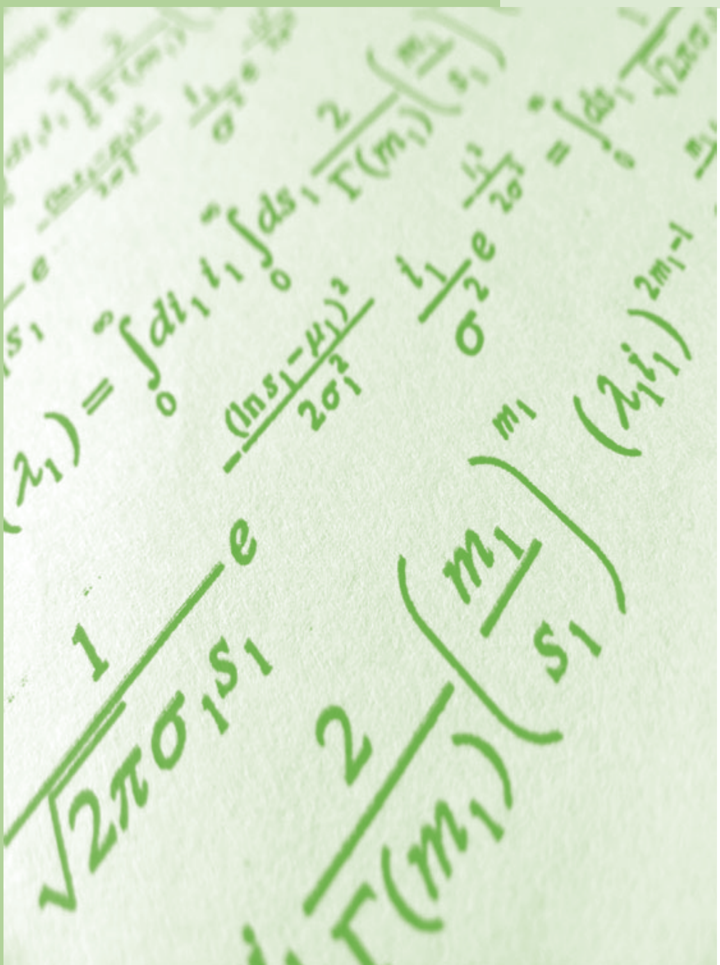
Chapter 17

Modular Arithmetic



REMEMBER

Before beginning this chapter, you should be able to:

- Have basic knowledge of numbers and their types
- Apply operations on numbers

KEY IDEAS

After completing this chapter, you would be able to:

- Find congruency of numbers and set of residues
- Understand modular addition and multiplication to solve problems
- Represent modular arithmetic system by Caley's table
- Study linear congruence and its solutions

INTRODUCTION

Suppose you have set out for a drive from your place. Let your wrist watch show time as 5 O'clock. After 6 hours of driving, the time would be 11 O'clock. We can deduce that the time is 11 O'clock by adding 6 hours to 5 O'clock. Now, if the journey continues, after another 7 hours, the time would be 6 O'clock.

It is not going to be 11 O'clock + 7 = 18 O'clock.

Thus, 18 have to be replaced with 6.

How did we get this 6 from 18?

How is 6 related to 18?

The number 6 is the remainder obtained, when 18 is divided by 12. We say that 18 is congruent to 6 modulo 12.

Similarly, 21 is congruent to 9 modulo 12.

CONGRUENCE

Let a and b be two integers, and m be a positive integer. We say that a is congruent to b modulo m if m is a factor of $(a - b)$.

It is denoted by $a \equiv b \pmod{m}$ or $(a - b) \equiv 0 \pmod{m}$.

Notes

1. If m is not a factor of $a - b$, then $a \neq b \pmod{m}$.
2. If m is a factor of a, then $a \equiv 0 \pmod{m}$.
   Example: 5 is a factor of 15. So, $15 \equiv 0 \pmod{5}$.
3. If r is the remainder on dividing a by m, then $a \equiv r \pmod{m}$.
   Example: $125 \equiv 5 \pmod{6}$ as 125 leaves a remainder of 5 on being divided by 6.
4. If $a \equiv b \pmod{m}$ and c is an integer, then $a + c \equiv b + c \pmod{m}$.
5. If $a \equiv b \pmod{m}$ and c is an integer, then $a \cdot c \equiv b \cdot c \pmod{m}$.
6. If $a \equiv b \pmod{m}$ and c is a positive integer, then $a^c \equiv b^c \pmod{m}$.
7. If p is a prime number, then $x^p \equiv x \pmod{p}$.

