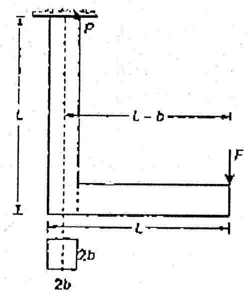
# 5 CHAPTER

# Bending Stresses in Beams

**Q.1** Two beams, one having circular section and other having square cross-section is subjected to the same amount of bending moment. If the cross-sectional area and material are the same, then
(a) Maximum bending stress developed in both the beam is the same
(b) The circular beam experiences more bending stress than the square beam
(c) The square beam experiences more bending stress than circular beam
(d) Both the beams will experience same deformation

**Q.2** For the component loaded with a force $F$ as shown in the figure, the axial stress at corner point $P$ is

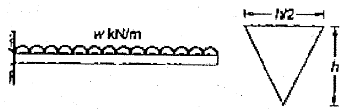



(a) $\frac{F(3L-b)}{4b^3}$ (b) $\frac{F(3L+b)}{4b^3}$
(c) $\frac{F(3L-4b)}{4b^3}$ (d) $\frac{F(3L-2b)}{4b^3}$

**Q.3** **Assertion (A):** In pure bending theory, section is considered prismatic and symmetrical in the loading plane.
**Reason (R):** Twisting will also occur along with bending if bending takes place in nonsymmetrical sections.
(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

**Q.4** **Assertion (A):** For structures subjected to bending steel I-beams are preferred to other shapes.
**Reason (R):** I-beams have large area of cross-section compared to other shapes.
(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

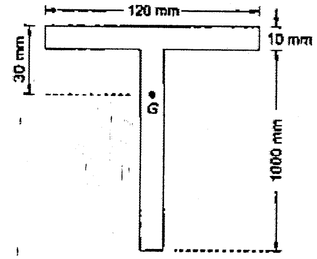
**Q.5** Consider the cantilever beam loaded as shown below:




The ratio of maximum tensile stress to maximum compressive stress is
(a) 1.0 (b) 0.5
(c) 2.0 (d) 0.33

**Q.6** A cantilever beam of rectangular cross-section is 0.6 m deep and 1 m thick. If the beam were to be 1 m deep and 0.6 m thick, then it would
(a) be weakened 0.6 times
(b) be strengthened 0.6 times
(c) be strengthened 1.67 times
(d) Have the same strength

**Q.7** If the bending stress in top fibre, of the *T*-beam shown in figure, is 30 MPa, then the stress in the bottom fiber would be
(*G* is the centroid of the beam)




(a) 30 MPa (b) −80 MPa
(c) 50 MPa (d) None of these

**Q.8** **Assertion (A):** A square section is more economical in bending than the circular section of same area of cross-section.
**Reason (R):** The modulus of the square section is less than that of circular section of same area of cross-section.
(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

**Q.9** A rectangular timber beam is cut out of a cylindrical log of diameter '$D$'. The depth of the strongest timber beam will be
(a) $D\sqrt{\frac{1}{2}}$ (b) $D\sqrt{\frac{2}{3}}$
(c) $D\sqrt{\frac{5}{8}}$ (d) $D\sqrt{\frac{3}{4}}$

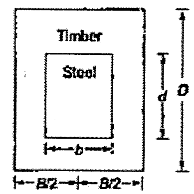
**Q.10** **Assertion (A):** The Poisson effect of adjacent layers do not affect the previous layer in pure bending theory.
**Reason (R):** In pure bending theory each layer of material is free to contract and expand longitudinally and laterally.
(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

**Q.11** Consider the following statements:
1. In pure bending, the Young's modulus in tension and compression is assumed to be equal.
2. Bending strain distribution is linear in pure bending.
3. Hooke's law is not valid in pure bending theory even through material is assumed to be elastic.

