Arithmetic Progressions

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1. Find the missing variable from a, d, n and a_n, where a is the first term, d is the common difference and a_n is the nth term of AP.
(i) a= 7, d = 3, n = 8
(ii) a = –18, n= 10, $a_n=0$
(iii) d = –3, n = 18, $a_n=-5$
(iv) a = –18.9, d = 2.5, $a_n=3.6$
(v) a = 3.5, d = 0, n = 105
Ans. (i) a= 7, d = 3, n = 8
We need to find a_n here.
Using formula $a_n=a+(n-1)d$
Putting values of a, d and n,
$a_n=7+(8-1)3=7+(7)3=7+21=28$
(ii) a = –18, n= 10, $a_n=0$
We need to find d here.
Using formula $a_n=a+(n-1)d$
Putting values of a, a_n and n,
0 = –18 + (10 – 1)d
⇒ $0=-18+9d$
⇒ $18=9d$
⇒ $d=2$
(iii) d = –3, n = 18, $a_n=-5$
We need to find a here.
Using formula $a_n=a+(n-1)d$
Putting values of d, a_n and n,
–5 = a + (18 – 1)(–3)
⇒ $-5=a+(17)(-3)$
⇒ $-5=a-51$
⇒ $a=46$
(iv) a = –18.9, d = 2.5, $a_n=3.6$
We need to find n here.
Using formula $a_n=a+(n-1)d$
Putting values of d, a_n and a,
3.6 = –18.9 + (n – 1) (2.5)
⇒ $3.6=-18.9+2.5n-2.5$
⇒ $2.5n=25$
⇒ $n=10$
(v) a = 3.5, d = 0, n = 105
We need to find a_n here.
Using formula $a_n=a+(n-1)d$

Putting values of d, n and a,
$a_n=3.5+(105-1)(0)$
$\Rightarrow a_n=3.5+104\times 0$
$\Rightarrow a_n=3.5+0$
$\Rightarrow a_n=3.5$

2. Choose the correct choice in the following and justify:
(i) 30th term of the AP: 10,7,4… is
(A) 97
(B) 77
(C) –77
(D) –87
(ii) 11th term of the AP: −3,−12,2…is
(A) 28
(B) 22
(C) –38
(D) $-48\frac{1}{2}$
Ans. (i) 10,7,4…
First term = a =10, Common difference = d = 7 – 10= 4 – 7= –3
And n = 30 {Because, we need to find 30th term}
$a_n=a+(n-1)d$
$\Rightarrow a_{30}=10+(30-1)(-3)=10-87=-77$
Therefore, the answer is (C).
(ii) −3,−½,2…

First term = a = –3, Common difference = d = −½ −(−3)=2−(−½)= $\frac{5}{2}$
And n = 11 (Because, we need to find 11th term)
$a_n=-3+(11-1)\frac{5}{2}=-3+25=22$

3. Which term of the AP: 3, 8, 13, 18 … is 78?
Ans. First term = $a=3$, Common difference = d = 8 – 3=13 – 8=5 and $a_n=78$
Using formula $a_n=a+(n-1)d$, to find nth term of arithmetic progression,
$a_n=3+(n-1)5$,
$\Rightarrow 78=3+(n-1)5$
$\Rightarrow 75=5n-5$
$\Rightarrow 80=5n$
$\Rightarrow n=16$
It means 16th term of the given AP is equal to 78.

4. Find the number of terms in each of the following APs:
(i) 7,13,19....,205

(ii) 18, $15\frac{1}{2}$,13..., −47

Ans. (i) 7,13,19..., 205
First term = a =7, Common difference = d = 13 – 7= 19 – 13= 6
And $a_n=205$
Using formula $a_n=a+(n-1)d$, to find nth term of arithmetic progression,
$205=7+(n-1)6=7+6n-6$
$\Rightarrow 205=6n+1$
$\Rightarrow 204=6n$
$\Rightarrow n=34$
Therefore, there are 34 terms in the given arithmetic progression.

