26

Wave Optics

IN THIS CHAPTER ....

Ray optics uses the geometry of straight lines to explain the phenomenon of rectilinear propagation of light, reflection and refraction of light, etc. However, wave optics deals light on the basis of waves, where light can bend around objects, can diffract and interfere, etc.

Huygens' Wave Theory

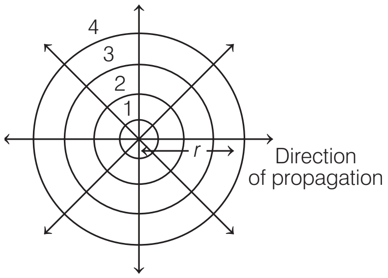
In 1678, a Dutch scientist, Christian Huygen propounded the wave theory of light. He assumed that, light travels in the form of waves. These waves travel in all the directions with the velocity of light. He also introduced the concept of wavefront.

Wavefront

It is the locus of points (wavelets) having same phase (a surface of constant phase) of oscillations. As the wave travels forward, the wavefront also moves forward. The perpendicular to the surface of a wavefront gives the direction of light ray. The energy transfers along the direction of rays.

Types of Wavefronts

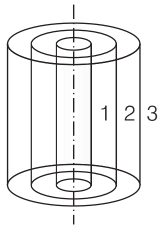
- **Light emerging from a point source** Wavefronts are spherical and eccentric with point source at their centre as shown in figure by circles 1, 2, 3 and 4. Rays are radial as shown in arrows.



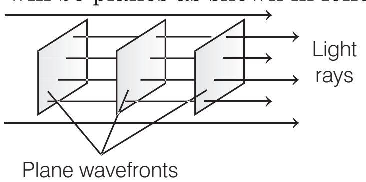
In spherical wavefront,

$$\text{amplitude} \propto \frac{1}{r} \quad \text{and} \quad \text{intensity} \propto \frac{1}{r^2}$$

- **Light coming out from a line source** Wavefronts 1, 2 and 3 are cylindrical and coaxial with the straight source as their common axis.



- **When light sources are emitting parallel rays or when the light is coming from a very far-off source** Wavefronts will be planes as shown in following figure.



Note Wavefronts parallel to XY-plane can be represented by Z = constant. Similarly, wavefronts parallel to YZ-plane and ZX-plane are X = constant and Y = constant, respectively.

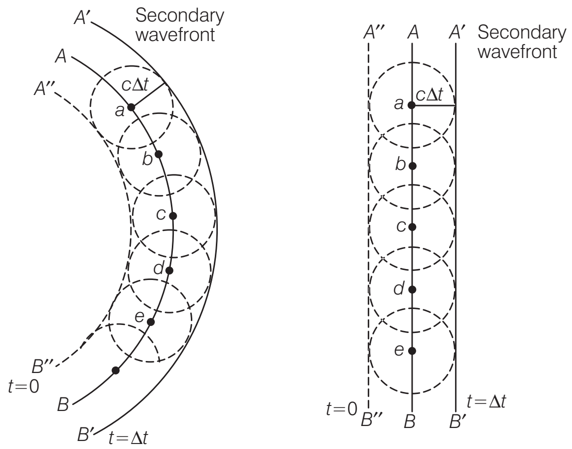
Huygens' Principle of Secondary Wavelets

In 1678, in order to explain the propagation of wave in a medium, Huygen propounded a principle known as Huygen's principle of secondary wavelets by which at any instant, we can geometrically obtain the position of wavefront. He made following three assumptions:

(i) Every point on a given wavefront (called **primary wavefront**) can be regarded as fresh source of new disturbance and sends out its own spherical wavelets called **secondary wavelets**.

(ii) The secondary wavelets spread in all directions with the velocity of wave (*i.e.* velocity of light).

(iii) A surface touching these secondary wavelets, tangentially in the forward direction at any instant gives the position and shape of the new wavefront at the instant. This is called **secondary wavefront**.



(a) Spherical wavefront (b) Plane wavefront

Principle of Superposition of Waves

According to superposition principle, at a particular point in the medium, the resultant displacement y produced by a number of waves is the vector sum of the displacements $(y_1, y_2, \ldots)$ produced by each of the waves.

i.e. $y = y_1 + y_2 + y_3 + y_4 + \ldots$

Note The superposition principle which was stated first for mechanical waves is equally applicable to the electromagnetic (light) waves.

Coherent and Incoherent Sources

Two sources are said to be **coherent**, if they have the same frequency and have constant phase difference with time.

However, two such sources of light, which do not emit light waves with constant phase difference are called **incoherent sources.**

Interference of Light

When two light waves of exactly equal frequency having phase difference which is constant with respect to time travelled in same direction and overlap each other, then at some points, the intensity of light is maximum, while at some points, the intensity of light is minimum.

Thus, formation of maximum intensity at some points and minimum intensity at some other points by the two identical light waves travelling in same direction is called the *interference of light*.

The interference at the points where the two waves meet in same phase, *i.e.* the intensity of light is maximum, is called the *constructive interference* while at the points where the two waves meet in opposite phase, *i.e.* the intensity of light is minimum is called the *destructive interference*.

Conditions of Maxima and Minima

For two waves from sources S_1 and S_2 interfering at a certain point such that $y_1 = A_1 \sin \omega t$ and $y_2 = A_2 \sin(\omega t + \phi)$

where, A_1 and A_2 are the respective amplitudes of the two waves and ϕ is the constant phase angle by which second wave leads the first wave.

By the principle of superposition, the magnitude of resultant displacement of the waves is

$$y = A \sin(\omega t + \theta)$$

where, $$A = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos \phi}$$

and $$\theta = \tan^{-1}\left[\frac{A_2 \sin \phi}{A_1 + A_2 \cos \phi}\right]$$

As, resultant intensity I is $I \propto A^2$

$$\Rightarrow \quad I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

For constructive interference (maximum intensity) The intensity I is maximum, when $\cos \phi = +1$, *i.e.* when phase difference $\phi = 2n\pi$; $n = 0, 1, 2, \ldots$, etc.

If Δx is the path difference between the interfering waves, then $\Delta x = n\lambda$.

So, $\quad I_{\max} = A_1^2 + A_2^2 + 2A_1A_2 = (A_1 + A_2)^2$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$$

When $\quad I_1 = I_2 = I_0$, then resultant intensity,

$$I_{\max} = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

For destructive interference (minimum intensity)
The intensity I is minimum, when $\cos\phi = -1$, *i.e.* when phase difference,

$$\phi = (2n - 1)\pi;\ n = 1, 2, \ldots, \text{etc.}$$

Here, path difference, $\Delta x = (2n - 1)\dfrac{\lambda}{2}$

So, $\quad I_{\min} = A_1^2 + A_2^2 - 2A_1A_2 = (A_1 - A_2)^2$

$$= I_1 + I_2 - 2\sqrt{I_1 I_2} = (\sqrt{I_1} - \sqrt{I_2})^2$$

When $\quad I_1 = I_2 = I_0$, then resultant intensity,

$$I_{\min} = 0$$

$\therefore$ We can write,

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2$$

Note Relation between path difference, phase difference and time difference

$$\frac{2\pi}{\text{Phase diff.}} = \frac{\lambda}{\text{Path diff.}} = \frac{T}{\text{Time diff.}}$$

Example 1. *Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is* 16. *The intensity of the waves are in the ratio* ***[JEE Main 2019]***

(a) 16 : 9 (b) 5 : 3
(c) 25 : 9 (d) 4 : 1

Sol. (c) Let the intensity of two coherent sources be I_1 and I_2, respectively.

It is given that, $\dfrac{\text{maximum intensity}}{\text{minimum intensity}} = \dfrac{I_{\max}}{I_{\min}} = \dfrac{16}{1}$

Since, we know, $\quad I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$

and $\quad I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$

$\therefore$ We can write, $\dfrac{I_{\max}}{I_{\min}} = \dfrac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = 16$

$$\Rightarrow \quad \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = \frac{\sqrt{16}}{1} = \frac{4}{1}$$

$$\sqrt{I_1} + \sqrt{I_2} = 4\sqrt{I_1} - 4\sqrt{I_2}$$

$$\Rightarrow \quad 5\sqrt{I_2} = 3\sqrt{I_1}\ , \frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{5}{3}$$

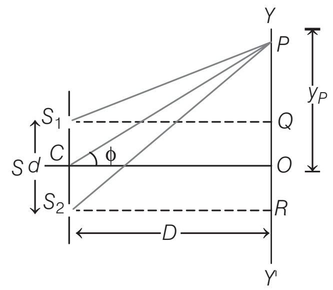
Squaring both the sides, we get $\dfrac{I_1}{I_2} = \dfrac{25}{9}$

Young's Double Slit Experiment

Young's experimental arrangement for double slit experiment consists of two monochromatic narrow slits S_1 and S_2 which are produced by light source S_1 as shown in the figure given below.

These S_1 and S_2 serves as the section of two coherent sources of light. A screen is placed in the path of light emerging out of the slits S_1 and S_2 on which alternate coloured fringes are observed. When the distance between the two slits was increased, it was observed that the bands or fringes disappeared. Therefore, an optimum distance between them is required to observe the well defined fringes pattern.

In this pattern, we get alternate dark (minima's) and bright (maxima's) fringes.



For light waves reaching at a point P, situated at a distance x from central point O, the path difference Δy is given by

$$\Delta y = S_2P - S_1P = \frac{xd}{D}$$

If $\dfrac{xd}{D} = n\lambda$, then we get nth bright fringe, where $n = 1, 2, 3, 4 \ldots$.

Similarly, if $\dfrac{xd}{D} = (2n - 1)\dfrac{\lambda}{2}$, then we get nth dark fringe, where $n = 1, 2, 3, 4 \ldots$.

Fringe Width

The separation between any two consecutive bright or dark fringes is called *fringe width* (β) and it is given by

$$\beta = \frac{D\lambda}{d}$$

where, λ = wavelength of wave,
D = distance between slit and screen
and $\quad d$ = distance between two slits.
or $\quad \beta \propto \lambda$

Important points related with fringe width are given below

- If YDSE apparatus is immersed in a liquid of refractive index μ, then wavelength of light decreases μ times and hence, fringe width decreases μ times.
- If white light is used in place of a monochromatic light, then coloured fringes are obtained on the screen with red fringes of larger size than that of violet because $\lambda_{\text{red}} > \lambda_{\text{violet}}$.

But note that centre is still white because path difference there is zero for all colours. Hence, all the wavelengths interfere constructively.

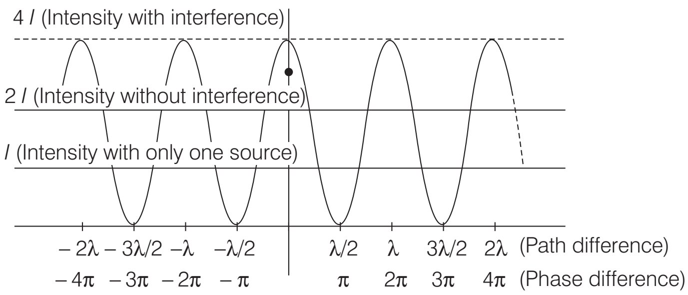
Angular width of a fringe

It is given as, $\theta = \theta_{n+1} - \theta_n = \dfrac{\lambda}{d}$ or $\theta = \dfrac{\lambda}{d} = \dfrac{\beta}{D}$

It is independent of n, *i.e.* angular widths of all fringes are same.

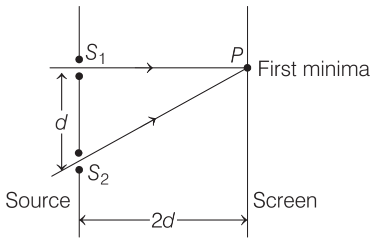
Distribution of intensity

The distribution of intensity in Young's double slit experiment is shown below.



Distribution of intensity in YDSE

Example 2. *Consider a Young's double slit experiment as shown in figure.*

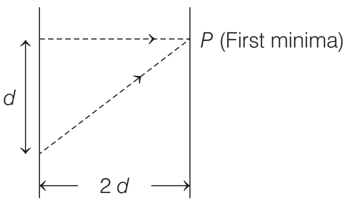


What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit S_1? **[JEE Main 2019]**

(a) $\dfrac{\lambda}{2(5-\sqrt{2})}$ (b) $\dfrac{\lambda}{(5-\sqrt{2})}$

(c) $\dfrac{\lambda}{2(\sqrt{5}-2)}$ (d) $\dfrac{\lambda}{(\sqrt{5}-2)}$

Sol. (c) In the given case, figure for first minima will be as shown below.



We know that condition for minima in Young's double slit experiment, path difference is

$$\Delta x = (2n-1)\,\lambda/2$$

For first minima, $n = 1$

$$\Rightarrow \quad \Delta x = \lambda/2 \qquad \text{... (i)}$$

Path difference between the rays coming from virtual sources S_1 and S_2 at point P will be

$$\Delta x = S_2P - S_1P \qquad \text{... (ii)}$$

From triangle S_1S_2P,

$$S_1P = 2d \qquad \text{... (iii)}$$

and $(S_2P)^2 = (S_1S_2)^2 + (S_1P)^2$ (from Pythagoras theorem)

$$= d^2 + (2d)^2$$

$\Rightarrow$ $(S_2P)^2 = 5d^2$

or $S_2P = \sqrt{5}\,d$ ...(iv)

Substituting the values from Eqs. (iii) and (iv) in Eq. (ii), we get

$$\Delta x = \sqrt{5}\,d - 2d \qquad \text{...(v)}$$

From Eqs. (i) and (v), we get

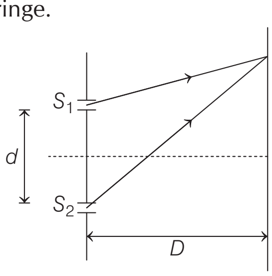
$$\sqrt{5}\,d - 2d = \lambda/2$$

$$\Rightarrow \quad d = \frac{\lambda}{2(\sqrt{5}-2)}$$

Example 3. *In a Young's double slit experiment, the separation between the slits is 0.15 mm. In the experiment, a source of light of wavelength 589 nm is used and the interference pattern is observed on a screen kept 1.5 m away. The separation between the successive bright fringes on the screen is* **[JEE Main 2020]**

(a) 4.9 mm (b) 3.9 mm (c) 6.9 mm (d) 5.9 mm

Sol. (d) In YDSE, separation between successive bright fringes is width of one dark fringe.



As, fringe width in YDSE, $\beta = \dfrac{\lambda D}{d}$ ...(i)

Here, $\lambda = 589\text{ nm} = 589 \times 10^{-9}$ m,

$D = 1.5$ m, $d = 0.15\text{ mm} = 0.15 \times 10^{-3}$ m

Substituting the given values in Eq. (i), we get

$$\beta = \frac{589 \times 10^{-9} \times 1.5}{0.15 \times 10^{-3}} = 589 \times 10^{-5}\text{ m}$$

$$= 5.89\text{ mm or } 5.9\text{ mm}$$

Example 4. *In a YDSE, if $D = 2$ m, $d = 6$ mm, $\lambda = 6000$Å, then the fringe width and the position of the 3rd maxima are*

(a) 0.2 mm, 0.6 mm (b) 0.8 mm, 0.1 mm

(c) 1.2 mm, 0.2 mm (d) None of these

Sol. (a) We know that,

$$\text{Fringe width, } \beta = \frac{\lambda D}{d} = \frac{6000 \times 10^{-10} \times 2}{6 \times 10^{-3}} = 0.2\text{ mm}$$

$$\text{Position of 3rd maxima, } y_3 = \frac{3\lambda D}{d} = 3\beta = 3 \times 0.2 = 0.6\text{ mm}$$

Example 5. *In Young's double slit experiment interference fringes 1° apart are produced on the screen, the slit separation is*

(a) 0.546 *mm* (b) 0.0337 *mm*
(c) 0.246 *mm* (d) 0.0927 *mm*

Sol. (*b*) The fringe width, $\beta = \dfrac{\lambda D}{d}$

The angular separation of the fringes is given by

$$\theta = \frac{\beta}{D} = \frac{\lambda}{d}$$

Given, $\theta = 1° = \dfrac{\pi}{180}$ rad

$\lambda = 589$ nm

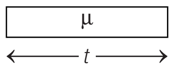
$\therefore \quad d = \dfrac{\lambda}{\theta} = \dfrac{589 \times 180 \times 10^{-9}}{\pi}$

$= 0.0337$ mm

Insertion of Transparent Slab in YDSE

If a thin transparent plate (or slab) of thickness t and refractive index μ is placed in the path of one of interfering waves in YDSE, then fringe pattern shifts. First we will discuss the optical path, path difference due to the slab and then shifting in pattern as follows:

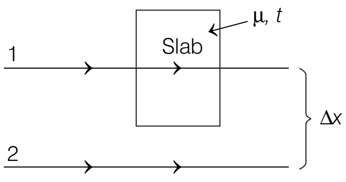
(i) **Optical path of a slab** If light travels a distance t in a slab (medium) of refractive index μ in a given time t_0, then in this same time, it travels a distance μt in vacuum.



Optical path of a slab

This distance μt is called optical path of the medium in air corresponding to distance t in the slab (medium).

(ii) **Path difference produced by a slab** For two light waves 1 and 2 moving in air parallel to each other, if a slab of refractive index μ and thickness t is inserted between the path of one of the rays, then a path difference



Path difference between rays 1 and 2

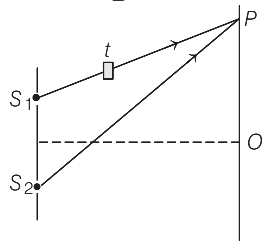
$$\Delta x = (\mu - 1)\, t$$

is produced among them.

(iii) **Displacement of fringes** If a thin transparent plate of thickness t and refractive index μ is placed in the path of one of interfering waves (say in path S_1P), then effective path in air is increased by an amount $(\mu - 1)\, t$ due to introduction of plate.

