Chapter 3

QUADRATIC EQUATIONS

A SINGLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

1. If $a(p+q)^2 + 2bpq + c = 0$ and $a(p+r)^2 + 2bpr + c = 0, (a \neq 0)$ then

(a) $qr = p^2 + \frac{c}{a}$ (b) $qr = p^2$
(c) $qr = -p^2$ (d) None or these.

2. If $p(x) = ax^2 + bx$ and $q(x) = \ell x^2 + mx + n$ with $p(1) = q(1)$; $p(2) - q(2) = 1$ and $p(3) - q(3) = 4$, then $p(4) - q(4)$ is

(a) 0 (b) 5 (c) 6 (d) 9

3. If α, β be the roots of $x^2 - a(x-1) + b = 0$, then the value of $\frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - b\beta} + \frac{2}{a+b}$ is

(a) $\frac{4}{a+b}$ (b) $\frac{1}{a+b}$
(c) 0 (d) None of these

4. If the roots of $ax^2 - bx - c = 0$ change by the same quantity then the expression in a, b, c that does not change is

(a) $\frac{b^2 - 4ac}{a^2}$ (b) $\frac{b - 4c}{a}$
(c) $\frac{b^2 + 4ac}{a^2}$ (d) $\frac{a^2 + b^2}{c^2}$

5. If the roots of the equation, $ax^2 + bx + c = 0$, are the form $\frac{\alpha}{\alpha - 1}$ and $\frac{\alpha + 1}{\alpha}$ then the value of $(a+b+c)^2$ is

(a) $2b^2 - ac$ (b) $b^2 - 2ac$
(c) $b^2 - 4ac$ (d) $4b^2 - 2ac$

6. The equation $(a+2)x^2 + (a-3)\ x = 2a - 1, a \neq -2$ has rational roots for

(a) all rational values of a except $a = -2$
(b) all real values of a except $a = -2$
(c) rational values of $a > \frac{1}{2}$ only
(d) None of these.

7. If $a, b, c \in \boldsymbol{R}$ and 1 is a root of equation $ax^2 + bx + c = 0$, then equation $4ax^2 + 3bx + 2c = 0,\ c \neq 0$ has

(a) imaginary root (b) real and equal roots
(c) real and unequal roots (d) rational roots.

8. If $a, b, c, d \in \boldsymbol{R}$ then the equation $(x^2 + ax - 3b)\ (x^2 - cx + b)\ (x^2 - dx + 2b) = 0$ has

(a) 6 real roots (b) 3 real roots
(c) 4 real roots (d) at least 2 real roots.

9. It the equation $ax^2 + 2bx - 3c = 0$ has non real roots and $\left(\frac{3c}{4}\right) < (a+b)$. Then c is always

(a) < 0 (b) > 0
(c) ≥ 0 (d) None of these.

10. Given that, for all $x \in \boldsymbol{R}$. The expression $\frac{x^2 - 2x + 4}{x^2 + 2x + 4}$ lies between $\frac{1}{3}$ and 3, the value between which the expression $\frac{9.3^{2x} + 6.3^x + 4}{9.3^{2x} - 6.3^x + 4}$ lies are

(a) 3^{-1} and 3 (b) –2 and 0
(c) – 1 and 1 (d) 0 and 2.

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ

$a(x+1)^2 + b(x^2 - 3x - 2) + x + 1 = 0 \ \forall\, x \in \boldsymbol{R}$ is

(a) 0 (b) 1
(c) 2 (d) infinite.

12. If $\cos\theta, \sin\phi, \sin\theta$ are in GP, then roots of $x^2 + 2\cot\phi . x + 1 = 0$ are

(a) equal (b) real
(c) imaginary (d) greater than 1

13. If α, β are the roots of $x^2 - 3x + a = 0, a \in \boldsymbol{R}$ and $\alpha < 1 < \beta$ then

(a) $a \in (-\infty, 2)$ (b) $a \in \left(-\infty, \frac{9}{4}\right]$
(c) $\left(-2, \frac{9}{4}\right]$ (d) None of these.

14. If the equations $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0, a \neq c$ have a negative common root, then the value of $a - b + c$ is

(a) 0 (b) 1
(c) 2 (d) None of these.

15. The number of values of k for which $[x^2 - (k-2)x + k^2]\,[x^2 + kx + (2k-1)]$ is a perfect square is

(a) 1 (b) 2
(c) 0 (d) None of these.

16. If the expression $\left(mx - 1 + \frac{1}{x}\right)$ is non-negative for all positive real x, then the minimum value of m must be

(a) $-\frac{1}{2}$ (b) 0 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$.

17. If $x \in \boldsymbol{R}$, then the maximum value of $y = 2(a-x)\ (x + \sqrt{x^2 + b^2})$ is

(a) $a^2 + b^2$ (b) $a^2 - b^2$
(c) $a^2 + 2b^2$ (d) None of these

18. If a, b, c are three distinct real numbers then the equation

$$\frac{(x-b)(x-c)}{(a-b)(a-c)}a^2 + \frac{(x-c)(x-a)}{(b-c)(b-a)}.\,b^2 + \frac{(x-a)(x-b)}{(c-a)(c-b)}c^2 - x^2 = 0$$ has

(a) exactly one root (b) exactly two roots
(c) no root (d) None of these

19. The integral values of m for which the roots of the equation $mx^2 + (2m-1)\ x + (m-2) = 0$ are rational are given by the expression [Where n is integer]

(a) $n(n+2)$ (b) $n(n+1)$
(c) n^2 (d) None of these

20. If both roots of the equation $x^2 - 2ax + a^2 - 1 = 0$ lie between –3 and 4, then $[a]$ can not be ($[a]$ is the integral part of a)

(a) 0 (b) -1 (c) 1 (d) 4

21. If t_n denotes the n^{th} term of an $A.P.$ and $t_p = \frac{1}{q}$ and $t_q = \frac{1}{p}$, then which of the following is necessarily a root of the equation

$$(p + 2q - 3r)x^2 + (q + 2r - 3p)x + (r + 2p - 3q) = 0$$

(a) t_p (b) t_q (c) t_{pq} (d) t_{p+q}

22. Let α and β be the real and distinct roots of the equation $ax^2 + bx + c = |c|$, ($a > 0\ c \neq 0$) and p, q be the real and distinct roots of the equation $ax^2 + bx + c = 0$. Then

(a) p and q lie between α and β
(b) p and q do not lie between α and β
(c) Only p lies between α and β
(d) Only q lies between α and β.

23. The roots of the equation $ax^2 + bx + c = 0$, where $a \in R_+$ are two consecutive odd positive integers, then

(a) $|b| \leq 4a$ (b) $|b| \geq 4a$
(c) $|b| = 2a$ (d) None of these.

24. Let $f(x) = ax^2 + bx + c$ and $g(x) = a\,f(x) + bf'(x) + cf''(x)$. If $f(x) > 0\ \forall x \in R$, then the sufficient condition for $g(x) > 0\ \forall\, x \in R$ is

(a) $c > 0$ (b) $b > 0$ (c) $b < 0$ (d) $c < 0$.

MARK YOUR RESPONSE					
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	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ
	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ	

25. Let $f(x) = ax^2 + bx + c$; $a, b, c \in R$. If $f(x)$ takes real values for real values of x and non-real values for non-real values of x, then a satisfies

(a) $a > 0$ (b) $a = 0$ (c) $a < 0$ (d) $a \in R$.

26. The integral values of x for which $x^2 + 19x + 92$ is perfect square are

(a) -8 and -11 (b) -8 and 11
(c) 8 and -11 (d) $\pm 8, \pm 18$.

27. If α, β are the roots of $x^2 + px + q = 0$, and also of $x^{2n} + p^n x^n + q^n = 0$ and $\dfrac{\alpha}{\beta}$ is a root of $x^n + 1 + (x+1)^n = 0$, $\alpha^n \neq \beta^n$, the n must be

(a) any integer (b) an even integer
(c) an odd integer (d) None of these

28. If α and β are the roots of the equation $x^2 - p(x+1) - q = 0$, then the value of $\dfrac{\alpha^2 + 2\alpha + 1}{\alpha^2 + 2\alpha + q} + \dfrac{\beta^2 + 2\beta + 1}{\beta^2 + 2\beta + q}$ is

(a) 2 (b) 1
(c) 0 (d) None of these

29. Let $f(x) = ax^2 + bx + c$ and $f(-1) < 1$, $f(1) > -1$, $f(3) < -4$ and $a \neq 0$, then

(a) $a > 0$
(b) $a < 0$
(c) sign of 'a' can not be determined
(d) $b > 0$

30. If a, b, c, d are four consecutive terms of an increasing AP then the roots of the equation $(x-a)(x-c) + 2(x-b)(x-d) = 0$ are

(a) real and distinct (b) nonreal complex
(c) real and equal (d) integers

31. If $0 < a < b < c$ and the roots α, β of the equation $ax^2 + bx + c = 0$ are imaginary, then

(a) $|\alpha| = |\beta| > 1$ (b) $|\alpha| = |\beta| < 1$
(c) $|\alpha| \neq |\beta|$ (d) None of these

32. If $x^2 + ax + b$ is an integer for every integer x, then

(a) 'a' is always an integer but 'b' need not be an integer
(b) 'b' is always an integer but 'a' need not be an integer
(c) a and b are non-integer but $a + b$ is always an integer
(d) a and b are always integer.