(ii) 18, $15\frac{1}{2}$,13..., −47

First term = a =18, Common difference = d = $15\frac{1}{2}-18=\frac{31}{2}-18=\frac{31-36}{2}=\frac{-5}{2}$
And $a_n=-47$
Using formula $a_n=a+(n-1)d$, to find nth term of arithmetic progression,
$-47=18+(n-1)\left(-\frac{5}{2}\right)=36-\frac{5}{2}n+\frac{5}{2}$
$\Rightarrow -94=36-5n+5$
$\Rightarrow 5n=135$
$\Rightarrow n=27$
Therefore, there are 27 terms in the given arithmetic progression

5. Check whether –150 is a term of the AP: 11,8,5,2...
Ans. Let −150 is the n^th^ of AP 11,8,5,2... which means that $a_n=-150$
Here, First term = a = 11, Common difference = d = 8 – 11= –3
Using formula $a_n=a+(n-1)d$, to find n^th^ term of arithmetic progression,
$-150=11+(n-1)(-3)$
$\Rightarrow -150=11-3n+3$
$\Rightarrow 3n=164$
$\Rightarrow n=\frac{164}{3}$
But, n cannot be in fraction.
Therefore, our supposition is wrong. −150 cannot be term in AP.

6. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Ans. An AP consists of 50 terms and the 50th term is equal to 106 and $a_3=12$

Using formula $a_n=a+(n-1)d$, to find nth term of arithmetic progression,

$a_{50}=a+(50-1)d$ And $a_3=a+(3-1)d$

$\Rightarrow 106=a+49d$ And $12=a+2d$

These are equations consisting of two variables.

Using equation $106=a+49d$, we get $a=106-49d$

Putting value of a in the equation $12=a+2d$,

$12=106-49d+2d$

$\Rightarrow 47d=94$

$\Rightarrow d=2$

Putting value of d in the equation, $a=106-49d$,

$a=106-49(2)=106-98=8$

Therefore, First term $=a=8$ and Common difference $=d=2$

To find 29th term, we use formula $a_n=a+(n-1)d$ which is used to find nth term of arithmetic progression,

$a_{29}=8+(29-1)2=8+56=64$

Therefore, 29th term of AP is equal to 64

7. How many multiples of 4 lie between 10 and 250?

Ans. The odd numbers between 0 and 50 are 1,3,5,7…49

It is an arithmetic progression because the difference between consecutive terms is constant.

First term = a = 1, Common difference = 3 – 1= 2, Last term = $l=49$

We do not know how many odd numbers are present between 0 and 50.

Therefore, we need to find n first.

Using formula $a_n=a+(n-1)d$, to find nth term of arithmetic progression, we get

$49=1+(n-1)2$

$\Rightarrow 49=1+2n-2$

$\Rightarrow 50=2n$

$\Rightarrow n=25$

Applying formula, $S_n=\frac{n}{2}(a+l)$ to find sum of n terms of AP, we get

$$S_{25}=\frac{25}{2}(1+49)=\frac{25}{2}\times 50$$

$$=25\times 25=625$$

8. Which term of the AP: 121, 117, 113, …..is its first negative term?

Ans. Given: 121, 117, 113, …….

Here $a = 121,\ \ d = 117 - 121 = 4$

Now, $a_n = a + (n-1)d$

$= 121 + (n-1)(-4)$ $= 121 - 4n + 4$ $= 125 - 4n$

For the first negative term, $a_n < 0$

$\Rightarrow 125 - 4n < 0$

$\Rightarrow 125 < 4n$

is an integer and

Hence, the first negative term is 32nd term

9. A ladder has rungs 25 cm apart (see figure). The rungs decrease uniformly in length

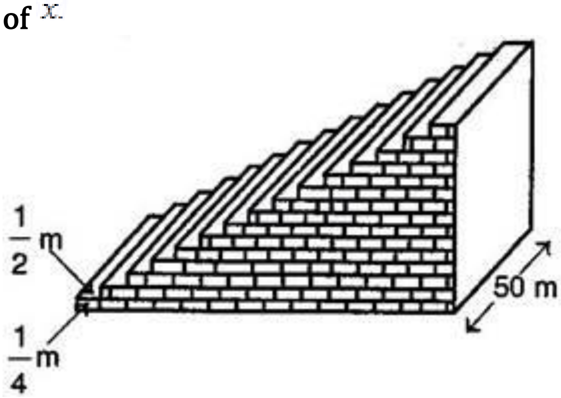
from 45 cm, at the bottom to 25 cm at the top. If the top and the bottom rungs are m apart, what is the length of the wood required for the rungs?

