9

Quadrilateral

9.01 Introduction

In chapters 5 and 6 you have studied, about many properties of triangles. You know that a triangle is formed by joining three non-collinear points.

Now, we mark the groups of 4-4 points on paper and join them one by one in some order and see that how many possible figures can be found?

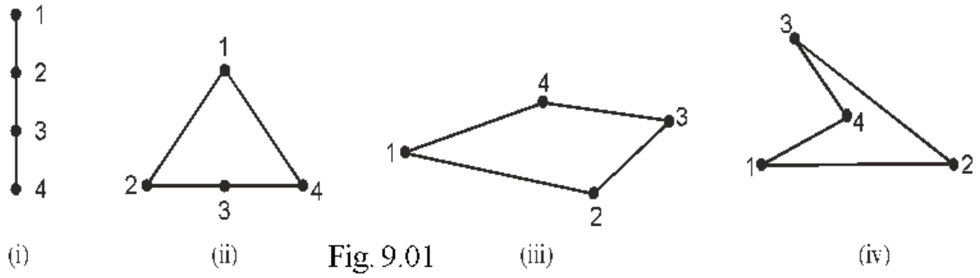


Fig. 9.01

Possible figures like 9.01 (i), (ii), (iii) and (iv) can be formed. In this chapter, we will study figures like 9.01 (iii), which we call quadrilateral.

9.02 Quadrilateral

A figure enclosed by four line segments is called a quadrilateral. A quadrilateral has four sides, four angles and four vertices. Like in Fig. 9.02, $PQRS$ is a quadrilateral where PQ, QR, RS and SP are four sides, P, Q, R and S are four vertices and $\angle P$, $\angle Q$, $\angle R$ and $\angle S$ are four angles.

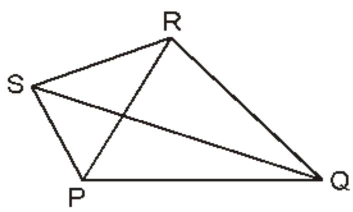


Fig. 9.02

Opposite Sides and Opposite Angles : In Fig. 9.02 RS is the opposite side of PQ and QR is the opposite side of PS. $\angle P$ is opposite of $\angle R$ and $\angle Q$

is opposite of $\angle$ S.

In Fig. 9. 02, pair of **adjacent sides** are PQ, QR and PS, SR. Similarly, SP, PQ and SR, RQ are also pair of adjacent sides.

Diagonal : Line joining opposite vertices is called diagonal. In Fig 9.02, PR and QS are the diagonals of quadrilateral $PQRS$.

9.03 Sum of the Angles of Quadrilateral

Sum of all four angles of a quadrilateral is 4 right angles (360°). We have learnt this property of quadrilateral in chapter 5 through Corollary 4.

9.04 Types of Quadrilateral

- **Kite** : In Fig 9.03 $WXYZ$ is a quadrilateral whose two pairs of adjecent sides *i. e.*, WX, XY and WZ, YZ are equal. It is called kite. Such quadrilateral whose two pair of adjacent sides are equal, is called kite.

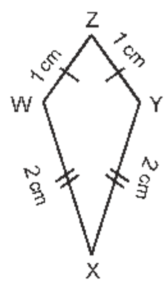


Fig. 9.03
Kite

- **Trapezium** : In Fig 9.04, $ABCD$ is a quadrilateral whose one pair of opposite sides AB and DC are parallel. This quadrilateral is known as trapezium.

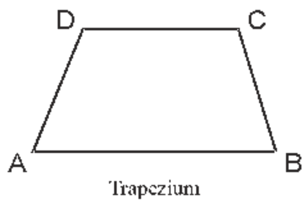


Fig. 9.04

- **Parallelogram :** In Fig. 9.05, $PQRS$ is a parallelogram whose two pair of opposite sides PQ, RS and PS, QR are parallel

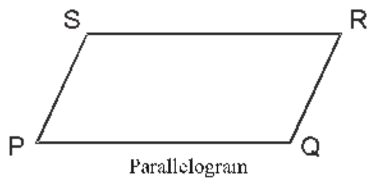


Fig. 9.05

A parallelogram is a trapezium but a trapezium is not a parallelogram.

- **Rectangle :** In Fig 9.06, $EFGH$ is a special parallelogram called rectangle whose each angle is 90°.

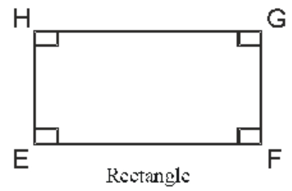


Fig. 9.06

A rectangle is a parallelogram but a parallelogram is not necessarily a rectangle.

A rectangle is a trapezium but a trapezium is not rectangle.

- **Rhombus :** In fig. 9.07, *TUVW* is a special parallelogram called rhombus, whose each side is equal in measure.

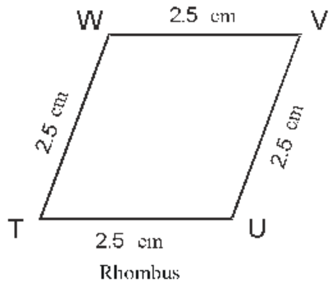


Fig. 9.07

Such parallelogram whose each side is equal, is known as rhombus.

A rhombus is a parallelogram but a parallelogram is not necessarily a rhombus.

A rhombus is a parallelogram, but a parallelogram is not a rhombus.

- **Square** In Fig, 9.08, *KLMN* is a special rectangle called square whose all sides are equal or a special parallelogram whose each side is equal and each angle is 90°.

A square is a trapezium but a trapezium is not a square. A square is a parallelogram but a parallelogram is not necessarily a square. A square is a rectangle but a rectangle is not necessarily a square. A square is a rhombus but a rhombus is not necessarily a square.

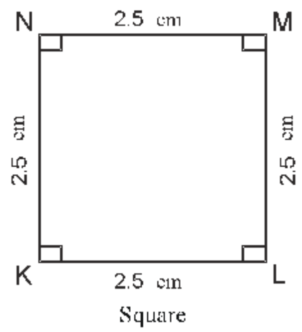


Fig. 9.08

9.05 Properties of Parallelogram

Theorem 9.1. ***The diagonal of a parallelogram divides it into two congruent triangles.***

Given : $ABCD$ is a parallelogram and BD its diagonal.

To Prove : $\Delta ABD \cong \Delta BCD$

Proof: In Fig. 9.09, $ABCD$ *is* a parallelogram.

$AB \parallel$ CD and BD is a transversal

$\therefore$ $\angle ABD = \angle CDB$ (Alternate interior angles) ...(i)

$AD \parallel BC$ and BD is a transversal

$\therefore$ $\angle ADB = \angle CBD$ (Alternate interior angles) ...(ii)

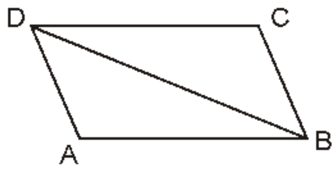


Fig. 9.09

Now, in ΔABD and ΔCDB

$\angle ABD - \angle CDB$ [From equ. (i)]

$BD = BD$ [Common]

$\angle ADB - \angle CBD$ [From equ (ii)]

$\therefore$ $\Delta ABD \cong \Delta CDB$ (By ASA congrunce rule)

Hence proved

Theorem 9.2. ***Opposite sides of a parallelogram are equal.***

Given : In Fig. 9.09, $ABCD$ is a parallelogram.

To Prove : $AB - CD$ and $AD - BC$

Construction : Draw a diagonal BD.

Proof : From theorem 9.1, $\Delta ABD \cong \Delta$CDB

Since, corresponding parts of a congruent triangle are equal.

$AB - CD$ and $AD - BC$ **Hence Proved**

Theorem 9.3. (Converse of Theorem 9.2)

If each pair of opposite sides of a quadrilateral be equal, then it is a parallelogram.

Given : $ABCD$ is a quadrilateral whose opposite sides $AB - CD$ and $BC - AD$.

To prove : $ABCD$ is a parallelogram.

Construction : Join A to C.

Proof: In ΔABC and ΔCDA,

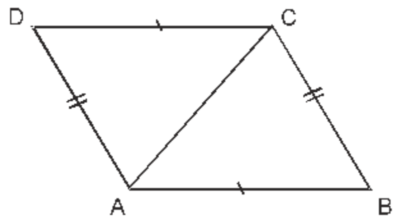


Fig. 9.10

$AB - CD$ (Given)

$BC - AD$ (Given)

$AC = AC$ (Common)

$\therefore$ $\Delta ABC \cong \Delta CDA$ [By SSS congruence rule]

or $\angle CAB = \angle ACD$ (CPCT)

Transversal line AC intersects two lines AB and CD such that alternate interior angles $\angle CAB$ and $\angle ACD$ are equal

$\therefore$ $AB \parallel DC$ ...(i)

and $\angle ACB - \angle CAD$ (CPCT)

Transversal line AC intersects two lines BC and AD such that interior angles $\angle ACB$ and $\angle CAD$ are equal.

$\therefore$ $BC \parallel AD$ ...(ii)

Thus, from (i) and (ii), quadrilateral $ABCD$ is a parallelogram. **Hence Proved**

Theorem 9.4. ***Opposite angles of a parallelogram are equal.***

Given : ABCD is a parallelogram.

To Prove : $\angle A = \angle C$ and $\angle B = \angle D$

Proof: $ABCD$ is a parallelogram.

$AB \parallel DC$ and $AD \parallel CB$

Since, sum of the interior angles on the same side of a transversal is 180°.

$\therefore$ $\angle A + \angle D = 180°$ ...(1)

and $\angle C + \angle D = 180^0$ ...(2)

From (1) and (2)

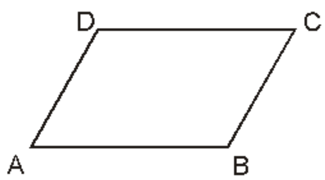


Fig. 9.11

$$\angle A + \angle D = \angle C + \angle D$$

$\therefore$ $\angle A = \angle C$

Similarly, we can prove $\angle B = \angle D$ **Hence Proved.**

Now, converse of this theorem is also true? Let us prove it.

Theorem 9.5. ***If opposite angles of a quadrilateral are equal, then it is a parallelogram.***

Given : A quadrilateral ABCD in which

$$\angle A - \angle C \quad \text{and} \quad \angle B = \angle D$$

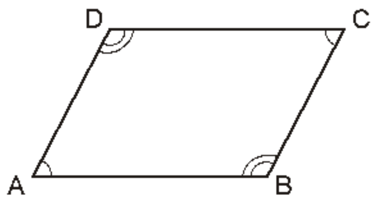


Fig. 9.12

To Prove : ABCD is a parallelogram.

