Chapter 4

ILD and Rolling Loads

CHAPTER HIGHLIGHTS

- *Introduction*
- *Influence line*
- *ILD for simply supported beams*
- *Simply supported beams with overhang*
- *Maximum shear at a given section*
- *Maximum bending moment at a given section*
- *Maximum bending moment under a chosen wheel load*
- *Absolute maximum bending moment*
- *Muller–Breslau principle*
- *Influence line diagram for bridge truss members*

Introduction

In this chapter, the structures subjected to live loads or moving loads or rolling loads are discussed. The common types of rolling loads are the axle loads of moving trucks or vehicles, wheel loads of a railway train or wheel loads of a gantry assembly on a gantry girder etc. The variation of the shear force and bending moment due to these moving loads are best described by using the influence line. Therefore influence lines have important application for the design of structures that resist large live loads. The determination of the absolute maximum shear and moment in a member is also discussed.

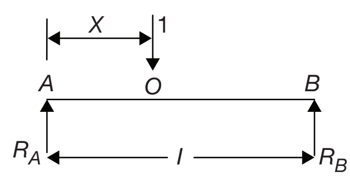
Influence Line

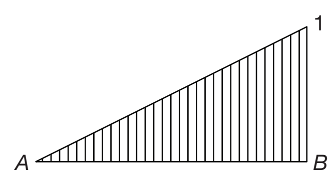
An Influence line represents the variation of either the reaction, shear, moment or deflection at a specific point in a member as a moving load moves from one end to another of member.

ILD for Simply Supported Beams

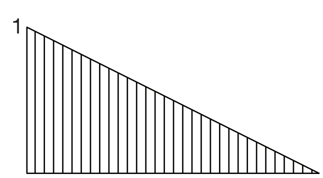
ILD for Reactions at the Supports

Let a unit load move from left end '*A*' to the right end '*B*' of the beam.



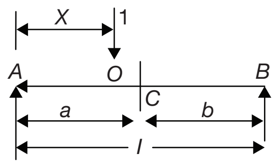


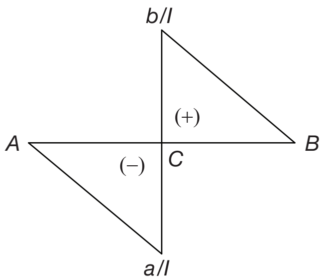
ILD for Reaction R_B



ILD for Reaction R_A

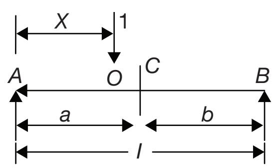
ILD for Shear Force at a Given Section

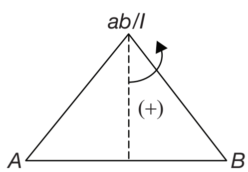




ILD for SF at '*C*'

ILD for Bending Moment at a Given Section

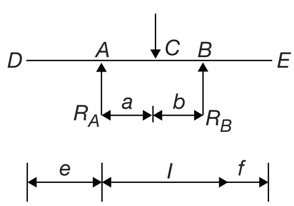


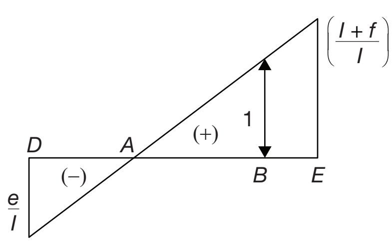


ILD for BM at *C*

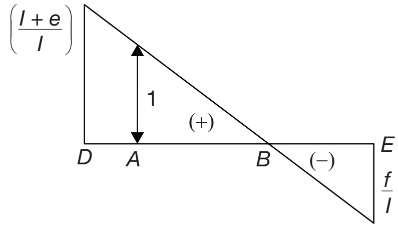
Simply Supported Beams with Overhang

A simply supported beam with overhangs on both left and right end of support is shown in the following figures:

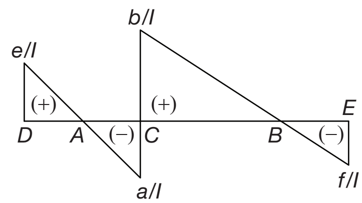




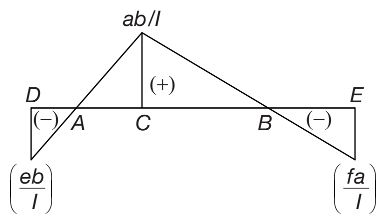
ILD for R_B



ILD for R_A



ILD for SF at '*C*'



ILD for BM at *C*

> **NOTE**
> The ILD for overhang beam is similar to that of a simply supported beam between supports and just extend the influence line to the side where overhang is present.