Effective path difference in air is

$$\Delta x = \frac{yd}{D} - (\mu - 1)t$$



$\therefore \quad \text{Shift} = \dfrac{(\mu - 1)\, t\, D}{d}$

Following three points are important with regard to above equation

(a) Shift is independent of n, (the order of the fringe), *i.e.* shift of zero order maximum = shift of 7th order maximum or shift of 5th order maximum = shift of 9th order minimum and so on.

(b) Shift is independent of λ, *i.e.* if white light is used, then shift of red colour fringe = shift of violet colour fringe.

(c) Number of fringes shifted $= \dfrac{\text{Shift}}{\text{Fringe width}}$

$$= \frac{(\mu - 1)\, t\, D/d}{\lambda D/d}$$

$$= \frac{(\mu - 1)t}{\lambda}$$

Example 6. *In a double slit experiment, when a thin film of thickness t having refractive index μ is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used)* ***[JEE Main 2019]***

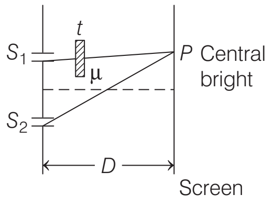
(a) $\dfrac{2\lambda}{(\mu - 1)}$ (b) $\dfrac{\lambda}{2(\mu - 1)}$

(c) $\dfrac{\lambda}{(\mu - 1)}$ (d) $\dfrac{\lambda}{(2\mu - 1)}$

Sol. (c) As we know,

Path difference introduced by thin film,

$$\Delta x = (\mu - 1)\, t \quad \text{...(i)}$$



and if fringe pattern shifts by one fringe width, then path difference,

$$\Delta x = 1 \times \lambda = \lambda \quad \text{...(ii)}$$

So, from Eqs. (i) and (ii), we get

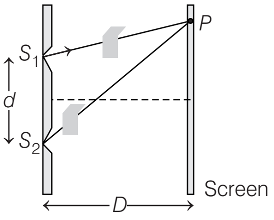
$$(\mu - 1)\, t = \lambda \Rightarrow t = \frac{\lambda}{\mu - 1}$$

Example 7. *In a double slit arrangement fringes are produced using light of wavelength 4800Å. One slit is covered by a thin plate of glass of refractive index 1.4 and the other with another glass plate of same thickness but of refractive index 1.7. By doing so, the central bright shifts to original 4th bright fringe from centre. What is the thickness of the glass plate?*

(a) 6.8 m (b) 9.2 m (c) 6.4 μm (d) 9.2 μm

Sol. (c) As, we know that, shift due to insertion of a slab is

$$\Delta x = \frac{\beta}{\lambda}(\mu - 1)t$$



Shift due to one plate, $\Delta x_1 = \frac{\beta}{\lambda}(\mu_1 - 1)t$

Shift due to another path, $\Delta x_2 = \frac{\beta}{\lambda}(\mu_2 - 1)t$

Net shift, $\Delta x = \Delta x_2 - \Delta x_1 = \frac{\beta}{\lambda}(\mu_2 - \mu_1)t$ ...(i)

Here, it is given that, $\Delta x = 4\beta$ ...(ii)

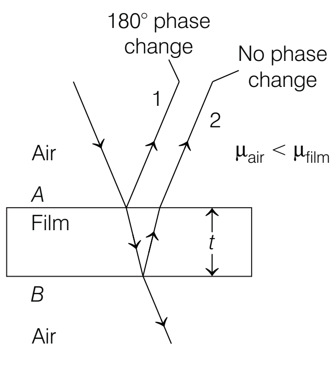
Hence, $4\beta = \frac{\beta}{\lambda}(\mu_2 - \mu_1)t$

$$\Rightarrow \quad t = \frac{4\lambda}{(\mu_2 - \mu_1)} = \frac{4 \times 4800 \times 10^{-10}}{(1.7 - 1.4)}$$

$$= 6.4 \times 10^{-6}\,\text{m} = 6.4\,\mu\text{m}$$

Interference in thin Films

Interference effects are commonly observed in thin films, such as thin layers of oil on water or the thin surface of a soap bubble. The varied colours observed when white light is incident on such films as the result of the interference of waves reflected from the two surfaces of the film. A film of uniform thickness t and index of refraction μ, is shown in the following figure.



Interference in light reflected from a thin film is due to a combination of rays reflected from the upper and lower surfaces of the film.

If a wave is reflected from a denser medium, then it undergoes a phase change of 180°. The path difference between the two rays 1 and 2 is $2t$, while the phase difference between them is 180°. Hence, condition of constructive interference will be

$$2\mu t = \left(n - \frac{1}{2}\right)\lambda \qquad \left(\because \lambda_\mu = \frac{\lambda}{\mu}\right)$$

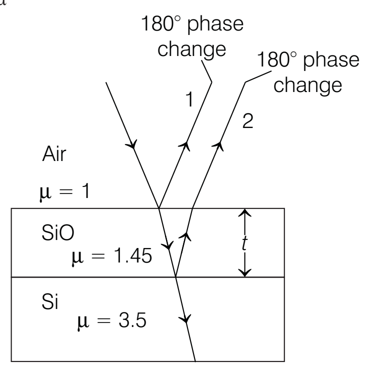
Similarly, condition of destructive interference will be

$$2\mu t = n\lambda \qquad (\text{where, } n = 0, 1, 2, \ldots)$$

Example 8. *In solar cells, a silicon solar cell ($\mu = 3.5$) is coated with a thin film of silicon monoxide SiO ($\mu = 1.45$) to minimise reflective losses from the surface. What is the minimum thickness of SiO that produces the least reflection at a wavelength of 550 nm, near the centre of the visible spectrum?*

(a) 94.8 nm (b) 6.28 nm (c) 9.26 m (d) 8.26 μm

Sol. (a) The reflected light is a minimum when rays 1 and 2 (shown in figure) meet the condition of destructive interference. Both rays undergo a 180° phase change upon reflection. The net change in phase due to reflection is therefore zero and the condition for a reflection to be minimum requires a path difference of $\lambda_\mu/2$.



Hence, $2t = \frac{\lambda}{2\mu}$

Thus, the minimum thickness of silicon monoxide,

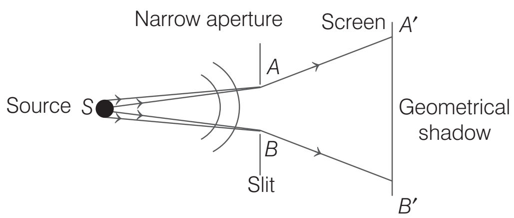
$$t = \frac{\lambda}{4\mu} = \frac{550}{4\,(1.45)} = 94.8\ \text{nm}$$

Fresnel's Biprism

It is a combination of two prisms of very small refracting angles placed base to base. It is used to obtain two coherent sources from a single light source.

Diffraction of Light

It is the phenomenon of bending of light around corners of an obstacle or aperture in the path of light. This bending light penetrates into the geometrical shadow of the obstacle. The light thus deviates from its linear path. This deviation is more effective when the dimensions of the aperture or the obstacle are comparable to the wavelength of light.



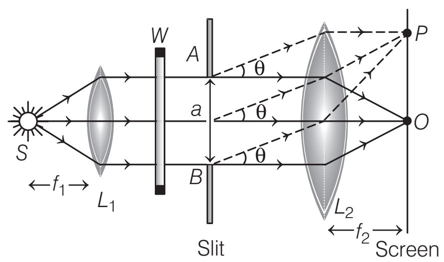
The phenomenon of diffraction is divided mainly in the following two classes

(a) Fresnel class (b) Fraunhoffer class

Fresnel class	Fraunhoffer class
The source is at a finite distance.	The source is at infinite distance.
No opticals are required.	Opticals in the form of collimating lens and focusing lens are required.
Fringes are not sharp and well defined.	Fringes are sharp and well defined.

Diffraction Due to Single Slit

Fraunhoffer's arrangement for studying diffraction at a single narrow slit (width = a) is shown in figure given below. Lenses, L_1 and L_2 are used to render incident light beam parallel and to focus parallel light beam.



Following are few important points related to diffraction.

- **nth order secondary minima** is obtained, when

$$a \sin\theta = n\lambda, \quad \text{where } n = 1, 2, 3, \dots.$$

- **nth order secondary maxima** is obtained, when

$$a \sin\theta = \left(n + \frac{1}{2}\right)\lambda, \text{ where } n = 0, 1, 2, 3, \dots.$$

- **Angular separation** of nth order minima,

$$\theta_n = \frac{n\lambda}{a}, \text{ where } n = 1, 2, 3, \dots.$$

- **Linear separation** of nth order secondary minima,

$$x_n = \frac{Dn\lambda}{a}$$

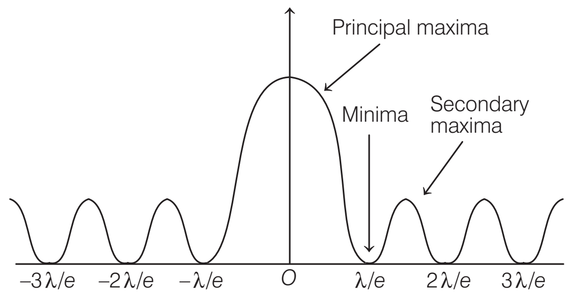
- **Angular position** of nth order secondary maxima,

$$\theta_n = \left(n + \frac{1}{2}\right)\frac{\lambda}{a}$$

- **Width of central maximum** $= 2y = \dfrac{2D\lambda}{a}$

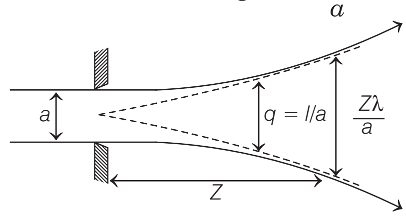
As the slit width a increases, width of central maximum decreases.

- **Angular width of central maxima,** $2\theta = \dfrac{2\lambda}{a}$.
- **Intensity of central (principal) maxima** is maximum and intensity of secondary maxima decreases with the increase of their order. The diffraction pattern is graphically shown below.



Fresnel's distance

When a slit or hole of size a is illuminated by a parallel beam, it is diffracted into an angle of $\approx \dfrac{\lambda}{a}$.



Diffraction of a parallel beam

In travelling a distance Z, size of beam is $Z\lambda / a$.

$$\Rightarrow \qquad Z \geq \frac{a^2}{\lambda}$$

Now, distance Z_F is called *Fresnel's distance,*

$$Z_F = a^2 / \lambda$$

So, image formation can be explained by ray optics for distance less than Z_F.

Example 9. *In a single slit diffraction experiment, first minima for $\lambda_1 = 660$ nm coincides with first maxima for wavelength λ_2, then λ_2 will be equal to*

(a) 240 *nm* (b) 345 *nm*
(c) 440 *nm* (d) 330 *nm*

Sol. (c) Given, $\lambda = 660$ nm

Position of minima in diffraction pattern is given by

$$a \sin\theta = n\lambda$$

For first minima ($n = 1$) of wavelength λ_1, we get

$$a \sin\theta_1 = 1\lambda_1 \text{ or } \sin\theta_1 = \frac{\lambda_1}{a}$$

For the first maxima approximately of wavelength λ_2,

$$a \sin\theta_2 = \frac{3}{2}\lambda_2, \ \sin\theta_2 = \frac{3\lambda_2}{2a}$$

The two will be considered if,

$$\theta_1 = \theta_2 \quad \text{or} \quad \sin\theta_1 = \sin\theta_2$$

$$\frac{\lambda_1}{a} = \frac{3\lambda_2}{2a}$$

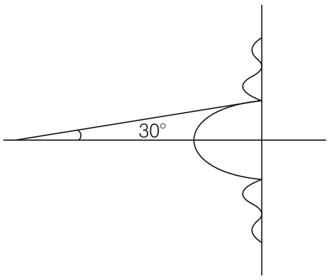
or $$\lambda_2 = \frac{2}{3}\lambda_1 = \frac{2}{3}\times 660 \text{ nm}$$

$$\lambda_2 = 440 \text{ nm}$$

Example 10. *The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance? (i.e. distance between the centres of each slit.)* **[JEE Main 2018]**

(a) 25 μm (b) 50 μm (c) 75 μm (d) 100 μm

Sol. (a) Angular width of diffraction pattern = 60°



For first minima,

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}, \text{ [here, } a = 10^{-6} \text{ m}, \theta = 30°]$$

$$\Rightarrow \quad \lambda = 10^{-6}\times\sin 30° \quad \Rightarrow \lambda = \frac{10^{-6}}{2}\text{ m}$$

Now, in case of interference caused by bringing second slit,

$$\therefore \text{ Fringe width, } \beta = \frac{\lambda D}{d}$$

$$\left[\text{here, } \lambda = \frac{10^{-6}}{2}\text{ m}, \beta = 1\text{ cm} = \frac{1}{100}\text{ m}, \; d = ? \text{ and } D = 50 \text{ cm} = \frac{50}{100}\text{ m}\right]$$

So, $$d = \frac{\lambda D}{\beta} = \frac{10^{-6}\times 50}{2\times\frac{1}{100}\times 100} = 25\times 10^{-6}\text{ m}$$

or $$d = 25\,\mu\text{m}$$

Resolving Power of Optical Instruments

It is the ability of the instrument to produce distinctly separate images of two close objects.

The least separation between two objects, so that they appear just separated is given by

$$d = \frac{\lambda}{2\mu\sin\theta}$$

where, μ is the refractive index of the medium between the objective of the microscope and the object. This distance is called **limit of resolution** of the microscope.

$$\therefore \text{ Resolving power of a microscope} = \frac{1}{d} = \frac{2\mu\sin\theta}{\lambda}$$

where, θ = half angle of the cone of light from the point object, μ sinθ = numerical aperture.

Similarly, the smallest angular separation between two objects, so that they appear just separated is found to be

$$d\theta = \frac{1.22\,\lambda}{D}$$

where, D is the diameter of objective.

$$\therefore \text{Resolving power of telescope} = \frac{1}{d\theta} = \frac{D}{1.22\,\lambda}$$

If d is the width of white strip (gap) which separates the two regions and D be the distance of screen from eye, then d/D is the limit of resolution of our eye.

Example 11. *A telescope is used to resolve two stars separated by 4.6×10^{-6} rad. If the wavelength of light is 5460Å, what should be the aperture of the objective of the telescope?*

(a) 0.0448 *m* (b) 0.1448 *m*

(c) 1.1448 *m* (d) 0.011 *m*

Sol. (b) Here, $d\theta = 4.6\times10^{-6}$ rad

$$\lambda = 5460\,\text{Å} = 5460\times10^{-10}\text{ m}$$

The aperture (D) of the telescope is given as

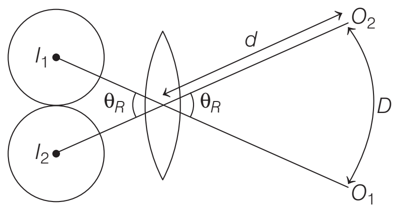
$$D = \frac{1.22\,\lambda}{d\theta} = \frac{1.22\times5460\times10^{-10}}{4.6\times10^{-6}} = 0.1448\text{ m}$$

Example 12. *The aperture of a telescope is 5m. The separation between the moon and the earth is 4×10^5 km. With light of wavelength of 5500 Å, the minimum separation between objects on the surface of moon, so that they are just resolved, is close to* **[JEE Main 2020]**

(a) 600 *m* (b) 20 *m* (c) 60 *m* (d) 200 *m*

Sol. (c) From Rayleigh's criteria, two objects will be just resolved when their angular separation, $\theta_R = 1.22\,\frac{\lambda}{a}$

where, λ = wavelength and a = aperture size.



Two objects are just resolved when their central maximas does not overlap.

From diagram, distance between objects in object plane,

$$D = \theta_R\cdot d = 1.22\,\frac{\lambda d}{a}$$

Here, $a = 5$ m, $d = 4 \times 10^5$ km $= 4 \times 10^8$ m,

$$\lambda = 5500 \text{ Å} = 5500 \times 10^{-10} \text{ m}$$

So, $$D = \frac{(1.22 \times 5500 \times 10^{-10} \times 4 \times 10^8)}{5} \approx 54 \text{ m}$$

From options, minimum separation distance is 60 m.

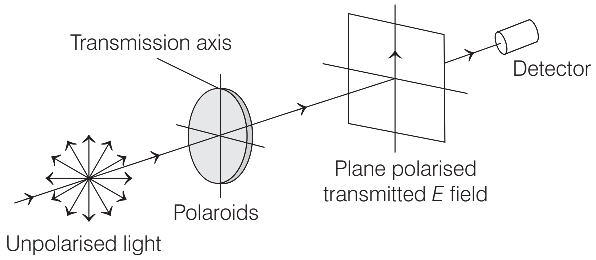
Polarisation

The phenomenon of restricting the vibrations of light (electric vector) in a particular direction, perpendicular to the direction of wave motion is called *polarisation of light*.The plane perpendicular to the plane of oscillation is called *plane of polarisation*.

Polaroids

These are thin and large sheets of crystalline polarising material (made artificially) capable of producing plane polarised beams of large cross-section.

A polaroid has a characteristic plane called *transmission plane*. When unpolarised light falls on a polaroid, only the vibrations parallel to the transmission plane get transmitted.



The crystal or polaroid on which unpolarised light is incident is called *polariser*. Crystal or polaroid on which polarised light is incident is called *analyser*.

Malus' Law

This law states that, when a beam of completely plane polarised light is incident on an analyser, the resultant intensity of light (I) transmitted from the analyser varies directly as the square of cosine of angle (θ) between plane of transmission of analyser and polariser.

i.e. $$I \propto \cos^2 \theta$$

If intensity of plane polarised light incidenting on analyser is I_0, then intensity of emerging light from analyser is $I_0 \cos^2 \theta$.

Brewster's Law

According to Brewster's law, when unpolarised light is incident at polarising angle (i_B) on an interface separating air from a medium of refractive index μ, then the reflected light is plane polarised (perpendicular to the plane of incidence), provided $\mu = \tan i_B$.

Here, the angle of incidence at which the reflected light is completely plane polarised is called **polarising angle** or **Brewster's angle** i_B.

At polarising angle, $$i_B + r = \frac{\pi}{2}$$

i.e. Reflected plane polarised light is at right angle to the refracted light.