Proof : In a quadrilateral $ABCD$

$$\angle A - \angle C \quad ...(i)$$

and $\angle B = \angle D$ ...(ii)

on adding (1) and (2), we get

$$\angle A + \angle B = \angle C + \angle D \quad ...(1)$$

But $\angle A + \angle B + \angle C + \angle D - 360^\circ$ ...(2)

From (1) and (2), we get

$$\angle A + \angle B - \angle C \quad \angle D - 180^\circ$$

$$\angle A + \angle B - 180^\circ$$

A transversal line AB intersects two lines AD and BC such that sum of consecutive interior angles is 180°

$\therefore$ $AD \mid BC$

$\angle C + \angle D - 180^\circ \Rightarrow \angle A + \angle D = 180^\circ \quad [\because \angle C = \angle A]$

A transversal line AD intersects two lines AB and CD such that sum of consecutive interior angles is 180°.

$$AB \parallel DC \quad ...(6)$$

From (5) and (6), we get

Thus, ABCD is a parallelogram. **Hence Proved**

Theorem 9.6. ***Diagonals of a parallelogram bisect each other.***

Given : A parallelogram $ABCD$ whose diagonals AC and BD intersect each other at O.

To Prove : $OA - OC$ and $OB = OD$

Proof : $\because$ $ABCD is$ a parallelogram.

$\therefore$ $AD \mid BC$ and tansversal BD intersects them.

$\angle ADB - \angle DBC$ (Alternate interior angles)

$\Rightarrow$ $\angle ADO = \angle OBC$ ...(1)

Again $AD \mid BC$ and transversal AC intersects them

$\angle DAC = \angle BCA$ (Alternate interior angle)

$\Rightarrow$ $\angle DAO = \angle BCO$ ...(2)

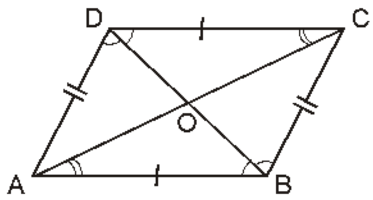


Fig. 9.13

In ΔAOD and ΔCOB

$\angle ADO - \angle CBO$ [From equ (1)]

$AD - BC$ [Opposite side of a parallelogram]

$\angle ADO = \angle BCO$ [From equ (2)]

$\Delta AOD \cong \Delta COB$ [By ASA congruency rule]

Since, corresponding parts of congruent triangles are equal,

so, $OD - OB$

and $OA = OC$ **Hence Proved**

Theorem 9.7. (Converse of Theorem 9.6)

If diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Given : A quadrilateral $ABCD$ whose diagonals AC and BD bisect each other at O, *i.e.*, $OA - OC$ and $OB - OD$

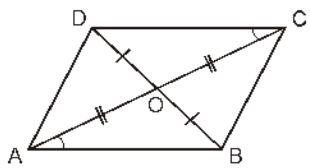


Fig. 9.14

To Prove : $ABCD$ is a parallelogram.

Proof : In ΔAOB and ΔCOD,

$OA - OC$ (Given)

$\angle AOB - \angle COD$ (Vertically opposite angles)

and $OB = OD$ (Given)

$\Delta AOB \cong \Delta COD$ (By SAS congruency rule)

Since, corresponding parts of congruent triangles are equal so

$\angle OAB = \angle OCD \Rightarrow \angle CAB - \angle ACD$

Transversal AC intersects two lines AB and DC such that alternate interior angles CAB and ACD are equal.

$$AB \parallel CD$$

Similarly, we can prove $\quad AD \parallel BC$

$ABCD$ is a parallelogram. **Hence Proved**

Theorem 9.8. ***A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.***

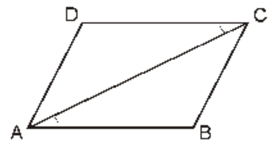


Fig. 9.15

Given : A quadrilateral $ABCD$, in which $AB \parallel DC$ and $AB = DC$

To Prove : $ABCD$ is a parallelogram.

Construction : Join A to C.

Proove : $AB \parallel DC$ and AC is a transversal

$\angle BAC = \angle DCA$ (Alternate interior angles) ...(1)

Now, in ΔABC and ΔCDA

$AB = DC$ (Given)

$\angle BAC = \angle DCA$ [From (1)]

$AC = AC$ (Common)

$\Delta ABC \cong \Delta CDA$ (By SAS congruency)

Since, corresponding parts of congruent triangles are equal,

so $\quad \angle ACB = \angle CAD$

Now, AD and BC both intersects by transversal AC such that alternate angles

$\angle ACB$ *and are equal* $\angle CAD$

$\therefore \quad AD \parallel BC$ ...(2)

and $\quad AB \parallel CD$ (Given)

Thus, $ABCD$ *is* a parallelogram. **Hence Proved**

Illustrative Examples

Example 1. **Two line segments AC and BD bisect each other at point P. Prove that $ABCD$ is a parallelogram.**

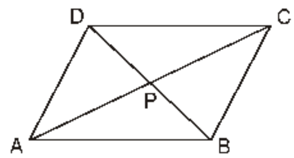


Fig. 9.16

Solution : **Given** AC and BD intersect each other at point P.

To Prove : $ABCD$ is a parallelogram.

Construction : Join AB, BC, CD and DA respectively.

Proof : In ΔAPB and ΔCPD

From figure $ABCD$ is a quadrilateral in which AC and BD are diagonals.

Since, $AP = PC$ and $BP = PD$ (Given)

So, AC and BD bisects each other.

$\therefore$ From theorem 9.7, $ABCD$ is a parallelogram. **Hence Proved**

Example 2. **In a parallelogram ABCD, it is not defined that bisectors of $\angle A$ and $\angle B$ intersect at point P. Prove that $\angle APB = 90°$.**

Solution : Given : In Fig. 9.17, bisectors of adjacent angles $\angle A$ and $\angle B$, intersect at P.

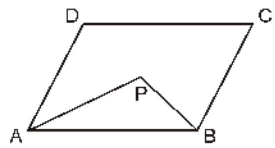


Fig. 9.17

To Prove : $\angle APB - 90°$

Proof : We know that the sum of the adjecent angles of a parallelogram is 180°

$\therefore$ $\angle A + \angle B - 180°$ ...(1)

$\angle PAB = \frac{1}{2}\angle A$ [AP is the bisector of $\angle A$]

and $\angle PBA = \frac{1}{2}\angle B$ [BP is the bisector of $\angle B$]

$\therefore$ $\angle PAB + \angle PBA = \frac{1}{2}(\angle A + \angle B)$ ...(2)

From (1) and (2), we have

$\angle PAB + \angle PBA = 90°$ ...(3)

In Δ PAB, sum of the angles of a triangle is 180°.

$\therefore$ $\angle PAB + \angle PBA + \angle APB = 180°$

From (3) and (4)

$\Rightarrow$ $\angle APB = 90°$ **Hence Proved**

Example 3. Two points *P* and *Q* are situated on diagonal *BD* of a parallelogram *ABCD* such that *DQ = BP*. Prove *APCQ* is a parallelogram.

Solution :

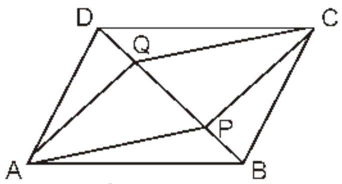


Fig. 9.18

Given : Two points P and Q are situated on diagonal BD of a parallelogram $ABCD$ such that $BP = QD$.

To Prove : $APCQ$ is a parallelogram.

Proof : In ΔAQD and Δ CPB.

$AD = BC$ [Opposite sides a of parallelogram]

$\angle ADQ = \angle CBP$ (Alternate interior angles)

$QD - BP$ (Given)

$\Delta AQD \cong \Delta CPB$ (By SAS congruency)

Since, corresponding parts of congruent triangles are equal

$AQ - CP$ ...(1)

Similarly, $\Delta CQD \cong \Delta APB$

and $CQ - AP$ ...(2)

From (1) and (2), $APCQ$ is a parallelogram. **Hence Proved**

Example 4. Diagonals of a quadrilateral $ABCD$ intersect each other at point O such that $OA : OC = 3 : 2$. Is $ABCD$ a parallelogram? Clearify with reason.

Solution : Given, $OA : OC = 3 : 2$

$\Rightarrow$ $$OA \neq OC$$

Thus, $ABCD$ is not a parallelogram because diagonals of a quadrilateral $ABCD$ do not bisect each other. **Hence Proved**

Example 5. The angle of a quadrilateral are in ratio 3:4:4:7. Find all the anlges of the quadrilateral.

Solution : Let the angles of a quadrilateral be $3x$, $4x$, $4x$ and $7x$.

Since, the sum of the angles of a quadrilateral is 360°,

$\therefore$ $$3x + 4x + 4x + 7x = 360^\circ$$

$\Rightarrow$ $$18x = 360^\circ,$$

$\Rightarrow$ $$x = 20^\circ$$

$\therefore$ Required angles are $3x = 3 \times 20^\circ = 60^\circ$

$4x = 4 \times 20^\circ = 80^\circ$

$4x = 4 \times 20^\circ = 80^\circ$

$7x = 7 \times 20^\circ = 140^\circ$

Thus, the angles are 60°, 80°, 80° and 140°.

Example 6. A diagonal of a parallelogram bisects its one angle. Prove that this diagonal will also bisect its apposite angle.

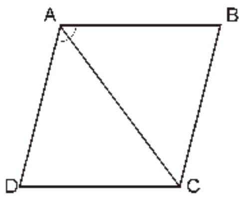


Fig. 9.19

Solution : Let us draw a Fig 9.19 as per given conditions. In which diagonal AC bisects $\angle BAD$ of a parallelogram $ABCD$.

Given: $\angle BAC = \angle DAC$

To prove : $\angle BCA = \angle DCA$

Proof : Given, $AB \parallel CD$ and AC is a transversal.

$\therefore$ $\angle BAC = \angle DCA$ (Alternate interior angles) ...(1)

Similarly, $\angle DAC = \angle BCA$ (Alternate interior angles, as $AD \parallel BC$)...(2)

But $\angle BCA = \angle DAC$ (Given) ...(3)

From (1), (2) and (3), we get

$$\angle BCA = \angle DCA$$ **Hence Proved**

Example 7. PQ and RS are two equal and parallel line segment. Point M which is not on side PQ or RS, is joined with Q and S respectively. A line parallel to QM and passing through P is drawn, and another line parallel to SM and passing through R is drawn. These lines meet at N. Prove tha line segments MN and PQ are equal and parallel to each other.

Solution : Draw a diagram according to the (Fig. 9.20) given conditions

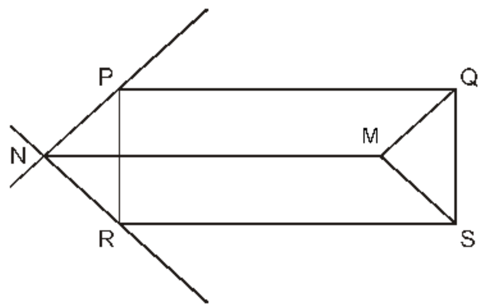


Fig. 9.20

Given that PQ $= RS$ and $PQ \,||\, RS$

$\therefore$ $PQSR$ is a parallelogram.