Set of Residues

When a positive integer is divided by 2, then the remainder will be either 0 or 1. Hence, 0 and 1 are called residues of 'modulo 2'.

$\therefore$ {0, 1} is the set of residues 'modulo 2'. It is denoted by Z_2.

So,

1. $Z_4 = \{0, 1, 2, 3\}$
2. $Z_m = \{0, 1, 2, \ldots, (m - 1)\}$

Modular Addition

Let a and b be two integers, and 'm' be a fixed positive integer. Then, an 'addition modulo m' b is denoted by $a \oplus_m b$ is defined as the remainder when $a + b$ is divided by m.

Examples:

1. $5 \oplus_6 4 = 3$
2. $3 \oplus_4 5 = 0$

Notes

1. If $a \oplus_m b = r$, then $a + b \equiv r \pmod{m}$.
2. $a \oplus_m b = b \oplus_m a$.

Modular Multiplication

Let a and b be two integers, and 'm' be a fixed positive integer.

Then, a 'multiplication modulo m' b is denoted by $a \otimes_m b$ is defined as the remainder when $a \cdot b$ is divided by m.

Examples:

1. $4 \otimes_5 4 = 1$
2. $8 \otimes_3 4 = 2$

Notes

1. If $a \otimes_m b = r$, then $ab \equiv r \pmod{m}$.
2. $a \otimes_m b = b \otimes_m a$.

Construction of Caley's Table

Consider, $Z_2 = \{0, 1\}$

1. All the possible results under addition modulo 2 are:

   $0 \oplus_2 0 = 0$, $1 \oplus_2 0 = 1$,

   $1 \oplus_2 0 = 0$ and $1 \oplus_2 1 = 0$

   These results can be tabulated as follows:

$\oplus_2$	0	1
0	0	1
1	1	0

2. All the possible results under multiplication modulo 2 are:

   $0 \otimes_2 0 = 0$, $1 \otimes_2 0 = 0$,

   $0 \otimes_2 1 = 0$ and $1 \otimes_2 1 = 1$

   These results can be tabulated as follows:

$\otimes_2$	0	1
0	0	0
1	0	1

Note Caley's table is the representation of modular arithmetic system.

EXAMPLE 17.1

(a) Construct Caley's table for $A = \{1, 2, 3\}$ under addition modulo 5.

SOLUTION

$\oplus_5$	1	2	3
1	2	3	4
2	3	4	0
3	4	0	1

(b) Construct Caley's table for the set $\{0, 1, 2, 3, 4\}$ under multiplication modulo 6.

$\otimes_6$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	0	2
3	0	3	0	3	0
4	0	4	2	0	4

Linear Congruence

A polynomial congruence of degree 1 is called a *linear congruence*.

A linear congruence is of the form $ax \equiv b \pmod{m}$, where $a \neq 0 \pmod{m}$.

Examples:

1. $5x \equiv 8 \pmod{4}$
2. $6x \equiv 3 \pmod{5}$

Solution of Linear Congruence

An integer x_1 is said to be a solution of linear congruence $ax \equiv b \pmod{m}$ if $ax_1 \equiv b \pmod{m}$. That is, $ax_1 - b$ is divisible by m.

1. Consider the linear congruence $5x \equiv 8 \pmod{4}$

   Clearly, $x = 0$ is a solution as $5(0) - 8 = -8$ is divisible by 4.

2. Consider the linear congruence $6x \equiv 3 \pmod{5}$

   Clearly, $x = 3$ is a solution as $6(3) - 3 = 15$ is divisible by 5.

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. $15 \equiv -3 \pmod 9$ (True/False).

2. $5 \equiv 2 \pmod 4$ (True/False).

3. $4 \otimes_3 9 =$ _______.

4. The 19th hour of the day is equivalent to _______ hour.

5. $6 \oplus_4 7 =$ _______.

6. In a certain month, the first Sunday falls on the fifth day of the month. In the same month, the fourth Sunday falls on the _______ day.