Example 13. *When a polaroid sheet is rotated between two crossed polaroids, the intensity of the transmitted light will be maximum, when angle θ between pass axes is*

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{4}$

(c) $\frac{\pi}{4}$ (d) $\frac{2\pi}{3}$

Sol. (c) Let I_0 be the intensity of polarised light after passing through the first polariser P_1. Then the intensity of light after passing through second polariser P_2 will be

$$I = I_0 \cos^2 \theta$$

where, θ is the angle between pass axes of P_1 and P_2. Since, P_1 and P_3 are crossed the angle between the pass axes of P_2 and P_3 will be $\left(\frac{\pi}{2} - \theta\right)$. Hence, the intensity of light emerging from P_2 will be

$$I = I_0 \cos^2 \theta \cos^2 \left(\frac{\pi}{2} - \theta\right)$$

$$I = I_0 \cos^2 \theta \sin^2 \theta = \left(\frac{I_0}{4}\right) \sin^2 2\theta$$

Intensity of transmitted light is maximum,

when $\sin^2 2\theta = 1$

$\Rightarrow$ $\sin 2\theta = 1 = \sin \frac{\pi}{2}$

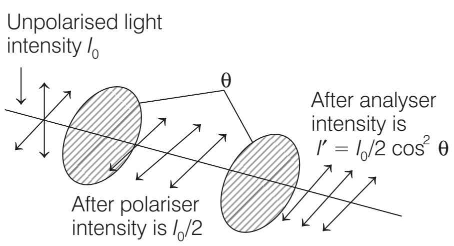
$\Rightarrow$ $2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$

Example 14. *A polariser-analyser set is adjusted such that the intensity of light coming out of the analyser is just 10% of the original intensity. Assuming that the polariser-analyser set does not absorb any light, the angle by which the analyser need to be rotated further to reduce the output intensity to be zero, is* **[JEE Main 2020]**

(a) 71.6° (b) 90°

(c) 45° (d) 18.4°

Sol. (d) A polariser-analyser set is as shown below.



By Malus' law,

Output intensity obtained from analyser, $I' = \frac{I_0}{2} \cos^2 \theta$ ...(i)

where, θ = angle between polariser and analyser.

Now, given $I' = 10\%$ of initial intensity

$$\Rightarrow \quad I' = \frac{1}{10} I_0 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\Rightarrow \quad \cos^2 \theta = \frac{1}{5}$$

$$\Rightarrow \quad \cos \theta = \frac{1}{\sqrt{5}}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right) \approx 63°$$

Now, output intensity is zero, when angle between polariser and analyser is 90°.

So, analyser must be rotated by $90° - 63° = 27°$ to reduce output intensity to zero.

From given options, we can see that output intensity is least when analyser rotation is 18.4°.

Example 15. *Light reflected from the surface of glass plate of refractive index 1.57 is linearly polarised. What is the angle of refraction of glass ?*

(a) 32.5° (b) 42.5° (c) 30.2° (d) 44.5°

Sol. (a) Given that, $\mu = 1.57$

According to Brewster's law,

$$\mu = \tan i_p = 1.57, i_p = \tan^{-1}(1.57) = 57.5°$$

As, $r = 90° - i_p$ or $r = 90° - 57.5° = 32.5°$

Doppler's Effect in Light

The phenomenon of apparent change in frequency (or wavelength) of the light due to the relative motion between the source of light and the observer is called Doppler's effect in light.

A stationary observer will receive ν wave per second. In case, the observer moves towards or away from the source of light with velocity v along the direction of propagation of light, then the number of waves received per second, *i.e. apparent frequency* of light will be

$$\nu' = \nu\left(1 \pm \frac{v}{c}\right)$$

Doppler shift is given as

$$\Delta\nu = \pm \frac{v}{c} \nu$$

The positive sign is considered, when the source and the observer approach each other and the negative sign considered is, when both source and observer move away from each other.

Doppler shift in terms of the wavelength of the light is given by

$$\Delta\lambda = \pm \frac{v}{c} \lambda$$

Example 16. *The spectral line for a given element in light received from a distant star is shifted towards the longer wavelength by 0.32%. Deduce the velocity of star in the line of sight.*

(a) 9.6×10^4 m/s (b) 11.6×10^{-4} m/s

(c) 12.6×10^{-4} m/s (d) None of the above

Sol. (a) Here, $\frac{\Delta\lambda}{\lambda} = \frac{0.032}{100}$ (position, as the shift is towards longer wavelength)

Now, Doppler shift is given by

$$\Delta\lambda = -\frac{v}{c}\lambda$$

$$v = -\frac{\Delta\lambda}{\lambda} c = -\frac{0.032}{100} \times 3 \times 10^8$$

$$= -9.6 \times 10^4 \text{ ms}^{-1}$$

The negative sign indicates that the star is receding.

Practice Exercise

ROUND I Topically Divided Problems

Wavefront and Huygens' Principle

1. For light diverging from a point source
(a) the wavefront is spherical
(b) the intensity increases in proportion to the distance squared
(c) the wavefront is parabolic
(d) the intensity at the wavefront does not depend on the distance

2. Consider a point at the focal point of a convergent lens. Another convergent lens of short focal length is placed on the other side. What is the nature of the wavefront emerging from the final image?
(a) Cylindrical (b) Elliptical
(c) Spherical (d) Square

3. Light waves travel in vacuum along the Y-axis. Which of the following may represent the wavefront?
(a) $y =$ constant (b) $x =$ constant
(c) $z =$ constant (d) $x + y + z =$ constant

4. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam
(a) become narrower **[JEE Main 2015]**
(b) goes horizontally without any deflection
(c) bends downwards
(d) bends upwards

Coherent and Incoherent Sources : Interference and YDSE

5. Two non-coherent sources emit light beams of intensities I and $4I$. The maximum and minimum intensities in the resulting beam are
(a) $9I$ and I (b) $9I$ and $3I$
(c) $5I$ and I (d) $5I$ and $3I$

6. Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point A and π at point B, then the difference between the resultant intensities at points A and B is
(a) $2I$ (b) $4I$
(c) $5I$ (d) $7I$

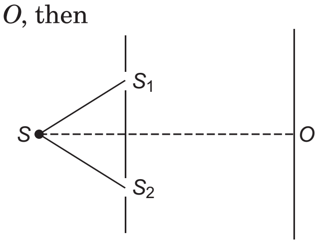
7. The maximum intensity in the case of n identical incoherent waves, each of intensity 2 Wm^{-2} is 32 Wm^{-2}. The value of n is
(a) 4 (b) 16 (c) 32 (d) 64

8. Three waves of equal frequency having amplitudes $10\mu m$, $4\mu m$, $7\mu m$ arrive at a given point with successive phase difference of $\frac{\pi}{2}$, the amplitude of the resulting wave (in μm) is given by
(a) 4 (b) 5 (c) 6 (d) 7

9. A parallel beam of light of intensity I_0 is incident on a glass plate, 25% of light is reflected by upper surface and 50% of light is reflected from lower surface. The ratio of maximum to minimum intensity in interference region of reflected rays is
(a) $\left(\dfrac{\frac{1}{2}+\sqrt{\frac{3}{8}}}{\frac{1}{2}-\sqrt{\frac{3}{8}}}\right)^2$ (b) $\left(\dfrac{\frac{1}{4}+\sqrt{\frac{3}{8}}}{\frac{1}{2}-\sqrt{\frac{3}{8}}}\right)^2$
(c) $\frac{5}{8}$ (d) $\frac{8}{5}$

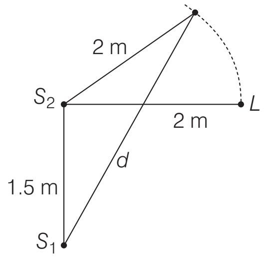
10. Two waves originating from sources S_1 and S_2 having zero phase difference and common wavelength λ will show complete destructive interference at a point P, if $(S_1P - S_2P) =$
(a) 5λ (b) $\frac{3\lambda}{4}$ (c) $\frac{4\lambda}{2}$ (d) $\frac{11\lambda}{2}$

11. In the setup shown in figure, the two slits S_1 and S_2 are not equidistant from the slit S. The central fringe at O, then



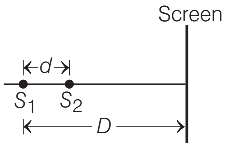
(a) always bright
(b) always dark
(c) Either dark or bright depending on the position of S
(d) Neither dark nor bright

12. Two coherent sources of sound S_1 and S_2, produce sound waves of the same wavelength $\lambda = 1$ m, in phase. S_1 and S_2 are placed 1.5 m apart (see figure). A listener, located at L, directly in front of S_2 finds that the intensity is at a minimum when he is 2 m away from S_2. The listener moves away from S_1, keeping his distance from S_2 fixed. The adjacent maximum of intensity is observed when the listener is at a distance d from S_1. Then, d is **[JEE Main 2020]**



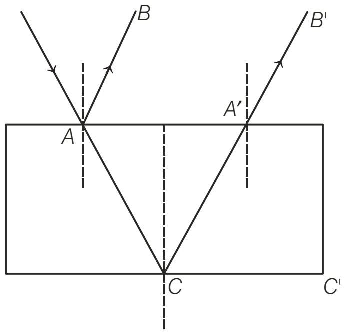
(a) 12 m (b) 5 m
(c) 2 m (d) 3 m

13. Two coherent point sources S_1 and S_2 are separated by a small distance d as shown. The fringes obtained on the screen will be **[JEE Main 2013]**



(a) points (b) straight lines
(c) semi-circle (d) concentric circles

14. A ray of light of intensity l is incident on a parallel glass slab at point A as shown in diagram. It undergoes partial reflection and refraction. At each reflection, 25% of incident energy is reflected. The rays AB and $A'B'$ undergo interference. The ratio of I_{max} and I_{min} is



(a) $49:1$ (b) $7:1$
(c) $4:1$ (d) $8:1$

15. In Young's experiment, the wavelength of red light is 7.8×10^{-5} cm and that of blue light is 5.2×10^{-5} cm. The value of n for which $(n+1)$th blue light band coincides with nth red bond is
(a) 4 (b) 2 (c) 3 (d) 1

16. In a Young's double slit experiment, the source is white light. One of the holes is covered by a red filter and another by a blue filter. In this case **[NCERT Exemplar]**
(a) there shall be alternate interference patterns of red and blue
(b) there shall be an interference pattern for red distinct from that for blue
(c) there shall be no interference fringes
(d) there shall be an interference pattern for red mixing with one for blue

17. In an experiment, the two slits are 0.5 mm apart and the fringes are observed to 100 cm from the plane of the slits. The distance of the 11th bright fringe from the 1st bright fringe is 9.72 mm. The wavelength is
(a) 4.86×10^{-5} cm
(b) 5.72×10^{-4} cm
(c) 5.87×10^{-4} cm
(d) 3.25×10^{-4} cm

18. In a Young's double slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 cm. Determine the wavelength of light used in the experiment. **[NCERT]**
(a) 4×10^{-9} m (b) 5×10^{-10} m
(c) 3×10^{-7} m (d) 6×10^{-7} m

19. Interference fringes are observed on a screen by illuminating two thin slits 1 mm apart with a light source ($\lambda = 632.8$ nm). The distance between the screen and the slits is 100 cm. If a bright fringe is observed on a screen at a distance of 1.27 mm from the central bright fringe, then the path difference between the waves, which are reaching this point from the slits is close to **[JEE Main 2020]**
(a) 2 nm (b) 1.27 μm
(c) 2.87 nm (d) 2.05 μm

20. In Young's double slit experiment, the 8th maximum with wavelength λ_1 is at a distance, d_1 from the central maximum and the 6th maximum with wavelength λ_2 is at a distance, d_2. Then, d_1/d_2 is equal to
(a) $\frac{4}{3}\left(\frac{\lambda_2}{\lambda_1}\right)$ (b) $\frac{4}{3}\left(\frac{\lambda_1}{\lambda_2}\right)$
(c) $\frac{3}{4}\left(\frac{\lambda_2}{\lambda_1}\right)$ (d) $\frac{3}{4}\left(\frac{\lambda_1}{\lambda_2}\right)$

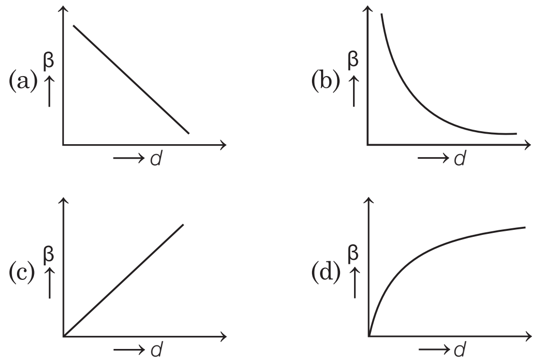
21. In a Young's double slit experiment, slits are separated by 0.5 mm and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide, is **[JEE Main 2017]**

(a) 7.8 mm (b) 9.75 mm
(c) 15.6 mm (d) 1.56 mm

22. In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\frac{1}{40}$ rad by using light of wavelength λ_1. When the light of the wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 nm to 740 nm), their values are **[JEE Main 2019]**

(a) 380 nm, 525 nm
(b) 400 nm, 500 nm
(c) 380 nm, 500 nm
(d) 625 nm, 500 nm

23. The correct curve between fringe width β and distance between the slits (d) in figure is



24. In Young's double slit experiment, the separation between slit is halved and the distance between the slits and screen is doubled. The fringe width is

(a) unchanged (b) halved
(c) double (d) quardrupled

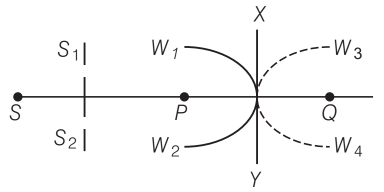
25. In Young's double slit arrangement, slits are separated by a gap of 0.5 mm, and the screen is placed at a distance of 0.5 m from them. The distance between the first and the third bright fringe formed when the slits are illuminated by a monochromatic light of 5890 Å is **[JEE Main 2021]**

(a) 1178×10^{-9} m (b) 1178×10^{-6} m
(c) 1178×10^{-12} m (d) 5890×10^{-7} m

26. In a Young's experiment, two coherent sources are placed 0.90 mm apart and the fringes are observed one metre away. If it produces the second dark fringe at a distance of 1 mm from the central fringe, the wavelength of monochromatic light used will be

(a) 60×10^{-4} cm (b) 10×10^{-4} cm
(c) 10×10^{-5} cm (d) 6×10^{-5} cm

27. Interference fringes are being produced on screen XY by the slits S_1 and S_2. In figure, the correct fringe locus is



(a) PQ (b) W_1W_2 (c) W_3W_4 (d) XY

28. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength $\lambda = 500$ nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-30° \leq \theta \leq 30°$ is **[JEE Main 2019]**

(a) 320 (b) 321 (c) 640 (d) 641

29. In a two slits experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the slits. If the screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-5} m. If separation between the slits is 10^{-3} m, the wavelength of light used is

(a) 4500 Å (b) 3000 Å
(c) 5000 Å (d) 6000 Å

30. The maximum number of possible interference maxima for slit separation equal to twice the wavelength in Young's double slit experiment is

(a) infinite (b) 5
(c) 3 (d) zero

31. If Young's double slit experiment, is performed in water
(assume that the water is still and clear)

(a) the fringe width will decrease
(b) the fringe width will increase
(c) the fringe width will remain unchanged
(d) there will be no fringe

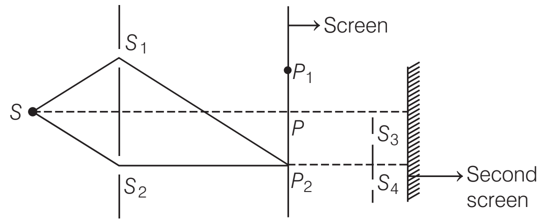
32. In Young's double slit experiment, when violet light of wavelength 4358 Å is used, 84 fringes are seen in the field of view, but when sodium light of certain wavelength is used, then 62 fringes are seen in the field of view, the wavelength of sodium light is

(a) 6893 Å (b) 5904 Å
(c) 5523 Å (d) 6429 Å

33. The Young's double slit experiment is performed with blue and with green light of wavelengths 4360 Å and 5460 Å, respectively. If, x is the distance of 4th maxima from the central one, then

(a) x (blue) = x (green)
(b) x (blue) > x (green)
(c) x (blue) < x (green)
(d) x (blue)/x (green) = 5400/4360

34. Figure shows a standard two slit arrangement with slits S_1, S_2. P_1 and P_2 are the two minima points on either side of P.



At P_2 on the screen, there is a hole and behind P_2 is a second 2-slit arrangement with slits S_3, S_4 and a second screen behind them. **[NCERT Exemplar]**

(a) There would be no interference pattern on the second screen but it would be lighted.
(b) The second screen would be totally dark.
(c) There would be a single bright point on the second screen.
(d) There would be a regular two slit pattern on the second screen.