$\therefore$ $PR = QS$ and $PR \quad QS$ ...(1)

Now $PR \,|\, QS$

$\therefore$ $\angle RPQ + \angle PQS = 180^\circ$ (Interior angles of the same side of a transversal)

$\Rightarrow$ $\angle RPQ + \angle PQM + \angle MQS = 180^\circ$ ...(2)

Also $PN \,||\, QM$ (By construction)

$\therefore$ $\angle NPR + \angle RPQ + \angle PQM = 180^\circ$ [From (2) and (3)] ...(3)

$\Rightarrow$ $\angle NPR - \angle RPQ + \angle PQM = \angle RPQ + \angle PQM + \angle MQS$

$\therefore$ $\angle NPR - \angle MQS$ ...(4)

Similarly, $\angle NRP = \angle MSQ$ ...(5)

In Δ PNR and Δ QMS

$PR - RS$ [From (1)]

$\angle NPR = \angle MQS$ [From (4)]

$\angle NRP - \angle MSQ$ [From (5)]

$\therefore$ $\Delta PNR \cong \Delta QMS$ [By ASA, (1), (4) and (5)]

So, $PN - QM$ and $NR =$ MS

Since, PN || QM, so $PQMN$ is a parallelogram.

Thus, $MN - PQ$ and MN || PQ. **Hence Proved**

Exercise 9.1

1. The angles of a quadrilateral are in the ratio 3:5:9:13. Find all the angles of the quadrilateral.
2. Diagonals AC and BD of a parallelogram $ABCD$ intersect each other at point O, where $OA = 3$ cm and $OD = 2$ cm. Find the lengths of AC and BD.

3. The diagonals of a parallelogram are perpendicular to each other. Is this satement true? Give reason in support of your answer.
4. State whether the angles 110°, 80°, 70° and 95°, are angles of a quadrilateral? Why and why not?
5. Are all the angles of a quadrilateral, be obtuse angles? Give reason for your answer.
6. One angle of a quadrilateral is 108° and other three angles are equal. Find the measure of each of the three equal angles.
7. $ABCD$ is a trapezium in which AB DC and $\angle A - \angle B - 45°$. Find $\angle C$ and $\angle D$ of this trapezium.
8. The angle between two altitudes, drawn from the vertex of an obtuse angle to the opposite sides of a parallelogram is 60°. Find all the angles of this parallelogram.
9. Points E and F lie on diagonal AC of a parallelogram $ABCD$ such that $AE = CF$. Show that $BFDE$ is a parallelogram.
10. In Fig. 9.21, $ABCD$ is a parallelogram. AQ and CP are the bisectors of $\angle$A and $\angle$C respectively. Prove that $APCQ$ is a parallelogram.

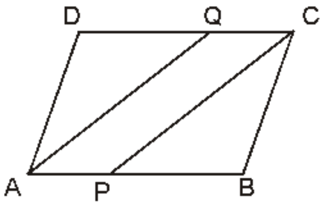


Fig. 9.21

11. In Fig. 9.22, $ABCD$ and $AFEB$ are parallelograms. Prove that $CDFE$ is a parallelogram.

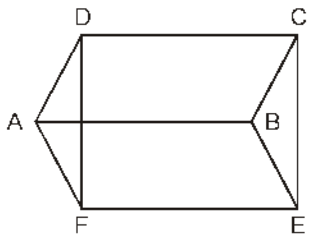


Fig. 9.22

12. The perpendiculars AP and CQ are drawn from points A and C respectively on diagonal BD of a parallelogram $ABCD$. Prove that $AP - CQ$.
13. In Fig. 9.23. $ABCD$ is a quadrilateral in which $AB \| DC$ and $AD - BC$. Prove that $\angle A - \angle B$.

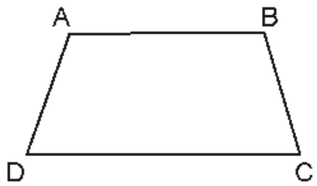


Fig. 9.23

14. In Fig. 9.24, $ABCD$ *is* a parallelogram. P and Q are the mid points of opposite sides AB and CD. Prove that $PRQS$ is a parallelogram.

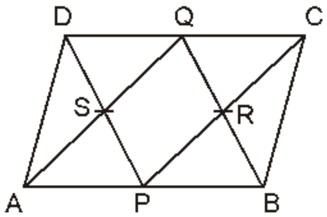


Fig. 9.24

9.06 Special Parallelogram and their Properties

You have studied different types of parallelogram in this chapter and you have also verified some properties.

Let us try to understand some existing properties of those specific parallelograms with the help of some theorems.

Theorem 9.9. ***If diagonals of a parallelogram are equal, then it is a rectangle.***

Given : $ABCD$ is a parallelogram in which $AC - BD$.

To Prove : $ABCD$ is a rectangle.

Proof : In ΔABC and ΔBAD

$BC - AD$ (Opp. sides of parallelogram)

$AB - AB$ (Common side)

$AC - BD$ (Given)

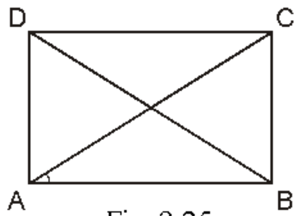


Fig. 9.25

By SSS property of congruency

$$\Delta ABC \cong \Delta BAD$$

Since, corresponding angles of congruent triangles are equal,

$\therefore \quad \angle DAB + \angle CBA = 180°$

$\Rightarrow \quad \angle DAB + \angle DAB - 180°$

$\therefore \quad \angle DAB - \angle CBA - 90°$

Thus, $ABCD$ *is* a rectangle. **Hence Proved**

Converse : ***Diagonals of a rectangular are equal to each other.***

Theorem 9.10. ***If the diagonals of a parallelogram are perpendicular to each other, then it is a rhombus.***

Given : Diagonals AC and BD of parallelogram $ABCD$ are perpendicular to each other.

To prove : $ABCD$ is a rhombus

Proof : In ΔAOB and ΔCOB

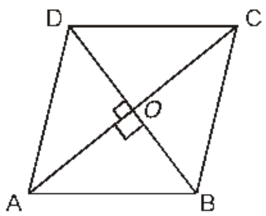


Fig. 9.26

$OB - OB$ (Common)

$\angle AOB = \angle BOC = 90°$ (Given)

$AO = CO$ (Diagonals of parallelogram bisect each other)

$\Delta AOB \cong \Delta COB$ (By SAS congruency)

Since, corresponding parts of congruent triangles are equal,

$AB = BC$

Thus, $ABCD$ *is* a rhombus. **Hence Proved**

Converse : ***Diagonals of a rhombus are perpendicular to each other.***

Theorem 9.11. ***If the diagonals of a parallelogram are equal and perpendicular to each other, then it is a square.***

Given : $ABCD$ is a parallelogram where $AC - BD$ and

$AC \perp BD$

To Prove : $ABCD$ *is* a square.

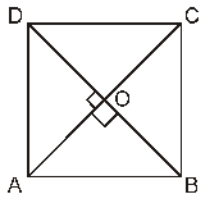


Fig. 9.27

Proof: $\because$ Diagonals of a parallelogram bisect each other

$BO - OD$ ...(1)

In ΔABO and ΔADO

$BO = OD$ [From (1)]

$\therefore$ $\angle AOB - \angle AOD - 90°$ (Given)

$AO - AO$ (Common)

$\Delta ABO - \Delta ADO$ (By SAS congruency)

Since, corresponding parts of congruent triangles are equal,

$\therefore$ $AB - AD$ ...(2)

Now, in ΔABD and ΔBAC

$BD - AC$ (Given)

$AB = AB$ (Common)

$AD = BC$ (Opposite sides of parallelogram)

$\Delta ABD \cong \Delta BAC$ (By SSS congruency)

Since, corresponding parts of congruent triangles are equal,

$\therefore$ $\angle DAB - \angle CBA$ ...(3)

We know that the sum of two adjacent angles of a parallelogram is 180°,

$\therefore$ $\angle DAB + \angle CBA = 180°$ ...(4)

From (3) and (4), we get

$\angle DAB - \angle CBA - 90°$ ...(5)

And from (2) and (5), we get

ABCD is a square. **Hence Proved**

Converse : ***Diagonals of a square are equal and perpendicular to each other.***

9.07 Mid-Poit Theorem

You have studied about some properties of triangles and quadrilaterals. Now let us study another result which is related to the mid-point theorem.

Theorem 9.12. ***A line segment joining the mid points of any two sides of a triangle, is half and parallel to its third side.***

Given : In ΔABC, points D and E are mid-points of sides AB and AC respectively.

To Prove : (i) $DE |\ BC$, and (ii) $DE = \frac{1}{2} BC$

Construction : Extend DE to F where $EF - DE$. Join C to F.

Proof : In Δ ADE and Δ CFE

$AE - CE$ (Given)

$\angle AED - \angle CEF$ (Vertically opposite angles)

$DE - EF$ (By construction)

$\Delta ADE \cong \Delta CFE$ (By SAS congruency rule]

Since, corresponding parts of congruent triangles are equal.

$\therefore$ $AD - CF$ ...(1)

and $\angle EAD - \angle ECF$

Transversal AC intersects lines AB and CF such that alternate angles EAD and ECF are equal.

$\therefore$ $AD || CF$ and $BD || CF$

But $AD - BD$ (Given) ...(3)

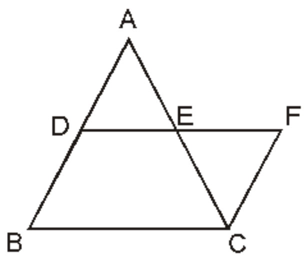


Fig. 9.28

From (1) and (3), we get

$BD - CF$ and $BD \quad CF$

$\therefore$ $BCFD$ is a paralellogram.

$\therefore$ DF = BC and $DF \quad BC$

$\Rightarrow$ $\frac{1}{2} DF = \frac{1}{2} BC$

$\Rightarrow$ $DE = EF = \frac{1}{2} DF$. (By construction)

$\Rightarrow \qquad \text{DE} = \frac{1}{2}\text{BC}$

Thus, $\qquad \text{DE} = \frac{1}{2}\text{BC}$ and DE || BC **Hence Proved**

Theorem 9.14. (Converse of Theorem 9.12)

The line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side.

Given : In ΔABC, D is the mid-point of side AB in Fig. 9.29 $DE \quad BC$ and it intersects AC at point E.

To Prove : $AE = EC$

Construction : Draw CF parallel to BD which intersects DE (extended) at point F.