7. In a certain non-leap year, 1st February is Wednesday. Then the last day of the month is also Wednesday. (True/False).

8. If $63 \equiv 2 \pmod a$ and $a > 1$, then a is _______.

9. If x belongs to the set of residues modulo 4 and $2 + x \equiv 5 \pmod 4$, then $x =$ _______.

10. If $x \equiv y \pmod m$, then $6x - 5 \equiv 6y - 5 \pmod m$. (True/False).

11. In the set of integers modulo 5, $16 \oplus_5 7 =$ _______.

12. In the set of integers modulo 6, $35 \otimes_6 5 =$ _______.

13. If $a + 2 \equiv 3 \pmod 6$, then a is _______.

14. If $6x \equiv 5 \pmod 7$, then find x.

15. If $x - 4 \equiv 8 \pmod 5$, then x is _______.

Short Answer Type Questions

16. If x belongs to the set of residues modulo 6 and $5 + x \equiv 3 \pmod 6$, then find x.

17. If x belongs to the set of residues modulo 4 and $6x - 3 \equiv -1 \pmod 4$, then find x.

18. If $46 \equiv 11 \pmod a$, and a is a prime number, then find the greatest possible value of a.

19. If 1st July 2006 was a Saturday, then what day of the week will be 18th July, 2007?

20. If you were born on March 8, 1990 and the day of the week was a Thursday, then on what day of the week did your birthday fall in 1991?

21. Find the remainder when $(26)^{31}$ is divided by 31.

22. Find the remainder when 8^{15} is divided by 5.

23. If $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then list out all the pairs of distinct numbers from set A which are congruent to each other under modulo 5.

24. Construct Caley's table for Z_7 under multiplication modulo 7.

25. Construct Caley's table for the set $A = \{2, 4, 6, 8, 10\}$ under addition modulo 12.

Essay Type Questions

26. Construct Caley's table for the set $B = \{1, 3, 5, 7, 9\}$ under multiplication modulo 10.

27. Construct Caley's table for Z_8 under addition modulo 8.

28. Find the remainder when 3^{31} is divided by 31.

29. If $a \otimes_m b = 1$, then b is called there ciprocal of a under modulo m. Find the reciprocal of 8 under modulo 17.

30. How many two digit numbers satisfy the equation, $3x \equiv 5 \pmod 7$?

CONCEPT APPLICATION

Level 1

1. In the set of integers modulo 8, $28 \otimes_8 2 =$ ______.
   (a) 0 (b) 1
   (c) 2 (d) 3

2. If $25 \equiv 4 \pmod{p}$, where p is a prime number, then p is:
   (a) 3 (b) 5
   (c) 7 (d) Either (a) or (c)

3. Solve for x, if $5x \equiv 0 \pmod{4}$.
   (a) 0 (b) 3
   (c) 2 (d) Both (a) and (b)

4. In the set of integers modulo 12, $38 \oplus_{12} 28 =$ ______.
   (a) 6 (b) 5
   (c) 4 (d) 3

5. The largest single-digit number that satisfies $14x \equiv 4 \pmod{3}$ is:
   (a) 5 (b) 7
   (c) 8 (d) 9

6. If 8th August of 2009 is a Saturday, then 15th August of 2010 falls on ________.
   (a) Saturday (b) Sunday
   (c) Wednesday (d) Thursday

7. If $23 \equiv 7 \pmod{x}$, then which of the following cannot be the value of x?
   (a) 4 (b) 6
   (c) 8 (d) 16

8. Find the remainder when 13^{15} is divided by 5.
   (a) 4 (b) 3
   (c) 2 (d) 1

9. Now the time is 1: 30 pm. If I woke up 8 hours ago, then I woke up at:
   (a) 4: 30 am (b) 5:30 am
   (c) 3:30 am (d) 6:30 am

10. If $37 \equiv 18 \pmod{p}$, where p is a prime number, then find p.
   (a) 3 (b) 7
   (c) 19 (d) Either (a) or (b)

11. If $15 \equiv 3 \pmod{x}$, then which of the following cannot be the value of x?
   (a) 3 (b) 4
   (c) 6 (d) 8

12. Find the remainder when 5^{18} is divided by 19.
   (a) 1 (b) 4
   (c) 11 (d) 17

13. The largest two-digit number that satisfies $5x \equiv 6 \pmod{4}$ is:
   (a) 96 (b) 97
   (c) 98 (d) 99

14. If you were born on April 15, 1993 which was a Tuesday, then on which day of the week did your birthday fall in 1994?
   (a) Tuesday (b) Wednesday
   (c) Thursday (d) Monday

