DPP No. # B18 (JEE–ADVANCED)

Total Marks : 29 **Max. Time : 24 min.**
Single choice Objective ('-1' negative marking) Q.1 to Q.3 **(3 marks 3 min.) [09, 09]**
Multiple choice objective ('-2' negative & Partial marking) Q.4 to Q.8 **(4 marks 3 min.) [20, 15]**

Question No.	1	2	3	4	5	6	7	8	Total
Marks Obtained									

1. In the expansion of $(7^{1/3} + 11^{1/9})^{6561}$, the number of terms free from radicals is:
(A) 730 (B) 729 (C) 725 (D) 750

2. The coefficient of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is the same, if α equals :
(A) $-\frac{5}{3}$ (B) $\frac{10}{3}$ (C) $-\frac{3}{10}$ (D) $\frac{3}{5}$

3. If the coefficients of r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms in the binomial expansion of $(1 + y)^m$ are in AP, then m and r satisfy the equation :
(A) $m^2 - m(4r - 1) + 4r^2 + 2 = 0$. (B) $m^2 - m(4r + 1) + 4r^2 - 2 = 0$.
(C) $m^2 - m(4r + 1) + 4r^2 + 2 = 0$. (D) $m^2 - m(4r - 1) + 4r^2 - 2 = 0$.

4. Difference of squares of two distinct odd natural numbers is always a multiple of.
(A) 4 (B) 3 (C) 6 (D) 8

5. If P and Q are sum and product respectively of all real values of x satisfying the equation $|4 - |x - 2|| = 3$, then
(A) $|P| + |Q| = 143$ (B) $|P| + |Q| = 127$ (C) $|P + Q| = 127$ (D) $|P| + |Q| = 142$

6. The set of all values of 'x which satisfies the inequation $\left| 1 - \frac{|x|}{1 + |x|} \right| \geq \frac{1}{2}$ belongs to
(A) $[-1, 1]$ (B) $(-\infty, -1]$ (C) $[1, \infty)$ (D) $[-1, 2]$

7. The value of x satisfying
$2\log_{\frac{1}{4}}(x+5) > \frac{9}{4}\log_{\frac{1}{3\sqrt{3}}}(9) + \log_{\sqrt{x+5}}(2)$ is/are
(A) (–5, –4) (B) (–3, –1) (C) (–4, –1) (D) (–5, –2)

8. The simultaneous equations, $y = x + 2|x|$ & $y = 4 + x - |x|$ have the solution set given by :
(A) $\left(\frac{4}{3}, \frac{4}{3}\right)$ (B) $\left(4, \frac{4}{3}\right)$ (C) $\left(-\frac{4}{3}, \frac{4}{3}\right)$ (D) $\left(\frac{4}{3}, 4\right)$

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Multiple choice objective ('-2' negative & Partial marking) Q.4 to Q.8 (4 marks 3 min.) [20, 15]

Question No.	1	2	3	4	5	6	7	8	Total
Marks Obtained									

1. In the expansion of $(7^{1/3} + 11^{1/9})^{6561}$, the number of terms free from radicals is:
$(7^{1/3} + 11^{1/9})^{6561}$ के प्रसार में करणी चिन्ह (radical sign) से रहित पदों की संख्या है –
(A*) 730 (B) 729 (C) 725 (D) 750

Sol. $T_{r+1} = {}^{6561}C_r (7)^{\frac{6561-r}{3}} (11^{1/9})^r$

Here r should be multiple of 9

r = 0, 9,18.....6561

Number of terms = 730

Hindi $T_{r+1} = {}^{6561}C_r (7)^{\frac{6561-r}{3}} (11^{1/9})^r$

यहाँ r, 9 का गुणज होना चाहिए

r = 0, 9,18.....6561

पदों की संख्या = 730

2. The coefficient of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is the same, if α equals :

द्विपद प्रमेय में $(1 + \alpha x)^4$ और $(1 - \alpha x)^6$ के प्रसार में x की घात में मध्य पद का गुणांक समान है, यदि α बराबर है–

