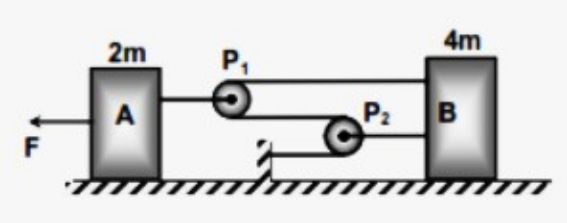
PHYSICS **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

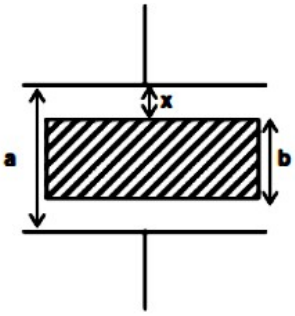
Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

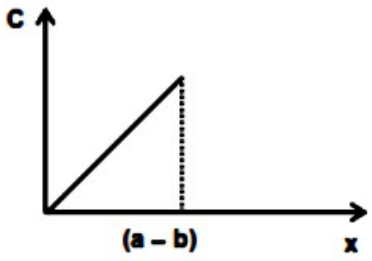
1. The acceleration of the block B shown in the figure will be (Assuming the surfaces and the light pulleys P_1 and P_2 all are smooth)



A) $\frac{F}{4m}$ B) $\frac{F}{6m}$ C) $\frac{F}{2m}$ D) $\frac{3F}{17m}$

2. The distance between two parallel plates of a capacitor is a. A conductor of thickness b (b < a) is inserted between the plates as shown in the figure. The variation of effective capacitance between the plates of the capacitors as a function of the distance (x) is best represented by



A)


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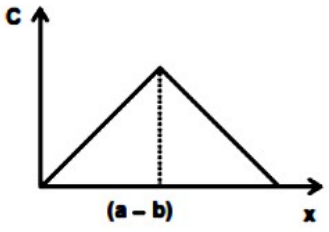

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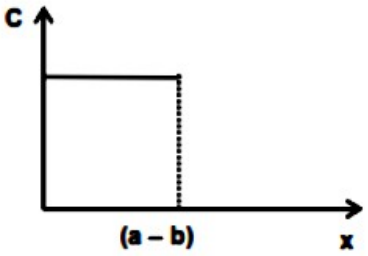

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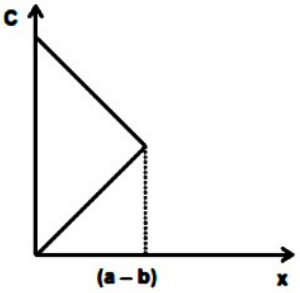
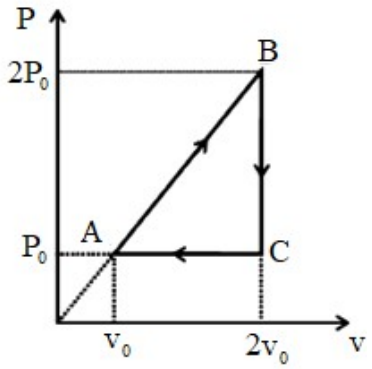
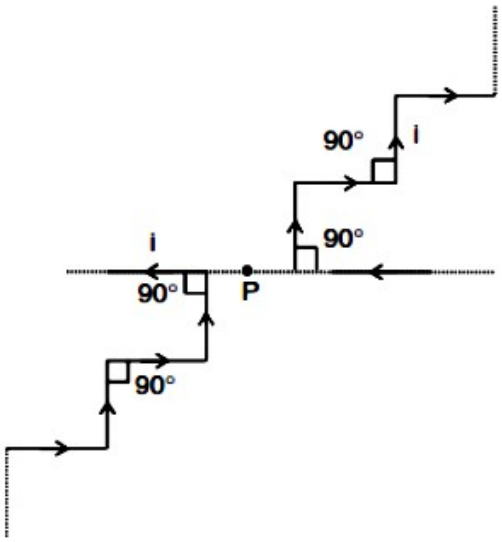


3. One mole of an ideal monoatomic gas is taken through a cyclic process ABCA as shown in the P-V diagram. The efficiency (in percentage) of the cyclic process is

A) 8.33 B) 12.33 C)16.33 D) 20.33



4. Two infinitely long conductors carrying equal currents are shaped as shown . The short sections are all of equal lengths. The point P is located symmetrically with respect to the two conductors. The magnetic field at point P due to any one conductor is B. The total magnetic field at point P is



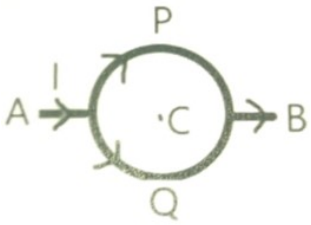
A) zero B) B C) $\sqrt{2}$B D) 2B

5. At the initial moment three point A,B and C are on a horizontal plane along a straight line such that AB = BC . Point A begins to move upward with a constant velocity 'v' and point C downward without any initial velocity at a constant acceleration 'a'. If the point begin to move simultaneously, then the initial velocity and acceleration of point B for all the three particles to be constantly on same straight line must be:

A) $\frac{v}{2}$upwards, $\frac{a}{2}$ downwards B) $\frac{v}{2}$upwards, $\frac{a}{2}$upwards

C) $\frac{v}{2}$downwards, $\frac{a}{2}$downwards D) $\frac{v}{2}$downwards, $\frac{a}{2}$upwards

6. Assertion (A): The magnetic field at the center of the current carrying circular coil shown in the fig. is zero.



Reason (R): The magnitudes of magnetic fields are equal and the directions of magnetic fields due to both the semicircles are opposite.

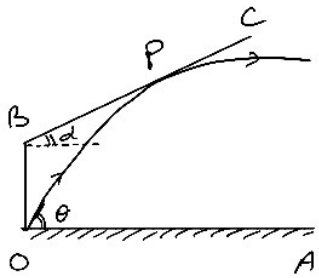
A) Both A and R are true but R is the correct explanation of A

B) Both A and R are true but R is not the correct explanation of A

C) A is true but R is false

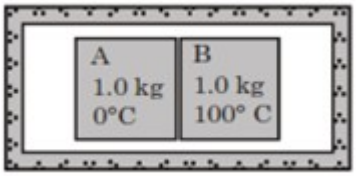
D) A is false but R is also false

7. A particle projected at an angle θ grazes the inclined surface BC at point P as shown. Find the time required to reach P from O.



A) $\frac{2u\sin\theta}{g\cos\alpha}$ B) $\frac{u\sin\theta}{g\cos\alpha}$ C) $\frac{u\sin(\theta-\alpha)}{g\cos\alpha}$ D) $\frac{2u\sin(\theta-\alpha)}{g\cos\alpha}$

8. Objects A and B that are initially separated from each other and well isolated from their surroundings are then brought into thermal contact . Initially temperature of A and B are 0^0C and 100^0C respectively. The specific heat of A is less than the specific heat of B. After some time, the system comes to an equilibrium state. The final temperatures are



A) $T_A = T_B > 50^0\text{C}$ B) $T_A > T_B > 50^0\text{C}$

C) $T_A = T_B < 50^0\text{C}$ D) $T_B > T_A > 50^0\text{C}$

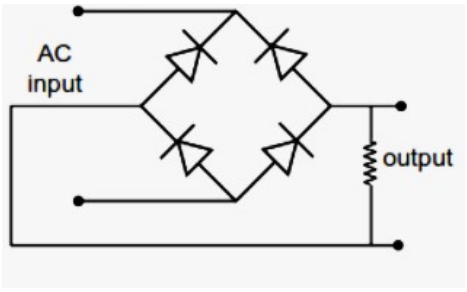
9. A large plane mirror with its bottom on the floor is tilted at an angle 30° to the vertical. A boy whose eyes are at height √3 m above the floor is standing in front of the mirror. At what maximum distance (in m) from mirror should the boy be to see his full image in mirror?

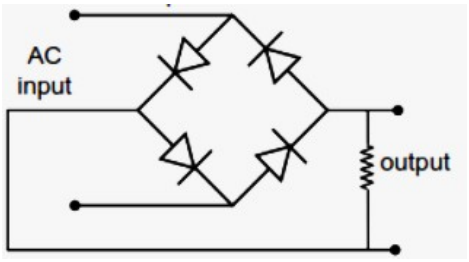
A) 1m B) 2m C) 2.3m D) 1.5m

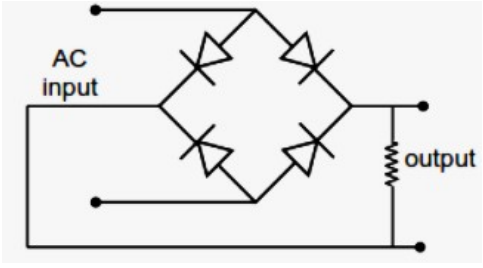
10. A heavy nucleus X having mass number 200 gets disintegrated into two small fragments Y and Z of mass numbers 80 and 120 respectively. If binding energy per nucleon for the parent atom X is 6.5 MeV and for daughter nuclei Y and Z are 7 MeV and 8 MeV respectively. Energy released in the decay will be

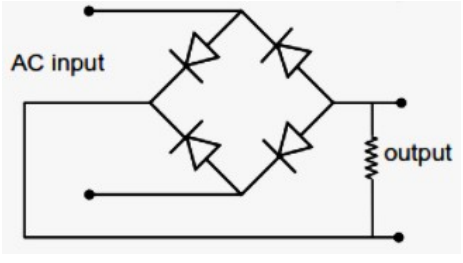
A) 200 MeV B) 240 MeV C) 220 MeV D) 180 MeV

11. Which of the following circuits will provide a full wave rectification of an AC input?

A)


B)


C)


D)


12. A vessel of volume of V_0 is evacuated by means of a piston air pump. One piston stroke captures the volume $\Delta V = 0.2V_0$. If process is assumed to be isothermal then find the minimum number of strokes after which pressure in the vessel becomes $\left(\frac{1}{1.728}\right)\left(P_{initial}\right)$.

A) 2 B) 3 C) 5 D) 7

13. A balloon of volume V, contains a gas whose density is σ and the density of the air at the earth's surface is 15σ. If the envelope of the balloon be of weight w but of negligible volume. Find the acceleration with which it will begin to ascend.

A) $\left[\frac{7Vg\sigma - w}{Vg\sigma + w}\right] \times g$ B) $\left[\frac{2Vg\sigma - w}{Vg\sigma + w}\right] \times g$

C) $\left[\frac{14Vg\sigma - w}{Vg\sigma + w}\right] \times g$ D) $\left[\frac{7Vg\sigma + w}{Vg\sigma - w}\right] \times g$

14. Assertion (A): A small body suspended by a light spring, perform SHM. When the entire system is immersed in a non-viscous liquid, the period of oscillation does not change.