35. In a Young's double slit experiment, the path difference at a certain point on the screen between two interfering waves is $\frac{1}{8}$th of wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to **[JEE Main 2019]**

(a) 0.80 (b) 0.74
(c) 0.94 (d) 0.85

36. In the Young's double slit experiment, the interference pattern is found to have an intensity ratio between bright and dark fringes as 9. This implies that

(a) the intensities at the screen due to two slits are 5 units and 4 units respectively
(b) the intensities at the screen due to two slits are 4 units and 1 unit respectively
(c) the amplitude ratio is 3
(d) the amplitude ratio is 2

37. A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincides with the 4th bright fringe of the unknown light. From this data, the wavelength of the unknown light is **[AIEEE 2009]**

(a) 393.4 nm (b) 885.0 nm
(c) 442.5 nm (d) 776.8 nm

38. Two identical radiators have a separation of $d = \lambda/4$, where λ is the wavelength of the waves emitted by either source. The initial phase difference between the source is $\pi/4$. Then the intensity on the screen at a distant point situated at an angle, $\theta = 30°$ from the radiators is (here, I_0 is intensity at that point due to one radiator alone)

(a) I_0 (b) $2I_0$
(c) $3I_0$ (d) $4I_0$

39. In a Young's double slit experiment, the ratio of the slit's width is 4 : 1. The ratio of the intensity of maxima to minima, close to the central fringe on the screen, will be **[JEE Main 2019]**

(a) 4 : 1
(b) 25 : 9
(c) 9 : 1
(d) $(\sqrt{3}+1)^4 : 16$

40. In Young's double slit experiment, the two slits act as coherent sources of equal amplitude A and wavelength λ. In another experiment with the same setup, the two slits are sources of equal amplitude A and wavelength λ but are incoherent. The ratio of the intensity of light at the mid-point of the screen in the first case to that in the second case is

(a) 2 : 1 (b) 1 : 2
(c) 3 : 4 (d) 4 : 3

41. A thin glass plate of thickness $\frac{2500}{3}\lambda$ (where, λ is wavelength of light used) and refractive index $\mu = 1.5$ is inserted between one of the slits and the screen in Young's double slit experiment. At a point on the screen equidistant from the slits, the ratio of the intensities before and after the introduction of the glass plate is **[JEE Main 2013]**

(a) 2 : 1 (b) 1 : 4
(c) 4 : 1 (d) 4 : 3

42. In a Young's double slit experiment, 16 fringes are observed in a certain segment of the screen when light of wavelength 700 nm is used. If the wavelength of light is changed to 400 nm, the number of fringes observed in the same segment of the screen would be **[JEE Main 2020]**

(a) 24 (b) 30
(c) 18 (d) 28

43. In an interference pattern, the position of zeroth order maxima is 4.8 mm from a certain point P on the screen. The fringe width is 0.2 mm. The position of second maxima from point P is

(a) 5.1 mm (b) 5 mm
(c) 40 mm (d) 5.2 mm

44. In a Young's experiment, one of the slits is covered with a transparent sheet of thickness 3.6×10^{-3} cm due to which position of central fringe shifts to a position originally occupied by 30th fringe. The refractive index of the sheet, if $\lambda = 6000$ Å, is

(a) 1.5 (b) 1.2
(c) 1.3 (d) 1.7

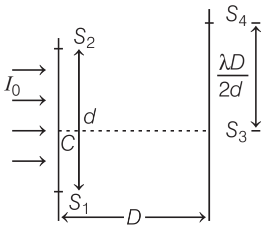
45. In Young's double slit experiment, distance between two sources is 0.1 mm. The distance of screen from the sources is 20 cm. Wavelength of light used is 5460 Å. Then angular position of first dark fringe is
(a) 0.08° (b) 0.16°
(c) 0.20° (d) 032°

46. In Young's double slit experiment, light of 500 nm is used to produce an interference pattern. When the distance between the slits is 0.05 mm, the angular width (in degree) of the fringes formed on the distance screen is close to **[JEE Main 2020]**
(a) 0.17° (b) 0.57°
(c) 1.7° (d) 0.07°

47. In Young's double slit experiment, the spacing between the slits is d and wavelength of light used is 6000 Å. If the angular width of a fringe formed on a distance screen is 1°, then value of d is
(a) 1 mm (b) 0.05 mm
(c) 0.03 mm (d) 0.01 mm

48. In double-slit experiment using light of wavelength 600 nm, the angular width of a fringe formed on a distant screen is 0.1°. What is the spacing between the two slits?
(a) 3.44×10^{-4} m (b) 3.03×10^{-4} m
(c) 4.03×10^{-4} m (d) 2.68×10^{-4} m

49. In the given figure, C is middle point of lines S_1S_2. A monochromatic light of wavelength λ is incident on slits. The ratio of intensities of S_3 and S_4 is



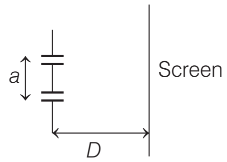
(a) zero (b) ∞
(c) 4 : 1 (d) 1 : 4

50. The maximum intensity of fringes in Young's experiment is I. If one of the slit is closed, then the intensity at that place becomes I_0. Which of the following relation is true?
(a) $I = I_0$ (b) $I = 2I_0$
(c) $I = 4I_0$ (d) $I = 0$

51. In a double slit experiment, at a certain point on the screen the path difference between the two interfering waves is $\frac{1}{8}$ th of a wavelength. The ratio of the intensity of light at that point to that at the centre of a bright fringe is **[JEE Main 2020]**
(a) 0.568 (b) 0.853
(c) 0.760 (d) 0.672

52. In a double slit experiment, the angular width of a fringe is found to be 0.2° on a screen placed 1 m away. The wavelength of light used is 600 nm. What will be the angular width of the fringe if the entire experimental apparatus is immersed in water? Take, refractive index of water to be 4/3.
(a) 0.15° (b) 0.30°
(c) 0.27° (d) 0.45°

53. The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index μ is put in front of one of the slits, the central maximum gets shifted by a distance equal to n fringe widths. If the wavelength of light used is λ, t will be **[JEE Main 2019]**



(a) $\frac{2nD\lambda}{a(\mu-1)}$ (b) $\frac{2D\lambda}{a(\mu-1)}$ (c) $\frac{D\lambda}{a(\mu-1)}$ (d) $\frac{nD\lambda}{a(\mu-1)}$

54. In a biprism experiment, 5th dark fringe is obtained at a point. If a thin transparent film is placed in the path of one of waves, then 7th bright fringes is obtained at the same point. The thickness of the film in terms of wavelength l and refractive index, μ will be
(a) $\frac{1.5\lambda}{(\mu-1)}$ (b) $1.5(\mu-1)\lambda$
(c) $2.5(\mu-1)\lambda$ (d) $\frac{2.5\lambda}{(\mu-1)}$

Diffraction of Light

55. The main difference between the phenomena of interference and diffraction is that
(a) diffraction is caused by reflected waves from a source whereas interference is caused due to refraction of waves from a source
(b) diffraction is due to interaction of waves derived from the same source, whereas interference is that bending of light from the same wavefront
(c) diffraction is due to interaction of light from wavefront, whereas the interference is the interaction of two waves derived from the same source
(d) diffraction is due to interaction of light from the same wavefront whereas interference is the interaction of waves from two isolated sources

56. When monochromatic light is replaced by white light in Fresnel's biprism arrangement, the central fringe is
(a) coloured (b) white
(c) dark (d) None of these

57. Consider sunlight incident on a slit of width 10^4 Å. The image seen through the slit shall **[NCERT Exemplar]**

(a) be a fine sharp slit white in colour at the centre
(b) a bright slit white at the centre diffusing to zero intensities at the edges
(c) a bright slit white at the centre diffusing to regions of different colours
(d) Only be a diffused slit white in colour

58. Estimate the distance for which ray optics is good approximation for an aperture of 4 mm and wavelength 400 nm. **[NCERT]**

(a) 40 m (b) 43 m
(c) 47 m (d) 38 m

59. A parallel beam of light of wavelength 3141.59 Å is incident on a small aperture. After passing through the aperture, the beam is no longer parallel but diverges at 1° to the incident direction. What is the diameter of the aperture?

(a) 180 m (b) 18 μm
(c) 1.8 m (d) 0.18 m

60. Plane microwaves are incident on a long slit having a width of 5 cm. The wavelength of the microwaves if the first minimum is formed at 30° is

(a) 2.5 cm (b) 2 cm
(c) 25 cm (d) 2 mm

61. A plane wave of wavelength 6250 Å is incident normally on a slit of width 2×10^{-2} cm. The width of the principal maximum on a screen distant 50 cm will be

(a) 312.5×10^{-3} cm
(b) 312.5×10^{-4} cm
(c) 312 cm
(d) 312.5×10^{-5} cm

62. The distance between the first and the sixth minima in the diffraction pattern of a single slit is 0.5 mm. The screen is 0.5 m away from the slit. If the wavelength of light used is 5000 Å. Then the slit width will be

(a) 5 mm
(b) 0.5 mm
(c) 1.25 mm
(d) 1.0 mm

63. The Fraunhoffer diffraction pattern of a single slit is formed in the focal plane of a lens of focal length 1 m. The width of slit is 0.3 mm. If third minimum is formed at a distance of 5 mm from central maximum, then wavelength of light will be

(a) 5000 Å
(b) 2500 Å
(c) 7500 Å
(d) 8500 Å

64. Light of wavelength λ is incident on a slit width d. The resulting diffraction pattern is observed on a screen at a distance D. The linear width of the principal maximum is equal to the width of the slit, if D equals

(a) $\dfrac{d^2}{2\lambda}$ (b) $\dfrac{d}{\lambda}$
(c) $\dfrac{2\lambda^2}{d}$ (d) $\dfrac{2\lambda}{d}$

65. The ratio of intensities of successive maxima in the diffraction pattern due to single slit is

(a) $1:4:9$ (b) $1:2:3$
(c) $1:\dfrac{4}{9\pi^2}:\dfrac{4}{25\pi^2}$ (d) $1:\dfrac{4}{\pi^2}:\dfrac{9}{\pi^2}$

66. Visible light of wavelength 6000×10^{-8} cm falls normally on a single slit and produces a diffraction pattern. It is found that the second diffraction minimum is at 60° from the central maximum. If the first minimum is produced at θ_1, then θ_1 is close to **[JEE Main 2020]**

(a) 25° (b) 30°
(c) 20° (d) 45°

Resolving Power of Optical Instruments

67. Diameter of the objective lens of a telescope is 250 cm. For light of wavelength 600 nm coming from a distant object, the limit of resolution of the telescope is close to **[JEE Main 2019]**

(a) 3.0×10^{-7} rad (b) 2.0×10^{-7} rad
(c) 1.5×10^{-7} rad (d) 4.5×10^{-7} rad

68. Calculate the limit of resolution of a telescope objective having a diameter of 200 cm, if it has to detect light of wavelength 500 nm coming from a star. **[JEE Main 2019]**

(a) 610×10^{-9} rad (b) 305×10^{-9} rad
(c) 457.5×10^{-9} rad (d) 152.5×10^{-9} rad

69. The value of numerical aperture of the objective lens of a microscope is 1.25. If light of wavelength 5000Å is used, the minimum separation between two points, to be seen as distinct, will be **[JEE Main 2019]**

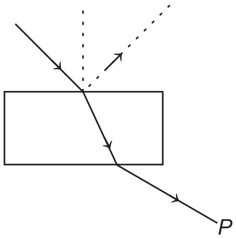
(a) 0.24 μm (b) 0.38 μm
(c) 0.12 μm (d) 0.48 μm

70. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is **[JEE Main 2015]**

(a) 1 μm (b) 30 μm
(c) 100 μm (d) 300 μm

Polarisation of Light

71. Consider a light beam incident from air to a glass slab at Brewster's angle as shown in the figure. **[NCERT Exemplar]**



A polaroid is placed in the path of the emergent ray at a point P and rotated about an axis passing through the centre and perpendicular to the plane of the polaroid.

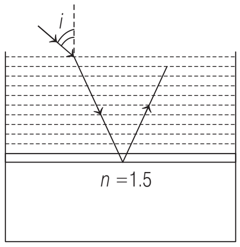
(a) For a particular orientation, there shall be darkness as observed through the polaroid.

(b) The intensity of light as seen through the polaroid shall be independent of the rotation.

(c) The intensity of light as seen through the polaroid shall go through a minimum but not zero for two orientations of the polaroid.

(d) The intensity of light as seen through the polaroid shall go through a minimum for four orientations of the polaroid.

72. Consider a tank made of glass (refractive index is 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarised.



For this to happen, the minimum value of μ is **[JEE Main 2019]**

(a) $\frac{3}{\sqrt{5}}$ (b) $\frac{5}{\sqrt{3}}$

(c) $\frac{4}{3}$ (d) $\sqrt{\frac{5}{3}}$

73. λ_a and λ_m are the wavelengths of a beam of light in air and medium respectively. If θ is the polarising angle, the correct relation between λ_a, λ_m and θ is

(a) $\lambda_a = \lambda_m \tan^2\theta$ (b) $\lambda_m = \lambda_a \tan^2\theta$

(c) $\lambda_a = \lambda_m \cot\theta$ (d) $\lambda_m = \lambda_a \cot\theta$

74. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is

(a) I_0 (b) $I_0/2$ (c) $I_0/4$ (d) $I_0/8$

75. An unpolarised beam of intensity $2a^2$ passes through a thin polaroid. Assuming zero absorption in the polaroid, the instensity of emergent plane polarised light is

(a) $2a^2$ (b) a^2 (c) $\sqrt{2}\,a^2$ (d) $\frac{a^2}{2}$

76. A beam of ordinary unpolarised light passes through a tourmaline crystal C_1 and then it passes through another tourmaline crystal C_2, which is oriented such that its principal plane is parallel to that of C_2. The intensity of emergent light is I_0. Now C_2 is rotated by 60° about the ray. The emergent ray will have an intensity

(a) $2I_0$ (b) $I_0/\sqrt{2}$ (c) $I_0/4$ (d) $I_0/\sqrt{2}$

77. Two Nicol prisms are first crossed and then one of them is rotated through 60°. The percentage of incident light transmitted is

(a) 1.25 (b) 25.0

(c) 37.5 (d) 50

78. Unpolarised light of intensity I passes through an ideal polariser A. Another identical polariser B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now, another identical polariser C is placed between A and B. The intensity beyond B is now found to be $\frac{1}{8}$. The angle between polariser A and C is **[JEE Main 2018]**

(a) 0° (b) 30°

(c) 45° (d) 60°

79. A system of three polarisers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at 60° to the pass axis of P_3. When a beam of unpolarised light of intensity I_0 is incident on P_1, the intensity of light transmitted by the three polarisers is I. The ratio (I_0 / I) equals (nearly) **[JEE Main 2019]**

(a) 5.33 (b) 16.00

(c) 10.67 (d) 1.80

80. The 6563 Å H_α sign line emitted by hydrogen in a star is found to be red-shifted by 15 Å. Estimate the speed with which the star is receding from the earth.

(a) 6.9 m/s approaching the earth

(b) 6.86 m/s receding the earth

(c) 7.9 m/s receding the earth

(d) 8.9 m/s receding the earth

ROUND II Mixed Bag

Only One Correct Option

1. Interference was observed in interference chamber when air was present, now the chamber is evacuated and if, the same light is used, a careful observer will see

(a) interference in which width of the fringe will be slightly increased
(b) interference with bright band
(c) interference with dark band
(d) All of the above

2. A beam of light of wavelength 600 nm from a distant source falls on a single slit 1.00 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. The distance between the first dark fringes on either side of the central bright fringe is

(a) 1.2 cm (b) 1.2 mm (c) 2.4 cm (d) 2.4 mm

3. Find the thickness of a plate which will produce a change in optical path equal to half the wavelength λ of the light passing through it normally. The refractive index of the plate μ is equal to

(a) $\frac{\lambda}{4(\mu-1)}$ (b) $\frac{2\lambda}{4(\mu-1)}$ (c) $\frac{\lambda}{(\mu-1)}$ (d) $\frac{\lambda}{2(\mu-1)}$

4. Red light differs from blue light as they have **[JEE Main 2021]**

(a) different frequencies and different wavelengths
(b) different frequencies and same wavelengths
(c) same frequencies and same wavelengths
(d) same frequencies and different wavelengths

5. Consider the diffraction pattern for a small pinhole. As the size of the hole is increased,

(a) the size decreases
(b) the intensity increases
(c) the size increases
(d) Both (a) and (b)

6. Two slits separated by a distance of 1 mm are illuminated with red light of wavelength 6.5×10^{-7} m. The interference fringes are observed on a screen placed at 1 m from the slits. The distance between the third dark fringe and the fifth bright fringe is equal to

(a) 0.65 mm (b) 1.63 mm
(c) 3.25 mm (d) 4.88 mm

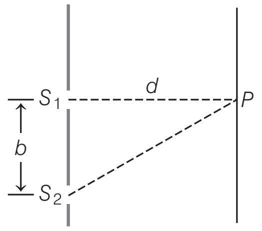
7. In a double slit experiment, green light (5303 Å) falls on a double slit having a separation of 19.44 μm and a width of 4.05 μm. The number of bright fringes between the first and the second diffraction minima is **[JEE Main 2019]**

(a) 5 (b) 10 (c) 9 (d) 4

8. A beam of unpolarized light having flux 10^{-3} W falls normally on a polarizer of cross-sectional area $3 \times 10^{-4}\ m^2$. The polarizer rotates with an angular frequency of 31.4 rads^{-1}. The energy of light passing through the polarizer per revolution will be

(a) 10^{-4} J (b) 10^{-3} J (c) 10^{-2} J (d) 10^{-1} J

9. White light is used to illuminate the two slits in a Young's double slit experiment. The separation between slits is b and the screen is at a distance $d (>> b)$ from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing, figure. Some of these missing wavelengths are



(a) $\lambda = \frac{b^2}{d}, \frac{2b^2}{3d}$ (b) $\lambda = \frac{b^2}{2d}, \frac{3b^2}{2d}$
(c) $\lambda = \frac{2b^2}{3d}$ (d) $\lambda = \frac{3b^2}{4d}$

10. Two sources S_1 and S_2 of intensity I_1 and I_2 are placed in front of a screen [Fig.(a)]. Then, which amongst the following is incorrect about the pattern of intensity distribution seen in the central portion is given by Fig. (b)? **[NCERT Exemplar]**

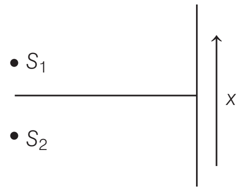


Fig. (a)

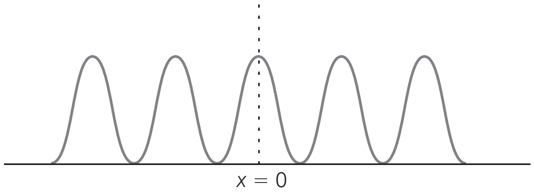


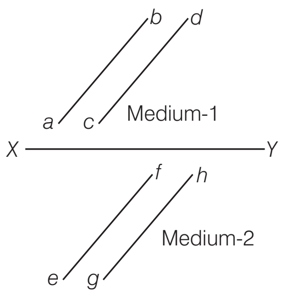
Fig. (b)

(a) S_1 and S_2 have the same intensities
(b) S_1 and S_2 have a constant phase difference
(c) S_1 and S_2 have the same phase
(d) S_1 and S_2 have the same wavelength

11. In Young's double slit experiment $\frac{d}{D} = 10^{-4}$ (d = distance between slits, D = distance of screen from the slits). At a point P on the screen, resultant intensity is equal to the intensity due to individual slit I_0. Then the distance of point P from the central maximum is ($\lambda = 6000$ Å)

(a) 0.5 mm (b) 2 mm
(c) 1 mm (d) 4 mm

12. The figure shows a surface XY separating two transparent media, medium-1 and medium-2. The lines ab and cd represent wavefronts of a light wave travelling in medium-1 and incident on XY. The lines ef and gh represent wavefronts of the light wave in medium-2 after refraction.



