

Introduction

Ratio and proportion is not only important for quantitative aptitude but also for data interpretation. Almost all topics in arithmetic (percentages, profit and loss, averages, mixtures, time and work, and time, speed, and distance, simple and compound interest, etc.) require the conceptual understanding of ratios. Also, it is one of the easiest topics to understand in quantitative aptitude.

Ratio

Simply, a ratio compares two similar quantities in terms of magnitude. The sign used to denote a ratio is '∶'. If the values of two quantities A and B are 20 and 30, respectively, then we can say that they are in the ratio of 2:3 (read as 2 is to 3 or 2 ratio 3).

Combining Ratios

Whenever we want to combine ratio, the objective is to make the value of the common variable same in both the ratios. Here, in the case given below, you can see that the variable Q is common in both the ratios, so we will make the value of Q same in both the ratios by multiplying the first ratio by 'x' and the second ratio by 'b'.

Let $P:Q = a:b$ or $ax:bx$ (x is the common ratio)
and $Q:R = x:y$ or $bx:by$ (b is the common ratio)
(Multiplying both elements of the ratio by the same number does not alter the ratio.)

Then $P:Q:R = ax:bx:by$

Graphically, we represent it with the help of an inverted 'N'

$$\begin{array}{lcl} P : Q & = & a : b \\ Q : R & = & x : y \end{array} \quad \begin{array}{l} \swarrow \quad \searrow \\ P : Q : R = ax : bx : by \end{array}$$

Example 1:

The ratio of the salary of Shahrukh and Salman is 3:5, and the ratio of the salary of

Salman and Aamir is 10:7. What is the ratio of the Salary of Shahrukh, Salman, and Aamir?

Solution: 6:10:7

The salary of Shahrukh : the salary of Salman
= 3:5 or 6:10

The salary of Salman : the salary of Aamir
= 10:7

Since, the salary of Salman is same in both cases. The ratio of the salaries of Shahrukh, Salman, and Aamir is 6:10:7.

Example 2:

Given that $A:B = 2:3$, $B:C = 4:3$, $C:D = 1:2$ and $D:E = 3:5$. Find $A:B:C:D:E$.

Solution: 8:12:9:18:30

Here, our objective will be to make the common variables same. So, first we will try to make B value same in both the ratios $A:B$ and $B:C$, then we will make C value same in both the ratios $B:C$ and $C:D$, and then finally we will make D value same in both the ratios $C:D$ and $D:E$.

$$A:B \quad 2:3 \text{ or } (2 \times 4:3 \times 4) \text{ or } 8:12$$

$$B:C \quad 4:3 \text{ or } (4 \times 3:3 \times 3) \text{ or } 12:9$$

$$C:D \quad 1:2 \text{ or } (9 \times 1:9 \times 2) \text{ or } 9:18$$

$$D:E \quad 3:5 \text{ or } (3 \times 6:5 \times 6) \text{ or } 18:30$$

Thus, $A:B:C:D:E = 8:12:9:18:30$.

Two Ways to Solve Problems Based on Ratios

For example, a student's marks in Physics, Chemistry, and Mathematics are in the ratio of 7:8:9 and his total marks in these three subjects are 480. Find his marks in Physics, Chemistry, and Mathematics.

Method 1

We can say that marks in Physics, Chemistry, and Mathematics are seven parts, eight parts and nine parts, respectively. Therefore, total parts will be $= 7 + 8 + 9 = 24$ parts.

$$\text{Marks in Physics} = (7/24) \times 480 = 140$$

Marks in Chemistry = $(8/24) \times 480 = 160$

Marks in Mathematics = $(9/24) \times 480 = 180$

Method 2

This method is very important and we will be using this method in solving most of the ratio-based questions.

Here, we can assume that students' marks in Physics, Chemistry, and Mathematics are 7K, 8K, and 9K, respectively.

Total marks = $7K + 8K + 9K = 24K$

Given total marks = $24K = 480$, which means $K = 20$.

Marks in Physics = $7K = 7 \times 20 = 140$

Marks in Chemistry = $8K = 8 \times 20 = 160$

Marks in Mathematics = $9K = 9 \times 20 = 180$

Comparison of Ratios

Comparison of ratios is useful for quantitative aptitude and data interpretation. Some of the useful methods to compare ratios are discussed further.

Cross Multiplication Method

In this method, we have to multiply the numerator of the first fraction with the denominator of the second fraction and the denominator of the first fraction with the numerator of the second fraction.

Example 3:

Compare $\frac{9}{13}$ and $\frac{11}{17}$

Solution:

$$\begin{array}{ccc} \frac{9}{13} & \begin{array}{c} \nearrow \searrow \\ \nwarrow \nearrow \end{array} & \frac{11}{17} \\ 9 \times 17 & & 13 \times 11 \\ 153 & & 143 \end{array}$$

Since 153 is greater than 143,

$$\left(\frac{9}{13} > \frac{11}{17} \right)$$

Denominator Comparison

The following steps are involved in this method.

Step 1: Write the given ratios as a fraction.

Step 2: Find the least common multiple LCM of the denominators of given ratios.

Step 3: Make the denominators of the given fractions equal to the value of LCM.

Step 4: After getting the same denominator for the given fractions, compare the numerators and decide which fraction is greater.

The fraction which has a larger numerator is greater in value.

Example 4:

Compare 2:3 and 4:7.

Solution: 2:3 > 4:7

Write the given ratio as a fraction.

$$2:3 = \frac{2}{3} \quad 4:7 = \frac{4}{7}$$

The LCM of the denominators (3, 7) is 21.

Make the fraction's denominator equal to 21 using a multiplier (7, 3).

$$\frac{2}{3} = \frac{2 \times 7}{3 \times 7} = \frac{14}{21}$$

$$\frac{4}{7} = \frac{4 \times 3}{7 \times 3} = \frac{12}{21}$$

Here, $14 > 12$

So, 2:3 is greater than 4:7.

Standard Results on Ratio

1. The ratio value does not change when both the numerator and the denominator are multiplied or divided by the same non-zero quantities.

$$\text{i.e., } \frac{P}{Q} = \frac{P \times x}{Q \times x} = \frac{P/a}{Q/a} \text{ etc.}$$

$$\frac{P}{Q} = \frac{P/x}{Q/x} = \frac{P/a}{Q/a}$$

2. If $\frac{a}{b}$ = original ratio,

$$\text{then } \frac{a^2}{b^2} = \text{Duplicate ratio of } \frac{a}{b}$$

$$\frac{a^3}{b^3} = \text{Triplicate ratio of } \frac{a}{b}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \text{Sub duplicate ratio of } \frac{a}{b}$$

6. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h} = \dots\dots\dots K$, then $\frac{\sqrt[3]{a}}{\sqrt[3]{b}} =$ Sub triplicate ratio of $\left(\frac{a}{b}\right)$

$$\frac{a+c+e+g+\dots\dots\dots}{b+d+f+h+\dots\dots\dots} = K$$

(Only if $b + d + f + h + \dots \neq 0$)

Forexample, $\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{3+6+9}{5+10+15} = \frac{18}{30}$

Example 5:

The salaries of Ravi and Kuber are in the ratio of 5:7. The sum of their salaries is ₹36,000. Find the salary of Kuber.

Solution: ₹21,000

Here, the ratio of their salaries is 5:7.

Kuber's salary is $\frac{7}{12}$ th of the total salary,

$$\text{Kuber's salary} = \frac{7}{12} \times 36,000 = ₹21,000.$$

Example 6:

What number shall be added to each term of the ratio of 5:8, so that it becomes equal to 19:22?

Solution: 19:22

Let x be the number to be added to each term of the ratio. Then we have, $\frac{5+x}{8+x} = \frac{19}{22}$
 $\Rightarrow 110 + 22x = 152 + 19x$

$$\therefore x = 14$$

So, 14 has to be added to each term to make it = 19:22.