Reason (R): The angular frequency of oscillation of the particle does not change.

A) Both A and R are true but R is the correct explanation of A

B) Both A and R are true but R is not the correct explanation of A

C) A is true but R is false

D) A is false but R is also false

15. A satellite is revolving around the earth in an orbit such that it time period of revolution as same as that of earth and it revolve in same sense as of earth. To make it escape from gravitational field of earth, its velocity must be increased by

A) 100% B) 41.4% C) 50% D) 59.6%

16. Match the List-I with the List-II.

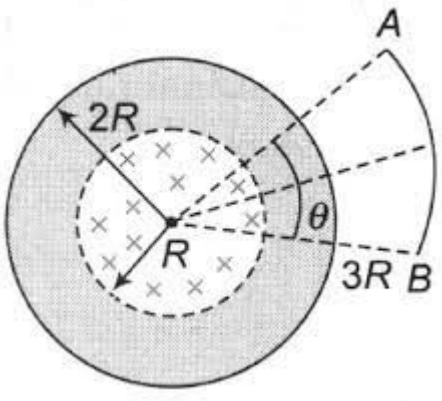
List-I		**List-II**	
A.	Phase difference between current and voltage in a purely resistive AC circuit	1.	$\frac{\pi}{2}$; current leads voltage
B.	Phase difference between current and voltage in a pure inductive AC circuit	2.	Zero
C.	Phase difference between current and voltage in a pure capacitive AC circuit	3.	$\frac{\pi}{2}$; current lags voltage
D.	Phase difference between current and voltage in an LCR series circuit	4.	$\tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

Choose the most appropriate answer from the options given below:

A) A-2, B-3, C-1, D-4 B) A-1, B-3, C-4, D-2

C) A-2, B-3, C-4, D-1 D) A-2, B-4, C-3, D-1

17. In the given figure two concentric cylindrical region in which time varying magnetic field is present as shown. From the centre to radius R magnetic field is perpendicular into the plane varying as $\frac{dB}{dt} = 2k_0$ and in a region from R to 2R magnetic field is perpendicular out of the plane varying as $\frac{dB}{dt} = 4k_0$. Find the induced emf across an arc AB of radius 3R.

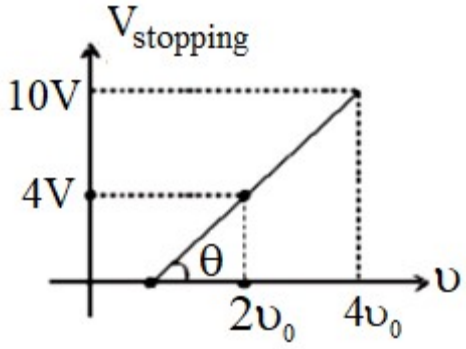


A) $6R^2k_0\theta$ B) $5R^2k_0\theta$ C) $7R^2k_0\theta$ D) None of these

18. If a string of length ℓ fixed at both ends vibrates with a standing wave $y = A\sin\left(\frac{2\pi}{\ell}x\right)\sin 2\pi t$ in resonance. Then the minimum time (from t=0) after which energy is maximum at mid-point of string will be –

A) $\frac{1}{4}$ sec B) $\frac{1}{5}$ sec C) $\frac{1}{8}$ sec D) $\frac{1}{6}$ sec

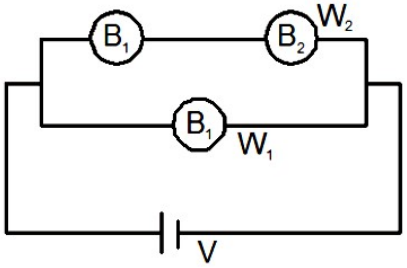
19. Figure shows the graph of stopping potential versus the frequency of a photosensitive metal. The plank's constant and work function of the metal are (V and υ_0 are two different constant.)



A) $W_c = (2V)e; h = \frac{(3V)e}{\upsilon_0}$ B) $W_c = (2V)e; h = \frac{(2V)e}{\upsilon_0}$

C) $W_c = (3V)e; h = \frac{(3V)e}{\upsilon_0}$ D) $W_c = (3V)e; h = \frac{(2V)e}{\upsilon_0}$

20. There are two bulbs $B_1(P,V), B_2(P,2V)$ their rated power and voltages are mentioned with them. Calculate the ratio of consumed power $\frac{W_1}{W_2}$?



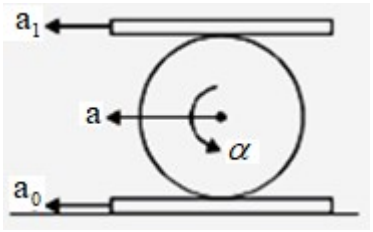
A) $\frac{25}{4}$ B) $\frac{4}{25}$ C) $\frac{10}{4}$ D) $\frac{4}{10}$

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

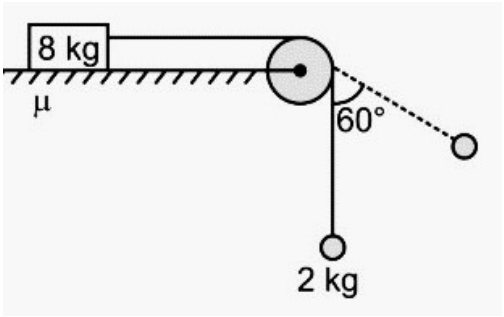
This section contains 10 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only. Have to Answer any 5 only out of 10 questions** and question will be evaluated according to the following marking scheme:

Marking scheme: +4 for correct answer, -1 in all other cases.

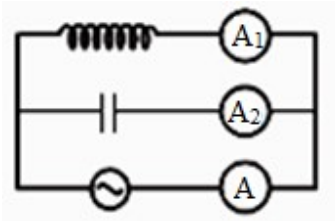
21. A system of two planks and a sphere of radius R is in motion as shown in figure. Radius of the sphere is R and there is no slipping anywhere. It is given that $R\alpha = 2a_0$ where α is angular acceleration of sphere and acceleration of upper block is $a_1 = ka_0$ where k is a +ve constant then the value of k will be



22. In the system shown, the mass $m = 2$ kg oscillates in a circular arc of amplitude 60^0. The minimum value of coefficient of friction between mass = 8 kg and surface of table to avoid slipping is μ. Then find 10μ.



23. Consider a circuit with an alternating source and contains inductor and capacitor. Given reading of A_1 and A_2 as 3 ampere and 5 ampere respectively. Find the magnitude of reading of A in ampere.

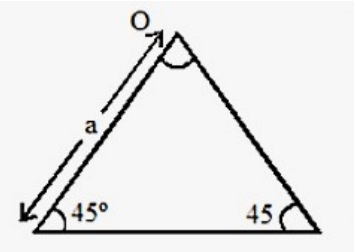


24. The electric field associated with e.m. waves in vacuum is given by $\vec{E} = \hat{i}40\cos\left(kz - 6\times10^8 t\right)$, where E, z and t are in volt/m, meter and seconds respectively. The value of wave vector k is _____per m

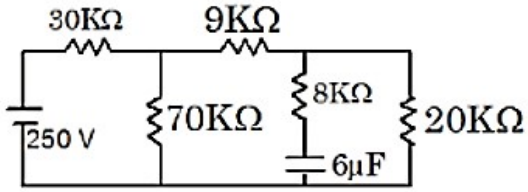
25. A point like sound source emits sound in all direction uniformly. An observer who is at a distance of 50 m from the source detects sound of intensity 10^{-2} watt/m². If the bulk modulus of air is 1.6 x 10^5 N/m² and velocity of sound is 340 m/s. Find pressure amplitude (in N/m²) at the position of observer in nearest integer.

26. An x-ray beam of monochromatic photon are incident on a metallic surface having a negligible work function $\left(\text{take } \phi = 0\right)$. It is seen that the wavelength of most energetic photoelectrons is equal to the wavelength of x-ray photons. Find the wavelength (in pm). Round off to nearest integer.

27. Figure shows a right angle solid prism $(< O = \frac{\pi}{2})$ and mass of prism is m. Moment of inertia of prism about axis perpendicular to plane passing through O is $\frac{ma^2}{k}$, find value of k.

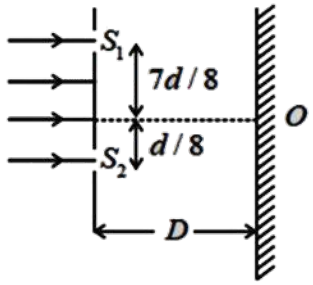


28. Calculate time constant (in milli-sec) of the circuit.



29. A vertical capillary tube with inside radius 0.25 mm is submerged into water so that the length of its part protruding over the water surface is equal to 25 mm. Surface tension of water is 73 x 10^{-3} N/m and angle of contact is zero degree for glass and water, acceleration due to gravity is 9.8 m/s². Then value of 10R approximately (in mm) is (where R is radius of meniscus and h is height of water in capillary tube)

30. In the figure, if a parallel beam of white light is incident on the plane of the slits S_1 & S_2 then the distance of the central maxima on the screen from O is $\frac{kd}{8}$. Find the value of k. $\left[\text{Assume } D >> d, d >> \lambda\right]$.



CHEMISTRY **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

31. There are four elements 'p', 'q', 'r' and 's' having atomic numbers Z-1, Z, Z+1 and Z+2 respectively. If the element 'q' is an inert gas, select the correct answers from the following statements.

(i) 'p' has most negative electron gain enthalpy in the respective period.

(ii) 'r' is an alkali metal.

(iii) 's' exists in +2 oxidation state.

A) (i) and (ii) only B) (ii) and (iii) only

C) (i) and (iii) only D) (i), (ii) and (iii)

32. The following are some statements related to VA group hydrides. **INCORRECT** statement is:

A) Reducing property increases from NH_3 to BiH_3.

B) Tendency to donate lone pair decreases from NH_3 to BiH_3.

C) Thermal stability of hydrides decreases from NH_3 to BiH_3.

D) Bond angle of hydrides increases from NH_3 to BiH_3.

33. The incorrect statement is

A) The first ionization enthalpy of K is less than that of Na and Li

B) Xe does not have the lowest first ionization enthalpy in its group

C) The first ionization enthalpy of element with atomic number 37 is lower than that of the element with atomic number 38.

D) The first ionization enthalpy of Ga is higher than that of the d-block element with atomic number 30.

34. Which one of the following statements is incorrect?

A) Highest oxidation state of manganese in fluorides is +4 (MnF_4) but highest oxidation state in oxides is +7 (Mn_2O_7).

B) $VO_2^+ < Cr_2O_7^{2-} < MnO_4^-$ is the correct order of oxidizing power.