Maximum Shear at a Given Section

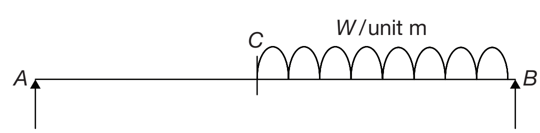
Wheel Loads

1. For a single concentrated load, maximum shear at a section occurs when the load is placed at the section itself.
2. For a series of wheel loads, maximum shear at a section occurs if it satisfies the following condition. That is,

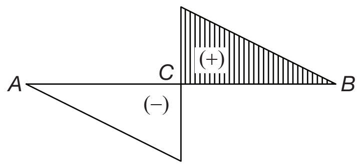
$$\text{If } \frac{\text{The load rolled past the section}}{\text{Succeeding wheel space}} < \frac{\text{Sum of all the loads}}{\text{Span}}$$

Uniformly Distributed Load

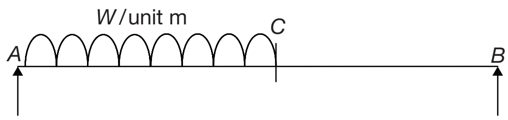
Maximum positive shear at *C*: For this condition, the tail of the UDL should be at the section itself.



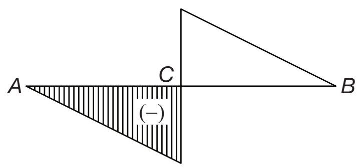
Load position for max +ve SF at *C*:



Maximum negative shear at *C*: For this condition the head of the load moving from the end *A* must arrive at *C* as shown below:



Load position for max –ve SF at *C*:

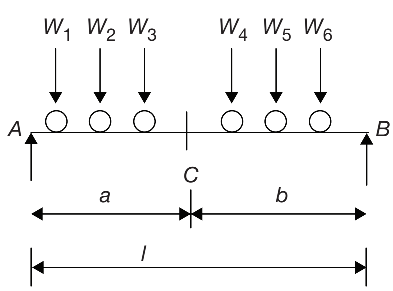


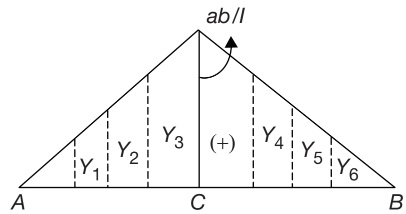
Maximum Bending Moment at a Given Section

Wheel Loads

1. For a single concentrated load, maximum bending moment at a section occurs when the load is at the section itself.
2. For two concentrated loads, maximum bending moment at any section may be the maximum of the following two cases.
   (a) When the leading is at the section
   (b) When the trailing load is at the section.
3. For several loads, the maximum bending moment at a given section of a beam occurs when the load system is so placed that the average load on the left segment is equal to the average load on the right segment.

For example: A simply supported beam subjected to a series of loads W_1, W_2, …, W_6 on the span. The maximum bending moment a section '*C*' occurs only when the loads are placed on span such that average load to LHS of section is equal to average loads to the RHS of section.





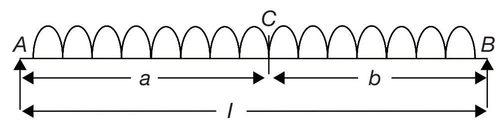
The following table form is used in order to easily find the average loads.

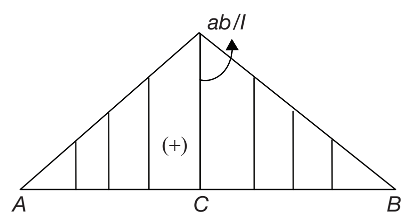
Load crossing the section *C*	Average load on *AC*	Average load on *CB*	Remarks

Uniformly Distributed Load

Two Cases

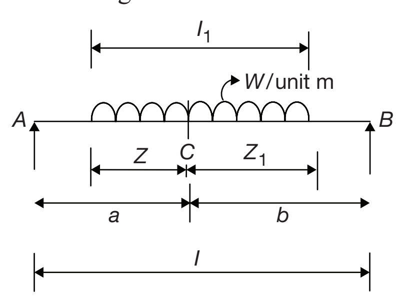
Case 1: When the length of the load is greater than the span. Since length of UDL is greater than the length of span, to get maximum bending moment, UDL should be entirely loaded on span as shown below.

