Application of Derivatives
Part - 1

Assertion–Reasoning MCQs

Directions (Q. Nos. 46-60) Each of these questions contains two statements : Assertion (A) and Reason (R). Each of these questions has four alternative choices in which any one of them is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) A is true, R is true; R is a correct explanation for A.

(b) A is true, R is true; R is not a correct explanation for A.

(c) A is true; R is false.

(d) A is false; R is true.

46. **Assertion (A)** The function $f(x) = x^2 - 4x + 6$ is strictly increasing in the interval $(2, \infty)$.

Reason (R) The function $f(x) = x^2 - 4x + 6$ is strictly decreasing in the interval $(-\infty, 2)$.

47. **Assertion (A)** $y = \dfrac{e^x + e^{-x}}{2}$ is an increasing function on $[0, \infty)$.

Reason (R) $y = \dfrac{e^x - e^{-x}}{2}$ is an increasing function on $(-\infty, \infty)$.

48. **Assertion (A)** The function $f(x) = \sin x$ decreases on the interval $(0, \pi / 2)$.

Reason (R) The function $f(x) = \cos x$ decreases on the interval $(0, \pi / 2)$.

49. **Assertion (A)** The tangents to curve $y = 7x^3 + 11$ at the points, where $x = 2$ and $x = -2$ are parallel.

Reason (R) The slope of the tangents at the points, where $x = 2$ and $x = -2$, are equal.

50. **Assertion (A)** The tangent at $x = 1$ to the curve $y = x^3 - x^2 - x + 2$ again meets the curve at $x = -2$.

Reason (R) When a equation of a tangent solved with the curve, repeated roots are obtained at point of tangency.

51. **Assertion (A)** The equation of tangent to the curve $y^2 = 9x$ at the point $(1, 1)$ is $9x - 2y = 7$.

Reason (R) Equation of tangent is $y - y_1 = m(x - x_1)$, where m is the slope at (x_1, y_1).

52. **Assertion (A)** The equation of the normal to the curve $y^2 = 4x$ at the point $(1, 2)$ is $x + y - 3 = 0$.

Reason (R) Equation of normal is $y - y_1 = \dfrac{-1}{\left(\dfrac{dy}{dx}\right)_{(x_1, y_1)}}(x - x_1)$.

53. **Assertion (A)** The equation of the tangent to the curve $x^{2/3} + y^{2/3} = 2$ at $(1, 1)$ is $y + x - 2 = 0$.

Reason (R) The equation of the normal to the curve $x^{2/3} + y^{2/3} = 2$ at $(1, 1)$ is $y + x = 0$.

54. **Assertion (A)** If two positive numbers are such that sum is 16 and sum of their cubes is minimum, then numbers are 8, 8.

Reason (R) If f be a function defined on an interval I and $c \in I$ and let f be twice differentiable at c, then $x = c$ is a point of local minima if $f'(c) = 0$ and $f''(c) > 0$ and $f(c)$ is local minimum value of f.

55. The function f be given by $f(x) = 2x^3 - 6x^2 + 6x + 5$.

Assertion (A) $x = 1$ is not a point of local maxima.

Reason (R) $x = 1$ is not a point of local minima.

56. **Assertion (A)** If manufacturer can sell x items at a price of ₹ $\left(5 - \dfrac{x}{100}\right)$ each. The cost price of x items is ₹ $\left(\dfrac{x}{5} + 500\right)$. Then, the number of items he should sell to earn maximum profit is 240 items.

Reason (R) The profit for selling x items is given by $\dfrac{24}{5}x - \dfrac{x^2}{100} - 300$.

57. **Assertion (A)** $f(x) = 2x^3 - 9x^2 + 12x - 3$ is increasing outside the interval $(1, 2)$.

Reason (R) $f'(x) < 0$ for $x \in (1, 2)$.

58. **Assertion (A)** The equation of all lines having slope 0 which are tangents to the curve $y = \frac{1}{x^2 - 2x + 3}$, is $y = \frac{1}{2}$.

Reason (R) The point at which tangent to the given curve having slope 0, is $\left(1, \frac{1}{2}\right)$.

59. **Assertion (A)** The absolute maximum value of the function $2x^3 - 24x$ in the interval $[1, 3]$ is 89.
Reason (R) The absolute maximum value of the function can be obtained from the value of the function at critical points and at boundary points.

