Chapter 8

Mechanical Properties of Solids

Solutions (Set-1)

Very Short Answer Type Questions :

1. Why are girders given I shape?

Sol. To reduce stress at top and bottom of girders.

2. The length of wire increases by 1 mm under 1 N. What will be increase in length under 2 N?

Sol. 2 mm

3. What is the SI unit of volumetric strain?

Sol. No unit

4. Which of the three Y, B or G is possible in all the three state of matter (solid, liquid and gas)?

Sol. Bulk modulus of elasticity (B) only.

5. Which type of strain is there, when a spiral spring is stretched by a force?

Sol. Shear strain

6. State Hooke's law.

Sol. Within elastic limit,

stress $\propto$ strain

$\Rightarrow$ stress = E × strain

7. What is the dimensional formula of Young's modulus?

Sol. $[ML^{-1}T^{-2}]$

8. Define bulk modulus of elasticity.

Sol. The ratio of volume stress to volume strain is called bulk modulus.

9. What do you mean by Poisson's ratio?

Sol. The ratio of the lateral strain to the longitudinal strain is called the Poisson's ratio.

10. What is the value of modulus of rigidity for a liquid?

Sol. Zero

Short Answer Type Questions :

11. Why is a spring made of steel, not of copper?

Sol. A spring will be better one, if a large restoring force is set up in it on being deformed, which in turn depends upon the elasticity of material of the spring. Since the Young's modulus of elasticity of steel is more than that of copper, hence steel is used.

12. An elastic wire is cut to half its original length. How would it affect the maximum load that the wire can support?

Sol. Breaking load = breaking stress × area, is free from the length of elastic wire.

13. Which is more elastic-rubber or steel? Explain.

Sol. Steel, because it sustains more deforming force.

14. Calculate the force required to increase length by 1% of a rod of area of cross-section 10^{-3} m^2. Modulus of elasticity is 1.2×10^{12} Nm^{-2}.

Sol. $F = YA \cdot \dfrac{\Delta l}{l}$

$F = 1.2 \times 10^4$ N

15. Explain the terms (i) Young's modulus (ii) Bulk modulus.

Sol. (i) Young's modulus : The ratio of the longitudinal stress to the longitudinal strain is called Young's modulus.

(ii) Bulk modulus : The ratio of volume stress to volume strain is called bulk modulus.

16. A thick rope of density ρ and length L is hung from a rigid support. The Young's modulus of the material of rope is Y. What is the increase in length of the rope due to its own weight?

Sol. Let A be the area of cross-section of the rope. Weight of the rope, $F = AL\rho g$. It will be acting at the centre of gravity of the rope which lies at a distance $\dfrac{L}{2}$ from rigid support. Therefore,

$$Y = \frac{\text{normal stress}}{\text{longitudinal strain}}$$

$$= \frac{AL\rho g/A}{\dfrac{\Delta L}{L/2}} \text{ or } \Delta L = \frac{L^2\rho g}{2Y}$$

17. Define elastic limit.

Sol. No body is perfectly elastic. However, a body behaves as a perfectly elastic body and recovers its original configuration completely when the deforming force does not exceed a particular limit. This limit is called the elastic limit.

18. When the pressure on a sphere is increased by 80 atm then its volume decreases by 0.01%. Find the Bulk modulus of elasticity of the material of sphere.

Sol. $P = 80 \times 1.013 \times 10^5$ N/m^2

$$\frac{\Delta V}{V} = \frac{0.01}{100}$$

$$\therefore \quad B = \frac{PV}{\Delta V} = \frac{80 \times 1.013 \times 10^5}{\dfrac{0.01}{100}} = 8.1 \times 10^{10} \text{ N/m}^2$$

19. A specimen of oil having an initial volume of 600 cm^3 is subjected to a pressure increase of 3.6×10^6 Pa and the volume is found to decrease by 0.45 cm^3. What is the Bulk modulus of the material?

Sol. $B = \dfrac{PV}{\Delta V}$

$= \dfrac{3.6 \times 10^6 \times 600}{0.45} = 4.8 \times 10^9$ N/m^2

20. Find the decrease in the volume of a sample of water from the following data. Initial volume = 1000 cm^3, initial pressure = 10^5 N m^{-2}, final pressure = 10^6 N m^{-2}, compressibility of water = 50×10^{-11} m^2 N^{-1}.

Sol. The change in pressure

$= \Delta P = 10^6 \text{ N m}^{-2} - 10^5 \text{ N m}^{-2}$

$= 9 \times 10^5 \text{ N m}^{-2}$.

Compressibility $= \dfrac{1}{\text{Bulk modulus}} = -\dfrac{\Delta V / V}{\Delta P}$

or, $50 \times 10^{-11} \text{ m}^2 \text{ N}^{-1} = -\dfrac{\Delta V}{(10^{-3} \text{ m}^3) \times (9 \times 10^5 \text{ N m}^{-2})}$

or, $\Delta V = -50 \times 10^{-11} \times 10^{-3} \times 9 \times 10^5 \text{ m}^3$

$= -4.5 \times 10^{-7} \text{ m}^3 = -0.45 \text{ cm}^3$

Thus the decrease in volume is 0.45 cm^3.

21. The material in human bones and elephant bones is essentially same, but an elephant has much thicker legs. Explain why, in terms of breaking stress.

Sol. $\text{Stress} = \dfrac{\text{force}}{\text{area}}$

22. If a metal wire has its length doubled and its diameter tripled, then by what factor does its Young's modulus change?

Sol. Young's modulus of material is independent of stress and strain.

23. Calculate the longest length of steel wire that can hang vertically without breaking. Breaking stress for steel = 7.982×10^8 N/m^2 and density for steel $d = 8.1 \times 10^3$ kg/m^3.