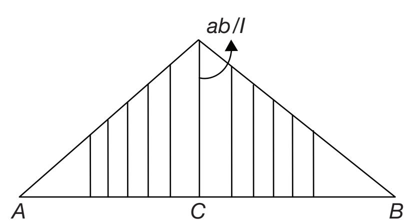




Maximum *BM* at *C* = Intensity of UDL × Area of ILD covered

Case 2: When the length of the load is shorter than span





To get maximum bending moment, UDL should be placed from section in such a way that section '*C*' divides the UDL in same ratio as it divides the span. 'C' divides the span in the ratio $a : b$.

UDL should also be divided by the section in the same ratio as of span

$$z : z_1 = a : b$$

Load on LHS of section, i.e.,

$$AC = z = \left[\frac{a}{a+b}\right] \times l_1$$

Load on RHS of section, i.e.,

$$CB = z_1 = \left[\frac{b}{a+b}\right] \times l_1$$

Maximum Bending Moment under a Chosen Wheel Load

The bending moment under a chosen load of a wheel load system will be a maximum, when the load system is so placed on the girder that the chosen load and the resultant of all the wheel loads are equidistant from the middle point of girder.

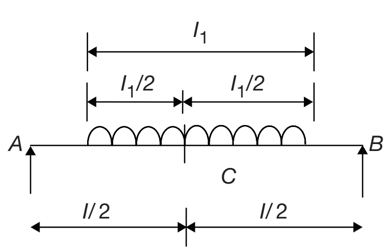
Absolute Maximum Bending Moment

Wheel Loads

1. For a single concentrated load, absolute maximum bending moment occurs when the load is at centre of the span.
2. In case of series of wheel loads, the wheel load should be selected by inspection such that Bending moment at mid span may be maximum and now the load system is so placed on the span that the resultant of all the loads and chosen wheel loads should be equidistant from mid-span.

Uniformly Distributed Load

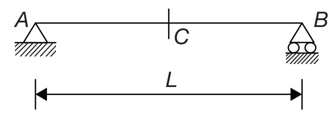
1. The absolute maximum bending moment occurs at mid-span when the length of the uniformly distributed load is greater than span and when the whole span is loaded.
2. When the length of the uniformly distributed load is shorter than the span, the absolute maximum bending moment occurs at the centre of span when the loading is symmetrically placed on span.



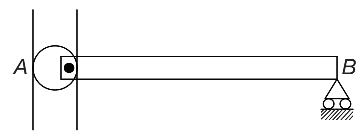
Muller–Breslau Principle

- Muller–Breslau principle states that the influence line for a function (reaction, shear, or moment) is to the same scale as the deflected shape of the beam when the beam is acted upon by the function.
- It is the quick method for establishing the shape of the influence line.
- Applicable for both determinate and indeterminate structures.
- The shape of influence line is linear for determinate structures and non-linear in case of indeterminate structures.
- The principle of virtual work can be used for establishing the proof of Muller–Breslau principle.
- Muller–Breslau principle can be explained with the help of an example as below:

Consider a simply supported beam of span 'L' with hinge at A and roller at B.

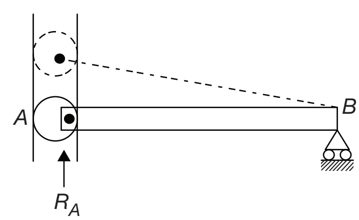


Influence line for R_A: First remove the reaction at 'A' so that the beam can deflect easily when the function (reaction) is applied.

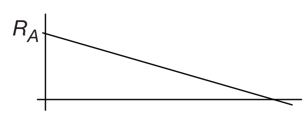


In the above figure, the hinge support at A is replaced a roller guide which can resist horizontal force but not vertical force.

The deflected shape obtained by applying the reaction at A gives directly the influence line for reaction at A.