Proportion

If $\frac{a}{b} = \frac{c}{d}$ or $a:b = c:d$, then we can say that a , b , c , and d are in proportion and written as $a:b :: c:d$.

Here, a and d are called extreme terms and $(a \times d)$ is called the product of extremes, b and c are called 'mean (middle) terms', and $(b \times c)$ is called the product of means.

Therefore, we can say, product of extremes = product of means.

Standard Results on Proportion

1. If $\frac{a}{b} = \frac{b}{c}$, then a , b , and c are in continued proportion. So, $b^2 = ac$.

Here, b is known as the mean proportion.

Similarly, if $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$ then a, b, c , and d are in continued proportion.

2. If $\frac{a}{b} = \frac{c}{d}$ then,

a) $\rightarrow \frac{b}{a} = \frac{d}{c}$ (Invertendo)

b) $\rightarrow \frac{a}{c} = \frac{b}{d}$ (Alternendo)

c) $\rightarrow \frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)

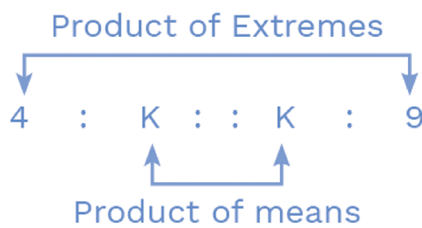
d) $\rightarrow \frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)

e) $\rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$ (Componendo and dividendo)

Example 7:

If 4, K , K , 9 are in proportion, find K .

Solution: ± 6



$$K \times K = 4 \times 9$$

$$K^2 = 36$$

$$K = \pm 6$$

Example 8:

What is the least possible natural number which must be subtracted from 11, 14, and 18 so that the resulting numbers are in continued proportion?

Solution: 2

Let the number be k , then $(11 - k)$, $(14 - k)$ and $(18 - k)$ are in continued proportion, i.e., $(11 - k):(14 - k) :: (14 - k):(18 - k)$.

Product of extremes = product of means.

$$(11 - k) \times (18 - k) = (14 - k) \times (14 - k).$$

Solving the above equation, we get $k = 2$.

Variation

If two or more quantities change each other according to a mathematical relation or condition, then they are said to be in variation.

For example, if the number of persons increases, then their time to complete a certain work decreases.

There are two types of variation:

1. Direct variation
2. Inverse variation

Direct Variation

A quantity A is said to vary directly with B, if both increase or decrease simultaneously by the same magnitude. It is expressed as $A \propto B$.

This means A is directly proportional to B. It is expressed as $A \propto B \Rightarrow A = KB$.

Where K is called the proportionality constant.

When two quantities A and B vary directly with each other, then $\frac{A}{B} = K$ (where K is the constant).

For example, when a vehicle's speed is doubled, then the total distance covered by the vehicle will also be twice for a constant time period.

Inverse Variation

Two quantities are in inverse variation, when one quantity increases, and the other quantity decreases.

It is expressed as $A \propto \frac{1}{B} \Rightarrow A = \frac{K}{B}$

where K is called proportionality constant.

When two quantities A and B vary inversely with each other, then product of A and B is constant.

For example, if the number of women doing a certain work increase, then the time taken

to do the work decrease. Conversely, as the number of women decreases, the time taken to do the work increase.

Joint Variation

A quantity A varies directly with the product of two variables, B and C.

$$A \propto (B \times C)$$

$A = K (B \times C)$, where K is a proportionality constant.

Example 9:

If x varies directly to y^2 and $x = 16$ when $y = 2$, find x when $y = 6$.

Solution: 144

Here, $x \propto y^2 \Rightarrow x = Ky^2$, where K is constant

Since, $x = 16$, when $y = 2$

$$16 = K (2)^2$$

$$\Rightarrow K = 4$$

When $y = 6$

$$x = Ky^2 = (4) \times 6^2$$

$$\Rightarrow x = 144$$

Example 10:

Variable A varies inversely with B. If $B = 2$, then $A = 4$. Find A when $B = 8$.

Solution: 1

$$\text{Here, } A \propto \frac{1}{B} \Rightarrow A = \frac{K}{B}$$

Since, $A = 4$, when $B = 2$

$$4 = \frac{K}{2}$$

$$K = 8$$

When, $B = 8$

$$A = \frac{K}{B} = \frac{8}{8} = 1$$

Example 11:

Variable A varies directly with the product of B and C. $A = 16$ when $B = 2$ and $C = 4$. Find A when $B = 4$ and $C = 6$.

Solution: 48

Here, $A \propto (B \times C)$

So, $A = K (B \times C)$

where K is a proportionality constant.

Since, $A = 16$ when $B = 2$ and $C = 4$

$$16 = K (2 \times 4)$$

$$\Rightarrow K = 2$$

When $B = 4$ and $C = 6$

then, $A = K (B \times C) = 2 (4 \times 6)$

$$\Rightarrow A = 48$$

Application of Ratio, Proportion, and Variation

Partnership

A partnership is a formal agreement between two or more parties to manage and operate a business and to share its profit. There are two types of partners in a business:

1. Sleeping partner or silent partner: A partner who just invests his/her money and is not involved in the day-to-day operation of the business.
2. Working partner: A partner who invests his/her money and is also involved in the business's day-to-day operations.

Rules of Partnership

1. When partners invest the same amount for a different time period, then the profits are shared in the ratio of their respective time period of investment.
2. When partners invest different amounts for the same time, then the profits are shared in the ratio of their respective investments.
3. When partners invest different amounts for different times, profits are shared in the ratio of the product of investment and the time period of investment.
(Profit's share = investment $\times$ time).

Example 12:

A and B started a business by investing an amount of ₹5,000 and ₹6,000, respectively.

Find the share of A, out of the annual profit of ₹33,000.

Solution: 15,000

Ratio of share of A and B

$$5,000:6,000 = 5:6$$

$$\text{Share of A} = \frac{5}{11} \times 33,000 \Rightarrow ₹15,000$$

Example 13:

A and B started a business by investing the same capital. At the end of the year, they shared the profit in the ratio of 4:3. If A invested his capital for the whole year, for how many months did B invest his capital?

Solution: 9

We know that when partners invest the same amount for a different time, the profits are shared in the ratio of their respective time period of investment.

The ratio of the share of A and B = 4:3.

A invested his capital for 12 months.

According to the ratio, A invested the money for $4K$ months, $4K = 12$ or $K = 3$.

B invested his capital for $3K$ months, $3K = 3 \times 3 = 9$ months.

Example 14:

The ratio of the present ages of Ravi and Kavi is 15:8. Three years ago, the ratio of their ages was 2:1. Find the present age (in years) of Ravi.

Solution: 45

Let the present age of Ravi and Kavi be 15 years and 8 years, respectively.

3 years ago, their ages were $(15x - 3)$ and $(8x - 3)$.

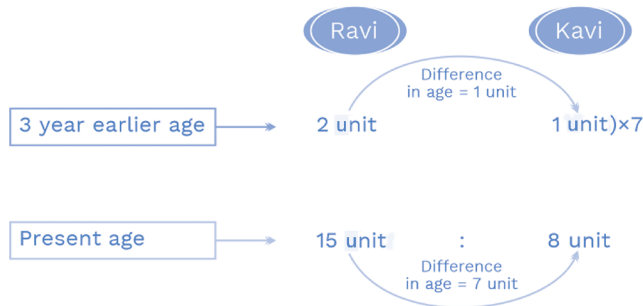
$$\text{So, } \frac{15x-3}{8x-3} = \frac{2}{1}$$

$$\Rightarrow 15x - 3 = 16x - 6$$

$$\therefore x = 3$$

Present age of Ravi = $15x = 15 \times 3 = 45$ years.