C) When I^- is oxidized by MnO_4^- in alkaline medium, I^- converts into I_2.

D) Silver nitrate on heating decomposes to give two types of paramagnetic gases along with a residue.

35. Consider the following co-ordination compounds.

(i) $Ni(CO)_4$ (ii) $[Co(CO)_4]^-$ (iii) $[Fe(CO)_4]^{2-}$

The M-C bond strength follows the order :

(A) (ii)>(iii)>(i) (B) (iii)>(ii)>(i) (C) (i)>(ii)>(iii) (D) (i)>(iii)>(ii)

36. **Assertion (A):** $\left[Fe(CN)_6\right]^{3-}$ is more stable than $\left[Fe(CN)_6\right]^{4-}$

Reason (R): Complexes where the iron is in the (III) oxidation state are generally more stable than those in (II) oxidation state.

A) Assertion is True, Reason is True; Reason is correct explanation for Assertion

B) Assertion is True, Reason is True; Reason is NOT a correct explanation for Assertion

C) Assertion is True, Reason is False

D) Assertion is False, Reason is True

37. Which of the following statements is **CORRECT**?

A) In the formation of dioxygen from oxygen atoms, 10 molecular orbitals will be formed.

B) All the molecular orbitals in the dioxygen will be completely filled.

C) Total number of bonding molecular orbitals will not be same as total number of anti-bonding orbitals in dioxygen.

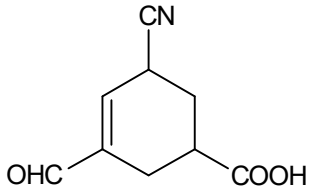
D) Number of filled bonding orbitals will be same as number of filled anti bonding orbitals.

38. Yellow coloured solution of metal ion [which belongs to group-III of basic radicals] + $SCN^- \rightleftharpoons$ Blood red colour solution.

The intensity of red colour will not change on addition of:

A) $Hg(NO_3)_2$ B) F^- ions C) $C_2O_4^{2-}$ ions D) Br^- ions

39. The IUPAC name of the following compound is

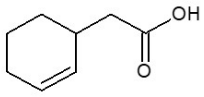


A) 5-cyano-3-formylcyclohex-3-en-1-carboxylic acid

B) 3-cyano-5-formylcyclohex-4-ene-1-carboxylic acid

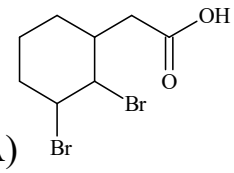
C) 5-cyano-3-oxocyclohex-3-ene-1-carboxylic acid

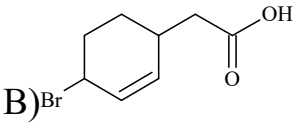
D) 5-carboxy-3-formylcyclohex-2-ene-1-carbonitrile

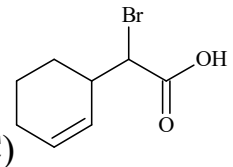


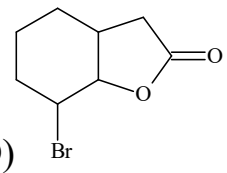
40 $\xrightarrow{Br_2, NaHCO_3}$ [A], Major organic product

The CORRECT structure of A is

A) 

B) 

C) 

D) 

41. Compound X($C_7H_{13}Br$) is a tertiary bromide. On treatment with potassium ethoxide in ethanol, X is converted into Y(C_7H_{12}). Ozonolysis of Y gives Z as the only product. Z can also be obtained by the oxidation of heptane-2,6-diol.

Select the **INCORRECT** statement from the following:

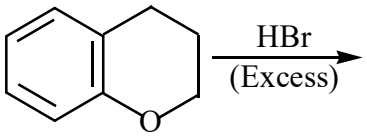
A) Z can also be obtained by oxidation of Y with conc. hot $KMnO_4$ solution.

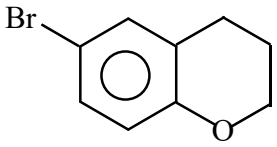
B) Z gives negative 2,4-DNP test.

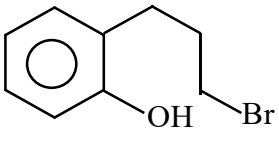
C) Z gives positive iodoform test.

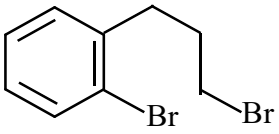
D) Z can be reduced to a diol with $NaBH_4$.

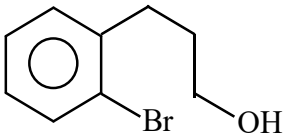
42. Find out correct product of reaction:



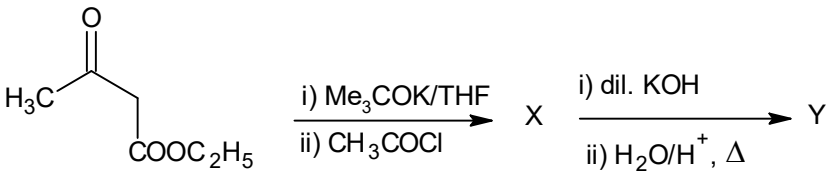
A) 

B) 

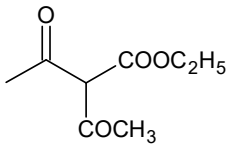
C) 

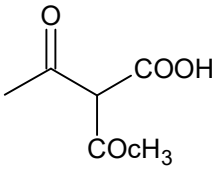
D) 

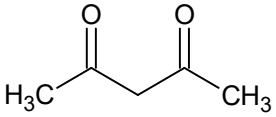
43. Analyse the following reaction sequence:

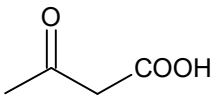


The compound Y is the above sequence of reaction is

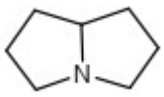
A) 

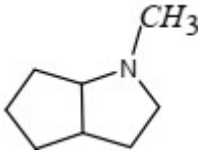
B) 

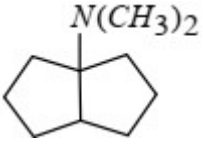
C) 

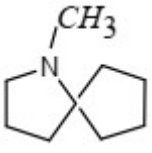
D) 

44. The nitrogen atom in each of the following tertiary amines may be removed as trimethyl amine by repeated Hoffmann eliminations (exhaustive methylation followed by heating with moist Ag_2O). Which of the amines requires the greater number of such Hoffmann sequences to accomplish complete removal of nitrogen.

A) 

B) 

C) 

D) 

45. Which of the following statement is **correct**?

A) α-D-glucose and β -D-glucose are enantiomers.

B) Glycine is optically active.

C) Cellulose on hydrolysis yields only α-D-glucose.

D) Arginine is most basic amino acid

46. One litre of a mixture of CO & CO_2, is burnt in excess of oxygen. The reduction in volume after combustion and cooling was found to be 100 ml at same temperature and pressure. Calculate the volume ratio of CO and CO_2 in initial mixture.

A) 1 : 9 B) 9 : 1 C) 1 : 4 D) 4 : 1

47. The entropy change can be calculated by using the expression $\Delta S = q_{rev} / T$. When water freezes in a glass beaker, choose the correct statement amongst the following.

A) ΔS (system) decreases but ΔS (surroundings) remains the same.

B) ΔS (system) increases but ΔS (surroundings) decreases.

C) ΔS (system) decreases but ΔS (surroundings) increases.

D) ΔS (system) decreases and ΔS (surroundings) also decreases.

48. A liquid mixture contain 10 moles of A($P°_A$ = 200 mmHg) and 10 moles of

B($P°_B$ = 100 mmHg). The vapour pressure over liquid mixture is 160 mm Hg. Which is correct statement?

A) ΔG_{mix} = +ve B) ΔV_{mix}= -ve

C) $\Delta S_{surrounding}$= +ve D) $\Delta S_{surrounding}$ = -ve

49. For the reaction $N_2O_4(g) \rightleftharpoons 2NO_2(g)$, the value of K is 50 at 400 K and 1700 at 500 K. Which of the following options is **INCORRECT**?

A) The reaction is endothermic.

B) The reaction is exothermic.

C) If NO_2 (g) and N_2O_4 (g) are mixed at 400 K at partial pressures 20 bar and 2 bar respectively, more N_2O_4 (g) will be formed.

D) The entropy of the system increases in the forward direction.

50. Match the items of Column I and Column II.

Column I	**Column II**
(i) Leclanche cell	(a) cell reaction $2H_2 + O_2 \rightarrow 2H_2O$
(ii) Ni–Cd cell	(b) does not involve any ion in solution and is used in hearing aids.
(iii) Fuel cell	(c) rechargeable
(iv) Mercury cell	(d) reaction at anode, $Zn \rightarrow Zn^{2+} + 2e^-$
	(e) converts energy of combustion into electrical energy

A) (i) →(d); (ii) →(c); (iii) →(a), (e); (iv) →(b)

B) (i) →(c); (ii) →(d); (iii) →(e); (iv) →(b)

C) (i) → (c); (ii) →(e); (iii) →(a), (c); (iv) →(b)

D) (i) → (d); (ii) →(d); (iii) →(a); (iv) →(c)

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

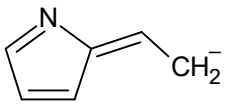
This section contains 10 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only.** **Have to Answer any 5 only out of 10 questions** and question will be evaluated according to the following marking scheme:
Marking scheme: +4 for correct answer, -1 in all other cases.

51. Total number of geometrical isomers for the square planar complex [RhCl(CO)$(PPh_3)(NH_3)$] is /are _____ .

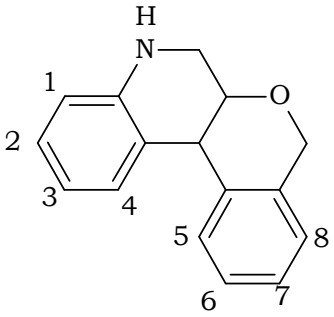
52. The total number of compounds having $p\pi-p\pi$ bonding among the molecules given below are _____ .

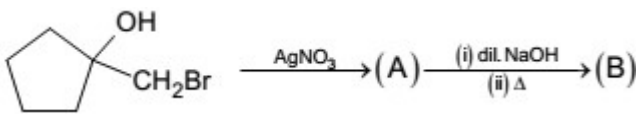
SO_2, SO_3, CO_2, C_3O_2, N_2O_5, Cl_2O_7 and Cl_2O_6

53. Find the number of resonating structures of the given carbanion where negative charge is on 2° carbon.



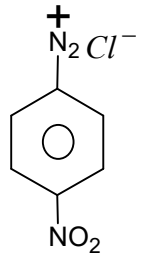
54. From which position does NO_2^+ replace a hydrogen from the following compound predominantly?