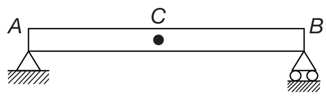
Deflected shape

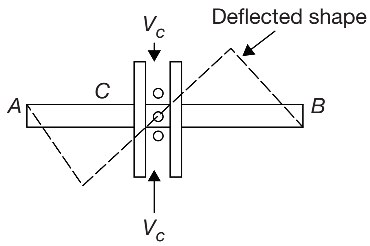


Influence line for R_A

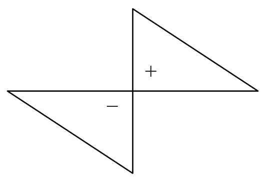
Influence line for shear at C: Follow the same producer as above i.e., at 'C', a roller guide is to be placed which can resist moment and axial force but not shear.

Now apply a shear force at C. The deflected shape gives directly the influence line diagram for shear at C.





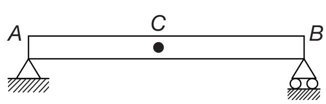
Deflected shape

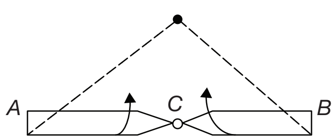


Influence line for V_c

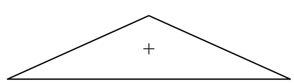
Influence line for BM at *C*: A hinge is to be placed at a section where influence line for BM is required. This can resist shear and axial force but not moment.

Then apply a positive moment at section and the deflected shape gives directly the influence line for bending moment at that section.





Deflected shape

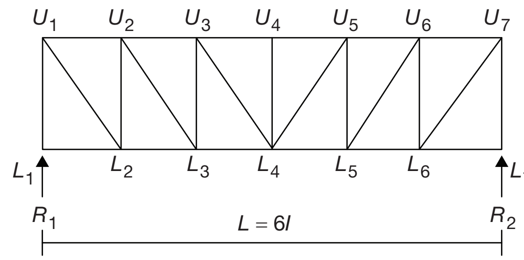


Influence line for M_c

Influence Line Diagram for Bridge Truss Members

- Trussed bridges are classified into deck and through type bridges.
- If the load is received at the top chord joints, the truss is called a deck type truss and if the load is received at the bottom chord joints, the truss is called a through type truss.

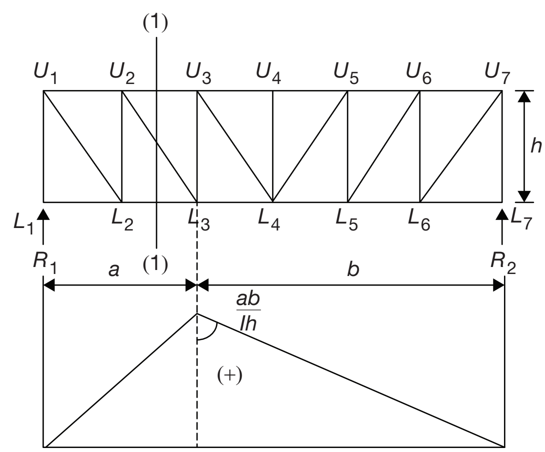
Pratt truss: Considering the case of through type bridges, i.e., the unit load travels from one end to other through bottom chord.



The Pratt truss

U_1U_2, U_2U_3, U_3U_4, U_4U_5, U_5U_6, U_6U_7—Top chord members
L_1L_2, L_2L_3, L_3L_4, L_4L_5, L_5L_6, L_6L_7—Bottom chord members
U_1L_1, U_2L_2, U_3L_3, U_4L_4, U_5L_5, U_6L_6 U_7L_7—Vertical members
U_1L_2, U_2L_3, U_3L_4, U_5L_4, U_6L_5, U_7L_6—Diagonal members

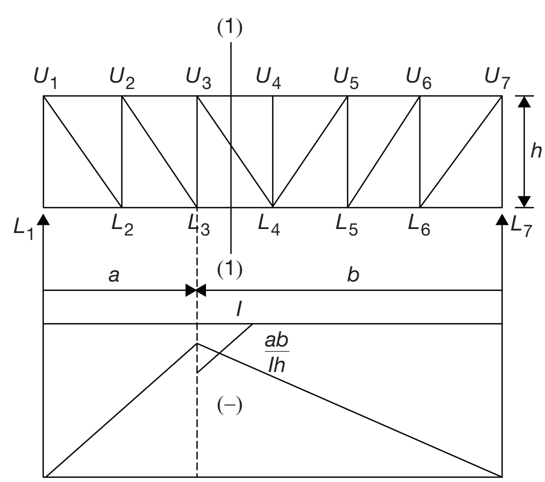
ILD for the top chord members: Consider a section (1)-(1) passing through top chord member U_2U_3 as shown below.