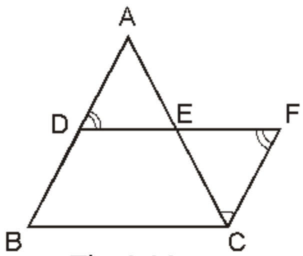


Fig. 9.29

Proof : $\because$ $\quad BC \,||\, DF$ (Given)

and $\quad BD \quad CF$ (By construction)

$\therefore$ $\quad BCFD$ is a parallelogram.

$\therefore$ $\quad BD = CF$

But $\quad BD = AD$ (Given)

$\therefore$ $\quad AD - CF$ ...(1)

Now, in ΔADE and ΔCFE

$\angle ADE - \angle CFE$ (Alternate interior angles)

$AD = CF$ [From (1)]

$\angle DAE - \angle ECF$ [Alternate interior angles]

$\Delta \text{ADE} \cong \Delta \text{CFE}$ (By ASA congruency rule)

$AE = CE$ **Hence Proved**

9.08. Intercept

If there are two lines l_1 and l_2 in a plane and line l_3 intersects these lines on different points A and B, then the line segment AB is called intercept made by lines l_1 and l_2 on line l_3

Here, will prove the following theorem for three parallel lines. This theorem can be extended for more than three lines.

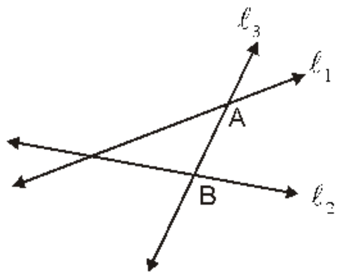


Fig. 9.30

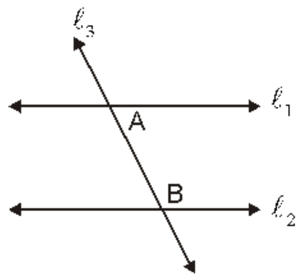


Fig. 9.31

Theorem 9.15. ***If there are three or more than three parallel lines and intercepts made by them on a transversal line are equal, then the corresponding intercepts on the other transversal will be also equal.***

Given : In Fig 9.32, l, m, n are three parallel lines and two transversals lines l_1 and l_2 intersect them on points A, B, C and D, E, F respectively and $AB = BC$.

To Prove : $DE = EF$

Construction : Draw $GH \parallel l_1$ which passes through E.

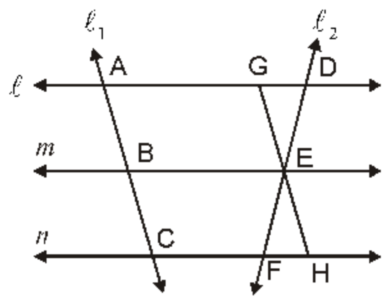


Fig. 9.32

Proof : $\because$ l and m are parallel lines

$\therefore$ $AG \parallel BE$

and $AB \parallel GE$ (By construction)

$\therefore$ $ABEG$ is a parallelogram.

$\therefore$ $AB = GE$ ...(1)

Similarly, $BCHE$ is a parallelogram.

$\therefore$ $BC = EH$ ...(2)

Given $AB = BC$

$\therefore$ $GE = EH$ ...(3)

Again $l \parallel n$ and transversal l_2 intersects them.

$\angle GDE = \angle EFH$ (Alternate interior angles)...(4)

Now, in ΔGDE and ΔHFE

$\angle GDE = \angle EFH$ [From (4)]

$GE = EH$ [From (3)]

$\angle GED = \angle HEF$ [Vertically opposite angles]

$\Delta GDE \cong \Delta HFE$ (By ASA congruency rule)

Since, corresponsding parts of congruent triangles are equal,

Thus, $DE = EF$ **Hence Proved**

Illustrative Examples

Example 8. ***D, E* and *F* are respectively mid-points of sides BC, CA and *AB* of are equilateral triangle *ABC*. Prove that ΔDEF is an equilateral triangle.**

Solution :

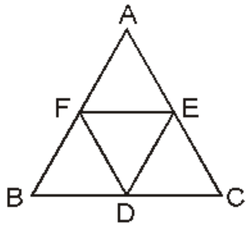


Fig. 9.33

Given : Δ ABC in which D, E, F are the mid-points of sides EC, CA and AB respectively.

To Prove : ΔDEF is an equilateral triangle.

Proof : In ΔABC, D, E and F are the mid - points of sides BC, CA and AB respectively.

$$\therefore \qquad DE = \frac{1}{2}AB \qquad ...(1)$$

$$EF = \frac{1}{2}BC \qquad ...(2)$$

$$FD = \frac{1}{2}AC \qquad ...(3)$$

But ΔABC is an equilateral triangle .

$$AB = BC = CA \qquad ...(4)$$

From (1), (2), (3) and (4)

$$DE - EF - FD$$

Thus, ΔDEF is an equilateral triangle. **Hence Proved**

Example 9. **Prove that a line, joining the mid-points of the diagonals of a trapezium, will be parallel to its parallel sides and half of their difference.**

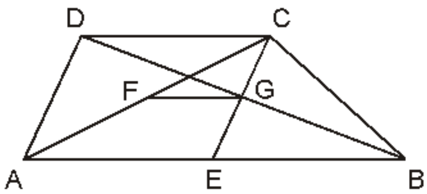


Fig. 9.34

Solution : Given : A trapezium $ABCD$ in which $AB \mid DC$, F and G are mid-points of diagonals AC and BD respectively.

To Prove : (i) $FG \quad AB$

(ii) $FG = \frac{1}{2}(AB - CD)$

Construction : Join CG and extend it such that it will meet AB at E.

Proof : In ΔCDG and ΔEBG

$\angle CDG = \angle EBG$	(Alternate interior angles)
$DG - GB$	(Given)
$\angle DCG - \angle BEG$	(Alternate interior angles)
$\Delta CDG \cong \Delta EBG$	(By ASA cong:uency rule)

Since, corresponding parts of two congruent triangles are equal.

$\therefore \qquad CG = EG \qquad ...(1)$

and $\qquad CD = EB \qquad ...(2)$

Now in ΔACE, F and G are the mid points of sides AC and CE respect ively.

$$FG \,||\, AE \text{ and } FG = \frac{1}{2}AE \qquad ...(3)$$

But $\qquad AE = AB - EB$

$\qquad AE = AB - CD \qquad$ [From (2)] $\qquad ... (4)$

From (3) and (4), we get

$$FG = \frac{1}{2}AE = \frac{1}{2}(AB - CD)$$

and $\qquad FG \,||\, AE$

Thus, $\qquad FG \,||\, AB$ and $FG = \frac{1}{2}(AB - CD)$ $\qquad$ **Honce Proved**

Example 10. Prove that the quadrilateral obtained by joining the mid points of consecutive sides of a quadrilateral is a parallelogram.

Solution :

Given : In Fig 9. 35 $ABCD$ is a quadrilateral where P, Q, R and S are the mid-points of its consecutive sides respectively.

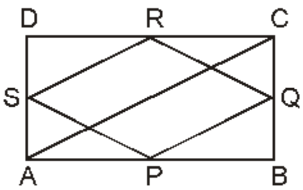


Fig. 9.35

To Prove : $PQRS$ is a parallelogram.

Construction : Join AC.

Proof : In ΔABC, P and Q are the mid-points of sides AB and BC respectively.

$$PQ \,||\, AC \text{ and } PQ = \frac{1}{2}AC \qquad ...(1)$$

In ΔADC, S and R are mid points of sides AD and DC respectively.

$$\therefore \qquad SR \,||\, AC \text{ and } SR = \frac{1}{2}AC \qquad ...(2)$$

From (1) and (2), we get

$$PQ \,||\, SR \text{ and } PQ = SR$$

Thus, in quadrilateral $PQRS$ one pair of opposite sides are equal and parallel.

Thus, $PQRS$ is a parallelogram

Example 11. In Fig 9.36, X and Y are respectively mid points of opposite sides AD and BC of a parallelogram $ABCD$. Also, BX and DY intersect line AC at points P and Q respectively. Show that AP = PQ = QC.

Solution : In fig. 9.36, X and Y are the mid-points of sides AD and BC respectively of a parallelogram $ABCD$.

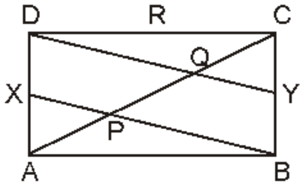


Fig. 9.36

$\therefore \quad DX = \frac{1}{2}AD$ and $BY = \frac{1}{2}CB$

But $ABCD$ is a parallelogram.

$\therefore$ AD = BC and AD || BC [Opposite sides of a parallelogram]

$\Rightarrow \quad \frac{1}{2}AD = \frac{1}{2}BC$ and AD || BC

$\Rightarrow$ DX = BY and DX || YB

$\because$ One pair of opposite sides of quadrilateral XBYD are equal and parallel

$\therefore$ XBYD is a parallelogram

$\Rightarrow$ PX || QD

We know that the segment drawn from the mid-point of one side of a triangle and parallel to the other side bisects the third side.

In Δ CBP, Y is the mid-point of BC and YQ || BP

$\therefore$ Q is the mid-point of CP

$\Rightarrow$ P is the mid-point of AQ

$\Rightarrow$ AP = PQ ...(i)

Simlilary CQ = PQ ...(ii)

AP = PQ = QC [From (i) and (ii)]

Example 12. In Fig. 9.37 AY and CX are respectively the bisectors of opposite angles A and C of parallelogram ABCD. Show that

AY || CX

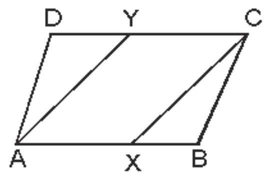


Fig. 9.37

Solution : $\angle A = \angle C$ (opposite angles of parallelogram)

$\Rightarrow \quad \frac{1}{2}\angle A = \frac{1}{2}\angle C$

[$\because$ AY and CX are the bisectors of $\angle$ A and $\angle$ C respectively]

$\therefore$ AB || CD (opposite sides of parallelogram)

Also AX || YC and transversal CX intersects them

$\angle YAX = \angle YCX$ ...(i)

A transversal line CX intersects two parallel lines AB and CD.

So, $\angle CXB = \angle YCX$ ($\because$ alternative angles) ... (ii)

From (i) and (ii) $\angle YAX = \angle CXB$

because the corresponding angles are equal.

$\therefore$ AX || YC (because sum of interior angles on one side of a transversal line is 180°)

$\therefore$ AY || CX **Hence Proved**

Example 13. Show that a quadrilateral, formed by joining the mid-points of the sides of a rhombus, in the same order, is a rectangle.

Solution : Let ABCD be a rhombus and P, Q, R, S are mid-points of sides AB, BC, CD and DA respectively. (Fig. 9.38). Join AC and BD.

