ALL INDIA TEST SERIES

JEE (Advanced)-2024

PART TEST – I

PAPER –2

Time Allotted: 3 Hours **Maximum Marks: 180**

General Instructions:

- The test consists of total 51 questions.
- Each subject (PCM) has 17 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Three Sections**: **Section-A, Section-B & Section-C**.

Section – A *(01 – 04, 18 – 21, 35 – 38)*: This section contains **TWELVE (12)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.

Section – A *(05 –07, 22 – 24, 39 – 41)*: This section contains **NINE (09)** questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).

Section – B *(08 – 13, 25 – 30, 42 – 47):* This section contains **EIGHTEEN (18)** numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.

Section – C *(14 –17, 31 – 34, 48 – 51)*: This section contains **SIX (06) paragraphs.** Based on each paragraph, there are **TWO (02)** questions of numerical answer type. The answer to each question is a **NUMERICAL VALUE (XXXXX.XX).** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

MARKING SCHEME

Section – A (Single Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct option is chosen.
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

Section – A (One or More than One Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If only (all) the correct option(s) is (are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial marks	:	+2	If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–2	In all other cases.

Section – B: Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If ONLY the correct integer is entered;
Zero Marks	:	0	Question is unanswered;
Negative Marks	:	0	In all other cases.

Section – C: Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct integer is entered;
Zero Marks	:	0	Question is unanswered;
Negative Marks	:	0	In all other cases.

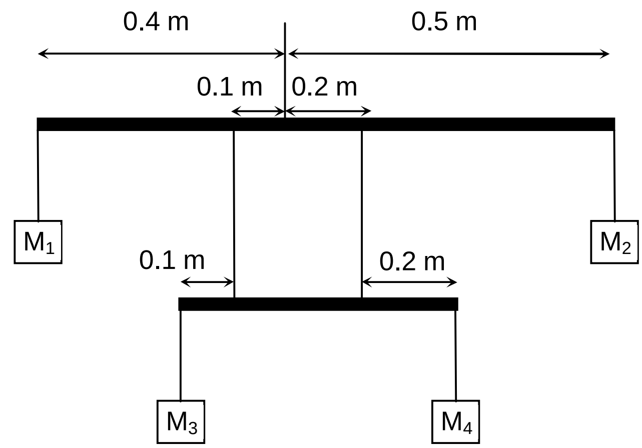
Physics

PART – I

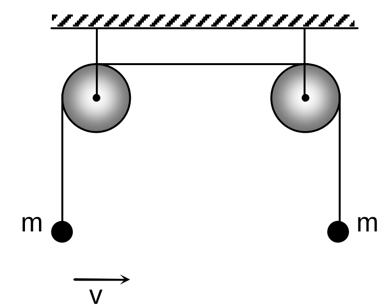
SECTION – A
(One Options Correct Type)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

1. In the diagram below, the two cross beams and the seven supporting strings are all massless. The hanging objects are M_1 = 400 g, M_2 = 200 g, and M_4 = 500 g. What is the value of M_3 for the system to be in static equilibrium?
(A) 400 g
(B) 500 g
(C) 600 g
(D) 700 g

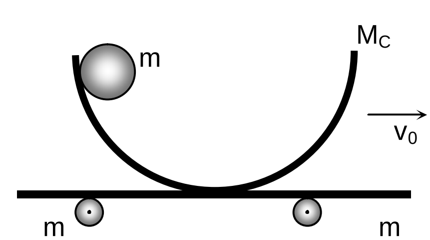


2. Two particles of mass m are connected by pulleys as shown. The mass on the left is given a small horizontal velocity, and oscillates back and forth. The mass on the right
(A) remains at rest
(B) oscillates vertically, and with a net upward motion
(C) oscillates vertically, and with a net downward motion
(D) oscillates vertically, with no net motion



3. Two particles with mass m_1 and m_2 are connected by a massless rigid rod of length L and placed on a horizontal frictionless table. At time t = 0, the first mass receives an impulse perpendicular to the rod, giving it speed v. At this moment, the second mass is at rest. The time after which the second mass is at rest again is
(A) $t = \frac{2\pi L}{v}$
(B) $t = \frac{\pi(m_1 + m_2)L}{m_2 v}$
(C) $t = \frac{2\pi m_2 L}{(m_1 + m_2)v}$
(D) $t = \frac{2\pi m_1 L}{(m_1 + m_2)v}$

4. A ball of mass m rolls in a bowl bolted on a cart. The bowl and cart move together and have a combined mass Mc, as shown in the figure. Initially, the cart moves to the right with speed v_0 with respect to the ground, while the ball is at rest with respect to the cart. The ball is released from the top of the bowl. When it reaches the bottom of the bowl, it moves with velocity v_b with respect to the cart. At this instant, how fast is the cart moving with respect to the ground?



(A) $v_0 - v_b$

(B) $v_0 - \frac{m}{M_c} v_b$

(C) $\frac{M_c v_0 - m v_b}{m + M_c}$

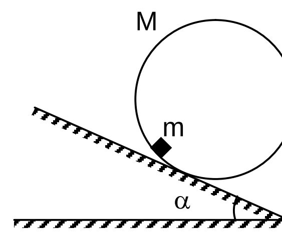
(D) $\frac{(m + M_c) v_0 - m v_b}{m + M_c}$

SECTION – A

(One or More than one correct type)

*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

5. On an incline with slope angle α there lies a cylinder with mass M, its axis being horizontal. A small block with mass m is placed inside it. The coefficient of friction between the block and the cylinder is μ; the incline is nonslippery. What can be the possible slope angle α for the cylinder to stay at rest . The block is much smaller than the radius of the cylinder. $\mu = \frac{3}{4}, m = 1 kg, M = \frac{1}{5} kg$.



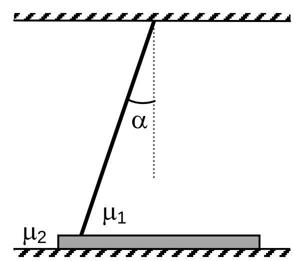
(A) 10^0

(B) 15^0

(C) 40^0

(D) 60^0

6. A rod with length $l = 5m$ is hinged to a ceiling with height $h = 4m$. Underneath, a board is being dragged on the floor. The rod is meant to block the movement the board in one direction while allowing it move in the opposite direction. The coefficient of friction is μ_1 between the board and the rod, and μ_2 between the board and the floor. What condition should be fulfilled for it to do its job?