Alternately:



We know that difference between the ages of two people remains constant.



Present age of Ravi = 15 unit = $15 \times 3 = 45$ years.

Example 15:

The ratio of Ajeet's present age and his mother's present age is 11:18. The difference in their age is 14 years. The ratio of their ages 6 years from now will be.

Solution: 2:3

Let their ages be $11x$ and $18x$.

$$18x - 11x = 14$$

$$7x = 14$$

$$\therefore x = 2$$

So, their present ages are 22 years and 36 years.

Their ages after 6 years will be 28 years and 42 years, respectively. The ratio of the ages of Ajeet and his mother after 6 years will be

$$28:42 = 2:3$$

Incomes, Expenses, and Savings

Example 16:

Who among Chulbul and Bulbul saves more money if the ratio of their incomes and their expenses are 3:5 and 1:3, respectively?

Solution: Cannot be determined

Let's understand the question with the help of two cases.

Case 1:

	Income	Expenses	Savings
Bulbul	3	1	2
Chulbul	5	3	2

In this case, savings of both of them are equal.

Case 2:

	Income	Expenses	Savings
Bulbul	300	150	150
Chulbul	500	450	50

In this case, Bulbul saves more than Chulbul.

Case 3:

	Income	Expenses	Savings
Bulbul	300	1	299
Chulbul	500	3	497

In this case, Chulbul saves more than Bulbul.

Hence, we cannot determine who is saving more.

Example 17:

The income of A and B are in the ratio of 4:5. If the expenditure of both of them is ₹35,000 each, then the ratio of savings of A and B is 1:3. Find the respective income (in ₹) of A and B.

Solution: ₹40,000 and ₹50,000

Let the income of A and B is $4x$ and $5x$, respectively.

We know that savings = Income - Expenses

$$\text{Savings of A} = 4x - 35,000$$

$$\text{Savings of B} = 5x - 35,000$$

The ratio of their savings is 1:3.

$$\frac{4x - 35,000}{5x - 35,000} = \frac{1}{3}$$

$$\Rightarrow 12x - 1,05,000 = 5x - 35,000$$

$$7x = ₹70,000$$

$$x = ₹10,000$$

Income of A = $4x = 4 \times 10,000 = ₹40,000$.

Income of B = $5x = 5 \times 10,000 = ₹50,000$.

Denomination of Coins

The denomination is a proper description of a currency amount for coins and banknotes. In any currency, there is a main unit and a sub unit; that is a fraction of the main unit. In our country, rupees is the main unit and paisa is the sub unit.

₹ = 100 paisa.

Example 18:

A bag contains two types of coins, ₹1 coins and 50 paisa coins. If the total value of each kind of coins is the same, what is the total amount (in ₹) in the bag? (It is known the total number of coins is 153.)

Solution: 102

Here, it is given that the total value of the coins of each type is the same. This means if the value of ₹ 1 coins is 'x', the value of 50 paise coins will also be 'x'.

So, number of ₹1 coins = x

Number of 50 paisa coins = $2x$

Total number of coins = 153

$$x + 2x = 153$$

$$3x = 153$$

$$x = 51$$

Total amount in the bag is = $2x = 2 \times 51 = 102$.

Practice Exercise – 1

Level of Difficulty – 1

1. P and Q together have ₹1,210. If $\frac{4}{15}$ of P's amount is equal to $\frac{2}{5}$ of Q's amount, how much amount (in ₹) does Q have?
2. Two quantities are, respectively, 30% and 60% more than a third quantity. What is their ratio?
3. The ratio of dogs and cats in a house is 3:2 and ratio of cats and birds in a house is 4:5. Find the ratio of dogs, cats, and birds in the same house.
4. ₹3,105 is to be divided among P, Q, and R in the ratio of $\frac{2}{3}:\frac{3}{4}:\frac{1}{2}$, respectively. How much amount (in ₹) will Q get?
5. Find the compounded ratio of (2:3), (5:11), and (11:2).

Level of Difficulty – 2

6. The value of a diamond varies directly with the square of its weight. A diamond broke into three pieces whose weights were in the ratio of 32:24:9. The loss caused due to the breakage was ₹25.44 lakhs. Find the initial value (in ₹) of the diamond.
7. Raghav distributed a certain amount of money among his friends Ankur, Ashwani, and Neeti in the ratio of 3:4:5. Neeti distributed the total amount with her among her friends Dheeraj, Sunil, and Surendra in the ratio of 6:7:8. If Dheeraj gets an amount of 3,000, find the amount (₹) that Ashwani got from his friend Raghav.
8. The ratio of income to the expenditure of P is 5:3 and that of Q is 7:6. If the savings of P are twice that of Q, then what is the

ratio of the total income of P and Q to the total expenditure of P and Q?

9. During Navratri, every devotee offers fruits to the orphans. Every orphan received bananas, oranges, and mangoes in the ratio of 3:2:7. The weight of a mango is 24 g and the weight of a banana and an orange are in the ratio of 4:5 and the weight of an orange is 150 g. Find the ratio of all the three fruits in terms of weight that an orphan gets.
10. ₹4,199 was intended to be divided among P, Q, R, and S in the ratio of 2:3:5:7, respectively. By mistake, the money got divided in the reciprocal of the intended ratio. Find the gain of P (in ₹) due to this mistake.
(A) 1,235
(B) 741
(C) 1,291
(D) None of the above.

Level of Difficulty – 3

11. Let A, B, C, D, and E be positive integers such that $\frac{2}{3}A = \frac{3}{4}B = \frac{5}{6}C$ and $\frac{7}{8}B = \frac{9}{10}D = \frac{11}{12}E$. Find the minimum value of $A + B + C + D + E$.
12. The number of mangoes in three containers is in the ratio of 3:4:5. In which ratio the number of mangoes in the first two containers must be increased, so the new ratio becomes 5:4:3?
(A) 5:4
(B) 3:1
(C) 2:1
(D) 4:1
13. Pyare Lal Ji sells gulab-jamun at ₹15 per kg. A gulab-jamun is made up of flour and sugar in the ratio of 5:3. The ratio of the price of sugar and flour is 7:3 (per kg).

He earns $66\frac{2}{3}\%$ profit. What is the cost price per kg (in ₹) of sugar?

- 14.** In an examination of 1,260 students, the ratio of the number of boys to the number of girls is 3:4. The ratio of number of students who passed the exam to those who failed the exam is in the ratio of 11:3. Among the girls, the ratio of the number of students who passed to those who failed the exam is in the ratio of 11:4. Find the number of boys who passed.
- (A) 462
(B) 528
(C) 990
(D) 270

- 15.** Three employees of an IT company started a business. Employee 1 invested ₹30,000 for 5 months. After this, he again invested ₹20,000 extra for the rest of the year. Employee 2 invested ₹60,000 after 6 months of the start of the business. Employee 3 invested ₹50,000 for the first 8 months, then he took out ₹20,000. Find the share (in ₹) of the second employee in the total profit, if the total profit at the end of 1 year is ₹7,65,900.

Solutions

1. 484

$$\text{Since, } \frac{4}{15}P = \frac{2}{5}Q$$

$$P = \frac{3}{2}Q$$

$$\frac{P}{Q} = \frac{3}{2} \Rightarrow (P : Q = 3 : 2)$$

$$Q\text{'s share} = \left(\frac{2}{5} \times 1,210 \right) = ₹484$$

2. 13:16

Let the third number be $100x$

Then, first number = 130% of $100x = 130x$

and second number = 160% of $100x = 160x$

$\therefore$ Ratio of first two numbers
= $130x:160x = 13:16$.