Which of the following statement(s) is/are correct?
(a) Only 1 (b) Only 2
(c) 1 and 2 (d) 1 and 3

**Q.12** **Assertion (A):** For a non-symmetrical section, elastic and plastic neutral axis will be different.
**Reason (R):** Elastic neutral axis, transverse centroidal axis, and plastic neutral axis are equal area axis.
(a) both A and R are true and R is the correct explanation of A
(b) both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

**Q.13** A section of a flitched beam is shown in figure below:




The main role of the steel plate placed between the two timber sections is to
(a) increase the axial resistance of the section
(b) increase the density

(c) increase shear and bending resistance of the section
(d) avoid development of shear stress in the two timber sections

**Q.14** Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I
A. Moment of inertia
B. Elongation
C. Neutral axis
D. Top fibre

List-II
1. Tensile stress
2. Modulus of rupture
3. Zero shear stress
4. Zero longitudinal stress

Codes:

| | A | B | C | D |
|---|---|---|---|---|
| (a) | 2 | 1 | 3 | 4 |
| (b) | 1 | 2 | 4 | 3 |
| (c) | 3 | 4 | 1 | 2 |
| (d) | 2 | 1 | 4 | 3 |

**Q.15** The strength of a beam depends on
1. centre of gravity of the section
2. its weight
3. section modulus

Which of these statements is/are correct?
(a) only 1 (b) only 3
(c) both 1 and 2 (d) both 2 and 3

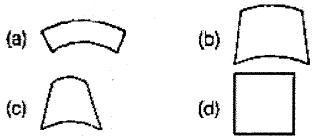
**Q.16** The bending stress on a beam section is zero at
(a) centroid of the section
(b) top fibre
(c) bottom fibre
(d) depends on the moment of inertia

**Q.17** The ratio of the maximum bending stress in the flange to that in the web of an I-section at any section on the beam is always
(a) less than one
(b) equal to one
(c) more than one
(d) no exact relation as above

**Q.18** The normal stress at a point in the beam subjected to bending and shear
(a) is independent of the shear stress
(b) is related to the shear stress
(c) is zero in most cases
(d) depends on the shear force

**Q.19** A rectangular cross-sectioned beam is made of one steel plate sandwiched between two aluminium plates of double the thickness of the steel plate. The ratio of the normal stress in the fibres of the steel to that in the aluminium plates at the same distance from the $CG$ is ($E_s > E_{Al}$)
(a) equal to one
(b) more than one
(c) less than one
(d) uncertain to define with the data given

**Q.20** For a square sectional beam bent as shown in the figure, the exaggerated view of the deformed cross section is

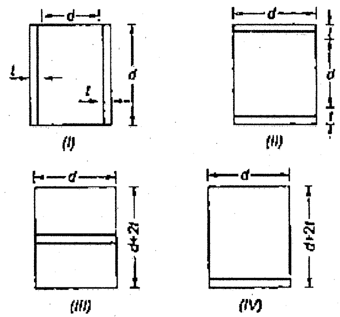




**Q.21** The statement that plane sections that are plane before the application of twisting moment remain plane after the application of twisting moment is valid for
(a) all types of cross section
(b) all types of cross section with curved boundary
(c) only solid circular cross section
(d) only circular cross sections, hollow or solid

**Q.22** The cross sections of the beams of equal length are a circle and a square whose permissible bending stress are same under same maximum bending. The ratio of their weights is
(a) 1.118 (b) 1.338
(c) 1.228 (d) 1.108

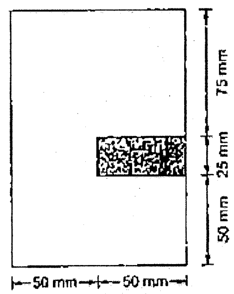
**Q.23** The width of a beam of uniform strength having a constant depth $d$ length $L$, simply supported at the ends with a central load $W$ is
(a) $\frac{2WL}{3fd^2}$ (b) $\frac{3WL}{2fd^2}$
(c) $\frac{2fL}{3Wd^2}$ (d) $\frac{3fL^2}{3Wd}$

**Q.24** The modular ratio of the materials used in the flitched beam is 10 and the ratio of the allowable stresses is also 10. Four different sections of the beam are shown in the given figures. The material shown hatched has larger modulus of elasticity and allowable stress than the rest of the material



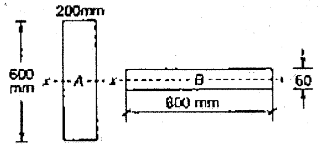

Which one of the following statement is true for the beam under consideration?
(a) All the given sections would support the same magnitude of load.
(b) Sections II, III and IV would support equal loads which is more than what section-I would support.
(c) Sections I and II would support equal loads which is more than what section-III and IV would support.
(d) Section II would support the greatest load.

