Unit Test 1

(Mechanics)

1 Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between the planet and the star is inversely proportional to $R^{5/2}$, then T^2 is proportional to

(a) R^3 (b) $R^{7/2}$
(c) $R^{3/2}$ (d) $R^{3.75}$

2 The escape velocity from the earth's surface is 11kms^{-1}. A certain planet has a radius twice that of the earth, but its mean density is the same as that of the earth. The value of the escape velocity from this planet would be

(a) 22 kms^{-1} (b) 11 kms^{-1}
(c) 5.5 kms^{-1} (d) 16.5 kms^{-1}

3 A ball projected from ground at an angle of 45° just across a wall in front. If point of projection is 4 m from the foot of wall and ball strikes the ground at a distance of 6 m on the other side of the wall, the height of the wall is

(a) 4.4 m (b) 2.4 m
(c) 3.2 m (d) 1.6 m

4 The product of pressure and volume have the same units as the product of

(a) charge and potential
(b) electric field strength and distance
(c) electromotive force and capacitance
(d) magnetic moment and magnetic induction

5 The radius of a ball is (5.2 ± 0.2) cm. The percentage error in the volume of the ball is approximately

(a) 11% (b) 4%
(c) 7% (d) 9%

6 At time $t = 0$ s, particle starts moving along the X-axis. If its kinetic energy increases uniformly with time t, then the net force acting on it must be proportional to

(a) $\sqrt{t}$ (b) t^{-1}
(c) t (d) $\frac{1}{\sqrt{t}}$

7 Suppose, the kinetic energy of a body oscillating with amplitude A and at a distance x is given by

$$\text{KE} = \frac{Bx}{x^2 + A^2}$$

The dimensions of B are the same as that of

(a) work /time (b) work $\times$ distance
(c) work /distance (d) work $\times$ time

8 A soap bubble oscillates with time period T, which in turn depends on the pressure (p), density (ρ) and surface tension (σ). Which of the following correctly represents the expression for T^2?

(a) $\rho\sigma^2/p^3$ (b) $\rho p^3/\sigma$ (c) $p^3\sigma/p$ (d) $\rho/p^3\sigma$

9 A boat which has a speed of 5 kmh^{-1} in still water crosses a river of width 1 km along the shortest path in 15 min. The velocity of the river (in kmh^{-1}) is

(a) 1 (b) 3 (c) 4 (d) $\sqrt{41}$

10 The velocity of a particle at an instant is 10 ms^{-1} and after 5 s the velocity of the particle is 20 ms^{-1}. The velocity 3 s before (in ms^{-1}) was

(a) 8 (b) 4 (c) 6 (d) 7

11 A particle moving in a straight line covers half the distance with speed of 3 ms^{-1}. The other half of the distance is covered in two equal time intervals with speed of 4.5 ms^{-1} and 7.5 ms^{-1}, respectively. The average speed of the particle during this motion is

(a) 4.0 ms^{-1} (b) 5.0 ms^{-1} (c) 5.5 ms^{-1} (d) 4.8 ms^{-1}

12 What will be the ratio of the distance moved by a freely falling body from rest in 4th and 5th seconds of journey?

(a) 4 : 5 (b) 7 : 9 (c) 16 : 25 (d) 1 : 1

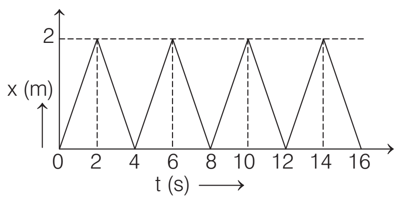
13 A particle moves in xy-plane according to the equation

$$x = 4t^2 + 5t + 16 \text{ and } y = 5t$$

The acceleration of the particle is

(a) 8 (b) 13 (c) 14 (d) 16

14 The figure shows the position-time $(x\text{-}t)$ graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is



(a) 0.4N - s (b) 0.8 N - s (c) 1.6N - s (d) 0.2 N - s

15 The minimum velocity (in ms^{-1}) with which a car driver must traverse a curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is

(a) 60 (b) 30 (c) 15 (d) 25

16 A particle is dropped vertically from rest, from a height. The time taken by it to fall through successive distances of 1 m each will be, then

(a) all equal, being equal to $\sqrt{2/g}$ second
(b) in the ratio of the square roots of the integers 1, 2, 3,...
(c) in the ratio of the difference in the square roots of integers, i.e. $\sqrt{1}, (\sqrt{2}-\sqrt{1}), (\sqrt{3}-\sqrt{2}), (\sqrt{4}-\sqrt{3}), \ldots$
(d) in the ratio of the reciprocals of square roots of the integers, i.e. $\frac{1}{\sqrt{1}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{4}}, \ldots$

17 Let $\mathbf{A} = \hat{\mathbf{i}} A\cos\theta + \hat{\mathbf{j}} A\sin\theta$ be any vector. Another vector $\mathbf{B}$ which is normal to $\mathbf{A}$ is

(a) $\hat{\mathbf{i}} B\cos\theta + \hat{\mathbf{i}} B\sin\theta$ (b) $\hat{\mathbf{i}} B\sin\theta + \hat{\mathbf{j}} B\cos\theta$
(c) $\hat{\mathbf{i}} B\sin\theta - \hat{\mathbf{j}} B\cos\theta$ (d) $\hat{\mathbf{i}} B\cos\theta - \hat{\mathbf{j}} B\sin\theta$