(A) $-\frac{5}{3}$ (B) $\frac{10}{3}$ (C*) $-\frac{3}{10}$ (D) $\frac{3}{5}$

Sol. Coefficient of middle term in $(1 + \alpha x)^4 = {}^4C_2 \alpha^2$

coefficient of middle term in $(1 - \alpha x)^6 = {}^6C_3 (-\alpha)^3$

${}^4C_2 \alpha^2 = - {}^6C_3 \alpha^3$

$-\frac{6}{20} = \alpha$

$\alpha = \frac{-3}{10}$

Hindi $(1 + \alpha x)^4$ में मध्य पद का गुणांक $\equiv {}^4C_2 \alpha^2$

$(1 - \alpha x)^6$ में मध्य पद का गुणांक $\equiv {}^6C_3 (-\alpha)^3$

${}^4C_2 \alpha^2 = - {}^6C_3 \alpha^3$

$-\frac{6}{20} = \alpha$

$\alpha = \frac{-3}{10}$

3. If the coefficients of r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms in the binomial expansion of $(1 + y)^m$ are in AP, then m and r satisfy the equation :

यदि $(1 + y)^m$ के द्विपद प्रसार में r वें, (r + 1) वें और (r + 2)वें पदों के गुणांक समान्तर श्रेढ़ी में हैं, तो m और r निम्न समीकरण को संतुष्ट करते हैं–

(A) $m^2 - m(4r - 1) + 4r^2 + 2 = 0$. (B*) $m^2 - m(4r + 1) + 4r^2 - 2 = 0$.

(C) $m^2 - m(4r+1) + 4r^2 + 2 = 0$. (D) $m^2 - m(4r - 1) + 4r^2 - 2 = 0$.

Sol. $(1 + y)^m$

$T_r = {}^mC_{r-1} \cdot y^{r-1}$

$T_{r+1} = {}^mC_r \cdot y^r$

$T_{r+2} = {}^mC_{r+1} \cdot y^{r+1}$

$\because \quad {}^mC_{r-1} + {}^mC_{r+1} = 2\,{}^mC_r$

$\Rightarrow \quad \frac{{}^mC_{r-1}}{{}^mC_r} + \frac{{}^mC_{r+1}}{{}^mC_r} = 2$

$\Rightarrow \quad m^2 - m(4r + 1) + 4r^2 - 2 = 04$

4. Difference of squares of two distinct odd natural numbers is always a multiple of.

दो विभिन्न विषम प्राकृत संख्याओं के वर्गों का अन्तर सदैव निम्न का गुणज होता है–

(A*) 4 (B) 3 (C) 6 (D*) 8

Sol. Let any two distinct odd number be (2n + 3) and (2n + 1) when $n \in W$

Now According to question $(2n + 3)^2 - (2n + 1)^2$

$= (4n^2 + 12n + 9) - (4n^2 + 4n + 1)$

$= 4n^2 + 12n + 9 - 4n^2 - 4n - 1$

$= 8n + 8 = 8(n + 1)$

Which is always divisible by 4 & 8.

Hindi माना कोई दो भिन्न–भिन्न विषम संख्याऐं (2n + 3) तथा (2n + 1) है, जबकि $n \in W$

अब प्रश्नानुसार $(2n + 3)^2 - (2n + 1)^2 = (4n^2 + 12n + 9) - (4n^2 + 4n + 1)$

$= 4n^2 + 12n + 9 - 4n^2 - 4n - 1 = 8n + 8 = 8(n + 1)$

यह हमेशा 4 तथा 8 से भाज्य है।

5. If P and Q are sum and product respectively of all real values of x satisfying the equation $|4 - |x - 2|| = 3$, then

यदि P तथा Q क्रमशः समीकरण $|4 - |x - 2|| = 3$ को सन्तुष्ट करने वाले x के सभी वास्तविक मानों के योग व गुणा को दर्शाता है, तो–