33. Let $p, q, r, s \in \boldsymbol{R}$ and $pr = 2(q+s)$ Consider two quadratic equations $x^2 + px + q = 0$ and $x^2 + rx + s = 0$. Then

(a) both the equations have real and equal roots
(b) both the equations have real and distinct roots
(c) at least one of the equations has real roots
(d) both the equations have imaginary roots

34. Let $[a]$ denote the greatest integer less than or equal to a. Given that the quadratic equation $x^2 + [a^2 - 5a + b + 4]x + b = 0$ has roots -5 and 1. Then the set of values of a is

(a) $\left(-1, \dfrac{5 - 3\sqrt{5}}{2}\right] \cup \left[\dfrac{5 + 3\sqrt{5}}{2}, 6\right)$

(b) $\left(\dfrac{5 - 3\sqrt{5}}{2}, \dfrac{5 + 3\sqrt{5}}{2}\right)$

(c) $(-\infty, -1] \cup [6, \infty)$

(d) $(-\infty, \infty)$

35. Let $y = \dfrac{\sin x \cos 3x}{\sin 3x \cos x}$. Then

(a) $y \in \left[\dfrac{1}{3}, 3\right]$ (b) $y < \dfrac{1}{3}$ or $y > 3$

(c) $y \leq -3$ or $y > \dfrac{1}{3}$ (d) None of these.

36. If α, β be the roots of $4x^2 - 16x + \lambda = 0, \lambda \in R$. Such that $1 < \alpha < 2$ and $2 < \beta < 3$, then the number of integral solutions of λ is

(a) 5 (b) 6 (c) 2 (d) 3

37. If $ax^2 + bx + 6 = 0$ does not have two distinct real roots where $a \in R, b \in R$, then the least value $3a + b$ is

(a) 4 (b) -1 (c) 1 (d) -2

38. If α and β are the roots of the equation $ax^2 + bx + c = 0$ and α^4 and β^4 are the roots of the equation $\ell x^2 + mx + n = 0$, then the roots of the equation $a^2 \ell x^2 - 4ac\ell x + 2c^2 \ell + a^2 m = 0$ are (all coefficients are real)

(a) always positive (b) always non-real
(c) opposite in sign (d) negative

MARK YOUR RESPONSE					
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	35. ⓐⓑⓒⓓ	36. ⓐⓑⓒⓓ	37. ⓐⓑⓒⓓ	38. ⓐⓑⓒⓓ	

39. If the inequality $\frac{mx^2+3x+4}{x^2+2x+2}<5$ is satisfied for all $x \in R$, then

(a) $1<m<5$ (b) $-1<m<1$

(c) $-5<m<\frac{11}{24}$ (d) $m<\frac{71}{24}$

40. The quadratic equation

$\frac{(x+b)(x+c)}{(b-a)(c-a)}+\frac{(x+c)(x+a)}{(c-b)(a-b)}+\frac{(x+a)(x+b)}{(a-c)(b-c)}=1$ has

(a) Two real and distinct roots
(b) Two equal roots
(c) Non real complex roots
(d) Infinite roots

41. If $ax^2+by^2+cz^2+2ayz+2bzx+2cxy$ can be resolved into linear factors, then $a^3+b^3+c^3=$

(a) abc (b) $2\,abc$

(c) $3\,abc$ (d) $\pm\sqrt{2}abc$

42. If $x^2-4cx+b^2>0\ \forall\ x \in R$ & $a^2+c^2<ab$, then the range of the function $\frac{x+a}{x^2+bx+c^2}$ is

(a) $(-\infty, 0)$ (b) $(0, \infty)$

(c) $(-\infty, \infty)$ (d) $\left[\frac{b^2}{a^2}, \frac{c^2}{a^2}\right]$

43. Let $p, q \in \{1, 2, 3, 4\}$. The number of equation of the form $px^2+qx+1=0$ having real roots is

(a) 15 (b) 9 (c) 7 (d) 8

44. If $b>a$, then the equation $(x-a)(x-b)-1=0$ has

(a) both roots in $[a, b]$
(b) both roots in $(-\infty, a)$
(c) both roots in (b, ∞)
(d) one root in $(-\infty, a)$ and other in (b, ∞)

45. If α and β $(\alpha<\beta)$ are the roots of the equation $x^2+bx+c=0$, where $c<0<b$, then

(a) $0<\alpha<\beta$ (b) $\alpha<0<\beta<|\alpha|$

(c) $\alpha<\beta<0$ (d) $\alpha<0<|\alpha|<\beta$

46. Let $F(x)=(1+b^2)x^2+2bx+1$ & let $m(b)$ be the minimum value of $f(x)$. As b varies, the range of $m(b)$ is

(a) $(0, 1]$ (b) $\left[0, \frac{1}{2}\right]$

(c) $\left[\frac{1}{2}, 1\right]$ (d) $[0, 1]$

47. If $f(x)=x^2+2bx+2c^2$ and $g(x)=-x^2-2cx+b^2$, such that minimum $f(x)$ > maximum $g(x)$, then the relation between b and c, is

(a) no real value of b & c (b) $0<c<b\sqrt{2}$

(c) $|c|<|b|\sqrt{2}$ (d) $|c|>|b|\sqrt{2}$

48. If the equation $|x^2+4x+3|-mx+2m=0$ has exactly three solutions, then the value of m is equal to

(a) $-8+2\sqrt{15}$ (b) $-8-2\sqrt{15}$

(c) 3 (d) $-\sqrt{15}$

49. Let $f(x)=x^2+ax+b$ be a quadratic polynomial in which a and b are integers. If for a given integer n. $f(n)f(n+1)=f(m)$ for some integer m, then the value of m is

(a) $(a+b)\,n+ab$ (b) n^2+an+b

(c) $n(n+1)+an+b$ (d) $n^2+n+a+b$

50. Let a, b, c are distinct positive numbers such that each of the quadratics ax^2+bx+c, bx^2+cx+a and cx^2+ax+b is non-negative for all $x \in R$. If $P=\frac{a^2+b^2+c^2}{ab+bc+ca}$, then

(a) $1 \le P \le 4$ (b) $1 \le P < 4$

(c) $1<P \le 4$ (d) $1<P<4$

51. If $\min\{x^2+(a-b)x+1-a-b\}>$ $\max.\{-x^2+(a+b)x-(1+a+b)\}$ then

(a) $a^2+b^2<2$ (b) $a^2+b^2<4$

(c) $a^2+b^2>4$ (d) $a+b>ab$

MARK YOUR RESPONSE	39. ⓐⓑⓒⓓ	40. ⓐⓑⓒⓓ	41. ⓐⓑⓒⓓ	42. ⓐⓑⓒⓓ	43. ⓐⓑⓒⓓ
	44. ⓐⓑⓒⓓ	45. ⓐⓑⓒⓓ	46. ⓐⓑⓒⓓ	47. ⓐⓑⓒⓓ	48. ⓐⓑⓒⓓ
	49. ⓐⓑⓒⓓ	50. ⓐⓑⓒⓓ	51. ⓐⓑⓒⓓ		

52. Consider the equation $x^2 + x - n = 0$, where n is an integer between 1 to 100. Total number of different values of 'n' so that the equation has integral roots is
(a) 6 (b) 4 (c) 9 (d) None

53. Number of positive integers n for which $n^2 + 96$ is a perfect square is
(a) 4 (b) 8 (c) 12 (d) infinite

54. A point (α, α^2) lies inside the triangle formed by the coordinate axes and the line $x + y = 6$. If α is a root of $f(x) = x^2 + ax + b = 0$ then which of the following is always true?
(a) $f(0) > 0$
(b) $f(2) > 0$
(c) $f(\beta) \le 0$ for atleast one $\beta \in (0, 2)$
(d) $-4 < a < 0$

55. If the equation $x^2 + nx + n = 0, n \in I$, has integral roots, then $n^2 - 4n$ can assume
(a) no integral value (b) one integral value
(c) two integral values (d) three integral values

56. If all the real solutions of the equation $4^x - (a-3)2^x + (a-4) = 0$ are non positive, then
(a) $4 < a \le 5$ (b) $0 < a < 4$
(c) $a > 4$ (d) $a < 3$

57. The equation $x^2 + nx + m = 0, n, m \in I$, can not have
(a) integral roots
(b) non-integral rational roots
(c) irrational roots
(d) complex roots

MARK YOUR RESPONSE					
	52. ⓐⓑⓒⓓ	53. ⓐⓑⓒⓓ	54. ⓐⓑⓒⓓ	55. ⓐⓑⓒⓓ	56. ⓐⓑⓒⓓ
	57. ⓐⓑⓒⓓ				

B COMPREHENSION TYPE

This section contains groups of questions. Each group is followed by some multiple choice questions based on a paragraph. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

PASSAGE-1

Consider a function $f(x) = \dfrac{ax^2 + bx + c}{px^2 + qx + r}$.

If we write $y = f(x)$ and simplify we get a quadratic in x and $x \in R$, so the discriminant ≥ 0, which leads to the range of y. However there may be limitations of discontinuity of $f(x)$, whenever $px^2 + qx + r$ vanishes. If y tends to $+\infty$ and $-\infty$ while crossing a finite value of x, say $x = a$ then the line $x = a$ is a vertical asymptote of the curve $y = f(x)$. Similarly the line $y = b$ is a horizontal asymptote if x tends to $+\infty$ and $-\infty$ while crossing $y = b$.

Now answer the following questions:

1. The curve $y = \dfrac{x+1}{x^2 + 2x + 2}$ has
(a) one horizontal and one vertical asymptote
(b) no horizontal but one vertical asymptote
(c) one horizontal but no vertical asymptote
(d) one horizontal and two vertical asymptote

2. The curve $y = \dfrac{x+1}{x^2 - 3x}$ has
(a) one horizontal and one vertical asymptote
(b) no horizontal but one vertical asymptote
(c) one horizontal but no vertical asymptote
(d) one horizontal and two vertical asymptote

3. If the range of the expression $\dfrac{x^2 + x + d}{x^2 + 2x + d}$ is $\left[\dfrac{5}{6}, \dfrac{3}{2}\right]$ for all real x for which it is defined then the value of d is
(a) 4 (b) -4 (c) 1 (d) -1

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ		

PASSAGE-2

The point of intersection of two curves $y = f(x)$ and $y = g(x)$ can be obtained by solving the equations of the curves simultaneously. Thus by eliminating y we get an equation $f(x) - g(x) = 0$, which on solving gives the abscissa of point of intersection. Suppose the equation $f(x) - g(x) = 0$ turns out to be a quadratic equation in x, we can apply the concepts of nature of roots and sign of quadratic expressions.