15. Find the remainder when 11^{12} is divided by 7.
   (a) 0 (b) 1
   (c) 3 (d) 5

Level 2

16. If $a \equiv b \pmod{m}$ and the remainder obtained when 'a' is divided by m is 2, then find the remainder when 'b' is divided by m.
   (a) 2 (b) 1
   (c) 0 (d) Cannot be determined

17. If $x \equiv y \pmod{2}$, then which of the following is/are correct?
   (A) x is even and y is odd.
   (B) Both x and y are odd.
   (C) Both x and y are even.

PRACTICE QUESTIONS

(a) Only (C)

(b) Only (A)

(c) Both (B) and (C)

(d) Both (A) and (B)

18. If January 1, 2010 is a Friday, then the fifth Sunday of January, 2011 will fall on

(a) 26th day (b) 27th day

(c) 29th day (d) 30th day

19. Anand started a work on Sunday at 9:30 am. He finished the work after 87 hours. Then he finished the work on

(a) Wednesday at 11:30 pm

(b) Thursday at 0:30 am

(c) Wednesday at 0:30 am

(d) Thursday at 11:30 pm

20. Which of the following are the common solutions of $3x \equiv 0 \pmod 6$ and $2x \equiv 0 \pmod 4$?

(A) 0 (B) 2 (C) 4

(a) Both (A) and (B)

(b) Both (A) and (C)

(c) Both (B) and (C)

(d) All of (A), (B) and (C)

21. If $15x \equiv 2 \pmod 3$, then which of the following is a possible value of x?

(a) 3 (b) 315

(c) 0 (d) None of these

22. Which of the following is a common solution for $6x \equiv 0 \pmod 8$ and $8x \equiv 0 \pmod{10}$?

(a) 0 (b) 4

(c) 6 (d) Both (a) and (b)

23. Find the remainder when 2^{24} is divided by 35.

(a) 2 (b) 31

(c) 1 (d) 29

24. Which of the following is correct?

(a) $5 \oplus_3 2 \equiv 3 \otimes_3 6 \pmod 4$

(b) $4 \oplus_3 2 \equiv 3 \otimes_4 5 \pmod 6$

(c) $5 \oplus_5 3 \equiv 6 \otimes_8 9 \pmod 3$

(d) None of these

25. Which of the following is/are correct?

(a) $6 \oplus_4 3 \equiv 7 \otimes_9 8 \pmod 5$

(b) $10 \oplus_5 4 \equiv 9 \otimes_{11} 9 \pmod{11}$

(c) $14 \oplus_8 8 \equiv 15 \otimes_{16} 12 \pmod 4$

(d) Both (a) and (c)

26. If x belongs to the set of residues modulo 10, then the common solution of $5 + x \equiv 0 \pmod 3$ and $6 + x \equiv 0 \pmod 5$ is ________.

(a) 1 (b) 2

(c) 4 (d) 5

27. By which of the following numbers should 3^5 be divided to obtain a remainder 3?

(a) 7 (b) 11

(c) 5 (d) None of these

28. Find the remainder when $6^{11} - 6$ is divided by 11.

(a) 5 (b) 1

(3) 0 (d) None of these

29. Find x, if $9x \equiv 2 \pmod 7$.

(a) 1 (b) 2

(c) 3 (d) 4

30. Find the remainder when 3^{19} is divided by 19.

(a) 3 (b) 15

(c) 16 (d) None of these

31. In the set of integers modulo 9, $15 \otimes_9 10 =$ _______.

(a) 3 (b) 6

(c) 0 (d) 1

32. If $7x \equiv 1 \pmod 5$, then which of the following is a possible value of x?

(a) 10 (b) 11

(c) 12 (d) None of these

33. In the set of integers modulo 17, $19 \oplus_{17} 15 =$ _______.

(a) 0 (b) 1

(c) 2 (d) 3

34. In order to enter her name in the Guinness Book of world records, Sangeeta started singing on

Monday at 10.30 am. If she sings continuously for 36 hours, then she will finish her singing on