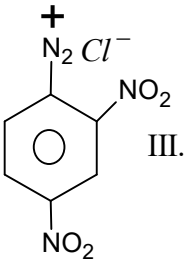


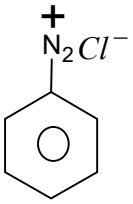
55. 

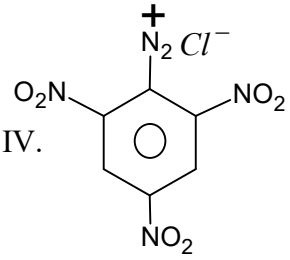
The degree of unsaturation in major product (B) will be:

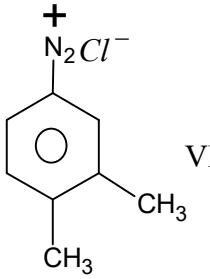
56. How many of the following diazonium salts are less reactive than (I) towards diazo coupling reaction?

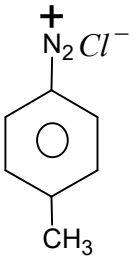
I. 

II. 

III. 

IV. 

V. 

VI. 

57. Analyse and complete the following reactions

(i) $CH_2CH{=}CH_2 + H_2$ –(Ni) →

(ii) $CH_3CH_2CH_2Cl + H_2$ –(Zn/H+) →

(iii) $CH_2CH_2CH_2COONa + NaOH$ –(CuO/Δ) →

Find the sum of molar masses (in g) of byproducts (if any) formed in above reactions

[Take atomic masses – H = 1, C = 12, O = 16, Na = 23, Cl = 35, Cu = 63, Zn =65

58. A light of wavelength 200nm falls upon a surface and two different wavelength photons λ = 800nm and λ =400nm are emitted from the surface. 80% of the energy absorbed is re-emitted in the form of photon. Number of photons emitted as λ = 800nm is 3 times that of number of photons emitted as λ = 400nm. If the ratio of total absorbed photon to total emitted photon is x. then find the numerical value of (12.8x)

59. When 1 mole of an ideal gas at 20 atm and 15 L volume expands such that the final pressure becomes 10 atm and volume is 60 L. The entropy change of the process in J/K/mole is (Round off your answer to nearest double digit integer)

Given that: Cp,m = 30.96 J/mole/K, R = 8.314 J/mol/K, ln 2 = 0.69

60. pH of 0.08 $moldm^{-3}$HOCl solution is 2.85. If its ionization constant is $A \times 10^{-B}$

Find the value of B. [antilog (0.15) = 1.413] (Given : $1 \le A < 10,\ \log 2 = 0.3$)

MATHEMATICS **MAX.MARKS: 100**

SECTION – I
(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.

61. Let C be the set of all complex numbers and

$$R=\left\{\left(z_1,z_2\right)\in C\times C:\frac{z_1-z_2}{z_1+z_2}\,is\ real\right\}$$

Then, on C, R is a

A) reflexive relation B) symmetric relation

C) transitive relation D) equivalence relation

62. Let A be a 3×3 real matrix such that $A=I-2BB^T$, where B^T is transpose of column matrix B, whose sum of the squares of elements is unity. Given the statements

I) $Tr\left(A\right)=1$ ($Tr\left(A\right)$ is sum of the elements in the principal diagonal of matrix A)

II) A is a symmetric matrix

III) A is an orthogonal matrix

Then the number of the correct statements among the above is

A) 0 B) 1 C) 2 D) 3

63. If λ is real and $\left(\lambda^2+\lambda-2\right)x^2+\left(\lambda+2\right)x<1$ for all real x, then λ belongs to the interval

A) $(-2,1)$ B) $(-2,2/5)$ C) $(2/5,1)$ D) $(1,2)$

64. Let $f\left(x\right)=x^2-3x+3$ and x_1,x_2,x_3 and x_4 be solutions of the equation $f\left(f\left(x\right)\right)=x$. Then the number of arrangements of x_1,x_2,x_3 and x_4 taken all at a time, is

A) 24 B) 4 C) 6 D) 1

65. Let $S=\{1,2,3,4,5,6,7\}$ .A subset of A is selected from 'S' and now keeping back elements in S, again a subset of S,B is selected. Let $E_1=$event that $A\cup B=\{1,2,3,4,5\}$, $E_2=$event that $A\cap B=\{1,2\}$, if $P\left(\frac{E_2}{E_1}\right)=\frac{a}{b}\left(a,b\in N\right)\left(G.C.D\ of\ \left(a,b\right)=1\right)$,then $\frac{1}{10}\left[\frac{b}{a}\right]$ equals___

([.] represents G.I.F)

A) 2 B) 3 C) 4 D) 5

66. Let $H_n:\frac{x^2}{1+n}-\frac{y^2}{3+n}=1,\ n\in N$. Let k be the smallest even value of n such that the eccentricity of H_k is a rational number. If l is the length of the latus rectum of H_k, then $21l$ is equal to

A) 101 B) 204 C) 102 D) 306

67. Two adjacent sides of parallelogram ABCD are given by $\overline{AB}=2\hat{i}+10\hat{j}+11\hat{k}$ and $\overline{AD}=-\hat{i}+2\hat{j}+2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by

A) $\frac{8}{9}$ B) $\frac{\sqrt{17}}{9}$ C) $\frac{1}{9}$ D) $\frac{4\sqrt{5}}{9}$

68. Consider a complex number z on the argand plane satisfying $\arg\left(z^2-\omega^2\right)=\frac{\pi}{2}+\arg\left(z^2-\omega\right)\left(\omega=e^{\frac{i2\pi}{3}}\right)$. If minimum value of $\left|z-2-2i\right|\left|z+2+2i\right|$ is $\frac{\sqrt{a}-\sqrt{b}}{2}\left(a,b\in N\right)$,then the value of $\frac{a+b}{52}=$

A) 2 B) 5 C) 10 D) 20

69. $\int_{1/e}^{\tan x}\frac{t}{1+t^2}dt+\int_{1/e}^{\cot x}\frac{dt}{t\left(1+t^2\right)}=$

A) $2\left(\tan e-1\right)$ B) $2\tan e$ C) 1 D) $\tan e+\cot e$

70. The area enclosed between the curves $y^2 = 4x$ and $x^2 = 4y$ inside the square formed by the lines $x = 1, y = 1, x = 4, y = 4$ is

A) $\frac{8}{3}$ B) $\frac{16}{3}$ C) $\frac{13}{3}$ D) $\frac{11}{3}$

71. The curve passing through the point $(0, \pi/4)$ satisfying the differential equation $\frac{dy}{dx} + x\sin 2y = x^3 \cos^2 y$ is

A) $\tan y = \frac{1}{2}\left(x^2 - 1\right) + \frac{3}{2}e^{-x^2/2}$ B) $\tan y = \frac{1}{2}\left(x^2 + 1\right) + \frac{3}{2}e^{-x^2/2}$

C) $\tan y = \frac{1}{2}\left(x^2 - 1\right) + \frac{3}{2}e^{-x^2}$ D) $\tan y = \frac{1}{2}\left(x^2 + 1\right) + \frac{3}{2}e^{-x^2}$

72. Statement – 1: If a,b,c are non zero real numbers such that $3\left(a^2 + b^2 + c^2 + 1\right) = 2\left(a + b + c + ab + bc + ca\right)$, then a,b,c are in A.P. as well as in G.P.

Statement – 2: A series is in A.P. as well as in G.P. if all the terms in the series are equal and non zero.

A) Both Statement - 1 and Statement - 2 are true

B) Both Statement - 1 and Statement - 2 are false

C) Statement - 1 is true, Statement - 2 is false

D) Statement - 1 is false, Statement - 2 is true

73. If a_1, a_2, a_3 and a_4 are the coefficients of any four consecutive terms in the expansion of $(1+x)^n$, then $\frac{a_1}{a_1 + a_2}, \frac{a_2}{a_2 + a_3}, \frac{a_3}{a_3 + a_4}$ are in

A) AP B) GP C) HP D) AGP

74. $x_1, x_2, \ldots\ldots\ldots x_{10}$ are ten observations such that $\sum x_i = 50$ and $\sum x_i x_j = 1100 \forall\ 1 \le i < j \le 10$, then standard deviation of $x_1, x_2 \ldots\ldots\ldots x_{10}$ is equal to

A) 5 B) 10 C) $\sqrt{5}$ D) $\sqrt{10}$

75. Match the following

	Column I		Column II
A)	If $\tan\theta$ is the G.M. between $\sin\theta$ and $\cos\theta$ then $2-4\sin^2\theta+3\sin^4\theta-\sin^6\theta$ can be	P)	1
B)	$\sqrt{3}\cot 20^0-4\cos 20^0=$	Q)	0
C)	$\cot 16^0\cot 44^0+\cot 44^0\cot 76^0-\cot 76^0\cot 16^0=$	R)	3
D)	$\sum_{r=1}^{9}\sin^2\left(\frac{r\pi}{18}\right)=$	S)	5

A) A – S; B – P; C – R; D – P B) A – P; B – R; C – P; D – S

C) A – P; B – P; C – R; D – S D) A – R; B – D; C – P; D – S

76. Let

$$P_n=\frac{2^3-1}{2^3+1}\cdot\frac{3^3-1}{3^3+1}.....\frac{n^3-1}{n^3+1};n=2,3,4.....$$

Then $\lim_{n\to\infty}P_n$ is equal to

A) $\frac{1}{2}$ B) $\frac{7}{11}$ C) $\frac{3}{4}$ D) $\frac{2}{3}$

77. Let $f(x)=a_0|x|^3+a_1|x|^2+a_2|x|+a_3$. Then

A) f is differentiable at x = 0 if $a_2\neq 0$

B) f is not differentiable at x=0, whatever be a_0,a_1,a_2,a_3

C) f is differentiable at $x=0$ if and only if $a_2=0$

D) If f is differentiable at $x=0$, then $a_0=0$ and $a_2=0$

78. Let A, B and C be finite sets such that $A\cap B\cap C=\phi$ and each one of the sets $A\Delta B, B\Delta C$ and $C\Delta A$ has 100 elements. The number of elements in $A\cup B\cup C$ is

A) 250 B) 200 C) 150 D) 300

79. Let Z be the set of all integers,

$A=\{(x,y)\in Z\times Z:(x-2)^2+y^2\le 4\}$

$B=\{(x,y)\in Z\times Z:x^2+y^2\le 4\}$ and

$C=\{(x,y)\in Z\times Z:(x-2)^2+(y-2)^2\le 4\}$

If the total number of relations from $A\cap B$ to $A\cap C$ is 2^p, then the value of p is

A) 16 B) 49 C) 25 D) 9

80. Let A,B and C be three events such that the probability that exactly one of A and B occurs is $(1-k)$, the probability that exactly one of B and C occurs is $(1-2k)$, the probability that exactly one of C and A occurs is $(1-k)$ and the probability of all A, B and C occur simultaneously is k^2, where $0<k<1$. Then the probability that at least one of A, B and C occur is

A) Greater than $\frac{1}{2}$ B) Exactly equal to $\frac{1}{2}$

C) Greater than $\frac{1}{8}$ but less than $\frac{1}{4}$ D) Greater than $\frac{1}{4}$ but less than $\frac{1}{2}$

SECTION-II
(NUMERICAL VALUE ANSWER TYPE)

This section contains 10 questions. The answer to each question is a Numerical value. If the Answer in the decimals , **Mark nearest Integer only. Have to Answer any 5 only out of 10 questions** and question will be evaluated according to the following marking scheme:
Marking scheme: +4 for correct answer, -1 in all other cases.