18 A particle moves from position $\mathbf{r}_1 = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$ to position $\mathbf{r}_2 = 14\hat{\mathbf{i}} + 13\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$ under the action of force $(4\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ N. The work done will be

(a) 100 J (b) 50 J (c) 200 J (d) 75 J

19 An elevator car whose floor to ceiling distance is equal to 2.7 m starts ascending with constant acceleration of 1.2 ms^{-2}. After 2 s of the start, a bolt begins falling from the ceiling of the car. The free fall time of the bolt is

(a) $\sqrt{6}$ s (b) 0.7 s (c) 1 s (d) $\sqrt{0.54}$ s

20 Two balls of masses m_1 and m_2 are separated from each other by a powder charge placed between them. The whole system is at rest on the ground. Suddenly, the powder charge explodes and masses are pushed apart. The mass m_1 travels a distance s_1 and stops. If the coefficients of friction between the balls and ground are same, the mass m_2 stops after travelling the distance

(a) $s_2 = \frac{m_1}{m_2} s_1$ (b) $s_2 = \frac{m_2}{m_1} s_1$ (c) $s_2 = \frac{m_1^2}{m_2^2} s_1$ (d) $s_2 = \frac{m_2^2}{m_1^2} s_1$

21 A block slides from an inclination of 45°. If it takes time twice with friction than to that without friction, then coefficient of friction for surface is given by

(a) 1 (b) 0.75 (c) 0.5 (d) 0.25

22 Three solids of masses m_1, m_2 and m_3 are connected with weightless string in succession and are placed on a frictionless table. If the mass m_3 is dragged with a force, T the tension in the string between m_2 and m_3 is

(a) $\frac{m_2}{m_1 + m_2 + m_3} T$ (b) $\frac{m_3}{m_1 + m_2 + m_3} T$
(c) $\frac{m_1 + m_2}{m_1 + m_2 + m_3} T$ (d) $\frac{m_2 + m_3}{m_1 + m_2 + m_3} T$

23 A body of mass m slides down an incline and reaches the bottom with a velocity v, if the same mass were in the form of a ring which rolls down this incline, the velocity of the ring at bottom would have been

(a) v (b) $\sqrt{2}v$ (c) $\frac{1}{\sqrt{2}}v$ (d) $\sqrt{2/5}\,v$

24 A bullet is shot from a rifle. As a result, the rifle recoils. The kinetic energy of rifle as compared to that of bullet is

(a) less (b) greater
(c) equal (d) cannot be concluded

25 Two bodies of masses m_1 and m_2 and having velocities v_1 and v_2 respectively, moving along a straight line are brought to rest by equal resistance forces. If one moves twice as that of the duration of other, but goes only 1/2 of the distance covered by the other before coming to rest, the ratio v_1/v_2 and m_1/m_2 are

(a) $\frac{1}{4}, 8$ (b) $\frac{1}{2}, \frac{1}{4}$ (c) $\frac{1}{8}, \frac{1}{2}$ (d) $\frac{1}{4}, \frac{1}{6}$

26 The angle between the two vectors $\mathbf{A} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\mathbf{B} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ will be

(a) 90° (b) 0° (c) 60° (d) 45°

27 If $|\mathbf{A} \times \mathbf{B}| = \sqrt{3}\,\mathbf{A} \cdot \mathbf{B}$, then the value of $|\mathbf{A} + \mathbf{B}|$ is

(a) $(A^2 + B^2 + AB)^{1/2}$ (b) $\left(A^2 + B^2 + \frac{AB}{\sqrt{3}}\right)^{1/2}$
(c) $(A + B)$ (d) $(A^2 + B^2 + \sqrt{3}\,AB)^{1/2}$

28 A force acts on a 3.0 g particle in such a way that the position of the particles as a function of time is given by $x = 3t - 4t^2 + t^3$, where x is in metre and t is in second. The work done during the first 4 s is

(a) 570 mJ (b) 450 mJ (c) 490 mJ (d) 530 mJ

29 The kinetic energy of a particle moving along a circle of radius R depends upon the distance s as $K = as^2$. Then, the force acting on particle is

(a) $2a\frac{s^2}{R}$ (b) $2as\left[1 + \frac{s^2}{R^2}\right]^{1/2}$
(c) $2as$ (d) $2a$

30 A flywheel rotating about a fixed axis has a kinetic energy of 360 J when its angular speed is 30 $rads^{-1}$. The moment of inertia of the flywheel about the axis of rotation is

(a) 0.15 kg-m^2 (b) 0.75 kg-m^2
(c) 0.60 kg-m^2 (d) 0.80 kg-m^2

31 A body of mass m is thrown upwards at an angle θ with the horizontal with velocity v. While rising up the velocity of the mass after t seconds will be

(a) $\sqrt{(v\cos\theta)^2 + (v\sin\theta)^2}$ (b) $\sqrt{(v\cos\theta - v\sin\theta)^2 - gt}$

(c) $\sqrt{v^2 + g^2t^2 - 2v\sin\theta\, gt}$ (d) $\sqrt{v^2 + g^2t^2 - 2v\cos\theta\, gt}$

32 A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is

(a) $M\omega^2L/2$ (b) $M\omega^2L$ (c) $M\omega^2L/4$ (d) $M\omega^2L^2/2$

33 Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that each makes a complete circle in the same time t. The ratio of the angular speeds of the first to the second car is

