10

Straight Lines and Pair of Straight Lines

Numerical

Q.1 A man starts walking from the point P(−3, 4), touches the x-axis at R, and then turns to reach at the point Q(0, 2). The man is walking at a constant speed. If the man reaches the point Q in the minimum time, then $5((PR)^2 + (RQ)^2)$ is equal to ___________.

1st Sep Evening Shift 2021

Q.2 Let the points of intersections of the lines x − y + 1 = 0, x − 2y + 3 = 0 and 2x − 5y + 11 = 0 are the mid points of the sides of a triangle ABC is ____________.

1st Sep Evening Shift 2021

Q.3 Consider a triangle having vertices A(−2, 3), B(1, 9) and C(3, 8). If a line L passing through the circum-centre of triangle ABC, bisects line BC, and intersects y-axis at point $\left(0, \frac{\alpha}{2}\right)$, then the value of real number α is _______________.

20th Jul Evening Shift 2021

Q.4 A square ABCD has all its vertices on the curve $x^2y^2 = 1$. The midpoints of its sides also lie on the same curve. Then, the square of area of ABCD is

18th Mar Morning Shift 2021

Q.5

Let tanα, tanβ and tanγ; $\alpha, \beta, \gamma \neq \frac{(2n-1)\pi}{2}$, n∈N be the slopes of three line segments OA, OB and OC, respectively, where O is origin. If circumcentre of ΔABC coincides with origin and its orthocentre lies on y-axis, then the value of $\left(\frac{\cos 3\alpha+\cos 3\beta+\cos 3\gamma}{\cos \alpha \cos \beta \cos \gamma}\right)^2$ is equal to

______________.

17th Mar Evening Shift 2021

Q.6 The maximum value of z in the following equation $z = 6xy + y^2$, where $3x + 4y \leq 100$ and $4x + 3y \leq 75$ for $x \geq 0$ and $y \geq 0$ is _________.

17th Mar Evening Shift 2021

Q.7 Let λ be an integer. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is ________.

24th Feb Evening Slot 2021

Numerical Answer Key

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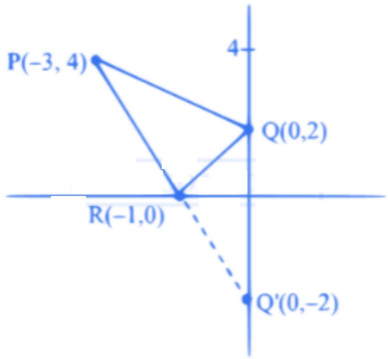
1. **Ans. (1250)**
2. **Ans. (6)**
3. **Ans. (9)**
4. **Ans. (80)**
5. **Ans. (144)**
6. **Ans. (904)**
7. **Ans. (1)**

Numerical Explanation

Ans. 1



$50(PR^2 + RQ^2)$

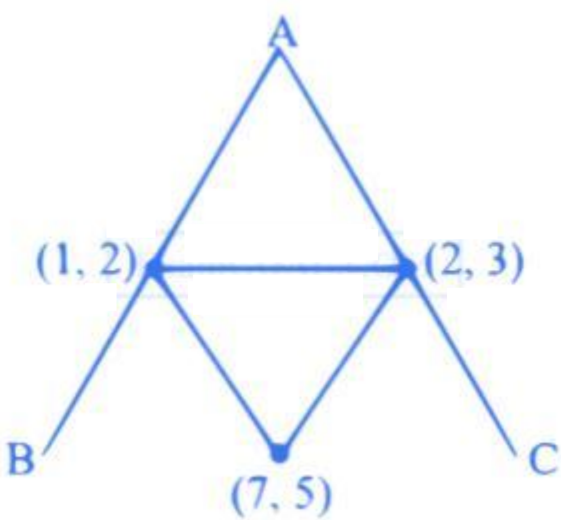
$= 50(20 + 5)$

$= 50(25)$

$= 1250$

Ans 2.

Intersection point of given lines are (1, 2), (7, 5), (2, 3)



$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 7 & 5 & 1 \\ 2 & 3 & 1 \end{vmatrix}$$

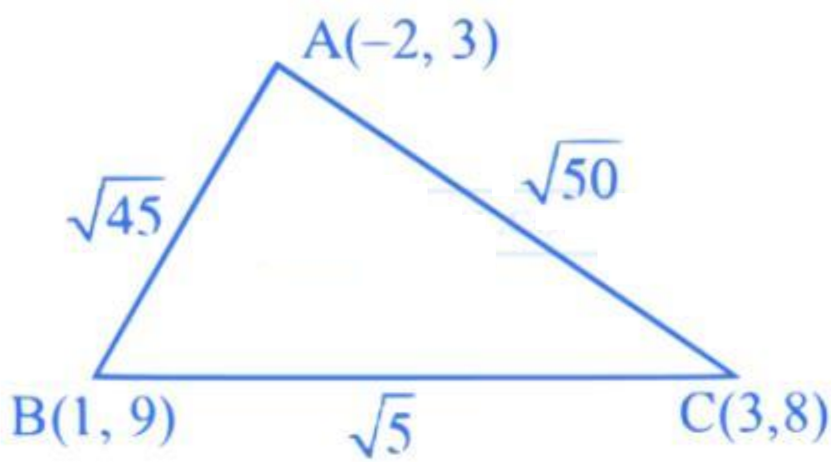
$$= \frac{1}{2}[1(5-3) - 2(7-2) + 1(21-10)]$$

$$= \frac{1}{2}[2 - 10 + 11]$$

$$\Delta\text{DEF} = \frac{1}{2}(3) = \frac{3}{2}$$

$$\Delta\text{ABC} = 4\Delta\text{DEF} = 4\left(\frac{3}{2}\right) = 6$$

Ans 3.



$$\left(\sqrt{50}\right)^2 = \left(\sqrt{45}\right)^2 + \left(\sqrt{5}\right)^2$$

$\angle B = 90^\circ$

Circum-center $= \left(\frac{1}{2}, \frac{11}{2}\right)$

Mid point of BC $= \left(2, \frac{17}{2}\right)$

Line : $\left(y - \frac{11}{2}\right) = 2\left(x - \frac{1}{2}\right) \Rightarrow y = 2x + \frac{9}{2}$

Passing through $\left(0, \frac{\alpha}{2}\right)$

$\frac{\alpha}{2} = \frac{9}{2} \Rightarrow \alpha = 9$

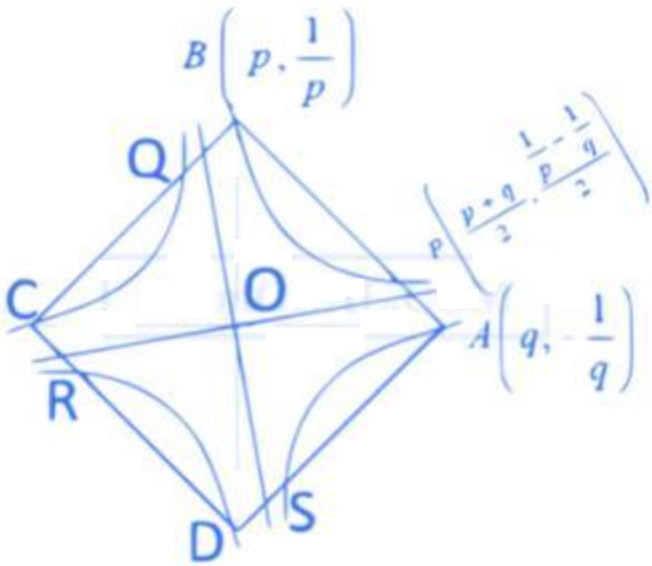
Ans 4.

$x^2y^2 = 1$

$\Rightarrow y^2 = \frac{1}{x^2}$

$\Rightarrow y = \pm\frac{1}{x}$

Graph of this equation,



$OA \perp OB$

$\Rightarrow \left(\frac{1}{p^2}\right)\left(-\frac{1}{q^2}\right) = -1$

$\Rightarrow p^2q^2 = 1$

$P\left(\frac{p+q}{2}, \frac{\frac{1}{p}-\frac{1}{q}}{2}\right)$ midpoint of AB lies

On $x^2y^2 = 1$

$\Rightarrow (p+q)^2\left(\frac{1}{p} - \frac{1}{q}\right)^2 = 16$

$\Rightarrow (p+q)^2(p-q)^2 = 16$

$\Rightarrow (p^2 - q^2)^2 = 16$

$\Rightarrow P^2 - \frac{1}{P^2} = \pm 4$

$\Rightarrow p^4 \pm 4p^2 - 1 = 0$

$$\Rightarrow p^2 = \frac{\pm 4 \pm \sqrt{20}}{2} = \pm 2 \pm \sqrt{5}$$

$$\Rightarrow p^2 = 2 + \sqrt{5} \text{ or } -2 + \sqrt{5}$$

$$OB^2 = p^2 + \frac{1}{p^2} = 2 + \sqrt{5} + \frac{1}{2+\sqrt{5}} \text{ or } -2 + \sqrt{5} + \frac{1}{-2+\sqrt{5}} = 2\sqrt{5}$$

$$\text{Area} = 4\left(\frac{1}{2}\right)(OA)(OB) = 2(OB)^2 = 4\sqrt{5}$$

Ans 5.

Since orthocentre and circumcentre both lies on y-axis.

⇒ Centroid also lies on y-axis.

⇒ $\sum \cos\alpha = 0$

$\cos\alpha + \cos\beta + \cos\gamma = 0$

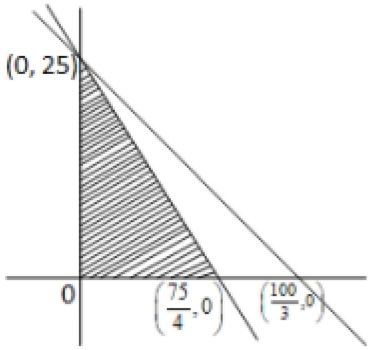
⇒ $\cos^3\alpha + \cos^3\beta + \cos^3\gamma = 3\cos\alpha\cos\beta\cos\gamma$

$$\therefore \frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha\cos\beta\cos\gamma}$$

$$= \frac{4(\cos^3\alpha + \cos^3\beta + \cos^3\gamma) - 3(\cos\alpha + \cos\beta + \cos\gamma)}{\cos\alpha\cos\beta\cos\gamma} = 12$$

$$\text{then, } \left(\frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha\cos\beta\cos\gamma}\right)^2 = 144$$

Ans 6.



3x + 4y ≤ 100

4x + 3y ≤ 75

x ≥ 0, y ≥ 0

Feasible region is shown in the graph

Let maximum value of $6xy + y^2 = c$

For a solution with feasible region,

$6xy + y^2 = c$ and $4x + 3y = 75$ must have at least one positive solution.

$$y^2 + 6y\left(\frac{75-3y}{4}\right) - c = 0$$

$$\Rightarrow \frac{7}{2}y^2 - \frac{225}{2}y + c = 0$$

$$\Rightarrow \left(\frac{225}{2}\right)^2 \geq 4.\frac{7}{2}.c$$

$$\Rightarrow c \leq \frac{225^2}{56} \approx 904$$

Ans 7.

$\frac{x-\lambda}{1}=\frac{y-\frac{1}{2}}{\frac{1}{2}}=\frac{z}{-\frac{1}{2}}$

$\frac{x-\lambda}{2}=\frac{y-\frac{1}{2}}{1}=\frac{2}{-1}$ .... (1)

Point on line = $\left(\lambda,\frac{1}{2},0\right)$

$\frac{x}{1}=\frac{y+2\lambda}{1}=\frac{z-\lambda}{1}$ .... (2)

Point on line = $(0,-2\lambda,\lambda)$

Distance between skew lines $=\frac{\left[\vec{a}_2-\vec{a}_1\,\vec{b}_1\,\vec{b}_2\right]}{\left|\vec{b}_1\times\vec{b}_2\right|}$

$$\begin{vmatrix}\lambda & \frac{1}{2}+2\lambda & -\lambda\\ 2 & 1 & -1\\ 1 & 1 & 1\end{vmatrix}$$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -1\\ 1 & 1 & 1\end{vmatrix}$$

$=\frac{\left|-5\lambda-\frac{3}{2}\right|}{\sqrt{14}}=\frac{\sqrt{7}}{2\sqrt{2}}$

$=|10\lambda+3|=7\Rightarrow\lambda=-1$

$\Rightarrow|\lambda|=1$

MCQ (Single Correct Answer)

Q.1 Let A be the set of all points (α, β) such that the area of triangle formed by the points (5, 6), (3, 2) and (α, β) is 12 square units. Then the least possible length of a line segment joining the origin to a point in A, is :

A $\frac{4}{\sqrt{5}}$

B $\frac{16}{\sqrt{5}}$

C $\frac{8}{\sqrt{5}}$

D $\frac{12}{\sqrt{5}}$

31st Aug Evening Shift 2021

Q.2 If p and q are the lengths of the perpendiculars from the origin on the lines,

x cosec α − y sec α = k cot 2α and

x sinα + y cosα = k sin2α

respectively, then k^2 is equal to :

A $4p^2 + q^2$

B $2p^2 + q^2$

C $p^2 + 2q^2$

D $p^2 + 4q^2$

31st Aug Morning Shift 2021

Q.3 The angle between the straight lines, whose direction cosines are given by the equations 2l + 2m − n = 0 and mn + nl + lm = 0, is :

(A) $\frac{\pi}{2}$

(B) $\pi - \cos^{-1}\left(\frac{4}{9}\right)$

(C) $\cos^{-1}\left(\frac{8}{9}\right)$

(D) $\frac{\pi}{3}$

27th Aug Evening Shift 2021

Q.4 Let A be a fixed point (0, 6) and B be a moving point (2t, 0). Let M be the mid-point of AB and the perpendicular bisector of AB meets the y-axis at C. The locus of the mid-point P of MC is :

(A) $3x^2 - 2y - 6 = 0$

(B) $3x^2 + 2y - 6 = 0$

(C) $2x^2 + 3y - 9 = 0$

(D) $2x^2 - 3y + 9 = 0$

27th Aug Morning Shift 2021

Q.5 Let ABC be a triangle with A(−3, 1) and ∠ACB = θ, 0 < θ < π/2. If the equation of the median through B is 2x + y − 3 = 0 and the equation of angle bisector of C is 7x − 4y −− 1 = 0, then tanθ is equal to :

A. $\frac{1}{2}$

B. $\frac{3}{4}$

C. $\frac{4}{3}$

D. 2

26th Aug Morning Shift 2021

Q.6 Let the equation of the pair of lines, y = px and y = qx, can be written as (y − px)(y − qx) = 0. Then the equation of the pair of the angle bisectors of the lines $x^2 - 4xy - 5y^2 = 0$ is :

A. $x^2 - 3xy + y^2 = 0$

B. $x^2 + 4xy - y^2 = 0$

C. $x^2 + 3xy - y^2 = 0$

D. $x^2 - 3xy - y^2 = 0$

25th Jul Evening Shift 2021

Q.7 Two sides of a parallelogram are along the lines 4x + 5y = 0 and 7x + 2y = 0. If the equation of one of the diagonals of the parallelogram is 11x + 7y = 9, then other diagonal passes through the point :

A. (1, 2)

B. (2, 2)

C. (2, 1)

D. (1, 3)

27th Jul Evening Shift 2021

Q.8 The point P (a, b) undergoes the following three transformations successively :

(a) reflection about the line y = x.

(b) translation through 2 units along the positive direction of x-axis.

(c) rotation through angle π/4 about the origin in the anti-clockwise direction.

If the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$, then the value of 2a + b is equal to :

A 13

B 9

C 5

D 7

27th Jul Evening Shift 2021

Q.9 Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line x + y = 3. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then (R + r) is equal to :

A $7\sqrt{2}$

B $\frac{9}{\sqrt{2}}$

C $2\sqrt{2}$

D $3\sqrt{2}$

18th Mar Evening Shift 2021

Q.10 The equation of one of the straight lines which passes through the point (1, 3) and makes an angles $\tan^{-1}(\sqrt{2})$ with the straight line, $y + 1 = 3\sqrt{2}x$ is

A $4\sqrt{2}x + 5y - \left(15 + 4\sqrt{2}\right) = 0$

B $5\sqrt{2}x + 4y - \left(15 + 4\sqrt{2}\right) = 0$

C $4\sqrt{2}x + 5y - 4\sqrt{2} = 0$

D $4\sqrt{2}x - 5y - \left(5 + 4\sqrt{2}\right) = 0$

18th Mar Morning Shift 2021

Q.11 The number of integral values of m so that the abscissa of point of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is :

A 1

B 2

C 3

D 0

18th Mar Morning Shift 2021

Q.12 In a triangle PQR, the co-ordinates of the points P and Q are (−2, 4) and (4, −2) respectively. If the equation of the perpendicular bisector of PR is $2x - y + 2 = 0$, then the centre of the circumcircle of the ΔPQR is :

A (−1, 0)

B (1, 4)

C (0, 2)

D (−2, −2)

17th Mar Morning Shift 2021

Q.13 Let A(−1, 1), B(3, 4) and C(2, 0) be given three points.
A line y = mx, m > 0, intersects lines AC and BC at point P and Q respectively. Let A_1 and A_2 be the areas of ΔABC and ΔPQC respectively, such that $A_1 = 3A_2$, then the value of m is equal to :

(A) 1

(B) 3

(C) 2

(D) $\frac{4}{15}$

16th Mar Evening Shift 2021

Q.14 The intersection of three lines x − y = 0, x + 2y = 3 and 2x + y = 6 is a :

(A) Right angled triangle

(B) Equilateral triangle

(C) None of the above

(D) Isosceles triangle

26th Feb Morning Shift 2021

Q.15 The image of the point (3, 5) in the line x − y + 1 = 0, lies on :

(A) $(x-4)^2 + (y-4)^2 = 8$

(B) $(x-4)^2 + (y+2)^2 = 16$

(C) $(x-2)^2 + (y-2)^2 = 12$

(D) $(x-2)^2 + (y-4)^2 = 4$

25th Feb Morning Slot 2021

Q.16 A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts 1/4 of this line on the coordinate axes is Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is/are on the path of the man ?