Now consider the following curves given in parametric forms :

$S_1 : x = at^2 + bt + c_1, y = at$ and

$S_2 : x = -(at^2 + bt + c_2),\ y = at, a \neq 0$

4. S_1 and S_2 intersect if

(a) $b^2 > 4a(c_1 + c_2)$ (b) $b^2 > 2a(c_1 + c_2)$

(c) $b^2 > a(c_1 + c_2)$ (d) $b^2 + 2a(c_1 + c_2) > 0$

5. If the curves S_1 and S_2 intersect orthogonally then in addition to the condition obtained in question (1), coefficients must satisfy the condition

(a) $a^2 - b^2 + 2a(c_1 + c_2) = 0$ (b) $a^2 + b^2 + 2a(c_1 + c_2) = 0$

(c) $a^2 - b^2 - 2a(c_1 + c_2) = 0$ (d) $a^2 + b^2 - 2a(c_1 + c_2) = 0$

6. If the curves S_1 and S_2 touch then the equation of common normal at the point of contact is

(a) $x = \dfrac{4ac_1 - b^2}{4a}$ (b) $x = \dfrac{b^2 - 4ac_2}{4a}$

(c) $y = -\dfrac{b}{2}$ (d) $y = x$

PASSAGE-3

Suppose two quadratic equations

$a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ have a common root α, then

$a_1\alpha^2 + b_1\alpha + c_1 = 0$ ...(1) and

$a_2\alpha^2 + b_2\alpha + c_2 = 0$ ...(2)

Eliminating α using cross-multiplication method gives us the condition for a common root. Solving two equations simultaneously, the common root can be obtained.

Now consider three quadratic equations,

$x^2 - 2rp_r x + r = 0\ ; r = 1, 2, 3.$

Given that each pair has exactly one root common.

7. The common root between the equations obtained by $r = 1$ and $r = 3$ is

(a) $-\sqrt{\dfrac{3}{2}}$ or $\sqrt{\dfrac{3}{2}}$ (b) $-\sqrt{\dfrac{2}{3}}$ or $\sqrt{\dfrac{2}{3}}$

(c) $-\sqrt{6}$ or $\sqrt{6}$ (d) 1

8. The sum of all the three roots is equal to

(a) 0 or $\sqrt{6}$ (b) $\dfrac{11}{\sqrt{6}}$ or $-\dfrac{11}{\sqrt{6}}$

(c) $\dfrac{11}{\sqrt{6}}$ or $-\sqrt{11}$ (d) 0 or $\pm\dfrac{11}{\sqrt{6}}$

9. The number of the triplets (p_1, p_2, p_3) for which such equations exist is

(a) 2 (b) 3 (c) ${}^6C_2$ (d) infinite

PASSAGE-4

Let $y = f(x)$ be a quadratic polynomial such that

$$f\left(\frac{a^2-3}{a} + x\right) = f\left(\frac{a^2-3}{a} - x\right);\ f\left(\frac{a^2-3}{a}\right) = -\frac{(a^2+3)^2}{a}$$

and the parabola $y^2 = \dfrac{1}{a}x$ is an equal parabola to that of $y = f(x)$ then,

10. The roots of the equation $y = f(x) = 0$ are

(a) Real and distinct for all $a \neq 0$

(b) Real and may be equal if $a > 0$

(c) Real and distinct only if $a > 0$

(d) non-real if $a < 0$

11. If exactly one root of the equation lies in the interval (0, 1), then the values of a is/are

(a) $a \in R$ (b) $[0, 1]$ (c) $\left[0, \dfrac{1}{2}\right)$ (d) $\left(0, \dfrac{1}{2}\right)$

12. Let α, β be the roots of the equation $y = f(x) = 0 \forall a \in R^+$. If λ be such that $\alpha < a\lambda < \beta$ and $a \in (0,1)$, then λ lies in

(a) $(0, 1)$ (b) $\left(0, \dfrac{1}{2}\right)$

(c) $\left(-\dfrac{6}{a^2}, 2\right)$ (d) $\left(\dfrac{6}{a^2}, 2\right)$

MARK YOUR RESPONSE	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ
	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	

C REASONING TYPE

In the following questions two Statements (1 and 2) are provided. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct. Mark your responses from the following options :

(a) Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.

(b) Both Statement-1 and Statement-2 are true and Statement-2 is not the correct explanation of Statement-1.

(c) Statement-1 is true but Statement-2 is false.

(d) Statement-1 is false but Statement-2 is true.

1. **Statement-1** : $\forall x \in R,\ x^2 + x + 1$ is positive

Statement-2 : If $\Delta < 0$, $ax^2 + bx + c$ and a have same sign $\forall x \in R$.

2. **Statement-1** : If $x \in (2, 3)$ then $x^2 - 5x + 6 > 0$

Statement-2 : If $\alpha < x < \beta$, $ax^2 + bx + c$ and a have opposite signs $(\alpha < \beta)$

3. **Statement-1** : The equation $a \sin x + \cos 2x = 2a - 7$ posseses a solution if $a \in [2, 6]$.

Statement-2 : $-1 \le \sin x \le 1\ \forall\ x \in R$

4. **Statement-1** : The set of all real numbers a such that $a^2 + 2a$, $2a + 3$ and $a^2 + 3a + 8$ are the sides of a triangle is $(5, \infty)$

Statement-2 : In a triangle sum of two sides is greater than the other and also sides are always positive.

5. **Statement-1** : Roots of the equation $(x^2 - 4x + 3) + \lambda (x^2 - 6x + 8) = 0$ are real and distinct for all $\lambda \in R$.

Statement-2 : If $P(x) = 0$ and $Q(x) = 0$ are two different quadratic equations, each having real and distinct roots then $P(x) + \lambda Q(x) = 0$ has real and distinct roots for all $\lambda \in R$.

6. Let $a \ne -2$ and $(a + 2)\, x^2 + 2\,(a + 1)\, x + a = 0$ has integral roots.

Statement-1 : a can take four distinct integral values.

Statment-2 : $\dfrac{a}{a+2}$ must be integer.

7. **Statement-1** : The function $y = \dfrac{ax^2 + bx + c}{px^2 + qx + r}$ has no asymptote if $q^2 - 4pr < 0$

Statement-2 : If $q^2 - 4pr < 0$, then $px^2 + qx + r \ne 0$ for any real value of x

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ			

D MULTIPLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

1. If A, G and H are the Arithmetic mean, Geometric mean and Harmonic mean between two unequal positive integers. Then the equation $Ax^2 - |G|x - H = 0$ has

(a) both roots fractions
(b) one negative fraction root
(c) exactly one positive root
(d) no root greater than 2

2. If a, b, c are positive rational numbers such that $a > b > c$ and the quadratic equation $(a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b) = 0$ has a root in the interval $(-1, 0)$, then

(a) $c + a < 2b$
(b) Both roots of the given equation are rational
(c) The equation $ax^2 + 2bx + c = 0$ has both negative real roots
(d) The equation $cx^2 + 2ax + b = 0$ has both negative real roots

3. If $a < b < c < d$, then for any real non-zero λ, the quadratic equation $(x - a)(x - c) + \lambda (x - b)\ (x - d) = 0$ has

(a) non - real roots
(b) one real root between a and c
(c) one real root between b and d
(d) irrational roots

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ		

4. Equation $\frac{\pi^e}{x-e}+\frac{e^\pi}{x-\pi}+\frac{\pi^\pi+e^e}{x-\pi-e}=0$ has

(a) one real root in (e,π) and other in $(\pi-e,e)$

(b) one real root in (e,π) and other in $(\pi,\pi+e)$

(c) Two real roots in $(\pi-e,\pi+e)$

(d) No real root

5. If $a<0$, then the root of the equation $x^2-2a|x-a|-3a^2=0$ is

(a) $a(-1-\sqrt{6})$ (b) $a(1-\sqrt{2})$

(c) $a(-1+\sqrt{6})$ (d) $a(1+\sqrt{2})$

6. The equation $x^2-6x+8+\lambda(x^2-4x+3)=0, \lambda\in R$, has

(a) real and unequal roots for all λ

(b) real roots for $\lambda<0$

(c) real roots for $\lambda>0$

(d) real and unequal roots for $\lambda=0$

7. If a, b, and c are odd integers, then the roots of $ax^2+bx+c=0$, if real, cannot be

(a) integers (b) rational numbers

(c) irrational (d) Equal

8. The roots of $ax^2+bx+c=0$. Where $a\neq 0$ and coefficients are real, are nonreal complex and $a+c<b$. Then

(a) $4a+c>2b$ (b) $4a+c<2b$

(c) $a+4c>2b$ (d) $a+4c<2b$

9. Which of the following is correct for the quadratic equation $x^2+2(a-1)x+a+5=0$

(a) The equation has positive roots, if $a\in(-5,-1]$

(b) The equation has roots of opposite sign, if $a\in(-\infty,-5)$

(c) The equation has negative roots, if $a\in[4,\infty)$

(d) None of these

10. Consider the quadratic equation $x^2-2px+p^2-1=0$ where p is parameter, then

(a) Both the roots of the equation are less than 4 if $p\in(-\infty,3)$

(b) Both the roots of the equation are greater than –2 if $p\in(-\infty,-1)$

(c) Exactly one root of the equation lies in the interval $(-2,4)$ if $p\in(-1,3)$