(a) 1 : 1 (b) $m_1 : m_2$ (c) $r_1 : r_2$ (d) $m_1m_2 : r_1r_2$

34 A wheel is rotating at 900 rpm about its axis. When the power is cut-off it comes to rest in 1 min. The angular retardation (in rad s^{-2}) is

(a) $\pi/2$ (b) $\pi/4$ (c) $\pi/6$ (d) $\pi/8$

35 A long horizontal rod has a bead which can slide along its length and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ and gravity is neglected, then the time after which the bead starts slipping is

(a) $\sqrt{\frac{\mu}{\alpha}}$ (b) $\frac{\mu}{\sqrt{\alpha}}$ (c) $\frac{1}{\sqrt{\mu\alpha}}$ (d) infinitesimal

36 A metallic sphere of diameter 10 cm and mass 0.5 kg is fixed to one end of a thin rod of length 50 cm and mass 1 kg as shown in the given figure. The rod with the sphere will balance horizontally, when it is supported at a point whose distance from the free end of the rod is

(a) 45 cm (b) 40 cm (c) 35 cm (d) 30 cm

37 Two bodies of mass 10 kg and 5 kg moving in concentric orbits of radius R and r such that their periods are the same. Then, the ratio between their centripetal acceleration is

(a) R/r (b) r/R
(c) R^2/r^2 (d) r^2/R^2

38 The mean radius of the earth is R, its angular speed on its own axis is ω and the acceleration due to gravity at earth's surface is g. What will be the radius of the orbit of a geostationary satellite?

(a) $(R^2g/\omega^2)^{1/3}$ (b) $(Rg/\omega^2)^{1/3}$
(c) $(R^2\omega^2/g)^{1/3}$ (d) $(R^2g/\omega)^{1/3}$

39 For a satellite, escape velocity is 11 kms^{-1}. If the satellite is launched at an angle of 60° with the vertical, then escape velocity will be

(a) 11 kms^{-1} (b) $11\sqrt{3}$ kms^{-1}
(c) $\frac{11}{\sqrt{3}}$ kms^{-1} (d) 33 kms^{-1}

40 Two bodies of masses m_1 and m_2 are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction with the escape velocity. Their relative velocity of approach at a separation distance r between them is

(a) $\left[2G\frac{(m_1 - m_2)}{r}\right]^{1/2}$ (b) $\left[2G\frac{(m_1 + m_2)}{r}\right]^{1/2}$

(c) $\left[\frac{r}{2G(m_1m_2)}\right]^{1/2}$ (d) $\left[\frac{2G}{r}(m_1m_2)\right]^{1/2}$

41 The moon's radius is 1/4 that of the earth and its mass is 1/80 times that of the earth. If g represents the acceleration due to gravity on the surface of the earth, then on the surface of the moon its value is

(a) $g_e/4$ (b) $g_e/5$ (c) $g_e/6$ (d) $g_e/8$

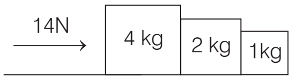
42 A circular road of radius 1000 m has banking angle 45°. Calculate the maximum safe speed of a car having mass 200 kg will be, if the coefficient of friction between tyre and road is 0.5.

(a) 172 ms^{-1} (b) 124 ms^{-1} (c) 99 ms^{-1} (d) 86 ms^{-1}

43 A body of mass m hangs at one end of a string of length l, the other end of which is fixed. It is given a horizontal velocity, so that the string would just reach, where it makes an angle of 60° with the vertical. The tension in the string at bottom most point position is

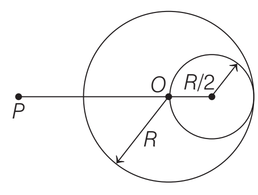
(a) $2mg$ (b) mg (c) $3mg$ (d) $\sqrt{3}\,mg$

44 Three blocks of masses 4 kg, 2 kg, 1 kg respectively are in contact on a frictionless table as shown in the figure. If a force of 14 N is applied on the 4 kg block, the contact force between the 4 kg and the 2 kg block will be



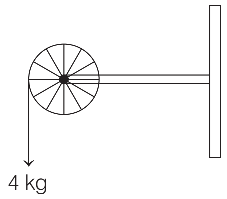
(a) 2 N (b) 6 N (c) 8 N (d) 14 N

45 A solid sphere of radius $R/2$ is cut-out of a solid sphere of radius R such that the spherical cavity, so formed touches, the surface on one side and the centre of the sphere on the other side, as shown in figure. The initial mass of the solid sphere was M. The gravitational field of the left over part of the sphere at a very far off point P located as shown below.