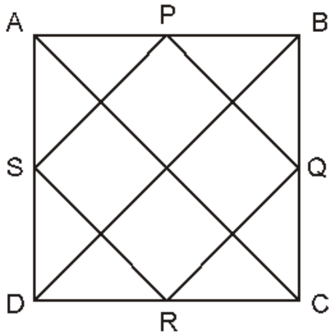


Fig. 9.38

$\therefore$ From ΔABD, we get

$$SP = \frac{1}{2} BD \text{ and } SP \parallel DB$$

(S and P are the mid points of sides AB and AD respectively)

Similarly, $RQ = \frac{1}{2} BD$ and RQ || BD

$\therefore$ SP = RQ and SP || RQ

$\therefore$ PQRS is a parallelogram.

Also $AC \perp BD$ (Diagonals of a rhombus are perpendicular to each other)

PQ || AC (In Δ BAC, P and Q are mid points of sides AB and BC respectively)

SP || BD ...(1)

In Δ ABC, P and Q are the mid point of AB and BC respectively

$\therefore$ PQ || AC ...(2)

From (1) and (2) $SP \perp PQ$ [$\because AC \perp BD$]

$\therefore$ $\angle SPQ = 90^\circ$

$\therefore$ PQRS is a rectangle. **Hence Proved**

Exercise 9.2

1. In Fig. 9.39 ABCD and AEFG are two parallelograms. If $\angle C = 55^\circ$, then determine $\angle F$.

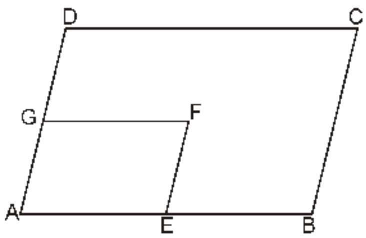


Fig. 9.39

2. Can all the angles of a quadrilateral be acute angles? Give reason for your answer.
3. Can all the angles of a quadrilateral be right angles? Give reason of your answer.
4. The diagonals of a quadrilateral ABCD bisect each other. If $\angle A = 35^\circ$, then determine $\angle B$.
5. Opposite angles of a quadrilateral ABCD are equal. If AB = 4 cm, then determine CD.
6. ABCD is a rhombus in which altitude from D on AB, bisects AB. Find the angles of the rhombus.
7. In a triangle ABC, lines RQ, PR and QP are respectively drawn parallel to lines BC, CA and AB passing through points A, B and C, as shown in fig. 9.40 Show that

$$BC = \frac{1}{2} = QR \ .$$

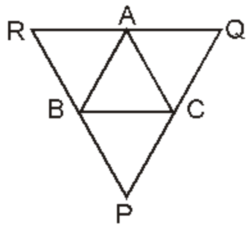


Fig. 9.40

8. D, E and F are respectively the mid points of sides BC, CA and AB respectively of an equilateral triangle ABC. Show that ΔDEF is also an equilateral triangle.
9. Points P and Q are respectively taken on opposite sides AB and CD of a parallelogram $ABCD$ such that $AP = CQ$ (Fig 9.41). Show that AC and PQ bisect each other.

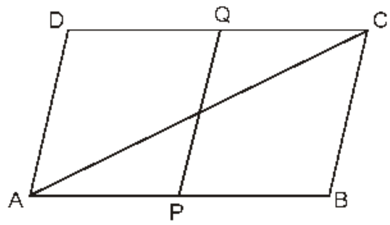


Fig. 9.41

10. E is the mid-point of side AD of a trapezium $ABCD$ in which $AB \parallel DC$. Through point E a line is drawn parallel to AB, intersects BC at F. Show that F is the mid-point of BC. [Hint: Join AC].
11. In ΔABC, $AB - 5$cm, BC = 8cm and CA = 7cm. If D and E are mid-points of AB and BC respectively, then find the length of DE.
12. In Fig 9.42, it is given that $BDEF$ and $FDCE$ are parallelograms. Can you say that $BD - $ CD? Why and why not?

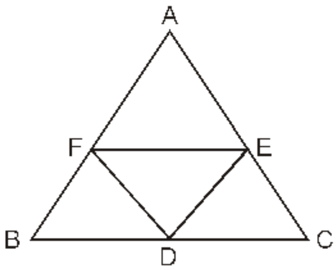


Fig. 9.42

13. In Fig. 9.43, D, E and F are mid points of sides BC, CA and AB respectively. If AB – 4.3 cm, BC – 5.6 cm and AC – 3.9 cm, then find the perimeter of the following :
    (i) ΔDEF (ii) quadrilateral $BDEF$

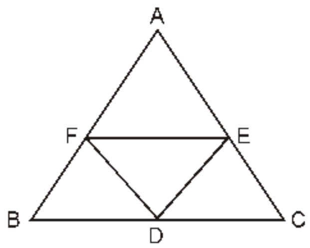


Fig. 9.43

14. Prove that a quadrilateral obtained by joining the mid points of consecutive sides of a square, is also a square.

15. The diagonals of a quadrilateral are perpendicular to each other. Prove that a quadrilateral obtained by joining the mid-points of its sides is a rectangle.

16. Prove that in a right-angled triangle, the bisecting median of hypotenuse is half of the hypotenuse.

17. Prove that a rhombus is obtained by joining the mid-points of the pairs of opposite sides of a rectangle.

Constructions of Quadrilaterals

9.09. Quadrilateral

A plane figure enclosed by four line segment, is called a quadrilateral. The line joining its opposite vertices is called its diagonal.

In Fig. 9.44 AB, BC, CD and DA are its four sides. A, B, C and D are its vertices and AC and BD are the diagonals of quadrilateral $ABCD$.

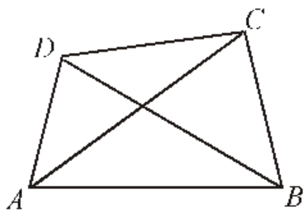


Fig. 9.44

9.10. Construction of Quadrilateral

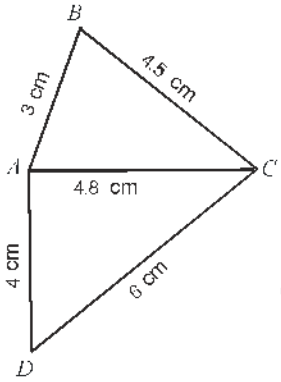
When we have to construct a quadrilateral, a rough sketch should be drawn and mark the given facts. Generally, there is specific importance of diagonal in construction of a quadrilateral. So it should be considered by drawing a diagonal in rough sketch definitly. Is any triangle can be construct by it? It should be seen by drawing quadrilateral after formation of triangle. Construction of a quadrilateral can be completed by drawing triangle. It is not necessary to draw a diagonal in each condition. Some times without drawing any diagonal, quadrilateral can be constructed.

It can be understand clearly by the constructions given in this chapter.

Construction 9.16 : Construction of a quadrilateral when four sides and a diagonal are given.

Construct a quadrilateral $ABCD$ in which $AB – 3$ cm, $BC – 4.5$ cm, CD = 6 cm, $DA = 4$ cm and $AC = 4.8$ *cm*.

Construction : First draw a rough sketch on the basis of given measures and mark them on it.



(Rough Figure)

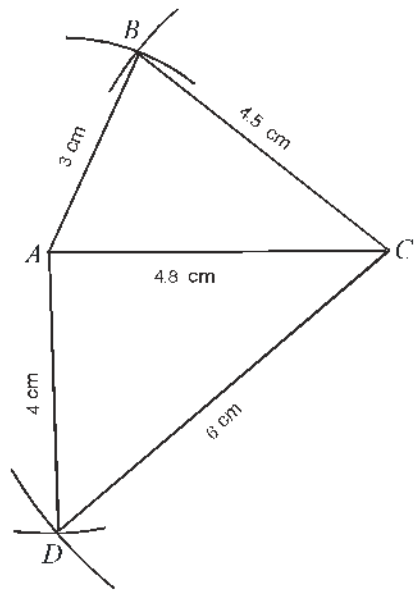


Fig. 9.45

According to the rough sketch, draw $AC – 4.8$ cm. Construct a triangle ABC by drawing an arc of radius 3 cm from A and arc of radius 4.5 cm from C. Similarly, complete Δ ACD by drawing arc of length equal to AD and CD.

Thus, $ABCD$ *is* the required quadrilateral.

Construction 9.17. ***Construction of a Quadrilateral in which four sides and one angle are given.***

Construct a quadrilateral $ABCD$ in which $AB - 4.8$ cm, $BC - 3.5$ cm, $CD - 4.5$ cm, DA = 4 cm and $\angle A = 60°$

Construction : Draw a rough sketch according to the given measures and mark them.

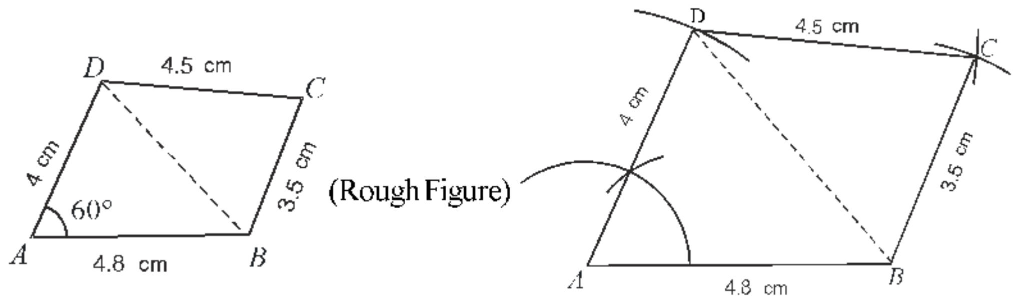


Fig. 9.46

Draw line segment $AB - 4.8$ cm. Draw $\angle DAB - 60°$ at point A and cut $AD = 4$ cm from A . Draw two arcs from point D and B of raddi 4.5 cm and 3.5 cm respectively they intersect at point C. Join DC and BC. Thus, $ABCD$ is a required quadrilateral.

Construction 9.18. ***Construction of a quadrilateral when three sides an two diagonals are given.***

Construct a quadrilateral $ABCD$ in which $AB - 5.5$ cm, $BC - 3.3$ cm, $AD - 4.6$ cm and diagonals $AC = 5.7$ cm and $BD = 6$ cm.

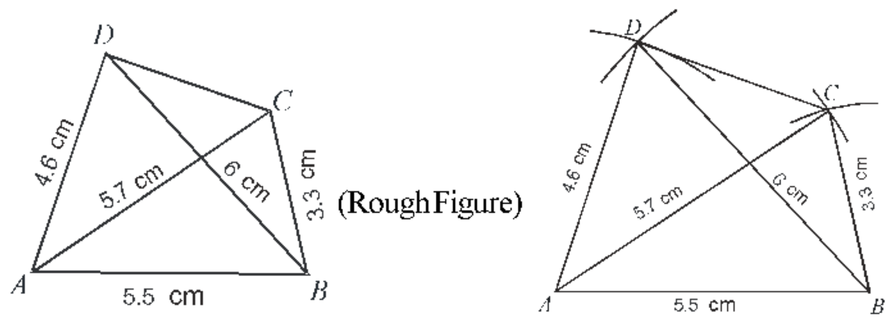


Fig. 9.47

Construction : Draw a rough sketch according to the given measures and mark them. According to rough sketch draw line segment $AB - 5.5$ cm. Construct a ΔABD by drawing the arcs of radii 4.6 cm and 6 cm from points A and B respectively. Similarly again, construct ΔABC by drawing arcs of radii 5. 7 cm and 3.3 cm from points A and B. Join C and D.