(d) 1 lies between the roots of the equation if $p\in(0,2)$

11. If α is a real root of the equation $ax^2+bx+c=0$, and β is a real root of the equation $-ax^2+bx+c=0$, then the equation $\frac{a}{2}x^2+bx+c=0$ has

(a) real roots

(b) none-real roots

(c) a root lying between α and β

(d) a root equal to α or β

12. If $\frac{x^2+ax+3}{x^2+x+a}$, takes all real values for possible real values of x, then

(a) $4a^3+39\geq 0$ (b) $4a^3+39<0$

(c) $a<\frac{1}{4}$ (d) $a\geq\frac{1}{4}$

13. The set of real values of k for which the equation $x^2-4|x|+3-|k-1|=0$ will have exactly four roots is

(a) $(-2,4)$ (b) $(-4,4)$

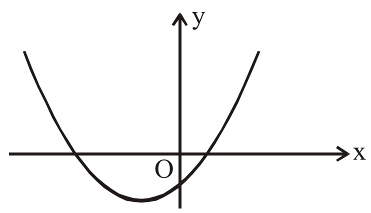
(c) $(-4,2)$ (d) $(-1,0)$

14. If the equation $x^2+ax+b=0$ has distinct real roots and $x^2+a|x|+b=0$ has only one real root, then

(a) $b=0$ (b) $a<0$

(c) $b>0$ (d) $a>0$

15. Consider the graph of $f(x) = ax^2+bx+c$ shown in the adjacent diagram. We can conclude that



(a) $a>0$ (b) $b>0$

(c) $c>0$ (d) $a-b+c<0$

16. If p and q be odd integers, then the equation $x^2+2px+2q=0$

(a) has no integral roots

(b) has no rational roots

(c) may have rational roots

(d) always have real roots

MARK YOUR RESPONSE	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ
	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ
	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ		

17. If the equation $x^2 - 4|x| + 3 - (k-1) = 0$ has exactly four real roots then k cannot be equal to
(a) -2 (b) 0 (c) 2 (d) 4

18. If the equation $ax^2 + bx + c = 0$ has distinct real roots and $ax^2 + b|x| + c = 0$ also has two distinct real roots then
(a) $a > 0$ (b) $\frac{c}{a} < 0$ (c) $\frac{b}{a} < 0$
(d) $x = 0$ cannot be a root of the first equation

19. If $x^2 - 2x + \sin^2\alpha = 0$, then x may lie in the set
(a) $[-1, 1]$ (b) $[0, 2]$
(c) $[-2, 2]$ (d) $[1, 2]$

20. Let $f(x) = ax^2 + bx + c$, $a, b, c \in R$ and $a \neq 0$.
If $f(x) > 0 \ \forall \ x \in R$ then
(a) $4a - 2b + c > 0$ (b) $2a - b + c > 0$
(c) $10a + 3b + c > 0$ (d) $2a + b + c > 0$

21. If $4ac > b^2$ and $a + c > b$ for real numbers a, b and c, then which of the following is true?
(a) $a > 0$ (b) $c > 0$
(c) $a + b + c > 0$ (d) $4a + c > 2b$

22. If $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0$ $(a, b, c \in R)$ have a common non-real root, then
(a) $-2|a| < b < 2|a|$ (b) $-2|c| < |b| < 2|c|$
(c) $a = \pm c$ (d) $a = c$

23. If the equation $cx^2 + bx - 2a = 0$ has no real roots and $a < \frac{b+c}{2}$ then
(a) $ac < 0$ (b) $a < 0$
(c) $\frac{c-a}{2} > a$ (d) $\frac{c+2b}{8} > a$

24. If the equation $ax^2 + bx + c = 0 \ (a < 0)$ has two roots α and β such that $\alpha < -3$ and $\beta > 3$, then
(a) $9a + 3|b| + c > 0$ (b) $c > 0$
(c) $4a + 2|b| + c > 0$ (d) $a + |b| + c < 0$

25. If α is one root of the equation $4x^2 + 2x - 1 = 0$, then the other root is
(a) $\frac{1}{2} - \alpha$ (b) $-\frac{1}{2} - \alpha$
(c) $4\alpha^3 + 3\alpha$ (d) $4\alpha^3 - 3\alpha$

26. If $y = f(x) = ax^2 + bx + c$, such that $c > 0$ and $4a + c < 2b$ then
(a) the roots of the equation are real
(b) one roots lie in the interval $(-2, 0)$
(c) if $a > 0$ then $b > 0$
(d) none of these

27. Let $6a < 4b < 3c$ then the equation $(2x-a)(3x-b) + (3x-b)(4x-c) + (4x-c)(2x-a) = 0$ has
(a) both roots real
(b) both roots imaginary
(c) one root lies between $\left(\frac{a}{2}, \frac{b}{3}\right)$
(d) other root lies between $\left(\frac{b}{3}, \frac{c}{4}\right)$

28. If exactly one root of the equation $x^2 - (k-1)x + k(k+4) = 0$ lies between the roots of the equation $x^2 - (k+3)x + k + 2 = 0$ then
(a) $k \in (-6, -3)$ (b) $(3, 6)$
(c) $k \in [-3, -1)$ (d) $[1, 6)$

29. If the expression $ax^2 + 8x - (20 + 4a) < 0 \ \forall x \in [0, 2]$ then a
(a) $a \in (-5, \infty)$ (b) $a \in (1, \infty)$
(c) $a \in (0, \infty)$ (d) $a \in [0, 1]$

30. The question $x^2 + a^2x + b^2 = 0$ has two roots each of which exceeds a number c, then
(a) $a^4 > 4b^2$ (b) $c^2 + a^2c + b^2 > 0$
(c) $-\frac{a^2}{2} > c$ (d) None of these

31. If the equation $ax^2 + 2bx + c = 0$ and $x^2 + 2p^2x + 1 = 0$ have one common root and a, b, c are in A.P. $(p^2 \neq 1)$, then the roots of the equation $x^2 + 2p^2x + 1 = 0$ are
(a) $-\frac{a}{c}$ (b) $-\frac{c}{a}$ (c) $\frac{b}{a}$ (d) $\frac{a}{b}$

MARK YOUR RESPONSE	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ
	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ
	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ	31. ⓐⓑⓒⓓ

E MATRIX-MATCH TYPE

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labeled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column -I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A–p, s and t; B–q and r; C–p and q; and D–s and t; then the correct darkening of bubbles will look like the given.

	p	q	r	s	t
A	(p)	(q)	(r)	(s)	(t)
B	(p)	(q)	(r)	(s)	(t)
C	(p)	(q)	(r)	(s)	(t)
D	(p)	(q)	(r)	(s)	(t)

1. Observe the following lists :

Column-I	**Column-II**
(A) $-7x^2+8x-9>0$	p. $R-\{2\}$
(B) $2x^2-4x+5>0$	q. $(-1, 6)$
(C) $x^2-4x+4>0$	r. $(-\infty,-1)\cup(6,\infty)$
(D) $x^2-5x-6<0$	s R
	t. ϕ

2. Observe the following lists :

Column-I	**Column-II**
(A) If a,b,c are unequal positive numbers and b is $A.M$ of a and c then the roots of $ax^2+2bx+c=0$ are	p. of opposite signs
(B) If $a \in R$, then the roots of the equation $x^2-(a+1)x-a^2-4=0$ are	q. rational numbers
(C) If a,b,c are unequal positive numbers and b is $H.M$ of a and c then the roots of $ax^2+2bx+c=0$ are	r. real and unequal
(D) If $\left|a \pm b\right| < c$ and $a=0$ then the roots of $a^2x^2+(b^2+a^2-c^2)x+b^2=0$ are	s. imaginary
	t. of same sign

MARK YOUR RESPONSE

1.

	p	q	r	s	t
A	(p)	(q)	(r)	(s)	(t)
B	(p)	(q)	(r)	(s)	(t)
C	(p)	(q)	(r)	(s)	(t)
D	(p)	(q)	(r)	(s)	(t)

2.

	p	q	r	s	t
A	(p)	(q)	(r)	(s)	(t)
B	(p)	(q)	(r)	(s)	(t)
C	(p)	(q)	(r)	(s)	(t)
D	(p)	(q)	(r)	(s)	(t)

F NUMERIC/INTEGER ANSWER TYPE

The answer to each of the questions is either numeric (eg. 304, 40, 3010 etc.) or a single-digit integer, ranging from 0 to 9.

The appropriate bubbles below the respective question numbers in the response grid have to be darkened.

For example, if the correct answers to a question is 6092, then the correct darkening of bubbles will look like the given.

For single digit integer answer darken the extreme right bubble only.

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

1. For any real x, the maximum value of

$$\frac{2}{k^2}(x-k)\left(x+\sqrt{x^2+k^2}\right) \text{ is equal to}$$

2. The number of integral values of a for which the inequality $3 - |x - a| > x^2$ is satisfied by at least one negative x, is equal to

3. Let a and b be the roots of the equation $x^2 - 10cx - 11d = 0$ and those of $x^2 - 10ax - 11b = 0$ are c, d then find the value of $a + b + c + d$, when $a \neq b \neq c \neq d \neq 0$

4. If $x + y + z = 5$ and $xy + yz + zx = 3$ then the greatest value of $f(3x)$ is

5. Let $f(x) = x^2 - ax + b$, a is odd positive integer and the roots of the equation $f(x) = 0$ are two distinct prime numbers. If $a + b = 35$ then find the value of $[f(1) + f(2) + f(3) + \ldots\ldots.. + f(10)]f(10)$

6. If the roots of equation $ax^2 + bx + c = 0 \quad (a \neq 0)$ are α and β, and the roots of the equation $a^5x^2 + ba^2c^2x + c^5 = 0$ are 4 and 8 then the numerical value of $\alpha\beta$ is _________ .