Sol. $\text{Breaking stress} = \dfrac{\text{force}}{\text{area}} = \dfrac{mg}{a}$

$\Rightarrow \quad \text{Breaking stress} = \dfrac{\rho l a g}{a}$

$l = \dfrac{\text{Breaking stress}}{\rho g}$

$l = \dfrac{7.982 \times 10^8}{8.1 \times 10^3 \times 9.8}$ m

$l = 1 \times 10^4$ m

24. A copper wire of length 2.2 m and a steel wire of length 1.6 m, both of diameter 3.0 mm, are connected end to end. When stretched by a load, the net elongation is found to be 0.70 mm. Obtain the load applied.

Sol. The copper and steel wires are under a tensile stress because they have the same tension (equal to the load w) and the same area of cross-section A. We have, stress = strain × Young's modulus. Therefore,

$$\frac{w}{A} = Y_c \cdot \frac{\Delta L_c}{L_c} = Y_s \cdot \frac{\Delta L_s}{L_s}$$

where the subscripts *c* and *s* refer to copper and stainless steel respectively, or,

$$\frac{\Delta L_c}{\Delta L_s} = \left(\frac{Y_s}{Y_c}\right) \times \left(\frac{L_c}{L_s}\right)$$

Given, L_c = 2.2 m, L_s = 1.6 m

$Y_c = 1.1 \times 10^{11}$ Nm^{-2} and $Y_s = 2.0 \times 10^{11}$ Nm^{-2}

$$\frac{\Delta L_c}{\Delta L_s} = \left(\frac{2.0 \times 10^{11}}{1.1 \times 10^{11}}\right) \times \left(\frac{2.2}{1.6}\right) = 2.5$$

The total elongation is given to be

$\Delta L_c + \Delta L_s = 7.0 \times 10^{-4}$ m

Solving the above equations,

$\Delta L_c = 5.0 \times 10^{-4}$ m and $\Delta L_s = 2.0 \times 10^{-4}$ m

Therefore, $w = \dfrac{(A \times Y_c \times \Delta L_c)}{L_c}$

$= \pi(1.5 \times 10^{-3})^2 \times [(5.0 \times 10^{-4} \times 1.1 \times 10^{11})/2.2]$

$= 1.8 \times 10^2$ N

25. A 2 m long wire is stretched by 0.5 cm. Calculate the elastic potential energy per unit volume if the Young's modulus of the material of the wire is $Y = 8 \times 10^{10}$ N/m^2.

Sol. $u = \dfrac{1}{2}(\text{stress}) \times \text{strain}$

$= \dfrac{1}{2}\text{Young's modulus} \times (\text{strain})^2 = \dfrac{1}{2} \times 8 \times 10^{10} \times \left(\dfrac{0.5 \times 10^{-2}}{20}\right)^2$

$= 4 \times 10^{10} \times \dfrac{25 \times 10^{-4}}{400}$ = 2.5×10^4 J/m^3

Long Answer Type Questions :

26. A metal wire 3.50 m long and 0.70 mm in diameter has given the following test. A load weighing 20 N was originally hung from the wire to keep it straight. The position of the lower end of the wire was read on a scale as load was added.

Added Load (N)	Scale Reading (cm)
0	3.02
10	3.07
20	3.12
30	3.17
40	3.22
50	3.27
60	3.32
70	4.27

(i) Graph these values, plotting the increases in length horizontally and the added load vertically.

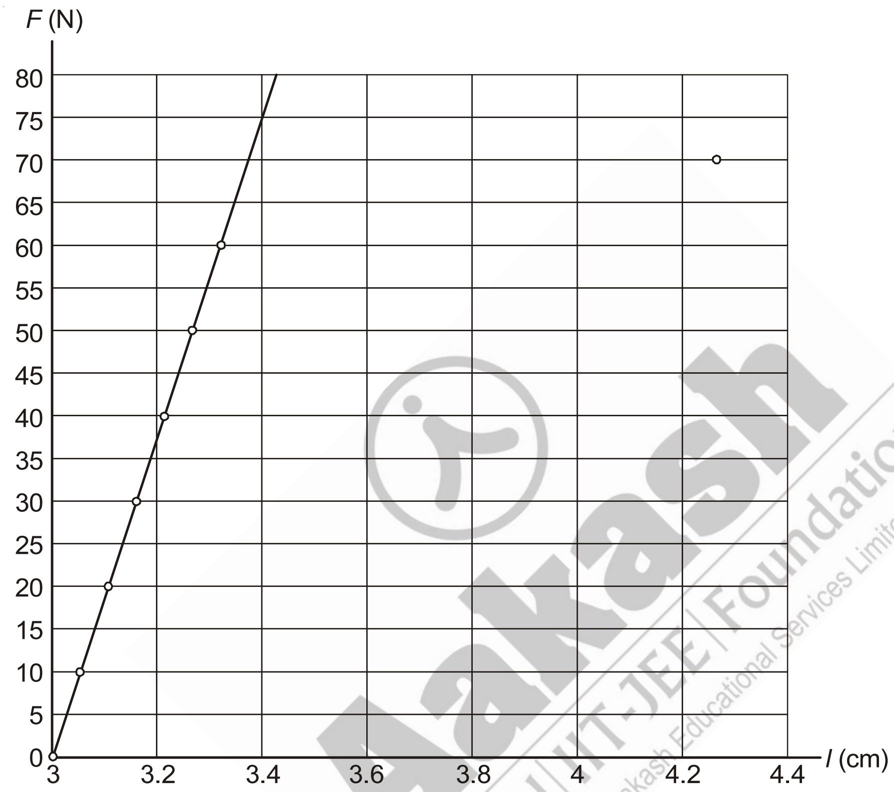
(ii) Calculate the value of Young's modulus.

(iii) The proportional limit occurred at a scale reading of 3.34 cm. What was the stress at this point?