(A) A only

(B) All the three

(C) C only

(D) B only

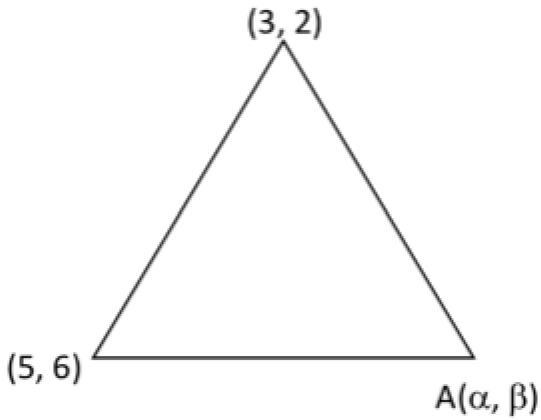
24th Feb Morning Slot 2021

MCQ Answer Key

1. **Ans.** (C)
2. **Ans.** (A)
3. **Ans.** (A)
4. **Ans.** (c)
5. **Ans.** (c)
6. **Ans.** (c)
7. **Ans.** (B)
8. **Ans.** (B)
9. **Ans.** (B)
10. **Ans.** (a)
11. **Ans.** (b)
12. **Ans.** (d)
13. **Ans.** (a)
14. **Ans.** (d)
15. **Ans.** (d)
16. **Ans.** (d)

MCQ Explanation

Ans 1.



$$\left|\frac{1}{2}\begin{vmatrix} 5 & 6 & 1 \\ 3 & 2 & 1 \\ \alpha & \beta & 1 \end{vmatrix}\right| = 12$$

$4\alpha - 2\beta = \pm 24 + 8$

$\Rightarrow 4\alpha - 2\beta = + 24 + 8 \Rightarrow 2\alpha - \beta = 16$

$2x - y - 16 = 0$ ..... (1)

$\Rightarrow 4\alpha - 2\beta = - 24 + 8 \Rightarrow 2\alpha - \beta = -8$

$2x - y + 8 = 0$ ...... (2)

perpendicular distance of (1) from (0, 0)

$\left|\frac{0-0-16}{\sqrt{5}}\right| = \frac{16}{\sqrt{5}}$

perpendicular distance of (2) from (0, 0)

$\left|\frac{0-0+8}{\sqrt{5}}\right| = \frac{8}{\sqrt{5}}$

Ans 2.

First line is $\frac{x}{\sin\alpha} - \frac{y}{\cos\alpha} = \frac{k\cos 2\alpha}{\sin 2\alpha}$

$\Rightarrow x\cos\alpha - y\sin\alpha = \frac{k}{2}\cos 2\alpha$

$\Rightarrow p = \left|\frac{k}{2}\cos\alpha\right| \Rightarrow 2p = |k\cos 2\alpha|$ .... (i)

second line is $x\sin\alpha + y\cos\alpha = k\sin 2\alpha$

$\Rightarrow q = |k\sin 2\alpha|$ .... (ii)

Hence, $4p^2 + q^2 = k^2$ (From (i) & (ii))

Ans 3.

n = 2 (l + m)

lm + n(l + m) = 0

lm + 2(l + m)^2 = 0

2l^2 + 2m^2 + 5ml = 0

$2\left(\frac{l}{m}\right)^2 + 2 + 5\left(\frac{l}{m}\right) = 0$

2t^2 + 5t + 2 = 0

(t + 2)(2t + 1) = 0

⇒ t = −2; − ½

(i) l/m = −2

n/m = −2

(−2m, m, −2m)

(−2, 1, −2)

(ii) l/m = − 1/2

n = −2l

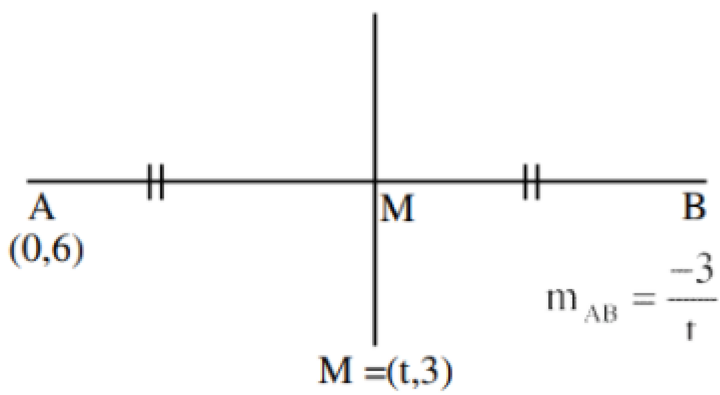
(l, −2l, −2l)

(1, −2, −2)

$$\cos\theta = \frac{-2-2+4}{\sqrt{9}\sqrt{9}} = 0 \Rightarrow 0 = \frac{\pi}{2}$$

Ans 4.

A(0, 6) and B(2t, 0)



Perpendicular bisector of AB is

$(y-3)=\frac{t}{3}(x-t)$

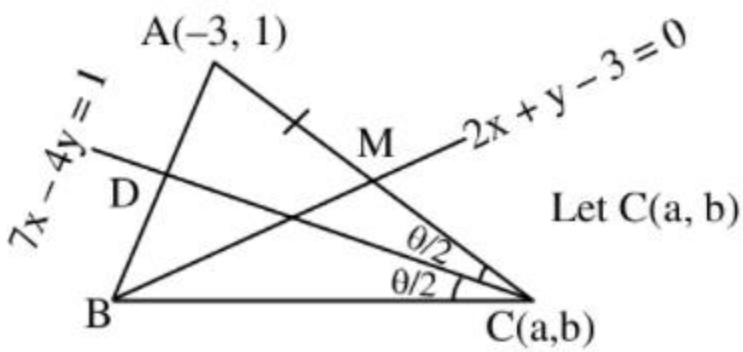
So, $C=\left(0,3-\frac{t^2}{3}\right)$

Let P be (h, k)

$h=\frac{t}{2}; k=\left(3-\frac{t^2}{6}\right)$

$\Rightarrow k=3-\frac{4h^2}{6} \Rightarrow 2x^2+3y-9=0$

Ans 5.



$\therefore M\left(\frac{a-3}{2}, \frac{b+1}{2}\right)$ lies on 2x + y – 3 = 0

$\Rightarrow$ 2a + b = 11 ....(i)

$\because$ C lies on 7x – 4y = 1

$\Rightarrow$ 7a – 4b = 1 .... (ii)

$\therefore$ by (i) and (ii) : a = 3, b = 5

$\Rightarrow$ C(3, 5)

$\therefore m_{AC} = 2/3$

Also, $m_{CD} = 7/4$

$\Rightarrow \tan\frac{\theta}{2} = \left|\frac{\frac{2}{3}-\frac{4}{4}}{1+\frac{14}{12}}\right| \Rightarrow \tan\frac{\theta}{2} = \frac{1}{2}$

$\Rightarrow \tan\theta = \frac{2.\frac{1}{2}}{1-\frac{1}{4}} = \frac{4}{3}$

Ans 6.

Equation of angle bisector of homogeneous

equation of pair of straight line $ax^2 + 2hxy + by^2$ is

$\frac{x^2-y^2}{a-b} = \frac{xy}{h}$

for $x^2 - 4xy - 5y^2 = 0$

a = 1, h = – 2, b = – 5

So, equation of angle bisector is

$\frac{x^2-y^2}{1-(-5)} = \frac{xy}{-2}$

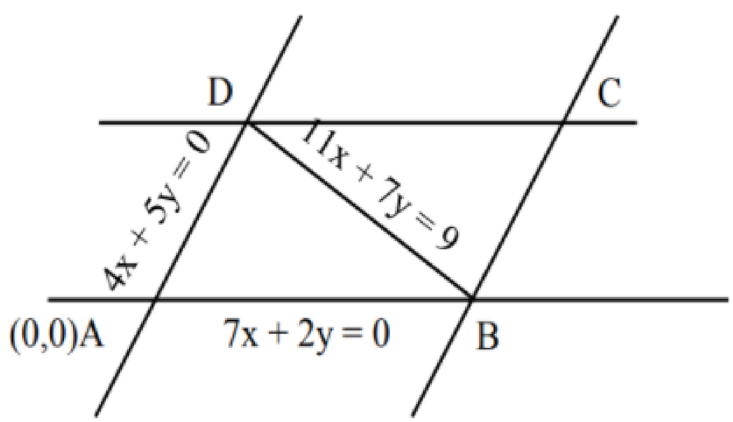
$\frac{x^2-y^2}{6} = \frac{xy}{-2}$

$\Rightarrow x^2 - y^2 = -3xy$

So, combined equation of angle bisector is $x^2 + 3xy - y^2 = 0$

Ans 7.

Both the lines pass through origin.



point D is equal to intersection of 4x + 5y = 0 & 11x + 7y = 9

So, coordinates of point D = (5/3, −4/3)

Also, point B is point of intersection of 7x + 2y = 0 and 11x + 7y = 9

So, coordinates of point B = (−2/3, 7/3)

diagonals of parallelogram intersect at middle let middle point of B, D

$$\Rightarrow \left(\frac{\frac{5}{3}-\frac{2}{3}}{2}, \frac{\frac{-4}{3}+\frac{7}{3}}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

equation of diagonal AC

$$\Rightarrow (y-0) = \frac{\frac{1}{\alpha}-0}{\frac{1}{\alpha}-0}(x-0)$$

$$y = x$$

diagonal AC passes through (2, 2)

Ans 8.

Image of A(a, b) along y = x is B(b, a). Translating it 2 units it becomes C(b + 2, a).

Now, applying rotation theorem

$$-\frac{1}{\sqrt{2}} + \frac{7}{\sqrt{2}}i = ((b+2)+ai)\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$-\frac{1}{\sqrt{2}} + \frac{7}{\sqrt{2}}i = \left(\frac{b+2}{\sqrt{2}} - \frac{a}{\sqrt{2}}\right) + i\left(\frac{b+2}{\sqrt{2}} + \frac{a}{\sqrt{2}}\right)$$

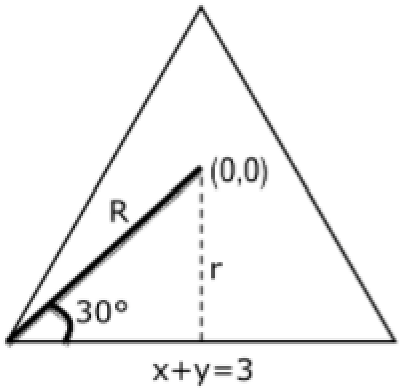
$\Rightarrow$ b − a + 2 = −1 ......(i)

and b + 2 + a = 7 ...... (ii)

$\Rightarrow$ a = 4; b = 1

$\Rightarrow$ 2a + b = 9

Ans 9.



$r = \left|\frac{0+0-3}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}}$

$\sin 30^\circ = \frac{r}{R} = \frac{1}{2}$

R = 2r

So, $r + R = 3r = 3 \times \left(\frac{3}{\sqrt{2}}\right) = \frac{9}{\sqrt{2}}$

Ans 10.

Let slope of line be m

$$\therefore \left|\frac{m-3\sqrt{2}}{1+3\sqrt{2}m}\right| = \sqrt{2}$$

$$\Rightarrow m - 3\sqrt{2} = \pm\sqrt{2} \pm 6m$$

$$\Rightarrow m \mp 6m = \pm\sqrt{2} + 3\sqrt{2}$$

$\Rightarrow m = -\frac{4\sqrt{2}}{5}$ or $\frac{2\sqrt{2}}{7}$

Hence line can be

$$y - 3 = \frac{-4\sqrt{2}}{5}(x-1)$$

$$\Rightarrow 5y - 15 = -4\sqrt{2}x + 4\sqrt{2}$$

$$\Rightarrow 4\sqrt{2}x + 5y - (15 + 4\sqrt{2}) = 0$$

Ans 11.

3x + 4(mx + 1) = 9

⇒ x(3 + 4m) = 5

$\Rightarrow x = \frac{5}{(3+4m)}$

$\Rightarrow (3 + 4m) = \pm1, \pm5$

$\Rightarrow 4m = -3 \pm 1, -3 \pm 5$

$\Rightarrow 4m = -4, -2, -8, 2$

$\Rightarrow m = -1, -\frac{1}{2}, -2, \frac{1}{2}$

$\therefore$ Two integral value of m.

Ans 12.

Mid point of $PQ \equiv \left(\frac{-2+4}{2}, \frac{4-2}{2}\right) \equiv (1, 1)$

Slope of $PQ = \frac{4+2}{-2-4} = -1$

Slope of perpendicular bisector of PQ = 1

Equation of perpendicular bisector of PQ

$y - 1 = 1(x - 1)$

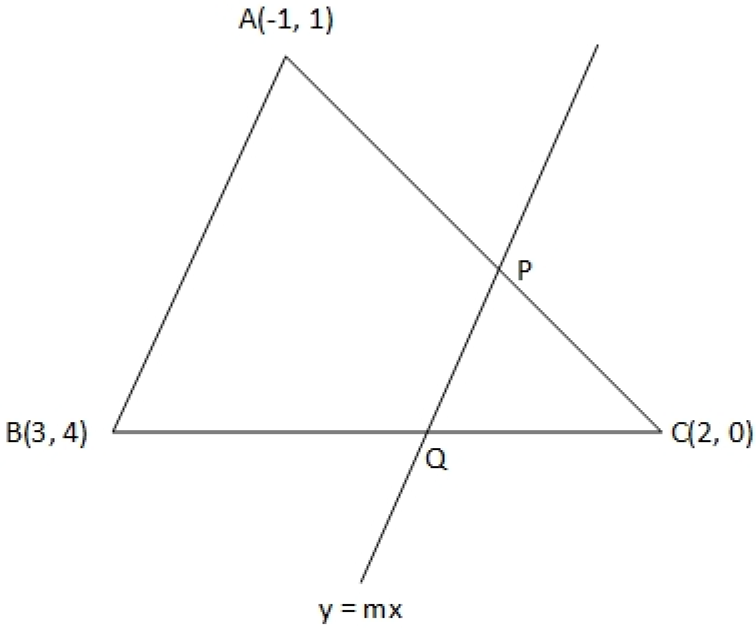
$\Rightarrow y = x$

Solving with perpendicular bisector of PR,

$2x - y + 2 = 0$

Circumcentre is $(-2, -2)$

Ans 13.



$A_1 = \Delta ABC = \frac{1}{2}\left|\begin{vmatrix} -1 & 1 & 1 \\ 2 & 0 & 1 \\ 3 & 4 & 1 \end{vmatrix}\right|$

$A_1 = \frac{13}{2}$

Equation of line AC is $y - 1 = -\frac{1}{3}(x + 1)$

Line AC intersect with line y = mx at P,

Solving we get $P\left(\frac{2}{3m+1}, \frac{2m}{3m+1}\right)$

Equation of line BC is y − 0 = 4(x − 2)

Line BC intersect with line y = mx at Q,

Solving we get $Q\left(\frac{-8}{m-4}, \frac{-8m}{m-4}\right)$

$$A_2 = \text{Area of } \Delta PQC = \frac{1}{2}\left|\begin{vmatrix} 2 & 0 & 1 \\ \frac{2}{3m+1} & \frac{2m}{3m+1} & 1 \\ \frac{-8}{m-4} & \frac{-8m}{m-4} & 1 \end{vmatrix}\right| = \frac{A_1}{3} = \frac{13}{6}$$

$$\Rightarrow \frac{1}{2}\left(2\left(\frac{2m}{3m+1}+\frac{8m}{m-4}\right)-1\left(\frac{-16m}{(3m+1)(m-4)}+\frac{16m}{(3m+1)(m-4)}\right)\right) = \pm\frac{13}{6}$$

$$\Rightarrow \frac{26m^2}{3m^2-11m-4} = \pm\frac{13}{6}$$

$$\Rightarrow 12m^2 = \pm(3m^2-11m-4)$$

taking +ve sign

$9m^2 + 11m + 4 = 0$ (Rejected ∵ m is imaginary)

taking −ve sign

$15m^2 - 11m - 4 = 0$

M = 1, −4/15

∴ m = 1 (As given m > 0)

Ans 14.

The given three lines are x − y = 0, x + 2y = 3 and 2x + y = 6 then point of intersection,

lines x − y = 0 and x + 2y = 3 is (1, 1)

lines x − y = 0 and 2x + y = 6 is (2, 2)

and lines x + 2y = 3 and 2x + y = 0 is (3, 0)

The triangle ABC has vertices A(1, 1), B(2, 2) and C(3, 0)

∴ AB = √2, BC = √5 and AC = √5

∴ ΔABC is isosceles

Ans 15.

So, let the image is (x, y)

So, we have

$$\frac{x-3}{1}=\frac{y-5}{-1}=-\frac{2(3-5+1)}{1+1}$$

$\Rightarrow$ x = 4, y = 4

$\Rightarrow$ Point (4, 4)

Which will satisfy the curve

$(x-2)^2+(y-4)^2=4$

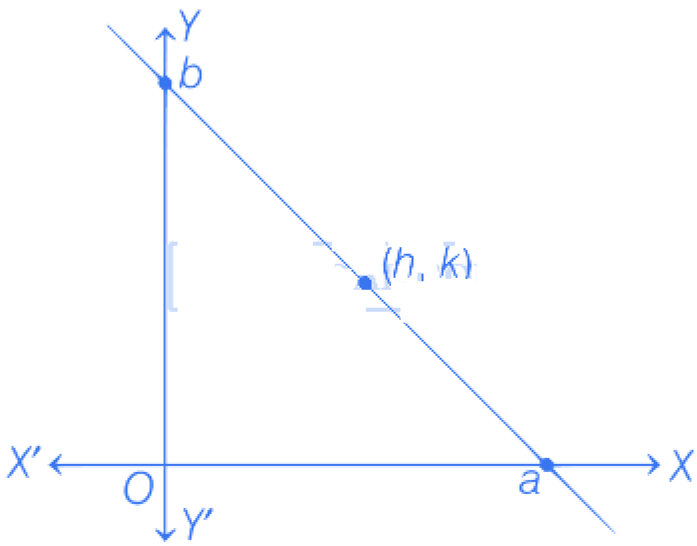
as $(4-2)^2+(4-4)^2=4+0=4$

Ans 16.

Given, position of A = (1, 1)

Position of B = (2, 2)

Position of C = (4, 4)



Let x-intercept be a and y-intercept be b.