81. P is a point satisfying $\arg z=\pi/4$, such that sum of its distances from two given points $(0,1)$ and $(0,2)$ is minimum, then P must be $\frac{k}{3}(1+i)$. Then numerical value of k is _____

82. The number of ways can 14 identical toys distributed among three boys so that each one gets atleast one toy and no two boys get equal number of toys is n, then $\frac{n}{10}$ is equal to____

83. Let P be an interior point of a triangle ABC such that $\overrightarrow{PA}+\overrightarrow{2PB}+\overrightarrow{3PC}=\vec{0}$. Then the ratio $\frac{\text{Area of triangle ABC}}{\text{Area of triangle APC}}$ is equal to _____

84. The number of values of x at which the function $f(x)=(x-1)x^{2/3}$ has extremum values is

85. If the complete solution set of the inequality $\left(\text{cosec}^{-1}x\right)^2-2\left(\text{cosec}^{-1}x\right)\geq\frac{\pi}{6}\text{cosec}^{-1}x-\frac{\pi}{3}$ is $(-\infty,a]\cup[b,\infty)$, then $(a+b)$ is equal to

86. A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinate axes at points P and Q. Then the absolute minimum value of OP+OQ as L varies (Where O is the origin) is

87. The sum of first 100 terms of the series is

$\tan^{-1}\left(\frac{1}{1+x+x^2}\right)+\tan^{-1}\left(\frac{1}{3+3x+x^2}\right)+\tan^{-1}\left(\frac{1}{7+5x+x^2}\right)+\tan^{-1}\left(\frac{1}{13+7x+x^2}\right)+....$(if x>0) is $\tan^{-1}\left(\frac{100}{1+ax^2+bx}\right)$, then $a+b=$____

88. Let the curve C be the mirror image of the parabola $y^2=4x$ with respect to the line $x+y+4=0$. If A and B are the points of intersection of C with the line $y=-5$, then the distance between A and B is

89. If $f(x)=\int\frac{5x^8+7x^6}{\left(x^2+1+2x^7\right)^2}dx,\ (x\geq 0)\ f(0)=0$ and $f(1)=\frac{1}{K}$, then the value of K is _____

90. Suppose x_1 and x_2 are the point of maximum and the point of minimum respectively of the function $f(x)=2x^3-9ax^2+12a^2x+1$ respectively, $(a>0)$ then for the equality $x_1^2=x_2$ to be true the value of 'a' must be

Sec: MODEL-A,B&C | Date: 05-01-24

Time: 3 HRS | Max. Marks: 300

JEE-MAIN

KEY SHEET

PHYSICS

1	**D**	2	**C**	3	**A**	4	**A**	5	**A**
6	**A**	7	**C**	8	**A**	9	**A**	10	**C**
11	**D**	12	**B**	13	**C**	14	**A**	15	**B**
16	**A**	17	**B**	18	**A**	19	**A**	20	**A**
21	**5**	22	**5**	23	**2**	24	**2**	25	**3**
26	**1**	27	**3**	28	**120**	29	**6**	30	**3**

CHEMISTRY

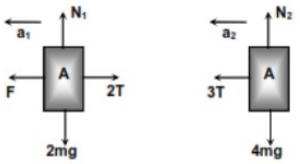
31	**D**	32	**D**	33	**D**	34	**C**	35	**B**
36	**A**	37	**A**	38	**D**	39	**A**	40	**D**
41	**B**	42	**B**	43	**C**	44	**A**	45	**D**
46	**C**	47	**C**	48	**D**	49	**B**	50	**A**
51	**3**	52	**5**	53	**4**	54	**3**	55	**4**
56	**3**	57	**4**	58	**5**	59	**27**	60	**5**

MATHEMATICS

61	**B**	62	**D**	63	**B**	64	**B**	65	**B**
66	**D**	67	**B**	68	**B**	69	**C**	70	**D**
71	**C**	72	**A**	73	**A**	74	**C**	75	**C**
76	**D**	77	**C**	78	**C**	79	**C**	80	**A**
81	**2**	82	**6**	83	**3**	84	**2**	85	**1**
86	**18**	87	**101**	88	**4**	89	**4**	90	**2**

SOLUTIONS
PHYSICS

1.



$F - 2T = 2ma_1$

$3T = 4ma_2$

$2a_1 = 3a_2$

solving we get

$$a_2 = \frac{3F}{17m}$$

2. $$c = \frac{c_1 c_2}{c_1 + c_2} = \frac{\frac{A\varepsilon_0}{x} \times \frac{A\varepsilon_0}{(a-b-x)}}{\frac{A\varepsilon_0}{x} + \frac{A\varepsilon_0}{(a-b-x)}}$$

$$c = \frac{A\varepsilon_0}{(a-b)} \text{independent of } x$$

3. $$\Delta W_{cyclic} = \frac{1}{2} V_0 P_0 = \frac{P_0 V_0}{2}$$

$\text{For the process AB}, P = KV \Rightarrow PV^{-1} = \text{constant}$

Molar heat capacity of the gas in the process AB,

$$C = C_v + \frac{R}{(1-x)} = \frac{3R}{2} + \frac{R}{2} = 2R$$

$$\Delta Q_{AB} = nC\Delta T = n2R(4T_0 - T_0) = 6nRT_0 = 6P_0V_0$$

$\Delta Q_{BC} < 0 \text{ and } \Delta Q_{CA} < 0$

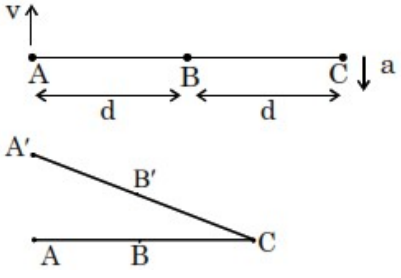
The efficiency of the cyclic process, $\eta = \frac{\Delta W_{cyclic}}{\Delta Q_{supplied}} \times 100 = \frac{P_0 V_0}{2 \times 6P_0 V_0} \times 100$

$$\eta = \frac{25}{3} = 8.33\%$$

4. By the symmetry, $\vec{B}_{total} = 0$

5. Let us observe the motion of A and B relative to C.

$$\frac{AA'}{2d} = \frac{BB'}{d}$$



$$BB' = \frac{AA'}{2} = \frac{vt + \frac{1}{2}at^2}{2}$$

$$BB' = \frac{Vt}{2} + \frac{1}{4}at^2$$

$\frac{V}{2}$ is initially speed of B w.r.t C as well as ground.

$\frac{a}{2}$ is acceleration of B w.r.t C

$$\vec{a}_{B/g} = \vec{a}_{B/C} + \vec{a}_{C/g}$$

$$= \frac{a}{2} - a = \frac{-1}{2}a$$

6. Conceptual

7. $v\cos\alpha = u\cos\theta$

$$v = \frac{u\cos\theta}{\cos\alpha}$$

$$v\sin\alpha = u\sin\theta - gt$$

$$\frac{u\cos\theta\sin\alpha}{\cos\alpha} - u\sin\theta = -gt$$

$$\frac{u\cos\theta\sin\alpha - u\sin\theta\cos\alpha}{\cos\alpha} = -gt$$

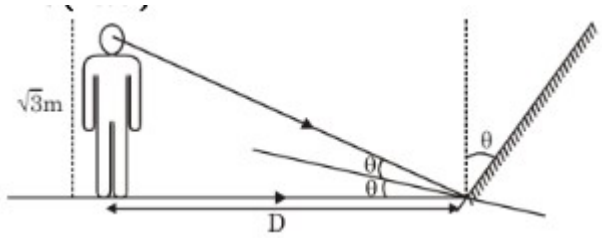
$$t = \frac{u\sin(\theta-\alpha)}{g\cos\alpha}$$

8. $m_1 s_1 \Delta T_1 = m_2 s_2 \Delta T_2$

Since $m_1 s_1 < m_2 s_2$

$\Delta T_1 > \Delta T_2$

9.



$$\tan 2\theta = \frac{\sqrt{3}}{D}$$

10. $Q = (80\times 7) + (120\times 8) - (200\times 6.5)\,\text{MeV} = 220\,\text{MeV}$

11. By the property of full wave rectifier

12. $$P_{final} = P_{initial}\left(\frac{V_0}{V_0 + \Delta V}\right)^n$$

13. $$15\sigma Vg - \sigma Vg - w = \left(\frac{w}{g} + \sigma V\right)a$$

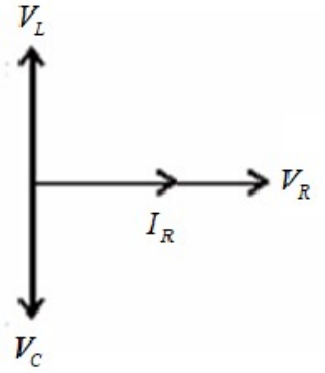
$$a = \left(\frac{14\sigma Vg - w}{w + \sigma Vg}\right)g$$

14. Time period of a spring mass system will remain constant when fluid is non-viscous.

15. $v_c = \sqrt{2}v_0 = 1.414\,v_0$ % increase in orbital

$$\text{velocity} = \frac{v_0 - v_0}{v_0} \times 100 = 41.4\%$$

16.



A) phase difference between current and voltage in a purely resistive AC circuit is zero.

B) phase difference between current and voltage in a pure inductive AC circuit is $\frac{\pi}{2}$; current lags voltage.

C) phase difference between current and voltage in a pure capacitive AC circuit is $\frac{\pi}{2}$; current leads voltage.

D) phase difference between current and voltage in an LCR series circuit is $\tan^{-1}\left(\frac{X_C - X_L}{R}\right)$.