Sol. $F_{\perp} = \left(\frac{YA}{l_0}\right)\Delta l$ so the slope of the graph in part (a) depends on Young's modulus.

$F_{\perp}$ is the total load, 20 N plus the added load.

(i) The graph is given in figure



(ii) The slope is $\frac{60\text{ N}}{(3.32-3.02)\times 10^{-2}\text{ m}} = 2.0\times 10^4$ N/m

$$Y = \left(\frac{l_0}{\pi r^2}\right)(2.0\times 10^4\text{ N/m}) = \left(\frac{3.50\text{ m}}{\pi[0.35\times 10^{-3}\text{ m}]^2}\right)(2.0\times 10^4\text{ N/m}) = 1.8\times 10^{11}\text{ Pa}$$

(iii) The stress is $F_{\perp}/A$. The total load at the proportional limit is 60 N + 20 N = 80 N

$$\text{stress} = \frac{80\text{ N}}{\pi(0.35\times 10^{-3}\text{ m})^2} = 2.1\times 10^8\text{ Pa}$$

The value of Y we calculated is close to the value for iron, nickel and steel.

27. A brass rod with a length of 1.40 m and a cross-sectional area of 2.00 cm^2 is fastened end to end to a nickel rod with length L and cross-sectional area 1.00 cm^2. The compound rod is subjected to equal and opposite pulls of magnitude 4.00×10^4 N at its ends.

(i) Find the length L of the nickel rod if the elongation of the two rods are equal.

(ii) What is the stress in each rod?

(iii) What is the strain in each rod?

Sol. Each piece of the composite rod is subjected to a tensile force of 4.00×10^4 N.

(i) $Y = \dfrac{F_{\perp} l_0}{A\Delta l}$ so $\Delta l = \dfrac{F_{\perp} l_0}{YA}$

$\Delta l_b = \Delta l_n$ gives that $\dfrac{F_{\perp} l_{0,b}}{Y_b A_b} = \dfrac{F_{\perp} l_{0,n}}{Y_n A_n}$ (b for brass and n for nickel); $l_{0,\,n} = L$

But the $F_{\perp}$ is same for both, so

$$l_{0,n} = \frac{Y_n}{Y_b}\frac{A_n}{A_b} l_{0,b}$$

$$L = \left(\frac{21\times10^{10}\ \text{Pa}}{9.0\times10^{10}\ \text{Pa}}\right)\left(\frac{1.00\ \text{cm}^2}{2.00\ \text{cm}^2}\right)(1.40\ \text{m}) = 1.63\ \text{m}$$

(ii) Stress $= \dfrac{F_{\perp}}{A} = \dfrac{T}{A}$

Brass : stress = $T/A = (4.00 \times 10^4\ \text{N})/(2.00 \times 10^{-4}\ \text{m}^2) = 2.00 \times 10^8$ Pa

Nickel : stress = $T/A = (4.00 \times 10^4\ \text{N})/(1.00 \times 10^{-4}\ \text{m}^2) = 4.00 \times 10^8$ Pa

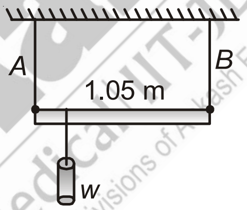
(iii) Y = stress/strain and strain = stress/Y

Brass : strain = $(2.00 \times 10^8\ \text{Pa})/(9.0 \times 10^{10}\ \text{Pa}) = 2.22 \times 10^{-3}$

Nickel : strain = $(4.00 \times 10^8\ \text{Pa})/(21 \times 10^{10}\ \text{Pa}) = 1.90 \times 10^{-3}$

Larger Y means less Δl and smaller A means greater Δl, so the two effects largely cancel and the lengths don't differ greatly. Equal Δl and nearly equal l means the strains are nearly the same. But equal tensions and A differing by a factor of 2 means the stresses differ by a factor of 2.

28. A 1.05 m long rod of negligible weight is supported at its ends by wires A and B of equal lengths as shown in figure. The cross-sectional area of A is 2.00 mm^2 and that of B is 4.00 mm^2. Young's modulus for wire A is 1.80×10^{11} Pa; and that for B is 1.20×10^{11} Pa. At what point along the rod should a weight w be suspended to produce



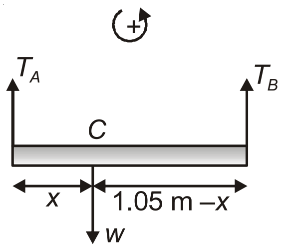
(i) Equal stress in A and B and

(ii) Equal strains in A and B?

Sol. (i) Stress = $F_{\perp}/A$, so equal stress implies T/A same for each wire.

$T_A/2.00\ \text{mm}^2 = T_B/4.00\ \text{mm}^2$ so $T_B = 2.00 T_A$

The question is where along the rod to hang the weight in order to produce this relation between the tensions in the two wires. Let the weight be suspended at point C, a distance x to the right of wire A. The free-body diagram for the rod is given in the figure.



$$\sum \vec{\tau}_C = 0$$

$$+T_B(1.05\ \text{m} - x) - T_A x = 0$$

But $T_B = 2.00T_A$ so $2.00T_A(1.05\text{ m} - x) - T_Ax = 0$

$2.10\text{ m} - 2.00x = x$ and $x = 2.10\text{ m}/3.00 = 0.70\text{ m}$ (measured from *A*).

(ii) Y = stress/strain gives that strain = stress/$Y = F_{\perp}/AY$.