Equation of line traced is

$\frac{x}{a} + \frac{y}{b} = 1$

This is the equation of path, let a point (h, k) lie on this path.

Then, $\frac{h}{a} + \frac{k}{b} = 1$

Also, AM of reciprocal of a and b = $\frac{1}{4}$

$\therefore \frac{\frac{1}{a}+\frac{1}{b}}{2} = \frac{1}{4}$

$\frac{1}{a} + \frac{1}{b} = \frac{1}{2}$

On comparing Eqs. (i) and (ii), we get (h, k) = (2, 2)

Hence, the required stone is B(2, 2).

TOPIC 1 Distance Formula, Section Formula, Results of Triangle, Locus, Equation of Locus, Slope of a Straight Line, Slope of a line joining two points, Parallel and Perpendicular Lines

1. A triangle ABC lying in the first quadrant has two vertices as $A(1,2)$ and $B(3,1)$. If $\angle BAC = 90°$, and ar $(\Delta ABC) = 5\sqrt{5}$ sq. units, then the abscissa of the vertex C is: **[Sep. 04, 2020 (I)]**

   (a) $1+\sqrt{5}$ (b) $1+2\sqrt{5}$ (c) $2+\sqrt{5}$ (d) $2\sqrt{5}-1$

2. If the perpendicular bisector of the line segment joining the points $P(1, 4)$ and $Q(k, 3)$ has y-intercept equal to -4, then a value of k is : **[Sep. 04, 2020 (II)]**

   (a) -2 (b) -4 (c) $\sqrt{14}$ (d) $\sqrt{15}$

3. If a ΔABC has vertices $A(-1, 7)$, $B(-7, 1)$ and $C(5, -5)$, then its orthocentre has coordinates : **[Sep. 03, 2020 (II)]**

   (a) $\left(-\frac{3}{5}, \frac{3}{5}\right)$ (b) $(-3, 3)$

   (c) $\left(\frac{3}{5}, -\frac{3}{5}\right)$ (d) $(3, -3)$

4. Let $A(1, 0)$, $B(6, 2)$ and $C\left(\frac{3}{2}, 6\right)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line segment

   PQ, where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$, is———.

   [NA Jan. 7, 2020 (I)]

5. A triangle has a vertex at $(1, 2)$ and the mid points of the two sides through it are $(-1, 1)$ and $(2, 3)$. Then the centroid of this triangle is : **[April 12, 2019 (II)]**

   (a) $\left(1, \frac{7}{3}\right)$ (b) $\left(\frac{1}{3}, 2\right)$ (c) $\left(\frac{1}{3}, 1\right)$ (d) $\left(\frac{1}{3}, \frac{5}{3}\right)$

6. Let O(0, 0) and A(0, 1) be two fixed points. Then the locus of a point P such that the perimeter of ΔAOP is 4, is : **[April 8, 2019 (I)]**

   (a) $8x^2 - 9y^2 + 9y = 18$ (b) $9x^2 - 8y^2 + 8y = 16$

   (c) $9x^2 + 8y^2 - 8y = 16$ (d) $8x^2 + 9y^2 - 9y = 18$

7. Two vertices of a triangle are (0, 2) and (4, 3). If its orthocentre is at the origin, then its third vertex lies in which quadrant? **[Jan. 10, 2019 (II)]**

   (a) third (b) second

   (c) first (d) fourth

8. Let the orthocentre and centroid of a triangle be A(–3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is : **[2018]**

   (a) $2\sqrt{10}$ (b) $3\sqrt{\frac{5}{2}}$ (c) $\frac{3\sqrt{5}}{2}$ (d) $\sqrt{10}$

9. A square, of each side 2, lies above the x-axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30° with the positive direction of the x-axis, then the sum of the x-coordinates of the vertices of the square is : **[Online April 9, 2017]**

   (a) $2\sqrt{3}-1$ (b) $2\sqrt{3}-2$ (c) $\sqrt{3}-2$ (d) $\sqrt{3}-1$

10. A ray of light is incident along a line which meets another line, $7x - y + 1 = 0$, at the point (0, 1). The ray is then reflected from this point along the line, $y + 2x = 1$. Then the equation of the line of incidence of the ray of light is : **[Online April 10, 2016]**

    (a) $41x - 25y + 25 = 0$ (b) $41x + 25y - 25 = 0$

    (c) $41x - 38y + 38 = 0$ (d) $41x + 38y - 38 = 0$

11. Let L be the line passing through the point P(1, 2) such that its intercepted segment between the co-ordinate axes is bisected at P. If L_1 is the line perpendicular to L and passing through the point (–2, 1), then the point of intersection of L and L_1 is : **[Online April 10, 2015]**

(a) $\left(\frac{4}{5},\frac{12}{5}\right)$ (b) $\left(\frac{3}{5},\frac{23}{10}\right)$

(c) $\left(\frac{11}{20},\frac{29}{10}\right)$ (d) $\left(\frac{3}{10},\frac{17}{5}\right)$

12. The points $\left(0,\frac{8}{3}\right)$, (1, 3) and (82, 30) : **[Online April 10, 2015]**

(a) form an acute angled triangle.
(b) form a right angled triangle.
(c) lie on a straight line.
(d) form an obtuse angled triangle.

13. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1) (1, 1) and (1, 0) is : **[2013]**

(a) $2+\sqrt{2}$ (b) $2-\sqrt{2}$ (c) $1+\sqrt{2}$ (d) $1-\sqrt{2}$

14. A light ray emerging from the point source placed at P(l, 3) is reflected at a point Q in the axis of x. If the reflected ray passes through the point R (6, 7), then the abscissa of Q is: **[Online April 9, 2013]**

(a) 1 (b) 3 (c) $\frac{7}{2}$ (d) $\frac{5}{2}$

15. Let A (h, k), B(1, 1) and C (2, 1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1square unit, then the set of values which 'k' can take is given by **[2007]**

(a) $\{-1, 3\}$ (b) $\{-3, -2\}$ (c) $\{1, 3\}$ (d) $\{0, 2\}$

16. If a vertex of a triangle is (1, 1) and the mid points of two sides through this vertex are (–1, 2) and (3, 2) then the centroid of the triangle is **[2005]**

(a) $\left(-1,\frac{7}{3}\right)$ (b) $\left(\frac{-1}{3},\frac{7}{3}\right)$ (c) $\left(1,\frac{7}{3}\right)$ (d) $\left(\frac{1}{3},\frac{7}{3}\right)$

17. If the equation of the locus of a point equidistant from the point (a_1, b_1) and (a_2, b_2) is $(a_1-b_2)x+(a_1-b_2)y+c=0$, then the value of `c` is **[2003]**

(a) $\sqrt{a_1^2+b_1^2-a_2^2-b_2^2}$ (b) $\frac{1}{2}a_2^2+b_2^2-a_1^2-b_1^2$

(c) $a_1^2-a_2^2+b_1^2-b_2^2$ (d) $\frac{1}{2}(a_1^2+a_2^2+b_1^2+b_2^2)$

18. Locus of centroid of the triangle whose vertices are $(a\cos t, a\sin t), (b\sin t, -b\cos t)$ and (1, 0), where t is a parameter, is **[2003]**

(a) $(3x+1)^2+(3y)^2=a^2-b^2$

(b) $(3x-1)^2+(3y)^2=a^2-b^2$

(c) $(3x-1)^2+(3y)^2=a^2+b^2$

(d) $(3x+1)^2+(3y)^2=a^2+b^2$

19. A triangle with vertices (4, 0), (–1, –1), (3, 5) is **[2002]**

(a) isosceles and right angled
(b) isosceles but not right angled
(c) right angled but not isosceles
(d) neither right angled nor isosceles

TOPIC 2 Various Forms of Equation of a Line

20. Let f: R → R be defined as

$$f(x)=\begin{cases} x^5\sin\left(\frac{1}{x}\right)+5x^2, & x<0 \\ 0, & x=0 \\ x^5\cos\left(\frac{1}{x}\right)+\lambda x^2, & x>0 \end{cases}$$

The value of λ for which $f''(0)$ exists, is ______. **[NA Sep. 06, 2020 (I)]**

21. Let C be the centroid of the triangle with vertices (3, –1), (1, 3) and (2, 4). Let P be the point of intersection of the lines $x+3y-1=0$ and $3x-y+1=0$. Then the line passing through the points C and P also passes through the point: **[Jan. 9, 2020 (I)]**

(a) (–9, –6) (b) (9, 7) (c) (7, 6) (d) (–9, –7)

22. Slope of a line passing through P(2, 3) and intersecting the line x + y = 7 at a distance of 4 units from P, is: **[April 9, 2019 (I)]**

(a) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$ (b) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$ (c) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$ (d) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$

23. A point on the straight line, $3x + 5y = 15$ which is equidistant from the coordinate axes will lie only in : **[April 8, 2019 (I)]**

(a) 4th quadrant (b) 1st quadrant
(c) 1st and 2nd quadrants (d) 1st, 2nd and 4th quadrants

24. Two vertical poles of heights, 20 m and 80 m stand apart on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is : **[April 08, 2019 (II)]**

(a) 15 (b) 18 (c) 12 (d) 16

25. If a straight line passing through the point P(–3, 4) is such that its intercepted portion between the coordinate axes is bisected at P, then its equation is : **[Jan. 12, 2019 (II)]**

(a) $3x-4y+25=0$ (b) $4x-3y+24=0$
(c) $x-y+7=0$ (d) $4x+3y=0$

26. If in a parallelogram ABDC, the coordinates of A, B and C are respectively (1, 2), (3, 4) and (2, 5), then the equation of the diagonal AD is : **[Jan. 11, 2019 (II)]**

(a) $5x-3y+1=0$ (b) $5x+3y-11=0$

(c) $3x-5y+7=0$ (d) $3x+5y-13=0$

27. A point P moves on the line $2x-3y+4=0$. If Q(1, 4) and R(3, –2) are fixed points, then the locus of the centroid of ΔPQR is a line: **[Jan. 10, 2019 (I)]**

(a) with slope $\frac{3}{2}$ (b) parallel to x-axis

(c) with slope $\frac{2}{3}$ (d) parallel to y-axis

28. If the line $3x+4y-24=0$ intersects the x-axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin, is: **[Jan. 10, 2019 (I)]**

(a) (3,4) (b) (2,2) (c) (4,3) (d) (4,4)

29. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is : **[2018]**

(a) $2x+3y=xy$ (b) $3x+2y=xy$

(c) $3x+2y=6xy$ (d) $3x+2y=6$

30. In a triangle ABC, coordianates of A are (1, 2) and the equations of the medians through B and C are $x+y=5$ and $x=4$ respectively. Then area of ΔABC (in sq. units) is **[Online April 15, 2018]**

(a) 5 (b) 9 (c) 12 (d) 4

31. Two sides of a rhombus are along the lines, $x-y+1=0$ and $7x-y-5=0$. If its diagonals intersect at (–1, –2), then which one of the following is a vertex of this rhombus? **[2016]**

(a) $\left(\frac{1}{3},-\frac{8}{3}\right)$ (b) $\left(-\frac{10}{3},-\frac{7}{3}\right)$

(c) (–3, –9) (d) (–3, –8)

32. A straight line through origin O meets the lines $3y=10-4x$ and $8x+6y+5=0$ at points A and B respectively. Then O divides the segment AB in the ratio : **[Online April 10, 2016]**

(a) 2 : 3 (b) 1 : 2 (c) 4 : 1 (d) 3 : 4

33. If a variable line drawn through the intersection of the lines $\frac{x}{3}+\frac{y}{4}=1$ and $\frac{x}{4}+\frac{y}{3}=1$, meets the coordinate axes at A and B, $(A \neq B)$, then the locus of the midpoint of AB is : **[Online April 9, 2016]**

(a) $7xy=6(x+y)$

(b) $4(x+y)^2-28(x+y)+49=0$

(c) $6xy=7(x+y)$

(d) $14(x+y)^2-97(x+y)+168=0$

34. The point (2, 1) is translated parallel to the line L : $x-y=4$ by $2\sqrt{3}$ units. If the new points Q lies in the third quadrant, then the equation of the line passing through Q and perpendicular to L is : **[Online April 9, 2016]**

(a) $x+y=2-\sqrt{6}$ (b) $2x+2y=1-\sqrt{6}$

(c) $x+y=3-3\sqrt{6}$ (d) $x+y=3-2\sqrt{6}$

35. A straight line L through the point (3, – 2) is inclined at an angle of 60° to the line $\sqrt{3}x+y=1$. If L also intersects the x-axis, then the equation of L is : **[Online April 11, 2015]**

(a) $y+\sqrt{3}x+2-3\sqrt{3}=0$

(b) $\sqrt{3}y+x-3+2\sqrt{3}=0$

(c) $y-\sqrt{3}x+2+3\sqrt{3}=0$

(d) $\sqrt{3}y-x+3+2\sqrt{3}=0$

36. The circumcentre of a triangle lies at the origin and its centroid is the mid point of the line segment joining the points (a^2+1, a^2+1) and $(2a, -2a)$, $a \neq 0$. Then for any a, the orthocentre of this triangle lies on the line: **[Online April 19, 2014]**

(a) $y-2ax=0$

(b) $y-(a^2+1)x=0$

(c) $y+x=0$

(d) $(a-1)^2x-(a+1)^2y=0$

37. If a line intercepted between the coordinate axes is trisected at a point A(4, 3), which is nearer to x-axis, then its equation is: **[Online April 12, 2014]**

(a) $4x-3y=7$ (b) $3x+2y=18$

(c) $3x+8y=36$ (d) $x+3y=13$

38. Given three points P, Q, R with P(5, 3) and R lies on the x-axis. If equation of RQ is $x-2y=2$ and PQ is parallel to the x-axis, then the centroid of ΔPQR lies on the line: **[Online April 9, 2014]**

(a) $2x+y-9=0$ (b) $x-2y+1=0$

(c) $5x-2y=0$ (d) $2x-5y=0$

39. A ray of light along $x+\sqrt{3}y=\sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is **[2013]**

(a) $y=x+\sqrt{3}$ (b) $\sqrt{3}y=x-\sqrt{3}$

(c) $y=\sqrt{3}x-\sqrt{3}$ (d) $\sqrt{3}y=x-1$

40. Let A (–3, 2) and B (–2, 1) be the vertices of a triangle ABC. If the centroid of this triangle lies on the line $3x+4y+2=0$, then the vertex C lies on the line : **[Online April 25, 2013]**

(a) $4x+3y+5=0$ (b) $3x+4y+3=0$

(c) $4x+3y+3=0$ (d) $3x+4y+5=0$

41. If the extremities of the base of an isosceles triangle are the points $(2a, 0)$ and $(0, a)$ and the equation of one of the sides is $x = 2a$, then the area of the triangle, in square units, is : **[Online April 23, 2013]**

(a) $\frac{5}{4}a^2$ (b) $\frac{5}{2}a^2$ (c) $\frac{25a^2}{4}$ (d) $5a^2$

42. If the x-intercept of some line L is double as that of the line, $3x + 4y = 12$ and the y-intercept of L is half as that of the same line, then the slope of L is : **[Online April 22, 2013]**

(a) -3 (b) $-3/8$ (c) $-3/2$ (d) $-3/16$

43. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1,1) and (2,4) in the ratio 3 :2, then k equals : **[2012]**

(a) $\frac{29}{5}$ (b) 5 (c) 6 (d) $\frac{11}{5}$

44. The line parallel to x-axis and passing through the point of intersection of lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is **[Online May 26, 2012]**

(a) above x-axis at a distance 2/3 from it
(b) above x-axis at a distance 3/2 from it
(c) below x-axis at a distance 3/2 from it
(d) below x-axis at a distance 2/3 from it

45. If the point $(1, a)$ lies between the straight lines $x + y = 1$ and $2(x + y) = 3$ then a lies in interval **[Online May 12, 2012]**

(a) $\left(\frac{3}{2}, \infty\right)$ (b) $\left(1, \frac{3}{2}\right)$ (c) $(-\infty, 0)$ (d) $\left(0, \frac{1}{2}\right)$

46. If the straight lines $x + 3y = 4$, $3x + y = 4$ and $x + y = 0$ form a triangle, then the triangle is **[Online May 7, 2012]**

(a) scalene
(b) equilateral triangle
(c) isosceles
(d) right angled isosceles

47. If A $(2, -3)$ and B $(-2, 1)$ are two vertices of a triangle and third vertex moves on the line $2x + 3y = 9$, then the locus of the centroid of the triangle is : **[2011RS]**

(a) $x - y = 1$ (b) $2x + 3y = 1$
(c) $2x + 3y = 3$ (d) $2x - 3y = 1$

48. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}$, $x > 0$ and $y = 3x$, $x > 0$, then a belong to **[2006]**

(a) $\left(0, \frac{1}{2}\right)$ (b) $(3, \infty)$ (c) $\left(\frac{1}{2}, 3\right)$ (d) $\left(-3, -\frac{1}{2}\right)$

49. A straight line through the point A (3, 4) is such that its intercept between the axes is bisected at A. Its equation is **[2006]**

(a) $x + y = 7$ (b) $3x - 4y + 7 = 0$
(c) $4x + 3y = 24$ (d) $3x + 4y = 25$

50. The line parallel to the x- axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is **[2005]**

(a) below the x - axis at a distance of $\frac{3}{2}$ from it
(b) below the x - axis at a distance of $\frac{2}{3}$ from it
(c) above the x - axis at a distance of $\frac{3}{2}$ from it
(d) above the x - axis at a distance of $\frac{2}{3}$ from it

51. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is -1 is **[2004]**

(a) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$
(b) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$
(c) $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{1} = 1$
(d) $\frac{x}{2} + \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

52. Let $A(2, -3)$ and $B(-2, 3)$ be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line **[2004]**

(a) $3x - 2y = 3$ (b) $2x - 3y = 7$
(c) $3x + 2y = 5$ (d) $2x + 3y = 9$

53. Locus of mid point of the portion between the axes of $x \cos\alpha + y \sin\alpha = p$ whre p is constant is **[2002]**

(a) $x^2 + y^2 = \frac{4}{p^2}$ (b) $x^2 + y^2 = 4p^2$
(c) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{2}{p^2}$ (d) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$