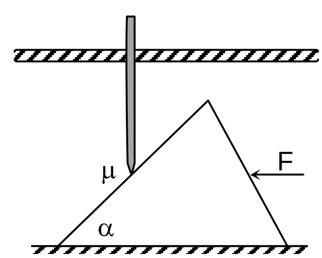
(A) $\mu_1 = 0.5, \mu_2 = 0.6$

(B) $\mu_1 = 0.7, \mu_2 = 0.8$

(C) $\mu_1 = 0.8, \mu_2 = 0.6$

(D) $\mu_1 = 0.9, \mu_2 = 0.8$

7. A wedge with the angle $\alpha = 45^0$ at the tip is lying on the horizontal floor. There is a hole with smooth walls in the ceiling. A light rod has been inserted snugly into that hole, and it can move up and down without friction, while its axis remains vertical. The rod is supported against the wedge. Assume that the only point which is rough is the contact point of the wedge and the rod. The friction coefficient at the point of contact is μ. For which values of μ it is possible to push the wedge through, behind the rod, by only applying a sufficiently large horizontal force?



(A) 0.5

(B) 0.6

(C) 0.7

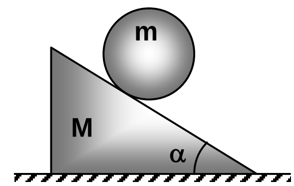
(D) 0.8

SECTION – B

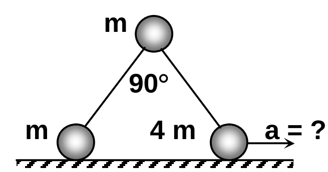
(Numerical Answer Type)

*This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.*

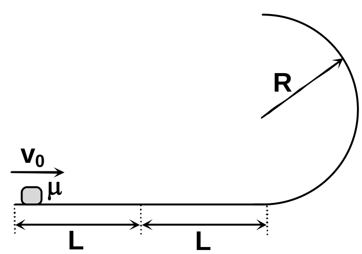
8. A wedge of mass M = 10 kg and inclination $\alpha = 30°$ is free to move on a smooth horizontal plane. A uniform solid cylinder of mass m = 20 kg is placed on the rough inclined face of the wedge. Find the acceleration (in m/s^2) of the centre of the cylinder relative to the wedge down the inclined face. There is no slipping between the cylinder and the wedge.



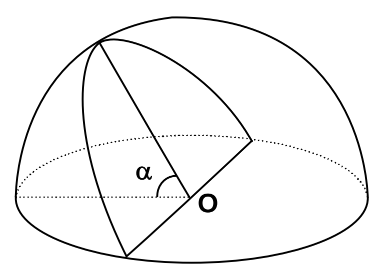
9. Three little cylinders are connected with weightless identical rods, where there is a hinge near the middle cylinder, so that the angle between the rods can change freely. Initially this angle is a right angle. Two of the cylinders have mass m, another one at the side has the mass 4m as shown in the figure. Find the acceleration of the heavier (in m/s^2) cylinder immediately after the motion begins. Ignore friction. Take $g = 9\ m/s^2$.



10. A small object is initially at rest at the starting point of a horizontal straight track of length 2L. The speed with which it should be pushed in order that after sliding along the vertical semicircular path of radius R at the end of the straight track it hit the midpoint of the horizontal track is v. Find value of $\frac{5v}{21}$. The coefficient of kinetic friction along the horizontal track is μ, the circular track is frictionless. (L=2 m; R=0.5 m; μ=0.4, g = $10 m/s^2$)



11. A part of the surface of hemisphere is isolated from hemisphere by cutting it by two planes passing through the same diameter and forming an angle with each other. The shape thus obtained is called a "mericarp".
The left part is referred as mericarp with central angle α. If mericarp is cut from solid hemisphere then it is called a solid mericarp . If mericarp is obtained from hemispherical shell it is called a hollow mericarp .
Find the diatance of centre of mass of a uniform solid mericarp of radius $R = \frac{16}{\pi}$ and central angle $\alpha = 10^0$, from the centre of sphere. For angles less than 7 degrees you may assume $\sin\theta = \theta$.



12. Find the diatance of centre of mass of a uniform hollow mericarp of radius $R = \frac{16}{\pi}$ and central angle $\alpha = 10^0$, from the centre of sphere. For angles less than 7 degrees you may assume $\sin\theta = \theta$.

13. At what speed (in m/s) must a pebble be thrown from a height of h = 3.6 m, and at an angle of 30^o, measured from the horizontal, if it is to hit the ground at an angle of 45^o. (Air drag can be neglected.) (Take $g = 10\ m/s^2$)

SECTION – C
(Numerical Answer Type)

This section contains **TWO (02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions of numerical answer type. The answer to each question is a **NUMERICAL VALUE (XXXXX.XX).** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

Paragraph for Question Nos. 14 and 15

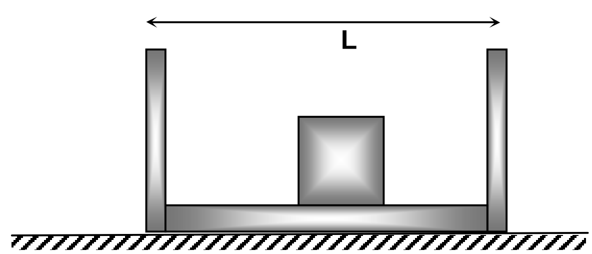
The 15.2 km M4 metro line connects Delhi railway station and Gurugram railway station, and between them it has 7 other stops. Trains leave each station (or stop) with a constant acceleration of 1.0 m/s^2, move with constant speed for some time and stop at next station with the same deceleration when brakes are applied. The maximum speed between stations is 72 km/h. At the stations the average time to change passengers is 0.5 minutes.

14 During one journey, how long (time in seconds) does the train travel at the speed of 72 km/h?

15. What is the journey time of the M4 metro between the two terminals, i.e. how much time elapses between the departure of a train from Delhi and its arrival at Gurugram railway station?