Equal strain thus implies

$$\frac{T_A}{(2.00\text{ mm}^2)(1.80\times10^{11}\text{ Pa})} = \frac{T_B}{(4.00\text{ mm}^2)(1.20\times10^{11}\text{ Pa})}$$

$$T_B = \left(\frac{4.00}{2.00}\right)\left(\frac{1.20}{1.80}\right)T_A = 1.333T_A$$

The $\sum\vec{\tau}_C = 0$ equation still gives $T_B(1.05\text{ m} - x) - T_Ax = 0$

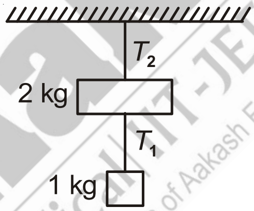
But now $T_B = 1.333T_A$ so $(1.333T_A)(1.05\text{ m} - x) - T_Ax = 0$

$1.40\text{ m} = 2.33x$ and $x = 1.40\text{ m}/2.33 = 0.60\text{ m}$ (measured from *A*)

Wire *B* has twice the cross-sectional area so it takes twice the tension to produce the same stress. For equal stress the moment arm for T_B (0.35 m) is half that for T_A (0.70 m), since the torques must be equal. The smaller *Y* for *B* partially compensates for the larger area in determining the strain and for equal strain the moment arms are closer to being equal.

29. One end of a metal wire is fixed to a ceiling and a load of 2 kg hangs from the other end. A similar wire is attached to the bottom of the load and another load of 1 kg hangs from this lower wire. Find the longitudinal strain in both the wires. Area of cross section of each wire is 0.005 cm^2 and Young's modulus of the metal is 2.0×10^{11} Nm^{-2}. (Take $g = 10$ ms^{-2}.)

Sol. The situation is described in figure. As the 1 kg mass is in equilibrium, the tension in the lower wire equals the weight of the load.



Figure

Thus, $T_1 = 10$ N

Stress = 10 N/0.005 cm^2

$= 2\times 10^7$ N m^{-2}.

$$\text{Longitudinal strain} = \frac{\text{stress}}{Y} = \frac{2\times10^7\text{ N m}^{-2}}{2\times10^{11}\text{ N m}^{-2}} = 10^{-4}.$$

Considering the equilibrium of the upper block, we can write,

$T_2 = 20\text{ N} + T_1$, or, $T_2 = 30$ N.

Stress = 30 N/0.005 cm^2

$= 6 \times 10^7$ N m^{-2}.

$$\text{Longitudinal strain} = \frac{6\times10^7\text{ N m}^{-2}}{2\times10^{11}\text{ N m}^{-2}} = 3\times10^{-4}.$$

30. Discuss experimental determination of Young's modulus of a metallic wire.

Sol. Construction

The apparatus used is shown in figure. Here A and B are two long straight wires of nearly equal leng area of cross-section and of same material. They are suspended side by side from a common rigid. The wire A is called reference wire. A pan is attached to its free end and known weights can be P it. A main scale M is fixed on the wire A near its lower end. A pointer P, attached to the lower end B, is connected to the Vernier scale V, which can slide on the main scale M. Here, the purpose of re wire is to compensate any change in length of wire B, which may occur due to change in room temperature.

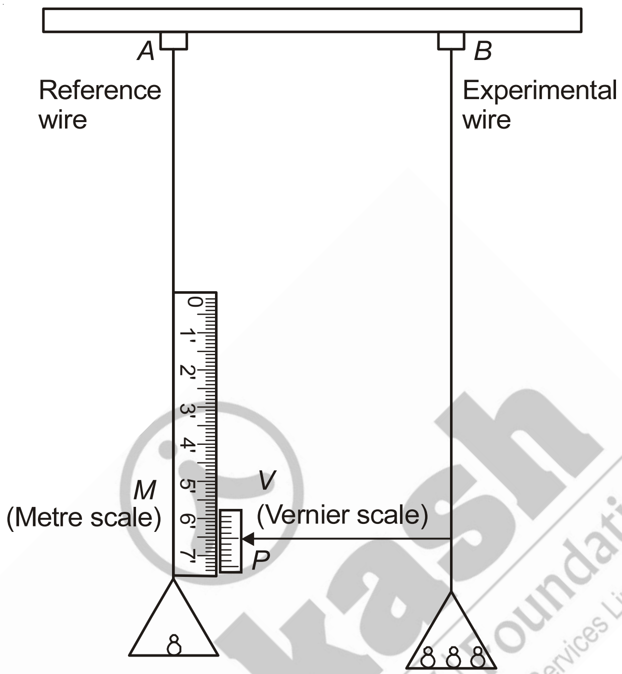


Fig.: An arrangement for the determination of Young's modulus of the material of a wire

Working and Theory

- Remove the kinks if any in the wires A and B and put small load in each pan so that both the wires A and B are straight and parallel. Note the reading of main scale M and Vernier scale V.
- Put some weight (say m) in pan of experimental wire B. The wire B. The wire B gets stretched. A tensile stress is set up in this wire. Note the reading of main scale and vernier scale. The difference of two readings will give us the extension in the wire B = (Δl say). For the weight mg, let r and l be the initial radius and length of the wire B.

  Area of cross-section of the wire $B = \pi r^2$

  Stretching force = mg

  Normal stress = $\frac{mg}{\pi r^2}$

  Longitudinal strain = $\frac{\Delta l}{l}$

  Young's modulus, $Y = \frac{mg/\pi r^2}{\Delta l/l} = \frac{mgl}{\pi r^2 \Delta l}$

  Thus, Y can be determined.

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Chapter 8

Mechanical Properties of Solids

Solutions (Set-2)

1. Select the correct alternative(s)

   (1) Elastic forces are not always conservative

   (2) Elastic forces are always conservative

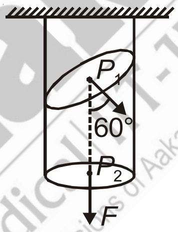
   (3) Elastic forces are conservative only when Hooke's law is obeyed

   (4) Elastic forces are not conservative

Sol. Answer (1)

Elastic forces are not always conservative.