The phases of the light wave at c, d, e and f are ϕ_c, ϕ_d, ϕ_e and ϕ_f respectively. It is given that $\phi_c \neq \phi_f$

(a) ϕ_c cannot be equal to ϕ_d
(b) ϕ_d can be equal to ϕ_e
(c) $(\phi_d - \phi_f)$ is equal to $(\phi_c - \phi_e)$
(d) $(\phi_d - \phi_c)$ is not equal to $(\phi_f - \phi_e)$

13. An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

The initial shape of the wavefront of the beam is **[AIEEE 2010]**

(a) planar
(b) convex
(c) concave
(d) convex near the axis and concave near the periphery

14. In Young's double slit experiment, one of the slit is wider than other, so that amplitude of light from one slit is double of that from other slit. If, I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ, is given by **[AIEEE 2012]**

(a) $\frac{I_m}{9}(4 + 5\cos\phi)$ (b) $\frac{I_m}{3}\left(1 + \cos^2\frac{\phi}{2}\right)$

(c) $\frac{I_m}{5}\left(1 + 4\cos^2\frac{\phi}{2}\right)$ (d) $\frac{I_m}{9}\left(1 + 8\cos^2\frac{\phi}{2}\right)$

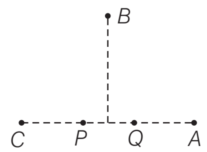
15. A beam of plane polarised light of large cross-sectional area and uniform intensity of $3.3\ \text{Wm}^{-2}$ falls normally on a polariser, which rotates about its axis with an angular speed of 31.4 rad/s. The energy of light passing through the polariser per revolution is close to
(Take, cross-sectional area = $3 \times 10^{-4}\ \text{m}^2$) **[JEE Main 2020]**

(a) 1.5×10^{-4} J
(b) 5.0×10^{-4} J
(c) 1.0×10^{-4} J
(d) 1.0×10^{-5} J

16. In an interference arrangement similar to Young's double slit experiment, the slits S_1 and S_2 are illuminated with coherent microwave sources each of frequency 10^6 Hz. The sources are synchronized to have zero phase difference. The slits are separated by distance $d = 150$ m. The intensity $I(\theta)$ is measured as a function of θ, where θ is defined as shown. If I_0 is maximum intensity, then $I(\theta)$ for $0 \leq \theta \leq 90°$ is given by

(a) $I(\theta) = I_0$ for $\theta = 30°$
(b) $I(\theta) = \frac{I_0}{2}$ for $\theta = 30°$
(c) $I(\theta) = \frac{I_0}{4}$ for $\theta = 90°$
(d) $I(\theta)$ is constant for all values of θ

17. In the given figure, P and Q are two equally intense coherent sources emitting radiation of wavelength 20 m. The separation between P and Q is 5 m and the phase of P is ahead of that of Q by 90°. A, B and C are three distinct points of observation, each equidistant from the mid-point of PQ. The intensities of radiation at A, B and C will be in the ratio



[JEE Main 2020]

(a) 0 : 1 : 4 (b) 2 : 1 : 0
(c) 0 : 1 : 2 (d) 4 : 1 : 0

18. Two light waves having the same wavelength λ in vacuum are in phase initially. Then, the first wave travels a path L_1 through a medium of refractive index n_1 while the second wave travels a path of length L_2 through a medium of refractive index n_2. After this, the phase difference between the two waves is **[JEE Main 2020]**

(a) $\frac{2\pi}{\lambda}(n_1 L_1 - n_2 L_2)$ (b) $\frac{2\pi}{\lambda}\left(\frac{L_1}{n_1} - \frac{L_1}{n_2}\right)$

(c) $\frac{2\pi}{\lambda}\left(\frac{L_2}{n_1} - \frac{L_1}{n_2}\right)$ (d) $\frac{2\pi}{\lambda}(n_2 L_1 - n_1 L_2)$

Numerical Value Questions

19. If amplitude ratio of two sources producing interference is 3:5. The ratio of intensities of maxima and minima is $x:1$, where the value of x is ....

20. In Young's double slit experiment, an interference pattern is obtained for $\lambda = 6000\mathring{A}$, coming from two coherent sources S_1 and S_2. At certain point P on the screen, third dark fringe is formed. Then the path difference $S_1P - S_2\ P$ (in micron) is ....

21. In a Young's double slit experiment, 15 fringes are observed on a small portion of the screen when light of wavelength 500 nm is used. Ten fringes are observed on the same section of the screen when another light source of wavelength λ is used. Then, the value of λ is (in nm) .... **[JEE Main 2020]**

22. Young's double slit experiment is carried out using microwaves of wavelength $\lambda = 3$ cm. Distance between the slits is $d = 5$ cm and the distance between the plane of slits and the screen is $D = 100$ cm, then what is the number of maximas on the screen?

23. Orange light of wavelength 6000×10^{-10} m illuminates a single slit of width 0.6×10^{-4} m. The maximum possible number of diffraction minima produced on both sides of the central maximum is .... **[JEE Main 2020]**

24. A Young's double slit experiment is performed using monochromatic light of wavelength λ. The intensity of light at a point on the screen, where the path difference is λ, is K units. The intensity of light at a point where the path difference is $\frac{\lambda}{6}$ is given by $\frac{nK}{12}$, where n is an integer. The value of n is .... **[JEE Main 2020]**

Answers

Round I

1. (a)	2. (c)	3. (a)	4. (d)	5. (a)	6. (b)	7. (b)	8. (b)	9. (a)	10. (d)
11. (c)	12. (d)	13. (d)	14. (a)	15. (b)	16. (c)	17. (a)	18. (d)	19. (b)	20. (b)
21. (a)	22. (d)	23. (b)	24. (d)	25. (b)	26. (d)	27. (c)	28. (d)	29. (d)	30. (b)
31. (a)	32. (b)	33. (c)	34. (d)	35. (d)	36. (b)	37. (c)	38. (b)	39. (c)	40. (a)
41. (c)	42. (d)	43. (a)	44. (a)	45. (b)	46. (b)	47. (c)	48. (a)	49. (b)	50. (c)
51. (b)	52. (a)	53. (*)	54. (d)	55. (d)	56. (b)	57. (a)	58. (a)	59. (b)	60. (a)
61. (a)	62. (b)	63. (a)	64. (a)	65. (c)	66. (a)	67. (a)	68. (b)	69. (a)	70. (b)
71. (c)	72. (a)	73. (d)	74. (c)	75. (b)	76. (c)	77. (c)	78. (c)	79. (c)	80. (b)

Round II

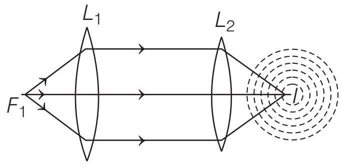
1. (a)	2. (d)	3. (d)	4. (a)	5. (d)	6. (b)	7. (a)	8. (a)	9. (a)	10. (c)
11. (b)	12. (c)	13. (a)	14. (d)	15. (c)	16. (b)	17. (b)	18. (a)	19. 16	20. 1.5
21. 750	22. 3	23. 198	24. 9						

Solutions

Round I

1. For light diverging from a point source, the wavefront is (diverging) spherical. The intensity varies inversely as the area of the wavefront ($=4\pi r^2$), *i.e.* intensity decreases in proportion to the distance squared.

2. The given situation is shown below



Consider a point F on focus of converging lens L_1.
The light rays from F, becomes parallel after refraction through L_1. When these parallel rays falls on converging lens L_2 placed co-axial on the other side of F of lens L_1, lens L_2 converges the rays at its focus at I. It now behaves like a point source of rays and hence form a spherical wavefront.

3. As, velocity of light is perpendicular to the wavefront and light is travelling in vacuum along the Y-axis, therefore the wavefront is represented by $y=$ constant.

4. According to Snell's law,

$$\mu \sin\theta = \text{constant}$$
$$\sin\theta \propto \frac{1}{\mu}$$

As, μ increases, θ decreases.
Hence, the light beam will bend upward.

5. Resultant intensity is given by

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Here, $I_1 = I$ and $I_2 = 4I$

For maximum intensity, $\cos\phi = 1$,

i.e. $\phi = 0°$

$\therefore \quad I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = I + 4I + 2\sqrt{I \cdot 4I}$

$\Rightarrow \quad I_{\max} = 9I$

For minimum intensity, $\cos\phi = -1$, *i.e.* $\phi = 180°$

$$\therefore \quad I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = I + 4I - 2\sqrt{I \cdot 4I} = 5I - 4I = I$$

6. Here, $I_1 = I, I_2 = 4I, \theta_1 = \pi/2, \theta_2 = \pi$

Resultant intensity at point A,

$$I_{\theta_1} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta_1 = I + 4I + 2\sqrt{I \times 4I}\cos\pi/2 = 5I$$

Resultant intensity at point B,

$$I_{\theta_2} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta_2 = I + 4I + 2\sqrt{I \times 4I}\cos\pi = 5I - 4I = I$$

$\therefore \quad I_{\theta_1} - I_{\theta_2} = 5I - I = 4I$

7. For incoherent waves,

$$I_{\max} = nI$$

$\therefore \quad n = \dfrac{I_{\max}}{I} = \dfrac{32}{2} = 16$

8. The amplitude of the waves are $a_1 = 10\,\mu\text{m}$, $a_2 = 4\,\mu\text{m}$ and $a_3 = 7\,\mu\text{m}$ and phase difference between Ist and IInd wave is $\dfrac{\pi}{2}$ and that between 2nd and 3rd wave is $\dfrac{\pi}{2}\cdot$ Then, phase difference between 1st and 3rd is π.

Combining Ist with 3rd, their resultant amplitude is given by

$$A_1^2 = a_1^2 + a_3^2 + 2a_1 a_3\cos\phi$$

or $\quad A_1 = \sqrt{10^2 + 7^2 + 2 \times 10 \times 7\cos\pi} = \sqrt{100 + 49 - 140} = \sqrt{9} = 3\,\mu\text{m}$

Now, combining this with 2nd wave, we have the resultant amplitude

$$A^2 = A_1^2 + a_2^2 + 2A_1 a_2 \cos\frac{\pi}{2}$$

or $\quad A = \sqrt{3^2 + 4^2 + 2 \times 3 \times 4 \times \cos 90°} = \sqrt{9 + 16} = \sqrt{25} = 5\,\mu\text{m}$

9. The intensity of light reflected from upper surface is

$$I_1 = I_0 \times 25\% = I_0 \times \frac{25}{100} = \frac{I_0}{4}$$

The intensity of transmitted light from upper surface is

$$I = I_0 - \frac{I_0}{4} = \frac{3I_0}{4}$$

$\therefore$ The intensity of reflected light from lower surface is

$$I_2 = \frac{3I_0}{4} \times \frac{50}{100} = \frac{3I_0}{8}$$

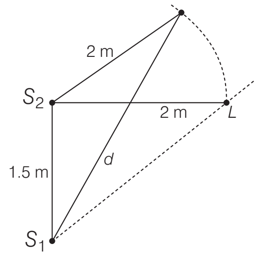
$\therefore \quad \dfrac{I_{\max}}{I_{\min}} = \dfrac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$

$\therefore \quad \dfrac{I_{\max}}{I_{\min}} = \dfrac{\left(\sqrt{\dfrac{I_0}{4}} + \sqrt{\dfrac{3I_0}{8}}\right)^2}{\left(\sqrt{\dfrac{I_0}{4}} - \sqrt{\dfrac{3I_0}{8}}\right)^2} = \dfrac{\left(\dfrac{1}{2} + \sqrt{\dfrac{3}{8}}\right)^2}{\left(\dfrac{1}{2} - \sqrt{\dfrac{3}{8}}\right)^2}$

10. Hence, path difference, $S_1P - S_2P =$ odd integral multiple of $\lambda/2 = 11\,(\lambda/2)$

11. Path difference, $x = (SS_1 + S_1O) - (SS_2 + S_2O)$. If, $x = n\lambda$, then central fringe at O will be bright. If, $x = (2n-1)\lambda/2$, the central fringe at O will be dark.

12. In the given figure, join the points S_1 and L.



From figure, $S_2L = 2$ m

$$S_1L = \sqrt{(2)^2 + (1.5)^2} = \sqrt{4 + 2.25} = 2.5 \text{ m}$$

Initial path difference where minima is observed,

$$\Delta x_1 = S_1L - S_2L = 2.5 - 2 = 0.5 \text{ m}$$

Path difference at new position, where maxima is observed,

$$\Delta x_2 = d - 2$$

Path difference between adjacent maxima and minima is $\frac{\lambda}{2}$.

i.e. $$\Delta x_2 - \Delta x_1 = \frac{\lambda}{2}$$

$\Rightarrow$ $$d - 2 - 0.5 = \frac{1}{2} \qquad (\because \lambda = 1 \text{ m})$$

$\Rightarrow$ $$d = 3 \text{ m}$$

13. After interference, in accordance to superposition principle, the fringes on the screen will be concentric circles.

14. A ray of light of intensity I is incident on a parallel glass slab, undergoes partial refraction and reflection.

According to figure given in question, the rays AB and $A'B'$ undergo interference.

Intensity of reflected ray AB,

$$I_{AB} = 25\% \text{ of } I = \frac{I}{4}$$

$\therefore$ Intensity of ray AC,

$$I_{AC} = I - I_{AB} = I - \frac{I}{4} = \frac{3I}{4}$$

Similarly, intensity of ray $A'C$ is

$$I_{A'C} = 25\% \text{ of } I_{AC} = \frac{25}{100} \times \frac{3I}{4} = \frac{3I}{16}$$

Similarly, $I_{A'B'} = 75\%$ of $I_{A'C} = \frac{75}{100} \times \frac{3I}{16} = \frac{9I}{64}$

$$\therefore \quad \frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{\frac{I_{A'B'}}{I_{AB}}} + 1}{\sqrt{\frac{I_{A'B'}}{I_{AB}}} - 1}\right)^2 = \left(\frac{\sqrt{\frac{9}{16}} + 1}{\sqrt{\frac{9}{16}} - 1}\right)^2 = \frac{49}{1}$$

$\therefore \quad I_{\max} : I_{\min} = 49 : 1$

15. Here, $n_1\lambda_1 = n_2\lambda_2$

$$n(7.8 \times 10^{-5}) = (n + 1)(5.2 \times 10^{-5})$$

$$\frac{n}{n+1} = \frac{5.2 \times 10^{-5}}{7.8 \times 10^{-5}}$$

$\therefore$ $$n = 2$$

16. As light from two slits of YDSE is of different colours/wavelength/frequencies, therefore there shall be no interference fringes.

17. Here, $2d = 0.5\text{mm} = 5 \times 10^{-2}\text{cm}$ and $D = 100$ cm

$$x_n = x_{11} - x_1 = 9.72 \text{ mm}$$

i.e. $$n = 10$$

We know that, $x_{11} - x_1 = \frac{n\lambda D}{2d}$

$\therefore$ $$\lambda = \frac{(x_n)(2d)}{nD}$$

$$= \frac{(0.972) \times (5 \times 10^{-2})}{10 \times 100}$$

$$= 4.86 \times 10^{-5} \text{ cm}$$

18. Given, separation between slits, $d = 0.28$ mm

$$= 0.28 \times 10^{-3} \text{ m}$$

Distance between screen and slit, $D = 1.4$ m,

Distance between central bright fringe and fourth bright fringe,

$$x = 1.2 \text{ cm} = 1.2 \times 10^{-2} \text{ m}$$

Number of fringes, $n = 4$

For constructive interference, $x = n\frac{D\lambda}{d}$

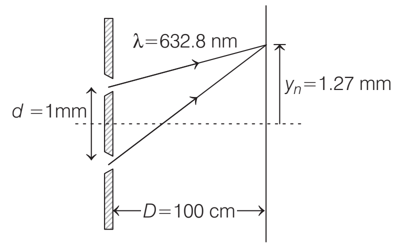
$$1.2 \times 10^{-2} = \frac{4 \times 1.4 \times \lambda}{0.28 \times 10^{-3}}$$

Wavelength, $$\lambda = \frac{1.2 \times 10^{-2} \times 0.28 \times 10^{-3}}{4 \times 1.4}$$

$$= 6 \times 10^{-7} \text{ m}$$

The wavelength of light is 6×10^{-7} m.