Paragraph for Question Nos. 16 and 17

There is a box A of length L = 1.0 m on the smooth horizontal surface. The center of mass of the box is located at its middle position. Inside the box, there is a block B with negligible size. Box A and block B have the same mass. Initially, box A is at rest and block B is located at the middle of A and moves to the right with an initial speed v_0 = 5 ms^{-1}. The coefficient of friction between A and B is μ = 0.05 and the collisions of B with left and right walls of A and B are elastic. (Take g = 9.8 m/s^2)



16. How many collisions occur between the walls of box A and block B?

17. What is the horizontal displacement of box A (in metre) from the initial moment to the moment when block B has just reached the relatively stationary position inside the box?

Chemistry PART – II

SECTION – A
(One Options Correct Type)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

18. Consider the following statements related to the compounds of alkali metals
(I) Na_3N is formed by heating sodium with N_2.
(II) CsO_2 is paramagnetic and coloured.
(III) LiCl is deliquescent.
(IV) LiH is thermally less stable than NaH.
The option with correct set of statements is
(A) (I) and (III) (B) (II) and (IV)
(C) (III) and (IV) (D) (II) and (III)

19. The **incorrect** statement among the statements given below is
(A) Dipole moment of NH_3 is greater than that of NF_3
(B) In the solid state of $KHCO_3$, HCO_3^- ions exist as dimeric anion
(C) In Borax all the Boron atoms are sp^3 hybridized
(D) In SO_4^{2-} all bond lengths are same

20. The compound in which the central atom exhibit same type of hybridization that is shown by Br in $\left[BrF_4\right]^-$ is
(A) SF_2Cl_2 (B) SF_4
(C) ClF_3 (D) $XeOF_4$

21. Consider the following statements related to alkaline earth metals and their compounds
(I) BeO is an amphoteric oxide.
(II) $BaSO_4$ is more soluble in water than $CaSO_4$.
(III) BeF_2 is soluble in water.
(IV) $Mg(NO_3)_2$ decomposes on heating producing $Mg(NO_2)_2$ and O_2.
The option with correct set of statement is
(A) (II) and (IV) (B) (I) and (III)
(C) (I) and (IV) (D) (III) and (IV)

SECTION – A
(One or More than one correct type)

*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

22. The correct statement(s) related to the equilibrium
$$NH_2COONH_4(s) \rightleftharpoons 2NH_3(g) + CO_2(g)$$
is/are
(A) At equilibrium addition of $NH_2COONH_4(s)$ increases the partial pressure of NH_3 at constant temperature
(B) Addition of $NH_3(g)$ at equilibrium decreases the value of K_c.
(C) Addition of inert gas at constant volume does not affect this equilibrium.
(D) Addition of catalyst does not affect the equilibrium state.

23. The solution (s) in which the solubility of AgCN will be greater than that in pure water is(are) $\left[K_{sp}(AgCN) = 4\times10^{-16}, K_a(HCN) = 5\times10^{-10}\right]$

(A) 0.01 M $AgNO_3$ solution
(B) A buffer solution of pH = 11
(C) 0.3 M NH_3 solution
(D) A buffer solution of pH = 5

24. Rate constant k varies with temperature by equation

$$\log k\left(\text{min}^{-1}\right) = 4 - \frac{3000}{T(k)}$$

The correct statement(s) related to the above equation is/are [Given : (R = 2 cal mole^{-1} K^{-1})]

(A) Pre exponential factor A is 4
(B) E_a is 13.818 Kcal
(C) E_a is 3000 cal
(D) When $T \to \infty,\ k = 10^4\ \text{min}^{-1}$

SECTION – B

(Numerical Answer Type)

This section contains ***SIX (06)*** *Numerical based questions. The answer to each question is a* ***NON-NEGATIVE INTEGER VALUE****.*

25. Among $SF_3Cl, XeO_2F_2, [ICl_4]^-, [NH_4]^+, [ICl_6]^-, XeO_3, SF_4, [XeF_5]^+$ and $[XeO_6]^{4-}$, the total number of species having sp^{3}d hybridized central atom is__________

26. Among $N_2^-, O_2^{2-}, H_2^-, Li_2, F_2, He_2, N_2^+, O_2^+, Be_2$ the number of species which are paramagnetic is______

27. Among $Zn(OH)_2, Sn(OH)_2, Cu(OH)_2, Fe(OH)_3, Pb(OH)_2, Al(OH)_3$ the number of hydroxide which are soluble in excess of NaOH solution is_________

28. 4.9 mg H_2SO_4, 3.65 mg of HCl and 3.15 mg of HNO_3 were taken in a flask of volume 'V' ml. the pH of the final solution is 4.3. The value of $\frac{V}{100}$ is______(log5 = 0.7)

29. At 200^{o}C, the gaseous reaction $A(g) \longrightarrow 2B(g) + C(g)$ is found to be of 1st order. Starting with pure A(g), if at the end of 5 minutes, the total pressure of the system is 128 mm and after infinite time it is 240 mm and the partial pressure of A(g) at the end of 5 min is 'y' mm. the value of y is_________

30. 9 moles of SO_2 reacts completely with acidified potassium dichromate solution. In this reaction the number of moles of potassium dichromate reacted is x and the number of moles of H_2O produced is y. the value of (x + y) is_______

SECTION – C

(Numerical Answer Type)

This section contains **TWO (02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions of numerical answer type. The answer to each question is a **NUMERICAL VALUE (XXXXX.XX).** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

Paragraph for Question Nos. 31 and 32

The photoelectric work function depends upon the nature of metal. A metal having a higher ionization energy has a greater value of photoelectric work function. The values of photoelectric work functions of Li, Na, K and Cu are 2.42 eV, 2.3 eV, 2.25 eV and 4.8 eV respectively.