3. 6:4:5

Ratio of dogs and cats = 3:2

Ratio of cats and birds = 4:5

In both the ratios, the number of cats
must be the same.

Ratio of dogs and cats = $(3:2) \times 2 = 6:4$

The ratio of cats and birds = 4:5

$\therefore$ Ratio of dogs:cats:birds = 6:4:5

4. 1,215

$$\text{Here, } P:Q:R = \frac{2}{3} : \frac{3}{4} : \frac{1}{2}$$

To simplify this ratio, take LCM of (3, 4, 2) = 12

Multiply the ratio by 12

$$P:Q:R = \frac{2}{3} \times 12 : \frac{3}{4} \times 12 : \frac{1}{2} \times 12$$

$$P:Q:R = 8:9:6$$

So, let's assume $P = 8K$, $Q = 9K$ and
 $R = 6K$

$$P + Q + R = 3,105$$

$$8K + 9K + 6K = 3,105$$

$$K = \frac{3,105}{23} = 135$$

$$Q\text{'s share} = 9K = 9 \times 135 = ₹1,215$$

5. 5:3

We know that compounded ratio of two
simple ratios

$$a:b \text{ and } c:d = \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

Similarly, the compounded ratio of (2:3),
(5:11), and (11:2) is

$$= \frac{2}{3} \times \frac{5}{11} \times \frac{11}{2} = 5:3$$

6. ₹42.25 lakhs

Let the weights of three pieces be $32x$,
 $24x$, and $9x$, respectively.

Let the value of the diamond be denoted
by 'v'

$$v \propto w^2 \quad (\text{where } w \text{ is the weight})$$

$$v = kw^2 \quad (\text{where } k \text{ is constant})$$

Total value of the broken pieces

$$= k\{(32x)^2 + (24x)^2 + (9x)^2\} = 1,681 kx^2$$

Value of the diamond initially

$$= k(65x)^2 = 4,225 kx^2$$

Given that $4,225 kx^2 - 1,681 kx^2 = 2,544 kx^2$

Now given that $2,544 kx^2 = ₹25.44 \text{ lakhs}$

$$\Rightarrow kx^2 = 1,000$$

The initial value of the unbroken diamond

$$= (4,225) \times 1,000 = ₹42.25 \text{ lakhs.}$$

7. 8,400

Let the amounts that Ankur, Ashwani,
and Neeti received be $3x$, $4x$, and $5x$,
respectively.

Let the amount that Dheeraj, Sunil, and
Surendra received be $6y$, $7y$, and $8y$,
respectively.

The amount with Neeti

$$= 5x = 6y + 7y + 8y = 21y$$

Given, $6y = 3,000$

$$y = 500$$

$$x = \frac{21}{5}y = \frac{21}{5} \times 500 = 2100$$

The amount that Ashwani received

$$= 4x = 4 \times 2,100 = ₹8,400.$$

8. 4:3

Let the income of P be $5x$. Then the ex-
penditure of P will be $3x$.

$$\therefore \text{ savings of P} = 5x - 3x = 2x$$

Let the income of Q = $7y$. Then the ex-
penditure of Q will be $6y$.

$\therefore$ saving of Q = $7y - 6y = y$
 Given savings of P is double that of Q
 $\therefore 2x = 2 \times y \Rightarrow x = y$
 $\therefore$ The ratio of total earning of P and Q to the total expenditure of P and Q

$$= \frac{5x+7y}{3x+6y} = \frac{12x}{9x} = 4:3$$

9. 30:25:14

The ratio of the weight of a banana and an orange = 4:5.

So, let's assume the weight of a banana and an orange be $4x$ and $5x$, respectively. Weight of an orange is 150 g.

$$\therefore 5x = 150$$

$$x = 30$$

Weight of a banana = $4x = 4 \times 30 = 120$ g

The ratio of number of fruits = 3:2:7

The ratio of the weight of fruits = 120:150:24

The ratio of fruits (combined) weight
 $= 3 \times 120 : 2 \times 150 : 7 \times 24$
 30:25:14

10. (C)

Intended ratio P:Q:R:S = 2:3:5:7

P's share by intention

$$= (2/17) \times 4,199 = ₹494$$

Mistaken ratio would be

$$= P:Q:R:S = 1/2:1/3:1/5:1/7$$

Multiply by all the elements of the ratio by LCM (210) of 2, 3, 5, and 7.

We will get P:Q:R:S = 105:70:42:30.

Now P's share = $(105/247) \times 4,199 = ₹1,785$

So, gain of P = $1,785 - 494 = ₹1,291$

Hence, option (C) is the correct answer.

11. 19,609

$$\frac{2}{3}A = \frac{3}{4}B = \frac{5}{6}C$$

Multiplying by 12 we will get

$$8A = 9B = 10C$$

$$\frac{7}{8}B = \frac{9}{10}D = \frac{11}{12}E$$

Multiplying by 120 we will get

$$105B = 108D = 110E$$

Now $8A = 9B = 10C$ ----- (i)

and $105B = 108D = 110E$ ----- (ii)

Multiplying equation (i) by 35 and (ii) by 3 we will get

$$280A = 315B = 350C \text{ and } 315B = 324D = 330E$$

OR

$$280A = 315B = 350C = 324D = 330E = K$$

(Let's assume) ----- (iii)

Given that A, B, C, D, and E are integers, which means that K must be divisible by 280, 315, 350, 324, and 330.

So minimum value of $K = \text{LCM}(280, 315, 350, 324, \text{ and } 330) = 1,247,400$ ----- (iv)

From equations (iii) and (iv), minimum values of A, B, C, D, and E will 4,455, 3,960, 3,564, 3,850, and 3,780, respectively.

Hence, minimum value of

$$A + B + C + D + E$$

$$= 4,455 + 3,960 + 3,564 + 3,850 + 3,780 = 19,609$$

12. 2:1

Here, the ratio of mangoes in three containers is:

$$C_1:C_2:C_3 = 3:4:5$$

So, $C_1 = 3x$, $C_2 = 4x$ and $C_3 = 5x$

After an increment in the number of mangoes in the first two containers, the ratio of mangoes in three containers becomes

$$C_1:C_2:C_3 = 5:4:3$$

So, $C_1 = 5y$, $C_2 = 4y$ and $C_3 = 3y$

The number of mangoes remains constant in container 3.

$$\therefore 5x = 3y$$

$$x = \frac{3y}{5}$$

Hence,

$$3x : 4x : 5x \quad C_1 : C_2 : C_3$$

$$\frac{9y}{5} : \frac{12y}{5} : \frac{15y}{5} \Rightarrow (9y : 12y : 15y)$$

$$5y : 4y : 3y \Rightarrow 25y : 20y : 15y$$

Increase in the number of mangoes in the first container (C_1) = $16y$.

Increase in the number of mangoes in the second container (C_2) = $8y$.

The required ratio = $16y:8y = (2:1)$.

13. ₹14

$$\text{Here, } 66\frac{2}{3}\% = \frac{2}{3}$$

Hence, $66\frac{2}{3}\%$ profit means there is a

profit of 2 units when CP is 3 units.

So, SP = 5 units

5 units = ₹15

1 unit = ₹3

CP of 1 kg of gulab-jamun = 3 units
= $3 \times 3 = ₹9$

Quantity of flour in 1 kg of gulab-jamun
= $(5/8) \times 1,000 = 625$ g.

Quantity of sugar in 1 kg of gulab-jamun
= $(3/8) \times 1,000 = 375$ g.

Given that cost prices of flour and sugar are in the ratio of 3:7, let's assume the cost price per gram of flour = 3K and cost price per gram of sugar = 7K.

So, the cost of flour = $625 \times 3K = 1,875K$
and the cost of sugar = $375 \times 7K = 2,625K$.