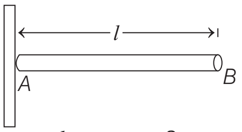
(a) $\frac{8}{5}\frac{GM}{R^2}$ (b) $\frac{7}{8}\frac{GM}{R^2}$ (c) $\frac{GM}{8R^2}$ (d) $\frac{6}{7}\frac{GM}{R^2}$

46 A wheel of radius 0.4 m can rotate freely about its axis as shown in the figure. A string is wrapped over its rim and a mass of 4 kg is hung. An angular acceleration of 8 rad s^{-2} is produced in it due to the torque. Then, moment of inertia of the wheel is (take, $g = 10\ ms^{-2}$)



(a) 2 kg-m^2 (b) 1 kg-m^2
(c) 4 kg-m^2 (d) 8 kg-m^2

47 A uniform rod AB of length l and mass m is free to rotate about point A. The rod is released from rest in horizontal position. Given that, the moment of inertia of the rod about A is $\frac{ml^2}{3}$, the initial angular acceleration of the rod will be



(a) $\frac{2g}{3l}$ (b) $mg\frac{l}{2}$ (c) $\frac{3}{2}gl$ (d) $\frac{3g}{2l}$

48 If the angular momentum of any rotating body increases by 200%, then the increase in its kinetic energy

(a) 400% (b) 800% (c) 200% (d) 100%

49 A body is rolling down an inclined plane. If KE of rotation is 40% of KE in translatory state, then the body is a

(a) ring (b) cylinder (c) hollow ball (d) solid ball

50 Two discs of moment of inertia I_1 and I_2 and angular speeds ω_1 and ω_2 are rotating along collinear axes passing through their centre of mass and perpendicular to their plane. If the two are made to rotate combindly along the same axis the rotational KE of system will be

(a) $\frac{I_1\omega_1 + I_2\omega_2}{2(I_1 + I_2)}$ (b) $\frac{(I_1 + I_2)(\omega_1 + \omega_2)^2}{2}$

(c) $\frac{(I_1\omega_1 + I_2\omega_2)^2}{2(I_1 + I_2)}$ (d) None of these

51 A cord is wound round the circumference of wheel of radius r. The axis of the wheel is horizontal and moment of inertia about it is I. A weight mg is attached to the end of the cord and falls from the rest. After falling through a distance h, the angular velocity of the wheel will be

(a) $\sqrt{\frac{2gh}{I + mr}}$ (b) $\left[\frac{2mgh}{I + mr^2}\right]^{1/2}$

(c) $\left[\frac{2mgh}{I + 2mr^2}\right]^{1/2}$ (d) $\sqrt{2gh}$

ANSWERS

1 (b)	**2** (a)	**3** (c)	**4** (a)	**5** (a)	**6** (d)	**7** (b)	**8** (a)	**9** (b)	**10** (b)
11 (d)	**12** (b)	**13** (a)	**14** (b)	**15** (b)	**16** (c)	**17** (c)	**18** (a)	**19** (c)	**20** (c)
21 (b)	**22** (c)	**23** (c)	**24** (a)	**25** (a)	**26** (a)	**27** (a)	**28** (d)	**29** (b)	**30** (d)
31 (c)	**32** (a)	**33** (a)	**34** (a)	**35** (a)	**36** (c)	**37** (a)	**38** (a)	**39** (a)	**40** (b)
41 (b)	**42** (a)	**43** (a)	**44** (b)	**45** (b)	**46** (a)	**47** (d)	**48** (b)	**49** (d)	**50** (c)
51 (b)									

Hints and Explanations

1 From universal law of gravitation,

$$\frac{mv^2}{R} = G\frac{Mm}{R^{5/2}}$$

Hence, $v^2 = \frac{GM}{R^{3/2}}$

$\therefore$ $T = \frac{2\pi R}{v}$

Hence,

$$T^2 = \frac{4\pi^2R^2}{v^2} = \frac{\frac{4\pi^2R^2}{GM}}{R^{3/2}} = \frac{4\pi^2}{GM}R^{\frac{7}{2}}$$

i.e. $T^2 \propto R^{7/2}$

2 $\therefore v_{es} = w\sqrt{2GM/R} = \sqrt{2G \times \frac{4}{3}\pi R^3\rho/R}$

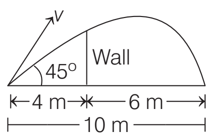
$= \sqrt{(8/3)\,G\pi R^2\rho}$

Hence, $v_{es} \propto R$

$\Rightarrow$ $v_{es} = 2 \times 11 = 22\ kms^{-1}$

3 As, range $= \frac{u^2 \sin 2\theta}{g} = 10$

$\Rightarrow$ $u^2 = \frac{10g}{\sin 2 \times 45°} \Rightarrow u^2 = 10\,g$



$\therefore$ $u = 10\ ms^{-1}$ (as, $g = 10\ ms^{-2}$)

$\because$ $y = x\tan\theta - \frac{1}{2}\frac{gx^2}{2v_0^2\cos^2\theta}$

$= 4\tan 45° - \frac{1}{2}\frac{g \times 16}{2\,v_0^2\cos^2 45°}$

$= 4 \times 1 - \frac{1}{2}\frac{10 \times 16}{2 \times 10 \times 10 \times \frac{1}{2}}$

$= 4 - 0.8 = 3.2$

4 $\therefore$ [Pressure $\times$ Volume]

$= \frac{N}{m^2} \times m^3 = N\text{-}m = J$

$\therefore$ [Charge × Potential] $= \frac{J}{C} \times C = J$

5 Radius of ball = 5.2 cm

$\therefore \quad V = \frac{4}{3}\pi R^3$

$$\left(\frac{\Delta V}{V}\right) = 3\left(\frac{\Delta R}{R}\right)$$

$$\left(\frac{\Delta V}{V}\right) \times 100 = 3\left(\frac{0.2}{5.2}\right) \times 100 = 11\%$$

6 Given, $\frac{dK}{dt} =$ constant $\Rightarrow K \propto t$

$\Rightarrow \quad v \propto \sqrt{t}$

Also, $\quad p = Fv = \frac{dK}{dt} =$ constant

$\Rightarrow \quad F \propto \frac{1}{v} \quad \Rightarrow F \propto \frac{1}{\sqrt{t}}$