17. $\phi = -\frac{1}{2}R^2B_1 + \frac{1}{2}B_2 3R^2\theta$

$$\therefore \frac{d\phi}{dt} = \varepsilon = \frac{R^2\theta}{2}\frac{dB_1}{dt} - \frac{3R^2\theta}{2}\frac{dB_2}{dt}$$

$$= \frac{R^2\theta}{2}.2K_0 + -\frac{3R^2}{2}\theta 4K_0$$

$$\therefore \text{emf} = 5R^2K_0\theta$$

18. At node, energy is maximum when all particle reach to there extreme position.

19. $V_s = \tan\theta.\,\upsilon + C$

$$4V = \tan\theta.\,2\upsilon_0 + C$$

$$10V = \tan\theta.\,4\upsilon_0 + C$$

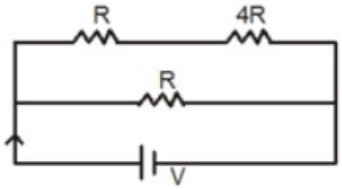
$$6V = 2\upsilon_0 \tan\theta \Rightarrow \tan\theta = \frac{3V}{\upsilon_0}$$

$$4V = 6V - \frac{W_c}{e}$$

$$W_c = (2V)e$$

$$h = \frac{(3V)e}{\upsilon_0}$$

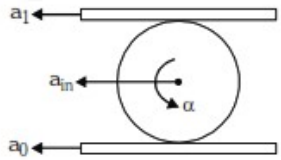
20.



$$w_1 = \frac{v^2}{R} = \frac{V^2}{\frac{v^2}{P}} = P$$

$$w_2 = \left(\frac{V}{5R}\right)^2 .4R = \frac{4V^2.P}{25V^2} = \frac{4P}{25}$$

21.



$$a_0 = a_{cm} - R\alpha.....(1)$$

$$a_1 = a_{cm} + R\alpha.....(2)$$

Solivng equation $(1)\&(2)$

$$a_1 - a_0 = 2R\alpha = 4a_0$$

$$a_1 = 5a_0$$

$$K = 5.00$$

22. Tension of rope is maximum at lowest point

$$T_{max} - mg = \frac{mv^2}{\ell}.....(1)$$

By energy conservation,

$$mg\frac{\ell}{2} = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{g\ell}$$

From (1),

$$T_{max} - mg = \frac{m(g\ell)}{\ell} \Rightarrow T_{max} = 2mg$$

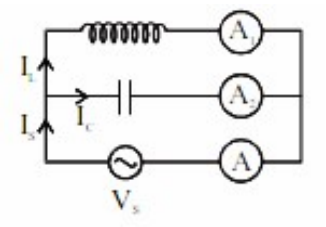
For 8 kg block, $T_{max} = f_L$

$$2mg = \mu(8g)$$

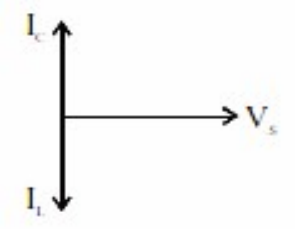
$$4g = \mu(8g)$$

$$\mu = 0.5$$

23.



Phasor diagram



$I_8 = I_C + I_L = 5 - 3 = 2A$

24. $E = E_0 \cos(kz - \omega t)\hat{i}$

$k = \frac{\omega}{V_{wave}} = \frac{6\times10^8}{3\times10^8} = 2$

25. $I = \frac{P_0^2 V}{2B}$

$10^{-2} = \frac{P_0^2 \times 340}{2\times1.6\times10^5}$

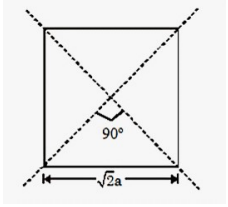
$P_0 = \sqrt{\frac{320}{34}} = \sqrt{\frac{160}{17}}$

$P_0 = 3$

26. $\frac{p^2}{2m} = \frac{hc}{\lambda} - \phi = \simeq \frac{hc}{\lambda} = \frac{h^2}{2m\lambda^2}$

$\lambda = \frac{h}{2mc} = \frac{6.60\times10^{-34}}{2\times9.1\times10^{-31}\times3\times10^8} = \frac{11}{91}\times10^{-11}\,m = 1.2\,pm$

27. Consider prism of mass 4 m by joining 4 prism given in question.Total MOI of this system will be



$I = (4I_{prism}) = \frac{(4m)(\sqrt{2}a)^2}{6} \Rightarrow I_{prism} = \frac{ma^2}{3}$

28. $\tau = R_{Th.}C, R_{Th}$ is the Thevenin's resistance at the capacitor terminals.

$R_{Th} = 8 + (20|(9 + (70|30)) = 20\,k\Omega$

$\tau = 0.12\ s$

29. $h = \frac{2T\cos\theta}{\rho rg} = 59.6mm$

Here h is greater than protruding part of tube hence water will rise to maximum length of tube such that radius of meniscus is given by $R = \frac{hr}{l}$

30. White spot on screen would be central maxima
Where

$\Delta x = 0\ y = \frac{d}{2} - \frac{d}{8} = \frac{3d}{8}$

CHEMISTRY

31. As 'q' is noble gas, p, r and s having atomic number Z-1, Z+1 and Z+2 should belong to halogen, alkali metal and alkaline earth metal respectively. As halogen has one electron less than stable noble gas configuration it has greater tendency to accept an additional electron forming anion. Alkaline earth metal having valence shell configuration ns2 exists in +2 oxidation state.

32. VA group hydrides: (a) reducing properties increases down the group.
(b) Basic nature decreases down the group.
(c) Thermal stability decreases down the group.
(d) Bond angles decreases down the group.

34. Mn_2O_7: multiple bonds

$$AgNO_3 \xrightarrow{\Delta} Ag\downarrow + NO_2\uparrow + O_2\uparrow$$

$$\overset{(+5)}{V}O_2^+ < \overset{(+6)}{Cr_2}O_7^{2-} < \overset{(+7)}{Mn}O_4^-$$

When I– is oxidized by MnO_4– in alkaline medium, I– converts into IO_3–

37. Conceptual

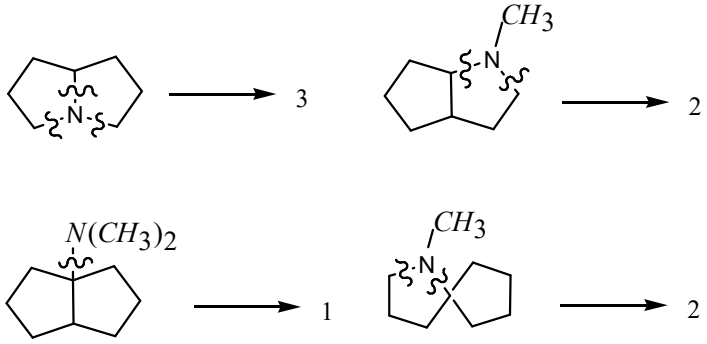
38. Yellow coloured solution of metal ion belonging to basic radicals of group-III → Fe^3 is metalion
$Fe^{+3} + SCN^- \rightarrow [Fe(SCN)_2]^+$ (Red solution) + other species
$Fe^{+3} + C_2O_4^{2-} \rightarrow [Fe(C_2O_4)_3]$ (stable complex)
$Fe^{3+} + F^- \rightarrow [FeF_6]^{3-}$ (stable complex)
F^- and $C_2O_4^{2-}$ are stronger ligand than SCN^-.
Hg^{2+} forms stable complex with SCN^-

39. Conceptual

41. X = 1-Bromo 1,2-dimethylcyclopentane, Y= 1,2 -dimethylcyclopentene,
Z = Heptane -2,6 –dione

43. Decarboxylation of betaketo acid after ester hydrolysis

44.



46. Initial volume of gas $= 1000 + V_{O_2}$

Final volume of gas $= 1000 + V_{O_2} - \frac{x}{2}$

$\therefore$ vol of $CO = 20\,ml$

47. Freezing is exothermic process. The heat released increases the entropy of surrounding.

48. $P_{solution} = 160$
Solution have positive deviation from Raoult's law.
$P_{total} = 200 \times 0.5 + 100 + 0.5 = 150$
$\Delta G_{mix} < 0 \quad \Delta V_{mix} > 0 \quad \Delta H_{mix} > 0$
$\Delta S_{surr} < 0$

49. Justification: K increases with increase in temperature.
Q > K, Therefore, reaction proceeds in the backward direction.
$\Delta n > 0$, Therefore, $\Delta S > 0$.

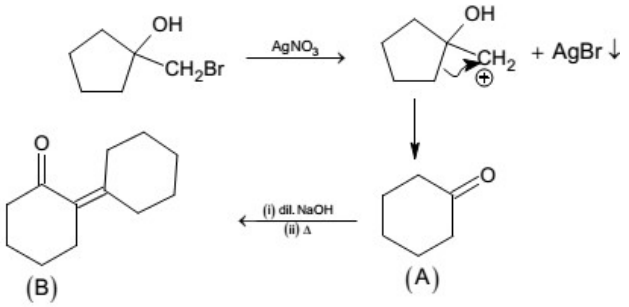
50. Conceptual

51. Two cis, one trans

52. Conceptual

53. Conceptual

55.



Degree of unsaturation = 4.

56. III, V & VI are less reactive

57. $\frac{[B]}{[C]}=\frac{3k_1}{8k_1}=\frac{3}{8}=\alpha \quad (\text{at any time})$

$\frac{[C]}{[D]}=\frac{8k_1}{7.5k_2}=\frac{8}{7.5}=\beta \,(\text{at any time})$

$\therefore \alpha\beta = 0.4$

58. $\frac{n_1}{2000}\times 0.8=\frac{3n_2}{8000}+\frac{n_2}{4000}$

$\Rightarrow \frac{n_1\times 0.8}{2000}=\frac{5n_2}{8000}$

$\Rightarrow \frac{n_1}{n_2}=\frac{5}{4\times 0.8}$

$\Rightarrow \frac{n_1}{4n_2}=\frac{5}{16\times 0.8}=\frac{5}{12.8}$

59. $\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2} \qquad \therefore \frac{T_2}{T_1}=\frac{6}{3}$

$\Delta S = 2.303\times n\left[C_P \log\frac{T_2}{T_1}+R\log\frac{P_1}{P_2}\right]$

$\Delta S = 27.22\ J/K/mole$

60. $pH \text{ of } HOCl = 2.85$

But, $-pH=\log\left[H^+\right]$

$\therefore -2.85=\log\left[H^+\right]$

$\bar{3}.15=\log\left[H^+\right]$

For weak mono basic acid $\left[H^+\right]=\sqrt{K_a\times C}$

$$K_a=\frac{\left[H^+\right]^2}{C}=\frac{\left(1.413\times 10^{-3}\right)^2}{0.08}$$
$$=24.957\times 10^{-6}=2.4957\times 10^{-5}$$

MATHS

61. Since $(0,0) \notin R, R$ is not reflexive, we have

$(z_1, z_2) \in R \Rightarrow \frac{z_1 - z_2}{z_1 + z_2}$ is real

$\Rightarrow \frac{z_2 - z_1}{z_1 + z_2}$ is real $\Rightarrow (z_2, z_1) \in R$

Therefore R is symmetric,

Since $(0,z) \in R$ and $(z,0) \in R$, but $(0,0) \notin R$ there fore R is not transitive. Hence R is not an equivalence relation.