$\left(h = 6.6\times10^{-34}\ \text{Joule sec}, 1\ \text{eV} = 1.6\times10^{-19}\ \text{J}\right)$

31. When sodium metal is exposed to an electromagnetic radiation, the maximum kinetic energy of the photoelectrons was found to be 0.70 eV. The wavelength of the incident radiation is $y \times 10^{-8}$ m. The value of y is_________

32. An electromagnetic radiation of wavelength $500\ A^{o}$ falls on the surface of copper metal. The maximum kinetic energy of ejected photoelectrons (in eV) is___________

Paragraph for Question Nos. 33 and 34

SO_2 (g) when circulated through suspension of Colemanite $(Ca_2B_6O_{11})$ powder, compound P and Q are formed. Compound Q on strong heating produced compounds R and S. R is an acidic oxide while S is a neutral oxide. Compound R reacts with P_2O_5 to form compound T. The difference in molar mass of P and Q is 140.2. Sum of molar mass of R and S is 87.6.
[Use molar mass (in g mol^{-1}) : H = 1, B = 10.8, O = 16, S = 32, P = 31]

33. Percentage by mass of oxygen in compound 'P' is_____________%

34. The total number of atoms in one molecule of compound 'T' is____________

Mathematics PART – III

SECTION – A
(One Options Correct Type)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

35. Suppose function f(x) is continuous and differentiable at x = 0, and function g(x) is continuous but non-differentiable at x = 0. If the function $h(x) = e^{g(x)} \cdot f^2(x) \cdot g(x)$ is differentiable at x = 0, then which of the following is CORRECT?
(A) f(0) must be equal to 0
(B) g(0) must be a particular negative value
(C) f(0) = 0 or g(0) = –1
(D) f(0) = g(0)

36. Let d(P, A) denotes the distance between the point P and point A. If a variable point P moves such that d(P, (–4, 0)) + d(P, (0, 0)) ≤ 6 and d(P, (4, 0)) + d(P, (0, 0)) ≥ 6, then the area of the region in which point P moves is equal to
(A) $\frac{20}{3} + \sin^{-1}\frac{2}{3}$
(B) $\frac{10}{3} + 6\sqrt{5}\sin^{-1}\left(\frac{2}{3}\right)$
(C) $\frac{20}{3} + 6\sqrt{5}\sin^{-1}\frac{2}{3}$
(D) $\frac{10}{3} + \sin^{-1}\left(\frac{2}{3}\right)$

37. If $f_n(x) = \tan^{-1}\left\{\left(\sum_{r=1}^{n}\operatorname{cosec}\frac{x}{2^{r-1}}\right) - \cot\frac{x}{2^n}\right\}$, then which of the following is CORRECT?
(A) $f_5\left(\frac{\pi}{4}\right) = \frac{3\pi}{4}$
(B) $f_5\left(\frac{3\pi}{4}\right) = \frac{3\pi}{4}$
(C) $f_5\left(\frac{3\pi}{4}\right) = \frac{\pi}{4}$
(D) $f_5\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$

38. Consider the function $f(x) = \frac{\log_e(1+|x|)}{|x|}$, then which of the following statement about f(x) is FALSE? (Where [.] represents greatest integer function)
(A) $\lim_{x\to 0}[f(x)] = 0$
(B) $\left[\lim_{x\to 0} f(x)\right] = 1$
(C) f(x) is a bounded function
(D) f(x) has a point of minima

SECTION – A
(One or More than one correct type)

*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

39. $f(x) = \frac{x^2 - 3x + 2}{|x^2 - 3x + 2|}$ and g(x) = sgn(x – 1)(x – 2), then which of the following is/are TRUE?
(A) $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} g(x)$
(B) $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} g(x)$
(C) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} g(x)$
(D) $\lim_{x\to 2} f(x) = \lim_{x\to 2} g(x)$

40. If a differentiable function f(x) satisfy the following relation $f(x) + f(x^2) + f(x^3) = f(x + x^2 + x^3)$ $\forall$ x $\in$ R, then which of the following is/are TRUE?
(A) $f'(-1) = f'(3)$
(B) $f'(-1) = f'(1)$
(C) $f'(1) = f'(3)$
(D) $3f(-1) = f(3)$

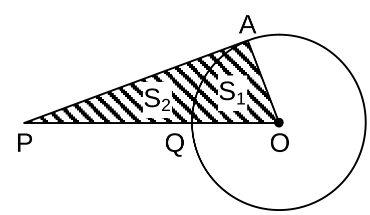
41. Which of the following(s) is/are CORRECT?
(A) If f(x) is periodic and g(x) is non-linear and non-periodic, then f(g(x)) is non-periodic
(B) If f(x) is periodic and g(x) is non-linear and non-periodic, then f(g(x)) can be periodic with a defined period
(C) If f(x) and g(x) both are non-periodic, then f(g(x)) can be periodic
(D) If f(x) and g(x) both the non-periodic, then f(g(x)) cannot be periodic with a defined period

SECTION – B

(Numerical Answer Type)

*This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.*

42. In the figure, OQ is fixed diameter of the circle, and AP is tangent to the circle which cut the line OQ at P. Let S_1 is area of sector AOQ, and area of curvilinear triangle APQ is S_2. Then $\lim_{A\to Q} \frac{S_2}{S_1}$ is equal to



43. A circle has center (0, 0) and variable radius r. If tangents are drawn from the point $\left(-3, \sqrt{7}\right)$ to the circle, then the value of r for which area of the triangle formed by the tangents and chord of contact is maximum, is equal to

44. The maximum value of the function $f(x) = \frac{4x^2 - 4x + 1}{8x^4 - 16x^3 + 12x^2 - 4x + 1}$ is equal to

45. Consider the function $f(x) = x\sqrt{1 + (x+1)\sqrt{1 + (x+2)\sqrt{1 + (x+3)\sqrt{1 + ...}}}}$, then $\lim_{n\to\infty} \sum_{x=1}^{n} \frac{1}{f(x)}$ is $\frac{p}{q}$ where p and q are coprime, then p + q is equal to

46. If $\int \frac{1 + \frac{1}{\lfloor 2\, x^2} + \frac{1}{\lfloor 4\, x^4} + ...}{x + \frac{1}{\lfloor 3\, x} + \frac{1}{\lfloor 5\, x^3} + ...} dx = f(x) + C$ where C is integration constant. Then $\lim_{x\to\infty} f(x)$ is equal to