TOPIC 3 **Distance Between two Lines, Angle Between two Lines and Bisector of the Angle Between the two Lines, Perpendicular Distance of a Point from a Line, Foot of the Perpendicular, Position of a Point with Respect to a Line, Pedal Points, Condition for Concurrency of Three Lines**

54. Let L denote the line in the xy-plane with x and y intercepts as 3 and 1 respectively. Then the image of the point $(-1, -4)$ in this line is: **[Sep. 06, 2020 (II)]**

(a) $\left(\frac{11}{5}, \frac{28}{5}\right)$ (b) $\left(\frac{29}{5}, \frac{8}{5}\right)$

(c) $\left(\frac{8}{5}, \frac{29}{5}\right)$ (d) $\left(\frac{29}{5}, \frac{11}{5}\right)$

55. If the line, $2x - y + 3 = 0$ is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines $4x - 2y + \alpha = 0$ and $6x - 3y + \beta = 0$, respectively, then the sum of all possible value of α and β is ______. **[NA Sep. 05, 2020 (I)]**

56. The locus of the mid-points of the perpendiculars drawn from points on the line, $x = 2y$ to the line $x = y$ is: **[Jan. 7, 2020 (II)]**

(a) $2x - 3y = 0$ (b) $5x - 7y = 0$

(c) $3x - 2y = 0$ (d) $7x - 5y = 0$

57. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60° with the line $x + y = 0$. Then an equation of the line L is: **[April 12, 2019 (II)]**

(a) $x + \sqrt{3}y = 8$

(b) $(\sqrt{3}+1)x + (\sqrt{3}-1)y = 8\sqrt{2}$

(c) $\sqrt{3}x + y = 8$

(d) None of these

58. Lines are drawn parallel to the line $4x - 3y + 2 = 0$, at a distance $\frac{3}{5}$ from the origin. Then which one of the following points lies on any of these lines ? **[April 10, 2019 (II)]**

(a) $\left(-\frac{1}{4}, \frac{2}{3}\right)$ (b) $\left(\frac{1}{4}, -\frac{1}{3}\right)$

(c) $\left(\frac{1}{4}, \frac{1}{3}\right)$ (d) $\left(-\frac{1}{4}, -\frac{2}{3}\right)$

59. If the two lines $x + (a-1)y = 1$ and $2x + a^2y = 1$ $(a \in R - \{0, 1\})$ are perpendicular, then the distance of their point of intersection from the origin is: **[April 09, 2019 (II)]**

(a) $\sqrt{\frac{2}{5}}$ (b) $\frac{2}{5}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{\sqrt{2}}{5}$

60. A rectangle is inscribed in a circle with a diameter lying along the line $3y = x + 7$. If the two adjacent vertices of the rectangle are $(-8, 5)$ and $(6, 5)$, then the area of the rectangle (in sq. units) is: **[April 09, 2019 (II)]**

(a) 84 (b) 98 (c) 72 (d) 56

61. Suppose that the points (h, k), $(1, 2)$ and $(-3, 4)$ lie on the line L_1. If a line L_2 passing through the points (h, k) and $(4, 3)$ is perpendicular on L_1, then equals : **[April 08, 2019 (II)]**

(a) $\frac{1}{3}$ (b) 0 (c) 3 (d) $-\frac{1}{7}$

62. If the straight line, $2x - 3y + 17 = 0$ is perpendicular to the line passing through the points $(7, 17)$ and $(15, \beta)$, then β equals : **[Jan. 12, 2019 (I)]**

(a) $\frac{35}{3}$ (b) -5 (c) $-\frac{35}{3}$ (d) 5

63. Two sides of a parallelogram are along the lines, $x + y = 3$ and $x - y + 3 = 0$. If its diagonals intersect at $(2, 4)$, then one of its vertex is: **[Jan. 10, 2019 (II)]**

(a) $(3, 5)$ (b) $(2, 1)$ (c) $(2, 6)$ (d) $(3, 6)$

64. Consider the set of all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$. Which one of the following statements is true? **[Jan. 9, 2019 (I)]**

(a) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$.

(b) Each line passes through the origin.

(c) The lines are all parallel.

(d) The lines are not concurrent.

65. Let the equations of two sides of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at $(1, 1)$, then the equation of its third side is: **[Jan. 09, 2019 (II)]**

(a) $122y - 26x - 1675 = 0$

(b) $122y + 26x + 1675 = 0$

(c) $26x + 61y + 1675 = 0$

(d) $26x - 122y - 1675 = 0$

66. The foot of the perpendicular drawn from the origin, on the line, $3x + y = \lambda(\lambda \neq 0)$ is P. If the line meets x-axis at A and y-axis at B, then the ratio $BP : PA$ is **[Online April 15, 2018]**

(a) $9:1$ (b) $1:3$ (c) $1:9$ (d) $3:1$

67. The sides of a rhombus $ABCD$ are parallel to the lines, $x - y + 2 = 0$ and $7x - y + 3 = 0$. If the diagonals of the rhombus intersect at $P(1, 2)$ and the vertex A (different from the origin) is on the y-axis, then the ordinate of A is **[Online April 15, 2018]**

(a) 2 (b) $\frac{7}{4}$ (c) $\frac{7}{2}$ (d) $\frac{5}{2}$

68. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then **[2014]**

(a) $3bc - 2ad = 0$ (b) $3bc + 2ad = 0$
(c) $2bc - 3ad = 0$ (d) $2bc + 3ad = 0$

69. Let PS be the median of the triangle vertices $P(2, 2)$, $Q(6, -1)$ and $R(7, 3)$. The equation of the line passing through $(1, -1)$ and parallel to PS is: **[2014]**

(a) $4x + 7y + 3 = 0$ (b) $2x - 9y - 11 = 0$
(c) $4x - 7y - 11 = 0$ (d) $2x + 9y + 7 = 0$

70. If a line L is perpendicular to the line $5x - y = 1$, and the area of the triangle formed by the line L and the coordinate axes is 5, then the distance of line L from the line $x + 5y = 0$ is: **[Online April 19, 2014]**

(a) $\frac{7}{\sqrt{5}}$ (b) $\frac{5}{\sqrt{13}}$ (c) $\frac{7}{\sqrt{13}}$ (d) $\frac{5}{\sqrt{7}}$

71. If the three distinct lines $x + 2ay + a = 0$, $x + 3by + b = 0$ and $x + 4ay + a = 0$ are concurrent, then the point (a, b) lies on a: **[Online April 12, 2014]**

(a) circle (b) hyperbola
(c) straight line (d) parabola

72. The base of an equilateral triangle is along the line given by $3x + 4y = 9$. If a vertex of the triangle is $(1, 2)$, then the length of a side of the triangle is: **[Online April 11, 2014]**

(a) $\frac{2\sqrt{3}}{15}$ (b) $\frac{4\sqrt{3}}{15}$ (c) $\frac{4\sqrt{3}}{5}$ (d) $\frac{2\sqrt{3}}{5}$

73. If the image of point P(2, 3) in a line L is Q(4, 5), then the image of point R(0, 0) in the same line is: **[Online April 25, 2013]**

(a) (2, 2) (b) (4, 5) (c) (3, 4) (d) (7, 7)

74. Let θ_1 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_2 = 0$ and θ_2 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_3 = 0$, where c_1, c_2, c_3 are any real numbers :

Statement-1: If c_2 and c_3 are proportional, then $\theta_1 = \theta_2$.
Statement-2: $\theta_1 = \theta_2$ for all c_2 and c_3. **[Online April 23, 2013]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation of Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation of Statement-1.
(c) Statement-1 is false; Statement-2 is true.
(d) Statement-1 is true; Statement-2 is false.

75. If the three lines $x - 3y = p$, $ax + 2y = q$ and $ax + y = r$ form a right-angled triangle then : **[Online April 9, 2013]**

(a) $a^2 - 9a + 18 = 0$ (b) $a^2 - 6a - 12 = 0$
(c) $a^2 - 6a - 18 = 0$ (d) $a^2 - 9a + 12 = 0$

76. Consider the straight lines
$L_1 : x - y = 1$
$L_2 : x + y = 1$
$L_3 : 2x + 2y = 5$
$L_4 : 2x - 2y = 7$
The correct statement is **[Online May 26, 2012]**

(a) $L_1 \parallel L_4$, $L_2 \parallel L_3$, L_1 intersect L_4.
(b) $L_1 \perp L_2$, $L_1 \parallel L_3$, L_1 intersect L_2.
(c) $L_1 \perp L_2$, $L_2 \parallel L_3$, L_1 intersect L_4.
(d) $L_1 \perp L_2$, $L_1 \perp L_3$, L_2 intersect L_4.

77. If $a, b, c \in$ R and 1 is a root of equation $ax^2 + bx + c = 0$, then the curve $y = 4ax^2 + 3bx + 2c$, $a \neq 0$ intersect x-axis at **[Online May 26, 2012]**

(a) two distinct points whose coordinates are always rational numbers
(b) no point
(c) exactly two distinct points
(d) exactly one point

78. Let L be the line $y = 2x$, in the two dimensional plane. **[Online May 19, 2012]**

Statement 1: The image of the point (0, 1) in L is the point $\left(\frac{4}{5}, \frac{3}{5}\right)$.

Statement 2: The points (0, 1) and $\left(\frac{4}{5}, \frac{3}{5}\right)$ lie on opposite sides of the line L and are at equal distance from it.

(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
(d) Statement 1 is false, Statement 2 is true.

79. If two vertices of a triangle are (5, –1) and (–2, 3) and its orthocentre is at (0, 0), then the third vertex is **[Online May 12, 2012]**

(a) (4, –7) (b) (–4, –7) (c) (–4, 7) (d) (4, 7)

80. If two vertical poles 20 m and 80 m high stand apart on a horizontal plane, then the height (in m) of the point of intersection of the lines joining the top of each pole to the foot of other is **[Online May 7, 2012]**

(a) 16 (b) 18 (c) 50 (d) 15

81. The point of intersection of the lines
$(a^3 + 3)x + ay + a - 3 = 0$ and
$(a^5 + 2)x + (a + 2)y + 2a + 3 = 0$ (a real) lies on the y-axis for **[Online May 7, 2012]**

(a) no value of a (b) more than two values of a
(c) exactly one value of a (d) exactly two values of a

82. The lines $x + y = |a|$ and $ax - y = 1$ intersect each other in the first quadrant. Then the set of all possible values of a in the interval : **[2011RS]**

(a) $(0, \infty)$ (b) $[1, \infty)$ (c) $(-1, \infty)$ (d) $(-1, 1)$

83. The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[2011]**

Statement-1: The ratio $PR : RQ$ equals $2\sqrt{2} : \sqrt{5}$

Statement-2: In any triangle, bisector of an angle divides the triangle into two similar triangles.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

84. The lines $p(p^2+1)x - y + q = 0$ and $(p^2+1)^2x + (p^2+1)y + 2q = 0$ are perpendicular to a common line for : **[2009]**

(a) exactly one values of p
(b) exactly two values of p
(c) more than two values of p
(d) no value of p

85. The shortest distance between the line $y - x = 1$ and the curve $x = y^2$ is : **[2009]**

(a) $\dfrac{2\sqrt{3}}{8}$ (b) $\dfrac{3\sqrt{2}}{5}$ (c) $\dfrac{\sqrt{3}}{4}$ (d) $\dfrac{3\sqrt{2}}{8}$

86. The perpendicular bisector of the line segment joining P (1, 4) and Q(k, 3) has y-intercept –4. Then a possible value of k is **[2008]**

(a) 1 (b) 2 (c) –2 (d) – 4

87. Let $P = (-1, 0)$, $Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three point. The equation of the bisector of the angle PQR is **[2007]**

(a) $\dfrac{\sqrt{3}}{2}x + y = 0$ (b) $x + \sqrt{3}y = 0$

(c) $\sqrt{3}x + y = 0$ (d) $x + \dfrac{\sqrt{3}}{2}y = 0$

88. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) **[2003]**

(a) are vertices of a triangle
(b) lie on a straight line
(c) lie on an ellipse
(d) lie on a circle.

89. A square of side a lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha \left(0 < \alpha < \frac{\pi}{4}\right)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is **[2003]**

(a) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$
(b) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$
(c) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$
(d) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$

TOPIC 4 Pair of Straight Lines

90. The equation $y = \sin x \sin(x + 2) - \sin^2(x + 1)$ represents a straight line lying in : **[April 12, 2019 (I)]**

(a) second and third quadrants only
(b) first, second and fourth quadrant
(c) first, third and fourth quadrants
(d) third and fourth quadrants only

91. If one of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines $xy = 0$, then m is **[2007]**

(a) 1 (b) 2 (c) –1/2 (d) –2

92. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then c equals **[2004]**

(a) –3 (b) 1 (c) 3 (d) 1

93. If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product c has the value **[2004]**

(a) –2 (b) –1 (c) 2 (d) 1

94. If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then **[2003]**

(a) $pq = -1$ (b) $p = q$ (c) $p = -q$ (d) $pq = 1$.

95. The pair of lines represented by $3ax^2 + 5xy + (a^2 - 2)y^2 = 0$ are perpendicular to each other for **[2002]**

(a) two values of a (b) $\forall\, a$
(c) for one value of a (d) for no values of a

96. If the pair of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ intersect on the y-axis then **[2002]**

(a) $2fgh = bg^2 + ch^2$ (b) $bg^2 \neq ch^2$
(c) $abc = 2fgh$ (d) none of these

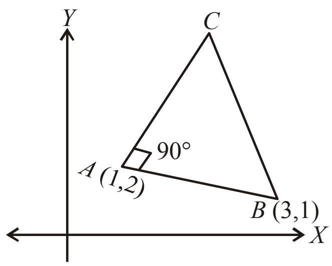
Hints & Solutions

1. **(b)** Let ΔABC be in the first quadrant

Slope of line $AB = -\frac{1}{2}$

Slope of line $AC = 2$

Length of $AB = \sqrt{5}$



It is given that ar$(\Delta ABC) = 5\sqrt{5}$

$\therefore \frac{1}{2} AB \cdot AC = 5\sqrt{5} \Rightarrow AC = 10$

$\therefore$ Coordinate of vertex $C = (1+10\cos\theta,\ 2+10\sin\theta)$

$\because \tan\theta = 2 \Rightarrow \cos\theta = \frac{1}{\sqrt{5}},\ \sin\theta = \frac{2}{\sqrt{5}}$

$\therefore$ Coordinate of C $= (1+2\sqrt{5},\ 2+4\sqrt{5})$

$\therefore$ Abscissa of vertex C is $1+2\sqrt{5}$.

2. **(b)** Mid point of line segment PQ be $\left(\frac{k+1}{2}, \frac{7}{2}\right)$.

$\therefore$ Slope of perpendicular line passing through

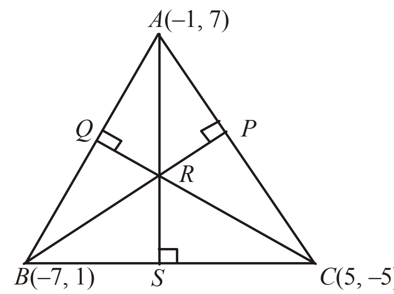
$(0, -4)$ and $\left(\frac{k+1}{2}, \frac{7}{2}\right) = \frac{\frac{7}{2}+4}{\frac{k+1}{2}-0} = \frac{15}{k+1}$

Slope of $PQ = \frac{4-3}{1-k} = \frac{1}{1-k}$

$\therefore \frac{15}{1+k} \times \frac{1}{1-k} = -1$

$1-k^2 = -15 \Rightarrow k = \pm 4$.

3. **(b)**



$m_{BC} = \frac{6}{-12} = -\frac{1}{2}$

$\therefore$ Equation of AS is $y-7 = 2(x+1)$

$y = 2x+9$ ...(i)

$m_{AC} = \frac{12}{-6} = -2$

$\therefore$ Equation of BP is $y-1 = \frac{1}{2}(x+7)$

$y = \frac{x}{2}+\frac{9}{2}$ ...(ii)

From equs. (i) and (ii),

$2x+9 = \frac{x+9}{2}$

$\Rightarrow 4x+18 = x+9$

$\Rightarrow 3x = 9 \Rightarrow x = -3$

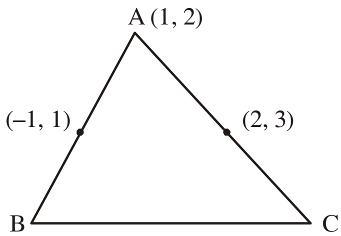
$\therefore y = 3$

4. **(5)** P will be centroid of ΔABC

$P\left(\frac{17}{6}, \frac{8}{3}\right) \Rightarrow PQ = \sqrt{(4)^2+(3)^2} = 5$

5. **(b)** From the mid-point formula co-ordinates of vertex B and C are $B(-3, 0)$ and $C(3, 4)$.
Now, centroid of the triangle

$G \equiv \left(\frac{3-3+1}{3}, \frac{0+4+2}{3}\right) \Rightarrow G \equiv \left(\frac{1}{3}, 2\right)$

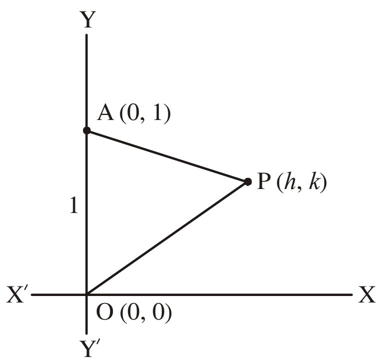


6. **(c)** Let point $P(h, k)$

$\because \quad OA = 1$

So, $OP + AP = 3$

$\Rightarrow \sqrt{h^2+k^2} + \sqrt{h^2+(k-1)^2} = 3$



$\Rightarrow h^2 + (k-1)^2 = 9 + h^2 + k^2 - 6\sqrt{h^2+k^2}$

$\Rightarrow 6\sqrt{h^2+k^2} = 2k + 8$

$\Rightarrow 9h^2 + 8k^2 - 8k - 16 = 0$

Hence, locus of point P is

$9x^2 + 8y^2 - 8y - 16 = 0$

7. **(b)** Since, $m_{QR} \times m_{PH} = -1$

$\Rightarrow m_{QR} = -\dfrac{1}{m_{PH}}$

$\Rightarrow m_{QR} = \dfrac{y-3}{x-4} = 0$

$\Rightarrow y = 3$

$m_{PQ} \times m_{RH} = -1$

$\Rightarrow \dfrac{1}{4} \times \dfrac{y}{x} = -1$

$\Rightarrow y = -4x$

$\Rightarrow x = -\dfrac{3}{4}$

Vertex R is $\left(\dfrac{-3}{4}, 3\right)$

Hence, vertex R lies in second quadrant.