Cost per kg of gulab-jamun
= $1,875K + 2,625K = 4,500K = ₹9$,
which means $K = 1/500$.

Thus, cost price per kg of sugar = $7,000K$
= $7,000 \times (1/500) = ₹14$.

Note: We assume cost price per gram of sugar = 7k, so cost price per kg (1,000 g) of sugar will be $7 \times 1,000 = 7,000K$.

14. (A)

Given:

The total number of students = 1,260.

The number of boys = $1,260 \times \frac{3}{7} = 540$.

The number of girls = $1,260 - 540 = 720$.

The total passed students in the examination = $1,260 \times \frac{11}{14} = 990$.

The total failed students in the examination = $1,260 - 990 = 270$.

Total number of girls who passed the examination = $\frac{11}{15} \times 720 = 528$.

The number of boys who have passed the examination = $990 - 528 = 462$.

Hence, option (A) is the correct answer.

15. 1,99,800

Employee 1 : Employee 2 : Employee 3

I × T =	30,000 × 5	:	60,000 × 6	:	50,000 × 8
	+ 50,000 × 7				+ 30,000 × 4
	50	:	36	:	52
Ratio of profit	25	:	18	:	26

Since total profit in the unit is (25 + 18 + 26) units, but in actuality, its total value is ₹7,65,900.

Thus, 69 unit = ₹7,65,900

1 unit = $\frac{7,65,900}{69} = ₹11,100$

18 units is the profit share of the second employee.

18 unit = $18 \times 11,100 = ₹1,99,800$.

Practice Exercise – 2

Level of Difficulty – 1

- Given that y is directly proportional to 8 less than the 5th power of x . Also, $y = 47$, when $x = 3$. Find x when $y = 4.8$.
- If $(2x + y) : (3x + 3y) =$ duplicate ratio of 2:3, then find $x:y$.
- An IAS aspirant took five papers in the exam, where the total maximum marks were the same for each paper. His marks in these papers were in the ratio of 6:7:8:9:10. In all the papers together, the candidate obtained 80% of the total maximum marks of all the subjects together. The number of papers in which he got more than 72% marks are:
(A) 1
(B) 2
(C) 3
(D) 4
- The mass of the liquid varies directly with its volume. The mass of a liquid is 12 g when its volume is 15 cm^3 . Find the mass of the liquid (in g) if its volume is 20 cm^3 .
- If $a + b - c : b + c - a : a + c - b = 7:8:9$, then find the value of $c:b:a$.
(A) 15:16:17
(B) 16:15:17
(C) 17:16:15
(D) 17:15:16
- X, Y, and Z divides ₹5,000 in such a way that if we take away ₹180 and ₹220 from X and Y, respectively, and give ₹200 to Z, then the new amounts with X, Y, and Z will be in the ratio of 4:5:7. Find Y's share in the original amount.
(A) ₹1,380
(B) ₹1,820
(C) ₹1,720
(D) ₹1,500
- The ratio of the number of chocolates with P, Q, R, and S is 6:7:9:10, respectively. How many of them has more than 25% of the total number of chocolates?
(A) 1
(B) 2
(C) 3
(D) 4
- In an exam, Ina's score was 37.5% of the sum of the scores of Mina and Dika. After a review, the score of each of them increased by 11. The reviewed scores of Ina, Mina, and Dika were in the ratio of 2:3:2. Mina's reviewed score was how much more than Dika's reviewed score?
(A) 22
(B) 18
(C) 15
(D) 11
- Amit, Sumit, Jimit, and Nimit have certain number of pencils in the ratio of $1/5:1/7:1/9:1/11$, respectively. What is the minimum number of pencils they must have altogether?
(A) 944
(B) 472
(C) 1,888
(D) 3,776
- If $a:b = 4:7$ and $b:c = 15:19$, then what is the value of $\frac{(c+a)}{3a+b-c}$?
(A) 11:7
(B) 117:113
(C) 152:193
(D) 193:152

Level of Difficulty – 2

- The ratio of incomes of A, B, C, and D are in the ratio of 5:6:7:8, respectively, and their expenditures are in the ratio of 9:10:11:12, respectively. If B saves one-third of his

income, then A's saving is how much per cent less than the saving of D?

- (A) 37.5%
- (B) 50%
- (C) 56.25%
- (D) 68.75%

12. Rohan divides all the chocolates with him, among his four sons A, B, C, and D such that five times the number of chocolates with A is equal to eight times the number of chocolates with B, three times the number of chocolates with B is equal to four times the number of chocolates with C and four times the number of chocolates with C is equal to five times the number of chocolates with D. What could be the minimum possible total number of chocolates with Rohan?

13. In an online test of data analytics, John and Joe scored in the ratio of 13:16. Since there were some wrong questions in the test, after an appeal to the board members, their scores increased by the same amount and their new scores were in the ratio of 55:64. Find the ratio of John's new score to that of his original score.

- (A) 55:39
- (B) 49:47
- (C) 57:44
- (D) 43:29

14. The value of a diamond varies directly with cube of its weight. The diamond broke into four pieces, whose weights were in the ratio of 1:2:3:4. When all the pieces were sold individually, the merchant got ₹65,700 less as compared to the value of the original unbroken diamond. Find the original value (in thousand ₹) of the diamond.

15. Rajan's present age is thrice his nephew's present age and four times his niece's present age. A few years from now, when Rajan's age will become twice his nephew's age, what will be the ratio of Rajan's age to his niece's age at that time?

- (A) 16:7
- (B) 4:7
- (C) 4:11
- (D) 12:11

16. The ratio of a two-digit number and the number formed by reversing the digits is 13:31. How many such numbers are possible?

- (A) 0
- (B) 2
- (C) 3
- (D) 6

17. A rope is cut into two parts so that the ratio of its length to the length of the larger part is the same as the ratio of the length of the larger to the smaller part. Find the ratio of the larger to the smaller part. (Take $\sqrt{5} = 2.2$)

- (A) 6/5
- (B) 7/5
- (C) 8/5
- (D) 9/5

18. Total cost of a wedding function is consisting of fixed cost and variable cost. Variable cost varies directly with the square of the number of guests attending the wedding. Total cost of function for 80 guests is ₹83,800 and the total cost of function for 120 guests is ₹1,19,800. Find the fixed cost (in ₹) of the wedding function.

19. In an innings of a cricket match, the ratio of the number of runs scored by P to the runs scored by Q is 1/5:1/9 and also, the ratio of the number of runs scored by Q to that scored by R is 1/5:1/9. If P scored 168 runs more than R, find the total number of runs scored by all the three players.

20. The Lucknow–Chennai express can move at 60 km/h without any wagons attached, and the decrement in speed directly varies with the square root of the number of wagons attached. If it is known that

with nine wagons, the train's speed is 45 km/h. What is the greatest number of wagons with which the train can just move?

Level of Difficulty – 3

- 21.** Virendra, Surya, and Pratap have started an Artificial Intelligence micro task-based company. Virendra invests ₹20 lakhs, Surya ₹14 lakhs, and Pratap invests ₹16 lakhs. In the first year, the company has made ₹6,60,000 profit. Some part of the profit is shared between Surya and Pratap in the ratio of 3:5, respectively. The remaining profit is divided among all the three in the ratio of their investments. The overall amount received by Pratap is $\frac{13}{10}$ times of the amount that Virendra received. Find the amount received by Surya.