(a) Tuesday at 10.30 am

(b) Wednesday at 10.30 am

(c) Tuesday at 10.30 pm

(d) Wednesday at 10.30 pm

35. Which of the following is a common solution of $3x \equiv 2 \pmod 5$ and $4x \equiv 0 \pmod 6$?

(a) 9

(b) 4

(c) 6

(d) None of these

Level 3

36. Find the remainder when 3^{215} is divided by 43.

(a) 35 (b) 28

(c) 33 (d) 30

37. Kishore reached his school on Monday at 8:30 a.m., and then immediately started on a tour to GOA. After $106\frac{1}{2}$ hours, he reached his house. Then, Kishore reached his home on

(a) Saturday at 7 pm

(b) Friday at 6 pm

(c) Saturday at 6 pm

(d) Friday at 7 pm

38. If August 1, 2012 is Wednesday, then find the day on which we shall celebrate our Independence Day in the year 2015.

(a) Saturday (b) Sunday

(c) Friday (d) Thursday

39. Find the remainder when 5^{97} is divided by 97.

(a) 5 (b) 97

(c) 92 (d) None of these

40. If $a \equiv b \pmod m$, then which of the following is not always true?

(a) $a^2 \equiv b^2 \pmod m$

(b) $a + m \equiv b + m \pmod{2m}$

(c) $am \equiv bm \pmod{m^2}$

(d) None of these

41. If $x^3 \equiv x \pmod 3$, then x can be ________.

(a) 2 (b) 5

(c) 4 (d) All of these

42. A part of Caley's table for $\otimes_6$ is given below. Find the values of x, y and z.

$\otimes_6$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	0	2
3	0	3	0	y	0
4	0	4	z	0	x

(a) $x = 4,\ y = 3,\ z = 2$

(b) $x = 2,\ y = 4,\ z = 3$

(c) $x = 3,\ y = 4,\ z = 2$

(d) $x = 4,\ y = 2,\ z = 3$

43. A part of Caley's table for $\oplus_7$ is given below. Find the value of p, q, x and y.

$\oplus_7$	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	p	x
4	5	Q	y	1

(a) $x = y = 0,\ p = q = 5$

(b) $x = y = 1,\ p = q = 6$

(c) $x = y = 3,\ p = q = 4$

(d) $x = y = 0,\ p = q = 6$

44. The Independence Day of India in 2007 was celebrated on a Wednesday, then Children's day in 2008 was celebrated on a ________.

(a) Friday (b) Saturday

(c) Sunday (d) Monday

45. If $13 \equiv 3 \pmod{p}$, then p can be ________.

(a) 2 (b) 5

(c) 10 (d) All of these

46. If the 1st January of a certain year, which was not a leap year, was a Thursday, then what day of the week was the 31st December of that year?

(a) Monday (b) Thursday

(c) Sunday (d) Saturday

47. If $x + 10 \equiv 1 \pmod{8}$, then x can be ________.

(a) 1 (b) 0

(c) 6 (d) 7

48. If x belongs to the set of residues modulo 6 and $3 + x \equiv 2 \pmod{6}$, then $x =$ ________.

(a) 1 (b) 3

(c) 4 (d) 5

49. The January 2, 2009 is a Friday. The fourth Sunday of January 2010 falls on the ______.

(a) 23rd day

(b) 24th day

(c) 25th day

(d) 26th day

50. Which of the following is/are correct?

(a) $5 \oplus_2 4 \equiv 17 \otimes_5 3 \pmod{7}$

(b) $6 \oplus_4 7 \equiv 19 \otimes_9 3 \pmod{3}$

(c) $9 \oplus_7 3 \equiv 8 \otimes_7 9 \pmod{9}$

(d) None of these

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. True
2. False
3. 0
4. 7th
5. 1
6. 26th
7. False
8. 61
9. 3
10. True
11. 3
12. 1
13. 1
14. 2
15. 2

Short Answer Type Questions

16. $x = 4$
17. 1 or 3
18. 7.
19. Wednesday.
20. Friday.
21. 31.
22. 2.
23. [0, 5], [0, 10], [1, 6], [2, 7] [3, 8], [4, 9], [5, 10]