**Q.25** A beam with the cross-section given below is subjected to a positive bending moment (causing compression at top) of 16 kNm acting around the horizontal axis. The tensile force acting on the hatched area of cross-section is




(a) Zero (b) 5.9 kN
(c) 8.9 kN (d) 17.8 kN

**Q.26**




Cross-sections of two beams A (600 mm × 200 mm) and B (800 mm × 60 mm) are shown in the figure given above. Both the beams have the same material. By how many times is the beam A stronger than the beam B in resisting bending?
(a) 80 (b) 60
(c) 50 (d) 25

**Q.27** A $MS$ beam is subjected to a bending moment, such that a stress of 100 MPa is developed in a layer at a distance of 100 mm from the neutral layer. If $E$ = 200 GPa, what is the radius of curvature of the beam?
(a) 400 m (b) 200 m
(c) 100 m (d) 50 m

**Q.28** A beam is of I-section with flanges 200 mm × 10 mm and web 180 mm × 10 mm. Due to the bending moment applied on the beam section, maximum stress developed in the beam section is 100 MPa. What is the stress developed at inner edge of the flange?
(a) 110 MPa (b) 100 MPa
(c) 90 MPa (d) 120 MPa

**Q.29** A cantilever of uniform strength $\sigma$, having rectangular section of constant breadth $b$ but variable depth d is subjected to a UDL throughout its length. If the depth of the section is 150 mm at the fixed end, then what is the depth of the middle of the length of cantilever?

(a) 150 mm (b) 100 mm

(c) 75 mm (d) 125 mm

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## Answers Bending Stress in Beams

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. (b) | 2. (d) | 3. (a) | 4. (c) | 5. (b) | 6. (c) | 7. (b) | 8. (c) | 9. (b) | 10. (a) |
| 11. (c) | 12. (a) | 13. (c) | 14. (d) | 15. (b) | 16. (a) | 17. (c) | 18. (b) | 19. (b) | 20. (a) |
| 21. (d) | 22. (a) | 23. (b) | 24. (d) | 25. (c) | 26. (d) | 27. (b) | 28. (c) | 29. (c) | |

## Explanations Bending Stress in Beams

**1. (b)**

$$\sigma = \frac{M}{I}y = \frac{M}{z}$$

For same cross-sectional area;

$Z_{square} > Z_{circular}$

$\therefore \sigma_{circular} > \sigma_{square}$

**2. (d)**

Total stress = Direct stress + Stress due to moment.

$$= \frac{P}{A} + \frac{My}{I}$$

$$= \frac{F}{4b^2} + \frac{F(L-b)b}{\frac{2b(2b)^3}{12}}$$

$$= \frac{F}{4b^2} + \frac{F(L-b)}{\frac{16b^2}{12}}$$

$$= \frac{Fb}{4b^3} + \frac{3F(L-b)}{4b^3}$$

$$= \frac{F(3L-2b)}{4b^3}$$

**4. (c)**

I-beams are preferred over other shapes because of the fact that large portion of their area is located away from neutral axis.

**5. (b)**

$$\sigma = \frac{My}{I}$$

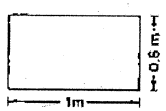
$$\sigma_{cmax} = \frac{M}{I} \times \left(\frac{2h}{3}\right)$$

($\because$ Maximum compressive stress occurs at bottom of beam for a cantilever)

$$\sigma_{tmax} = \frac{M}{I} \times \frac{h}{3}$$

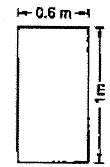
$$\frac{\sigma_{tmax}}{\sigma_{cmax}} = \frac{\left(\frac{h}{3}\right)}{\left(\frac{2h}{3}\right)} = \frac{1}{2}$$

**6. (c)**




$$Z_1 = \frac{I}{y} = \frac{\frac{1 \times 0.6^3}{12}}{\frac{0.6}{2}} = \frac{0.6^2}{6}$$

$= 6 \times 10^{-2}\ m^3$




$$Z_2 = \frac{I}{y} = \frac{\frac{0.6 \times 1^3}{12}}{\frac{1}{2}} = \frac{0.6}{6} = 10^{-1}\ m^3$$

$$\frac{Z_2}{Z_1} = \frac{10^{-1}}{6 \times 10^{-2}} = \frac{1}{0.6} = 1.667$$

**7. (b)**

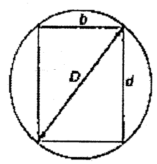
$$\frac{\sigma_1}{y_1} = \frac{\sigma_2}{y_2} \quad \left(= \frac{E}{R} = \frac{M}{I}\right)$$

$$\Rightarrow \sigma_2 = \sigma_1 \times \frac{y_2}{y_1} = 30 \times \left(\frac{80}{30}\right) = 80 \text{ MPa}$$

If top fibre is in tension, bottom fibre will be in compression and vice-versa.