47. Consider the differential equation $\frac{dy}{dx} = \frac{1 - \ln x}{x^2 + \ln^2 x}$. Given that y(1) = $\frac{\pi}{2}$, then $\left[y\left(\frac{1}{e}\right) \right]$ is equal to
(Where [.] represents greatest integer function)

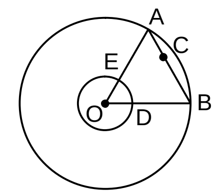
SECTION – C

(Numerical Answer Type)

This section contains **TWO (02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions of numerical answer type. The answer to each question is a **NUMERICAL VALUE (XXXXX.XX).** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

Paragraph for Question Nos. 48 and 49

There are two concentric circles (centred at point O) of radii 2 and 6 respectively, as shown in the figure. B is a fixed point while point A varies on the outer circle.
C is a point on AB such that AC : BC = 1 : 2
E and D lies on inner circle and on OA and OB
Let S is the area of ΔECD. Then answer the following question:



48. Maximum possible area of ΔECD i.e. S_{max} will be equal to

49. For S_{max}, the common area between ΔECD and inner circle is equal to

Paragraph for Question Nos. 50 and 51

Consider the following functions f(x) = sin x and g(x) = x – sin x and answer the following questions:

50. If fundamental periods of f(g(x)) and g(f(x)) are T_1 and T_2, then $T_1 + T_2$ will be equal to

51. $\int_0^{2\pi}\left(f\left(g(x)\right)+g\left(f(x)\right)\right)dx$ is equal to

ALL INDIA TEST SERIES
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PART TEST – I
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ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. D

Sol. balance the torques.

2. B

Sol. The mass on the right will oscillate because the tension in the string oscillates; however, it cannot oscillate horizontally because there is no torque exerted on it. Suppose it oscillates vertically with no net motion. Then the average y-component of the tension acting on the mass on the left is equal to mg, but the x-component of the tension is nonzero; then the average magnitude of the tension is greater than mg, a contradiction. Thus the right mass must instead oscillate and move upward.

3. A

Sol. The motion is the superposition of two motions: uniform translation of both masses with speed $\frac{m_1 v}{(m_1+m_2)}$ and circular motion about the common center of mass, where the first mass has speed $\frac{m_2 v}{(m_1+m_2)}$. The second mass comes to rest again after one period of the circular motion. The radius is $\frac{Lm_2 v}{(m_1+m_2)}$, giving a period of $t=\frac{2\pi L}{v}$.

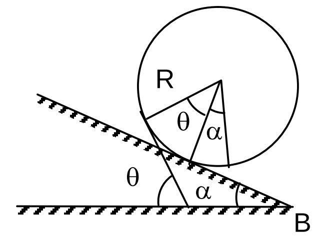
4. D

Sol. The problem can be solved using only momentum conservation, without knowing the friction between the ball and bowl. Work in the frame originally moving with the cart, where the momentum is zero. When the ball is at the bottom of the bowl, the ball and bowl have relative velocity v_b. This is only consistent with momentum conservation if the velocity of the bowl and ball are $-\frac{m}{m+M_c}v_b, \frac{M_c}{m+M_c}v_b$ respectively. Transforming back to the original frame, the final velocity of the bowl is $\frac{(m+M_c)v_0 - mv_b}{m+M_c}$

5. A, B

Sol. Let the angle formed from the mass, the center of the cylinder O, and the vertical be θ. By summing forces on the mass, we get

$$mg\sin\theta - \mu mg\cos\theta = 0 \Rightarrow \mu = \tan\theta$$



This is unsurprising, as it is the typical condition for an object to not slip. You can verify yourself that the effective angle of the incline is equal to the angle the normal force makes with the vertical, θ. Next, we sum up the torques with respect to the contact point between the ramp and the cylinder. The moment arm for the cylinder is R sin α and the moment arm for the block is R sin θ −R sin α. Therefore, we can write the torque balance equation as:

$$(M+m)g\sin\alpha = mg\sin\theta$$

Because tan θ = μ, we have $\sin\theta = \frac{\mu}{\sqrt{\mu^2+1}}$ Substituting this result into our equation of sum of torques at point P gives us

$$(M+m)g\sin\alpha = mg\frac{\mu}{\sqrt{\mu^2+1}}$$

Which gives

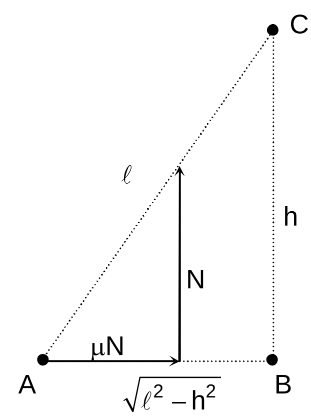
$$\alpha = \sin^{-1}\left(\frac{m}{(M+m)}\frac{\mu}{\sqrt{\mu^2+1}}\right) = 30°$$

So, angle must be less than 30°

6. C, D

Sol. Consider what happens when the applied force approaches infinity. To maintain equilibrium, the friction force between the rod and the board must also increase. This friction force will also approach infinity. When dealing with large forces, we can ignore constant forces such as the weight of both the board and the rod.

As a result, since the weight of the rod is negligible we can pretend it's a mass-less rod. We also know that the forces at the ends of a massless rod will always point along the rod. For example, the force exerted on the rod by the board must point along the rod as well. The angle of this force is solely dependent on the friction coefficient μ_1. Therefore:



$$\tan\alpha < \frac{\mu_1 N}{N}, \quad \mu_1 > \frac{\sqrt{l^2-h^2}}{h}$$

7. A, B, C, D

Sol. The key insight is noting if the net vertical forces of the normal and friction forces were directed downwards then the stopper would be blocked. Let us then try to calculate the vertical components of forces that are involved. Let the normal force directed on the wedge be N.