ILD for U_2U_3

ILD for bottom chord member: Top chord members will be under compression (+) while bottom chord members will be under tension (–).

The ILD for bottom chord member L_3L_4 is shown below.



ILD for L_3L_4

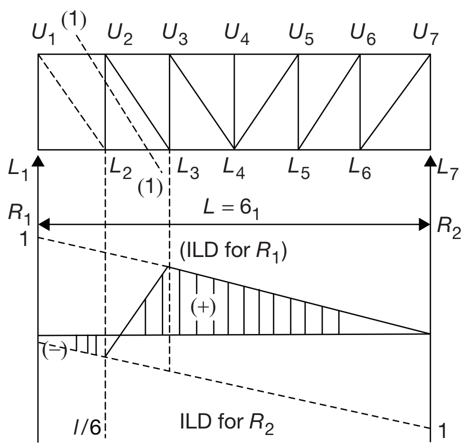
Procedure:

1. The member under consideration (L_3L_4) is cut by a section (1)–(1) as shown in the figure.
2. Take the meeting of the other members as a moment centre, i.e. at a joint U_3 in this case.
3. Draw a triangle with maximum ordinate as $\frac{ab}{lh}$ at moment centre U_3.
4. These members will be in tension.

NOTE

The ILD for top chord members may also be drawn in similar lines as that of bottom chord members.

ILD for vertical members: The ILD for the vertical member U_2L_2 is shown below.



ILD for U_2U_2

Procedure:

1. Cut the vertical member U_2L_2 under consideration as shown above.
2. Consider the RHS of cut part when the unit load to left of L_2.

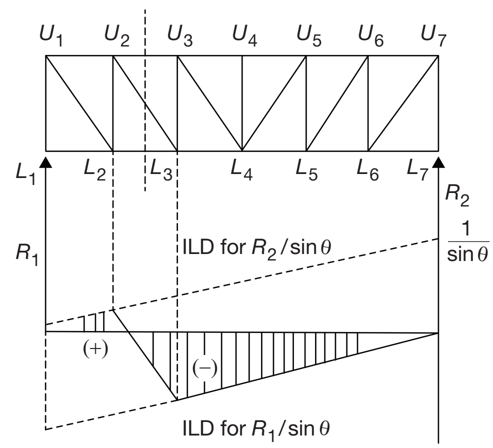
   From equilibrium; $\Sigma Fy = 0$

   $P_{U_2L_2} = R_2$ (tensile)
3. The ILD for R_2 is drawn below the reference line.
4. When the unit load is on the right side of L_3. Consider the LHS of cut part.

   From equilibrium; $\Sigma Fy = 0$

   $P_{U_2L_2} = R_1$ (Compressive)
5. The ILD for reaction R_1 is drawn above the reference line.
6. As the unit load moves from L_2 to L_3 force in U_2L_2 will change from tension to compression.
7. The hatched part shown in the figure is ILD for the vertical member U_2L_2.

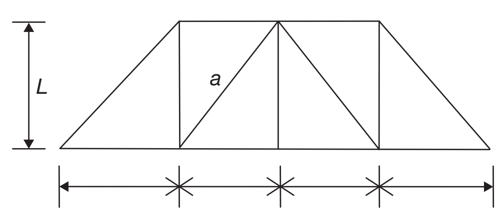
ILD for diagonal member:



Rolling loads and influence lines

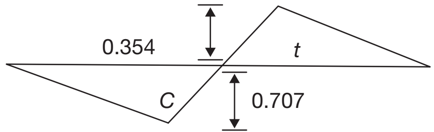
EXERCISES

1. For which one of the following cases is the Muller–Breslau principle applicable to get influence line?
   (A) Reaction at the ends of a simple beam.
   (B) Bending moment at a section.
   (C) Shear force at a section.
   (D) Forces and moments at any section.
2. The influence line diagram for the force in member '*a*' of the truss shown in the figure is given by