7 $\therefore$ $[K]$ = work

and $[Bx] = [K]\,[x^2 + A^2]$

or $\quad [B] = [K]\,\frac{[x^2 + A^2]}{x}$

$\quad$ = work × distance

8 $\therefore \quad T^2 = p^a \rho^b \sigma^c$

$= [ML^{-1}T^{-2}]^a [ML^{-3}]^b [MT^{-2}]^c$

$= [M^{a+b+c} L^{-a-3b} T^{-2a-2c}]$

Hence, $a + b + c = 0, -a - 3b = 0$

and $\quad -2a - 2c = 2$

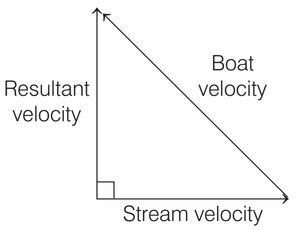
On solving, we get

$a = -3, b = 1$ and $c = 2$

As, $\quad T^2 = p^a \rho^b \sigma^c$

$\Rightarrow \quad T^2 = \frac{\rho\sigma^2}{p^3}$

9 According to the given condition,



Resultant velocity of boat

$= \frac{1 \text{ km}}{0.25 \text{ h}} = 4 \text{ kmh}^{-1}$

Also, (velocity of river)^2

+ (resultant velocity)^2

= (velocity of boat)^2

$\therefore$ Velocity of river

$= \sqrt{(5)^2 - (4)^2} = 3 \text{ kmh}^{-1}$

10 Using equation of motion, $v = u + at$

Here, $\quad 20 = 10 + a \times 5$

This gives $a = 2 \text{ ms}^{-2}$

and hence $v' = 10 + (2)(-3) = 4 \text{ ms}^{-1}$

11 As, $t_1 = \frac{x/2}{3}$; $x_1 = 4.5 \times t_2$,

$x_2 = 7.5 \times t_2$

Also, $\quad x_1 + x_2 = \frac{x}{2} = (4.5 + 7.5)\, t_2$

$\Rightarrow \quad t_2 = \frac{x}{24}$

$t = t_1 + t_2 = \frac{x}{6} + \frac{x}{24} = \frac{5}{24}x$

$v = \frac{x}{t} = \frac{24}{5} \text{ ms}^{-1} = 4.8 \text{ ms}^{-1}$

12 From the relation,

$x_n = \frac{a}{2}(2n - 1)$

So, $\frac{x_4}{x_5} = \frac{\frac{g}{2}(2 \times 4 - 1)}{\frac{g}{2}(2 \times 5 - 1)} = \frac{7}{9}$

13 Acceleration of the particle is given as

$a = \sqrt{a_x^2 + a_y^2}$

Here, $\quad a_x = \frac{d^2x}{dt^2} = 8$

and $\quad a_y = d^2y/dt^2 = 0$

$\Rightarrow \quad a = \sqrt{a_x^2 + a_y^2} = \sqrt{(8)^2 + 0} = 8$

14 From the graph, it is a straight line, so motion is uniform, because of impulse direction of velocity changes as can be seen from the slope of the graph.

Initial velocity, $v_1 = \frac{2}{2} = 1 \text{ ms}^{-1}$

Final velocity, $v_2 = -2/2 = -1 \text{ ms}^{-1}$

$\therefore \quad p_i = mv_1 = 0.4$ N - s

and $\quad p_f = mv_2 = -0.4$ N - s

Now, impulse,

$\mathbf{J} = p_f - p_i = -0.4 - 0.4$

$= -0.8$ N - s

$\Rightarrow \quad |\mathbf{J}| = 0.8$ N - s

15 Using the relations,

$\frac{mv^2}{r} = \mu R, R = mg$

$\Rightarrow \quad \frac{mv^2}{r} = \mu mg$ or $v^2 = \mu rg$

$\therefore \quad v^2 = 0.6 \times 150 \times 10$

or $\quad v = 30 \text{ ms}^{-1}$

16 Time taken to cover n metre is given by

$n = \frac{1}{2} g t_n^2$ or $t_n = \sqrt{\frac{2n}{g}}$

Time taken to cover $(n + 1)$ metre is given by

$t_{n+1} = \sqrt{\frac{2(n+1)}{g}}$

$t_{n+1} - t_n = \sqrt{\frac{2(n+1)}{g}} - \sqrt{\frac{2n}{g}}$

$= \sqrt{\frac{2}{g}}\left(\sqrt{n+1} - \sqrt{n}\right)$

This gives ratio as

$\sqrt{1}, (\sqrt{2} - \sqrt{1}), (\sqrt{3} - \sqrt{2})$, etc.