- (A) ₹2,00,000
- (B) ₹1,50,000
- (C) ₹1,60,000
- (D) ₹1,20,000

- 22.** Suppose A, B, C, D, and E are five companies. The profits made by A and B, B and C, C and D, and D and E are in the ratio of 2:3, 4:5, 2:1, and 3:4, respectively. If the profit made by company D is ₹432 crores less than the profit made by company B, then the total profit (in crore ₹) made by companies A and E together is:

- (A) 512
- (B) 1,240
- (C) 1,680
- (D) 1,728

- 23.** L and M started a business. They made a certain annual profit. M being a working partner, received 20% of the annual profit as his salary and the remaining profit was divided in the ratio of their investments. If the entire profits were divided in the ratio of their investments, L

would have received ₹12,000 more than he actually got. Find the actual amount received by L.

- (A) ₹48,000
- (B) ₹42,000
- (C) ₹36,000
- (D) ₹30,000

- 24.** If $\frac{9A - 18B + 8C}{9A - 18B - 8C} = \frac{9A + 18B - 8C}{9A - 18B + 8C}$, then

which of the following is true?

- (A) $9AB - 18B^2 = 8AC + 8BC$
- (B) $9AB + 8BC = 8AC + 18B^2$
- (C) $8AB + 18B^2 = 9AC + 8BC$
- (D) $9AB - 18B^2 = 8BC - 8AC$

- 25.** The ratio of the ages of the wife and husband among two couples are 2:3 and 4:5, respectively. The oldest person is 40 years old. The difference between the ages of any two persons cannot be more than 20. Find the age (in years) of the youngest person, given that any male person is not the second oldest. Also, the age of every person is an integer.

- (A) 20
- (B) 22
- (C) 24
- (D) Data insufficient

Direction for Questions 26 to 27: Read the following information carefully and answer the questions.

In the game of 'Free Cells', three participants Bibo, Joe, and Paes, start playing with amounts in the ratio of 3:4:6. They bet as follows: if Bibo wins a game, he gets an amount equal to what he has at that moment from each of the other two. If Joe wins, he gets an amount equal to one-fourth of what he has at that moment from each of the other two. If Paes wins, he gets an amount equal to one-fifth of what he has at that moment from each of the other two. In a series of four games, the order of the winners is Joe, Paes, Bibo, and Joe, respectively. At the end of the fourth game, Paes loses ₹20.

- 26.** After the third game, what is the ratio of the amounts with Bibo and Paes?
(A) 2:1
(B) 1:3
(C) 1:2
(D) 3:1
- 27.** Who among Bibo, Joe, and Paes did not make a loss on his/her investment?
- 28.** Four natural numbers are in proportion. The sum of the means is 28 and the sum of the extremes is 32. If the positive difference between the extremes is four times the positive difference between the means, find the difference between the extremes.
- 29.** A said to B, "I am 4 times as old as you were when I was one-third of your age 2 years from now". If the sum of their ages is 46 years, find A's age (in years).
- 30.** Two classes X and Y consist of American and Russian students. Each American in class X has five times the number of Americans as classmates as Russians. In class Y, each American has five times the number of Russians as classmates as Americans. Every Russian in class X has seven times the number of Americans as classmates as Russians. Class X and class Y have the same number of students. How many students are there in class Y?

Solutions

1. 2

$y \propto (x^5 - 8)$ (given)

Let $y = k(x^5 - 8)$ (i)

where k is the proportionality constant.

Put $x = 3$ and $y = 47$ in (i)

$$47 = k(243 - 8)$$

$$\Rightarrow k = \frac{47}{235} = \frac{1}{5}$$

$$\Rightarrow y = \frac{1}{5}(x^5 - 8)$$

$$\Rightarrow x^5 = 5y + 8$$

$$x = \sqrt[5]{5y + 8}$$

$$\text{Put } y = 4.8: x = \sqrt[5]{24 + 8} = \sqrt[5]{32} = 2$$

2. 1:2

$$\frac{2x + y}{3x + 3y} = \left(\frac{2}{3}\right)^2$$

$$\frac{2x + y}{3x + 3y} = \frac{4}{9}$$

$$18x + 9y = 12x + 12y$$

$$6x = 3y$$

$$x:y = 1:2$$

3. (C)

Let's assume that each paper's maximum marks are 100. So, five papers' total maximum marks will be 500. As given in the question, that student got 80% of the total maximum marks = 80% of 500 = 400. Let marks obtained by IAS aspirants in five papers are 6K, 7K, 8K, 9K, and 10K.

Total marks in all five papers

$$= 6K + 7K + 8K + 9K + 10K = 40K.$$

According to the question

$$40K = 400, \text{ which means } K = 10.$$

So, marks obtained by the student in five papers will be $6 \times 10 = 60$, $7 \times 10 = 70$, $8 \times 10 = 80$, $9 \times 10 = 90$ and $10 \times 10 = 100$.

Thus, in three papers, he got more than 72% of the marks.

Hence, option (C) is the correct answer.

4. 16

Let the mass of liquid is M and volume be V .

$M \propto V \Rightarrow M = KV$ where K is a constant

$$\frac{M_1}{M_2} = \frac{V_1}{V_2} \text{ taking } \begin{matrix} M_1 = 12 \text{ g} & M_2 = x \\ V_1 = 15 \text{ cm}^3 & V_2 = 20 \text{ cm}^3 \end{matrix}$$

$$\frac{12}{x} = \frac{15}{20}$$

$$x = 16 \text{ g}$$

5. (D)

$$\text{Let } a + b - c = 7K \quad \dots(i)$$

$$b + c - a = 8K \quad \dots(ii)$$

$$a + c - b = 9K \quad \dots(iii)$$

Adding equations (i), (ii), and (iii), we get

$$a + b + c = 24K \quad \dots(iv)$$

Now, subtracting equations (iv) from (i)

$$\text{we will get } c = \frac{17}{2}K$$

Subtracting equations (iv) from (ii) we will get $a = 8K$

Subtracting equations (iv) from (iii) we will get $b = \frac{15}{2}K$

$$\text{Now, } c:b:a = 17/2:8:15/2 = 17:16:15.$$

Hence, option (D) is the correct answer.

6. (C)

Total amount = ₹5,000

₹180 and ₹220 are taken away and ₹200 are added, so the net change is $180 + 220 - 200 = ₹200$.

So, the balance amount = ₹4,800.

Now, Y's share in this can be calculated using ratio of 4:5:7.

Therefore, Y's share

$$= (5/16) \times 4,800 = 1,500$$

But ₹1,500 is share from ₹4,800

So, Y's share in the original amount

$$= ₹1,500 + 220 = ₹1,720.$$

Hence, option (C) is the correct answer.

7. (B)

Let us say we have total $6 + 7 + 9 + 10 = 32$ chocolates.

$$\text{Chocolates with P} = (6/32) \times 32 = 6.$$

$$\text{Chocolates with Q} = (7/32) \times 32 = 7.$$

Chocolates with R = $(9/32) \times 32 = 9$.
 Chocolates with S = $(10/32) \times 32 = 10$.
 Now 25% of total chocolates = 8.
 We can clearly see that only R and S have more than eight chocolates.
 Hence, option (B) is the correct answer.

8. (A)

Let's assume the initial scores of Ina, Mina, and Dika as A, B, and C, respectively. Revised scores of Ina, Mina, and Dika will be $A + 11$, $B + 11$ and $C + 11$, respectively, and given that they are in the ratio of 2:3:2, respectively.
 So, $(A + 11)/(C + 11) = 2/2 = 1$, solving which we will get $A = C$.
 And $(A + 11)/(B + 11) = 2/3$, solving which we will get $2B - 3A = 11$ ----- (i)
 Also, given that $A = 37.5\%$ of $(B + C) = 3/8 (B + C)$ ----- (ii)
 On solving equations (i) and (ii) we will get $B = 55$ and $A = 33$.
 As $A = C$, means $C = 33$.
 Required answer = $55 - 33 = 22$.
 Hence, option (A) is the correct answer.