Ans. Number of rungs $= 10$

The length of the wood required for rungs = sum of 10 rungs

$= \ = 5 \times 70 = 350$ cm

10. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.



Ans. Here $a=1$ and $d=1$

$\therefore$ $S_{x-1}=\frac{x-1}{2}\left[2\times 1+(x-1-1)\times 1\right]$

$=\frac{x-1}{2}(2+x-2)$

$=\frac{(x-1)x}{2}$ $=\frac{x^2-x}{2}$

$S_x=\frac{x}{2}\left[2\times 1+(x-1)\times 1\right]$

$=\frac{x}{2}(x+1)$ $=\frac{x^2+x}{2}$

$S_{49}=\frac{49}{2}\left[2\times 1+(49-1)\times 1\right]$

$=\frac{49}{2}(2+48)$ = 49 x 25

According to question,

$S_{x-1}=S_{49}-S_x$

$\Rightarrow$ $\frac{x^2-x}{2}$ = 49 x 25 – $\frac{x^2+x}{2}$

$\Rightarrow$ $\frac{x^2-x}{2}$ + $\frac{x^2+x}{2}$ = 49 x 25

$\Rightarrow$ $\frac{x^2-x+x^2+x}{2}=49\times 25$

$\Rightarrow$ x^2 = 49 x 25

$\Rightarrow$ $x=\pm 35$

Since, x is a counting number, so negative value will be neglected.

$\therefore$ $x=35$

11. Find the first term and the common difference $\frac{1}{3},\frac{5}{3},\frac{9}{3},\ldots\ldots$

Ans. $\frac{1}{3},\frac{5}{3},\frac{9}{3},\ldots\ldots$

$a=\frac{1}{3}$

$d=\frac{5}{3}-\frac{1}{3}=\frac{4}{3}$

12. Is $\sqrt{3}, \sqrt{6}, \sqrt{9}, \ldots\ldots$ form an AP?

Ans. $a_1 = \sqrt{3}, a_2 = \sqrt{6}, a_3 = \sqrt{9}$

$d_1 = \sqrt{6} - \sqrt{3}$

$= \sqrt{3}\left(\sqrt{2} - 1\right)$

$d_2 = \sqrt{9} - \sqrt{6}$

$= 3 - \sqrt{6}$

Since $d_1 \neq d_2$

Hence, it is not an AP.

13. Which is the next term of the AP $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots\ldots$

Ans. $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$

$d = \sqrt{8} - \sqrt{2}$

$= 2\sqrt{2} - \sqrt{2}$

$= \sqrt{2}$

a_5 = a + (5 – 1)d

$= \sqrt{2} + 4 \times \sqrt{2} = 5\sqrt{2}$

Next term is $5\sqrt{2}$ or $\sqrt{50}$

14. Find the 11th term from the last term of the AP 10, 7, 4,………,-62.

Ans. $a = -62,\ d = -(7 - 10) = 3$

$a_{11} = a + 10d$

$= -62 + 10(3)$

$= -32$

15. If $x+1, 3x$ and $4x+2$ are in A.P, find the value of x.

Ans. Since $x+1,\ 3x$ and $4x+2$ are in AP

$$2(3x) = x+1+4x+2$$
$$\Rightarrow 6x = 5x+3$$
$$\Rightarrow 6x-5x = 3$$
$$\Rightarrow x = 3$$

16. Find the sum of first n odd natural numbers.

Ans. $1, 3, 5, 7, \ldots\ldots\ldots$

$a = 1, d = 3-1 = 2$

$$Sn = \frac{n}{2}\left[2a+(n-1)d\right]$$
$$= \frac{n}{2}\left[2\times1+(n-1)2\right]$$
$$= \frac{n}{2}\left[2+2n-2\right]$$
$$= n^2$$