63 Suppose that $(\lambda^2 + \lambda - 2)x^2 + (\lambda + 2)x - 1 < 0$

For all real x,

$\lambda^2 + \lambda - 2 < 0$ and $(\lambda + 2)^2 + 4(\lambda^2 + \lambda - 2) < 0$

$(\lambda + 2)(\lambda - 1) < 0$ and $5\lambda^2 + 8\lambda - 4 < 0$

$-2 < \lambda < 1$ and $(\lambda + 2)(5\lambda - 2) < 0$

$-2 < \lambda < 1$ and $-2 < \lambda < \frac{2}{5}$

These inequalities imply

$\lambda \in \left(-2, \frac{2}{5}\right)$

64. Note that every solution of $f(x) = x$ is also a solution of $f(f(x)) = x$

$f(x) = x \Rightarrow x^2 - 4x + 3 = 0 \Rightarrow x = 3$ or 1

Therefore, 3 and 1 are roots of $f(x) = x$, also

$f(f(x)) = x \Rightarrow (x^2 - 3x + 3)^2 - 3(x^2 - 3x + 3) + 3 = x$

$\Rightarrow x^4 - 6x^3 + 12x^2 - 10x + 3 = 0$

Since 3 and 1 are roots of $f(x) = x$, then are roots of $f(f(x)) = x$ also and therefore.

$f(f(x)) - x = (x-3)(x-1)(x^2 - 2x + 1) = (x-3)(x-1)^3$

Therefore 3, 1, 1, 1 are solutions of $f(f(x)) = x$. Hence

The number of arrangements of the solutions is

$\frac{4!}{3!} = 4$

66. $e = \sqrt{1 + \frac{3+n}{1+n}} = \sqrt{\frac{2n+4}{n+1}}$

Put $n = 48$

$e = \frac{10}{7}$ is a rational number

$\frac{x^2}{49} - \frac{y^2}{51} = 1$

$l = \frac{2b^2}{a} = \frac{102}{7}$

69. Let

$$F(x)=\int_{1/e}^{\tan x}\frac{t}{1+t^2}dt+\int_{1/e}^{\cot x}\frac{dt}{t\left(1+t^2\right)}$$

Then

$$F'(x)=\left(\frac{\tan x}{1+\tan^2 x}\right)\sec^2 x+\frac{1}{\cot x\left(1+\cot^2 x\right)}\left(-\operatorname{cosec}^2 x\right)$$

$$=\tan x-\left(1/\cot x\right)=0$$

Therefore F is a constant function. Now

$$F\left(\frac{\pi}{4}\right)=\int_{1/e}^{1}\frac{t}{1+t^2}dt+\int_{1/e}^{1}\frac{1}{t\left(1+t^2\right)}dt$$

$$=\int_{1/e}^{1}\frac{t^2+1}{t\left(1+t^2\right)}dt=\left[\log_e t\right]_{1/e}^{1}$$

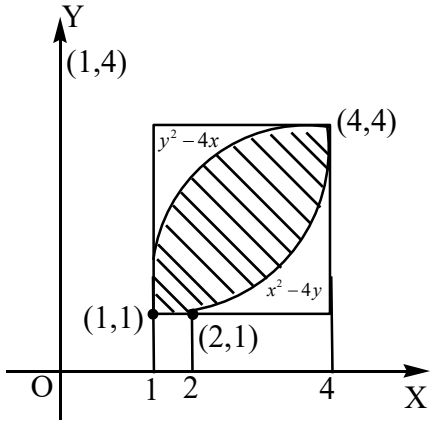
$$=0-\left(0-\log_e e\right)=1$$

Hence $F(x)=1$

70. The two curves intersect at $(4,4)$ which is a vertex of the given square. Therefore

Required area (Shaded portion) = $\int_1^4 2\sqrt{x}$

$$-\int_1^4\frac{x^2}{4}dx-(2-1)\times 1=2\times\frac{2}{3}\left[x^{3/2}\right]_1^4-\frac{1}{12}\left[x^3\right]_2^4-1$$



$$=\frac{4}{3}\times[8-1]-\frac{1}{12}(64-8)-1=\frac{28}{3}-\frac{56}{12}-1=\frac{112-56-12}{12}$$

$$=\frac{44}{12}=\frac{11}{3}$$

71. The given equation is

$$\sec^2 y\frac{dy}{dx}+x\left(2\tan y\right)=x^3$$

Put $\tan y=z$, Therefore

$$\frac{dz}{dx}+(2x)z=x^3 \qquad \text{(Linear in z)}$$

The integrating factor is

$$\text{I.F}=e^{\int 2xdx}=e^{x^2}$$

Therefore

$$ze^{x^2}=\int x^3e^{x^2}dx+c$$

$$= \int x^2 e^{x^2} x dx + c = \frac{1}{2}\int x^2 e^{x^2} (2x)\, dx + c$$

$$= \frac{1}{2}\int te^t dt + c \text{ where } t = x^2 = \frac{1}{2}e^t (t-1) + c$$

$$= \frac{1}{2}e^{x^2}\left(x^2 - 1\right) + c$$

So

$$\tan y = \frac{1}{2}\left(x^2 - 1\right) + ce^{-x^2}$$

The curve passes through $(0, \pi/4)$. This implies

$$1 = -\frac{1}{2} + c \Rightarrow c = \frac{3}{2}$$

Therefore $\tan y = \frac{1}{2}\left(x^2 - 1\right) + \frac{3}{2}e^{-x^2}$

72. $3\left(a^2 + b^2 + c^2 + 1\right) - 2\left(a + b + c + ab + bc + ca\right) = 0$

$\Rightarrow \quad (a-1)^2 + (b-1)^2 + (c-1)^2 + (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$

$\Rightarrow \quad a = b = c = 1$

73. Let a_1, a_2, a_3, a_4 be the coefficients of rth, $(r+1)th$, $(r+2)th$ and $(r+3)th$ terms, respectively.

Then

$a_1 = {}^n C_{r-1}, a_2 = {}^n C_r, a_3 = {}^n C_{r+1}, a_4 = {}^n C_{r+2}$

We know that

$$\frac{{}^n C_K}{{}^n C_{K-1}} = \frac{n-K+1}{K}$$

Therefore

$$\frac{a_2}{a_1} = \frac{n-r+1}{r} \Rightarrow 1 + \frac{a_2}{a_1} = \frac{n+1}{r}$$

$$\frac{a_3}{a_2} = \frac{n-r}{r+1} \Rightarrow 1 + \frac{a_3}{a_2} = \frac{n+1}{r+1}$$

$$\frac{a_4}{a_3} = \frac{n-r-1}{r+1} \Rightarrow 1 + \frac{a_4}{a_3} = \frac{n+1}{r+2}$$

And hence

$$\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{r}{n+1} + \frac{r+2}{n+1} = 2\left(\frac{r+1}{n+1}\right) = 2\left(\frac{a_2}{a_2 + a_3}\right)$$

74. $\sum x_1^2 = \left(\sum x_i\right)^2 - 2\sum x_1 x_j = 300; \frac{\sum x_1^2}{10} = 30$

$$\sigma = \sqrt{\frac{\Sigma x_1^2}{10} - \left(\frac{\Sigma x_1}{10}\right)^2}; \sigma = \sqrt{30 - 25} = \sqrt{5}$$

75. (A) $\tan^2\theta = \sin\theta\cos\theta \Rightarrow \sin\theta = \cos^3\theta$

$\therefore (1 - \sin^2\theta) + (1 - 3\sin^2\theta) + 3\sin^4\theta - \sin^6\theta$

$= \cos^2\theta + (1 - \sin^2\theta)^3 = \cos^2\theta + \cos^6\theta = \cos^2\theta + \sin^2\theta = 1$

(B) $\sin 40^0 = \sin(60^0 - 20^0)$

$$2\sin 20^0 \cos 20^0 = \frac{\sqrt{3}}{2}\cos 20^0 - \frac{1}{2}\sin 20^0$$

$4\cos 20^0 = \sqrt{3}\cot 20^0 - 1$

(C) $\dfrac{3+\cot 76^0 \cot 16^0}{\cot 76^0 + \cot 16^0} = \dfrac{3\sin 76^0 \sin 16^0 + \cos 76^0 \cos 16^0}{\sin(76^0+16^0)}$

$= \dfrac{2\sin 76^0 \sin 16^0 + \cos(76^0 - 16^0)}{\sin(76^0+16^0)} = \dfrac{\cos 60^0 - \cos 92^0 + \cos 60^0}{\sin 92^0} = \dfrac{1-\cos 92^0}{\sin 92^0} = \tan 46^0 = \cot 44^0$

(D) $\sin^2\left(\dfrac{\pi}{18}\right) + \sin^2\left(\dfrac{2\pi}{18}\right) + ... + \sin^2\left(\dfrac{\pi}{2}\right) = 5$

76. We have

$\dfrac{k^3-1}{k^3+1} = \dfrac{(k-1)\left(k^2+k+1\right)}{(k+1)\left(k^2-k+1\right)} = \left(\dfrac{k-1}{k+1}\right)\left(\dfrac{k^2+k+1}{(k-1)^2+(k-1)+1}\right)$

For $k = 2,3,.....n$ Therefore

$P_n = \left(\dfrac{2-1}{2+1}, \dfrac{3-1}{3+1}, \dfrac{4-1}{4+1} ..... \dfrac{n-2}{n} \cdot \dfrac{n-1}{n+1}\right) \Rightarrow \left(\dfrac{7}{3} \cdot \dfrac{13}{7} \cdot \dfrac{21}{13} ..... \dfrac{n^2+n+1}{(n-1)^2+(n-1)+1}\right)$

$= \left(\dfrac{1}{3} \cdot \dfrac{2}{4} \cdot \dfrac{3}{5} ..... \dfrac{n-2}{n} \cdot \dfrac{n-1}{n+1}\right)\left(\dfrac{7}{3} \cdot \dfrac{13}{7} \cdot \dfrac{21}{13} ..... \dfrac{n^2+n+1}{(n-1)^2+(n-1)+1}\right)$

$= \left(\dfrac{2}{n(n+1)}\right)\left(\dfrac{n^2+n+1}{3}\right) = \dfrac{2}{3}\left(1 + \dfrac{1}{n(n+1)}\right)$