60. **Assertion (A)** If x is real, then the minimum value of $x^2 - 8x + 17$ is 1.

Reason (R) If $f''(x) > 0$ at critical point, then the value of the function at critical point will be the minimum value of the function.

ANSWER KEY

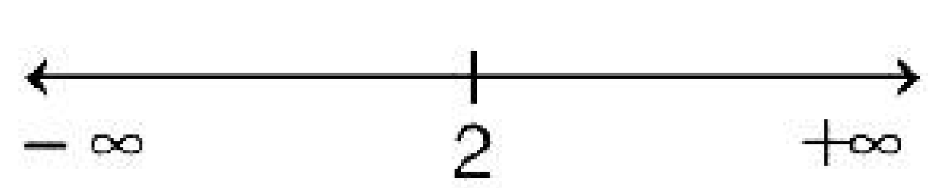
Assertion-Reasoning MCQs

46. (b)	47. (b)	48. (d)	49. (a)	50. (d)	51. (a)	52. (a)	53. (c)	54. (a)	55. (b)
56. (c)	57. (b)	58. (b)	59. (d)	60. (a)					

SOLUTION

46. We have, $f(x) = x^2 - 4x + 6$

or $f'(x) = 2x - 4 = 2(x - 2)$



Therefore, $f'(x) = 0$ gives $x = 2$.

Now, the point $x = 2$ divides the real line into two disjoint intervals namely, $(-\infty, 2)$ and $(2, \infty)$.

In the interval $(-\infty, 2)$, $f'(x) = 2x - 4 < 0$. Therefore, f is strictly decreasing in this interval.

Also, in the interval $(2, \infty)$, $f'(x) > 0$ and so the function f is strictly increasing in this interval.

Hence, both the statements are true but Reason is not the correct explanation of Assertion.

47. **Assertion** Let $f(x) = \dfrac{e^x + e^{-x}}{2}$

$$\Rightarrow \quad f'(x) = \frac{e^x - e^{-x}}{2} = \frac{1}{2}\left(e^x - \frac{1}{e^x}\right)$$

$$= \frac{1}{2}\left(\frac{e^{2x} - 1}{e^x}\right) \quad \ldots(\text{i})$$

Now, for $x \geq 0$, we have

$$2x \geq 0 \Rightarrow e^{2x} \geq e^0$$

[$\because e^x$ is an increasing function]

$$\Rightarrow \quad e^{2x} \geq 1$$

Also, for $x \geq 0$

$$\Rightarrow \quad e^x \geq 1$$

$\therefore$ From Eq. (i), we have

$$f'(x) = \frac{1}{2}\left(\frac{e^{2x} - 1}{e^x}\right) \geq 0$$

So, $f(x)$ is an increasing function on $[0, \infty)$.

Reason Let $g(x) = \dfrac{e^x - e^{-x}}{2}$

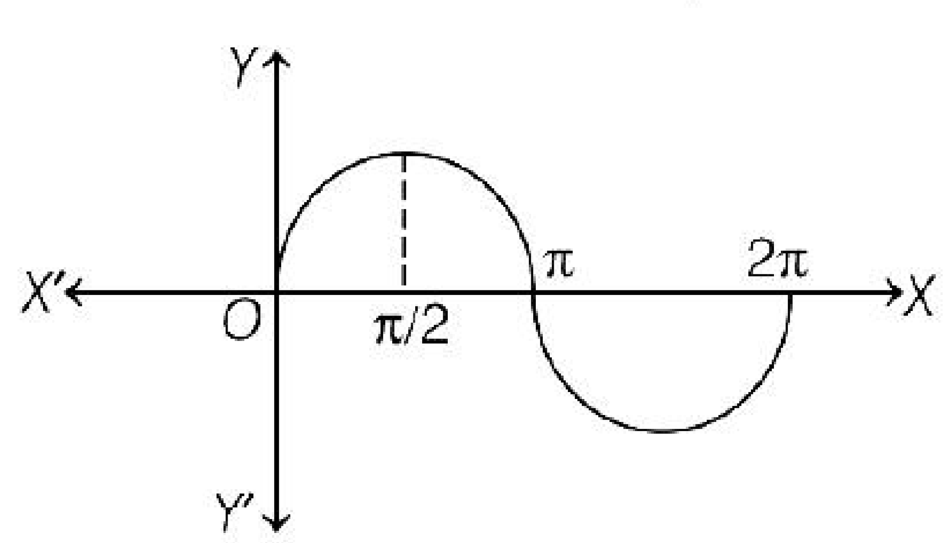
$$\Rightarrow \quad g'(x) = \frac{e^x + e^{-x}}{2} > 0$$

[$\therefore e^x$ and e^{-x} both are greater than zero in $(-\infty, \infty)$]

So, $g(x)$ is an increasing function on $(-\infty, \infty)$.