8. **(b)** Since Orthocentre of the triangle is A(–3, 5) and centriod of the triangle is B(3, 3), then

$AB = \sqrt{40} = 2\sqrt{10}$

Centroid divides orthocentre and circumcentre of the triangle in ratio 2 : 1

$\therefore AB : BC = 2 : 1$

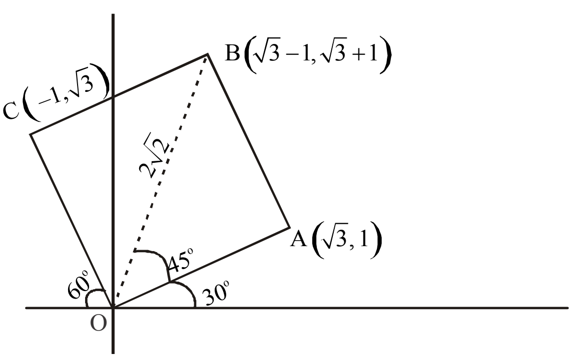
Now, $AB = \dfrac{2}{3}AC$

$\Rightarrow AC = \dfrac{3}{2}AB = \dfrac{3}{2}(2\sqrt{10}) \quad \Rightarrow AC = 3\sqrt{10}$

$\therefore$ Radius of circle with AC as diametre

$= \dfrac{AC}{2} = \dfrac{3}{2}\sqrt{10} = 3\sqrt{\dfrac{5}{2}}$

9. **(b)**



For A;

$\dfrac{x}{\cos 30^\circ} = \dfrac{y}{\sin 30^\circ} = 2$

$\Rightarrow x = \sqrt{3}$ and $y = 1$

For C,

$\dfrac{x}{\cos 120^\circ} = \dfrac{y}{\sin 120^\circ} = 2$

$\Rightarrow x = -1, y = \sqrt{3}$

For B,

$\dfrac{x}{\cos 75^\circ} = \dfrac{y}{\sin 75^\circ} = 2\sqrt{2}$

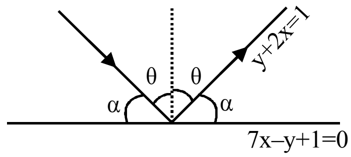
$\Rightarrow x = \sqrt{3} - 1$

and $y = \sqrt{3} + 1$

$\therefore$ Sum $= 2\sqrt{3} - 2$

10. **(c)** Let slope of incident ray be m.

$\therefore$ angle of incidence = angle of reflection



$\therefore \left|\frac{m-7}{1+7m}\right| = \left|\frac{-2-7}{1-14}\right| = \frac{9}{13}$

$\Rightarrow \frac{m-7}{1+7m} = \frac{9}{13}$ or $\frac{m-7}{1+7m} = -\frac{9}{13}$

$\Rightarrow 13m - 91 = 9 + 63m$ or $13m - 91 = -9 - 63m$

$\Rightarrow 50m = -100$ or $76m = 82$

$\Rightarrow m = -\frac{1}{2}$ or $m = \frac{41}{38}$

$\Rightarrow$ $y - 1 = -\frac{1}{2}(x - 0)$ or $y - 1 = \frac{41}{38}(x - 0)$

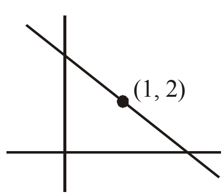
i.e $x + 2y - 2 = 0$ or $38y - 38 - 41x = 0$

$\Rightarrow 41x - 38y + 38 = 0$

11. **(a)** Equation of line L

$\frac{x}{2} + \frac{y}{4} = 1$

$2x + y = 4$ ...(i)



For line

$x - 2y = -4$ ...(ii)

solving equation (i) and (ii); we get point of intersection

$\left(4/5, \frac{12}{5}\right)$

12. **(c)** $A\left(0, \frac{8}{3}\right) B(1,3) C(89,30)$

Slope of AB $= \frac{1}{3}$

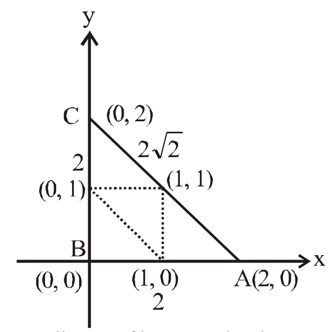
Slope of BC $= \frac{1}{3}$

So, lies on same line

13. **(b)** From the figure, we have

$a = 2, b = 2\sqrt{2}, c = 2$

$x_1 = 0, x_2 = 0, x_3 = 2$



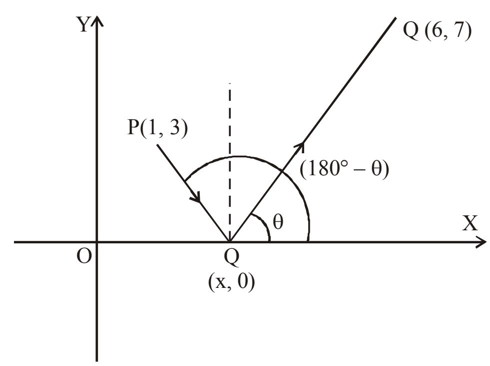
Now, x-co-ordinate of incentre is given as

$\frac{ax_1 + bx_2 + cx_3}{a + b + c}$

$\Rightarrow$ x-coordinate of incentre

$= \frac{2 \times 0 + 2\sqrt{2}.0 + 2.2}{2 + 2 + 2\sqrt{2}} = \frac{2}{2 + \sqrt{2}} = 2 - \sqrt{2}$

14. **(d)** Let abcissa of Q = x



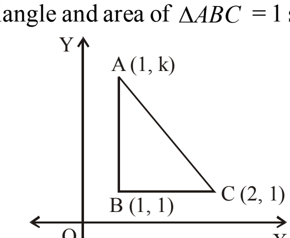
$\therefore$ Q = $(x, 0)$

$\tan\theta = \frac{0-7}{x-6}$, $\tan(180° - \theta) = \frac{0-3}{x-1}$

Now, $\tan(180° - \theta) = -\tan\theta$

$\therefore \frac{-3}{x-1} = \frac{-7}{x-6} \Rightarrow x = \frac{5}{2}$

15. **(a)** **Given :** A(l, k), B(1, 1) and C(2, 1) are vertices of a right angled triangle and area of ΔABC = 1 square unit

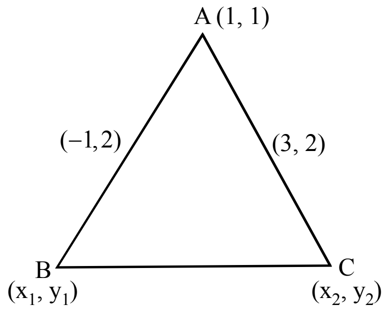


We know that, area of right angled triangle

$= \frac{1}{2} \times BC \times AB = 1 = \frac{1}{2}(a)|(k-1)|$

$\Rightarrow \pm(k-1) = 2 \Rightarrow k = -1, 3$

16. **(c)** Vertex of triangle is (1, 1) and midpoint of sides through this vertex is (– 1, 2) and (3, 2)



$$\frac{1+x_1}{2}=-1,\ \frac{1+y_1}{2}=2$$

$\Rightarrow$ B(–3, 3)

$$\frac{1+x_2}{2}=3,\frac{1+y_2}{2}=2$$

$\Rightarrow$ C (–5, 3)

$\therefore$ Centroid is $\frac{1-3+5}{3},\frac{1+3+3}{3}$

$\Rightarrow \left(1,\frac{7}{3}\right)$

17. **(b)** $(x-a_1)^2+(y-b_1)^2 = (x-a_2)^2+(y-b_2)^2$

$(a_1-a_2)x+(b_1-b_2)y+\frac{1}{2}(a_2{}^2+b_2{}^2-a_1{}^2-b_1{}^2)=0$

Comparing with given eqn. we get

$c=\frac{1}{2}(a_2{}^2+b_2{}^2-a_1{}^2-b_1{}^2)$

18. **(c)** We know that centroid

$$(x,y)=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

$$x=\frac{a\cos t+b\sin t+1}{3}$$

$\Rightarrow a\cos t+b\sin t=3x-1$

$$y=\frac{a\sin t-b\cos t}{3}$$

$\Rightarrow a\sin t-b\cos t=3y$

Squaring and adding,

$(3x-1)^2+(3y)^2=a^2+b^2$

19. **(a)** $AB=\sqrt{(4+1)^2+(0+1)^2}=\sqrt{26}$;

$BC=\sqrt{(3+1)^2+(5+1)^2}=\sqrt{52}$

$CA=\sqrt{(4-3)^2+(0-5)^2}=\sqrt{26}$;

$\because AB=CA$

$\therefore$ Isosceles triangle

$\because (\sqrt{26})^2+(\sqrt{26})^2=52$

$BC^2=AB^2+AC^2$

$\therefore$ right angled triangle,

So, the given triangle is isosceles right angled .

20. **(5)**

$$f'(x)=\begin{cases}5x^4\cdot\sin\left(\frac{1}{x}\right)-x^3\cos\left(\frac{1}{x}\right)+10x, & x<0\\ 0, & x=0\\ 5x^4\cos\left(\frac{1}{x}\right)+x^3\sin\left(\frac{1}{x}\right)+2\lambda x, & x>0\end{cases}$$

$$f''(x)=\begin{cases}(20x^3-x)\sin\left(\frac{1}{x}\right)-8x^2\cos\left(\frac{1}{x}\right)+10, & x<0\\ 0, & x=0\\ (20x^3-x)\cos\left(\frac{1}{x}\right)+8x^2\sin\left(\frac{1}{x}\right)+2\lambda, & x>0\end{cases}$$

Now, $f''(0^+)=f''(0^-)\Rightarrow 2\lambda=10\Rightarrow\lambda=5$

21. **(a)** Coordinates of centroides

$$C=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

$$=\left(\frac{3+1+2}{3},\frac{-1+3+4}{3}\right)=(2,2)$$

The given equation of lines are

$x+3y-1=0$ ...(i)

$3x-y+1=0$ ...(ii)

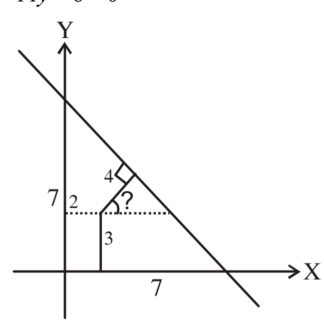
Then, from (i) and (ii)

point of intersection $P\left(-\frac{1}{5},\frac{2}{5}\right)$

equation of line DP

$8x-11y+6=0$

22. **(b)**



Since point at 4 units from P (2, 3) will be

A (4 cosθ + 2, 4 sinθ + 3) and this point will satisfy the equation of line x + y = 7

$$\Rightarrow\cos\theta+\sin\theta=\frac{1}{2}$$

On squaring

$$\Rightarrow \sin 2\theta - \frac{3}{4} \Rightarrow \frac{2\tan\theta}{1+\tan^2\theta} = -\frac{3}{4}$$

$$\Rightarrow 3\tan^2\theta + 8\tan\theta + 3 = 0$$

$$\Rightarrow \tan\theta = \frac{-8 \pm 2\sqrt{7}}{6} \qquad \text{(ignoring –ve sign)}$$

$$\Rightarrow \tan\theta = \frac{-8 \pm 2\sqrt{7}}{6} = \frac{1-\sqrt{7}}{1+\sqrt{7}}$$

23. **(c)** A point which is equidistant from both the axes lies on either $y = x$ and $y = -x$.

Since, point lies on the line $3x + 5y = 15$

Then the required point

$$\begin{array}{r} 3x + 5y = 15 \\ x + y = 0 \\ \hline x = -\frac{15}{2} \end{array}$$

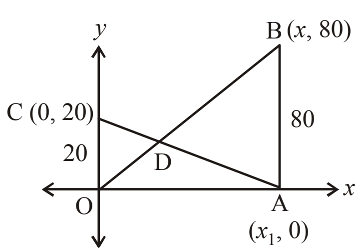
$$y = \frac{15}{2} \Rightarrow (x, y) = \left(-\frac{15}{2}, \frac{15}{2}\right) \quad \{2^{nd} \text{ quadrant}\}$$

$$\text{or} \quad \begin{array}{r} 3x + 5y = 15 \\ x - y = 0 \\ \hline x = \frac{15}{8} \end{array}$$

$$y = \frac{15}{8} \Rightarrow (x, y) = \left(\frac{15}{8}, \frac{15}{8}\right) \quad \{1^{st} \text{ quadrant}\}$$

Hence, the required point lies in 1^{st} and 2^{nd} quadrant.

24. **(d)**



Equations of lines OB and AC are respectively

$$y = \frac{80}{x_1}x \qquad \text{...(i)}$$

$$\frac{x}{x_1} + \frac{y}{20} = 1 \qquad \text{...(ii)}$$

$\because$ equations (i) and (ii) intersect each other

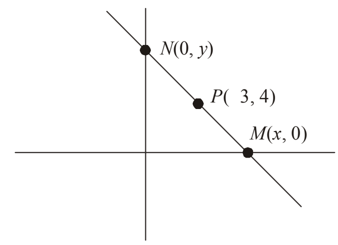
$\therefore$ substitute the value of x from equation (i) to equation (ii), we get

$$\frac{y}{80} + \frac{y}{20} = 1$$

$$\Rightarrow y + 4y = 80 \Rightarrow y = 16 \text{ m}$$

Hence, height of intersection point is 16 m.

25. **(b)** Since, P is mid point of MN



Then, $\frac{0+x}{2} = -3$

$$\Rightarrow x = -3 \times 2 \Rightarrow x = -6$$

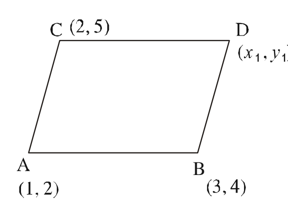
and $\frac{y+0}{2} = 4 \Rightarrow y + 0 = 2 \times 4 \Rightarrow y = 8$

Hence required equation of straight line MN is

$$\frac{x}{-6} + \frac{y}{8} = 1 \Rightarrow 4x - 3y + 24 = 0$$

26. **(a)** Since, in parallelogram mid points of both diagonals coinsides.

$\therefore$ mid-point of AD = mid-point of BC



$$\left(\frac{x_1+1}{2}, \frac{y_1+2}{2}\right) = \left(\frac{3+2}{2}, \frac{4+5}{2}\right)$$

$\therefore \quad (x_1, y_1) = (4, 7)$

Then, equation of AD is,

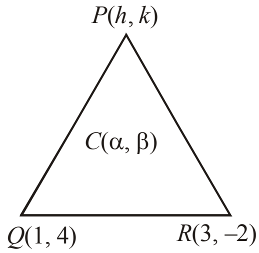
$$y - 7 = \frac{2-7}{1-4}(x-4)$$

$$y - 7 = \frac{5}{3}(x-4)$$

$$3y - 21 = 5x - 20$$

$$5x - 3y + 1 = 0$$

27. **(c)**



Let centroid C be (α, β)

we have $\alpha = \dfrac{1+3+h}{3} \Rightarrow h = 3\alpha - 4$

$\beta = \dfrac{4-2+k}{3} \Rightarrow k = 3\beta - 2$

but $P(h, k)$ lies on $2x - 3y + 4 = 0$

$\Rightarrow\ 2(3\alpha - 4) - 3(3\beta - 2) + 4 = 0$

$\Rightarrow\ 6\alpha - 9\beta - 8 + 6 + 4 = 0$

$\Rightarrow\ 6\alpha - 9\beta + 2 = 0$

Locus: $6x - 9y + 2 = 0$

$\Rightarrow\ y = \dfrac{6}{9}x + \dfrac{2}{9}\ \therefore$ its slope $= \dfrac{6}{9} = \dfrac{2}{3}$

28. **(b)** Equation of the line is:

$3x + 4y = 24$

or $\dfrac{x}{8} + \dfrac{y}{6} = 1$

$\therefore$ coordinates of A, B & O are (8, 0), (0, 6) & (0, 0) respectively.

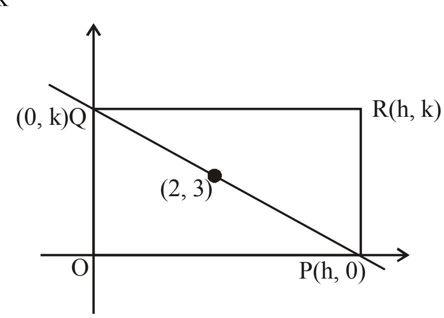
$\Rightarrow OA = 8, OB = 6\ \&\ AB = 10.$

$\therefore$ Incentre of ΔOAB is given as:

$$I \equiv \left(\frac{8\times 0 + 6\times 8 + 10\times 0}{8+6+10}, \frac{8\times 6 + 6\times 0 + 10\times 0}{8+6+10}\right) \equiv (2, 2).$$

29. **(b)** Equation of PQ is

$\dfrac{x}{h} + \dfrac{y}{k} = 1$ ...(i)



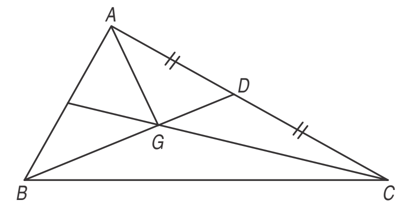
Since, (i) passes through the fixed point (2, 3) Then,

$\dfrac{2}{h} + \dfrac{3}{k} = 1$

Then, the locus of R is $\dfrac{2}{x} + \dfrac{3}{y} = 1$ or $3x + 2y = xy$.

30. **(b)** Median through C is $x = 4$

So the x coordianate of C is 4. let $C \equiv (4, y)$, then the midpoint of A (1, 2) and C (4, y) is D which lies on the median through B.