17. Find the 12th term of the AP $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}\ldots$

Ans. $a = \sqrt{2}, d = 3\sqrt{2}-\sqrt{2} = 2\sqrt{2}$

$$a_{12} = a+11d$$
$$= \sqrt{2}+11\left(2\sqrt{2}\right)$$
$$= \sqrt{2}+22\sqrt{2}$$
$$= 23\sqrt{2}$$

18. Find the sum of first 11 terms of AP 2, 6, 10…

Ans. $2, 6, 10, \ldots$

$a = 2, d = 6-2 = 4$

$$S_{11} = \frac{11}{2}\left[2\times2+(11-1)\times4\right]$$
$$= \frac{11}{2}\left[4+40\right]$$
$$= 11\times22 = 242$$

19. Find the sum of first hundred even natural numbers divisible by 5.

Ans. Even natural no. divisible by 5 are 10, 20, 30…

$a = 10,\ d = 10$

$n = 100$

$S_{100} = \frac{100}{2}\left[2(10) + (100-1).10\right]$

$= 50\left[20 + 99 \times 10\right]$

$= 50500$

20. Find $a_{30} - a_{20}$ for the A.P $-9, -14, -19, -24, \ldots$

Ans.

$a = -9,$

$d = (-14) - (-9) = -14 + 9 = -5$

$a_{30} - a_{20} = a + 29d - a - 19d = 10d$

$= 10 \times (-5) = -50$

21. Find the common difference and write the next two terms of the AP $1^2, 5^2, 7^2, 73, \ldots$

Ans. $1^2,\ 5^2,\ 7^2,\ 73, \ldots\ldots$

$\Rightarrow 1, 25, 49, 73, \ldots\ldots\ldots$

$d = a_2 - a_1 = 25 - 1 = 24$

$d = 49 - 25 = 24$

$d = 73 - 49 = 24$

Hence, it is AP.

$a_5 = 73 + 24 = 97$

$a_6 = 97 + 24 = 121$

22. Show that sequence defined by $a_n = 3 + 2n$ is an AP.

Ans. $a_n = 3 + 2n$

So $a_1 = 5,\ a_2 = 7,\ a_3 = 9,\ a_4 = 11$

$7-5=9-7=11-9=2$

Hence, it is AP.

23. The first term of an AP is -7 and common difference 5. Find its general term.

Ans. $a=-7,\ d=5$

$a_n=a+(n-1)d$

$=-7+(n-1)(5)$

$=-7+5n-5$

$\therefore \quad a_n=5n-12$

24. How many terms are there in A.P? $18, 15\frac{1}{2}, 13, \ldots\ldots, -47$

Ans. $a=18,\ d=\frac{31}{2}-\frac{18}{1}=\frac{-5}{2}$

$a_n=-47$

$a_n=a+(n-1)d$

$-47=18+(n-1)\left(\frac{-5}{2}\right)$

$\Rightarrow \quad -47-18=\frac{-5}{2}n+\frac{5}{2}$

$\Rightarrow \quad n=27$

25. In an AP, the sum of first n terms is $\frac{3n^2}{2}+\frac{13}{2}n.$ find its 2nd term.

Ans. $S_n=\frac{3n^2}{2}+\frac{13}{2}n$

Put $n=1,2,3\ldots\ldots$

$S_1=\frac{16}{2}=8$

$S_2=19$

$a_1=s_1=8$

$a_2=S_2-S_1=19-8=11$

26. Show that the progression $4, 7\frac{1}{4}, 10\frac{1}{2}, 13\frac{3}{4}, 17, \ldots\ldots$ **is an AP.**

Ans. $\frac{29}{4} - \frac{4}{1} = \frac{29-16}{4} = \frac{13}{4}$

And $\frac{21}{2} - \frac{29}{4} = \frac{13}{4}$

And $\frac{17}{1} - \frac{55}{4} = \frac{13}{4}$

Hence, it is an AP.