MARK YOUR RESPONSE

1.	2.	3.	4.	5.	6.
0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0
1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2
3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3
4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4
5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5
6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6	6 6 6 6
7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7
8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8	8 8 8 8
9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9	9 9 9 9

Answerkey

A SINGLE CORRECT CHOICE TYPE

1	(a)	**11**	(a)	**21**	(c)	**31**	(a)	**41**	(c)	**51**	(b)
2	(d)	**12**	(b)	**22**	(a)	**32**	(d)	**42**	(c)	**52**	(c)
3	(c)	**13**	(a)	**23**	(b)	**33**	(c)	**43**	(c)	**53**	(a)
4	(c)	**14**	(a)	**24**	(d)	**34**	(a)	**44**	(d)	**54**	(c)
5	(c)	**15**	(a)	**25**	(b)	**35**	(b)	**45**	(b)	**55**	(b)
6	(a)	**16**	(c)	**26**	(a)	**36**	(d)	**46**	(a)	**56**	(a)
7	(c)	**17**	(a)	**27**	(b)	**37**	(d)	**47**	(d)	**57**	(b)
8	(d)	**18**	(d)	**28**	(b)	**38**	(c)	**48**	(a)		
9	(a)	**19**	(b)	**29**	(b)	**39**	(d)	**49**	(c)		
10	(a)	**20**	(d)	**30**	(a)	**40**	(d)	**50**	(d)		

B COMPREHENSION TYPE

1	(c)	**3**	(a)	**5**	(a)	**7**	(a)	**9**	(a)	**11**	(d)
2	(d)	**4**	(b)	**6**	(c)	**8**	(b)	**10**	(a)	**12**	(c)

C REASONING TYPE

1	(a)	**3**	(a)	**5**	(c)	**7**	(d)
2	(d)	**4**	(a)	**6**	(a)		

D MULTIPLE CORRECT CHOICE TYPE

1	(b,c,d)	**6**	(a,b,c,d)	**11**	(a,c)	**16**	(a, b)	**21**	(a, b, c, d)	**26**	(a, b, c)
2	(a,b,c,d)	**7**	(a,b,d)	**12**	(b,c)	**17**	(a,b,d)	**22**	(a, b, d)	**27**	(a, c, d)
3	(b,c)	**8**	(b,d)	**13**	(a, d)	**18**	(b,d)	**23**	(a, b, c, d)	**28**	(a, c)
4	(b,c)	**9**	(a,b,c)	**14**	(a, d)	**19**	(b,d)	**24**	(a, b, c)	**29**	(b, c, d)
5	(b,c)	**10**	(a,d)	**15**	(a, b)	**20**	(a,b,c,d)	**25**	(b, d)	**30**	(a, b, c)
										31	(a, b)

E MATRIX-MATCH TYPE

1. **A - t; B - s; C - p; D - q**

2. **A - r, t; B - p, r; C - s; D - r, t**

F NUMERIC/INTEGER ANSWER TYPE

1	2	**2**	6	**3**	1210	**4**	13	**5**	880	**6**	2

Solutions

A SINGLE CORRECT CHOICE TYPE

1. **(a)** Given $a(p+q)^2+2bpq+c=0$ and

$a(p+r)^2+2bpr+c=0$

$\therefore$ q and r satisfy the equation

$a(p+x)^2+2bpx+c=0$

$\therefore$ q and r are the roots of $a(p+x)^2+2bpx+c=0$

i.e. q and r are the roots of

$ax^2+2(ap+bp)x+c+ap^2=0$

$\therefore$ $qr=$ product of roots $=\dfrac{c+ap^2}{a}=p^2+\dfrac{c}{a}$

CAUTION : *Watch the concept. Never try to evaluate qr by solving.*

2. **(d)** We have, $p(1)-q(1)=0 \Rightarrow (a+b)-(\ell+m+n)=0$ ...(1)

$p(2)-q(2)=1 \Rightarrow (4a+2b)-(4\ell+2m+n)=1$ ...(2)

$p(3)-q(3)=4 \Rightarrow (9a+3b)-(9\ell+3m+n)=4$ ...(3)

Now use the operation $3\times(3)+1\times(1)-3\times(2)$, we get

$(16a+4b)-(16\ell+4m+n)=9 \Rightarrow p(4)-q(4)=9.$

COMMON MISTAKE : *Don't try to find values of a, b, ℓ, m, n.*

3. **(c)** α,β are the roots of

$x^2-a(x-1)+b=0 \Rightarrow \alpha^2-a\alpha+a+b=0$ and

$\beta^2-a\beta+a+b=0$.

$\therefore \alpha^2-a\alpha=\beta^2-a\beta=-a-b$

Now, $\dfrac{1}{\alpha^2-a\alpha}+\dfrac{1}{\beta^2-a\beta}$

$+\dfrac{2}{a+b}=\dfrac{1}{-(a+b)}+\dfrac{1}{-(a+b)}+\dfrac{2}{a+b}=0$

NOTE : *Consider the expression $\alpha^2-a\alpha$ and $\beta^2-a\beta$. Applying $\alpha+\beta$ and $\alpha\beta$ can complicate the solution*

4. **(c)** Let α,β be the roots of $ax^2-bx-c=0$ and let α',β' be the roots of $a'x^2-b'x-c'=0$ such that $|\alpha-\beta|=|\alpha'-\beta'|$,

i.e., $(\alpha+\beta)^2-4\alpha\beta=(\alpha'+\beta')^2-4\alpha'\beta'$

$\Rightarrow \dfrac{b^2+4ac}{a^2}=\dfrac{b'^2+4a'c'}{a'^2}$.

Hence, the expression $\dfrac{b^2+4ac}{a^2}$ does not vary in value.

5. **(c)** From the given condition,

$\dfrac{\alpha}{\alpha-1}+\dfrac{\alpha+1}{\alpha}=-\dfrac{b}{a}$ and $\dfrac{\alpha}{\alpha-1}.\dfrac{\alpha+1}{\alpha}=\dfrac{c}{a}$

$\Rightarrow \dfrac{2\alpha^2-1}{\alpha^2-\alpha}=-\dfrac{b}{a}$ and $\alpha=\dfrac{c+a}{c-a}$

$\Rightarrow (c+a)^2+4ac$

$=-2b(c+a) \Rightarrow (c+a)^2+2b(c+a)+b^2$

$=b^2-4ac \Rightarrow (a+b+c)^2=b^2-4ac$.

6. **(a)** Sum of the coefficients = 0. So, $x=1$ is a root and the other root is $-\dfrac{2a-1}{a+2}$ = rational number, if a is rational, $a\neq-2$.

7. **(c)** $\because$ 1 is a root of $ax^2+bx+c=0 \Rightarrow a+b+c=0$ ...(1)

Now discriminant of equation $4ax^2+3bx+2c=0$ is

$D=9b^2-32ac=9(a+c)^2-32ac$ [using (1).]

$=9a^2-14ac+9c^2$

$=7(a^2-2ac+c^2)+2(a^2+c^2)$

$=7(a-c)^2+2(a^2+c^2)>0$

Hence, roots of the equation are real and unequal.

8. **(d)** The discriminants of the quadratic factors are,

$D_1=a^2+12b$; $D_2=c^2-4b$ and $D_3=d^2-8b$

$\therefore D_1+D_2+D_3=a^2+c^2+d^2\geq 0$

$\Rightarrow$ At least one of D_1, D_2, D_3 is non-negative. Hence, the equation has at least two real roots.

9. **(a)** Let $f(x)=ax^2+2bx-3c$. $\because f(x)=0$ has non real roots, $f(x)$ will have the same sign for all real values of x.

Given : $\dfrac{3c}{4}<a+b \Rightarrow 4a+4b-3c>0 \Rightarrow f(2)>0$

$\Rightarrow f(0)>0 \Rightarrow c<0.$

10. **(a)** Given $\frac{1}{3} < \frac{x^2-2x+4}{x^2+2x+4} < 3$, replacing x by 3^{x+1}, then

$$\frac{1}{3} < \frac{3^{2x}.9-6.3^x+4}{3^{2x}.9+6.3^x+4} < 3$$

$$\Rightarrow 3 > \frac{9.3^{2x}+6.3^x+4}{9.3^{2x}-6.3^x+4} > \frac{1}{3}$$

or $\frac{1}{3} < \frac{9.3^{2x}+6.3^x+4}{9.3^{2x}-6.3^x+4} < 3$.

11. **(a)** The equation is

$(a+b)x^2+(2a-3b+1)x+(a-2b+1)=0$.

Above is identity in x if

$a+b=0;\ 2a-3b+1=0$ and $a-2b+1=0$.

Solving first and second, we get $a=-\frac{1}{5}$ and $b=\frac{1}{5}$.

But these values do not satisfy third. Hence equation can not be identity for any value of a and b.

12. **(b)** We have $\sin^2\phi=\cos\theta.\sin\theta$. The discriminant of the given equation is

$$D=4\cot^2\phi-4=4\left[\frac{\cos^2\phi-\sin^2\phi}{\sin^2\phi}\right]$$

$$=\frac{4(1-2\sin^2\phi)}{\sin^2\phi}=\frac{4(1-2\sin\theta\cos\theta)}{\sin^2\phi}$$

$$D=\left[\frac{2(\sin\theta-\cos\theta)}{\sin\phi}\right]^2\geq 0.$$

Hence, the roots of the equation are always real.

13. **(a)** According to ques., 1 lies between the roots, therefore,

$f(1)<0\Rightarrow\ 1-3+a<0$

$\Rightarrow a<2$ or $a\in(-\infty,2)$.

14. **(a)** As the coefficients of two equations are in reverse order, if the roots of $ax^2+bx+c=0$ are α and β then the roots of second equation are $\frac{1}{\alpha},\frac{1}{\beta}$. Given that one negative root is common, two possibilities may arise.