Thus, $ABCD$ is the required quadrilateral.

Construction 9.19. ***Construction of a quadrilateral when three sides and two angles between them are given.***

Construct a quadrilateral $ABCD$ in which AB – 3.5 cm, BC – 5 cm, CD = 5.5 cm, $\angle$B = 120° and $\angle$C = 60°.

Construction : Draw a rough sketch according to given measures and mark them.

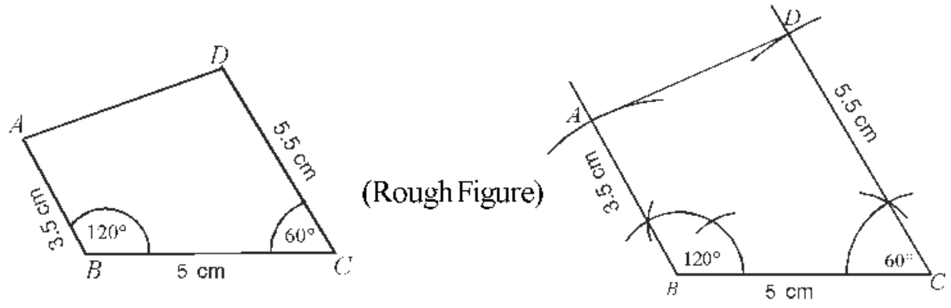


Fig. 9.48

Draw line segment BC = 5 cm. Draw $\angle$B = $\angle$120° and $\angle C$ = $\angle$ 60° at points B and C respectively with BC and cut the given lengths AB – 3.5 cm and CD – 5.5 cm from B and C respectively. Join A and D to obtained the quadrilateral.

Thus $ABCD$ is required quadrilateral.

Construction 9.20. ***Construction of a quadrilateral when two consecutive sides and angle between them and other two angles are given.***

Construct a quadrilateral $ABCD$ in which AB – 5 cm, AD – 5.3 cm, $\angle$ A= 60°, $\angle$C = 105° and $\angle$ D = 90°.

Construction : $\angle$A + $\angle$B + $\angle$C + $\angle$D = 360°

$\Rightarrow$ 60° + 105° + $\angle$B + 90° = 360° $\Rightarrow$ 255° + $\angle$B = 360°

Draw a rough sketch according to given measures and mark them.

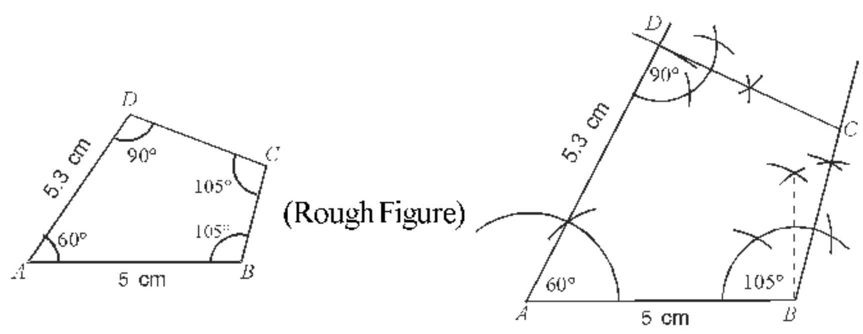


Fig. 9.49

Draw line segment $AB = 5$ cm. At points A and B, draw $\angle A = 60°$ and $\angle B = 105°$ with AB. From A cut, $AD = 5.3$ cm and at D, draw $\angle D = 90°$ with AD.

Thus, $ABCD$ is the required quadrilateral.

Exercise 9.3

Construct quadrilaterals for the following given facts with description of steps of construction.

1. In a quadrilateral $ABCD$, $AB – 3.5 cm$, $BC = 4.8$ cm, $CD – 5.1$ cm, $AD = 4.4$ cm and a diagonal $AC – 5.9$ cm.
2. In a quadrilateral $PQRS$, $PQ – 4$ cm, $QR = 3$ cm, $QS – 4.8$ cm, PS = 3.5 cm and $PR – 5$ cm.
3. In, quadrilateral $ABCD$, $AB – 4$ cm, $BC = 4.5$ cm, $CD – 3.5$ cm, $AD – 3$ cm and $\angle A = 60^{\circ}$
4. In, quadrilateral $ABCD$, $AB – 3.5$ cm, $BC – 3$ cm, $AD – 2.5$ cm, $AC = 4.5$ cm and BD = 4 cm.
5. In, quadrilateral PQRS, $PQ – 3$ cm, QR = 4 cm, PS = 4.5 cm, $PR – 6$ cm and QS = 5.5 cm.
6. In, quadrilateral $ABCD$, $AB – BC – 3.0$ cm, $AD – 5$ cm, $\angle A = 90°$ and $\angle B – 120°$.
7. In, quadrilateral $ABCD$, $AB – 3.8$ cm, $BC – 2.5$ cm, $CD – 4.5$ cm and $\angle B – 30°$ and $\angle C = 150°$.
8. In, quadrilateral $PQRS$, $PQ – 3$ cm, QR = 3.5 cm, $\angle Q = 90°$ and $\angle P = 105°$ and $\angle R = 120^{\circ}$.
9. In, quadrilateral $PQRS$, $PQ = 2.5$ cm, $QR – 3.7$ cm, $\angle Q = 120°$, $\angle S = 60°$ and $\angle R = 90^{\circ}$.

9.11 Constructions of Parallelograms and Rectangles

Before the construction of parallelogram, rectangle, square and rhombus, it is necessary to know the following facts.

1. In a parallelogram : (i) Opposite sides are equal,
   (ii) opposite angles are equal,
   (iii) Diagonals bisect each other,
   (iv) Each diagonal bisects a parallelogram into two congruent triangles.
2. In a rectangle : (i) Each angle is right angle,
   (ii) Opposite sides are equal.
   (iii) Diagonals are equal.
   (iv) Diagonals bisect each other.
3. In a square : (i) All four sides are equal.
   (ii) Each angle is right angle.
   (iii) Diagonals are equal.

(iv) Diagonals bisect each other at right angle,

(v) Each diagonal makes an angle of $45°$ with sides.

4. In a rhombus : (i) All four sides are equal.

(ii) Opposite angles are equal.

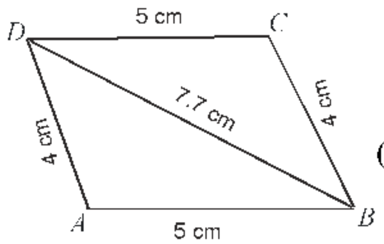
(iii) Diagonals bisect each other at right angle.

(iv) Diagonals are bisectors of the vertex angles.

Construction 9.21 . ***Construction of a parallelogram when two sides and one diagonal are given.***

Construct a parallelogram $ABCD$ *if* $AB = 5$ cm, $BC = 4$ cm and BD – 7.7 cm.

Construction : Draw rough sketch according to given measures and mark them.



(Rough Figure)

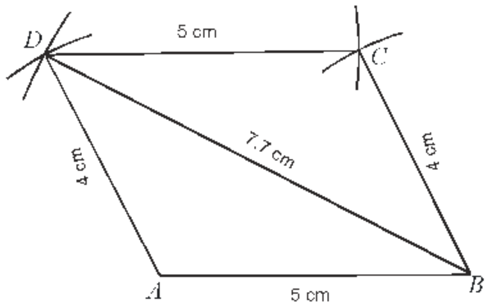


Fig. 9.50

Draw a line segment AB – 5 cm. Construct ΔABD by drawing arcs of radii 4 cm and 7.7 cm from points A and B respectively. Similarly, construct Δ BCD by drawing arcs of radii 4 cm and 5 cm from points B and D respectively.

Thus, $ABCD$ *is* the required parallelogram.

Construction 9.22. ***Construction of a parallelogram when a side and two diagonals are given.***

Construct a parallelogram $ABCD$ in which $AB = 5$ cm, diagonal AC – 7.6 cm and diagonal $BD = 5.6$ cm.

Hint: In a parallelogram, diagonal bisect each other

$$\therefore \quad AO = OC = \frac{1}{2}AC \text{ and } BO = OD = \frac{1}{2}BD.$$

$$AO = \frac{1}{2} \times 7.6 = 3.8\,cm \text{ and } BO = \frac{1}{2} \times 5.6 = 2.8\,cm.$$

Construction: Draw a rough sketch according to given measures and mark them.

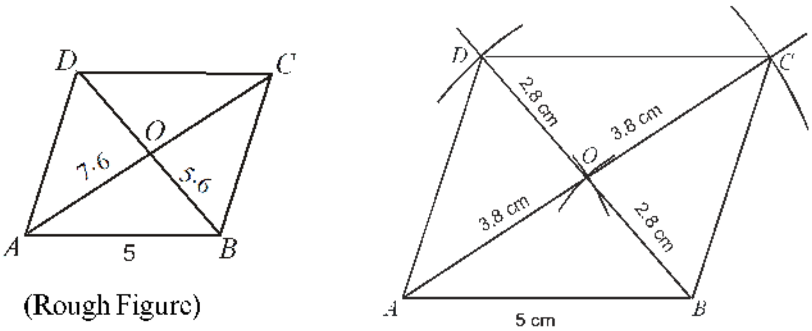


Fig. 9.51

Draw a line segment AB – 5 cm. Construct a ΔAOB by drawing arcs of radii 3.8 cm or half of diagonal AC and 2.8 cm or, half of diagonal BD. Extend AO and BO so that AC – 7.6 cm and BD = 5.6 cm. Join BC, CD and AD. Thus, $ABCD$ *is* the required parallelogram.

Construction 9.23. ***Construction of a parallelogram in which two adjecent sides and angle between them are given.***

Construct a parallelogram ABCD where AB – 5.5 cm, BC – 3.7 cm and $\angle A$ – 60°.

Construction : Draw a rough sketch according to given measures and mark them.