17 For normal vectors $\mathbf{A} \cdot \mathbf{B} = 0$. This is the case with the vector in option (c).

Given, $A = \hat{\mathbf{i}}\, A\cos\theta + \hat{\mathbf{j}}\, A\sin\theta$. From the options we can choose,

$B = \hat{\mathbf{i}}\, B\sin\theta - \hat{\mathbf{j}}\, B\cos\theta$

$\Rightarrow \mathbf{A} \cdot \mathbf{B} = (\hat{\mathbf{i}} A\cos\theta + \hat{\mathbf{j}} A\sin\theta)$

$\cdot\, (\hat{\mathbf{i}} B\sin\theta - \hat{\mathbf{j}} B\cos\theta)$

$= AB\sin\theta\cos\theta - AB\sin\theta\cos\theta = 0$

$[\because \hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = 1]$

18 Work done is given by

$W = \mathbf{F} \cdot \mathbf{r}$

Substituting $\quad \mathbf{F} = 4\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

and $\quad \mathbf{r} = (\mathbf{r}_2 - \mathbf{r}_1)$

$= [(14\hat{\mathbf{i}} + 13\hat{\mathbf{j}} + 9\hat{\mathbf{k}}) - (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 6\hat{\mathbf{k}})]$

$= 11\hat{\mathbf{i}} + 11\hat{\mathbf{j}} + 15\hat{\mathbf{k}}$, we get

$W = (4\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}) \cdot (11\hat{\mathbf{i}} + 11\hat{\mathbf{j}} + 15\hat{\mathbf{k}})$

$= 44 + 11 + 45 = 100$ J

19 Apparent acceleration,

$a = 9.8 + 1.2 = 11 \text{ ms}^{-2}$

Initial velocity of the car or bolt

$= 0 + 1.2 \times 2 = 2.4 \text{ ms}^{-1}$

Using $x = ut + \frac{1}{2}at^2$

As, $\quad u = -2.4 \text{ ms}^{-1}$, $a = 11 \text{ ms}^{-2}$

and $x = 2.7$ m, we get

$2.7 = -2.4 \times t + \frac{1}{2} \times 11 \times t^2$

$\Rightarrow \quad 5.5t^2 - 2.4t - 2.7 = 0$

$\Rightarrow \quad t \simeq 1$ s

20 Here, $\quad m_1 v_1 = m_2 v_2$

or $\quad m_1\left[\frac{1}{2} m_1 v_1^2\right] = m_2\left[\frac{1}{2} m_2 v_2^2\right]$

Also, $\frac{1}{2} m_1 v_1^2 = \mu m_1 g s_1$

and $\frac{1}{2} m_2 v_2^2 = \mu m_2 g s_2$

$\because \quad \frac{m_2 s_2}{m_1 s_1} = \frac{m_1}{m_2} \Rightarrow s_2 = \frac{m_1^2}{m_2^2} \cdot s_1$

21 Acceleration without friction,
$a_1 = g \sin\theta$.
Acceleration with friction,
$a_2 = g \sin\theta - \mu g \cos\theta$

Also, $t = \sqrt{\frac{2s}{a}}$

Therefore, $\frac{t_1}{t_2} = \sqrt{\frac{a_2}{a_1}}$

Thus, $\frac{2}{1} = \sqrt{\frac{g \sin\theta}{g \sin\theta - \mu g \cos\theta}}$

This gives, $\mu = \frac{3}{4} \tan\theta = \frac{3}{4} \times \tan 45°$

$= \frac{3}{4} \times 1 = \frac{3}{4} = 0.75$

22 The tension pulls m_2 and m_1 with acceleration, $a =$
$T/(m_1 + m_2 + m_3)$

Hence, $F = \frac{m_1 + m_2}{m_1 + m_2 + m_3} T$

23 When the body slides down the incline, its kinetic energy is given as

$\text{KE} = \frac{1}{2} mv^2$

However, in the case of ring, total kinetic energy will be equal to the sum of rotational and translational kinetic energies $= \frac{1}{2} I\omega^2 + \frac{1}{2} mv'^2$

$= \frac{1}{2} mR^2\omega^2 + \frac{1}{2} mv'^2 \quad [\because I = mR^2]$

$= \frac{1}{2} mv^2 + \frac{1}{2} mv'^2$

Since, kinetic energy due to translational will be equal to that due to rotation.

$\therefore \quad \frac{1}{2} mv^2 = mv'^2$ or $v' = \frac{v}{\sqrt{2}}$

24 In the given case, momentum would be conserved.

Hence, $p_b = p_r$ or $p_b^2 = p_r^2$

Since, $p^2 = 2MK$, we have

$M_b K_b = M_r K_r$

i.e. $K_r = \frac{M_b}{M_r} K_b$

Because, $M_b < M_r$. Hence, $K_r < K_b$

25 Change in energy = Work done

Hence, $\frac{1}{2} m_1 v_1^2 = \frac{F\,s}{2}$ and

$\frac{1}{2} m_2 v_2^2 = F\,s$

This gives, $\frac{m_1 v_1^2}{m_2 v_2^2} = \frac{1}{2}$ ...(i)

Also, change in momentum = impulse

i.e. $m_1 v_1 = F \times 2t$

and $m_2 v_2 = F \times t$

This gives, $\frac{m_1 v_1}{m_2 v_2} = 2$ ...(ii)

On solving Eqs. (i) and (ii), we get

$\frac{v_1}{v_2} = \frac{1}{4}$ and $\frac{m_1}{m_2} = 8$

26 $\therefore \cos\theta = \frac{\mathbf{A} \cdot \mathbf{B}}{|A| \cdot |B|}$

$= \frac{(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) \cdot (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}})}{\sqrt{9 + 16 + 25}\,\sqrt{9 + 16 + 25}}$

$= \frac{9 + 16 - 25}{50} = 0$

$\Rightarrow \quad \cos\theta = 0$

$\therefore \quad \theta = 90°$

27 $\because \quad |\mathbf{A} \times \mathbf{B}| = \sqrt{3}\, \mathbf{A} \cdot \mathbf{B}$