Either $\alpha=\frac{1}{\alpha}<0\Rightarrow\alpha=-1$ or

$\alpha=\frac{1}{\beta}<0\Rightarrow\alpha\beta=1\Rightarrow\frac{c}{a}=1\Rightarrow c=a$ (not possible)

$\therefore\ \alpha=-1$ is the common root. Put $\alpha=-1$ in any of the equations, we get $a-b+c=0$.

CAUTION : *Finding common root by solving two equations may result into lengthy calculation.*

15. **(a)** $x^2-(k-2)x+k^2=0$ and $x^2+kx+2k-1=0$ should have both roots, common or each should have equal roots.

$\therefore$ (i) $\frac{1}{1}=\frac{-(k-2)}{k}=\frac{k^2}{2k-1}\Rightarrow k=-k+2$

and $2k-1=k^2\Rightarrow k=1$

or (ii) $(k-2)^2-4k^2=0$ and

$k^2-4(2k-1)=0\Rightarrow(3k-2)(-k-2)=0$ and

$k^2-8k+4=0,$ no common value.

$\therefore\ k=1$, is the only possible value.

16. **(c)** Given $mx-1+\frac{1}{x}\geq 0$

$\Rightarrow\frac{mx^2-x+1}{x}\geq 0\Rightarrow mx^2-x+1\geq 0$

as $x>0$. It will hold if $m>0$ and $D\leq 0\Rightarrow m>0$ and $1-4m\leq 0\Rightarrow m\geq\frac{1}{4}$

$\therefore$ The minimum value of m is $\frac{1}{4}$.

17. **(a)** Let $t=x+\sqrt{x^2+b^2}$

$$\Rightarrow\frac{1}{t}=\frac{1}{x+\sqrt{x^2+b^2}}=\frac{\sqrt{x^2+b^2}-x}{b^2}$$

$$\Rightarrow t-\frac{b^2}{t}=2x\ \&\ t+\frac{b^2}{t}=2\sqrt{x^2+b^2}$$

$$\therefore\ 2(a-x)(x+\sqrt{x^2+b^2})$$

$$=\left(2a-t+\frac{b^2}{t}\right)(t)=2at-t^2+b^2$$

$$\Rightarrow a^2+b^2-(a^2-2at+t^2)$$
$$=a^2+b^2-(a-t)^2\leq a^2+b^2$$

18. **(d)** The given equation is a quadratic but satisfied by three values of x, $x=a$, $x=b$ and $x=c$. Hence, it must be an identity.

$\Rightarrow$ The equation has infinite roots.

19. **(b)** Discriminant $D=(2m-1)^2-4(m-2)m=4m+1$ must be perfect square

$\Rightarrow 4m+1=k^2$, say for some $k\in\mathbf{I}$

$\Rightarrow m=\frac{(k-1)(k+1)}{4}$, clearly k must be odd.

Let $k=2n+1$.

$$\therefore\ m=\frac{2n(2n+2)}{4}=n(n+1),n\in I$$

20. **(d)** Solving the given equation,

$x^2 - 2ax + a^2 - 1 = 0 \Rightarrow (x-a)^2 = 1 \Rightarrow x - a = \pm 1$

$\therefore \quad x = a+1$ or $a-1$.

$\therefore \quad a-1 > -3$ and $a+1 < 4 \Rightarrow a > -2$ and $a < 3$

$\Rightarrow -2 < a < 3$

$\therefore \quad [a] = -2, -1, 0, 1, 2 \Rightarrow [a] \neq 4.$

21. **(c)** The sum of the coefficients of the equation $= 0$

$\therefore \; x = 1$ is a root of the equation. Let a be the first term and d be the common difference of given $A.P.$

$$t_p = a + (p-1)d = \frac{1}{q} \qquad ...(1)$$

and $$t_q = a + (q-1)d = \frac{1}{p} \qquad ...(2)$$

Solving (1) and (2), $a = d = \dfrac{1}{pq}$

$\therefore \; t_{pq} = a + (pq-1)d = 1$

$\therefore \; t_{pq}$ is the root of the given equation.

22. **(a)** α and β are roots of $ax^2 + bx + c = |c|$.

$\Rightarrow a\alpha^2 + b\alpha + c = |c| \qquad ...(1)$

and $a\beta^2 + b\beta + c = |c| \qquad ...(2)$

and $-\dfrac{b}{2a}$ lies between α and β.

Now, Let $f(x) = ax^2 + bx + c$

$\Rightarrow f(\alpha) = a\alpha^2 + b\alpha + c = |c|$ and

$f(\beta) = a\beta^2 + b\beta + c = |c|$. [from (1) and (2)]

$\therefore \; f(\alpha) > 0, \; f(\beta) > 0$ and $\dfrac{-b}{2a}$ lies between α and β.

$\therefore$ Roots of the equation $f(x) = 0$, i.e. p and q lie between α and β.

23. **(b)** Let the roots be α and $\alpha + 2$, where α is an odd positive integer. Then $a\alpha^2 + b\alpha + c = 0 \qquad ...(1)$

and $a(\alpha+2)^2 + b(\alpha+2) + c = 0$

$\Rightarrow a\alpha^2 + b\alpha + c + (4a\alpha + 4a + 2b) = 0$

$\Rightarrow 2a(1+\alpha) + b = 0$ [From (1)]

$\Rightarrow b = -2a(1+\alpha)$

$\Rightarrow b^2 = 4a^2(1+\alpha)^2 \Rightarrow b^2 \geq 4a^2(1+1)^2$

[$\because \alpha \geq 1$ as α is odd positive integer]

$\Rightarrow b^2 \geq 16a^2 \Rightarrow |b| \geq 4a$

24. **(d)** We have, $g(x) = af(x) + bf'(x) + cf''(x)$,

where $f(x) = ax^2 + bx + c$

$\therefore \; g(x) = a^2x^2 + 3abx + b^2 + 3ac \qquad ...(1)$

$g(x) > 0 \; \forall x \in \mathbf{R}$ if Discriminant < 0

Now discriminant of (1);

$D = 9a^2b^2 - 4a^2(b^2 + 3ac) = a^2(5b^2 - 12ac)$

$D = a^2(5b^2 - 20ac) + 8a^3c = 5a^2(b^2 - 4ac) + 8a^3c$

D is always negative if $a^3c < 0 \Rightarrow c < 0$

[$\because \; b^2 - 4ac < 0$ and $a > 0$ as $f(x) > 0 \forall x \in \mathbf{R}$]

25. **(b)** $f(x)$ is non-real for non-real values of x. Let $x = \alpha + i\beta$, then $f(\alpha + i\beta)$ is non-real, where α and β are real $(\beta \neq 0)$.

Now $f(\alpha + i\beta) = a(\alpha + i\beta)^2 + b(\alpha + i\beta) + c$

$= a(\alpha^2 - \beta^2) + 2a\alpha\beta i + b\alpha + bi\beta + c$

$= a(\alpha^2 - \beta^2) + b\alpha + c + (2a\alpha + b)i\beta$

Now, $f(\alpha + i\beta)$ will be non real if $2a\alpha + b \neq 0$

$(\because \; \beta \neq 0)$

Now if we choose $\alpha = -\dfrac{b}{2a}$ [Assuming $a \neq 0$]

then $2a\alpha + b = 0 \Rightarrow f(\alpha + i\beta)$ is real, which is contrary to the hypothesis. Hence $a = 0$.

26. **(a)** Let $x^2 + 19x + 92 = n^2$, $n \in \mathbf{I}$, Then

$x^2 + 19x + 92 - n^2 = 0 \Rightarrow x = \dfrac{-19 \pm \sqrt{4n^2 - 7}}{2}$.

If x is an integer

$\Rightarrow -19 \pm \sqrt{4n^2 - 7}$ are even integers

$\Rightarrow \sqrt{4n^2 - 7}$ is an odd integer.

Let $\sqrt{4n^2 - 7} = 2k + 1, k \in \mathbf{I}$. Squaring both the sides,

$4n^2 - 7 = (2k+1)^2$

$\Rightarrow 4n^2 - (2k+1)^2 = 7$

$\Rightarrow (2n - 2k - 1)(2n + 2k + 1) = 7$

$\because$ 7 is a prime integer, thus one factor should be 1 and other 7 or one factor should be – 1 and other – 7

Case 1:

If $2n - 2k - 1 = 1$ and $2n + 2k + 1 = 7$, then $n = 2$.

Case 2 :

If $2n - 2k - 1 = 7$ and $2n + 2k + 1 = 1$, then $n = 2$.

$\therefore\ n = 2 \Rightarrow x = -8$ or -11

Similarly taking one factor – 1 and other – 7, we get $n = -2$, then again $x = -8$ or -11.

27. **(b)** α, β are the roots of

$x^2 + px + q = 0 \Rightarrow \alpha + \beta = -p$ and $\alpha\beta = q$ ...(1)

Again, α, β are also roots of $x^{2n} + p^n x^n + q^n = 0$

$\Rightarrow\ \alpha^{2n} + p^n \alpha^n + q^n = 0$ and $\beta^{2n} + p^n \beta^n + q^n = 0$

$\Rightarrow\ (\alpha^{2n} - \beta^{2n}) + p^n(\alpha^n - \beta^n) = 0$

$\Rightarrow \alpha^n + \beta^n = -p^n$ ...(2)

Now $\dfrac{\alpha}{\beta}$ is a root of $(x^n + 1) + (x+1)^n = 0$

$$\Rightarrow \left(\frac{\alpha^n}{\beta^n} + 1\right) + \left(\frac{\alpha}{\beta} + 1\right)^n = 0$$

$\Rightarrow (\alpha^n + \beta^n) + (\alpha + \beta)^n = 0 \ \Rightarrow\ -p^n + (-p)^n = 0,$

Which holds only if n is an even integer.

28. **(b)** We have $\alpha + \beta = p$ and $\alpha\beta = -(p+q)$.