19. Given, setup of Young's double slit experiment is as shown



Distance of a bright fringe from central bright fringe is

$$y_n = \frac{n\lambda D}{d}$$

where, $n\lambda$ = path difference between rays from lower and upper slit $= \frac{y_n d}{D} = \frac{1.27 \times 10^{-3} \times 10^{-3}}{100 \times 10^{-2}}$

$$= 1.27 \times 10^{-6} \text{m} = 1.27\, \mu\text{m}$$

20. Position of nth maxima from central maxima is given by $$x_n = \frac{n\lambda D}{d}$$

For 8th maxima, $$x_8 = \frac{8\lambda_1 D}{d_1}$$

and for 6th maxima, $x_6 = \dfrac{6\lambda_2 D}{d_2}$

Now, $\quad x_6 = x_8$

$\Rightarrow \quad \dfrac{d_1}{d_2} = \dfrac{8\lambda_1}{6\lambda_2} = \dfrac{4}{3}\left(\dfrac{\lambda_1}{\lambda_2}\right)$

21. Given, $d = 0.5 \text{ mm} = 5 \times 10^{-4}\text{m}$

$$D = 150 \text{ cm} = 1.5 \text{ m}$$
$$\lambda_1 = 650 \text{ nm} = 6.5 \times 10^{-7} \text{ m}$$
$$\lambda_2 = 520 \text{ nm} = 5.2 \times 10^{-7}\text{m}$$

Fringe width pattern due to wavelength λ_1 only,

$$\beta_1 = \frac{D\lambda_1}{d} \quad \text{...(i)}$$

Fringe width pattern due to wavelength λ_2 only,

$$\beta_2 = \frac{D\lambda_2}{d} \quad \text{...(ii)}$$

If n_1 and n_2 are number of bright fringes due to fringe patterns using λ_1 and λ_2 respectively, then

$$n_1\beta_1 = n_2\beta_2$$

$\Rightarrow \quad n_1 \dfrac{D\lambda_1}{d} = n_2 \dfrac{D\lambda_2}{d}$

$\Rightarrow \quad \dfrac{n_1}{n_2} = \dfrac{\lambda_2}{\lambda_1} = \dfrac{5.2 \times 10^{-7}}{6.5 \times 10^{-7}}$

$\Rightarrow \quad \dfrac{n_1}{n_2} = \dfrac{4}{5}$

$\Rightarrow \quad n_1 = 4, n_2 = 5$

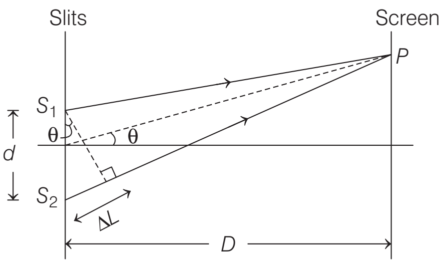
If x be the least distance where both the bright fringes coincides, then

$$\frac{xd}{D} = n_1\lambda_1$$

$\Rightarrow \quad x = \dfrac{n_1\lambda_1 D}{d} = \dfrac{4 \times 6.5 \times 10^{-7} \times 1.5}{5 \times 10^{-4}}$

$$= 7.8 \times 10^{-3}\text{m} = 7.8 \text{ mm}$$

22. In a YDSE, path difference between 2 rays, reaching at some common point P located at angular position θ as shown in the figure below is



$$\Delta L = d \sin\theta$$

For small value of $\theta, \sin\theta \approx \theta$

So, path difference $= \Delta L = d\theta$

For a bright fringe at same angular position θ, both of the rays from slits S_1 and S_2 are in phase.

Hence, path difference is an integral multiple of wavelength of light used.

i.e. $\quad \Delta L = n\lambda$

or $\quad d\theta = n\lambda$

$\Rightarrow \quad \lambda = \dfrac{d\theta}{n}$

Here, $\theta = \dfrac{1}{40}$ rad, $d = 0.1$ mm

Hence, $\quad \lambda = \dfrac{0.1}{40n} \text{ mm} = \dfrac{0.1 \times 10^{-3} \text{ m}}{40n}$

$$= \frac{0.1 \times 10^{-3} \times 10^{9}}{40n} \text{ nm}$$

$\Rightarrow \quad \lambda = \dfrac{2500}{n}$ nm

So, with light of wavelength λ_1, we have

$$\lambda_1 = \frac{2500}{n_1} \text{ nm}$$

and with light of wavelength λ_2, we have

$$\lambda_2 = \frac{2500}{n_2} \text{ nm}$$

Now, choosing different integral values for n_1 and n_2, (*i.e.* $n_1, n_2 = 1, 2, 3$ ..., etc) we find that for $n_1 = 4$,

$$\lambda_1 = \frac{2500}{4} = 625 \text{ nm}$$

and for $n_2 = 5$,

$$\lambda_2 = \frac{2500}{5} = 500 \text{ nm}$$

These values lie in given interval 500 nm to 625 nm.

23. As, $\beta = \dfrac{\lambda D}{d}$

$\therefore \quad \beta \propto 1/d$

This means that fringe width (β) will decrease with increase in distance between the slits (d) or *vice-versa*. This is clearly depicted in the graph given in option (b).

24. In Young's double slit experiment,

fringe width, $\quad \beta = \dfrac{D\lambda}{d} \quad$ ...(i)

when $d' = \dfrac{d}{2}$ and $D' = 2D$, then

fringe width, $\beta' = \dfrac{D'\lambda}{d'} = \dfrac{2D \cdot \lambda}{d/2}$

$$= \frac{4D\lambda}{d} = 4\beta \quad \text{[from Eq. (i)]}$$

Hence, fringe width is quardrupled.

25. Fringe width, $\beta = \dfrac{\lambda D}{d} = \dfrac{5890 \times 10^{-10} \times 0.5}{0.5 \times 10^{-3}} = 589 \times 10^{-6}$ m

Distance between first and third bright fringe is

$$2\beta = 2 \times 589 \times 10^{-6} \text{ m}$$
$$= 1178 \times 10^{-6} \text{ m}$$

26. As, $x = (2n - 1)\dfrac{\lambda}{2}\dfrac{D}{d}$

$\therefore \quad \lambda = \dfrac{2xd}{(2n-1)D}$

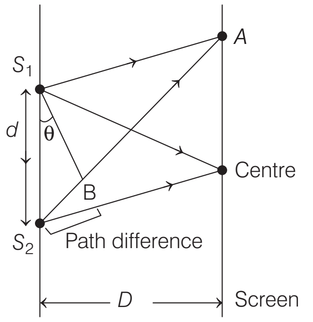
$$= \frac{2 \times 10^{-3} \times 0.9 \times 10^{-3}}{(2 \times 2 - 1) \times 1}$$
$$= 6 \times 10^{-7} \text{ m} = 6 \times 10^{-5} \text{ cm}$$

27. Interference fringes are the bands on screen XY running parallel to the lengths of slits. Therefore, the locus of fringes is represented correctly by W_3W_4.

28. In Young's double slit experiment, the condition of bright fringe and dark fringe are discussed below.

For bright fringes (maxima), path difference $= n\lambda$
$$d \sin \theta = n\lambda$$
for dark fringes (minima),

path difference $= (2n - 1)\dfrac{\lambda}{2}$
$$d \sin \theta = (2n - 1)\frac{\lambda}{2}$$



For the given question, distance between slits $(d) = 0.320$ mm

Wavelength of light used $(\lambda) = 500$ nm

Angular range for bright fringe
$$(\theta) = -30^\circ \text{ to } 30^\circ$$
Hence, for bright fringe,
$$n\lambda = d \sin \theta$$
$$n = \frac{d \sin \theta}{\lambda} = \frac{0.320 \times 10^{-3} \times \sin 30^\circ}{500 \times 10^{-9}}$$
$$n_{max} = 320$$
$\therefore$ Total number of maxima between the two lines are
$$n = (n_{max} \times 2) + 1$$
Here, $\quad n = (320 \times 2) + 1$
$$n = 641$$

29. As, $\beta = \dfrac{\lambda D}{d}$

and $\quad \beta' = \dfrac{\lambda D'}{d}$

$\therefore \quad \beta - \beta' = \dfrac{\lambda (D - D')}{d}$
$$3 \times 10^{-5} = \frac{\lambda \times 5 \times 10^{-2}}{10^{-3}}$$
or $\quad \lambda = \dfrac{3 \times 10^{-5}}{50} = 6 \times 10^{-7} \text{ m} = 6000 \text{ Å}$

30. For maxima on the screen,
$$d \sin \theta = n\lambda \quad \text{...(i)}$$
Given, $\quad d = 2\lambda$

$\therefore \quad 2\lambda \sin \theta = n\lambda$
$$2 \sin \theta = n$$
The maximum value of $\sin \theta = +1$

$\therefore \quad n = 2$

$\therefore$ Eq. (i) is satisfied by $-2, -1, 0, 1, 2$.

31. As we know,
$$\beta = \frac{D}{d}\lambda \quad \text{...(i)}$$
and $\quad \lambda \propto \dfrac{1}{\mu} \quad$ ...(ii)

From Eqs. (i) and (ii), we get
$$\beta \propto \lambda \propto \frac{1}{\mu}$$
$\therefore \quad \beta \propto \dfrac{1}{\mu}$

The refractive index of water is greater than air, therefore fringe width will decrease.

32. As, field of view is same in both cases
$$n_1\beta_1 = n_2\beta_2$$
or $\quad n_1\left(\dfrac{D\lambda_1}{d}\right) = n_2\left(\dfrac{D\lambda_2}{d}\right)$ or $\lambda_2 = \left(\dfrac{n_1}{n_2}\right)\lambda_1$

$\therefore \quad \lambda_2 = \left(\dfrac{84}{62}\right) \times 4358$
$$\lambda_2 = 5904 \text{ Å}$$

33. Distance of nth maxima, $x = n\lambda \dfrac{D}{d} \propto \lambda$

$\Rightarrow \quad \lambda_b < \lambda_g$

$\therefore \quad x_{\text{blue}} < x_{\text{green}}$

34. As there is a hole at minima point P_2, the hole will act as a source of fresh light for the slits S_3 and S_4. Therefore, there will be a regular two slit pattern on the second screen. Hence, option (d) is correct.

35. Let intensity of each wave be I_0, then intensity at the centre of bright fringe will be $4I_0$.

Given, path difference, $\Delta x = \lambda / 8$

$\therefore$ Phase difference, $\phi = \Delta x \times \dfrac{2\pi}{\lambda}$

$\Rightarrow \quad \phi = \dfrac{\lambda}{8} \times \dfrac{2\pi}{\lambda}$ or $\phi = \pi / 4$

Intensity of light at this point,
$$I' = I_0 + I_0 + 2I_0 \cos (\pi / 4)$$
$$= 2I_0 + \sqrt{2}\, I_0 = 3.41\, I_0$$
Now, $\quad \dfrac{I'}{4I_0} = \dfrac{3.41 I_0}{4I_0} = \dfrac{3.41}{4} = 0.85$

36. As, $\dfrac{I_{\max}}{I_{\min}} = \dfrac{(a+b)^2}{(a-b)^2} = 9$ or $\dfrac{a+b}{a-b} = 3$

or $\quad 3a - 3b = a + b$

or $\quad 2a = 4b$

$\Rightarrow \quad a = 2b$

$\therefore \quad \dfrac{I_1}{I_2} = \dfrac{a^2}{b^2} = \dfrac{4b^2}{b^2} = 4:1$

37. $$3\lambda_1 = 4\lambda_2 \qquad \left[\because \frac{3D\lambda_1}{d} = 4\frac{D\lambda_2}{d}\right]$$

$\Rightarrow$ $$\lambda_2 = \frac{3}{4}\lambda_1 = \frac{3}{4} \times 590$$

$$= \frac{1770}{4} = 442.5 \text{ nm}$$

38. The intensity at a point on screen is given by

$$I = 4I_0 \cos^2(\phi/2)$$

where, ϕ is the phase difference. In this problem, ϕ arises (i) due to initial phase difference of $\frac{\pi}{4}$ and (ii) due to path difference for the observation point situated at $\theta = 30°$.

Thus, $$\phi = \frac{\pi}{4} + \frac{2\pi}{\lambda}(d\sin\theta) = \frac{\pi}{4} + \frac{2\pi}{\lambda} \times \frac{\lambda}{4}(\sin 30°)$$

$$= \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$

Hence, $$\frac{\phi}{2} = \frac{\pi}{4}$$

and $$I = 4I_0 \cos^2\left(\frac{\pi}{4}\right) = 2I_0$$

39. Amplitude of light is directly proportional to area through which light is passing.

For same length of slits,

$$\text{amplitude} \propto (\text{width})^{1/2}$$

Also, $$\text{intensity} \propto (\text{amplitude})^2$$

In YDSE, ratio of intensities of maxima and minima is given by

$$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

where, I_1 and I_2 are the intensities obtained from two slits, respectively.

$\Rightarrow$ $$\frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

where, a_1 and a_2 are light amplitudes from slits 1 and 2, respectively.

$\Rightarrow$ $$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{W_1} + \sqrt{W_2})^2}{(\sqrt{W_1} - \sqrt{W_2})^2}$$

where, W_1 and W_2 are the widths of slits, respectively.

Here, $$\left(\frac{W_1}{W_2}\right) = \left(\frac{a_1}{a_2}\right)^2 = \frac{4}{1}$$

$\Rightarrow$ $$\sqrt{\frac{W_1}{W_2}} = \frac{2}{1}$$

So, we have

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{W_1} + \sqrt{W_2}}{\sqrt{W_1} - \sqrt{W_2}}\right)^2 = \left(\frac{\sqrt{\frac{W_1}{W_2}} + 1}{\sqrt{\frac{W_1}{W_2}} - 1}\right)^2$$

$$= \left(\frac{2+1}{2-1}\right)^2 = 9:1$$

40. When sources are coherent, then

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

At middle point of the screen $\phi = 0°$, then

$$I_R = I + I + 2\sqrt{I \cdot I}\cos 0° = 4I$$

When sources are incoherent, then

$$I_R' = I_1 + I_2 = I + I = 2I$$

$\therefore$ $$\frac{I_R}{I_R'} = \frac{4I}{2I} = 2$$

41. Here, path difference,

$$\Delta x = (l_1 - 1)t = (1.5 - 1) \times \frac{2500}{3}\lambda$$

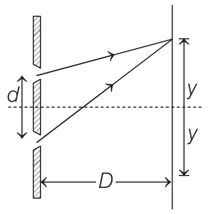
$\therefore$ $$\phi = \frac{2\pi}{\lambda}\Delta x$$

$\Rightarrow$ $$\phi = \frac{2\pi}{\lambda} \times \frac{2500}{6}\lambda = \frac{2500\pi}{3}$$

$$I = I_1 + I_2 + 2\sqrt{l_2 I_2}\cos\phi$$

$\therefore$ Intensity ratio $= 4:1$

42. In YDSE, let segment length is $2y$ as shown in the following figure.



Then, $$2y = 2 \times N \times \frac{\lambda D}{d}$$

where, $N =$ number of fringes in segment length $2y$.

$\Rightarrow$ $$N\lambda = \frac{yd}{D}$$

So, for same segment length $N\lambda$ is constant.

$\Rightarrow$ $$N_1\lambda_1 = N_2\lambda_2$$

Given, $$N_1 = 16, \; \lambda_1 = 700 \text{ nm}$$

$$N_2 = ?, \lambda_2 = 400 \text{ nm}$$

$\Rightarrow$ $$N_2 = \frac{N_1\lambda_1}{\lambda_2} = 16 \times \frac{700}{400} = 28$$

$\therefore$ 28 fringes are observed in same segment length.

43. The distance between zeroth order maxima and second order minima is

$$y_1 = \frac{\beta}{2} + \beta = \frac{3}{2}\beta = \frac{3}{2} \times 0.2 \text{ mm} = 0.3 \text{ mm}$$

$\therefore$ The distance of second maxima from point P is

$$y = (4.8 + 0.3) \text{ mm} = 5.1 \text{ mm}$$

44. The position of 30th bright fringe,

$$y_{30} = \frac{30\lambda D}{d}$$

Now, position shift of central fringe is

$$y_0 = \frac{30\lambda D}{d}$$

But we know, $y_0 = \frac{D}{d}(\mu - 1)t$

$$\frac{30\lambda D}{d} = \frac{D}{d}(\mu - 1)t$$

$$\Rightarrow \quad (\mu - 1) = \frac{30\lambda}{t} = \frac{30 \times (6000 \times 10^{-10})}{(3.6 \times 10^{-5})} = 0.5$$

$$\therefore \quad \mu = 1.5$$

45. Here, $d = 0.1\text{ mm} = 10^{-4}\text{m}, D = 20\text{ cm} = \frac{1}{5}\text{m}$

$$\lambda = 5460\text{ Å} = 5.46 \times 10^{-7}\text{ m}$$

Angular position of first dark ($n = 1$) fringe is

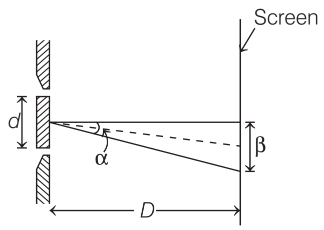
$$\theta = (2n - 1)\frac{\lambda}{2d} = (2 \times 1 - 1)\frac{\lambda}{2d} = \frac{\lambda}{2d} = \frac{5.46 \times 10^{-7}}{2 \times 10^{-4}}$$

$$= 2.73 \times 10^{-3}\text{ rad}$$

$$= 2.73 \times 10^{-3} \times \frac{180°}{\pi}$$

$$= 0.156° \simeq 0.16°$$

46.



Angular width of central maxima in YDSE is given by

$$\alpha = \frac{\beta}{D} = \frac{\lambda}{d}\text{ (in radian)}$$

$$= \frac{\lambda}{d} \times \frac{180°}{\pi}\text{ (in degree)}$$

Here, $\lambda = 500\text{ nm} = 500 \times 10^{-9}\text{m}$

and $d = 0.05\text{ mm} = 0.05 \times 10^{-3}\text{ m}$.

Hence, angular width is

$$\alpha = \frac{500 \times 10^{-9}}{0.05 \times 10^{-3}} \times \frac{180°}{\pi} = 0.57°$$

47. Here, $\sin\theta = \left(\frac{y}{D}\right)$

So, $\Delta\theta = \frac{\Delta y}{D}$

Angular fringe width, $\theta_0 = \Delta\theta$ (width $\Delta y = \beta$)

$$\Rightarrow \quad \theta_0 = \frac{\beta}{D} = \frac{D\lambda}{d} \times \frac{1}{D} = \frac{\lambda}{d}$$

Now, $\theta_0 = 1° = \pi/180$ rad

and $\lambda = 6 \times 10^{-7}$ m

$$\therefore \quad d = \frac{\lambda}{\theta_0} = \frac{180}{\pi} \times 6 \times 10^{-7}$$

$$= 3.44 \times 10^{-5}\text{ m}$$

$$\approx 0.03\text{ mm}$$

48. Given, wavelength of light, $\lambda = 600\text{ nm} = 600 \times 10^{-9}$ m

Angular width of fringe, $\theta = 0.1° = \frac{0.1\,\pi}{180}$ rad

Using the formula, $\theta = \frac{\lambda}{d}$

Spacing between the slits $d = \frac{\lambda}{\theta} = \frac{600 \times 10^{-9} \times 180}{0.1 \times \pi}$

$$d = 3.44 \times 10^{-4}\text{ m}$$

Thus, the spacing between the two slits is 3.44×10^{-4} m.