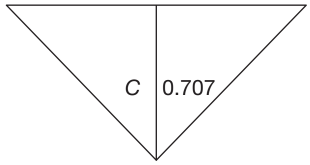


4 spans of 'L' each

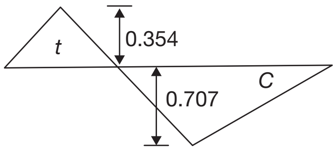
(A)



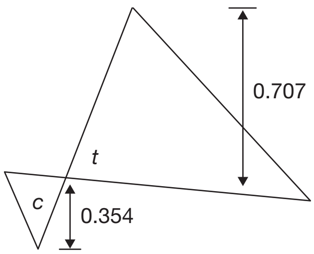
(B)



(C)



(D)



3. Identify the FALSE statement from the following, pertaining to the methods of structural analysis.

(A) Influence lines for stress resultants in beams can be drawn using Muller–Breslau's principle.

(B) The moment distribution method is a force method of analysis, not a displacement method.

(C) The principle of virtual displacements can be used to establish a condition of equilibrium.

(D) The substitute frame method is not applicable to frames subjects to significant side sway.

4. Muller–Breslau principle in structural analysis is used

(A) drawing influence line diagram for any force function.

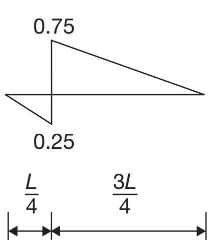
(B) writing virtual work equation.

(C) superposition of load effects.

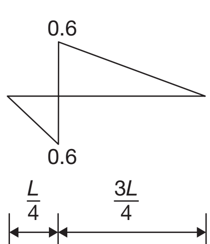
(D) None of these

5. In a beam of length L, four possible influence line diagrams for shear force at a section located at a distance of $\frac{L}{4}$ from the left end support (marked as P, Q, R and S) are shown in the following figures. The correct influence line diagram is

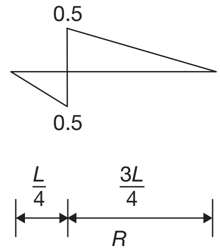
(A)



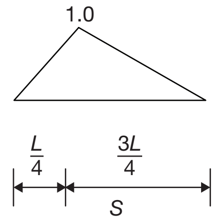
(B)



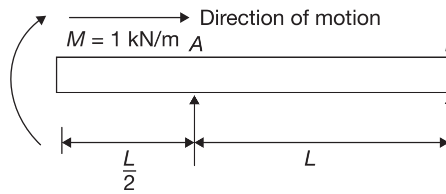
(C)



(D)

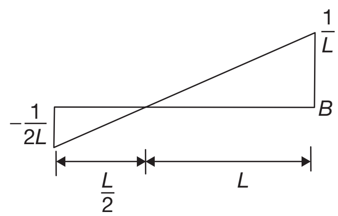


6. A simply supported beam with an overhang is traversed by a unit concentrated moment from the left to the right as shown below:

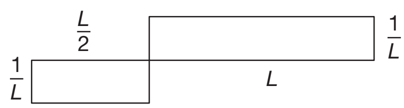


The influence line for reaction at B is given by

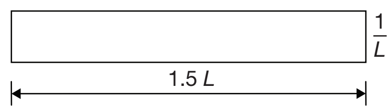
(A)



(B)

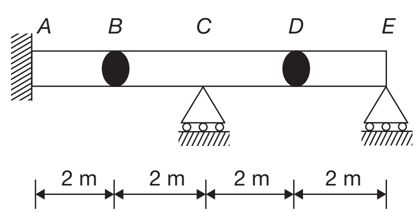


(C)



(D) zero every where.

7. Identify, from the following, the correct value of the bending moment M_A (in kN/m units) at the fixed end A in the statically determinate beam shown in the figure (with internal hinges at B and D), when a uniformly distributed load of 10 kN/m is placed on the spans. (**Hint:** Sketching the influence line for M_A or applying the principle of virtual displacements makes the solution easy).



(A) −80
(B) −40
(C) 0
(D) +40

Direction for questions 8 to 10:

A beam *PQRS* is 18 m long and is simply supported at points *Q* and *R* 10 m apart overhangs *PQ* and *RS* are 3 m and 5 m respectively. A train of two point loads of 150 kN and 100 kN, 5 m apart, crosses this beam from left to right with 100 kN load leading.