(A*) $|P| + |Q| = 143$ (B) $|P| + |Q| = 127$ (C*) $|P + Q| = 127$ (D) $|P| + |Q| = 142$

Sol. $|4 - |x - 2|| = 3$

$4 - |x - 2| = -3$

$|x - 2| = 7$

$x - 2 = 7$ $\qquad$ $x - 2 = -7$

$\boxed{x = 9}$ $\qquad$ $\boxed{x = -5}$

or या $4 - |x - 2| = 3$

$|x - 2| = 1$

$x - 2 = 1$ or या $x - 2 = -1$

$\boxed{x = 3}$ $\boxed{x = 1}$

$P = 3 + 1 + 9 - 5$ $\qquad$ $Q = -135$

$= 8$

$|P| + |Q| = 143$

6. The set of all values of 'x which satisfies the inequation $\left| 1 - \frac{|x|}{1 + |x|} \right| \geq \frac{1}{2}$ belongs to

असमिका $\left| 1 - \frac{|x|}{1 + |x|} \right| \geq \frac{1}{2}$ को संतुष्ट करने वाले x के सभी मानों का समुच्चय किस अन्तराल मे स्थित है –

(A*) $[-1, 1]$ (B) $(-\infty, -1]$ (C) $[1, \infty)$ (D*) $[-1, 2]$

Sol. $\left| \frac{1+|x|-|x|}{1+|x|} \right| \geq \frac{1}{2}$

$\frac{1}{1+|x|} \geq \frac{1}{2}$

$1 + |x| \leq 2$

$|x| \leq 1$

$x \in [-1, 1]$

7. The value of x satisfying

$2\log_{\frac{1}{4}}(x+5) > \frac{9}{4}\log_{\frac{1}{3\sqrt{3}}}(9) + \log_{\sqrt{x+5}}(2)$ is/are

असमिका $2\log_{\frac{1}{4}}(x+5) > \frac{9}{4}\log_{\frac{1}{3\sqrt{3}}}(9) + \log_{\sqrt{x+5}}(2)$ को सन्तुष्ट करने वाले x के मान है।

(A*) (–5, –4) (B*) (–3, –1) (C) (–4, –1) (D) (–5, –2)

Sol. $2\log_{1/4}(x+5) > \frac{9}{4}\log_{\frac{1}{3\sqrt{3}}}9 + \log_{\sqrt{x+5}}2$

$-1\log_2(x+5) > \frac{9}{4} \times \left(\frac{-2}{3}\right) \times 2\log_3 3 + 2\log_{(x+5)}2$

$-\log(x+5) > -3 + 2\log_{x+5}2$

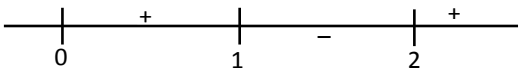
$-\log_{(x+5)}2 > -3 + 2\log_{x+5}2$

Let $\log_2(x+5) = t$

$-t > -3 + \frac{2}{t}$

$\frac{t^2-3t+2}{t} < 0$

$\frac{(t-1)(t-2)}{t} < 0$



$\log_2 x + 5 < 0$

$X + 5 < 1$ then $x < -4$

$1 < \log_2 x + 5 < 2$

$2 < x + 5 < 4$

$-3 < x < -1$

8. The simultaneous equations, $y = x + 2|x|$ & $y = 4 + x - |x|$ have the solution set given by :
समीकरण निकाय $y = x + 2|x|$ एवं $y = 4 + x - |x|$ का हल समुच्चय होगा–

(A) $\left(\frac{4}{3}, \frac{4}{3}\right)$ (B) $\left(4, \frac{4}{3}\right)$ (C*) $\left(-\frac{4}{3}, \frac{4}{3}\right)$ (D*) $\left(\frac{4}{3}, 4\right)$

Sol. $x + 2|x| = 4 + x - |x|$

$3|x| = 4$; $x = \pm\frac{4}{3}$ $\Rightarrow$ $x = \frac{4}{3}, y = 4$ $\Rightarrow$ $x = -\frac{4}{3}, y = \frac{4}{3}$