2. A massless uniform rod is subjected to force F at its free end as shown in figure. The ratio of tensile stress at plane P_1 to stress at P_2 is



(1) 1 : 2 (2) $\sqrt{2}:1$ (3) 1 : 4 (4) 3 : 2

Sol. Answer (3)

At P_2, stress = $S_2 = \frac{F}{A}$

At P_1, stress = $S_1 = \frac{F\cos 60^\circ}{\frac{A}{\cos 60^\circ}} = \frac{F}{4A}$

$\Rightarrow \frac{S_1}{S_2} = \frac{1}{4}$

3. A wire 2 m in length suspended vertically stretches by 10 mm when mass of 10 kg is attached to the lower end. The elastic potential energy gain by the wire is (take g = 10 m/s^2)

(1) 0.5 J (2) 5 J (3) 50 J (4) 500 J

Sol. Answer (1)

4. A wire of length L and cross-sectional area A is made of material of Young's modulus Y. The work done in stretching the wire by an amount x is

(1) $\frac{YAx^2}{L}$ (2) $\frac{YAx^2}{2L}$ (3) $\frac{2YAx^2}{L}$ (4) $\frac{4YAx^2}{L}$

Sol. Answer (2)

$$W = \Delta U = \left(\frac{1}{2}\right)\left(\frac{YA}{L}\right)x^2$$

5. What is the percentage increase in length of a wire of diameter 2.5 mm, stretched by a force of 100 kg wt? Young's modulus of elasticity of wire = 12.5 × 10^{11} dyne/cm^2

(1) 0.16% (2) 0.32% (3) 0.08% (4) 0.12%

Sol. Answer (1)

Here, $2r = 2.5$ mm or $r = 0.125$ cm

$F = 100 \times 1000 \times 980$ dyne

As, $Y = \frac{F \times l}{A \times \Delta l}$

$$\frac{\Delta l}{l} = \frac{F}{AY}$$

∴ Percentage increase in length $= \frac{\Delta l}{l} \times 100 = \frac{F}{AY} \times 100$

6. Two exactly similar wires of steel and copper are stretched by equal forces. If the total elongation is 2 cm, then how much is the elongation in steel and copper wire respectively? Given, $Y_{steel} = 20 \times 10^{11}$ dyne/cm^2, $Y_{copper} = 12 \times 10^{11}$ dyne/cm^2.

(1) 1.25 cm; 0.75 cm (2) 0.75 cm; 1.25 cm

(3) 1.15 cm; 0.85 cm (4) 0.85 cm; 1.15 cm

Sol. Answer (2)

$$Y_s = \frac{F \times l}{A \times \Delta l_s}; \quad Y_{Cu} = \frac{Fl}{A \times \Delta l_{Cu}}$$

$$\therefore \quad \frac{Y_S}{Y_{Cu}} = \frac{\Delta l_{Cu}}{\Delta l_S}$$

$$\Rightarrow \quad \Delta l_{Cu} = \frac{5}{3}\Delta l_S \quad \ldots(i)$$

According to the question,

$$\Delta l_S + \Delta l_{Cu} = 2 \text{ cm} \quad \ldots(ii)$$

From equation (i) & (ii), we get

$$\Delta l_S = 0.75 \text{ cm}, \ \Delta l_{Cu} = 1.25 \text{ cm}$$

7. A steel rod has a radius 10 mm and a length of 1.0 m. A force stretches it along its length and produces a strain of 0.32%. Young's modulus of the steel is 2.0 × 10^{11} Nm^{-2}. What is the magnitude of the force stretching the rod?

(1) 100.5 kN (2) 201 kN (3) 78 kN (4) 150 kN

Sol. Answer (2)

Given, $r = 10 \text{ mm} = 10 \times 10^{-3}$ m

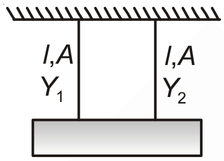
$l = 1.0$ m; Strain $= \dfrac{0.32}{100}$

$Y = 2.0 \times 10^{11}$ N/m^2; F = ?

$\therefore \quad F = Y \cdot \pi r^2 \cdot \dfrac{\Delta l}{l}$

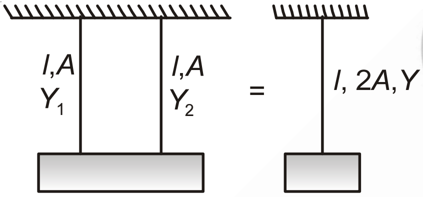
F = 201 kN

8. Two wires of equal length and cross-sectional area are suspended as shown in figure. Their Young's modulus are $Y_1 = 2 \times 10^{11}$ Pa and $Y_2 = 0.90 \times 10^{11}$ Pa respectively. What will be the equivalent Young's modulus of combination?



(1) 2.90×10^{11} Pa (2) 1.45×10^{11} Pa (3) 1.34×10^{11} Pa (4) 4.25×10^{11} Pa

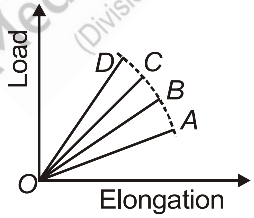
Sol. Answer (2)



$$\frac{Y \cdot 2A}{l} = \frac{Y_1 A}{l} + \frac{Y_2 A}{l}$$

$$Y = \frac{Y_1 + Y_2}{2} = 1.45 \times 10^{11} \text{ Pa}$$

9. The load versus elongation graph for four wires of the same material is shown in figure. The thinnest wire is represented by line

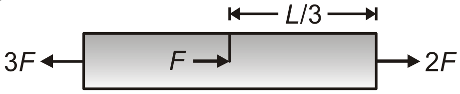


(1) OC (2) OD (3) OA (4) OB

Sol. Answer (3)

For a thinnest wire, the elongation in wire will be maximum for a given load, which is so corresponding to line OA.