$\therefore\ D \equiv \left(\dfrac{1+4}{2}, \dfrac{2+y}{2}\right)$

Now, $\dfrac{1+4+2+y}{2} = 5 \Rightarrow y = 3.$

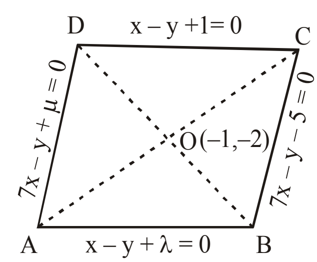
So, $C \equiv (4, 3)$.

The centroid of the triangle is the intersection of the mesians. Here the medians $x = 4$ and $x + 4$ and $x + y = 5$ intersect at G (4, 1).

The area of triangle $\Delta\, ABC = 3 \times \Delta\, AGC$

$= 3 \times \dfrac{1}{2}\,[1\,(1-3) + 4\,(3-2) + 4(2-1)] = 9.$

31. **(a)**



Let other two sides of rhombus are

$x - y + \lambda = 0$

and $7x - y + \mu = 0$

then O is equidistant from AB and DC and from AD and BC

$\therefore\ |-1+2+1| = |-1+2+\lambda| \Rightarrow \lambda = -3$

and $|-7+2-5| = |-7+2+\mu| \Rightarrow \mu = 15$

$\therefore$ Other two sides are $x - y - 3 = 0$ and $7x - y + 15 = 0$

$\therefore$ On solving the eq^{n}s of sides pairwise, we get the vertices as

$\left(\dfrac{1}{3}, \dfrac{-8}{3}\right), (1, 2), \left(\dfrac{-7}{3}, \dfrac{-4}{3}\right), (-3, -6)$

32. **(c)** Length of ⊥ to $4x+3y=10$ from origin (0, 0)

$$P_1=\frac{10}{5}=2$$

Length of ⊥ to $8x+6y+5=0$ from origin (0, 0)

$$P_2=\frac{5}{10}=\frac{1}{2}$$

∵ Lines are parallel to each other ⇒ ratio will be 4 : 1 or 1 : 4

33. **(a)** $L_1: 4x+3y-12=0$

$L_2: 3x+4y-12=0$

$L_1+\lambda L_2=0$

$(4x+3y-12)+\lambda(3x+4y-12)=0$

$x(4+3\lambda)+y(3+4\lambda)-12(1+\lambda)=0$

Point A $\left(\frac{12(1+\lambda)}{4+3\lambda}, 0\right)$

Point B $\left(0, \frac{12(1+\lambda)}{3+4\lambda}\right)$

mid point ⇒ $h=\frac{6(1+\lambda)}{4+3\lambda}$ ..... (i)

$k=\frac{6(1+\lambda)}{3+4\lambda}$ ..... (ii)

Eliminate λ from (i) and (ii), then

$6(h+k)=7hk$

$6(x+y)=7xy$

34. **(d)** $x-y=4$

To find equation of R

slope of L = 0 is 1

⇒ slope of QR = – 1

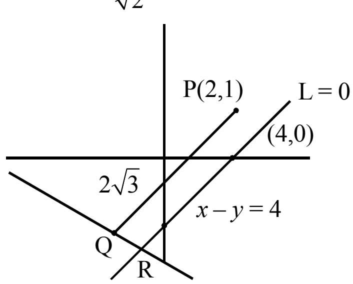
Let QR is $y=mx+c$

$y=-x+c$

$x+y-c=0$

distance of QR from (2, 1) is $2\sqrt{3}$

$$2\sqrt{3}=\frac{|2+1-c|}{\sqrt{2}}$$



$2\sqrt{6}=|3-c|$

$c-3=\pm 2\sqrt{6}$ $c=3\pm 2\sqrt{6}$

Line can be $x+y=3\pm 2\sqrt{6}$

$x+y=3-2\sqrt{6}$

35. **(c)** Given eqn of line is $y+\sqrt{3}x-1=0$

⇒ $y=-\sqrt{3}x+1$

⇒ (slope) $m_2=-\sqrt{3}$

Let the other slope be m_1

$$\therefore \tan 60°=\left|\frac{m_1-(-\sqrt{3})}{1+(-\sqrt{3}m_1)}\right|$$

⇒ $m_1=0, m_2=\sqrt{3}$

Since line L is passing through (3, –2)

$\therefore y-(-2)=+\sqrt{3}(x-3)$

⇒ $y+2=\sqrt{3}(x-3)$

$y-\sqrt{3}x+2+3\sqrt{3}=0$

36. **(d)** Circumcentre = (0, 0)

$$\text{Centroid}=\left(\frac{(a+1)^2}{2}, \frac{(a-1)^2}{2}\right)$$

We know the circumcentre (O), Centroid (G) and orthocentre (H) of a triangle lie on the line joining the O and G.

Also, $\frac{HG}{GO}=\frac{2}{1}$

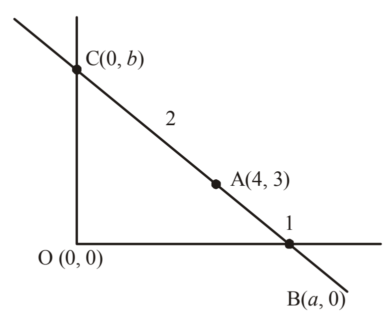
⇒ Coordinate of orthocentre = $\frac{3(a+1)^2}{2}, \frac{3(a-1)^2}{2}$

Now, these coordinates satisfies eqn given in option (d)

Hence, required eqn of line is

$(a-1)^2x-(a+1)^2y=0$

37. **(b)**



A divides CB in 2 : 1

⇒ $4=\left(\frac{1\times 0+2\times a}{1+2}\right)=\frac{2a}{3}$

⇒ $a=6$ ⇒ coordinate of B is B (6, 0)

$3=\left(\frac{1\times b+2\times 0}{1+2}\right)=\frac{b}{3}$

⇒ $b=9$ and C (0, 9)

Slope of line passing through (6, 0), (0, 9)

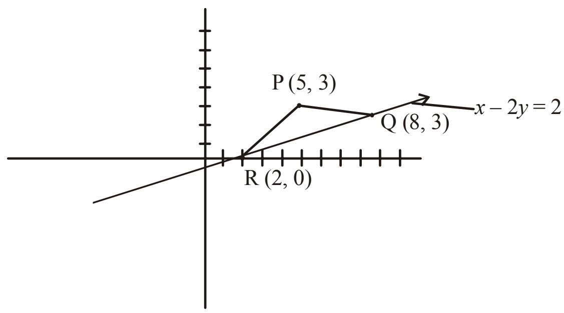
slope, $m=\frac{9}{-6}=-\frac{3}{2}$

Equation of line $y - 0 = \frac{-3}{2}(x-6)$

$2y = -3x + 18$

$3x + 2y = 18$

38. **(d)**



Equation of RQ is $x - 2y = 2$ ...(i)

at $y = 0$, $x = 2$ [R (2, 0)]

as PQ is parallel to x, y-coordinates of Q is also 3

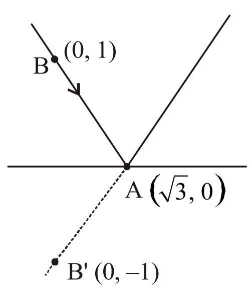
Putting value of y in equation (i), we get

Q (8, 3)

Centroid of $\Delta PQR = \left(\frac{8+5+2}{3}, \frac{3+3}{3}\right) = (5, 2)$

Only $(2x - 5y = 0)$ satisfy the given co-ordinates.

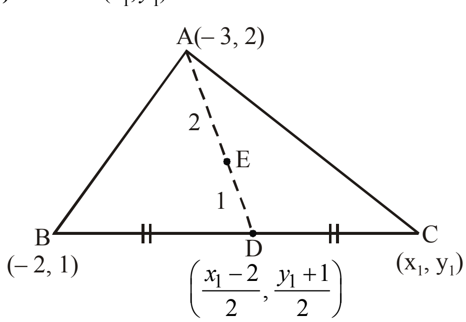
39. **(b)** Suppose B(0, 1) be any point on given line and co-ordinate of A is $(\sqrt{3}, 0)$. So, equation of



Reflected ray is $\frac{-1-0}{0-\sqrt{3}} = \frac{y-0}{x-\sqrt{3}}$

$\Rightarrow \sqrt{3}y = x - \sqrt{3}$

40. **(b)** Let $C = (x_1, y_1)$



Centroid, $E = \left(\frac{x_1 - 5}{3}, \frac{y_1 + 3}{3}\right)$

Since centroid lies on the line

$3x + 4y + 2 = 0$

$\therefore \quad 3\left(\frac{x_1 - 5}{3}\right) + 4\left(\frac{y_1 + 3}{3}\right) + 2 = 0$

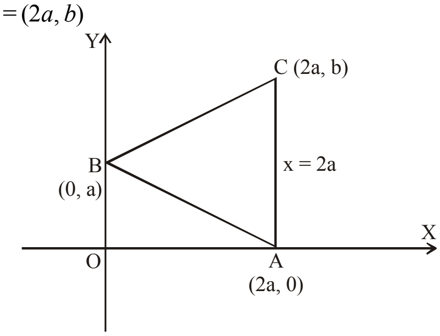
$\Rightarrow \quad 3x_1 + 4y_1 + 3 = 0$

Hence vertex (x_1, y_1) lies on the line

$3x + 4y + 3 = 0$

41. **(b)** Let y-coordinate of $C = b$

$\therefore \ C = (2a, b)$



$AB = \sqrt{4a^2 + a^2} = \sqrt{5}a$

Now, $AC = BC \Rightarrow \ b = \sqrt{4a^2 + (b-a)^2}$

$\Rightarrow b^2 = 4a^2 + b^2 + a^2 - 2ab$

$\Rightarrow 2ab = 5a^2 \Rightarrow b = \frac{5a}{2}$

$\therefore \ C = \left(2a, \frac{5a}{2}\right)$

Hence area of the triangle

$$= \frac{1}{2}\begin{vmatrix} 2a & 0 & 1 \\ 0 & a & 1 \\ 2a & \frac{5a}{2} & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2a & 0 & 1 \\ 0 & a & 1 \\ 0 & \frac{5a}{2} & 0 \end{vmatrix}$$

$$= \frac{1}{2} \times 2a\left(-\frac{5a}{2}\right) = -\frac{5a^2}{2}$$

Since area is always +ve, hence area

$= \frac{5a^2}{2}$ sq. unit

42. **(d)** Given line $3x + 4y = 12$ can be rewritten as

$\frac{3x}{12} + \frac{4y}{12} = 1 \Rightarrow \frac{x}{4} + \frac{y}{3} = 1$

$\Rightarrow x$-intercept $= 4$ and y-intercept $= 3$

Let the required line be

L : $\frac{x}{a}+\frac{y}{b}=1$ where

$a = x$-intercept and $b = y$-intercept

According to the question

$a = 4 \times 2 = 8$ and $b = 3/2$

$\therefore$ Required line is $\frac{x}{8}+\frac{2y}{3}=1$

$\Rightarrow 3x + 16y = 24$

$\Rightarrow y=\frac{-3}{16}x+\frac{24}{16}$

Hence, required slope $=\frac{-3}{16}$.

43. **(c)** Let the points be A (1,1) and B (2,4).
Let point C divides line AB in the ratio 3 : 2.
So, by section formula we have

$$C=\left(\frac{3\times 2+2\times 1}{3+2},\frac{3\times 4+2\times 1}{3+2}\right)=\left(\frac{8}{5},\frac{14}{5}\right)$$

Since Line $2x + y = k$ passes through $C\left(\frac{8}{5},\frac{14}{5}\right)$

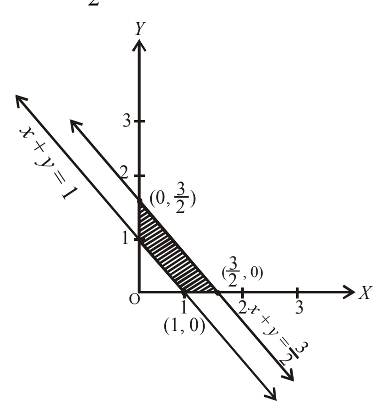
$\Rightarrow \frac{2\times 8}{5}+\frac{14}{5}=k \Rightarrow k=6$

44. **(c)** Given lines are
$ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$
Since, required line is || to x-axis
$\therefore x = 0$
We put $x = 0$ in given equation, we get

$2by = -3b \Rightarrow y=-\frac{3}{2}$

This shows that the required line is below x-axis at a distance of $\frac{3}{2}$ from it.

45. **(d)**



Since, $(1, a)$ lies between $x + y = 1$
and $2(x + y) = 3$
$\therefore$ Put $x = 1$ in $2(x + y) = 3$.
We get the range of y. Thus,

$2(1+y)=3 \Rightarrow y=\frac{3}{2}-1=\frac{1}{2}$

Thus 'a' lies in $\left(0,\frac{1}{2}\right)$

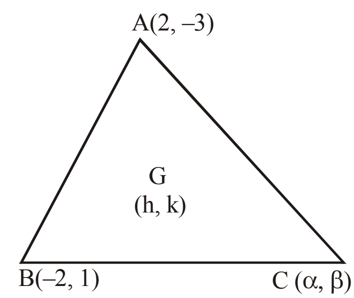
46. **(c)** Let equation of $AB : x + 3y = 4$
Let equation of $BC : 3x + y = 4$
Let equation of $CA : x + y = 0$
Now, By solving these equations we get
$A = (-2, 2), B = (1, 1)$ and $C = (2, -2)$
Now, $AB=\sqrt{9+1}=\sqrt{10}$,

$BC=\sqrt{1+9}=\sqrt{10}$

and $CA=\sqrt{16+16}=\sqrt{32}$

Since, length of AB and BC are same therefore triangle is isosceles.

47. **(b)**



Centroid $(h,k)=\left(\frac{2-2+\alpha}{3},\frac{-3+1+\beta}{3}\right)$

$\therefore \quad \alpha = 3h$

$\beta - 2 = 3k$

$\beta = 3k + 2$

Third vertex (α, β) lies on the line

$2x + 3y = 9$

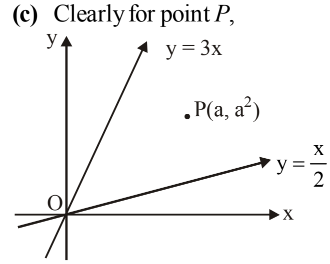
$2\alpha + 3\beta = 9$

$2(3h) + 3(3k + 2) = 9$

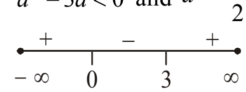
$2h + 3k = 1$

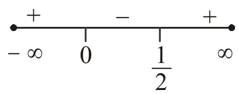
$2x + 3y = 1$

48. **(c)** Clearly for point P,



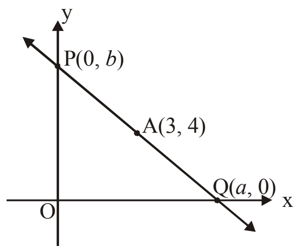
$a^2 - 3a < 0$ and $a^2-\frac{a}{2}>0$





$\Rightarrow \frac{1}{2} < a < 3$

49. **(c)**



$\because$ A is the mid point of PQ,

$\therefore \frac{a+0}{2} = 3, \frac{0+b}{2} = 4 \Rightarrow a = 6, b = 8$

$\therefore$ Equation of line is $\frac{x}{6} + \frac{y}{8} = 1$

or $4x + 3y = 24$

50. **(a)** The eqn. of line passing through the intersection of lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$ is

$ax + 2by + 3b + \lambda(bx - 2ay - 3a) = 0$

$\Rightarrow (a + b\lambda)x + (2b - 2a\lambda)y + 3b - 3\lambda a = 0$

Required line is parallel to x-axis.

$\therefore a + b\lambda = 0 \Rightarrow \lambda = -a/b$

$\Rightarrow ax + 2by + 3b - \frac{a}{b}(bx - 2ay - 3a) = 0$

$\Rightarrow ax + 2by + 3b - ax + \frac{2a^2}{b}y + \frac{3a^2}{b} = 0$

$y\left(2b + \frac{2a^2}{b}\right) + 3b + \frac{3a^2}{b} = 0$

$y\left(\frac{2b^2 + 2a^2}{b}\right) = -\left(\frac{3b^2 + 3a^2}{b}\right)$

$y = \frac{-3(a^2 + b^2)}{2(b^2 + a^2)} = \frac{-3}{2}$

So it is 3/2 units below x-axis.

51. **(a)** Let the required line be $\frac{x}{a} + \frac{y}{b} = 1$ ....(i)

then $a + b = -1 \Rightarrow b = -a - 1$ ....(ii)

(i) passes through (4, 3), $\Rightarrow \frac{4}{a} + \frac{3}{b} = 1$

$\Rightarrow 4b + 3a = ab$ ....(iii)

Putting value of b from (ii) in (iii), we get

$a^2 - 4 = 0 \Rightarrow a = \pm 2 \Rightarrow b = -3$ or 1

$\therefore$ Equations of straight lines are

$\frac{x}{2} + \frac{y}{-3} = 1$ or $\frac{x}{-2} + \frac{y}{1} = 1$

52. **(d)** Let the vertex C be (h, k), then the centroid of ΔABC is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

$= \left(\frac{2 - 2 + h}{3}, \frac{-3 + 1 + k}{3}\right)$

$= \left(\frac{h}{3}, \frac{-2 + k}{3}\right)$. It lies on $2x + 3y = 1$

$\Rightarrow \frac{2h}{3} - 2 + k = 1 \Rightarrow 2h + 3k = 9$

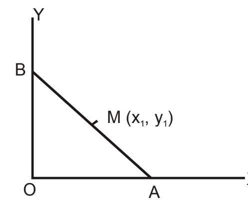
$\Rightarrow$ Locus of C is $2x + 3y = 9$

53. **(d)** Equation of AB is

$x \cos\alpha + y \sin\alpha = p$;

$\Rightarrow \frac{x\cos\alpha}{p} + \frac{y\sin\alpha}{p} = 1$;

$\Rightarrow \frac{x}{p/\cos\alpha} + \frac{y}{p/\sin\alpha} = 1$



So, co-ordinates of A and B are

$\left(\frac{p}{\cos\alpha}, 0\right)$ and $\left(0, \frac{p}{\sin\alpha}\right)$;

So, coordinates of midpoint of AB are

$M(x_1, y_1) = \left(\frac{p}{2\cos\alpha}, \frac{p}{2\sin\alpha}\right)$

$x_1 = \frac{p}{2\cos\alpha}$ & $y_1 = \frac{p}{2\sin\alpha}$;

$\Rightarrow \cos\alpha = p/2x_1$ and $\sin\alpha = p/2y_1$;

$\because \cos^2\alpha + \sin^2\alpha = 1$

Locus of (x_1, y_1) is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$.