$\Rightarrow \quad AB \sin\theta = \sqrt{3}\, AB \cos\theta$

$\Rightarrow \quad \tan\theta = \sqrt{3} \quad \Rightarrow \theta = 60°$

$\therefore |\mathbf{A} + \mathbf{B}| = \sqrt{A^2 + B^2 + 2AB \cos 60°}$

$= \sqrt{A^2 + B^2 + 2AB \times \frac{1}{2}}$

$= (A^2 + B^2 + AB)^{1/2}$

28 $\therefore$ Acceleration $= \frac{d^2x}{dt^2} = -8 + 6t$

$\therefore W = \int F dx = \int m \frac{d^2x}{dt^2} dx$

$= \int m \frac{d^2x}{dt^2} \frac{dx}{dt} dt$

$\therefore W = \int_0^4 \frac{3}{1000} (-8 + 6t)$

$(3 - 8t + 3t^2)\, dt$

On solving, we get

$W = 528 \text{ mJ} \approx 530 \text{ mJ}$

29 Here, $K = \frac{1}{2} mv^2 = as^2 \Rightarrow mv^2 = 2as^2$

On differentiating w.r.t. time t,

$\Rightarrow 2mv \cdot \frac{dv}{dt} = 4as \frac{ds}{dt} = 4asv$

$\Rightarrow \quad m \cdot \frac{dv}{dv} = 2as$

Centripetal force, $F_c = \frac{mv^2}{R} = \frac{2as^2}{R}$

$\therefore$ Force acting on the particle is

$F = \sqrt{F_t^2 + F_c^2} = \sqrt{(2as)^2 + \left(\frac{2as}{R}\right)^2}$

$= 2as\sqrt{1 + s^2 / R^2}$

30 As, $K_r = \frac{1}{2} I\omega^2$. Here, $K_r = 360$ J and

$\omega = 30 \text{ rads}^{-1} \Rightarrow I = 0.80 \text{ kg-m}^2$

31 Instantaneous velocity rising mass after t second will be

$v_t = \sqrt{v_x^2 + v_y^2}$

where, $v_x = v\cos\theta$ = horizontal Component of velocity,

$v_y = v\sin\theta - gt$ = vertical Component of velocity,

$v_t = \sqrt{(v\cos\theta)^2 + (v\sin\theta - gt)^2}$

$v_t = \sqrt{v^2 + g^2t^2 - 2v\sin\theta\, gt}$

32 The centre of mass is at $\frac{L}{2}$ distance from the axis. Hence, centripetal force,

$F_c = M\left(\frac{L}{2}\right)\omega^2$

So, the reaction at the other end will be equal to F_c.

33 As we know, angular speed,

$\omega = \frac{2\pi}{T} \quad \Rightarrow \frac{\omega_1}{\omega_2} = \frac{2\pi / T_1}{2\pi / T_2}$.

Here, $T_1 = T_2 = t$

$\therefore \quad \frac{\omega_1}{\omega_2} = 1$

34 Using $\omega = \omega_0 + \alpha t$

Here, $\omega_0 = 900$ rpm

$= (2\pi \times 900)/60$ rad/s

$\omega = 0$ and $t = 60$ s

So, $\omega_0 = \pi/2$

35 Bead starts moving when the centripetal force is equal to the friction. Hence, friction, $f = \mu m\, a_t$.
where, a_t is tangential acceleration.
It will be given by

$a_t = \frac{v}{t}$, where v is the linear speed.

$= L\omega$

Hence, $f = \mu m \frac{L\omega}{t}$

Also, centripetal force $= m L\omega^2$

Hence, $mL\omega^2 = \frac{\mu\, m\, L\omega}{t}$

i.e. $t = \dfrac{\mu}{\omega}$, also $\omega = \alpha\, t$

This gives, $t = \sqrt{\mu/\alpha}$

36 For balance of moments about G, (centre of mass), which is x cm form free end is

$$(50 - x + 5) \times 0.5 \times g = (x - 25) \times 1 \times g$$

$\Rightarrow x = 35$

37 $\because \dfrac{a_R}{a_r} = \dfrac{v_R^2}{R} \cdot \dfrac{r}{v_r^{\,2}} = \dfrac{v_R^2}{R^2} \cdot \dfrac{r^2}{v_r^{\,2}} \cdot \dfrac{R}{r}$

$$= \left(\frac{v_R}{2nR}\right)^2 \cdot \left(\frac{2nr}{v_r}\right)^2 \cdot \frac{R}{r}$$

$\Rightarrow \dfrac{a_R}{a_r} = \dfrac{T_R^2}{T_r^2} \cdot \dfrac{R}{r} = \dfrac{R}{r}$ $[\because T_R = T_r]$

38 $\therefore T = \dfrac{2\pi r}{v_0} = \dfrac{2\pi r}{(gR^2/r)^{1/2}} = \dfrac{2\pi r^{3/2}}{\sqrt{gR^2}} = \dfrac{2\pi}{\omega}$

Hence, $r^{3/2} = \dfrac{\sqrt{g R^2}}{\omega}$ or $r^3 = \dfrac{g R^2}{\omega^2}$

or $r = (gR^2/\omega^2)^{1/3}$

39 Escape velocity is same for all angles of projection.

40 Velocity of each body will be equal to the escape velocity w.r.t. the other. Hence, $v_1 = (2\,Gm_2/r)^{1/2}$ and $v_2 = (2\,Gm_1/r)^{1/2}$.