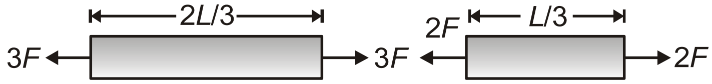
10. A uniform cylindrical rod of length L, cross-sectional area A and Young's modulus Y is acted upon by the forces as shown in figure. The elongation of the rod is



(1) $\dfrac{2FL}{5AY}$ (2) $\dfrac{3FL}{8AY}$ (3) $\dfrac{3FL}{5AY}$ (4) $\dfrac{8FL}{3AY}$

Sol. Answer (4)

The free body diagram of two parts are shown in figure



Total elongation is

$$\Delta l = \Delta l_1 + \Delta l_2 = \frac{3F\left(\frac{2L}{3}\right)}{AY} + \frac{2F\left(\frac{L}{3}\right)}{AY}$$

$$\Delta l = \frac{8FL}{3AY}$$

11. One end of a wire of length L and weight w is attached rigidly to a point in roof and a weight w_1 is suspended from its lower end. If A is the area of cross-section of the wire then the stress in the wire at a height $\frac{3L}{4}$ from its lower end is

(1) $\dfrac{w_1}{A}$ (2) $\dfrac{w_1 + \frac{w}{4}}{A}$ (3) $\dfrac{w_1 + \frac{3w}{4}}{A}$ (4) $\dfrac{w_1 + w_2}{A}$

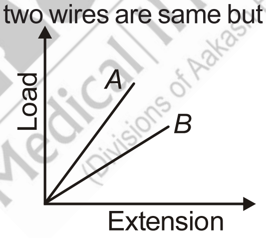
Sol. Answer (3)

Tension in the wire at length $\frac{3L}{4}$ from lower end is

T = suspended load + $\frac{3}{4}$ × weight of wire = $w_1 + \frac{3w}{4}$

$\therefore$ Stress $= \dfrac{w_1 + \frac{3w}{4}}{A}$

12. In the given figure, if the dimensions of the two wires are same but materials are different, then Young's modulus is



(1) More for A than B (2) More for B than A (3) Equal for A and B (4) None of these

Sol. Answer (1)

Slope of graph $\propto$ Young's modulus (Y)

13. Two wires A and B of same material have radii in the ratio 2 : 1 and lengths in the ratio 4 : 1. The ratio of the normal forces required to produce the same change in the lengths of these two wires is

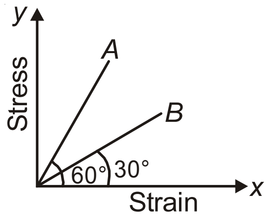
(1) 1 : 1 (2) 2 : 1 (3) 1 : 2 (4) 1 : 4

Sol. Answer (1)

$$\frac{F_1}{F_2} = \frac{A_1 \cdot l_2}{A_2 \cdot l_1} = 1$$

$\therefore$ $F_1 : F_2 = 1 : 1$

14. The stress versus strain graph for wires of two materials *A* and *B* are as shown in the figure. If Y_A and Y_B are the Young's moduli of the materials, then



(1) $Y_B = 2Y_A$ (2) $Y_A = 3Y_B$ (3) $Y_B = 3Y_A$ (4) $Y_A = Y_B$

Sol. Answer (2)

Slope $\propto$ Young's modulus

15. The following four wires are made of same material. Which of these will have the largest extension when the same tension is applied?

(1) Length 50 cm, diameter 0.5 mm
(2) Length 100 cm, diameter 1 mm
(3) Length 200 cm, diameter 2 mm
(4) Length 300 cm, diameter 3 mm

Sol. Answer (1)

$$Y = \frac{Fl}{A\Delta l}$$

$$\Rightarrow \quad \Delta l = \frac{Fl}{AY}$$

Force is same, *Y* is same.

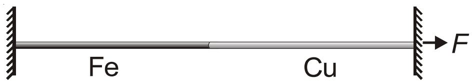
$$\Delta l \propto \frac{l}{A}$$

$$\Delta l_1 \propto \frac{\frac{1}{2}}{\pi\left(\frac{0.5\times10^{-3}}{2}\right)^2}, \Delta l_2 \propto \frac{1}{\pi\left(0.5\times10^{-3}\right)^2}, \Delta l_3 \propto \frac{2}{\pi\left(1\times10^{-3}\right)^2} \text{ and } \Delta l_4 \propto \frac{3}{\pi\left(1.5\times10^{-3}\right)^2}$$

So $$\Delta l_1 \propto \frac{2}{\pi\left(0.5\times10^{-3}\right)^2}, \Delta l_2 \propto \frac{1}{\pi\left(0.5\times10^{-3}\right)^2}, \Delta l_3 \propto \frac{1}{2\pi\left(0.5\times10^{-3}\right)^2}, \Delta l_4 \propto \frac{1}{3\pi\left(0.5\times10^{-3}\right)^2}$$

So Δl_1 will be largest.

16. As shown in figure, by combining together copper and steel wires of same length and same diameter, a force *F* is applied at one of their end. The combined length is increased by 2 cm. The wires will have



(1) Same stress and same strain
(2) Different stress and different strain
(3) Different stress and same strain
(4) Same stress and different strain

Sol. Answer (4)

Force on each cross section is same and since cross sectional area is same so stress will be same.

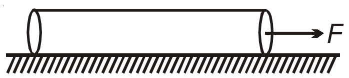
and Y is different for Fe and Cu.

So, $Y = \dfrac{\text{Stress}}{\text{Strain}}$

$\Rightarrow$ Strain $= \dfrac{\text{Stress}}{Y}$

Clearly strain will be different.