We then know that the vertical component of the normal force is clearly either N cos α or N sin α. We can figure the component by chasing angles around, but an easier way is to imagine α → 0. In this case, the horizontal component of the normal force also goes to zero, which is the behavior of a sine function, so the horizontal component is N sin α. This in turn means that the vertical component of force involved is N cos α. We now try to find the vertical component of friction involved. The friction force directed downwards on the direction of the wedge is μN (because N is already perpendicular, you do not have to manipulate it with trigonometric functions). This means that the vertical component of friction is μN sin α. We now equate these, with an inequality where the vertical component of the normal force greater than the friction force for the wedge to pass through. N cos α > μN sin α

SECTION – B

8. 5

Sol. When the wedge has moved through a distance y, let the point of contact of cylinder have moved down the plane through x (with respect to wedge). Let F be the friction between the cylinder and the plane. Since there is no horizontal force on (wedge + cylinder system)

$$M\frac{d^2y}{dt^2}+m\left(\frac{d^2y}{dt^2}-\cos\alpha\frac{d^2x}{dt^2}\right)=0 \qquad (1)$$

Also ,

$$m\left(\frac{d^2y}{dt^2}\cos\alpha-\frac{d^2x}{dt^2}\right)=F-mg\sin\alpha$$

$$\frac{ma}{2}\frac{d^2x}{dt^2}=\frac{ma^2}{2}\frac{d^2\theta}{dt^2}=Fa$$

This gives : $2\frac{d^2y}{dt^2}\cos\alpha-3\left(\frac{d^2x}{dt^2}\right)=-2g\sin\alpha \qquad (2)$

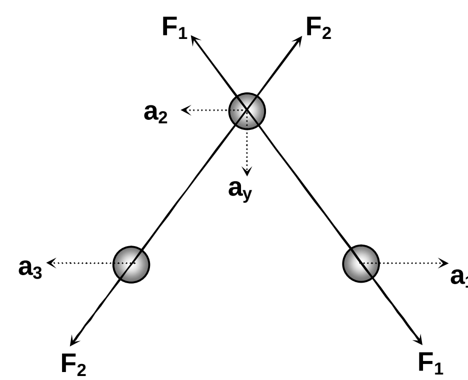
Equation (1) and (2) implies acceleration of the centre of the cylinder down the face, and relative to the wedge, is

$$\frac{2g\sin\alpha(M+m)}{3M+m+2m\sin^2\alpha}=5\frac{m}{s^2}$$

9. 1

Sol. There is no external force in the horizontal direction, therefore:

$$ma_2+ma_3=4ma_1$$



The component of acceleration of the top mass along the left rod must be the same as the component of acceleration of the left mass along the left rod. This means:

$$a_2\cos45^0+a_y\sin45^0=a_3\cos45^0$$

$$\Rightarrow a_2+a_y=a_3$$

The same is true along the right rod, giving

$$a_y-a_2=a_1$$

Solving these three equations, we get:

$$a_y=2a_1$$

$$a_2=a_1$$

$$a_3=3a_1$$

Using Newton's Second Law on the rightmost mass, we have

$$F_1\cos45^0=4ma_1$$

For the left mass

$$F_2\cos45^0=ma_3=3ma_1$$

For the top mass in the vertical direction. We get:

$$mg-(F_1\cos45^0+F_2\cos45^0)=ma_y=2ma_1$$

Substituting in F_1 and F_2 from above gives

$$mg-7ma_1=2ma_1$$

$$\Rightarrow a_1=\frac{g}{9}=1$$

10. 2

Sol. Let's denote the speed of the small body at the top point of the semicircle v_1 .Since leaving the half circle, the body makes a horizontal throw starting from height 2 R , during this time it travels a distance L in horizontal direction, it can be written that

$$2R = \frac{gt^2}{2} \text{ and } L = v_1 t$$

from where $v_1^2 = \frac{L_1 g}{4R}$

Comment. This is the speed at the moment before leaving the forced orbit

$$a = \frac{v_1^2}{R} = \left(\frac{L}{2R}\right)^2 g = 4g$$

corresponds to centripetal acceleration. Since a > g , the body neither then nor before leaves the semicircular path, it slides along it.

applying the work energy theorem:

$$\frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 = -2\mu mgL - 2Rmg \Rightarrow v_0 = \sqrt{\left(\frac{L^2}{4R} + 4\mu L + 4R\right)g} \approx 8.4\frac{m}{s}$$

11. 3

Sol. COM will be at $\frac{3\pi R}{16}$

12. 4

Sol. COM will be at $\frac{\pi R}{4}$ from centre

13. 12

Sol. $v_o \cos 30^0 = v \cos 45^0$

$$\Rightarrow v = v_o\sqrt{\frac{3}{2}}$$

Now using conservation of energy

$$\frac{1}{2}m\left(v^2 - v_0^2\right) = mgh$$

SECTION – C

14. 600.00

15. 1130.00

Sol. (Q.14-15):

Cruising speed of train is 72 km/h = 20 m/s .Train moving with an acceleration of 1 m/s^2 reaches its maximum speed in time t_1 = v_0/a =20 sec and needs the same amount of time to decelerate. During this time, the train travels 400 m .

Total number of stops = 7

Total number of journeys = 8

Total distance covered by the train while accelerating or decelerating = 400 x 8 = 3200m = 3.2 km

Distance travelled at cruising speed = 15.2 km – 3.2 km = 12 km

(During the calculation, we did not assume that the distance between the stations is the same, which is not true anyway.)

Total time covered by the train while accelerating or decelerating = 40x8 = 320 sec

Time for which the train travelled at cruising speed = $\frac{12000}{20} = 600\,\text{sec}$

Total time of journey between Delhi and Gurugram = 600+320+(30x7)= 1130 sec

16. 13.00

Sol. When the block is at rest relative to the box, they have the same (final) velocity v relative to the lab frame. Since there are no external forces, the total momentum is conserved and we have, $2mv = mv_0 \Rightarrow v = \frac{v_0}{2}$

Since the box and the block have the same mass and the collision is elastic, the relative velocity of the block relative to the box change sign after every collision and the block decelerates between collisions due to the friction.

Hence, the total distance travelled by the block before it stops (relative to the box) is, $\mu mgs = \frac{1}{2}mv_0^2 - 2x\frac{1}{2}mv^2 \Rightarrow s = \frac{v_0^2}{4\mu g} = 12.76m$

The block collides with the box at distances 0.5 m, 1.5 m, ..., 12.5 m. Hence the number of collisions is 13.

17. 12.64

Sol. Since there are no external forces acting on the system, the center of mass (CM) always moves with the constant velocity v. Hence the distance travelled by the CM is $x_{cm} = vt = \frac{v_0 t}{2}$. where t is the time for the block to decelerate from v_0 to v.