Therefore $\lim\limits_{n\to\infty} P_n = \dfrac{2}{3}(1+0) = \dfrac{2}{3}$

77. We have seen that $|x|$ is not differentiable at $x = 0$, Whereas $|x|^3$ is differentiable at $x = 0$. Also $|x|^2 = x^2$ is differentiable for all real x. If $a_2 = 0$, then

$f(x) = a_0|x|^3 + a_1|x|^2 + a_3$

Is differentiable at $x = 0$. Conversely, if $f(x)$ is differentiable at $x = 0$, then

$a_2|x| = f(x) - a_0|x|^3 - a_1|x|^2 - a_3$

Is differentiable at $x = 0$ which is possible when $a_2 = 0$

78. Let $n(X)$ denote the number of elements in X

Then,

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$

$-n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$= \Sigma n(A) - \Sigma n(A \cap B)$

(since $A \cap B \cap C = \phi$)

Now

$A\Delta B = (A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

Therefore

$n(A\Delta B) = n(A \cup B) - n(A \cap B)$

$n(A) + n(B) - 2n(A \cap B)$

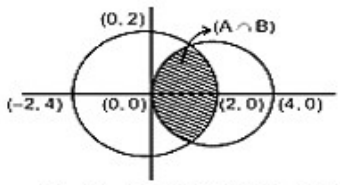
And

$300 = \Sigma n(A\Delta B) = \Sigma\left[n(A) + n(B) - 2n(A \cap B)\right] = 2\left[\Sigma n(A) - \Sigma n(A \cap B)\right]$

Therefore $n(A \cup B \cup C) = \Sigma n(A) - \Sigma n(A \cap B) = 300/2 = 150$

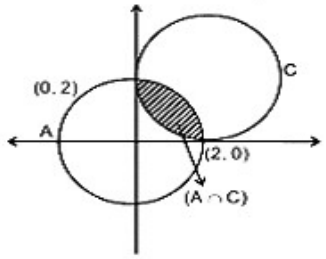
79.

The set A and set B are represented as



$\therefore\ A \cap B = \{(0, 0), (1, 0), (2, 0), (1, 1), (1, -1)\}$

The set A and set C are represented as



$\therefore\ A \cap C = \{(1, 1), (2, 0), (2, 1), (2, 2), (3, 2)\}$

$\therefore$ Total number relations from $A \cap B$ to $A \cap C = 2^{5 \cdot 5}$

$\therefore\ p = 25$

80.

$$P(A) + P(B) - 2P(A \cap B) = 1 - k \quad \text{...(i)}$$

$$P(B) + P(C) - 2P(B \cap C) = 1 - 2k \quad \text{...(ii)}$$

$$P(C) + P(A) - 2P(C \cap A) = 1 - k \quad \text{...(iii)}$$

(i) + (ii) + (iii)

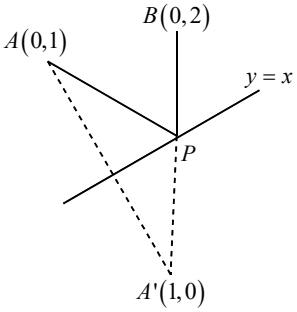
$$\Rightarrow \sum P(A) - \sum P(A \cap B) = \frac{3-4k}{2}$$

$$P(A \cup B \cup C) = \sum P(A) - \sum P(A \cap B) + P(A \cap B \cap C)$$

$$= \frac{3-4k}{2} + k^2$$

$$= (k-1)^2 + \frac{1}{2} > \frac{1}{2}$$

81.



$PA + PB$ will be minimum

Where A and A' are mirror image

A', P, B are collinear equation of line A'B : 2x+y=2 Solve A'B with y =x

$$\therefore x = \frac{2}{3}, y = \frac{2}{3}$$

$$\therefore P = \frac{2}{3}(1+i)$$

$$\therefore k = 2$$

82. Number of ways to distribute at least one toy to each ${}^{14-1}C_{3-1} = {}^{13}C_2 = 78$

If toys are distributed in the following way then two will get equal number of toys

	No.of ways
1 1 12	$\frac{3!}{2!} = 3 ways$
2 2 10	3 ways
3 3 8	3 ways
4 4 6	3 ways
5 5 4	3 ways
6 6 2	3 ways

$\therefore$ Required number of ways = 78-18= 60

83.

$$\frac{Ar\ (\Delta ABC)}{Ar(\Delta APC)} = \frac{\frac{1}{2}|\vec{a}\times\vec{b}\times\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}{\frac{1}{2}|\vec{a}\times\vec{b}|}$$

Now $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$

$\Rightarrow \ (\vec{a}+2\vec{b}+3\vec{c})\times\vec{b}=\vec{0}$

$\Rightarrow \ 2\vec{a}\times\vec{b}+3\vec{a}\times\vec{c}=\vec{0}$

and $\vec{a}\times\vec{b}+3\vec{c}\times\vec{b}=\vec{0}$

$\Rightarrow \ \vec{a}\times\vec{b}=3(\vec{b}\times\vec{c})$

Let $\vec{c}\times\vec{a}=\vec{\alpha}$, then $\vec{a}\times\vec{b}=\frac{3\vec{\alpha}}{2}, \vec{b}\times\vec{c}=\frac{\vec{\alpha}}{2}$

$\therefore$ Required ratio $= \frac{|\frac{3\vec{\alpha}}{2}+\frac{\vec{\alpha}}{2}+\vec{\alpha}|}{|\vec{\alpha}|} = 3$

84. Differentiating the given function we have

$$f'(x) = x^{2/3} + (x-1)\frac{2}{3}x^{-1/3}$$

$$= \frac{3x+2(x-1)}{3x^{1/3}} = \frac{5x-2}{3x^{1/3}}$$

Now $f'(x)=0 \Leftrightarrow x=2/5$. Also $f(x)$ is defined and continuous at $x=0$, f is not differentiable at $x=0$. Thus, zero is a critical point. Therefore, 0 and 2/5 are critical points of $f(x)$

(i) $x<0 \Rightarrow f'(x)>0$ and $x>0 \rightarrow f'(x)<0$. Therefore at $x=0$, f is maximum and the maximum value = $f(0)=0$

ii) $x<2/5 \Rightarrow f'(x)<0$ and $x>2/5 \rightarrow f'(x)>0$. Thus f is minimum at $x=2/5$ and the minimum value

$$f\left(\frac{2}{5}\right) = -\frac{3}{5}\left(\frac{4}{25}\right)^{1/3}$$

86. Let the equation of the line L, by hypothesis, be $y-2=m(x-8)$

Where $m<0$. Therefore

$p=\left(8-\frac{2}{m},0\right)$ and $Q=(0,2-8m)$

Now,

$$OP+OQ=\left(8-\frac{2}{m}\right)+(2-8m)\left(\because m<0\right)$$

$$10-\left(\frac{2}{m}+8m\right)\geq 10+2\sqrt{\frac{-2}{m}\times(-8m)}$$

$$=10+8\left(\because AM\geq GM\right)$$

And equality occurs if and only if

$$-\frac{2}{m}=-8m \text{ or } m=-\frac{1}{2}$$

Hence, the absolute minimum of $OP+OQ$ is 12 + 6 = 18

87.

Consider the series

$$\tan^{-1}\frac{1}{1+x+x^2}+\tan^{-1}\frac{1}{3+3x+x^2}+\tan^{-1}\frac{1}{(7+5x+x^2)}+\text{.....100 term}$$

$$=\tan^{-1}\left[\frac{(x+1)-x}{1+(x+1)\cdot x}\right]+\tan^{-1}\left[\frac{(x+2)-(x+1)}{1+(x+2)(x+1)}\right]+\tan^{-1}\left[\frac{(x+3)-(x+2)}{1+(x+3)(x+2)}\right]\ldots\ldots+\tan^{-1}\left[\frac{(x+100)-(x+99)}{1+(x+99)(x+100)}\right]$$

$= [\tan^{-1}(x+1)-\tan^{-1}x]+[\tan^{-1}(x+2)-\tan^{-1}(x+1)]+[\tan^{-1}(x+3)-\tan^{-1}(x+2)]+\ldots\ldots\ldots+\tan^{-1}(x+100)-\tan^{-1}(x+99)$

$=\tan^{-1}(x+100)-\tan^{-1}x$

$$=\tan^{-1}\left(\frac{100}{1+x^2+100x}\right)$$

88.

Equation of AA' is

$y+5=1(x-\alpha)$

$\Rightarrow\ y=x-\alpha-5$

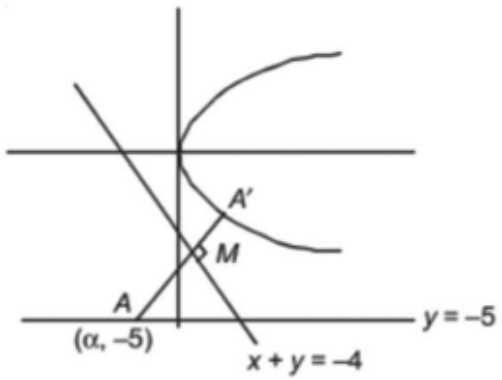
$y-x=-\alpha-5$ ... (i)

Also,

$x+y=-4$

$$y=\frac{-\alpha-9}{2},\qquad 2x=\alpha+1$$

$$\Rightarrow\ x=\frac{\alpha+1}{2}$$



$$M\equiv\left(\frac{\alpha+1}{2},\frac{-\alpha-9}{2}\right)$$

$A' = (1, -\alpha-4)$

A' lies on parabola

Now,

$\alpha^2+16+8\alpha=4$

$\Rightarrow\ \alpha^2+8\alpha+12=0$

$\Rightarrow\ \alpha=-2,-6$

AB = distance = 4 units

89. $\int \frac{5x^8+7^6}{x^{14}\left(\frac{1}{x^5}+\frac{1}{x^7}+2\right)}dx \Rightarrow \int \frac{5x^{-6}+7x^{-8}}{\left(2+\frac{1}{x^5}+\frac{1}{x^7}\right)^2}dx$

Put $2+\frac{1}{x^5}+\frac{1}{x^7}=t$

$\left(-5x^{-6}-7x^{-8}\right)dx=dt$

$\Rightarrow \int \frac{-dt}{t^2}=\frac{1}{t}+c$

$f(x)=\frac{x^7}{2x^7+x^2+1}+c$

$f(0)=0 \Rightarrow C=0 \Rightarrow f(1)=\frac{1}{4} \Rightarrow k=4$

90. $6\left(x^2-3ax+2a^2\right)$

$= 6(x-a)(x-2a);\ \ a>0$

x = a is point of maxima

x = 2a is point of minima

$\therefore a^2=2a$

$\Rightarrow a=0$ or $a=2$

But $a>0 \Rightarrow a=2$