9. (C)

Let the number of pencils with Amit, Sumit, Jimit, and Nimit be a , b , c , and d , respectively.
 Thus, $a:b:c:d$
 $= 1/5:1/7:1/9:1/11 = 693:495:385:315$
 Let $a = 693k$, $b = 495k$, $c = 385k$, and $d = 315k$ (where 'k' is a natural number).
 Thus, the total number of pencils with all of them = $a + b + c + d = 1,888k$.
 For the total number of pencils to be minimum, 'k' must be 1.
 Thus, the minimum number of pencils they have altogether = 1,888.
 Hence, option (C) is the correct answer.

10. (D)

Since	a:b	b:c
	$[4:7] \times 15$	$7 \times [15:19]$
	$[60:105]$	$[105:133]$
	a : b : c	
	60 : 105 : 133	

Let $a = 60x$, $b = 105x$, $c = 133x$

$$\frac{(c + a)}{3a + b - c} = \frac{133x + 60x}{180x + 105x - 133x} = \frac{193x}{152x} = \frac{193}{152}$$

Hence, option (D) is the correct answer.

11. (C)

Let's assume the incomes of A, B, C, and D are $5y$, $6y$, $7y$, and $8y$ respectively, and the expenditures are $9z$, $10z$, $11z$, and $12z$ respectively.
 According to the question B's saving = $1/3$ of $6y = 2y$.
 We know that
 income = expenditure + savings
 $6y = 10z + 2y$, which means $4y = 10z$ or $y = 2.5z$
 D's savings
 $= 8y - 12z = 8 \times 2.5z - 12z = 8z$
 A's savings
 $= 5y - 9z = 5 \times 2.5z - 9z = 3.5z$
 So, we need to find out A's saving is how much percent less than D's saving
 $= (4.5z/8z) \times 100 = 56.25\%$
 Hence, option (C) is the correct answer.

12. 89

The ratio of number of chocolates with A and B = 8:5.
 The ratio of number of chocolates with B and C = 4:3.
 The ratio of number of chocolates with C and D = 5:4.
 After combining ratios, we will get the ratio of the number of chocolates with A, B, C, and D as 32:20:15:12.
 Now, let's assume the number of chocolates with A, B, C, and D are $32k$, $30k$, $15k$, and $12k$, respectively.
 So, total number of chocolates distributed by Rohan.
 $= 32k + 30k + 15k + 12k = 89k$
 Hence, the minimum number of chocolates with Rohan can be 89 (putting $k = 1$).

13. (A)

Let the original score of John and Joe are $13x$ and $16x$.
 After an appeal, their scores are increased to the same extent.
 Let 'a' marks are increased for both.

According to the question

$$\therefore \frac{13x + a}{16x + a} = \frac{55}{64}$$

$$832x + 64a = 880x + 55a$$

$$48x = 9a$$

$$a = 48x/9$$

Now we have to find the ratio of new scores of John to the original score of John.

$$\frac{13x + a}{13x} = \frac{13x + \frac{48}{9}x}{13x} = \frac{117x + 48x}{13x}$$

$$\frac{165x}{117x} = \frac{165}{117} = \frac{55}{39}$$

Hence, option (A) is the correct answer.

14. 73

Let's assume that the weight of the pieces are 1, 2, 3, and 4 g. So, the weight of the unbroken diamond would be 10 g.

According to the question

Value $\propto$ (weight)³ or value = K(weight)³

Values of the pieces and unbroken diamond will be K (1)³, K (2)³, K (3)³, K (4)³, and K (10)³ respectively.

Now combined value of all four broken pieces would be K + 8K + 27K + 64K = 100K and the value of unbroken diamond = 1,000K.

Loss in value = 1,000K - 100K = 900K = ₹65,700 or K = 73.

Hence, the value of unbroken diamond = 1,000K = 1,000 × 73,000 = ₹73,000 = ₹73 thousand.

15. (A)

Let the present age of Rajan, his nephew and his niece be x, y, and z years, respectively.

According to the question,

$$x = 3y \text{ and } x = 4z \quad \dots(i)$$

Also, after say 'a' years, Rajan's age will be twice of his nephew's age

$$x + a = 2(y + a)$$

Put x = 3y in the above equation

$$3y + a = 2y + 2a$$

$$y = a \quad \dots(ii)$$

$$\text{Required ratio} = \frac{x+a}{z+a} = \frac{3y+y}{\frac{3}{4}y+y} = \frac{4y}{\frac{7}{4}y} = \frac{16}{7}$$

Hence, option (A) is the correct answer.

16. (C)

Let the original number be 10x + y, and the number formed by reversing the digit be 10y + x.

$$\frac{10x + y}{10y + x} = \frac{13}{31}$$

$$310x + 31y = 130y + 13x$$

$$297x = 99y$$

$$x/y = 99/297 = 1/3$$

$$y = 3x$$

Possible combinations:

a) when x = 1, y = 3, then number = 13

b) when x = 2, y = 6, then number = 26

c) when x = 3, y = 9, then number = 39

Hence, option (C) is the correct answer.

17. (C)

Let's assume the length of the larger and smaller part be 'a' and 'b' units, respectively.

$$\text{So, } (a + b)/a = a/b$$

$$1 + b/a = a/b$$

$$\text{Let } a/b = x$$

$$1 + 1/x = x$$

$$x^2 - x - 1 = 0$$

Comparing the above mentioned equation with the general form of the quadratic equation, where a = 1, b = (-1) and c = (-1), we can obtain the value of x

$$x = (1 + \sqrt{5})/2$$

$$x = 1.6 = 8/5$$

18. 55,000

Let's assume the fixed cost of the function = ₹F

Let's assume the variable cost = V

According to the question

V is directly proportional to the square of the number of guests attending the function.

V = K (N²), where N is the number of people and K is the constant.

According to the question

Total cost = fixed cost + variable cost
 $= F + K (N^2)$
 $83,800 = F + K (80^2) \quad \dots(i)$
 $1,19,800 = F + K (120^2) \quad \dots(ii)$
 Solving equations (i) and (ii) we will get
 $F = ₹55,000$.

19. 453

Ratio of the number of runs scored by P to that of Q = $1/5:1/9$ or $9:5$.
 Ratio of the number of runs scored by Q to that of R = $1/5:1/9$ or $9:5$.
 Ratio of numbers of runs scored by P, Q, and R, respectively = $81:45:25$.
 So, let's assume the runs scored by P, Q, and R to be $81K$, $45K$, and $25K$.
 According to the question $81K - 25K = 56K = 168$.
 Or $K = 3$
 Now $81K + 45K + 25K = 151K = 151 \times 3 = 453$.

20. 143

Since the given relation is $D \propto \sqrt{W}$
 where D = Decrement in speed.
 W = No. of wagons.
 $D = K\sqrt{W}$
 When the number of wagons = 9, then the speed is 45 km/h.
 $D = K\sqrt{W}$
 $15 = K\sqrt{9}$
 $15 = 3K$
 $K = 5$
 When the speed of the train becomes, zero the train will stop. Then decrement in speed is = 60 km/h.
 $D = K\sqrt{W}$
 $60 = 5\sqrt{W}$
 $12 = \sqrt{W}$
 $W = 144$
 So, when the 144 wagons will attach to the Lucknow–Chennai express, it will not move.
 Since $144 - 1 = 143$ number of wagons allows the train to move.

21. (A)

It is given that some part of the profit is divided between Surya and Pratap in the ratio of $3:5$. Let's assume that Surya got $3K$ and Pratap got $5K$.

It is given that the remaining profit is divided in the ratio of their investment, i.e., $20:14:16$ or $10:7:8$.

Let's assume that Virender, Surya, and Pratap got $10x$, $7x$ and $8x$, respectively.