Relative velocity,

$$v_1 - v_2 = \sqrt{\frac{2G(m_1 + m_2)}{r}}$$

41 Acceleration due to gravity,

$g_e = \dfrac{GM_e}{R_e^2}$, $g_m = \dfrac{GM_e/80}{(R_e/4)^2}$

$\therefore$ $g_m = \dfrac{g_e}{5}$

42 The maximum velocity for a banked road with friction,

$$v^2 = gr\left(\frac{\mu + \tan\theta}{1 - \mu\,\tan\theta}\right)$$

$\Rightarrow v^2 = 9.8 \times 1000 \times \left(\dfrac{0.5 + 1}{1 - 0.5 \times 1}\right)$

$\Rightarrow v = 172\ \text{ms}^{-1}$

43 $\therefore$ Velocity at mean position,

$$v = \sqrt{2gl\,(1 - \cos\theta)}$$

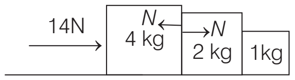
$\therefore$ Tension in the string,

$$T = mg + \frac{mv^2}{l}$$

$$= mg + \frac{m}{l}\,[2gl\,(1 - \cos 60°)]$$

$$= mg + mg = 2mg$$

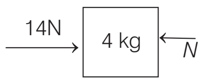
44 According to the given condition,



$\therefore$ $F = ma \Rightarrow 14 = 7a$

$\Rightarrow$ $a = 2\ \text{m/s}^2$

FBD of 4 kg block,



$14 = N + ma$

$\therefore$ Contact force, $N = 14 - 4 \times 2 = 6$ N

45 We have, $M' = \dfrac{M}{\frac{4\pi}{3}R^3} \times \dfrac{4\pi}{3}\left(\dfrac{R}{2}\right)^3 = \dfrac{M}{8}$

$\therefore$ Gravitational field at P

$$= \frac{GM}{R^2} - \frac{GM}{8R^2} = G \times \frac{M}{R^2}\left(1 - \frac{1}{8}\right)$$

$$= \frac{7}{8}\frac{GM}{R^2}$$

46 Given, $r = 0.4$ m, $\alpha = 8$ rad s^{-2},

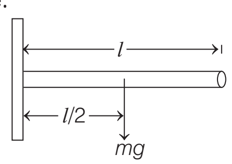
$m = 4$ kg, $I = ?$

$\therefore$ Torque, $\tau = I\alpha = mgr$

$\Rightarrow$ $4 \times 10 \times 0.4 = I \times 8$

$\Rightarrow$ $I = \dfrac{16}{8} = 2$ kg-m^2

47 Weight of the rod will produce the torque.



$\tau = I\alpha$

$\Rightarrow mg \times \dfrac{l}{2} = \dfrac{ml^2}{3} \times \alpha$

Angular acceleration, $\alpha = \dfrac{3g}{2l}$

48 As, $E = \dfrac{L^2}{2I}$

$\therefore$ $E \propto L^2 \Rightarrow \dfrac{E_2}{E_1} = \left(\dfrac{L_2}{L_1}\right)^2$

$$\frac{E_2}{E_1} = \left[\frac{L_1 + 200\% \text{ of } L_1}{L_1}\right]^2$$

$$= \left[\frac{L_1 + 2L_1}{L_1}\right]^2 = (3)^2$$

$\Rightarrow$ $E_2 = 9\,E_1$

Increment in kinetic energy,

$\Delta E = E_2 - E_1 = 9\,E_1 - E_1$

$\Delta E = 8\,E_1$

$\therefore$ $\dfrac{\Delta E}{E_1} = 8$

or percentage increases = 800%

49 $\dfrac{1}{2}mv^2\left(\dfrac{K^2}{R^2}\right) = 40\%\ \dfrac{1}{2}mv^2$

$\Rightarrow$ $\dfrac{K^2}{R^2} = \dfrac{40}{100} = \dfrac{2}{5}$

i.e. The body is solid sphere (ball).

50 Conservation of angular momentum,

$$I_1\,\omega_1 + I_2\,\omega_2 = (I_1 + I_2)\,\omega$$

Angular velocity of system,

$$\omega = \frac{I_1\,\omega_1 + I_2\,\omega_2}{I_1 + I_2}$$

$\therefore$ Rotational kinetic energy

$$= \frac{1}{2}(I_1 + I_2)\,\omega^2$$

$$= \frac{1}{2}(I_1 + I_2)\left(\frac{I_1\,\omega_1 + I_2\,\omega_2}{I_1 + I_2}\right)^2$$

$$= \frac{(I_1\,\omega_1 + I_2\,\omega_2)^2}{2(I_1 + I_2)}$$

51 We know that, $v = \sqrt{\dfrac{2gh}{1 + \dfrac{K^2}{r^2}}}$

$\therefore$ $\omega = \dfrac{v}{r} = \sqrt{\dfrac{2gh}{r^2 + K^2}}$

$\Rightarrow$ $\omega = \sqrt{\dfrac{2mgh}{mr^2 + mK^2}} = \sqrt{\dfrac{2mgh}{mr^2 + I}}$

$$= \sqrt{\frac{2mgh}{I + mr^2}}$$