Essay Type Questions

28. 2
29. 3
30. 4

CONCEPT APPLICATION

Level 1

1. (a)	**2.** (d)	**3.** (a)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (c)	**9.** (b)	**10.** (c)
11. (d)	**12.** (a)	**13.** (c)	**14.** (b)	**15.** (b)					

Level 2

16. (a)	**17.** (c)	**18.** (d)	**19.** (b)	**20.** (d)	**21.** (d)	**22.** (a)	**23.** (c)	**24.** (c)	**25.** (b)
26. (c)	**27.** (c)	**28.** (c)	**29.** (a)	**30.** (a)	**31.** (b)	**32.** (d)	**33.** (a)	**34.** (c)	**35.** (a)

Level 3

36. (b)	**37.** (d)	**38.** (a)	**39.** (a)	**40.** (b)	**41.** (d)	**42.** (a)	**43.** (d)	**44.** (a)	**45.** (d)
46. (b)	**47.** (d)	**48.** (d)	**49.** (b)	**50.** (a)					

CONCEPT APPLICATION

Level 1

1. Recall the concept of modular multiplication.
2. Check from the options.
3. Check from the options.
4. Recall the concept of modular addition.
5. Check from the options.
6. Use the concept, '365 under modulo 7'.
7. Check from the options.
8. Use the concept of congruence modulo.
9. Check from the options.
10. Check from the options.
11. Check from the options.
12. Start with the step $5^2 \equiv 6 \pmod{19}$ and proceed.
13. Check from the options.
14. Use the concept '365 under modulo 7'.
15. Use the concept of congruence modulo.
16. If $a \equiv b \pmod{m}$, then the remainder obtained when a is divided by m is equal to the remainder obtained when b is divided by m.
17. If $a - b$ is divisible by 2, then both a and b are either even numbers or odd numbers.
18. January 1, 2010 is a Friday. January 1, 2011 is a Saturday. First Sunday in 2011 is 2nd.
19. 87 hours = 3 days + 15 hours.
20. Verify whether the given options are common solutions are not.
21. $15x$ is always divisible by 3.
22. Verify whether each option is a solution of both the equations or not.
23. Use, if p is prime then $a^p \equiv a \pmod{p}$.
24. $a \oplus_m b$ means 'the remainder when $a + b$ is divided by m' and $a \otimes_m b$ means.

    'The remainder when ab is divided by m'.
25. (i) Use the definitions of addition modulo m and multiplication modulo m.

    (ii) Substitute the values in the options in the given inequations.

    (iii) The point which satisfies the given inequations is the required point.
26. $Z_{10} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $x \in Z_{10}$

    $5 + x \equiv 0 \pmod{3} \Rightarrow x = 1, 4$ and 7

    $6 + x \equiv 0 \pmod{5} \Rightarrow x = 4$ and 9

    $\therefore$ Common solution is $x = 4$.
27. $3^5 \equiv 3 \pmod{x} \Rightarrow 243 = 3 \pmod{x}$

    $\Rightarrow$ $243 - 3$ is divisible by x

    $\Rightarrow$ 240 is divisible by x.

    $\therefore$ From the options, $x = 5$.
28. We have $a^p \equiv a \pmod{p}$ where p is a prime number.

    11 is prime number, $6^{11} \equiv 6 \pmod{11}$

    That is, $6^{11} - 6$ is divisible by 11. The remainder when $6^{11} - 6$ is divided by 11 is zero.
29. Given, $9x \equiv 2 \pmod{7} \Rightarrow 9x - 2$ is divisible by 7.

    From the options, if $x = 1$, then $9 - 2$ is divisible by 7.
30. We have $a^p \equiv a \pmod{p}$, where p is a prime number.

    When 3^{19} is divided by 19, the remainder is 3.
31. The remainder when 15×10, i.e., 150 is divided by 9 is 6.
32. Given, $7x \equiv 1 \pmod{5}$.

    $7x - 1$is divisible by 5.

    $\therefore$ $x = 3$ or 8 or 13 or … satisfies the above relation.
33. The remainder when $19 + 15$, i.e., 34 is divided by 17 is 0.
34. Monday, 10.30 am to Tuesday, 10.30 am is 24 hours.

    Tuesday, 10.30 am to Tuesday, 10.30 pm = 12 hours.