$\therefore$ Profit ratio now becomes

Virendra : Surya : Pratap
 Profit $\rightarrow$ $10x$: $7x + 3K$: $8x + 5K$
 Now, according to the condition given in the question that

$$8x + 5K = \frac{13}{10} \times 10x$$

$$8x + 5K = 13x$$

$$5K = 5x$$

$$K = x$$

$\therefore$ Profit share becomes now

Virendra : Surya : Pratap
 Profit $\rightarrow$ $10x$: $7x + 3K$: $8x + 5K$
 $10x$: $10x$: $13x$

$\therefore$ Total profit share of Surya

$$= (10x/33x) \times 66,0000 = ₹2,00,000.$$

Hence option (A) is the correct answer.

22. (D)

The profits made by A and B and B and C are in the ratio of $2:3$ and $4:5$, so the profit made by A, B, and C will be in the ratio = $8:12:15$.

The profits made by A, B, and C are in the ratio of $8:12:15$ and profits made by C and D are in the ratio of $2:1$, so profits made by A, B, C, and D will be in the ratio of $16:24:30:15$.

Similarly, the profits made by A, B, C, D, and E will be in the ratio of $16:24:30:15:20$. Or the profits made by A, B, C, D, and E will be $16K$, $24K$, $30K$, $15K$, and $20K$, respectively.

According to the question, $24K - 15K = 9K = ₹432$ crores, which means $K = ₹48$ crores.

Hence, the total profits made by companies A and E together = $36K = 36 \times 48 = ₹1,728$ crores.

Hence, option (D) is the correct answer.

23. (A)

Let's assume total profit = $100y$ and ratio of investment of L and M as $P:Q$

So, M salary = $20y$

$$\text{So, L's share} = \frac{P}{P+Q} \times 80y$$

Now, if entire profit would have been divided in the ratio $P:Q$,

$$\text{then L's profit} = \frac{P}{P+Q} \times 100y$$

Now according to the question then

L's profit

$$\text{is } \frac{P}{P+Q} \times 100y - \frac{P}{P+Q} \times 80y = 12,000$$

$$\text{or, } \frac{P}{P+Q} \times 20y = 12,000$$

Now L's actual profit

$$= \frac{P}{P+Q} \times 80y$$

$$= 4 \times \frac{P}{P+Q} \times 20y = 4 \times 12,000 = ₹48,000$$

Hence, option (A) is the correct answer.

24. (B)

We know that if $\frac{A}{B} = \frac{C}{D}$, then $\frac{A+B}{A-B} = \frac{C+D}{C-D}$

$$\begin{aligned} &\Rightarrow \frac{(9A - 18B + 8C) + (9A - 18B - 8C)}{(9A - 18B + 8C) - (9A - 18B - 8C)} \\ &= \frac{(9A + 18B - 8C) + (9A - 18B + 8C)}{(9A + 18B - 8C) - (9A - 18B + 8C)} \end{aligned}$$

$$\Rightarrow \frac{2(9A - 18B)}{2(8C)} = \frac{2(9A)}{2(18B - 8C)}$$

$$\Rightarrow \frac{A - 2B}{4C} = \frac{A}{9B - 4C}$$

$$\Rightarrow 9AB - 4AC - 18B^2 + 8BC = 4AC$$

$$\Rightarrow 9AB + 8BC = 8AC + 18B^2$$

Hence, option (B) is the correct answer.

25. (A)

Let's assume the age of pair 1 be $2x$ and $3x$.

Let's assume the age of pair 2 be $4y$ and $5y$.

$3x$ cannot be 40, as the ages of all persons should be integers.

So $5y$ is oldest

$$5y = 40. \text{ So, } 4y = 32$$

Suppose $4y$ is the youngest. In this case we won't be able to find integers $2x$ and $3x$ between 32 and 40. Thus, $2x$ is the youngest person.

Maximum age possible for $3x$ is a multiple of 3 less than 40, i.e., 39, making maximum age possible for $2x$ as 26.

Considering that the age of the youngest person is 20 less than oldest person, $40 - 20 = 20$. Thus, the age of $2x$ can be an even number from 20 to 26.

Also, since age of $3x$ has to be less than 32, i.e., ≤ 30 ,

$$\text{So, } 2x \leq 20.$$

Thus, the only case possible is that the age of youngest person ($2x$) is 20.

Hence, option (A) is the correct answer.

26. (C)

Let the amounts with Bibos, Joe, and Paes be 3K, 4K, and 6K. The amounts with Bibo, Joe, and Paes after each game is as follows:

	Bibos	Joe	Paes
Initial Amounts	3K	4K	6K
After I game	2K	6K	5K
After II game	K	5K	7K
After III game	3K	4K	6K
After IV game	2K	6K	5K

$$\text{Also } 6k - 5k = 20 \Rightarrow K = 20.$$

After the third game, the ratio of the amounts with Bibo and Paes is 1:2.

27. Joe

Let the amounts with Bibos, Joe, and Paes be 3k, 4k, and 6k. The amounts with Bibo, Joe, and Paes after each game is as follows:

	Bibos	Joe	Paes
Initial Amounts	3K	4K	6K
After I game	2K	6K	5K
After II game	K	5K	7K
After III game	3K	4K	6K
After IV game	2K	6K	5K

Also $6K - 5K = 20 \Rightarrow K = 20$.

Joe does not lose on his investment.

28. 16

The sum of extremes is given as 32.

We require the difference of extremes.

Let this difference be $2x$.

Thus, the extremes will be $(16 + x)$ and $(16 - x)$.

The positive difference between the extremes = $2x$, which is four times the difference between the means.

Since $2x$ is not a multiple of 4, let's re-assume the extremes as $(16 + 4x)$ and $(16 - 4x)$.

Thus, the positive difference between extremes = $8x$.

And hence the difference between the means = $2x$

And the means can be now assumed as $(14 + x)$ and $(14 - x)$.

And 'product of extremes' = 'product of means' results in

$$(16 + 4x)(16 - 4x) = (14 + x)(14 - x)$$

$$\text{i.e., } 256 - 16x^2 = 196 - x^2$$

$$\text{Let } 15x^2 = 60, \text{ i.e., } x = 2.$$

and the four numbers are 24, 16, 12, and 8.

Hence, the difference between the extremes is 16.

29. 24

	A	B
Past	K	a
Present	4a	$3K - 2$
After 2 years		3K

Let the present age of A be $4a$ years and the age of B after 2 years be $3K$ years.

The difference in the age of two persons does not change with time.

$$K - a = 4a - (3K - 2)$$

$$\text{Or, } a = \frac{4K - 2}{5}$$

Sum of their present ages

$$4\left(\frac{4K - 2}{5}\right) + 3K - 2 = 46$$

$$\frac{16K - 8 + 15K - 10}{5} = 46$$

$$\text{Or } K = 8$$

$$\text{Present age of A is } 4\left(\frac{4K - 2}{5}\right) = 24 \text{ years.}$$

30. 25

Let the number of Americans in class X be X_A

Let the number of Russians in class X be X_R

Let the number of Americans in class Y be Y_A

Let the number of Russians in class Y be Y_R

In class X:

Each American has five times the number of Americans as classmates as Russians.

$$\therefore X_A - 1 = 5X_R \quad \dots(i)$$

Also, every Russian in class X has seven times the number of Americans as classmates as he has Russians as classmates.

$$\therefore 7(X_R - 1) = X_A \quad \dots(ii)$$

In class Y:

Each American has five times the number of Russians as classmates as he has Americans as classmates.

$$\therefore 5(Y_A - 1) = Y_R \quad \dots(iii)$$

Solving equations (i) and (ii)

$$X_R = 4 \text{ and } X_A = 21$$

Number of students in class X is $X_R + X_A = 4 + 21 = 25$.

Total number of students in X = total number of students in Y = 25.