17. A rod of length L kept on a smooth horizontal surface is pulled along its length by a force F. The area of cross-section is A and Young's modulus is Y. The extension in the rod is



(1) $\dfrac{FL}{AY}$ (2) $\dfrac{2FL}{AY}$ (3) $\dfrac{FL}{2AY}$ (4) Zero

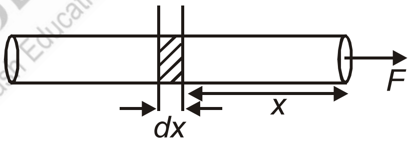
Sol. Answer (3)

$a = \dfrac{F}{m}$

$T(x) = \dfrac{m}{L}(L-x)\times\dfrac{F}{m}$

Stress $= \dfrac{F}{A} = \dfrac{F(L-x)}{LA}$

Strain $= \dfrac{\text{Strees}}{Y} = \dfrac{F\{L-x\}}{YAL}$



$\dfrac{dL}{dx} = \dfrac{F\{L-x\}}{YAL}$

$\int dL = \dfrac{F}{YAL}\int_0^L (L-x)dx \Rightarrow \Delta L = \dfrac{F}{YAL}\times\dfrac{L^2}{2} = \dfrac{FL}{2YA}$

18. When a load W is hung from a wire, it extends by Δl. The heat produced in the process is

(1) $W \times \Delta l$ (2) $\dfrac{1}{2}W\times\Delta l$ (3) $\dfrac{W\times\Delta l}{4}$ (4) Zero

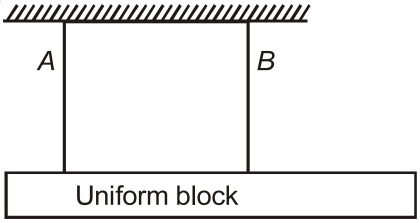
Sol. Answer (2)

Strain stored in the wire $= \dfrac{1}{2}$ stress $\times$ strain $\times$ volume $= \dfrac{1}{2}\times\dfrac{W}{A}\times\dfrac{\Delta l}{l}\times A\times l = \dfrac{1}{2}W\Delta l$

Work done by gravity = $W = \Delta l$

Heat produced = $W.\Delta l - \dfrac{1}{2}W\Delta l = \dfrac{1}{2}.W.\Delta l$

19. The figure shows a horizontal block that is suspended by two wires *A* and *B* which are identical except their original length. Which wire was originally shorter?



(1) *A* (2) *B*

(3) Both had equal length (4) Any one of them could be shorter

Sol. Answer (2)

Since $T_B > T_A$

$\Rightarrow \Delta L_B > \Delta L_A \Rightarrow (L_B)_i < (L_A)_i$

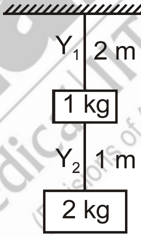
20. A force of one newton doubles the length of a cord having cross-sectional area 1 mm^2. The Young's modulus of the material of the cord is

(1) 1 Nm^{-2} (2) 5 × 10^5 Nm^{-2} (3) 10^6 Nm^{-2} (4) 2 × 10^6 Nm^{-2}

Sol. Answer (3)

$$Y = \frac{F.L}{A.\Delta L} = \frac{1N}{10^{-6}m^2} = 10^6 \text{Nm}^{-2}$$

21. Two blocks of masses 1 kg and 2 kg are suspended with the help of two wires having same area of cross section. If the ratio of Young's moduli *i.e.*, $Y_1 : Y_2 = 1 : 2$, then the ratio of extensions produced in wires is



(1) 2 : 1 (2) 4 : 1 (3) 6 : 1 (4) 8 : 1

Sol. Answer (3)

$$\Delta l = \frac{Fl}{AY} \Rightarrow \Delta l \propto \frac{Fl}{Y}$$

$$\Rightarrow \frac{\Delta l_1}{\Delta l_2} = \frac{F_1 l_1}{F_2 l_2}.\frac{Y_2}{Y_1} = \frac{3}{2}.\frac{2}{1}.\frac{2}{1} = \frac{6}{1}$$

22. An iron wire of negligible mass, length *L* and cross-section area *A* has one end fixed. A ball of mass *m* is attached to the other end of wire. The wire and ball are rotating with an angular velocity ω in a horizontal plane. If Δ*L* is extension produced in wire, then the Young's modulus of wire is

(1) $\frac{mL^2\omega}{A\Delta L}$ (2) $\frac{m\omega^2 L^2}{A\Delta L}$ (3) $\frac{m\omega^2 L}{A\Delta L}$ (4) $\frac{m\omega L}{A\Delta L}$

Sol. Answer (2)

Internal force in the wire due to strain is providing the necessary centripetal force

$\Rightarrow \quad F = \frac{YA\Delta L}{L} = m\omega^2 L$

$\Rightarrow \quad Y = \frac{m\omega^2 L^2}{A\Delta L}$

23. A body of mass M is attached to lower end of a metal wire whose upper end is fixed. The elongation is l. The ratio of loss of gravitational potential energy to the energy stored in the wire is

(1) 1 (2) 2 (3) $\frac{1}{2}$ (4) $\frac{4}{3}$

Sol. Answer (2)

24. A heavy rope is suspended from the ceiling of a room. If ρ is the density of the rope, L be its original length and Y be its Young's modulus, then increase ΔL in the length of the rope due to its own weight is

(1) $\Delta L = \frac{\rho g L^2}{Y}$ (2) $\Delta L = \frac{2\rho g L^2}{Y}$ (3) $\Delta L = \frac{\rho g L^2}{2Y}$ (4) $\Delta L = \frac{\rho g L}{Y}$

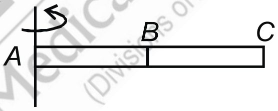
Sol. Answer (3)

25. If s is stress and Y is Young's modulus of material, the energy stored per unit volume is

(1) $\frac{2Y}{s}$ (2) $\frac{s}{2Y}$ (3) $2s^2Y$ (4) $\frac{s^2}{2Y}$

Sol. Answer (4)