54. **(a)** The line in xy-plane is,

$\frac{x}{3} + y = 1 \Rightarrow x + 3y - 3 = 0$

Let image of the point $(-1, -4)$ be (α, β), then

$\frac{\alpha + 1}{1} = \frac{\beta + y}{3} = -\frac{2(-1 - 12 - 3)}{10}$

$\Rightarrow \alpha + 1 = \frac{\beta + 4}{3} = \frac{16}{5}$

$\Rightarrow \alpha = \frac{11}{5}, \beta = \frac{28}{5}$

55. **(30)**

$L_1 : 2x - y + 3 = 0$

$L_1 : 4x - 2y + \alpha = 0 \Rightarrow 2x - y + \frac{\alpha}{2} = 0$

$L_1 : 6x - 3y + \beta = 0 \Rightarrow 2x - y + \frac{\beta}{3} = 0$

Distance between L_1 and L_2;

$$\left|\frac{\alpha-6}{2\sqrt{5}}\right| = \frac{1}{\sqrt{5}} \Rightarrow |\alpha-6| = 2$$

$\Rightarrow \alpha = 4, 8$

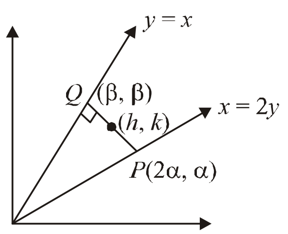
Distance between L_1 and L_3:

$$\left|\frac{\beta-9}{3\sqrt{5}}\right| = \frac{2}{\sqrt{5}} \Rightarrow |\beta-9| = 6$$

$\Rightarrow \beta = 15, 3$

Sum of all values $= 4+8+15+3 = 30$.

56. **(b)**



Since, slope of $PQ = \dfrac{k-\alpha}{h-2\alpha} = -1$

$\Rightarrow k-\alpha = -h+2\alpha$

$\Rightarrow \alpha = \dfrac{h+k}{3}$ ...(i)

Also, $2h = 2\alpha+\beta$ and

$2k = \alpha+\beta$

$\Rightarrow 2h = \alpha+2k$

$\Rightarrow \alpha = 2h-2k$ ...(ii)

From (i) and (ii), we have

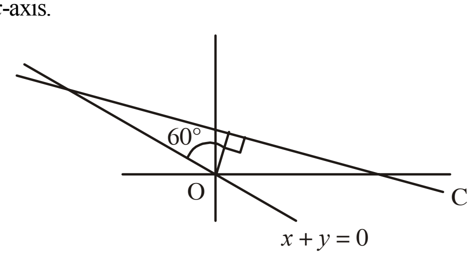
$$\frac{h+k}{3} = 2(h-k)$$

So, locus is $6x-6y = x+y$

$\Rightarrow 5x = 7y \Rightarrow 5x-7y = 0$

57. **(b)** $\because$ perpendicular makes an angle of 60° with the line $x+y=0$.

$\therefore$ the perpendicular makes an angle of 15° or 75° with x-axis.



Hence, the equation of line will be

$x\cos 75° + y\sin 75° = 4$

or $x\cos 15° + y\sin 15° = 4$

$(\sqrt{3}-1)x + (\sqrt{3}+1)y = 8\sqrt{2}$

or $(\sqrt{3}+1)x + (\sqrt{3}-1)y = 8\sqrt{2}$

58. **(a)** Let straight line be $4x-3y+\alpha = 0$

$\because$ distance from origin $= \dfrac{3}{5}$

$\therefore \dfrac{3}{5} = \left|\dfrac{\alpha}{5}\right| \Rightarrow \alpha = \pm 3$

Hence, line is $4x-3y+3=0$ or $4x-3y-3=0$

Clearly $\left(-\dfrac{1}{4}, \dfrac{2}{3}\right)$ satisfies $4x-3y+3=0$

59. **(a)** $\because$ two lines are perpendicular $\Rightarrow m_1 m_2 = -1$

$\Rightarrow \left(\dfrac{-1}{a-1}\right)\left(\dfrac{-2}{a^2}\right) = -1$

$\Rightarrow 2 = a^2(1-a) \Rightarrow a^3-a^2+2 = 0$

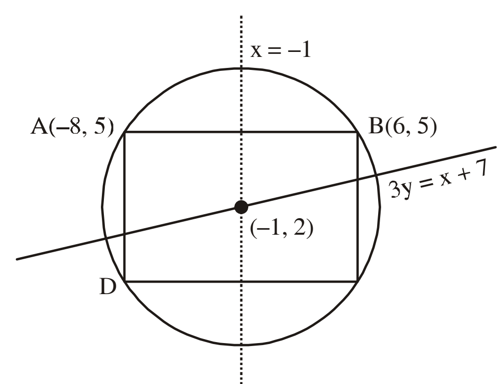
$\Rightarrow (a+1)(a^2+2a+2) = 0 \Rightarrow a = -1$

Hence equations of lines are $x-2y=1$ and $2x+y=1$

$\therefore$ intersection point is $\left(\dfrac{3}{5}, \dfrac{-1}{5}\right)$

Now, distance from origin $= \sqrt{\dfrac{9}{25}+\dfrac{1}{25}} = \sqrt{\dfrac{10}{25}} = \sqrt{\dfrac{2}{5}}$

60. **(a)** Given situation



$\because$ perpendicular bisector of AB will pass from centre.

$\therefore$ equation of perpendicular bisector $x = -1$

Hence centre of the circle is $(-1, 2)$

Let co-ordinate of D is (α, β)

$\Rightarrow \dfrac{\alpha+6}{2} = -1$ and $\dfrac{\beta+5}{2} = 2$

$\Rightarrow \alpha = -8$ and $\beta = -1$, $\therefore D \equiv (-8, -1)$

$|AD| = 6$ and $|AB| = 14$

Area $= 6 \times 14 = 84$

61. **(a)** $\because (h, k), (1, 2)$ and $(-3, 4)$ are collinear

$$\therefore \begin{vmatrix} h & k & 1 \\ 1 & 2 & 1 \\ -3 & 4 & 1 \end{vmatrix} = 0 \Rightarrow -2h - 4k + 10 = 0$$

$\Rightarrow h + 2k = 5$ ...(i)

Now, $m_{L_1} = \dfrac{4-2}{-3-1} = -\dfrac{1}{2} \Rightarrow m_{L_2} = 2 \quad [\because L_1 \perp L_2]$

By the given points (h, k) and $(4, 3)$,

$m_{L_2} = \dfrac{k-3}{h-4} \Rightarrow \dfrac{k-3}{h-4} = 2 \Rightarrow k - 3 = 2h - 8$

$2h - k = 5$ ...(ii)

From (i) and (ii)

$h = 3, k = 1 \Rightarrow \dfrac{k}{h} = \dfrac{1}{3}$

62. **(d)** $\because$ Equation of straight line can be rewritten as,

$y = \dfrac{2}{3}x + \dfrac{17}{3}$.

$\therefore$ Slope of straight line $= \dfrac{2}{3}$

Slope of line passing through the points $(7, 17)$ and $(15, \beta)$

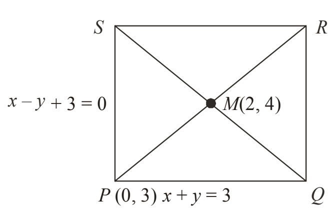
$= \dfrac{\beta - 17}{15 - 7} = \dfrac{\beta - 17}{8}$

Since, lines are perpendicular to each other.

Hence, $m_1 m_2 = -1$

$\Rightarrow \left(\dfrac{2}{3}\right)\left(\dfrac{\beta - 17}{8}\right) = -1 \Rightarrow \beta = 5$

63. **(d)**



Since, $x - y + 3 = 0$ and $x + y = 3$ are perpendicular lines and intersection point of $x - y + 3 = 0$ and $x + y = 3$ is $P(0, 3)$.

$\Rightarrow$ M is mid-point of $PR \Rightarrow R(4, 5)$

Let $S(x_1, x_1 + 3)$ and $Q(x_2, 3 - x_2)$

M is mid-point of SQ

$\Rightarrow x_1 + x_2 = 4, x_1 + 3 + 3 - x_2 = 8$

$\Rightarrow x_1 = 3, x_2 = 1$

Then, the vertex D is $(3, 6)$.

64. **(a)** The given equations of the set of all lines

$px + qy + r = 0$ ...(i)

and given condition is :

$3p + 2q + 4r = 0$

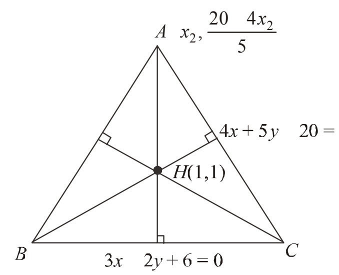
$\Rightarrow \dfrac{3}{4}p + \dfrac{2}{4}q + r = 0$ ...(ii)

From (i) & (ii) we get :

$\therefore x = \dfrac{3}{4}, y = \dfrac{1}{2}$

Hence the set of lines are concurrent and passing through the fixed point $\left(\dfrac{3}{4}, \dfrac{1}{2}\right)$

65. **(d)**



$\left(x_1, \dfrac{3x_1 + 6}{2}\right)$

Since, AH is perpendicular to BC

Hence, $m_{AH} \cdot m_{BC} = -1$

$$\left(\frac{\frac{20 - 4x_2}{5} - 1}{x_2 - 1}\right) \times \frac{3}{2} = -1$$

$\dfrac{15 - 4x_2}{5(x_2 - 1)} = -\dfrac{2}{3}$

$45 - 12x_2 = -10x_2 + 10$

$2x_2 = 35 \Rightarrow x_2 = \dfrac{35}{2}$

$\Rightarrow A\left(\dfrac{35}{2}, -10\right)$

Since, BH is perpendicular to CA.

Hence, $m_{BH} \times m_{CA} = -1$

$$\left(\frac{\frac{3x_1}{2} + 3 - 1}{x_1 - 1}\right)\left(-\frac{4}{5}\right) = -1$$

$\dfrac{(3x_1 + 4)}{2(x_1 - 1)} \times 4 = 5$

$\Rightarrow\ 6x_1 + 8 = 5x_1 - 5 \Rightarrow x_1 = -13 \Rightarrow \left(-13, \frac{-33}{2}\right)$

$\Rightarrow$ Equation of line AB is

$$y + 10 = \left(\frac{-\frac{33}{2}+10}{-13-35}\right)\left(x - \frac{35}{2}\right)$$

$\Rightarrow\ -61y - 610 = -13x + \frac{455}{2}$

$\Rightarrow\ -122y - 1220 = -26x + 455$

$\Rightarrow\ 26x - 122y - 1675 = 0$

66. **(d)** Equation of the line, which is perpendicular to the line, $3x + y = \lambda(\lambda \neq 0)$ and passing through origin, is given by

$\frac{x-0}{3} = \frac{y-0}{1} = r$

For foot of perpendicular

$r = \frac{-((3\times 0)+(1\times 0)-\lambda)}{3^2+1^2} = \frac{\lambda}{10}$

So, foot of perpendicular $P = \left(\frac{3\lambda}{10}, \frac{\lambda}{10}\right)$

Given the line meets X-axis at $A = \left(\frac{\lambda}{3}, 0\right)$ and meets Y-axis at $B = (0, \lambda)$

So, $BP = \sqrt{\left(\frac{3\lambda}{10}\right)^2 + \left(\frac{\lambda}{10} - \lambda\right)^2} \Rightarrow BP = \sqrt{\frac{9\lambda^2}{100} + \frac{81\lambda^2}{100}}$

$\Rightarrow BP = \sqrt{\frac{90\lambda^2}{100}}$

Now, $PA = \sqrt{\left(\frac{\lambda}{3} - \frac{3\lambda}{10}\right)^2 + \left(0 - \frac{\lambda}{10}\right)^2}$

$\Rightarrow PA\sqrt{\frac{\lambda^2}{900} + \frac{\lambda^2}{100}} \Rightarrow PA = \sqrt{\frac{10\lambda^2}{900}}$

Therefore $BP : PA = 3 : 1$

67. **(d)** Let the coordinate A be $(0, c)$

Equations of the given lines are

$x - y + 2 = 0$ and

$7x - y + 3 = 0$

We know that the diagonals of the rhombus will be parallel to the angle bisectors of the two given lines; $y = x + 2$ and $y = 7x + 3$

$\therefore$ equation of angle bisectors is given as:

$\frac{x-y+2}{\sqrt{2}} = \pm \frac{7x-y+3}{5\sqrt{2}}$

$5x - 5y + 10 = \pm\ (7x - y + 3)$

$\therefore$ Parallel equations of the diagonals are $2x + 4y - 7 = 0$ and $12x - 6y + 13 = 0$

$\therefore$ slopes of diagonals are $\frac{-1}{2}$ and 2.

Now, slope of the diagonal from $A(0, c)$ and passing through $P(1, 2)$ is $(2 - c)$

$\therefore\ 2 - c = 2 \Rightarrow c = 0$ (not possible)

$\therefore\ 2 - c = \frac{-1}{2} \Rightarrow c = \frac{5}{2}$

$\therefore$ ordinate of A is $\frac{5}{2}$.

68. **(a)** Given lines are

$4ax + 2ay + c = 0$

$5bx + 2by + d = 0$

The point of intersection will be

$\frac{x}{2ad - 2bc} = \frac{-y}{4ad - 5bc} = \frac{1}{8ab - 10ab}$

$\Rightarrow\ x = \frac{2(ad - bc)}{-2ab} = \frac{bc - ad}{ab}$

$\Rightarrow\ y = \frac{5bc - 4ad}{-2ab} = \frac{4ad - 5bc}{2ab}$

$\because$ Point of intersection is in fourth quadrant so x is positive and y is negative.

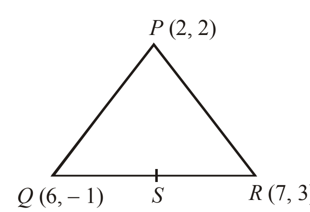
Also distance from axes is same

So $x = -y$

($\because$ distance from x-axis is $-y$ as y is negative)

$\frac{bc - ad}{ab} = \frac{5bc - 4ad}{2ab} \Rightarrow 3bc - 2ad = 0$

69. **(d)** Let P, Q, R, be the vertices of ΔPQR



Since PS is the median

S is mid-point of QR

So, $S = \left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$

Now, slope of $PS = \frac{2-1}{2-\frac{13}{2}} = -\frac{2}{9}$

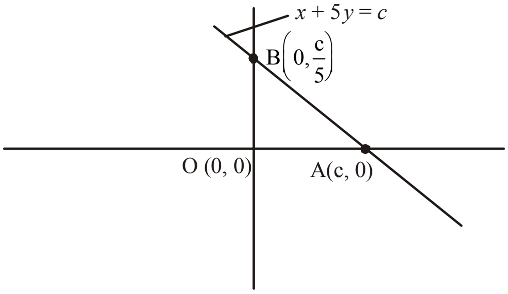
Since, required line is parallel to PS therefore slope of required line = slope of PS

Now, eqn. of line passing through $(1, -1)$ and having slope $-\frac{2}{9}$ is

$$y-(-1)=-\frac{2}{9}(x-1)$$

$$9y+9=-2x+2 \Rightarrow 2x+9y+7=0$$

70. **(b)** Let equation of line L, perpendicular to $5x - y = 1$ be $x + 5y = c$



Given that area of ΔAOB is 5.
We know

$$\left\{\text{area, } A=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]\right\}$$

$$\Rightarrow 5 = \frac{1}{2}\left[c\left(\frac{c}{5}\right)\right]$$

$$\left(\begin{array}{l}\because (x_1, y_1)=(10,0),(x_3, y_3)=\left(0,\frac{c}{5}\right)\\ (x_2, y_2)=(c,0)\end{array}\right)$$

$$\Rightarrow c = \pm\sqrt{50}$$

$\therefore$ Equation of line L is $x + 5y = \pm\sqrt{50}$
Distance between L and line $x + 5y = 0$ is

$$d = \left|\frac{\pm\sqrt{50}-0}{\sqrt{1^2+5^2}}\right| = \frac{\sqrt{50}}{\sqrt{26}} = \frac{5}{\sqrt{13}}$$

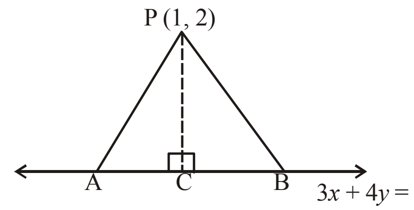
71. **(c)** $x+2ay+a = 0$ ...(i)

$x+3by+b = 0$ ...(ii)

$x+4ay+a = 0$ ...(iii)

Subtracting equation (iii) from (i)
$-2ay = 0$
$ay = 0$ χ $y = 0$
Putting value of y in equation (i), we get
$x + 0 + a = 0$
$x = -a$
Putting value of x and y in equation (ii), we get
$-a + b = 0 \Rightarrow a = b$
Thus, (a, b) lies on a straight line

72. **(b)**



Shortest distance of a point (x_1, y_1) from line

$ax + by = c$ is $d = \left|\frac{ax_1+by_1-c}{\sqrt{a^2+b^2}}\right|$

Now shortest distance of P (1, 2) from $3x + 4y = 9$ is

$$PC = d = \left|\frac{3(1)+4(2)-9}{\sqrt{3^2+4^2}}\right| = \frac{2}{5}$$

Given that ΔAPB is an equilateral triangle
Let 'a' be its side

then PB = a, CB = $\frac{a}{2}$
Now, In ΔPCB, $(PB)^2 = (PC)^2 + (CB)^2$
(By Pythagoras theoresm)

$$a^2 = \left(\frac{2}{5}\right)^2 + \frac{a^2}{4}$$

$$a^2 - \frac{a^4}{4} = \frac{4}{25} \Rightarrow \frac{3a^2}{4} = \frac{4}{25}$$

$$a^2 = \frac{16}{75} \Rightarrow a = \sqrt{\frac{16}{75}} = \frac{4}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{15}$$

$\therefore$ Length of Equilateral triangle $(a) = \frac{4\sqrt{3}}{15}$

73. **(d)** Mid-point of P(2, 3) and Q(4, 5) = (3, 4)
Slope of PQ = 1
Slope of the line L = – 1
Mid-point (3, 4) lies on the line L.
Equation of line L,
$y-4=-1(x-3) \Rightarrow x+y-7=0$ ...(i)
Let image of point R(0, 0) be $S(x_1, y_1)$

Mid-point of RS = $\left(\frac{x_1}{2}, \frac{y_1}{2}\right)$

Mid-point $\left(\frac{x_1}{2}, \frac{y_1}{2}\right)$ lies on the line (i)

$\therefore \quad x_1+y_1=14$ ...(ii)

Slope of RS = $\frac{y_1}{x_1}$

Since RS $\perp$ line L

$\therefore\ \frac{y_1}{x_1}\times(-1)=-1$

$\therefore\ x_1=y_1$ ...(iii)

From (ii) and (iii),

$x_1=y_1=7$

Hence the image of R = (7, 7)

74. **(a)** Two lines $-x+5y+c_2=0$ and $-x+5y+c_3=0$ are parallel to each other. Hence statement-1 is true, statement-2 is true and statement-2 is the correct explanation of statement-1.