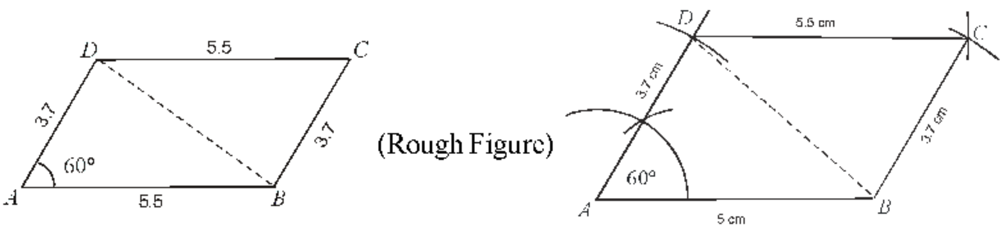


Fig. 9.52

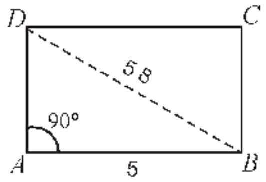
Draw a line segment AB – 5.5 cm. Construct $\angle BAM$ – 60° at point A and make a ΔABD by taking AD = 3.7 cm. Similarly, by taking BC = 3.7 cm and DC = 5.5 cm construct Δ BDC.

Thus, $ABCD$ *is* a required parallelogram.

Construction 9.24. ***Construction of a rectangle whose diagonal and one side are given.***

Construct a rectangle in which diagonal BD – 5.8 cm and one side AB – 5 cm.

Construction : Draw a rough sketch according to given measures and mark them.



(Rough Figure)

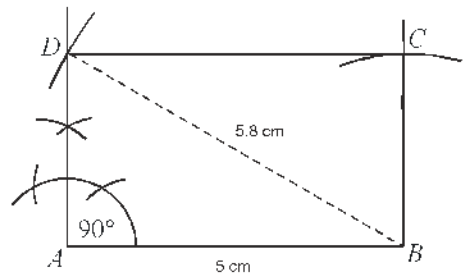


Fig. 9.53

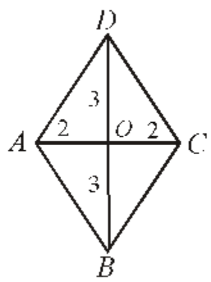
Draw a line segment AB – 5 cm. At point A, draw $\angle A$ – 90°. Taking B as centre. draw an arc of radius BD – 5.8 cm which intersect at D. Now taking D and B as centres draw two arcs of radii AB and AD respectively which intersect, at C. Join CD and BC.

Thus, ABCD is a required rectangle.

Construction 9.25: ***Construction of rhombus when two diagonals are given.***

Construct a rhombus whose diagonals are 4 cm and 6 cm respectively.

Construction : Draw a rough sketch of given measures and mark them.



(Rough Figure)

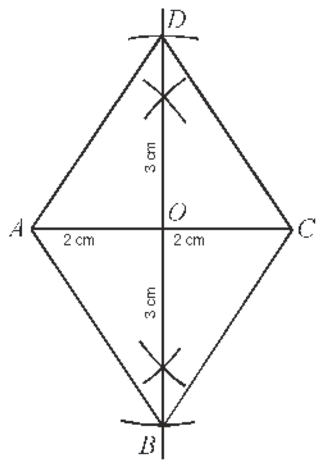


Fig. 9.54

Draw diagonal AC – 4 cm. Draw perpendicular bisector of AC which meets AC at O. Take O as centre and draw two arcs .of radius equal to half of the length of other diagonal $BD = \left(\frac{1}{2}\times 6 = 3cm\right)$ both sides of AC. These arcs intersect perpendicular bisector at B and D. Join AB, BC, CD and AD.

Thus ABCDis a required rhombus.

Construction 9.26 : ***Construction of a square whose diagonals are given.***

Construct a square whose diagonal is 5 cm.

Construction : Draw a rough sketch of given measures and mark them.

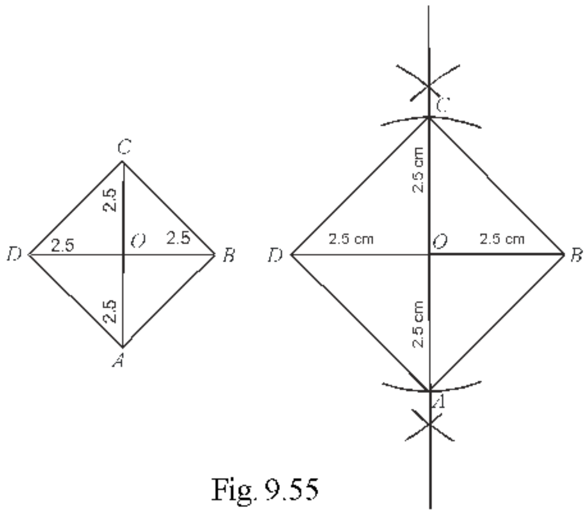


Fig. 9.55

Draw a diagonal $BD = 5$ cm. Draw its perpendicular bisector which meets BD at O. Take O as centre and cut $OC = OA = 2.5$ cm. Join AB, BC, CD and DA.

Thus, $ABCD$ is the required square.

Exercise 9.4

1. Construct a parallelogram $ABCD$ in which AB = 4.7 cm, $BC = 3.5$ cm and $AC = 7$cm.
2. Construct a parallelogram $PQRS$ in which $PQ = 5$ cm, diagonal $PR = 7.6$ cm and diagonal $QS = 5.6$ cm.
3. Construct a parallelogram ABCD whose two sides are 4.6 cm and 3 cm respectively and angle between them is 60°.
4. Construct a rectangle $ABCD$ in which $AB = 6$ cm and diagonal $AC = 10$ cm.
5. Construct a rhombus $ABCD$ whose diagonals $AC = 7$ cm and $BD = 5$ cm.
6. Construct a square $ABCD$ whose diagonal is 6 cm.

Construction 9.27 : ***Construction of a Trapezium***

(a) ***When four sides of a trapezium are given and in which two sides are parallel.***

Construct a trapezium $ABCD$ in which $AB = 7$ cm, $BC = 6$ cm, $CD = 4$ cm, $DA = 5$ cm and AB || CD.

Construction : Draw a rough sketch based on given measures and mark all the lengths.

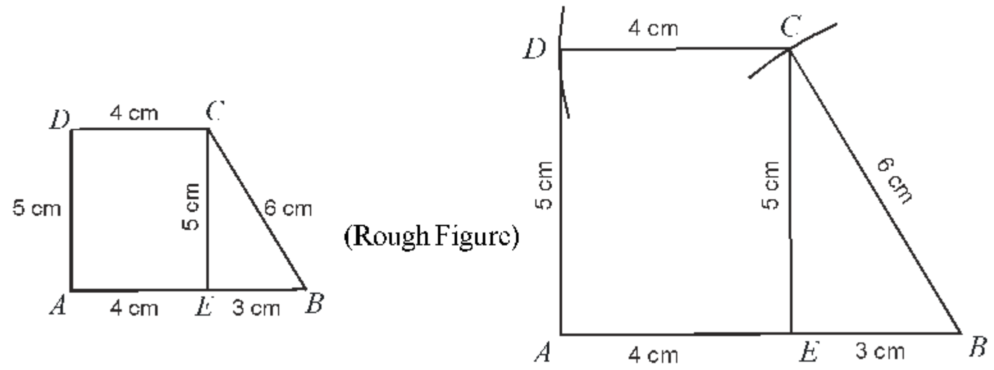


Fig. 9.56

Mark a point E on line AB such that $AE - DC - 4$ cm. Draw a line segment AB = 7 cm and mark point E such that $AE - 4$ cm. Take E and B as centres draw two arcs radii 5 cm (=AD) and 6 cm (= BC) respectively which intersect each other at point C. Again taking A and C as centres, draw two arcs of radii 5 cm and 4 cm such that they intersect each other at D. Join B to C, C to D and A to D to construct a complete quadrilateral.

Thus, $ABCD$ is the required trapezium.

(b) ***Construction of Trapezium if three sides and one angle are given and it is also given that which sides are parallel.***

Construct a trapezium $ABCD$ in which $AB \parallel CD$, $\angle B - 90°$, AB = 4 cm, BC = 2.8 cm, $AD - 3.5$ cm.

Construction : Draw a rough sketch of trapeÐium and mark all the given measures.

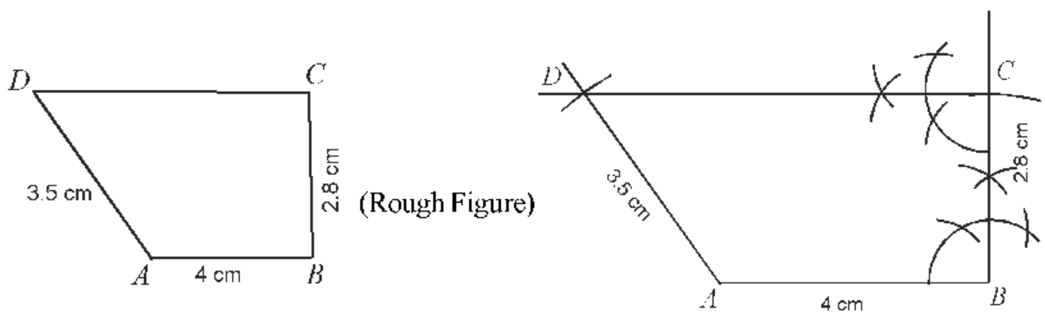


Fig. 9.57

For fair construction, take $AB - 4$ cm. Construct $\angle B = 90°$ at B. Cut a point at a distance of 2.8 cm from line making right angle and mark it as C. Again construct $\angle C - 90°$ at point C. ($\because AB \quad CD$ and $\angle B - 90°$, $\angle C - 90°$)

Taking A as a, centre, cut a point on a line drawn perpendicular at point C and mark it as D. Join A and D to construct a quadrilateral $ABCD$.

Thus, $ABCD$ is a required trapezium.

Exercise 9.5

1. Construct a trapezium $ABCD$ in which $AB \mid CD, AB = 4$ cm, $BC - 2.3$ cm, $CD - 2.8$ cm and $DA - 1.9$ cm.
2. Construct a trapezium $PQRS$ in which $PQ \mid SR, PQ - 6$ cm, $RS = 3$ cm, PS = 3 cm and $QR - 5$ cm.
3. Construct a trapezium $ABCD$ in which $AB \mid CD$, $AB - 8$ cm, $BC - 6$ cm, $CD = 4$ cm and $\angle B = 75°$.
4. Construct a trapezium $ABCD$ in which $AB \quad CD$, $AB - 4$ cm, $BC = 4$ cm, $AD = 5$ cm and $\angle B = 90°$.

IMPORTANT POINTS

1. Sum of all the angles of a quadrilateral is 360°.
2. A diagonal of a parallelogram divides it into two congruent triangles.
3. In a parallelogram :

   (i) opposite angles are equal.

   (ii) opposite sides are equal.

   (iii) diagonals bisect each other.
4. Any quadrilateral is a parallelogram, if :

   (i) its opposite angles are equal,

   (ii) its opposite sides are equal.

   (iii) its diagonals bisect each other.