Hence $\sigma_2 = -80$ MPa

**9. (b)**




Let b is width d is depth of strongest section cut from a log of wood having diameter $D$.

$\therefore b^2 + d^2 = D^2$

or $d = \sqrt{D^2 - b^2}$

Strength of section is $z$

where $z = \frac{1}{6} b(D^2 - b^2)$

For maximum $z$.

$$\frac{dz}{db} = 0 \quad \Rightarrow \quad b = \frac{D}{\sqrt{3}}$$

or $d = \frac{\sqrt{2}}{\sqrt{3}} D$

or $\frac{b}{d} = \frac{1}{\sqrt{2}}$

**10. (a)**

The assumption involved in pure bending theory that each layer is free to expand and contract longitudinally and laterally also implies that pressure of adjacent layers is ignored.

**11. (c)**

Hooke's law is valid in pure bending theory.

**15. (b)**

Strength means moment carrying capacity.

$\because \quad M = fZ$

$\therefore \quad M \propto Z$

**16. (a)**

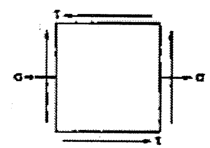
The bending stress on a beam section is zero at neutral axis and in elastic analysis neutral axis lies at centroidal axis

**17. (c)**

Bending stress is linear from neutral axis and flange is at greater distance than web from neutral axis.

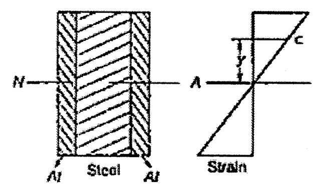
**18. (b)**

Due to bending normal stresses are set up while due to shear force, shear stresses are induced.




Under normal and shear stresses at any plane, normal stress is a function of shear stress.

**19. (b)**




At any distance $y$ from NA, the strains in steel and $Al$ are equal.

But $E_s > E_{Al}$

Therefore within the elastic limit,

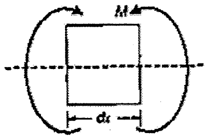
$$\sigma_s = E_s \epsilon$$

and $\sigma_{Al} = E_{Al} \epsilon$

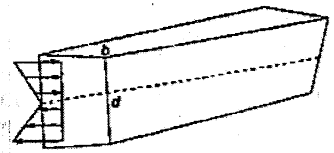
Since $E_s > E_{Al}$

$\Rightarrow \quad \sigma_s > s_{Al}$

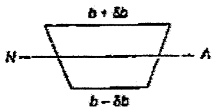
20. (a)




Due to sagging bending moment, top fibres above the neutral axis are in compression while bottom fibres below neutral axis are in tension.




Above the NA due to compression in axial direction, elongation takes place in lateral direction while below the neutral axis, contraction in lateral direction occurs.




22. (a)

If σ is same then

$$M = \sigma_1 Z_1 = \sigma_2 Z_2$$

$\Rightarrow \quad Z_1 = Z_2 \qquad (\because \sigma_1 = \sigma_2)$

For circle

$$Z_1 = \frac{\pi}{32} D^3 \qquad \text{...(i)}$$

For square

$$Z_2 = \frac{x^3}{6} \qquad \text{...(ii)}$$

$$\therefore \frac{\text{Weight of circle}}{\text{Weight of square}} = \frac{\gamma . A_c . L}{\gamma A_s L}$$

$$\Rightarrow \frac{W_c}{W_s} = \frac{(\pi/4) D^2}{x^2} = \frac{\pi}{4}\left(\frac{D}{x}\right)^2$$

From (i) and (ii)

$$\frac{\pi}{32} D^3 = \frac{x^3}{6}$$

$$\Rightarrow \quad \frac{D}{x} = \left(\frac{16}{3\pi}\right)^{1/3}$$

$$\therefore \quad \frac{W_c}{W_s} = \frac{\pi}{4} \times \left(\frac{16}{3\pi}\right)^{2/3} = 1.118$$

23. (b)

$$M_{max} = \frac{WL}{4} = f \cdot \frac{bd^2}{6}$$

$$b = \frac{3}{2} \frac{WL}{fd^2}$$

24. (d)

In the flitched beams there are two principles to be followed in analysis viz.