Consider the frictional force acting on the block, we have

$$-\mu mgt = mv - mv_0 = -\frac{1}{2}mv_0 \Rightarrow t = \frac{v_0}{2\mu g} = 5.1s$$

$$x_{cm} = \frac{v_0 t}{2} = 12.76m$$

Initially, B is at the midpoint of A. At distances 0.5 m, 2.5 m, ..., 12.5 m, the block collides with the right wall of the box. Hence when the block stops, it lies at 0.26 m from the right wall. So finally it will stop at 0.24 m to the right of the midpoint. Hence the CM is shifted to 0.12 m to the right of the midpoint when the block B is at rest relative to the box. The distance of the box travelled is

$$x_A = x_{cm} = 0.12 = 12.64m$$

Chemistry PART – II

SECTION – A

18. D

Sol. (I) Na does not react with N_2 to form Na_3N.
(II) LiH is more stable than NaH.

19. C

Sol. In Borax two Boron atoms are sp^3 hybridised and two Boron atoms are sp^2 hybridised.

20. D

Sol. Hybridisation of Br in $\left[BrF_4\right]^-$ is sp^3d^2.

21. B

Sol. (I) $BaSO_4$ is less soluble in water than $CaSO_4$.
(II) $2Mg\left(NO_3\right)_2 \xrightarrow{\Delta} 2MgO + 4NO_2 + O_2$

22. C, D

Sol. $K_P = \left(P_{NH_3}\right)^2 \left(P_{CO_2}\right)$

23. C, D

Sol. (I) Solubility of AgCN in NH_3 solution will be higher due to complex formation.
(II) AgCN is more soluble in acidic buffer solution.

24. B, D

Sol. $\log k = \log A - \frac{E_a}{2.303RT}$

log A = 4,
$A = 10^4$

$\frac{E_a}{2.303R} = 3000$

$E_a = 3000 \times 2.303 \times 2$
= 13818 cal
= 13.818 Kcal

SECTION – B

25. 3

Sol. SF_3Cl, XeO_2F_2, SF_4

26. 4

Sol. N_2^-, H_2^-, N_2^+, O_2^+ are paramagnetic.

27. 4

Sol. $Zn\left(OH\right)_2$, $Al\left(OH\right)_3$, $Pb\left(OH\right)_2$ and $Sn\left(OH\right)_2$ are amphoteric, hence they are soluble in excess of NaOH solution.

28. 50

Sol. Milli moles of $H_2SO_4 = \frac{4.9}{98} = 0.05$

Milli moles of $HCl = \frac{3.65}{36.5} = 0.1$

Milli moles of $HNO_3 = \frac{3.15}{63} = 0.05$

$\therefore \left[H^+\right] = \frac{(2\times 0.05)+0.1+0.05}{V(ml)}$

$5\times 10^{-5} = \frac{0.25}{V(ml)}$

$V(ml) = \frac{0.25}{5\times 10^{-5}} = 5000\text{ ml}$

$\therefore \frac{V}{100} = \frac{5000}{100} = 50$

29. 56

Sol.

$A(g) \longrightarrow 2B(g) + C(g)$

$t = 0$	P^o	0	0
$t = 5$ min	$P^o - x$	$2x$	x
$t = \infty$	0	$2P^o$	P^o

$\therefore$ when $t = \infty$, $3P^o = 240$ mm

$\therefore$ $P^o = 80$ mm

At t = 5, $P^o - x + 2x + x = 128$

$P^o + 2x = 128$

$80 + 2x = 128$

x = 24

Partial pressure of A(g) at the end of 5 min $y = P^o - x = 80 - 24 = 56$ mm

$\therefore y = 56$

30. 6

Sol. $9SO_2 + 3K_2Cr_2O_7 + 3H_2SO_4 \longrightarrow 3K_2SO_4 + 3Cr_2(SO_4)_3 + 3H_2O$

$x = 3, \quad y = 3$

$x + y = 3 + 3 = 6$

SECTION – C

31. 41.25

Sol. $\frac{hC}{\lambda} = (2.3 + 0.70)\times 1.6\times 10^{-19}$ J

$\lambda = \frac{6.6\times 10^{-34}\times 3\times 10^{8}}{3\times 1.6\times 10^{-19}}$

$\lambda = 41.25\times 10^{-8}$ m

$\therefore y = 41.25$

32. 19.95

Sol. $\frac{hC}{\lambda} = h\nu_0 + KE_{max}$

$KE_{max} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{500 \times 10^{-10} \times 1.6 \times 10^{-19}} - 4.8$

$= 24.75 - 4.8$

= 19.95 eV

33. 47.52

Sol. $Ca_2B_6O_{11} + 11H_2O + 4SO_2 \longrightarrow \underset{(P)}{2Ca(HSO_3)_2} + \underset{(Q)}{6H_3BO_3}$

$2H_3BO_3 \xrightarrow{\text{Strong heating}} \underset{(R)}{B_2O_3} + \underset{(S)}{3H_2O}$

$B_2O_3 + P_2O_5 \longrightarrow \underset{(T)}{2BPO_4}$

Compound P is $Ca(HSO_3)_2$

% mass of oxygen $= \frac{96}{202} \times 100 = 47.52$

34. 6.00

Sol. Compound 'T' is BPO_4.