8. The maximum sagging moment under the 150 kN anywhere is
(A) 500 kN-m (B) 450 kN-m
(C) 400 kN-m (D) 375 kN-m

9. During the passage of the loads, the maximum and the minimum reactions at supports '*R*' in kN, are respectively
(A) 300 and –30 (B) 300 and –25
(C) 225 and –30 (D) 225 and –25

10. The maximum hogging in the beam anywhere is
(A) 300 kN-m (B) 450 kN-m
(C) 500 kN-m (D) 750 kN-m

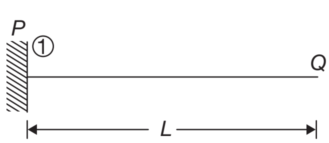
11. Influence line for redundant structures can be obtained by
(A) Castigliano's theorem.
(B) Muller–Breslou principle.
(C) Unit load theorem.
(D) Maxwell–Betti reciprocal theorem.

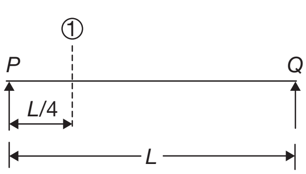
12. **Assertion (A):** Whether it is maximum BM at a section or absolute maximum BM, the moving UDL should cover the entire span of a simple beam if span is less than load length.

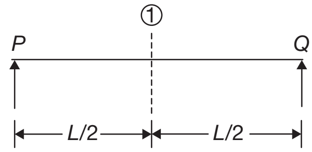
Reason (R): Whether it is maximum BM at a section or absolute maximum BM, the moving UDL should be divided by the section in the same ratio in which the section divides the span, if the span is greater than load length.
(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is not a correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

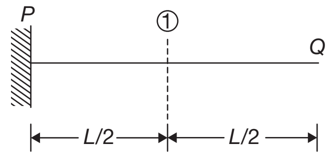
13. Match List I (Beam) with List II (Influence line for BM) and select the correct answer using the given codes:

List I

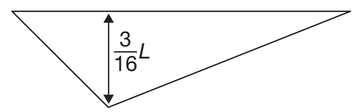
a.


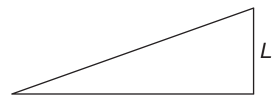
b.


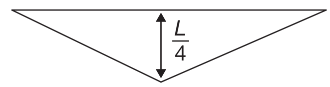
c.


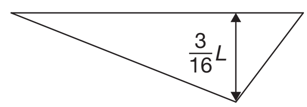
d.


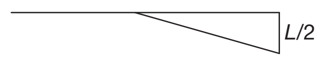
List II

1.


2.


3.


4.


5.


Codes:

	a	b	c	d
(A)	2	1	3	5
(B)	3	1	4	5
(C)	2	5	3	4
(D)	1	3	5	4

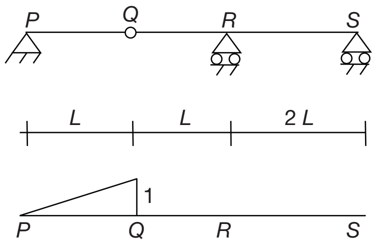
14. Which one of the following statements is correct? The influence line diagram (ILD) for bending moment at a section in a cantilever is a triangle extending between the section and the
(A) fixed end with maximum ordinate under the section.
(B) fixed end with maximum ordinate under the fixed end.
(C) unsupported end with maximum ordinate at the section.
(D) unsupported end with maximum ordinate at the unsupported end.

15. What is the variation of influence line for stress function in a statically determinate structure?
(A) Parabolic
(B) Bilinear
(C) Linear
(D) Uniformly rectangular

16. What is/are the use(s) of influence lines?
(A) To study the effect of moving loads on the structure.
(B) To calculate the value of stress function with the critical load condition.

(C) To find the position of live load for a maximum value of particular stress function.
(D) Towards all the above purposes.

17. Consider the beam *PQRS* and the influence line as shown in the figure. The influence line pertains to

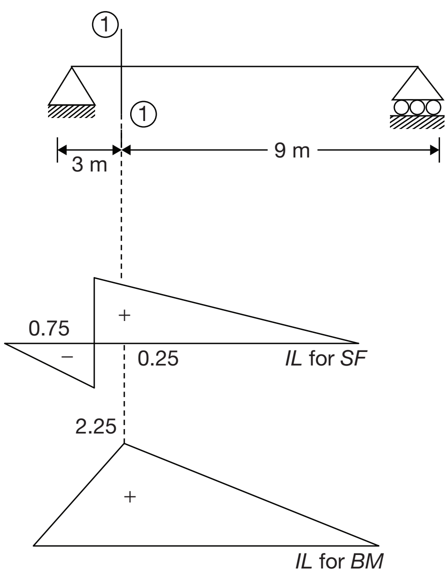


(A) reaction of P, R_P.
(B) shear force Q, V_Q.
(C) shear force on left of Q, V_Q^-.
(D) shear force on right of Q, V_Q^+.