(i) strain diagram is linear

(ii) the radius of curvature of both materials is same at all the points.

Allowable stress in weak material = $f$

Allowable stress in strong material = $10f$

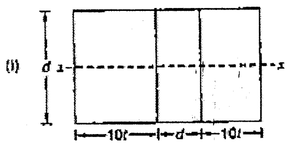
Modulus of elasticity of weak material = $E$

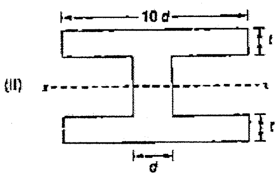
Modulus of elasticity of strong material = $10E$

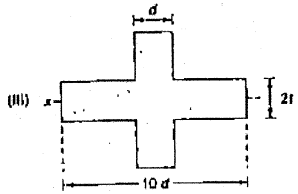
Maximum strain in weak material = $f/E$

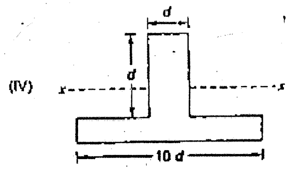
Maximum strain in strong material = $10f/10E = f/E$

The hatched material can be replaced by equivalent weak material when its width is multiplied by modular ratio. So the equivalent sections are:



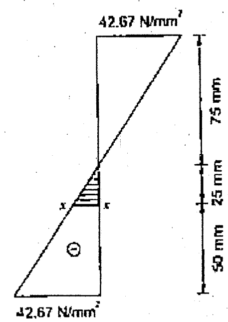

It is very clear that section modulus of section-II is more than that of section-III as more area is away from the neutral axis in section-II. Section-IV is unequal and its section modulus in tension and compression will be different. Since the strain is $(f/E)$ at top most and at bottom most fibre it is less than $f/E$.

So its capacity will be less than that of section-II.

Comparing section-I and section-II, more area is away from the neutral axis in section-II, so the moment capacity of section-II is more than that of section-I. Thus section-II has higher capacity.

25. (c)

The stress at the extreme fibres can be calculated as




$$f_c = \frac{M}{I} y = \frac{6M}{bd^2} = \frac{6 \times 16 \times 10^6}{100 \times (150)^2}$$

$$= 42.67 \text{ N/mm}^2$$

To calculate tensile force acting on hatched area, let us calculate stress on section $x$ - $x$

$$\frac{42.67}{75} = \frac{f_{xx}}{25}$$

$\Rightarrow \quad f_{xx} = 14.22 \text{ N/mm}^2$

$\therefore$ Tensile force on hatched area

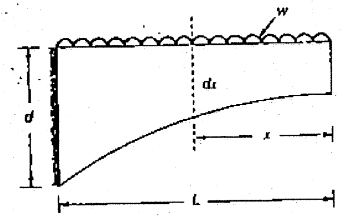
$$= \left(\frac{1}{2} \times 50 \times 25 \times 14.22\right) \text{N} = 8.9 \text{ kN}$$

27. (b)

$$\frac{E}{R} = \frac{\sigma}{y}$$

$$\Rightarrow \quad R = \frac{Ey}{\sigma} = \frac{200 \times 10^3 \times 100}{100} = 200 \text{ m}$$

29. (c)




Let the depth vary along the length such that depth at distance $x$ from RH support is $d_x$.

$$M_x = \sigma Z_x = \frac{\sigma b d_x^2}{6}, \text{ but } M_x = \frac{wx^2}{2}$$

$$\Rightarrow \quad \frac{wx^2}{2} = \frac{\sigma b d_x^2}{6} \quad \Rightarrow \quad \frac{3wx^2}{b\sigma} = d_x^2$$

$\Rightarrow$ or, $\quad d_x \propto x$

$$\therefore \quad \frac{d_1}{d} = \frac{L/2}{L} = \frac{1}{2}$$

$$\Rightarrow \quad d_1 = \frac{150}{2} = 75 \text{ mm}$$

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