75. **(a)** Since three lines x – 3y = p,

ax + 2y = q and ax + y = r

form a right angled triangle

∴ product of slopes of any two lines = –1

Suppose ax + 2y = q and x – 3y = p are ⊥ to each other.

$\therefore\ \frac{-a}{2}\times\frac{1}{3}=-1\Rightarrow a=6$

Now, consider option one by one

$a=6$ satisfies only option (a)

∴ Required answer is $a^2-9a+18=0$

76. **(d)** Consider the lines

$L_1: x-y=1$

$L_2: x+y=1$

$L_3: 2x+2y=5$

$L_4: 2x-2y=7$

$L_1\perp L_2$ is correct statement

(∵ Product of their slopes = – 1)

$L_1\perp L_3$ is also correct statement

(∵ Product of their slopes = – 1)

Now, $L_2: x+y=1$

$L_4: 2x-2y=7$

$\Rightarrow 2x-2(1-x)=7$

$\Rightarrow 2x-2+2x=7$

$\Rightarrow x=\frac{9}{4}$ and $y=\frac{-5}{4}$

Hence, L_2 intersects L_4.

77. **(d)** Given $ax^2+bx+c=0$

$\Rightarrow ax^2=-bx-c$

Now, consider

$y=4ax^2+3bx+2c$

$=4[-bx-c]+3bx+2c$

$=-4bx-4c+3bx+2c=-bx-2c$

Since, this curve intersects x-axis

∴ put $y=0$, we get

$-bx-2c=0\Rightarrow -bx=2c$

$\Rightarrow x=\frac{-2c}{b}$

Thus, given curve intersects x-axis at exactly one point.

78. **(c)** Statement - 1

Let $P'(x_1, y_1)$ be the image of (0, 1) with respect to the line $2x-y=0$ then

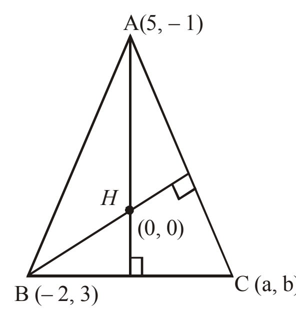
$\frac{x_1}{2}=\frac{y_1-1}{-1}=\frac{-4(0)+2(1)}{5}$

$\Rightarrow x_1=\frac{4}{5},\ y_1=\frac{3}{5}$

Thus, statement-1 is true.

Also, statement-2 is true and correct explanation for statement-1.

79. **(b)**



Let the third vertex of ΔABC be (a, b).

Orthocentre = $H(0,0)$

Let $A(5,-1)$ and $B(-2,3)$ be other two vertices of ΔABC.

Now, (Slope of AH) × (Slope of BC) = – 1

$\Rightarrow\ \left(\frac{-1-0}{5-0}\right)\left(\frac{b-3}{a+2}\right)=-1$

$\Rightarrow\ b-3=5(a+2)$ ...(i)

Similarly,

(Slope of BH) × (Slope of AC) = – 1

$\Rightarrow\ -\left(\frac{3}{2}\right)\times\left(\frac{b+1}{a-5}\right)=-1$

$\Rightarrow\ 3b+3=2a-10$

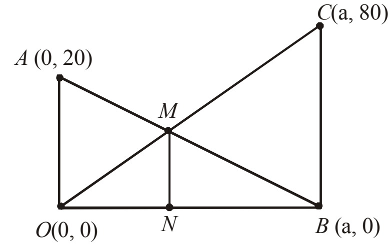
$\Rightarrow\ 3b-2a+13=0$ ...(ii)

On solving equations (i) and (ii) we get

$a=-4, b=-7$

Hence, third vertex is (– 4, – 7).

80. **(a)**



We put one pole at origin.

$BC = 80\,\text{m}$, $OA = 20\,\text{m}$

Line OC and AB intersect at M.

To find: Length of MN.

Eqn of OC: $y = \left(\frac{80-0}{a-0}\right)x$

$\Rightarrow \quad y = \frac{80}{a}x$ ...(i)

Eqn of AB: $y = \left(\frac{20-0}{0-a}\right)(x-a)$

$\Rightarrow y = \frac{-20}{a}(x-a)$ ...(ii)

At M: (i) = (ii)

$\Rightarrow \quad \frac{80}{a}x = \frac{-20}{a}(x-a)$

$\Rightarrow \quad \frac{80}{a}x = \frac{-20}{a}x + 20 \Rightarrow x = \frac{a}{5}$

$\therefore \quad y = \frac{80}{a} \times \frac{a}{5} = 16$

81. **(a)** Given equation of lines are

$(a^3+3)x + ay + a - 3 = 0$ and

$(a^5+2)x + (a+2)y + 2a + 3 = 0$ (a real)

Since point of intersection of lines lies on y-axis.

$\therefore$ Put $x = 0$ in each equation, we get

$ay + a - 3 = 0$ and

$(a+2)y + 2a + 3 = 0$

On solving these we get

$(a+2)(a-3) - a(2a+3) = 0$

$\Rightarrow \quad a^2 - a - 6 - 2a^2 - 3a = 0$

$\Rightarrow \quad -a^2 - 4a - 6 = 0 \Rightarrow a^2 + 4a + 6 = 0$

$\Rightarrow \quad a = \frac{-4 \pm \sqrt{16-24}}{2} = \frac{-4 \pm \sqrt{-8}}{2}$

(not real)

This shows that the point of intersection of the lines lies on the y-axis for no value of 'a'.

82. **(b)** Given that $x + y = |a|$

and $ax - y = 1$

Case I : If $a > 0$

$x + y = a$ .... (i)

$ax - y = 1$ .... (ii)

On adding equations (i) and (ii), we get

$x(1+a) = 1 + a \Rightarrow x = 1$

$y = a - 1$

Since given that intersection point lies in first quadrant

So, $a - 1 \geq 0$

$\Rightarrow \quad a \geq 1$

$\Rightarrow \quad a \in [1, \infty)$

Case II : If $a < 0$

$x + y = -a$ .... (iii)

$ax - y = 1$ .... (iv)

On adding equations (iii) and (iv), we get

$x(1+a) = 1 - a$

$x = \frac{1-a}{1+a} > 0 \Rightarrow \frac{a-1}{a+1} < 0$

Since $a - 1 < 0$

$\therefore \quad a + 1 > 0$

$\Rightarrow \quad a > -1$ .... (v)

$y = -a - \frac{1-a}{1+a} > 0 \quad = \frac{-a - a^2 - 1 + a}{1+a} > 0$

$\Rightarrow \quad -\left(\frac{a^2+1}{a+1}\right) > 0 \Rightarrow \frac{a^2+1}{a+1} < 0$

Since $a^2 + 1 > 0$

$\therefore \quad a + 1 < 0$

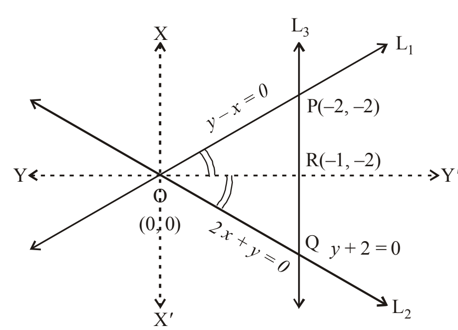
$\Rightarrow \quad a < -1$ .... (vi)

From (v) and (vi), $a \in \phi$

Hence, Case-II is not possible.

So, correct answer is $a \in [1, \infty)$

83. **(b)**



$L_1 : y - x = 0$

$L_2 : 2x + y = 0$

$L_3 : y + 2 = 0$

On solving the equation of line L_1 and L_2 we get their point of intersection (0, 0) i.e., origin O.

On solving the equation of line L_1 and L_3,

we get $P = (-2, -2)$.

Similarly, solving equation of line L_2 and L_3, we get

$Q = (-1, -2)$

We know that bisector of an angle of a triangle, divide the opposite side the triangle in the ratio of the sides including the angle [Angle Bisector Theorem of a Triangle]

$\therefore \frac{PR}{RQ} = \frac{OP}{OQ} = \frac{\sqrt{(-2)^2 + (-2)^2}}{\sqrt{(-1)^2 + (-2)^2}} = \frac{2\sqrt{2}}{\sqrt{5}}$

$\therefore$ Statement 1 is true but $\angle OPR \neq \angle OQR$

So ΔOPR and ΔOQR not similar

$\therefore$ Statement 2 is false.

84. **(a)** Given that the lines $p(p^2+1)x - y + q = 0$ and $(p^2+1)^2 x + (p^2+1)y + 2q = 0$ are perpendicular to a common line then these lines must be parallel to each other,

$\therefore \quad m_1 = m_2$

$\Rightarrow -\frac{p(p^2+1)}{-1} = -\frac{(p^2+1)^2}{p^2+1}$

$\Rightarrow (p^2+1)^2\ (p+1) = 0$

$\Rightarrow p = -1$

$\therefore$ p can have exactly one value.

85. **(d)** Let (a^2, a) be the point of shortest distance on $x = y^2$ Then distance between (a^2, a) and line $x - y + 1 = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right| = \frac{|a^2 - a + 1|}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left|(a - \frac{1}{2})^2 + \frac{3}{4}\right|$$

It is min when $a = \frac{1}{2}$ and

$D_{min} = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$

86. **(d)** Slope of $PQ = \frac{3-4}{k-1} = \frac{-1}{k-1}$

$\therefore$ Slope of perpendicular bisector of PQ $= (k-1)$

Also, mid point of PQ $\left(\frac{k+1}{2}, \frac{7}{2}\right)$.

$\therefore$ Equation of perpendicular bisector of PQ is

$$y - \frac{7}{2} = (k-1)\left(x - \frac{k+1}{2}\right)$$

$\Rightarrow 2y - 7 = 2(k-1)x - (k^2-1)$

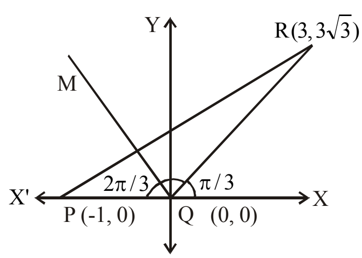
$\Rightarrow 2(k-1)x - 2y + (8 - k^2) = 0$

Given that y-intercept

$= \frac{8-k^2}{2} = -4$

$\Rightarrow 8 - k^2 = -8$ or $k^2 = 16 \Rightarrow k = \pm 4$

87. **(c)** **Given :** The coordinates of points P, Q, R are $(-1, 0)$, $(0, 0)$, $(3, 3\sqrt{3})$ respectively.



Slope of QR $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{3\sqrt{3}}{3}$

$\Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$

$\Rightarrow \angle RQX = \frac{\pi}{3}$

$\therefore \angle RQP = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Let QM bisects the $\angle PQR$,

$\therefore \angle MQR = \frac{\pi}{3} \Rightarrow \angle MQX = \frac{2\pi}{3}$

$\therefore$ Slope of the line QM $= \tan\frac{2\pi}{3} = -\sqrt{3}$

$\therefore$ Equation of line QM is $(y-0) = -\sqrt{3}(x-0)$

$\Rightarrow y = -\sqrt{3}x \Rightarrow \sqrt{3}x + y = 0$

88. **(b)** Taking co-ordinates as $A\left(\frac{x}{r}, \frac{y}{r}\right)$; $B(x, y)$ and $C(xr, yr)$.

Then slope of line joining

$$A\left(\frac{x}{r}, \frac{y}{r}\right), B(x, y) = \frac{y\left(1 - \frac{1}{r}\right)}{x\left(1 - \frac{1}{r}\right)} = \frac{y}{x}$$

and slope of line joining $B(x, y)$ and $C(xr, yr)$

$= \frac{y(r-1)}{x(r-1)} = \frac{y}{x}$

$\therefore m_1 = m_2$

$\because$ Slope of AB and BC are same and one point B common.

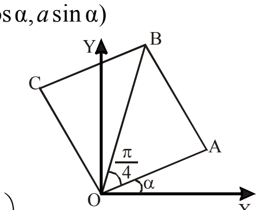
$\Rightarrow$ Points lie on the straight line.

89. **(a)** Co-ordinates of $A = (a\cos\alpha, a\sin\alpha)$

Equation of OB,

$y = \tan\left(\frac{\pi}{4} + \alpha\right)x$

$CA \perp^r$ to OB



$\therefore$ Slope of CA $= -\cot\left(\frac{\pi}{4} + \alpha\right)$

Equation of CA

$$y - a\sin\alpha = -\cot\left(\frac{\pi}{4} + \alpha\right)(x - a\cos\alpha)$$

$$\Rightarrow (y - a\sin\alpha)\left(\tan\left(\frac{\pi}{4} + \alpha\right)\right) = (a\cos\alpha - x)$$

$$\Rightarrow (y - a\sin\alpha)\left(\frac{\tan\frac{\pi}{4} + \tan\alpha}{1 - \tan\frac{\pi}{4}\tan\alpha}\right) = (a\cos\alpha - x)$$

$\Rightarrow (y - a\sin\alpha)(1 + \tan\alpha) = (a\cos\alpha - x)(1 - \tan\alpha)$

$\Rightarrow (y - a\sin\alpha)(\cos\alpha + \sin\alpha)$

$= (a\cos\alpha - x)(\cos\alpha - \sin\alpha)$

$\Rightarrow y(\cos + \sin\alpha) - a\sin\alpha\cos\alpha - a\sin^2\alpha$

$= a\cos^2\alpha - a\cos\alpha\sin\alpha - x(\cos\alpha - \sin\alpha)$

$\Rightarrow y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

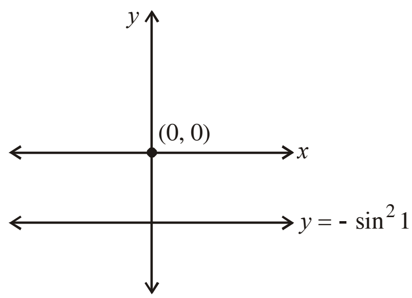
$y(\sin\alpha + \cos\alpha) + x(\cos\alpha - \sin\alpha) = a.$

90. **(d)** Consider the equation,

$y = \sin x.\ \sin(x+2) - \sin^2(x+1)$

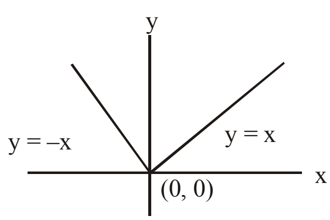
$$= \frac{1}{2}\cos(-2) - \frac{\cos(2x+2)}{2} - \left[\frac{1-\cos(2x+2)}{2}\right]$$

$$= \frac{(\cos 2) - 1}{2} = -\sin^2 1$$



By the graph y lies in III and IV quadrant.

91 **(a)** From figure equation of bisectors of lines, xy = 0 are y= ± x



$\therefore$ Put y = ± x in the given equation

$my^2 + (1 - m^2)xy - mx^2 = 0$

$\therefore\ mx^2 \pm (1 - m^2)x^2 - mx^2 = 0$

$\Rightarrow\ 1 - m^2 = 0 \Rightarrow\ m = \pm 1$

92. **(a)** $3x + 4y = 0$ is one of the line of the pair equations. of lines

$6x^2 - xy + 4cy^2 = 0$, Put $y = -\frac{3}{4}x$,

we get, $6x^2 + \frac{3}{4}x^2 + 4c\left(-\frac{3}{4}x\right)^2 = 0$

$\Rightarrow 6 + \frac{3}{4} + \frac{9c}{4} = 0 \Rightarrow c = -3$

93. **(c)** Let the lines be $y = m_1x$ and $y = m_2x$ then

$m_1 + m_2 = -\frac{2c}{7}$ and $m_1m_2 = -\frac{1}{7}$

Given that $m_1 + m_2 = 4\ m_1m_2$

$\Rightarrow -\frac{2c}{7} = -\frac{4}{7} \Rightarrow c = 2$

94. **(a)** Equation of bisectors of second pair of straight lines is,

$qx^2 + 2xy - qy^2 = 0$ ....(i)

It must be identical to the first pair

$x^2 - 2pxy - y^2 = 0$ ....(ii)

from (i) and (ii)

$\frac{q}{1} = \frac{2}{-2p} = \frac{-q}{-1} \Rightarrow pq = -1$.

95. **(a)** We know that pair of straighty lines

$ax^2 + 2hxy + by^2 = 0$ are perpendicular when $a + b = 0$

$3a + a^2 - 2 = 0 \Rightarrow a^2 + 3a - 2 = 0.;$

$\Rightarrow a = \frac{-3 \pm \sqrt{9+8}}{2} = \frac{-3 \pm \sqrt{17}}{2}$

96. **(a)** Put $x = 0$ in the given equation

$\Rightarrow by^2 + 2fy + c = 0.$

For unique point of intersection, $f^2 - bc = 0$

$\Rightarrow af^2 - abc = 0.$

We know that for pair of straight line

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow 2fgh - bg^2 - ch^2 = 0$