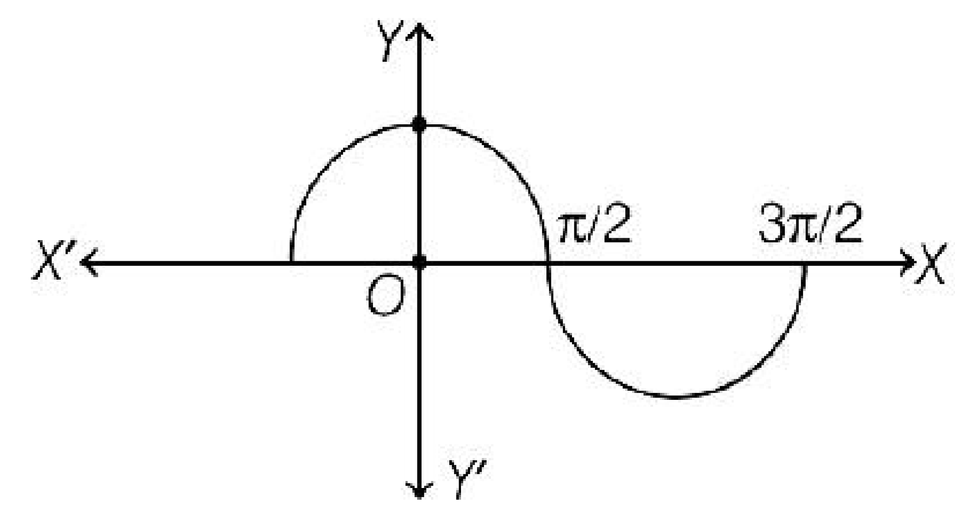
Hence, both Assertion and Reason are true.

48. **Assertion** Given, function $f(x) = \sin x$



From the graph of sin x, we observe that $f(x)$ increases on the interval $(0, \pi/2)$.

Reason Given function is $f(x) = \cos x$.



From the graph of cos x, we observe that, $f(x)$ decreases on the interval $(0, \pi/2)$.

Hence, Assertion is false and Reason is true.

49. The equation of the given curve is

$$y = 7x^3 + 11 \quad \ldots(\text{i})$$

$$\Rightarrow \quad \frac{dy}{dx} = 7 \times 3x^2 = 21x^2$$

[differentiating w.r.t. x]

$\because$ The slope of the tangent to the curve at (x_0, y_0) is $\left(\dfrac{dy}{dx}\right)_{(x_0, y_0)}$

$\therefore$ Slope of tangent at $x = 2$ is

$$\left(\frac{dy}{dx}\right)_{x=2} = 21(2)^2 = 84$$

Slope of tangent at $x = -2$ is

$$\left(\frac{dy}{dx}\right)_{x=-2} = 21(-2)^2 = 84$$

It is observed that the slopes of the tangents at the points where, $x = 2$ and $x = -2$ are equal. Hence, the two tangents are parallel.

Hence, both Assertion and Reason are true and Reason is the correct explanation of Assertion.

50. When $x = 1$, then $y = (1)^3 - (1)^2 - 1 + 2 = 1$

$$\therefore \quad \frac{dy}{dx} = 3x^2 - 2x - 1 \Rightarrow \left.\frac{dy}{dx}\right|_{x=1} = 0$$

$\therefore$ Equation of tangent at point (1, 1) is

$$y - 1 = 0(x - 1) \Rightarrow y = 1$$

Solving with the curve,

$$x^3 - x^2 - x + 2 = 1$$

$$\Rightarrow \quad x^3 - x^2 - x + 1 = 0$$

$$\Rightarrow \quad (x-1)(x^2-1) = 0 \Rightarrow = 1, 1, -1$$

[here, 1 is repeated root]

$\therefore$ Tangent meets the curve again at $x = -1$

$\therefore$ Assertion is false, Reason is true.