18. The ordinate of the influence line diagram for bending moment have dimension of ______.
(A) length (B) force
(C) length/force (D) None of these

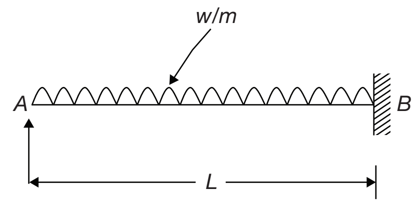
19. Muller–Breslau principle used for ______.
(A) to super impose the load effects.
(B) to draw influence line diagrams for any force function.
(C) to write virtual work equation.
(D) All of these

20. The given figure shows a beam with its influence line for shear force and bending moment at section '1'.



The value of shear force and bending movement at section '1' due to concentrated road of 30 kN placed at mid-span will be ______.
(A) 30 kN and 30 kN-m
(B) 15 kN and 30 kN-m
(C) 15 kN and 45 kN-m
(D) 30 kN and 15 kN-m

21. For a given beam, match the following.

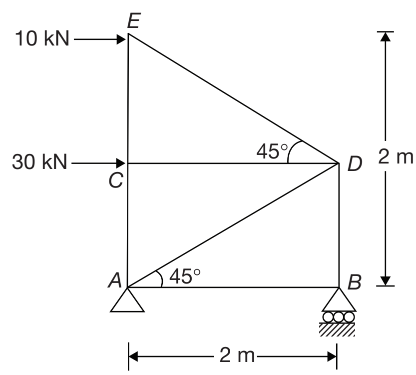


	List I		List II
a.	Moment at *B*	**1.**	$\frac{3}{8}wL$
b.	Slope at *A*	**2.**	$\frac{4EI}{L}$
c.	Reaction of *A*	**3.**	1.5 *M*, where $M = \frac{wl^2}{12}$
d.	Stiffness of *AB*	**4.**	$\frac{ML}{4EI}$, where $M = \frac{wl^2}{12}$

Codes:

	a	b	c	d
(A)	3	4	1	2
(B)	3	2	4	1
(C)	2	1	3	4
(D)	1	2	4	3

22. A pin jointed truss is loaded as shown in the figure. Match List I (member) with List II (Force induced).



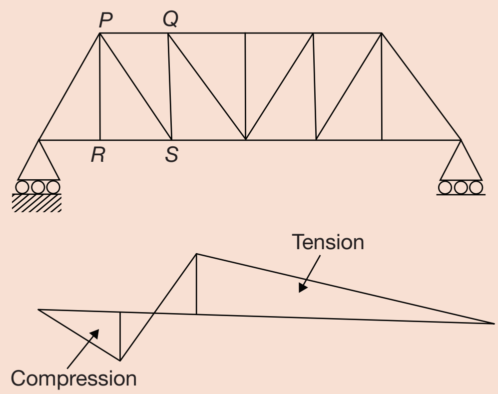
	List I		List II
a.	Member *AB*	**1.**	30 kN
b.	Member *AC*	**2.**	50 kN
c.	Member *BD*	**3.**	Zero
d.	Member *CD*	**4.**	10 kN

Codes:

	a	b	c	d
(A)	3	2	1	4
(B)	4	3	2	1
(C)	3	4	2	1
(D)	4	3	1	2

Previous Years' Questions

1. The influence line diagram (ILD) shown, is for the member **[GATE, 2007]**



(A) *PS* (B) *RS* (C) *PQ* (D) *QS*

Answer Keys

Exercises

1. D **2.** C **3.** B **4.** A **5.** A **6.** C **7.** C **8.** C **9.** A **10.** D
11. B **12.** B **13.** A **14.** D **15.** C **16.** D **17.** B **18.** A **19.** B **20.** C
21. A **22.** C

Previous Years' Questions

1. A