    $\therefore$ She will finish her singing on Tuesday night at 10.30 pm
35. Given $3x \equiv 2 \pmod{5}$

    $\Rightarrow$ $3x - 2$ is divisible by 5.

    $x = 4$ or 9 or 14, or … satisfies the above relation.

    $4x \equiv 0 \pmod{6}$

    $\Rightarrow$ $4x$ is divisible by 6

$x = 3$ or 9 or 15 … satisfies the above relation.

$\therefore$ The common solution of $3x \equiv 2 \pmod 5$ and $4x \equiv 0 \pmod 6$ is 9. (From the options)

37. $106\frac{1}{2}$ hours = 4 days + $10\frac{1}{2}$ hours.

38. The number of days in between August 15, 2015 and August 1, 2012 is 1109.

39. We know that, when p is a prime number $a^p \equiv a \pmod p$.

40. (i) Use the condition $a \equiv b \pmod m \Rightarrow a - b$ is divisible by m.

(ii) Verify whether each option satisfies the above condition or not.

(iii) Refer the properties of congruence modulo.

41. Given $x^3 \equiv x \pmod 3$

$x^3 - x$ is divisible by 3.

$x(x^2 - 1)$ is divisible by 3.

$(x - 1)\, x\, (x + 1)$ is divisible by 3.

$x - 1, x, x + 1$ are three consecutive numbers.

We know that the product of three consecutive numbers is always divisible by 3.

$\therefore$ For any integer of x (where $x \geq 2$), the given relation is true.

42. x is the remainder when 4×4 is divided by 6.

$\therefore x = 4$

y is the remainder when 3×3 is divided by 6.

$\therefore y = 3$

z is the remainder when 4×2 is divided by 6.

$\therefore z = 2$.

43. p is the remainder when $3 + 3$ is divided by $7 = 6$

q is the remainder when $4 + 2$ is divided by $7 = 6$

x is the remainder when $3 + 4$ is divided by $7 = 0$

y is the remainder when $4 + 3$ is divided by $7 = 0$

$\therefore x = y = 0$ and $p = q = 6$.

44. The number of days from 15.10.2007 to 14.11.2008 is 458 days.

Remainder when 458 is divided by 7 is 3.

Since the first day is Wednesday, the third day is Friday.

$\therefore$ Children's day in 2008 was celebrated on Friday.

45. Given $13 \equiv 3 \pmod p$

$\Rightarrow$ $13 - 3$ is divisible by p.

$\Rightarrow$ 10 is divisible by p,

i.e., p is a factor of 10.

$\therefore p = 1, 2, 5$ or 10.

$\therefore$ All the options are true.

46. The given year is a non-leap year and contains 365 days.

(The number of days from 1st January to 31st December is 365)

$365 \equiv 1 \pmod 7$

The 365th day is equivalent to the first day of the year.

$\therefore$ But the first day of the week is Thursday.

$\therefore$ Hence, 31st December is Thursday.

47. Given $x + 10 = 1 \pmod 8$

$\Rightarrow$ $x + 10 - 1$ is divisible by 8

$\Rightarrow$ $x + 9$ is divisible by 8

From the options, if $x = 7$ then $7 + 19$, i.e., 16 is divisible by 8.

$\therefore x = 7$.

48. $Z_6 = \{0, 1, 2, 3, 4, 5\}$ and $x \in Z_6$.

Given, $3 + x \equiv 2 \pmod 6$

$\Rightarrow$ $3 + x - 2$ is divisible by 6

$\Rightarrow$ $x + 1$ is divisible by 6

$x = 5$ satisfies the above conditions.

49. The number of days from 2.1.2009 to 1.1.2010 is 365 days

$\Rightarrow$ $365 \equiv 1 \pmod 7$

$\therefore$ 1.1.2010 is Friday ($\because$ 2nd January, 2009 is Friday)

$\therefore$ 1st Sunday falls on 3.01.2010

$\therefore$ 2nd Sunday falls on 10.01.2010

$\therefore$ 3rd Sunday falls on 17.01.2010

$\therefore$ 4th Sunday falls on 24.01.2010.

50. $5 \oplus_2 4 \equiv 17 \otimes_5 3 \pmod 7$

$1 \equiv 1 \pmod 7$

$\therefore 1 - 1$ is divisible by 7 (true).