51. The equation of the given curve is $y^2 = 9x$

$\Rightarrow \quad y^2 = 9x$

Differentiating w.r.t. x, we get

$$2y\frac{dy}{dx} = 9 \Rightarrow \frac{dy}{dx} = \frac{9}{2y}$$

$\therefore$ Slope of tangent at (1, 1) is

$$\left(\frac{dy}{dx}\right)_{(1,1)} = \frac{9}{2\times 1} = \frac{9}{2}$$

$$\Rightarrow \quad m = \frac{9}{2}$$

$\therefore$ Equation of tangent at (1, 1) is

$$y - 1 = \frac{9}{2}(x-1)$$

$$\Rightarrow \quad 2(y-1) = 9(x-1)$$

$$\Rightarrow \quad 2y - 2 = 9x - 9$$

$$\Rightarrow \quad 0 = 9x - 9 - 2y + 2$$

$$\Rightarrow \quad 9x - 2y - 7 = 0$$

$$\Rightarrow \quad 9x - 2y = 7$$

Hence, both Assertion and Reason are true and Reason is the correct explanation of Assertion.

52. The equation of the given curve is $y^2 = 4x$

On differentiating w.r.t. x, we get

$$2y\frac{dy}{dx} = 4$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{4}{2y} = \frac{2}{y}$$

$\therefore$ Slope of tangent at (1, 2), is $\left(\frac{dy}{dx}\right)_{(1,\,2)} = \frac{2}{2} = 1$

Slope of normal at the point $(1, 2) = -\frac{1}{1} = -1$

$\therefore$ Equation of the normal at (1, 2) is

$$y - 2 = -1(x-1)$$

$$\Rightarrow \quad y - 2 = -x + 1$$

$$\Rightarrow \quad x + y - 3 = 0$$

So, both Assertion and Reason are true and Reason is the correct explanation of Assertion.

53. Assertion Differentiating $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$ with respect to x, we get

$$\frac{2}{3}x^{\frac{-1}{3}} + \frac{2}{3}y^{\frac{-1}{3}}\frac{dy}{dx} = 0 \quad \Rightarrow \frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$$

Therefore, the slope of the tangent at (1, 1) is $\left.\frac{dy}{dx}\right|_{(1,\,1)} = -1.$

So, the equation of the tangent at (1, 1) is

$$y - 1 = -1(x-1) \Rightarrow y + x - 2 = 0$$

Reason Also, the slope of the normal at (1, 1) is given by

$$\frac{-1}{\text{Slope of the tangent at } (1,\ 1)} = 1$$

Therefore, the equation of the normal at (1, 1) is

$$y - 1 = (x-1) \Rightarrow y - x = 0$$

Hence, Assertion is true and Reason is false.

54. Let one number be x, then the other number will be $(16 - x)$.

Let the sum of the cubes of these numbers be denoted by S.

Then, $\quad S = x^3 + (16-x)^3$

On differentiating w.r.t. x, we get

$$\frac{dS}{dx} = 3x^2 + 3(16-x)^2(-1)$$

$$= 3x^2 - 3(16-x)^2$$

$$\Rightarrow \quad \frac{d^2S}{dx^2} = 6x + 6(16-x) = 96$$

For minima put $\frac{dS}{dx} = 0.$

$$\therefore \quad 3x^2 - 3(16-x)^2 = 0$$

$$\Rightarrow \quad x^2 - (256 + x^2 - 32x) = 0$$

$$\Rightarrow \quad 32x = 256$$

$$\Rightarrow \quad x = 8$$

At $x = 8, \left(\frac{d^2S}{dx^2}\right)_{x=8} = 96 > 0$

By second derivative test, $x = 8$ is the point of local minima of S.

Thus, the sum of the cubes of the numbers is the minimum when the numbers are 8 and $16 - 8 = 8$.

Hence, the required numbers are 8 and 8.

55. We have,

$$f(x) = 2x^3 - 6x^2 + 6x + 5$$

$$\Rightarrow \quad f'(x) = 6x^2 - 12x + 6 = 6(x-1)^2$$

and $f''(x) = 12(x-1)$

Now, $f'(x) = 0$ gives $x = 1$.
Also, $f''(1) = 0$.
Therefore, the second derivative test fails in this case.
So, we shall go back to the first derivative test.
Using first derivatives test, we get $x = 1$ is neither a point of local maxima nor a point of local minima and so it is a point of inflexion.