49. Path difference, $\Delta x = S_1S_3 - S_2S_3 = 0$

$$\therefore \quad \phi = \frac{2\pi}{\lambda}\Delta x = 0$$

$$\therefore \quad I_3 = I_0 + I_0 + 2\sqrt{I_0 \cdot I_0}(\cos 0°)$$

$$\therefore \quad I_3 = 4I_0$$

The path difference at S_4 is

$$\Delta x' = S_1S_4 - S_2S_4 = \frac{xd}{D} \quad \left(\text{Here, } x = \frac{\lambda D}{2d}\right)$$

$$= \frac{d}{D} \times \frac{\lambda D}{2d} = \frac{\lambda}{2}$$

$$\therefore \quad \phi' = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$$

$$\therefore \quad I_4 = I_0 + I_0 + 2I_0\cos\pi = 0$$

$$\therefore \quad \text{Ratio} = \frac{I_3}{I_4} = \frac{4I_0}{0} = \infty$$

50. Consider that the slit widths are equal, so they produce waves of equal intensity say, I'. Resultant intensity at any point $I_R = 4I'\cos^2\phi$, where ϕ is the phase difference between the wave at the point observation.

For maximum intensity,

$$\phi = 0° \Rightarrow I_{max} = 4I' = I \quad \ldots\text{(i)}$$

If one of the slits is closed, resultant intensity at the same point will be I' only, *i.e.*

$$I' = I_0 \quad \ldots\text{(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$I = 4I_0$$

51. In YDSE, intensity at a point is given by

$$I_{net} = 2I_0(1 + \cos\phi) \quad \ldots\text{(i)}$$

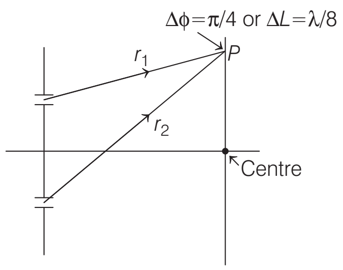
where, ϕ = phase difference between interfering waves.

Here, $\phi = \frac{1}{8}\lambda = \frac{1}{8} \times 2\pi$

(a path difference of one wavelength is equivalent to a phase difference of 2π radians)

$$\Rightarrow \quad \phi = \frac{\pi}{4} \Rightarrow \cos\phi = \frac{1}{\sqrt{2}} \approx 0.7$$

So, I_{net} at given point P is



Also, intensity maxima at centre of a bright fringe is obtained by substituting $\phi = 0, 2\pi \ldots n\pi$ in Eq. (i).

So, $\quad I_{net} = I_2 = 2I_0(1 + \cos 0^\circ) = 4I_0$

Hence, required ratio is

$$\frac{I_1}{I_2} = \frac{2 \times 1.7 I_0}{4I_0} = \frac{1.7}{2} = 0.853$$

52. Given, angular width, $\theta = 0.2^\circ$

Distance between screen and slit, $D = 1$ m

Wavelength of light, $\lambda = 600$ nm $= 600 \times 10^{-9}$ m

Refractive index of water, $\mu_w = \frac{4}{3}$

Using the formula of angular width,

$$\theta = \frac{\lambda}{D} \quad \ldots\text{(i)}$$

and $$\theta' = \frac{\lambda'}{D} \quad \left(\text{where, } \lambda' = \frac{\lambda}{\mu}\right) \quad \ldots\text{(ii)}$$

Dividing Eq.(ii) from Eq.(i), we get

$$\frac{\theta'}{\theta} = \frac{\lambda'}{\lambda} = \frac{\lambda}{\mu\lambda}$$

or $$\theta' = \frac{\theta}{\mu} = \frac{0.2 \times 3}{4} = 0.15^\circ \quad \left(\because \mu = \frac{4}{3}\right)$$

Thus, the angular fringe width is 0.15° as the apparatus is immersed in water.

53. Path difference introduced by a slab of thickness t and refractive index μ is given by

$$\Delta x = (\mu - 1)\, t$$

Position of the fringe is $x = \frac{\Delta x D}{d} = \frac{(\mu - 1)\, t\, D}{d}$

Also, fringe width is given by

$$\beta = \frac{\lambda D}{d}$$

According to the question, $n\beta = x$

$$\Rightarrow \quad \frac{n\lambda D}{d} = (\mu - 1)\, t \frac{D}{d}$$

$$\Rightarrow \quad n\lambda = (\mu - 1)\, t$$

or $$t = \frac{n\lambda}{(\mu - 1)}$$

54. For 5th ($n = 5$) dark fringe, $x_1 = (2n - 1)\frac{\lambda}{2}\frac{D}{d} = \frac{9\lambda D}{2d}$

For 7th ($n = 7$) bright fringe, $x_2 = n\lambda \frac{D}{d} = \frac{7\lambda D}{d}$

Now, $$x_2 - x_1 = (\mu - 1)\, t \frac{D}{d}$$

$$\Rightarrow \quad \frac{\lambda D}{d}\left[7 - \frac{9}{2}\right] = (\mu - 1)\, t \frac{D}{d}$$

or, $$t = \frac{2.5\lambda}{(\mu - 1)}$$

55. We know that, separate wavefront originating from two sources produce interference whereas secondary wavelets originating from different parts of same wavefront constitute diffraction. Hence, option (d) is correct.

56. At the centre, all colours meet in phase, hence central fringe is white.

57. Here, the width of the slit is 10^4 Å, *i.e.* 10000 Å. The wavelength of (visible) sunlight varies from 4000 Å to 8000 Å. As width of slit $a > \lambda$ (wavelength of light), therefore no diffraction occurs. The image seen through the slit shall be a fine sharp slit white in colour at the centre.

58. Given, aperture, $a = 4$ mm $= 4 \times 10^{-3}$ m

Wavelength, $\lambda = 400$ nm $= 400 \times 10^{-9}$ m

Ray optics is good approximation upto a distance equal to Fresnel's distance (Z_F).

$$Z_F = \frac{a^2}{\lambda} = \frac{4 \times 10^{-3} \times 4 \times 10^{-3}}{400 \times 10^{-9}}$$

$$Z_F = 40 \text{ m}$$

59. From diffraction at a single slit,

size of aperture, $a = \frac{\lambda}{\sin\theta}$

or $$a = \frac{3141.59 \times 10^{-10}}{\sin 1^\circ}$$

$$= 18 \times 10^{-6} \text{ m} = 18 \mu\text{m}$$

60. As, $a \sin\theta = n\lambda$

$$\Rightarrow \quad \lambda = \frac{a \sin\theta}{n} = \frac{5 \sin 30^\circ}{1} = 2.5 \text{ cm}$$

61. Here, $\lambda = 6250$ Å $= 6250 \times 10^{-10}$ m

$a = 2 \times 10^{-2}$ cm $= 2 \times 10^{-4}$ m

$D = 50$ cm $= 0.5$ m

$\therefore$ Width of central maxima $= \frac{2\lambda D}{a}$

$$= \frac{2 \times 6250 \times 10^{-10} \times 0.5}{2 \times 10^{-4}}$$

$$= 312.5 \times 10^{-3} \text{ cm}$$

62. Distance between first and sixth minima,

$$x = \frac{5\lambda D}{d}$$

For 1st maxima,

$$\therefore \quad d = \frac{n\lambda D}{x} = \frac{1 \times 5000 \times 10^{-10} \times 0.5}{0.5 \times 10^{-3}}$$

$$d = \frac{5000 \times 10^{-10} \times 0.5}{0.5 \times 10^{-3}}$$

$$\Rightarrow \quad d = 5 \times 10^{-4} = 0.5 \times 10^{-3} = 0.5 \text{ mm}$$

63. As, $a \sin\theta = n\lambda$

$$\Rightarrow \quad \frac{ax}{D} = 3\lambda \quad \text{(Hence, focal length is used as } D\text{)}$$

or $$\lambda = \frac{ax}{3D}$$

$$= \frac{0.3 \times 10^{-3} \times 5 \times 10^{-3}}{3 \times 1}$$

$$= 5 \times 10^{-7} \text{ m} = 5000 \text{ Å}$$

64. The linear width of central principal maximum $= \dfrac{2\lambda D}{d}$

If it is equal to width of slit (d), then

$$\frac{2\lambda D}{d} = d \text{ or } D = \frac{d^2}{2\lambda}$$

65. The intensity of maxima in the diffraction pattern due to single slit is given by

$$I = \frac{I_0}{\left[\left(\frac{2n+1}{2}\right)\pi\right]^2} \quad \text{...(i)}$$

$\therefore$ For I_1, put $n = 1$ in Eq. (i), we have

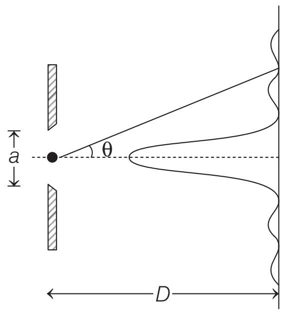
$$I_1 = \frac{I_0}{\left[\left(\frac{2\times 1+1}{2}\right)\pi\right]^2} = \frac{I_0}{\frac{9\pi^2}{4}} = \frac{4I_0}{9\pi^2}$$

Similarly, for I_2, put $n = 2$ in Eq. (ii), we have

$$I_2 = \frac{I_0}{\left[\left(\frac{2\times 2+1}{2}\right)\pi\right]^2} = \frac{I_0}{\frac{25\pi^2}{4}} = \frac{4}{25\pi^2} I_0$$

$$\therefore \quad I_0 : I_1 : I_2 = I_0 : \frac{4I_0}{9\pi^2} : \frac{4}{25\pi^2} I_0 = 1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2}$$

66. In case of a single slit diffraction pattern as shown below



angular position of nth minima is given by

$$\sin\theta_n = \frac{n\lambda}{D} \quad \text{...(i)}$$

Given, $\theta_2 = 60°$ and $\lambda = 6000 \times 10^{-8}$ cm

As, $\quad \sin\theta_2 = \dfrac{2\lambda}{D}$

We have, $\quad \sin 60° = \dfrac{2\lambda}{D}$

or $\quad \dfrac{\lambda}{D} = \dfrac{\sin 60°}{2} = 0.43 \approx \sin 25°$

[**Note** $\sin 30° = 0.5$, $\sin 25° = 0.422$ and $\sin 20° = 0.34$]

Now, if θ_1 = angular position of first minima, then

$$\sin\theta_1 = \frac{\lambda}{D} \quad \text{[from Eq. (i), } n = 1\text{]}$$

$\Rightarrow \quad \sin\theta_1 = 0.43 \approx \sin 25°$ or $\theta_1 \approx 25°$

67. Limit of resolution for a telescope from Rayleigh's criteria is

$$\theta_R = \frac{1.22\,\lambda}{D}$$

Here, $\quad D = 250 \text{ cm} = 250 \times 10^{-2}$ m

and $\quad \lambda = 600 \text{ nm} = 600 \times 10^{-9}$m

So, limit of resolution is

$$\theta_R = \frac{1.22 \times 600 \times 10^{-9}}{250 \times 10^{-2}} = 2.93 \times 10^{-7} \text{ rad}$$

$$\approx 3 \times 10^{-7} \text{rad}$$

68. For a telescope, limit of resolution is given by

$$\Delta\theta = \frac{1.22\lambda}{D}$$

Here, $\quad \lambda = 500 \text{ nm} = 500 \times 10^{-9}$ m,

$D = 200 \text{ cm} = 200 \times 10^{-2}$ m

So, limit of resolution is $\Delta\theta = \dfrac{1.22 \times 500 \times 10^{-9}}{200 \times 10^{-2}}$

$$= 305 \times 10^{-9} \text{ rad}$$

69. Given, $\lambda = 5000\text{Å} = 5000 \times 10^{-10}$ m and

$\text{NA} = 1.25$

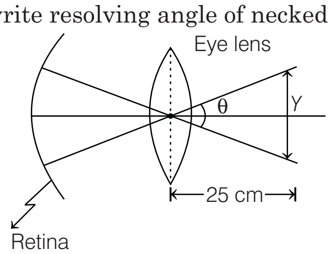
Now, $\quad d = \dfrac{0.61\,\lambda}{\text{NA}} = \dfrac{0.61 \times 5000 \times 10^{-10}}{1.25}$

or $\quad d = \dfrac{3.05}{1.25} \times 10^{-7}$ m

$= 2.4 \times 10^{-7}$ m

or $\quad d = 0.24\,\mu$m

70. We can write resolving angle of necked eye as



$$\theta = 1.22\,\frac{\lambda}{D}$$

where, D is the diameter of eye lens.

or $\quad \dfrac{Y}{25 \times 10^{-2}} = \dfrac{1.22 \times 500 \times 10^{-9}}{0.25 \times 2 \times 10^{-2}}$

$Y = 30 \times 10^{-6}$ m $= 30\,\mu$m

71. According to Brewster's law, the light reflected from the top of glass slab gets polarised. The light refracted into the glass slab and the light emerging from the glass slab is only partially polarised. Therefore, when a polaroid is held in the path of emergent light at P, and rotated about an axis passing through the centre and perpendicular to plane of polaroid, the intensity of light shall go through a minimum but not zero for two orientations of the polaroid.

72. When a beam of unpolarised light is reflected from a transparent medium of refractive index μ, then the reflected light is completely plane polarised at a certain angle of incidence i_B, which is known as Brewster's angle.

In the given condition, the light reflected irrespective of an angle of incidence is never completely polarised.
So, $i_C > i_B$
where, i_C is the critical angle.

$\Rightarrow \quad \sin i_C < \sin i_B \quad$ ...(i)

From Brewster's law, we know that

$$\tan i_B = {}^{w}\mu_g = \frac{\mu_{\text{glass}}}{\mu_{\text{water}}} = \frac{1.5}{\mu} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1}{\mu} < \frac{1.5}{\sqrt{(1.5)^2 + (\mu)^2}}$$

$$\Rightarrow \quad \sqrt{(1.5)^2 + \mu^2} < 1.5\,\mu$$

$$\mu^2 + (1.5)^2 < (1.5\,\mu)^2 \text{ or } \mu < \frac{3}{\sqrt{5}}$$

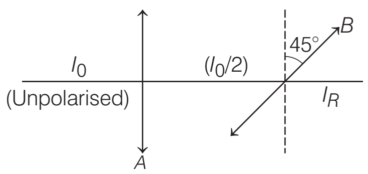
$\therefore$ The minimum value of μ should be $\frac{3}{\sqrt{5}}$.

73. We know, $\lambda_m = \frac{\lambda_a}{\mu}$

and $\quad \mu = \tan\theta$

$$\therefore \quad \lambda_m = \frac{\lambda_a}{\tan\theta} = \lambda_a \cot\theta$$

74. Relation between intensities is



$$I_R = \left(\frac{I_0}{2}\right)\cos^2(45^\circ)$$

$$= \frac{I_0}{2} \times \frac{1}{2} = \frac{I_0}{4}$$

75. The intensity of plane polarised light $= 2a^2$.

$\therefore$ Intensity of polarised light from thin polaroid

$$= \frac{I_0}{2} = \frac{1}{2} \times 2a^2 = a^2$$

76. Intensity of light from $C_2 = I_0$

On rotating the crystal through 60°,

$$I = I_0 \cos^2 60^\circ \quad \text{(Malus' law)}$$

$$= I_0\left(\frac{1}{2}\right)^2 = I_0/4$$

77. If I_0 is intensity of unpolarized light,

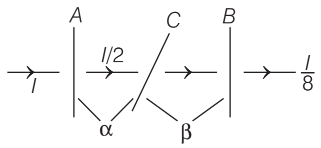
then from 1st Nicol prism, $I_1 = \frac{I_0}{2}$

From 2nd Nicol prism, $I_2 = I_1 \cos^2(90^\circ - 60^\circ)$

$$= \frac{I_0}{2}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{8} I_0$$

$$\Rightarrow \quad \frac{I_2}{I_0} = 0.375 = 37.5\ \%$$

78.



Using Malus's law, intensity available after polariser C

$$I' = \frac{I}{2} \times \cos^2\alpha$$

and intensity available after polariser

$$B = I'\cos^2\beta = \frac{I}{2}\cos^2\alpha \times \cos^2\beta = \frac{I}{8} \quad \text{(given)}$$

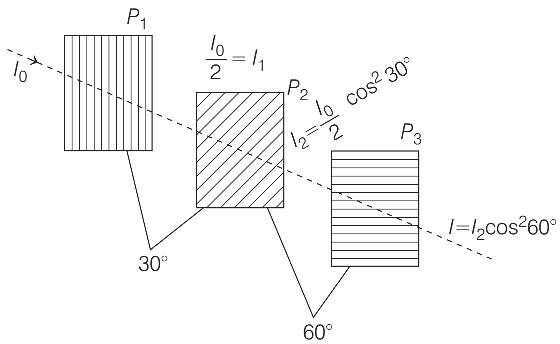
So, $\frac{I}{2} \times \cos^2\alpha \cdot \cos^2\beta = \frac{I}{8}$

$$\Rightarrow \quad \cos^2\alpha \cdot \cos^2\beta = \frac{1}{4}$$

This is satisfied with $\alpha = 45^\circ$ and $\beta = 45^\circ$

So, angle between A and C is 45°.

79.



When unpolarised light pass through polaroid P_1, intensity obtained is

$$I_1 = \frac{I_0}{2}$$

where, I_0 = intensity of incident light.

Now, this transmitted light is polarised and it passes through polariser P_2. So, intensity I_2 transmitted is obtained by Malus' law.

$$\Rightarrow \quad I_2 = I_1 \cos^2\theta$$

As angle of pass axis of P_1 and P_3 is 90° and angle of pass axis of P_2 and P_3 is 60°, so angle between pass axis of P_1 and P_2 is $(90^\circ - 60^\circ) = 30^\circ$.

So, $\quad I_2 = \frac{I_0}{2}\cos^2 30^\circ$

$$= \frac{I_0}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{8} I_0$$

When this light pass through third polariser P_3, intensity I transmitted is again obtained by Malus' law.

