear this test.'

   I. $\sim$ T

3. "Both Sita and Gita are students."

   S. G

4. "Not both Sita and Gita are Students."

   $\sim$ (S.G)

5. "Sita and Gita both are not students".

   $\sim$ S. $\sim$ G

6. "Neither Sita nor Gita are students."

   $\sim$ S. $\sim$ G

Disjunctive : (inclusive sense) :

When two simple propositions are combined by 'either, or' then the compound form is called disjunctive proposition. For example :

1. "Either I write to him or I will talk to him on telephone" is symbolized as :

   W v T.

   W, T are called disjuncts.

2. "Either the government will reduce the price of oil or it will have to increase the salary".

   R v I

3. "Either it rains in time or the crops will not be good".

   Rv ~ G

4. "Either S is not guilty or M is not telling truth".

   ~ Sv ~ M

If 'unless 'occurs between two components of a sentence, then they can be symbolically joined by the disjunctive sign. For example:

"I do not drive in night unless it is very necessary."

~ D v N.

~ D - "I do not drive in night".

N - "It is very necessary."

'Neither, nor' relationship between two propositions can also be symbolized by using wedge (v) sign. For example:

"Neither the teachers nor the students are the members of this library."

~ (T VS)

'neither, nor' relationship can be symbolized in two ways. The above example can be symbolized in an other way also such as:

~ T. ~ S

Thus ~ (TvS) = (~ T. ~ S)

Alternative: (Exclusive sense)

Two simple propositions are joined by 'either, or' in exclusive sense also.

"Either I go to library or I will stay in the hostel."

$L \wedge H$. (L, H are alternates)

Implicative :

Simple propositions when combined by 'If, then' relation are called implicative, hypothetical or conditional propositions. Such propositions are symbolized by using '$\supset$' sign and the sign is read as 'horse-shoe' for it looks like shoe of a horse.

Example :

1. "If I get ticket of plane, then I will attend board meeting in Bombay", is symbolized as :

   $T \supset A$.

2. "If you are not regular visitor to library, then you are not a serious student."

   $\sim L \supset \sim S$

   In $p \supset q$ proposition 'p' is antecedent, 'q' is consequent. Sometimes instead of 'if, then' some other words or phrases are used in implicative propositions.

   For instance: the following propositions are implicative propositions and they all are symbolized as '$p \supset q$'.

1.	p, only if q	$p \supset q$
2.	q if p	$p \supset q$
3.	q provided that p	$p \supset q$
4.	q on condition that p	$p \supset q$
5.	q in case p	$p \supset q$
6.	p hence q	$p \supset q$
7.	p implies q	$p \supset q$

8. Since p, so q $p \supset q$

9. "q is necessary condition for p" $p \supset q$

10. "p is sufficient condition for q" $p \supset q$

11. "Only if q, p" $p \supset q$

Equivalent propositions:

When two propositions have same values then they are called equivalent propositions; and this is symbolized as '$\cong$'

For example:

"You will catch the train if and only if you reach in time" is symbolized as $C \cong R$

'$\cong$' is symbol use for 'if and only if'.

There are different formulations for three similar looking propositions.

i. p only if q - $p \supset q$

ii. p if q - $q \supset p$

iii. p if and only if q - $p \cong q$

Symbolization of arguments

Now we are in a position to symbolize arguments. Each premise of an argument is symbolized separately. For example:

"If the demand stays constant and prices are lowered, then the business will increase. If business increases, then the prices are lowered and we can control the market. Demand will stay constant. Therefore, we can control the market.

The demand stays constant - D

Prices are lowered - L

Business will increase - I

We can control the market - C

1st premise $(D.L) \supset I$

2nd premise $I \supset (L.C)$

3rd premise D

Conclusion: C

Truth Table Method

There is two-fold task in elementary symbolic logic: First, to symbolize the propositions and arguments; second, to test the validity / invalidity of arguments and also the logical status of the propositions.

Logicians have provided many methods for this purpose. These methods are called decision procedures. Here we are dealing with the truth table method only. This is the most elementary and popular method to test the validity/invalidity of arguments and also to test the logical status of the propositions. By the logical status of the propositions we mean whether a proposition is tautology, contingent or contradictory (self-contradictory). A proposition is called tautology when it is always true. For example, "The sum of all the angles in a triangle is equivalent to 180°". A proposition is contingent when sometimes it is true and some other time it is false. For example, "It is very pleasant today." It is true now but it may not be true some other time. A proposition is contradictory when it is always false. For instance, "Two plus two is five."

Truth Table Method is a decision procedure because it helps us to take a decision whether an argument is valid/invalid and also whether a proposition is tautological or not, and also whether propositions are logically equivalent or not.

Let us see how the method is applied.