56. Let $S(x)$ be the selling price of x items and let $C(x)$ be the cost price of x items.
Then, we have

$$S(x) = \left(5 - \frac{x}{100}\right)x = 5x - \frac{x^2}{100}$$

and $C(x) = \frac{x}{5} + 500$

Thus, the profit function $P(x)$ is given by

$$P(x) = S(x) - C(x) = 5x - \frac{x^2}{100} - \frac{x}{5} - 500$$

i.e. $P(x) = \frac{24}{5}x - \frac{x^2}{100} - 500$

On differentiating both sides w.r.t. x, we get

$$P'(x) = \frac{24}{5} - \frac{x}{50}$$

Now, $P'(x) = 0$ gives $x = 240$.

Also, $P''(x) = \frac{-1}{50}$.

So, $P''(240) = \frac{-1}{50} < 0$

Thus, $x = 240$ is a point of maxima.
Hence, the manufacturer can earn maximum profit, if he sells 240 items.

57. Assertion We have, $f(x) = 2x^3 - 9x^2 + 12x - 3$

$\Rightarrow \quad f'(x) = 6x^2 - 18x + 12$

For increasing function, $f'(x) \geq 0$

$\therefore \quad 6(x^2 - 3x + 2) \geq 0$

$\Rightarrow \quad 6(x-2)(x-1) \geq 0$

$\Rightarrow \quad x \leq 1$ and $x \geq 2$

$\therefore f(x)$ is increasing outside the interval (1, 2), therefore it is true statement.

Reason Now, $f'(x) < 0$

$\Rightarrow \quad 6(x-2)(x-1) < 0$

$\Rightarrow \quad 1 < x < 2$

$\therefore$ Assertion and Reason are both true but Reason is not the correct explanation of Assertion.

58. The equation of the given curve is

$$y = \frac{1}{x^2 - 2x + 3} \quad \ldots(i)$$

The slope of the tangent to the given curve at any point (x, y) is given by

$$\frac{dy}{dx} = \frac{-1}{(x^2 - 2x + 3)^2}\frac{d}{dx}(x^2 - 2x + 3)$$

$$= \frac{-(2x-2)}{(x^2 - 2x + 3)^2} = \frac{-2(x-1)}{(x^2 - 2x + 3)^2}$$

For all tangents having slope 0, we must have $\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{-2(x-1)}{(x^2 - 2x + 3)^2} = 0$

$\Rightarrow \quad -2(x-1) = 0 \Rightarrow x = 1$

From Eq. (i), we get

$$y = \frac{1}{1^2 - 2 \times 1 + 3} = \frac{1}{2}$$

$\therefore$ The equation of tangent to the given curve at point $\left(1, \frac{1}{2}\right)$ having slope = 0 is

$$y - \frac{1}{2} = 0(x-1) \Rightarrow y = \frac{1}{2}$$

Hence, the equation of the required line is $y = \frac{1}{2}$.
Hence, both Assertion and Reason are true.

59. Let $f(x) = 2x^3 - 24x$

$\Rightarrow \quad f'(x) = 6x^2 - 24 = 6(x^2 - 4)$

$= 6(x+2)(x-2)$

For maxima or minima put $f'(x) = 0$.

$\Rightarrow \quad 6(x+2)(x-2) = 0$

$\Rightarrow \quad x = 2, -2$

We first consider the interval [1, 3].

So, we have to evaluate the value of f at the critical point $x = 2 \in [1, 3]$ and at the end points of [1, 3].

At $x = 1$, $f(1) = 2 \times 1^3 - 24 \times 1 = -22$

At $x = 2$, $f(2) = 2 \times 2^3 - 24 \times 2 = -32$

At $x = 3$, $f(3) = 2 \times 3^3 - 24 \times 3 = -18$

$\therefore$ The absolute maximum value of $f(x)$ in the interval [1, 3] is -18 occurring at $x = 3$.
Hence, Assertion is false and Reason is true.

60. Let $f(x) = x^2 - 8x + 17$

$\therefore \quad f'(x) = 2x - 8$

So, $\quad f'(x) = 0$, gives $x = 4$

Here $x = 4$ is the critical number

Now, $f''(x) = 2 > 0, \forall\ x$

So, $x = 4$ is the point of local minima.

$\therefore$ Minimum value of $f(x)$ at $x = 4$,

$$f(4) = 4 \times 4 - 8 \times 4 + 17 = 1$$

Hence, we can say that both Assertion and Reason are true and Reason is the correct explanation of the Assertion.