Also, $(\alpha+1)(\beta+1) = (\alpha+\beta) + \alpha\beta + 1 = 1 - q$

$$\text{Now, } \frac{\alpha^2 + 2\alpha + 1}{\alpha^2 + 2\alpha + q} + \frac{\beta^2 + 2\beta + 1}{\beta^2 + 2\beta + q}$$

$$= \frac{(\alpha+1)^2}{(\alpha+1)^2 + (q-1)} + \frac{(\beta+1)^2}{(\beta+1)^2 + (q-1)}$$

$$= \frac{2(\alpha+1)^2(\beta+1)^2 + (q-1)\{(\alpha+1)^2 + (\beta+1)^2\}}{(\alpha+1)^2(\beta+1)^2 + (q-1)^2 + (q-1)\{(\alpha+1)^2 + (\beta+1)^2\}}$$

$$= \frac{2(1-q)^2 + (q-1)\{(\alpha+1)^2 + (\beta+1)^2\}}{(1-q)^2 + (q-1)^2 + (q-1)\{(\alpha+1)^2 + (\beta+1)^2\}} = 1$$

29. **(b)** We have, $f(-1) < 1 \Rightarrow a - b + c < 1$ ...(1)

$f(1) > -1 \Rightarrow a + b + c > -1 \Rightarrow -a - b - c < 1$ ...(2)

and $f(3) < -4 \Rightarrow 9a + 3b + c < -4$ ...(3)

Now $(1) \times 3 + (3)$ gives, $12a + 4c < -1$ ...(4)

Again $(2) \times 3 + (3)$ gives, $6a - 2c < -1$ ...(5)

Apply $(4) + (5) \times 2,$ we get $24a < -3 \Rightarrow a < -\dfrac{1}{8}$

Hence, a is negative.

Also, $(1) + (2)$ gives $-2b < 2 \Rightarrow b > -1$

30. **(a)** If $k(>0)$ be the common difference then the equation is

$3x^2 - (6a + 10k)x + a(a + 2k) + 2(a + k)(a + 3k) = 0$

$[\because b = a + k, c = a + 2k, d = a + 3k]$

Its discriminant

$D = (6a + 10k)^2 - 4.3.\{a^2 + 2ak + 2a^2 + 8ak + 6k^2\}$

$= 28k^2 > 0.$

Hence, the roots are real and distinct.

$\therefore$ Roots are real and unequal for all k.

ALTERNATIVE SOLUTION:

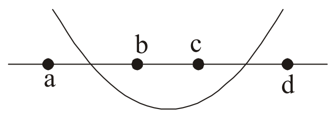
Let $f(x) = (x-a)(x-c) + 2(x-b)(x-d)$

then $f(a) = 2(a-b)(a-d) > 0,$

$f(b) = (b-a)(b-c) < 0$

$f(c) = 2(c-b)(c-d) < 0$

$f(d) = (d-a)(d-c) > 0$



So, $f(x) = 0$ has a root in (a, b) and other in (c, d)

31. **(a)** Since, the roots are imaginary, $D = b^2 - 4ac < 0$

$$\therefore \text{ Roots are } \alpha = \frac{-b + i\sqrt{4ac - b^2}}{2a}$$

$$\text{and } \beta = \frac{-b - i\sqrt{4ac - b^2}}{2a}$$

Clearly $\bar{\beta} = \alpha \Rightarrow |\alpha| = |\bar{\beta}| = |\beta|$

$$\text{Also, } |\alpha| = \sqrt{\frac{b^2}{4a^2} + \frac{4ac - b^2}{4a^2}} = \sqrt{\frac{c}{a}} > 1$$

$[\because c > a > 0]$

$\Rightarrow |\alpha| > 1 \ \Rightarrow\ |\alpha| = |\beta| > 1.$

32. **(d)** Let $f(x) = x^2 + ax + b$. Clearly $f(0) = b \Rightarrow b$ is an integer.

Also $f(1) = 1 + a + b \Rightarrow a$ is an integer [$\because b$ is integer]

$\therefore$ a+b is also an integer

33. **(c)** Let D_1 and D_2 be the discriminants of the equations such that $D_1 = p^2 - 4q$ and $D_2 = r^2 - 4s$

Then

$D_1 + D_2 = p^2 + r^2 - 4(q + s) = p^2 + r^2 - 2pr = (p - r)^2$

$[\because 2(q + s) = pr]$

$\therefore\ D_1 + D_2 \ge 0 \Rightarrow$ at least one of the equations has real roots.

34. **(a)** Since – 5 and 1 are the roots. Product of roots $= -5 \times 1 = b \Rightarrow b = -5$ and

Sum of roots $= -5+1 = -[a^2-5a+b+4]$

$\Rightarrow [a^2-5a-1]=4 \Rightarrow 4 \le a^2-5a-1<5$

$[\because [x]=n \Rightarrow n \le x < n+1]$

$\Rightarrow a^2-5a-5 \ge 0$ and $a^2-5a-6<0$

$\Rightarrow a \le \dfrac{5-\sqrt{45}}{2}$ or $a \ge \dfrac{5+\sqrt{45}}{2}$ and $-1<a<6$

$\Rightarrow -1 < a \le \dfrac{5-3\sqrt{5}}{2}$ or $\dfrac{5+3\sqrt{5}}{2} \le a < 6$

$\Rightarrow a \in \left(-1, \dfrac{5-3\sqrt{5}}{2}\right] \cup \left[\dfrac{5+3\sqrt{5}}{2}, 6\right)$

35. **(b)** Let $y = \dfrac{\sin x \cos 3x}{\sin 3x \cos x}$

$\Rightarrow y(3\sin x - 4\sin^3 x)\cos x = \sin x(4\cos^3 x - 3\cos x)$

$\Rightarrow y(3-4\sin^2 x) = 4\cos^2 x - 3$

$[\because \sin x \ne 0, \cos x \ne 0]$

$\Rightarrow 4(y-1)\sin^2 x + (1-3y) = 0 \Rightarrow \sin^2 x = \dfrac{3y-1}{4(y-1)}$

$\because 0 < \sin^2 x < 1 \Rightarrow 0 < \dfrac{3y-1}{4(y-1)} < 1$

$\Rightarrow \dfrac{3y-1}{4(y-1)} > 0$ and $\dfrac{3y-1}{4(y-1)} - 1 < 0$

$\Rightarrow (3y-1)(y-1) > 0$ and $(y-3)(y-1) > 0$

$\Rightarrow y < \dfrac{1}{3}$ or $y < 1$ and $y<1$ or $y > 3$

$\Rightarrow y < \dfrac{1}{3}$ or $y > 3$

36. **(d)** Let $f(x) = 4x^2 - 16x + \lambda$

then $f(1) > 0, f(2) < 0$

and $f(3) > 0$

$\Rightarrow \lambda > 12,\ \lambda < 16$ and $\lambda > 12$

$\therefore\ 12 < \lambda < 16$

$\therefore$ Integral values of $\lambda = 13, 14, 15$

37. **(d)** Since the equation does not have two distinct roots, therefore, has either equal roots are non-real complex roots. That is, $D \le 0$

$\Rightarrow b^2 - 24a \le 0$. Let $3a+b=\lambda$.

Then $b^2 - 8(\lambda - b) \le 0 \Rightarrow b^2 + 8b - 8\lambda \le 0$ ...(1)

$\because\ b \in \mathbf{R}$, therefore discriminant of (1) must be non-negative.

$\therefore\ 64 + 32\lambda \ge 0 \Rightarrow \lambda \ge -2$

$\therefore$ Least Value of λ and hence of $3a+b$ is -2.

ALTERNATIVELY: Since, $D \le 0 \Rightarrow f(x) \ge 0$ or $f(x) \le 0$ for all real x. But $f(0) = 6 > 0 \Rightarrow f(x) \ge 0$ for all real x. In particular,

$f(3) \ge 0 \Rightarrow 9a + 3b + 6 \ge 0 \Rightarrow 3a + b \ge -2$.

38. **(c)** $\dfrac{\text{absolute term}}{\text{coefficient of } x^2} = \dfrac{2c^2\ell + a^2 m}{a^2 \ell} = 2\left(\dfrac{c}{a}\right)^2 + \dfrac{m}{\ell}$

$= 2\alpha^2\beta^2 - \alpha^4 - \beta^4 = -(\alpha^2 - \beta^2)^2 < 0$

$\therefore$ Coefficient of x^2 and absolute term have opposite signs. So, roots have opposite signs.

39. **(d)** $\because x^2 + 2x + 2 = (x+1)^2 + 1 > 0\ \forall\ x \in \mathbf{R}$

$\therefore\ \dfrac{mx^2 + 3x + 4}{x^2 + 2x + 2} < 5$

$\Rightarrow (m-5)x^2 - 7x - 6 < 0\ \forall\ x \in \mathbf{R}$

$\Rightarrow m - 5 < 0$ and $D < 0 \Rightarrow m < 5$

and $49 + 24(m-5) < 0 \Rightarrow m < \dfrac{71}{24}$

40. **(d)** The given quadratic equation is satisfied by $x=-a, x=-b$ and $x=-c$. Hence the quadratic equation has three roots, which is only possible if it is an identity, hence it has infinite roots.