   (iv) a pair of opposite sides is equal and parallel.
5. Diagonals of a rectangle bisect each other and are equal and vice-versa.
6. Diagonals of a rhombus, bisect each other at right angles and vice-versa.
7. Diagonals of a square bisect each other at right angles and are equal and vice-versa.
8. A line segment joining mid-points of two sides of a triangle, is parallel to third side and half of it.
9. A line passing through the mid-point of one side of a triangle side and parallel to another side, bisects the third side.
10. A quadrilateral, obtained by joining the mid-points of the sides of a quadrilateral, in a order, is a parallelogram.
11. It is necessary for the construction of a quadrilateral:

    (i) four sides and a diagonal are given.

    (ii) four sides and an angle are given.

    (iii) three sides and two diagonals are given,

(iv) three sides and angle between them are given.

(v) two adjecent sides and angle between them and other two angles are given.

12. It is necessary for the construction of a parallelogram :

(i) two adjecent sides and a diagonal are given.

(ii) one side and two diagonals are given.

(iii) two adjecent sides and two angle between them are given.

13. It is necessary for the construction of a rectangle :

(i) two adjecent sides are given.

(ii) a side and a diagonal are given.

14. For construction of square, it is necessary that:

(i) a side is given.

(ii) diagonal is given.

15. For the construction of rhombus, it is necessary that :

(i) measure of a side and an angle between two adjecent sides,

(ii) diagonals are given.

16. For the construction of a trapezium, it is necessary that :

(i) four sides are given and parallel sides are known.

(ii) three sides and an angle are given and parallel sides are known.

17. Opposite sides of a parallelogram are equal and parallel and opposite angles a equal.

18. Four sides of a square are equal and each angle is right angle. Diagonals of a square are equal and bisect each other at right angles.

19. Opposite sides of a rectangle are equal and each angle is right angle.

20. In a rhombus, four sides are equal and opposite angles are equal. Diagonals bisect each other at right angles.

21. In a trapezium, only one pair of opposite sides be parallel.

Miscellaneous Exercise-9

Write the correct answers of each of the following questions (From question 1 to 15)

1. Three angles of a quadrilateral are 75°, 90° and 75°. Its fourth angle is :

(a) 90° (b)95° (c) 105° (d) 120°

2. A diagonal of rectangle is inclined at an angle of 25° with a side. The acute angle between its diagonals is :

(a) 55° (b) 50° (c) 40° (d) 25°

3. $ABCD$ is a rhombus in which $\angle ACB = 40°$. Then $\angle ADB$ is :

(a) 40° (b) 45° (c) 50° (d) 60°

4. A quadrilateral, formed by joining the mid-points of the sides of a quadrilateral $PQRS$, in a order, is a rectangle if :

(a) $PQRS$ is a rectangle

(b) $PQRS$ is a parallelogram

(c) Diagonals of $PQRS$ are perpendicular to each other.

(d) Diagonals $of\ PQRS$ are equal.

5. A quadrilateral, formed by joining the mid-points of the sides of a quadrilateral $PQRS$, in a order, is a rhombus if:

(a) $PQRS\ is\ a$ rhombus

(b) $PQRS$ is a parallelogram

(c) Diagonals $of\ PQRS$ are perpendicular to each other

(d) Diagonals $of\ PQRS$ are equal.

6. If the ratio of angles A, B, C and D of a quadrilateral ABCD, taking in this order is 3 : 7:6:4. Then $ABCD$ is a :

(a) rhombus (b) parallelogram (c) trapezium (d) kite

7. In a quadrilateral $ABCD$, bisectors of $\angle A$ and $\angle B$, intersect each other at P. bisectors of $\angle B$ and $\angle C$ at Q, bisectors of $\angle$C and $\angle$ D at R, and bisectors of $\angle$ D and $\angle A$ intersect each other at S. Then $PQRS$ is a :

(a) rectangle (b) rhombus (c) parallelogram

(d) quadrilateral whose opposite angles are supplementry.

8. If AP and CQD are two parallel lines, then bisectors of $\angle APQ$, $\angle BPQ$, $\angle CQP$ and $\angle$ PQD make:

(a) a square (b) a rhombus (c) a rectangle (d) any other parallelogram

9. Joining the mid-points of the sides of rhombus, in a order, obtained a figure is :

(a) a rhombus (b) a rectangle (c) a square (d) any parallelogram

10. D and E are mid points of side AB and AC respectively of a ΔABC and O is any point on side BC. O is joined with A. If P and Q are mid points of OB and OC respectively, then $DEQP$ is a :

(a) square (b) rectangle (c) rhombus (d) parallelogram

11. The figure obtained by joining the mid-points of the sides of a quadrilateral ABCD in order, is only a square if:

(a) $ABCD$ is a rhombus

(b) Diagonals of $ABCD$ are equal

(c) Diagonals of $ABCD$ are equal and perpendicular to each other

(d) Diagonals of $ABCD$ are perpendicular to each other

12. Diagonals AC and BD of a parallelogram $ABCD$ intersect each other at point O. If $\angle DAC = 32°$ and $\angle AOB = 70°$, then $\angle$DBC is :

(a) 24° (b) 86° (c) 38° (d) 32°

13. Which of the following statements is not true for a parallelogram :

(a) Opposite sides are equal

(b) Opposite angles are equal

(c) Opposite angles are bisected by diagoanls

(d) Diagonals bisect each other.

14. D and E are mid points of sides AB and AC respectively of a ΔABC. DE is extended upto F. To prove that CF is equal to the line segment DA and parallel, we are required an other information, which is :

(a) $\angle DAE = \angle EFC$ (b) $AE = EF$

(c) $DE = EF$ (d) $\angle ADE = \angle ECF$

15. Diagonals of a parallelogram $ABCD$ intersect at point O. If $\angle$ BOC = 90° and $\angle BDC = 50°$, then $\angle OAB$ is :

(a) 90° (b)50° (c)40° (d) 10°

16. $ABCD$ *is* a parallelogram. If its diagonals are equal, then find the value of $\angle ABC$.

17. Diagonals of a rhombus are equal and perpendicular to each other. Is this statement true? Given reason for your answer.

18. Three angles of a quadrilateral $ABCD$ are equal. Is this a parallelogram?

19. In quadrilateral $ABCD$, $\angle A + \angle D = 180°$. What specific name can be given to this quadrilateral?

20. All angles of a quadrilateral are equal. What specific name is given to this quadrilateral?

21. Diagonals of rectangle are equal and perpendicular to each other. Is this statement true? Give reason for your answer.

22. Any square, inside an isosceles right-angled triangle is such that one angle is common in both square and triangle. Show that the vertex of the square, which opposite to the vertex of the common angle, bisects the hypotenuse.

23. In a parallelogram $ABCD$, AB = 10 cm and AD = 6 cm. Bisector of $\angle A$ meets DC at E and producing AE and BC meet at F. Find the length of CF.

24. P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively in which $AC = BD$ and $AC \perp BD$. Prove that $PQRS$ is a square.

25. A diagonal of a parallelogram, bisects one of its angle. Prove that this parallelogram is a rhombus.

26. $ABCD$ is a quadrilateral in which $AB \mid DC$ and $AD = BC$. Prove that $\angle A = \angle B$ and $\angle C = \angle D$.

27. E is the mid point of median AD of ΔABC. BE is extended to meet AC at F. Show that

$$AF = \frac{1}{3} AC.$$

28. Show that a quadrilateral formed by joining the mid points of the consecutive side of a square is also a square.

29. Prove that a quadrilateral formed by the bisectors of the angles of a parallelogram is a rectangle.

30. P and Q are two points on opposite sides AD and BC of parallelogram $ABCD$ such that diagonal PQ passes through O, the point of intersection of diagonals AC and BD. Prove that PQ is bisected at point O.

31. $ABCD$ *is a* rectangle whose diagonal BD bisects $\angle B$. Show that $ABCD$ *is* a square.

32. D, E and F are respectively the mid-points of sides AB, BC and CA of a ΔABC. Prove that by joining the points D, E and F triangle ABC divided into four congruent triangles.

33. Prove that the line joining the mid-points of diagonals of a trapezium, is parallel to the parallel sides of that trapezium.

34. P is the mid point of side CD of a parallelogram $ABCD$. A line drawn passing through C and parallel to PA meets AB *at* Q and extended DA at R. Prove that $DA - AR$ and $CQ - QR$.

35. Construct a quadrilateral $ABCD$ in which $AB - 3.7$ cm, $BC - 3$ cm, $CD - 5$ cm, AD = 4 cm and $\angle A = 90°$.

36. Construct a quadrilateral $ABCD$ in which $AB - AD - 3.2$ cm, $BC - 2.5$ cm, $AC - 4$ cm and $BD = 5$ cm.

37. Construct a quadrilateral $PQRS$ in which PQ = 3.5 cm, $QR = 3.5$ cm, $\angle P = 60°$, $\angle Q = 105^0$ and $\angle S = 75°$.

38. Construct a rhombus whose one side is 3.6 cm and one angle is $60°$.

39. Construct a square in which $AB + BC + CD - DA - 12.8$ cm.

40. Construct a trapezium in which $AB \mid CD$, $AB - 5$ cm, $BC - 3$ cm, AD = 3.3 cm and distance between parallel sides is 2.5 cm.

41. Construct a rhombus $ABCD$ in which $AB = 6$ cm and $\angle A = 120°$.

42. Construct a trapezium where $AB - 2.3$ cm, $BC - 3.4$ cm, $CD = 5.4$ cm, $DA - 3.7$ cm and AB || CD.

43. Construct a rhombus $ABCD$ whose diagonals are 5.6 cm and 7.2 cm.

44. Construct a rectangle $ABCD$ in which $AB - 4.5$ cm and $BD = 6$ cm.

Answers

EXERCISE 9.1

1. 36°, 60°, 108°, 156°
2. AC = 6 cm, BD = 4 cm
3. No, diagonals of a parallelogram bisects each other.
4. No, sum of the angles of a quadrilateral should be 360°.
5. No, sum of angles will be greater than 360° which is not possible for a quadrilateral.
6. 84°
7. Each angle is 135°
8. 120°, 60°, 120°, 60°

EXERCISE 9.2

1. 55°
2. No, sum of all angles will be less than 360°.
3. Yes, that will be rectangle or square.
4. 145°
5. 4 cm
6. 60, 120, 60, 120
13. (i) 6.9 cm (ii) 9.9 cm

Miscellaneous Exercise 9

1. D	2. B	3. C	4. D
5. C	6. B	7. D	8. C
9. B	10. D	11. C	12. C
13. C	14. C	15. C	

16. 90°, this quadrilateral is a rectangle.
17. This statement is not true because diagoanls of a rhombus are perpendicular to each other but they are not equal.
18. It is not necessary to be parallelogram.
19. Parallelogram.
20. Rectangle or square
21. No, diagonals of rectangle are equal but not perpendicular to each other.
23. 4 cm