Mathematics

PART – III

SECTION – A

35. C

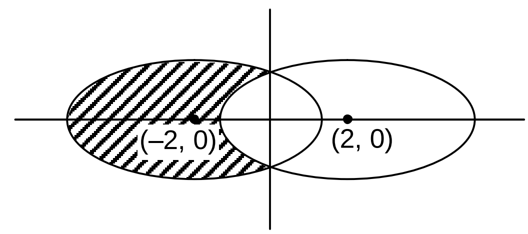
Sol. $h'(0)=\left(\lim_{h\to0}\frac{g(0+h)-g(0)}{h}\right)\left(e^{g(0)}\cdot f^2(0)g(0)+e^{g(0)}f^2(0)\right)+2e^{g(0)}f(0)f'(0)g(0)$

For h′(0) to be exist either f(0) = 0 or g(0) = –1

36. C

Sol. Point P moves inside the ellipse $\frac{(x+2)^2}{9}+\frac{y^2}{5}=1$

and outside the ellipse $\frac{(x-2)^2}{9}+\frac{y^2}{5}=1$



Required area is shown in the figure

$=3\sqrt{5}\pi-4\int_0^1\sqrt{5}\sqrt{\left(1-\frac{(x+2)^2}{9}\right)}dx$

$=3\sqrt{5}\pi-\frac{4}{3}\sqrt{5}\int_0^1\sqrt{9-(x+2)^2}dx=3\sqrt{5}\pi+\frac{20}{3}-3\sqrt{5}\pi+6\sqrt{8}\sin^{-1}\frac{2}{3}=\frac{20}{3}+6\sqrt{5}\sin^{-1}\left(\frac{2}{3}\right)$

37. C

Sol. $\sum_{r=1}^{n}\text{cosec}\frac{x}{2^{r-1}}=\cot\frac{x}{2^n}-\cot x$

Hence, $f_n(x)=\tan^{-1}(-\cot x)$

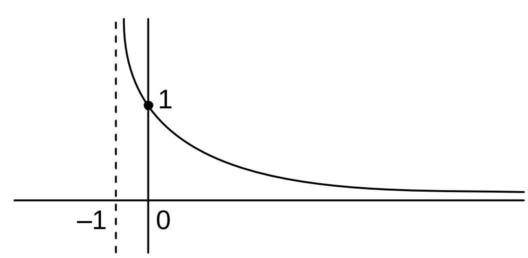
$f_5\left(\frac{\pi}{4}\right)=\tan^{-1}(-1)=-\frac{\pi}{4}$

$f_5\left(\frac{3\pi}{4}\right)=\tan^{-1}(1)=\frac{\pi}{4}$

38. D

Sol. $\frac{\log_e(1+x)}{x}$

is a decreasing function and its graph is as follows



39. A, B, C

Sol. $\lim_{x\to2^+}f(x)=1,\ \lim_{x\to2^-}f(x)=-1,\ \lim_{x\to1^+}f(x)=-1$

40. A, B, C

Sol. $f(x)+f(x^2)+f(x^3)=f(x+x^2+x^3)$ differentiate w.r.t x, we get

$f'(x)+2xf'(x^2)+3x^2f'(x^3)=(1+2x+3x^2)f'(x+x^2+x^3)$

Put x = 1 ⇒ 6f′(1) = 6f′(3) ⇒ f′(1) = f′(3)

Put x = –1 ⇒ 4f′(–1) – 2f′(1) = 2f′(–1) ⇒ f′(–1) = f′(1)

Put x = 1 in given relation 3f(1) = f(3)
Put x = –1 in given equation 2f(–1) + f(1) = f(–1) ⇒ f(–1) + f(1) = 0
∴ 3f(–1) = –f(3)

41. B, C

Sol. f(x) = sin x → periodic
g(x) = x + sin x → non-linear and non-periodic
f(g(x)) = sin(x + sin x) → periodic with period 2π
Hence, option (A) is incorrect and option (B) is correct
$f(x) = \sin x^{\frac{1}{3}}$ non-periodic
$g(x) = x^3$ non-periodic
f(g(x)) = sin x periodic with period 2π
Hence, option (C) is correct but option (D) is incorrect

SECTION – B

42. 0

Sol. Let ∠AOQ = θ and radius of circle = r
Area of sector AOQ = $\frac{1}{2}r^2\theta$
Area of ΔAOP = $\frac{1}{2}r \cdot r\tan\theta = \frac{r^2}{2}\tan\theta$ ⇒ Area of curvilinear triangle APQ = $\frac{r^2}{2}(\tan\theta - \theta)$
Also, A → Q ⇒ θ → 0. Now, $\lim_{A\to Q}\frac{S_2}{S_1} = \lim_{\theta\to 0}\frac{\tan\theta - \theta}{\theta} = \lim_{\theta\to 0}\left(\frac{\tan\theta}{\theta} - 1\right) = 0$

43. 2

Sol. Area of triangle will be $A = \frac{r}{16}(16 - r^2)^{\frac{3}{2}}$ for maximum area $\frac{dA}{dr} = 0$ ⇒ r = 2

44. 1

Sol. By using AM ≥ GM, we get maximum value of f(x) = 1

45. 7

Sol. $f(x) = x\sqrt{1 + f(x+1)}$ by solving, we get f(x) = x(x + 2)

46. 0

Sol. $f(x) = \ln\left|\left(e^{1/x} - e^{-1/x}\right)\right|$
$\lim_{x\to\infty} f(x) = 0$

47. 0

Sol. $\frac{dy}{dx} = \frac{1 - \ln x}{x^2 + \ln^2 x} \Rightarrow y = \int \frac{1 - \ln x}{x^2 + \ln^2 x}dx$
$\Rightarrow y = \int \frac{1}{1 + \left(\frac{\ln x}{x}\right)^2} \cdot \left(\frac{1 - \ln x}{x^2}\right)dx \Rightarrow y = \tan^{-1}\left(\frac{\ln x}{x}\right) + c$
Given $y(1) = \frac{\pi}{2}$ ∴ $\frac{\pi}{2} = c$

Now, $y\left(\frac{1}{e}\right) = \tan^{-1}(-e) + \frac{\pi}{2}$ = $-\tan^{-1}e + \frac{\pi}{2}$

$\Rightarrow \left[y\left(\frac{1}{e}\right)\right] = 0$

SECTION – C

48. 4.00

49. 1.14

Sol. (Q.48.-49.):

If $\angle AOB = \theta$, then for S_{max} $\theta = \frac{\pi}{2}$ and $S_{max} = 4$

Also, in this case area which is common to ΔECD and inner circle will be = $\frac{4\pi}{4} - 2 = 1.14$

50. 12.56

51. 0.00

Sol. (Q.50.-51.):

f(g(x)) = sin(x – sin x) period = 2π

g(f(x)) = sin x – sin(sin x) period = 2π

Hence, $T_1 + T_2 = 12.56$

$$\int_0^{2\pi}\left(\sin(x - \sin x) + \sin x - \sin(\sin x)\right)dx$$