26. A rigid rod of mass m and lengths l, is being rotated in horizontal plane about a vertical axis, passing through one end A. If T_A, T_B and T_C are the tensions in rod at point A, mid-point B and point C of rod respectively, then

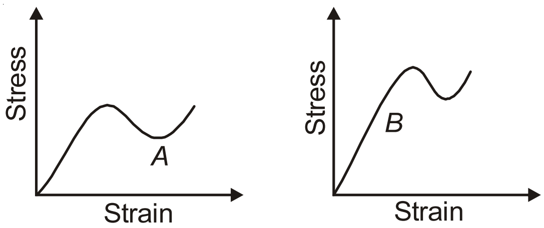


(1) $T_C = 0$ (2) $T_B = \frac{3}{4}T_A$ (3) $T_B = \frac{T_A}{2}$ (4) $T_A = m\omega^2 l$

Sol. Answer (2)

$T_{(x)} = \frac{m\omega^2}{2l}(l^2 - x^2), \; T_A = \frac{m\omega^2 l}{2}, T_B = \frac{3m\omega^2 l}{8}$

27. The stress-strain graphs for materials A and B are shown in the figure. Choose the correct alternative



(1) Material *A* is stronger than material *B*

(2) Material *B* is stronger than material *A*

(3) The Young's modulus of *A* is greater than or equal to that of *B*

(4) The Young's modulus of *B* is greater than that of *A*

Sol. Answer (2)

28. A steel wire of diameter 2 mm has a breaking strength of 4×10^5 N. What is the breaking force of similar steel wire of diameter 1.5 mm?

(1) 2.3×10^5 N (2) 2.6×10^5 N (3) 3×10^5 N (4) 1.5×10^5 N

Sol. Answer (1)

Breaking strength, F = breaking stress × area of cross-section

$$= \frac{S \times D^2}{4}$$

or $F \propto D^2$

$$\therefore \quad \frac{F_2}{F_1} = \left(\frac{D_2}{D_1}\right)^2$$

29. What is the greatest length of copper wire that can hang without breaking?

Breaking stress = 7.2×10^7 N/m^2. Density of copper = 7.2 g/cc, g = 10 m/s^2

(1) 100 m (2) 1000 m (3) 150 m (4) 1500 m

Sol. Answer (2)

$S = 7.2 \times 10^7$ N/m^2

$\rho = 7.2 \times 10^3$ kg/m^3

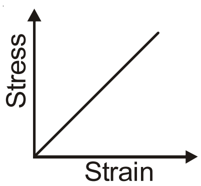
Breaking stress

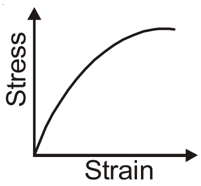
$$S = \frac{\text{Weight of wire}}{\text{Area of cross-section of wire}}$$

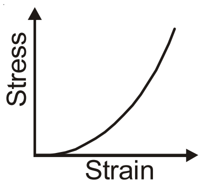
$$S = \frac{Al\rho g}{A}$$

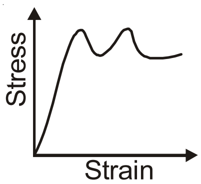
$$\Rightarrow \quad l = \frac{S}{\rho g}$$

30. Which of the following is the graph showing stress-strain variation for elastomers?

(1) 

(2) 

(3) 

(4) 

Sol. Answer (3)

31. A spherical ball contracts in volume by 0.01% when subjected to a normal uniform pressure of 100 atm. The Bulk modulus of its material is

(1) 1.01×10^{11} Nm^{-2} (2) 1.01×10^{12} Nm^{-2} (3) 1.01×10^{10} Nm^{-2} (4) 1.0×10^{13} Nm^{-2}

Sol. Answer (1)

32. Select the incorrect option

(1) The Young's modulus and modulus of rigidity exist only for solids

(2) The value of modulus of elasticity is independent of the magnitude of stress and strain

(3) The value of modulus of elasticity depends only on the nature of material of the body

(4) Only gases cannot exhibit a shearing stress

Sol. Answer (4)

Both gases and liquids (collectively called fluids) cannot exhibit shearing stress.

33. A horizontal rod is supported at both ends and loaded at the middle. If L and Y are length and Young's modulus respectively, then depression at the middle is directly proportional to

(1) L (2) L^2 (3) Y (4) $\frac{1}{Y}$

Sol. Answer (1)

34. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ± 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of ± 0.01 mm. Take g = 9.8 m/s^2 (exact). The Young's modulus obtained from the reading is

(1) $(2.0 \pm 0.3) \times 10^{11}$ N/m^2 (2) $(2.0 \pm 0.2) \times 10^{11}$ N/m^2

(3) $(2.0 \pm 0.1) \times 10^{11}$ N/m^2 (4) $(2.0 \pm 0.05) \times 10^{11}$ N/m^2

Sol. Answer (2)

$$\frac{\Delta Y}{Y} = \left(\frac{2\Delta D}{D} + \frac{\Delta\delta}{\delta}\right) \quad \because \frac{\Delta D}{D} = \frac{.01}{0.4}; \frac{\Delta\delta}{\delta} = \frac{0.05}{0.8}$$

$$\frac{\Delta Y}{2\times 10^{11}} = \left(2\times\frac{1}{40} + \frac{5}{80}\right)$$

$\Delta Y = 0.225 \times 10^{11}$

$(2.0 \pm 0.2) \times 10^{11}$ N/m^2

35. One end of a horizontal thick copper wire of length $2L$ and radius $2R$ is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is

(1) 0.25 (2) 0.50 (3) 2.00 (4) 4.00

Sol. Answer (3)

Force will be same.

Now, $\ell = \frac{FL}{AY}$

$$\frac{\ell_1}{\ell_2} = \frac{L}{R^2} \times \frac{(2R)^2}{2L} = 2$$

❑ ❑ ❑