Number of propositional variables:

First of all the number of simple propositions in the argument are counted. If a simple proposition is repeated in the argument or in the expression, then it is counted only once. In the expression $p \supset (q \vee p)$, the number of propositional variables are 2, since p occurs twice, it will be considered only once. In the expression $\{p \vee [(q.\ r) \supset \sim r]\}$ the propositional variables are three, p, q and r. In the expression $(p.\ q) \supset (r \vee s)$ propositional variables are four.

Number of rows

After knowing the number of propositional variables, the number of rows are decided. This depends on the number of propositional variables. If the numbers of propositional variables are three, the numbers of rows are $2^3 = 8$; and if the propositional variables are four, then the number of rows are $2^4 = 16$.

The base is always two, because we are dealing with two-valued logic only. A proposition has two values, true or false. The power of '2' or the exponent of '2' is according to the number of propositional variables an argument or an expression has. In the expression $\{\sim p \supset [q \vee (r\ .s)]\}$ since the number of propositional variables are four, the number of rows are $2^4 = 16$. After counting the variables and deciding the number of rows, the truth table is constructed. Before that we must know the characteristics of various logical constants such as :

.,V, ∧, ⊃, ≡, ~

p. q is true when both or all the conjuncts (p, q etc.) are true.

p ∨ q is true when at least one of the disjuncts (p, q etc.) is true,

p Λ q is true when one alternate is true and the other is false.

p ⊃ q is true in all cases except when antecedent (p) is true and consequent (q) is false.

p ≡ q is true when either both p, q are true or both p, q are false.

~ p is true when p is false.

Let us work them out in detail :

T = True

F = False.

p.q	p ∨ q	p Λ q	p ⊃ q
TTT	TTT	TFT	TTT
TFF	TTF	TTF	TFF
FFT	FTT	FTT	FTT
FFF	FFF	FFF	FTF

P ≡ Q	~ P
TTT	FT
TFF	TF
FFT	
FTF	

In a conjunctive proposition p. q; p is one conjunct and q is another conjunct. There are two variables p, q. So the number of rows are $2^2 = 4$. Under the first variable (p) we assign 2 Ts and 2Fs, (half of total number of rows are Ts and half of total number of rows Fs). Under the second variable (q). we keep one T and one F (total number of rows are 4)

For instance in the conjunctive proposition p. q we work out like this:

p	q	p .q
T	T	T
T	F	F

F	T	F
F	F	F

In the 1st row since both p. q are true p . q is also true. In the 2nd row though p is true but q is false so p . q is also false. In the 3rd row p is false though q is T hence p.q is false. In the 4th row p is false and q is also false, hence p .q is false.

Disjunctive proposition: p ∨ q: p, q are two disjuncts.

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

In the 1st row both p, q are true hence p ∨ q is true; in the 2nd row p is T though q is F, p ∨ q is true. In the 3rd row p is false but q is T therefore p ∨ q is T. But in the 4th row p is false, q is also false, therefore p ∨ q is false.

Alternative proposition: p Λ q (p , q are two alternates)

p	q	p Λ q
T	T	F
T	F	T
F	T	T
F	F	F

In the 1st row p is T and q is also T, therefore p Λ q is F. In the 2nd row p is T but q is F therefore p Λ q is T. In the 3rd row p is F, q is T hence p Λ q is T. But in 4th row p is F and q is also F, therefore p Λ q is F.

Implicative proposition: p ⊃ q

p	q	p ⊃ q	(p is antecedent q is consequent)
T	T	T	
T	F	F	
F	T	T	
F	F	T	

Implicative proposition is false only when antecedent (p) is true and consequent (q) is false. (This incidentally is the main characteristic of deductive logic) . In other combinations $p \supset q$ is true.

Equivalent proposition: $p \equiv q$

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

In the 1st row both p and q are T therefore $p \equiv q$ is T also, In 2nd row p is T but q is F $p \equiv q$ is F. In 3rd row p is F, q is T hence $p \equiv q$ is F. In 4th row p is F, q is also F hence $p \equiv q$ is T.

Negative proposition: $\sim p$

p	$\sim p$
T	F
F	T

In the first row p is T hence $\sim p$ is F and in the second row p is F hence $\sim p$ is T.

Logical status of propositions

Let us see some questions and their solutions for a better understanding. Determine the logical status of the following propositions with the help of truth table method.

Q1. $p \supset (q \vee r)$

Since there are 3 propositional variables we will make $2^3 = 8$ rows like this.

Solution:

p	q	r	$q \vee r$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	T

F	F	F	F
1	2	3	4

$p \supset (q \vee r)$
T
T
T
F
T
T
T
T
5

Since in the main column (No. 5) there are both Ts and Fs therefore the logical status of the above expression is contingent.