So, $\quad I = I_2 \cos^2 60^\circ = \left(\frac{3}{8} I_0\right)\cos^2 60^\circ$

$$= \frac{3}{8} I_0 \times \left(\frac{1}{2}\right)^2 = \frac{3}{32} I_0$$

So, ratio $\frac{I_0}{I} = \frac{32}{3} = 10.67$.

80. Given, wavelength of H_α, $\lambda = 6563\ Å = 6563 \times 10^{-10}$ m
Red- shift $\Delta\lambda = 15\ Å = 15 \times 10^{-10}$m
Since, the star is found to be red-shifted, hence star is receding away from earth and doppler's shift is negative.

$$\Delta\lambda = -v\frac{\lambda}{c}$$

$$v = -\frac{\Delta\lambda \cdot c}{\lambda} = -\frac{15 \times 10^{-10} \times 3 \times 10^8}{6563 \times 10^{-10}}$$

$$v = -6.86 \times 10^5 \text{ m/s}$$

Negative sign shows that the star is receding (away) from earth.

Round II

1. The refractive index of air is slightly more than 1. When chamber is evacuated, refractive index decreases and hence, the wavelength increases and fringe width also increases.

2. Given, $\lambda = 600 \text{ nm} = 6 \times 10^{-7}$ m,

$$a = 1 \text{ mm} = 10^{-3} \text{m}, D = 2\text{m}$$

Distance between the first dark fringes on either side of central bright fringe = width of central maximum

$$= \frac{2\lambda D}{a} = \frac{2 \times 6 \times 10^{-7} \times 2}{10^{-3}}$$

$$= 24 \times 10^{-4} \text{ m} = 2.4 \text{ mm}$$

3. From, $(\mu - 1)\, t = n\lambda$, we get,

$$t = \frac{n\lambda}{(\mu - 1)}$$

When $n\lambda = \frac{\lambda}{2}$,

then $t = \frac{\lambda}{2\,(\mu - 1)}$

4. Red light and blue light have different wavelengths and different frequencies.

5. As size of hole (a) is increased, width of central maximum of diffraction pattern of hole $\left(= \frac{2\,f\,\lambda}{a}\right)$ decreases. As the same amount of light emerge is now distributed over a smaller area, the intensity increases.

6. Here, $d = 1 \text{ mm} = 10^{-3}$ m,

$$\lambda = 6.5 \times 10^{-7} \text{ m}$$

$$D = 1 \text{ m}$$

Now, $x_5 = n\lambda\frac{D}{d} = 5 \times 6.5 \times 10^{-7} \times \frac{1}{10^{-3}}$

$$= 32.5 \times 10^{-4} \text{ m}$$

and $x_3 = (2n - 1)\frac{\lambda}{2}\frac{D}{d}$

$$= \frac{(2 \times 3 - 1) \times 6.5 \times 10^{-7}}{2 \times 10^{-3}}$$

$$= 16.25 \times 10^{-4} \text{ m}$$

$\therefore$ $x_5 - x_3 = (32.5 - 16.25)10^{-4}$ m

$$= 16.25 \times 10^{-4} \text{ m} = 1.63 \text{ mm}$$

7. Here, wavelength of light $(\lambda) = 5303\ Å$.
Distance between two slit (d) = 19.44 μm
Width of single slit (a) = 4.05 μm
Here, angular width between first and second diffraction minima, $\theta \simeq \frac{\lambda}{a}$
and angular width of a fringe due to double slit is

$$\theta' = \frac{\lambda}{d}$$

$\therefore$ Number of fringes between first and second diffraction minima, $n = \frac{\theta}{\theta'} = \frac{\frac{\lambda}{a}}{\frac{\lambda}{d}}$

$$= \frac{d}{a} = \frac{19.44}{4.05} = 4.81$$

or $n \simeq 5$

$\therefore$ 5 interfering bright fringes lie between first and second diffraction minima.

8. Here, $\omega = 31.4 \text{ rads}^{-1}$
$\therefore$ Time period of revolution,

$$T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{31.4} = 0.2 \text{ s}$$

Energy transmitted/revolution,

$$= (IA)T = \left(\frac{I_0}{2}A\right)T$$

$$= \frac{\phi_0 T}{2} = \frac{10^{-3} \times 0.2}{2} = 10^{-4} \text{ J}$$

9. Distance, $S_2P = (d^2 + b^2)^{1/2} = d\left(1 + \frac{b^2}{d^2}\right)^{1/2}$

$$= d\left(1 + \frac{b^2}{2d^2}\right)$$

$$= d + \frac{b^2}{2d}$$

and $S_1P = d$

$\therefore$ Path difference $= S_2P - S_1P$

$$x = d + \frac{b^2}{2d} - d = \frac{b^2}{2d}$$

For missing wavelengths, $(2n - 1)\frac{\lambda}{2} = x = \frac{b^2}{2d}$

For $n = 1$, $\lambda = \frac{b^2}{d}$

For $n = 2$, $\lambda = \frac{2b^2}{3d}$

10. From the pattern of intensity distribution seen in the central portion, we find that
(a) as intensity of successive maxima is the same, S_1 and S_2 have the same intensity.

(b) as width of successive maxima appears to increase slightly, S_1, S_2 must have a constant phase difference.

(d) as minimum intensity is zero, S_1, S_2 must have the same wavelength.

11. The resultant intensity at any point P is

$$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

$\therefore$ $$I_0 = 4I_0 \cos^2 \phi/2$$

or $$\cos\frac{\phi}{2} = \frac{1}{2}$$

$\therefore$ $$\frac{\phi}{2} = \frac{\pi}{3} \text{ or } \phi = \frac{2\pi}{3}$$

If Δx is the corresponding value of path difference at P, then

$$\phi = \frac{2\pi}{\lambda}(\Delta x)$$

$\Rightarrow$ $$\frac{2\pi}{3} = \frac{2\pi}{\lambda}\Delta x$$

As, $$\Delta x = \frac{xd}{D}$$

$\therefore$ $$\frac{1}{3} = \frac{1}{\lambda}\frac{xd}{D}$$

or $$x = \frac{\lambda}{3d/D} = \frac{6\times10^{-7}}{3\times10^{-4}} = 2\times10^{-3} \text{ m}$$

$$x = 2 \text{ mm}$$

This is the distance of point P from central maximum.

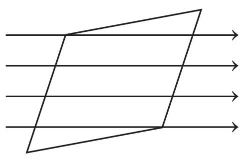
12. Points c and d are on same wavefront.

i.e. $$\phi_c = \phi_d$$

Also, $$\phi_e = \phi_f$$

Thus, $$\phi_d - \phi_f = \phi_c - \phi_e$$

13. Parallel cylindrical beam gives planar wavefront.



14. Here, $a_1 = 2a_2 \Rightarrow I_1 = 4I_2 = 4I_0$

As, $$I_m = (\sqrt{I_1} + \sqrt{I_2})^2 = (3\sqrt{I_2})^2 = 9I_2 = 9I_0$$

$\therefore$ $$I_0 = \frac{I_m}{9} \quad \text{...(i)}$$

Now, $$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$
$$= 4I_0 + I_0 + 2\sqrt{4I_0 I_0}\cos\phi$$
$$= 5I_0 + 4I_0\cos\phi$$
$$= \frac{I_m}{9}(5 + 4\cos\phi) \quad \text{[from Eq. (i)]}$$
$$= \frac{I_m}{9}[1 + 4(1+\cos\phi)]$$
$$= \frac{I_m}{9}\left(1 + 8\cos^2\frac{\phi}{2}\right)$$

15. Intensity falling on polariser, $I_0 = 3.3 \text{ W/m}^2$

Area of polariser, $A = 3\times10^{-4} \text{ m}^2$

Angular speed of rotation of polariser,

$$\omega = 31.4 \text{ rad/s}$$

Now, average energy is given by

$$< E > = < IAt >$$
$$< E > = < I_0 \cos^2\theta \times A \times t >$$
$$< E > = I_0 At < \cos^2\theta >$$
$$< E > = I_0 At\left(\frac{1}{2}\right) \quad \left[\because < \cos^2\theta > = \frac{1}{2}\right]$$

Now, the calculation is about one revolution, then

Time taken = Time period

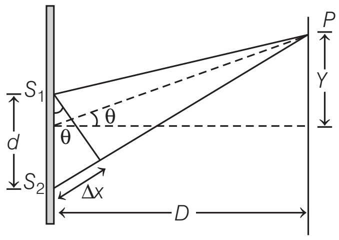
$$t = T = \frac{2\pi}{\omega} = \frac{2\pi}{31.4} = \frac{2\pi}{10\pi} = \frac{1}{5} \text{ s}$$

Now, $$< E > = I_0 At\left(\frac{1}{2}\right)$$

Substituting all the values in it, we get

$$< E > = 3.3 \times 3\times10^{-4} \times \frac{1}{5} \times \left(\frac{1}{2}\right)$$
$$= 0.99\times10^{-4} \text{ J}$$
$$\simeq 1\times10^{-4} \text{ J}$$

16. For microwave, $\lambda = \dfrac{c}{v} = \dfrac{3\times10^8}{10^6} = 300 \text{ m}$



As, $$\Delta x = d\sin\theta$$

Phase difference, $\phi = \dfrac{2\pi}{\lambda}$ (Path difference)

$$= \frac{2\pi}{\lambda}(d\sin\theta)$$
$$= \frac{2\pi}{300}(150\sin\theta) = \pi\sin\theta$$

$\therefore$ $$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Here, $I_1 = I_2$ and $\phi = \pi\sin\theta$

$\therefore$ $$I_R = 2I_1[1 + \cos(\pi\sin\theta)] = 4I_1\cos^2\left(\frac{\pi\sin\theta}{2}\right)$$

I_R will be maximum, when $\cos^2\left(\dfrac{\pi\sin\theta}{2}\right) = 1$

$\therefore$ $$(I_R)_{\max} = 4I_1 = I_0$$

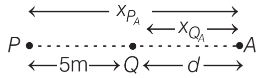
Hence, $$I = I_0\cos^2\left(\frac{\pi\sin\theta}{2}\right)$$

If $\theta = 0$, then $I = I_0\cos^2 0° = I_0$

If $\theta = 30°$, then $I = I_0\cos^2(\pi/4) = I_0/2$

If $\theta = 90°$, then $I = I_0\cos^2(\pi/2) = 0$

17. **For point A**



Path difference, $\Delta x = x_{P_A} - x_{Q_A} = 5$ m

Phase difference due to path difference,

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{20} \times 5$$

$$\Rightarrow \qquad \Delta\phi = \frac{\pi}{2}$$

Also, phase of P is ahead of Q by 90° or $\frac{\pi}{2}$.

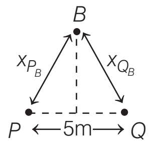
So, total phase difference at point A,

$$\Delta\phi = \frac{\pi}{2} - \frac{\pi}{2} = 0°$$

$\therefore$ Intensity at point A,

$$\begin{aligned} I_A &= I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi \\ &= I_0 + I_0 + 2I_0 \cos 0° \\ &= 4\,I_0 \end{aligned}$$

For point B



$$\Delta x = x_{P_B} - x_{Q_B} = 0$$

$$\therefore \qquad \Delta\phi = 0 \qquad \text{(due to path difference)}$$

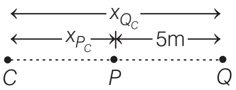
So, total phase difference at point B,

$$\Delta\phi = \frac{\pi}{2} + 0 = 90°$$

$\therefore$ Intensity at point B,

$$\begin{aligned} I_B &= I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi \\ &= I_0 + I_0 + 2I_0 \cos 90° = 2I_0 \end{aligned}$$

For point C



$$\Delta x = x_{Q_C} - x_{P_C} = 5 \text{ m}$$

$\Delta\phi$ due to path difference,

$$\Delta\phi = \pi/2$$

Total phase difference at point C,

$$\Delta\phi = \frac{\pi}{2} + \frac{\pi}{2} = \pi = 180°$$

($\because$ phase of P is ahead of Q by 90°)

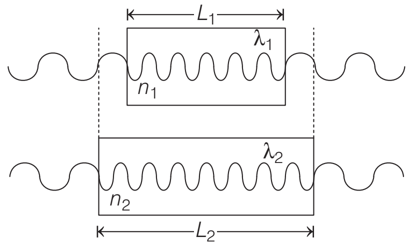
$\therefore$ Intensity at point C,

$$\begin{aligned} I_C &= I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi \\ &= I_0 + I_0 + 2I_0 \cos 180° = 0 \end{aligned}$$

Ratio of intensities,

$$I_A : I_B : I_C = 4I_0 : 2I_0 : 0$$
$$I_A : I_B : I_C = 2 : 1 : 0$$

18.



Let λ_1 be the wavelength inside medium of refractive index n_1 and it travels a length L_1. Let λ_2, n_2 and L_2 be the parameters for second wave.

$$\text{So,} \qquad \frac{\lambda_1}{v} = \frac{\lambda}{c}$$

$$\Rightarrow \qquad \lambda_1 = \frac{\lambda}{n_1} \text{ and } \lambda_2 = \frac{\lambda}{n_2}$$

Let there are N_1 number of wavelengths in medium 1 of length L_1, then

$$N_1 = \frac{L_1}{\lambda_1} = \frac{L_1 n_1}{\lambda} \text{ and } N_2 = \frac{L_2}{\lambda_2} = \frac{L_2 n_2}{\lambda}$$

So, difference in number of waves, ΔN

$$= N_1 - N_2 = \frac{L_1 n_1}{\lambda} - \frac{L_2 n_2}{\lambda} = \frac{L_1 n_1 - L_2 n_2}{\lambda}$$

As each wavelength is equivalent to a phase of 2π, we can write

$$\begin{aligned} \Delta Q &= 2\pi\,\Delta N = 2\pi(N_1 - N_2) \\ &= 2\frac{\pi}{\lambda}(L_1 n_1 - L_2 n_2) \end{aligned}$$

19. We know in interference,

$$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2 \text{ while } I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

$$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$$

According to problem,

$$\frac{A_1}{A_2} = \frac{3}{5}, \frac{I_{\max}}{I_{\min}} = \frac{(3+5)^2}{(3-5)^2} = \frac{16}{1}$$

$$\left(\because \frac{I_{\max}}{I_{\min}} = \frac{x}{1}, \text{given}\right)$$

$$\Rightarrow \qquad x = 16$$

20. For dark dringe at P, $S_1 P - S_2 P = \frac{(2n-1)\lambda}{2}$

$$S_1 P - S_2 P = \frac{5\lambda}{2}$$

$$= 5 \times \frac{6000}{2} = 15000 = 1.5 \text{ micron}$$

21. Given, in a YDSE,

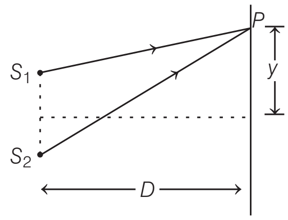
Fringe width of 15 fringes for wavelength of light of 500 nm

Fringe width of 10 fringes for wavelength of light of λ nm

$$\Rightarrow \qquad \frac{15 \times 500 \times D}{d} = \frac{10 \times \lambda \times D}{d}$$

$$\Rightarrow \qquad \lambda = \frac{15 \times 500}{10} = 750 \text{ nm}$$

22.



The maximum path difference that can be produced = distance between the sources, *i.e.* 5 cm.

Hence, in this condition, we can have only 3 maximas, *i.e.* one central maxima and two on its either sides for a path difference of λ or 3 cm.

Now, for maximum intensity at P

$$S_2P - S_1P = \lambda$$

or $$\sqrt{(y + d/2)^2 + D^2} - \sqrt{(y - d/2)^2 + D^2} = \lambda$$

Putting, $d = 5$ cm, $D = 100$ cm

and $\lambda = 3$ cm, we get

$$y = \pm 75 \text{ cm}$$

Thus, the three maximas will be at

$$y = 0 \text{ and } y = \pm 75 \text{ cm}$$

23. In single slit diffraction, if width of slit is a, wavelength of light is λ and angular position on screen is θ, then for nth minima, the condition is

$$a\sin(\theta) = n\lambda$$

$$\Rightarrow \quad \sin(\theta) = \frac{n\lambda}{a} \quad \ldots\text{(i)}$$

$$\because \quad \sin(\theta) < 1$$

$$\Rightarrow \quad \frac{n\lambda}{a} < 1$$

$$\Rightarrow \quad n < \frac{a}{\lambda}$$

$$\Rightarrow \quad n < \frac{0.6 \times 10^{-4}}{6000 \times 10^{-10}}$$

$$\Rightarrow \quad n < \frac{10^{-4}}{10^4 \times 10^{-10}}$$

$$\Rightarrow \quad n < 10^2 \quad \Rightarrow \quad n < 100$$

So, maximum possible number of minima on one side of central maximum = 99.

Therefore, maximum possible number of minima on both sides of central maximum

$$= 2 \times 99 = 198$$

24. For path difference of λ, *i.e.* $\Delta x = \lambda$

Phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{\lambda} \times \lambda = 2\pi$$

Intensity, $I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi$

As, light is monochromatic, so

$$I_1 = I_2 = I_0 \text{ (say)}$$

and $$I = K \text{ (given)}$$

$$\Rightarrow \quad K = I_0 + I_0 + 2\sqrt{I_0}\sqrt{I_0}\cos 2\pi$$

$$K = 4I_0 \text{ or } I_0 = \frac{K}{4}$$

For path difference of $\frac{\lambda}{6}$, *i.e.* $\Delta x = \frac{\lambda}{6}$.

Phase difference, $\Delta\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{6} = \frac{\pi}{3}$

So, intensity, $I' = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi$

$$= I_0 + I_0 + 2\sqrt{I_0}\sqrt{I_0}\cos\frac{\pi}{3}$$

$$= \frac{K}{4} + \frac{K}{4} + 2 \times \frac{K}{4} \times \frac{1}{2}$$

$$= \frac{3K}{4} = \frac{9K}{12}$$

Given, $$I' = \frac{nK}{12}$$

On comparing the both, we get

$$n = 9$$