Q.2 $\sim(\sim p \supset q) \vee q$

Solution: Since there are two propositional variables, p, q, we have $2^2 = 4$ rows.

p	q	$\sim p$	$\sim p \supset q$	$\sim(\sim p \supset q)$
T	T	F	T	F
T	F	F	T	F
F	T	T	T	F
F	F	T	F	T
1	2	3	4	5

$\sim(\sim p \supset q) \vee q$
T
F
T
T
6

Since in the column No. 6 there are both Ts and Fs, it is contingent.

Q3. $(p \supset q) \vee p$

Solution:

p	q	p ⊃ q	(p ⊃ q) ∨ p
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T
1	2	3	4

Since there are all Ts in the main column No. 4, therefore the logical status is tautology.

Q4. $(p \supset q) \supset \sim(\sim q \supset \sim p)$

Solution:

p	q	~p	~q	p ⊃ q
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T
1	2	3	4	5

~q ⊃ ~p	~(~q ⊃ ~p)
T	F
F	T
T	F
T	F
6	7

(p ⊃ q) ⊃ ~ (~q ~p)
F
T

F

F It is contingent

8

Q5. [(p .q)⊃ r)] ⊃ ~ [p⊃(q⊃r)]

Solution:

p	q	r	p. q	(p. q)⊃ r
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T
1	2	3	4	5

(q⊃r)	p⊃(q⊃r)	~[p⊃(q⊃r)]
T	T	F
F	F	T
T	T	F
T	T	F
T	T	F
F	T	F
T	T	F
T	T	F
6	7	8

$[(p. q) \supset r] \supset \sim [p \supset (q \supset r)]$

F

T

F

F

F

F

F

F — Since there are T and Fs in the main column

9 — It is contingent.

Questions

1. Symbolize the following propositions using given notations:

   a. I sit on the chair but my cat sits on the floor.(C, F)

   b. We will miss bus unless we run.(B, R)

   c. Two triangles are formed if a square is divided diagonally.(T, S)

   d. He is both fool and knave.(F, K)

   e. A man is both rationalist as well as hedonist. (R, H)

   f. Either the taxes rise or the inflation continues.(T, I)

   g. A doctor can diagnose this disease if and only if he has experience in the tropics. (D, T)

   h. If the market drops, Simon will borrow more money.(M, B)

   i. A will dance only if B sings.(A, B)

   j. Mr X can secure majority of votes if Mr X cares for the people.(S, C)

   k. She prepares to earn a good living only if her school days are well spent.(P, S)

   l. Neither the cost of living will rise nor will the public agitate.(C, A)

   m. We will go if it does not rain.(G, R)

n. Be neither a barrower nor a lender.(B, L)

o. Vinod failed though he tried. (F, T)

p. A implies B, and C implies D. (A, B, C, D)

q. If mathematics is difficult, then you will pass only if you work hard. (M, P, Q)

r. Either she is too good and innocent, or she is pretending to be so.(G, I, P)

s. Ram will go provided he is invited. (G,I)

t. Geeta is intelligent as well as hard working (I,H)

2. Make the truth tables of the following and determine their logical status as tautology, contingent or contradictory

a. $p \supset \sim q$

b. $\sim p \vee q$

c. $\sim (p \cdot q)$

d. $(p \vee q) \supset r$

e. $p \supset (p \vee q)$

f. $(p \cdot \sim p) \supset q$

g. $(p \vee q) \supset \sim p$

h. $(p . q) \equiv \sim p$

i. $p \supset (q \cdot \sim q)$

j. $p \vee (\sim p \supset p)$

k. $p \supset (\sim p \supset \sim q)$

l. $\sim (p \supset q) \vee \sim r$

m. $p \supset \sim (q \vee r)$

n. $(p \cdot q) \supset (r \vee p)$

o. $(\sim p \cdot \sim q) . \vee r$

p. $p \supset \sim (q \supset \sim r)$

q. $(p \vee q) \bullet (\sim p \bullet \sim q)$

r. $\sim (p \bullet q) \bullet (q \supset p)$

s. $(p \supset q) \supset \sim (\sim q \supset \sim p)$

t. $(\sim p \vee \sim q) \equiv (p \equiv q)$

u. $\sim (p . \sim q) \supset (q . \sim p)$

v. $(p \equiv q) \equiv [(p \supset q) \bullet (q \supset p)]$

w. $(p \supset q) \supset [(\sim p \vee q) \vee r)]$

x. $[(p \bullet q) \supset \sim r)] \supset (p \supset r)$

y. $[(p \supset q) . (q \supset r)] \supset (p \supset r)$

3. Give a brief history of symbolic logic.
4. Explain and illustrate the notions of ideograms and phonograms.
5. What are the various uses of symbols in logic? Explain.
6. Explain various types of symbols used in symbolic logic.
7. Point out difference between disjunction and alternation.
8. Explain Truth Table Method as a decision procedure.
9. What is the main characteristic of constant symbol? Explain with the help of examples.