41. **(c)** The given expression is

$ax^2 + by^2 + cz^2 + 2ayz + 2byz + 2bzx + 2cxy$

$= z^2\left[a\left(\dfrac{x}{z}\right)^2 + b\left(\dfrac{y}{z}\right)^2 + 2c\left(\dfrac{x}{z}\right)\left(\dfrac{y}{z}\right) + 2b\left(\dfrac{x}{z}\right) + 2a\left(\dfrac{y}{z}\right) + c\right]$

$= z^2[aX^2 + bY^2 + 2cXY + 2bX + 2aY + c]$,

where $\dfrac{x}{z} = X$ and $\dfrac{y}{z} = Y$

The expression within the brackets can be resolved into linear factors if

$abc + 2abc - a.a^2 - b.b^2 - c.c^2 = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

42. **(c)** $\because\ x^2 - 4cx + b^2 > 0\ \forall x \in \mathbf{R} \Rightarrow D < 0$

$[\because$ cofficient of $x^2 > 0]$

$\therefore\ 16c^2 - 4b^2 < 0 \Rightarrow 4c^2 - b^2 < 0$ ...(1)

Now, Let

$y = \dfrac{x+a}{x^2 + bx + c^2} \Rightarrow yx^2 + (by-1)x + c^2y - a = 0$

$\because\ x \in \mathbf{R} \Rightarrow D \ge 0 \Rightarrow (by-1)^2 - 4y(c^2y - a) \ge 0$

$\Rightarrow (b^2 - 4c^2)y^2 + (4a - 2b)y + 1 \ge 0$

The coefficient of y^2 is $b^2-4c^2>0$ [from (1)]

The discriminant of the quadratic in LHS of (2) is $(4a-2b)^2-4(b^2-4c^2).1=16(a^2+c^2-ab)<0$

[$\because a^2+c^2<ab$]

Hence the inequality (2) holds for all $y\in\mathbf{R}$.

$\therefore\quad -\infty<y<+\infty \Rightarrow -\infty<\dfrac{x+a}{x^2+bx+c^2}<\infty$

43. **(c)** The equation will have real roots, if $p^2\geq 4q$

If $q=1$, then $p^2\geq 4q$ for $p=2, 3$ and 4

If $q=2$, then $p^2\geq 4q$ for $p=3$ and 4

If $q=3$, then $p^2\geq 4q$ for $p=4$

If $q=4$, then $p^2\geq 4q$ for $p=4$

Hence, total 7 possibilities are there for $p^2\geq 4q$.

$\therefore$ 7 such equations with real roots are possible

44. **(d)** Let $f(x)=(x-a)(x-b)-1$

$\Rightarrow f(a)=-1$ and $f(b)=-1$.

Also, The coefficient $x^2=1>0$.

Hence a and b both lie between the roots of the equation $f(x)=0$.

$\therefore$ The equation $(x-a)(x-b)-1=0$ has one root in $(-\infty, a)$ and other in (b,∞) [$\because b>a$]

Note that $f\left(\dfrac{a+b}{2}\right)<0$

$\therefore f(x)=0$ has distinct real roots.

45. **(b)** $\because c<0$ and coefficient of $x^2=1>0$.

$\therefore$ Roots of the equation are of opposite sign.

$\therefore \alpha<0$ and $\beta>0$ [$\because \alpha<\beta$]

Now the roots of the equation are given by

$$x=\frac{-b\pm\sqrt{b^2-4c}}{2}$$

Obviously, $\alpha=\dfrac{-b-\sqrt{b^2-4c}}{2}$

and $\beta=\dfrac{-b+\sqrt{b^2-4c}}{2}$ [$\therefore b>0, c<0$]

From above it is clear that

$|\alpha|>\beta \Rightarrow \alpha<0<\beta<|\alpha|$

ALTERNATIVELY, $\alpha+\beta=-b<0$

$\Rightarrow$ the negative root has higher magnitude.

46. **(a)** The minimum value of $f(x)=(1+b^2)x^2+2bx+1$

$m(b)=\dfrac{4(1+b^2)-4b^2}{4(1+b^2)}=\dfrac{1}{1+b^2}$ [Minimum value of ax^2+bx+c is $\dfrac{4ac-b^2}{4a}$ if $a>0$]

Now $1+b^2\geq 1$ for all $b \Rightarrow 1\leq 1+b^2<\infty$

$\Rightarrow 1\geq\dfrac{1}{1+b^2}>0$

$\Rightarrow 0<m(b)\leq 1\Rightarrow$ range of $m(b)$ is (0, 1]

47. **(d)** $f(x)=x^2+2bx+c^2 \Rightarrow$ minimum

$f(x)=\dfrac{4.2c^2-4b^2}{4}=2c^2-b^2$

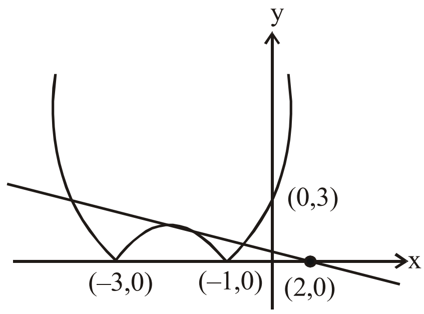
$g(x)=-x^2-2cx+b^2 \Rightarrow$ maximum

$g(x)=\dfrac{-4b^2-4c^2}{4(-1)}=b^2+c^2$

minimum $f(x)>$ maximum $g(x)$

$\Rightarrow 2c^2-b^2>b^2+c^2 \Rightarrow c^2>2b^2 \Rightarrow |c|>|b|\sqrt{2}$

48. **(a)**



If $|x^2+4x+3|=mx-2m$ has exactly three solution, then the curves $y=|x^2+4x+3|$ and $y=m(x-2)$ intersect at exactly three points $\Rightarrow y=m(x-2)$ is tangent to $y=-x^2-4x-3$

$\Rightarrow\quad m(x-2)=-x^2-4x-3$ has equal roots

$\Rightarrow\quad (m+4)^2-4(3-2m)=0$

$\Rightarrow\quad m=-8\pm 2\sqrt{15}$

$\therefore\quad m=-8+2\sqrt{15}\ \left(m\neq -8-2\sqrt{15}\right)$

49. **(c)** Let $f(x) = (x-\alpha)\ (x-\beta)$ ...(1)

Now $f(n)f(n+1)$

$= (n-\alpha)\ \ (n-\beta)\ \ (n+1-\alpha)\ \ (n+1-\beta)$

$= (n-\alpha)\ \ (n+1-\beta)\ \ (n-\beta)\ \ (n+1-\alpha)$

$= \{n(n+1) - n(\alpha+\beta) - \alpha + \alpha\beta\}$

$\{n(n+1) - n(\alpha+\beta) - \beta + \alpha\beta\}$

$= \{n(n+1) + na + b - \alpha\}$

$\{n(n+1) + \ na + b - \beta\} = (m-\alpha)(m-\beta)$

[where $m = n\,(n+1) + an + b$]

50. **(d)** Given $b^2 \le 4ac$, $c^2 \le 4ab$ and $a^2 \le 4ac$

Equality cannot hold simultaneously

[$\because a, b, c$ are different]

$\therefore\ a^2 + b^2 + c^2 < 4(ab + bc + ca) \Rightarrow P < 4$

Also, $a^2 + b^2 + c^2 - ab - bc - ca = \dfrac{1}{2}$

$[(b-c)^2 + (c-a)^2 + (a-b)^2] > 0 \Rightarrow P > 1$

51. **(b)** $\min \left\{ \{x^2 + (a-b)x + 1 - a - b\} = -\dfrac{(a-b)^2 - 4(1-a-b)}{4} \right.$

$\max \{-x^2 + (a+b)x - (1+a+b)\} = -\dfrac{(a+b)^2 - 4(1+a+b)}{-4}$

$\therefore$ From the given condition,

$-\dfrac{(a-b)^2 - 4\ \ (1-a-b)}{4} > \dfrac{(a+b)^2 - 4(1+a+b)}{4}$

$\Rightarrow a^2 + b^2 < 4$

52. **(c)** $x^2 + x - n = 0$,

Discrminent = $1 + 4n$ = odd number = D (say)

Now given equation would have a integral solution if D is a perfect square.

Let $D = (2\lambda + 1)^2$

$\Rightarrow\ n = \lambda + \lambda^2 = \lambda(\lambda + 1) =$ even number

$\Rightarrow\ n$ can be 2, 6, 12, 20, 30, 42, 56, 72, 90.

53. **(a)** Let m be a positive integer for which $n^2 + 96 = m^2$

$\Rightarrow\ \ m^2 - n^2 = 96\ \Rightarrow\ (m+n)(m-n) = 96$

$\Rightarrow\ \ (m+n)\{(m+n) - 2n\} = 96$

$\Rightarrow\ \ m + n$ and $m - n$ must be both even

$96 = 2 \times 48$ or 4×24 or 6×16 or 8×12

Number of solution = 4.

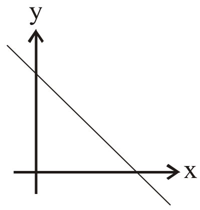
54. **(c)** We have $\alpha > 0$ and

$\alpha + \alpha^2 - 6 < 0 \Rightarrow -3 < \alpha < 2$

$\therefore\ 0 < \alpha < 2.$

So the equation

$x^2 + ax + b = 0$ has a root in $(0, 2)$



55. **(b)** The roots of the equation

$x = \dfrac{-n \pm \sqrt{n^2 - 4n}}{2}$ may be integers if $n^2 - 4n = I^2$

where I is an integer.

$\Rightarrow\ \ n^2 - 4n - I^2 = 0$ ...(1)

$\Rightarrow\ \ n = 2 \pm \sqrt{4 + I^2}$

Now n is an integer.

$\Rightarrow\ \ \sqrt{4 + I^2}$ should be an integer.

$\Rightarrow\ \ 4 + I^2 = k^2$ where k is an integer.

$\Rightarrow\ \ k^2 - I^2 = 4$

which is possible only when $k = \pm 2, I = 0.$

putting $I = 0$ in (1),

$n^2 - 4n = 0\ \Rightarrow\ n = 0, 4$

For both these values,

$x^2 + nx + n = 0$ has integral roots.

$\therefore\ n = 0, 4\ \Rightarrow\ n^2 - 4n = 0.$

56. **(a)** $2^x = \dfrac{a - 3 \pm \sqrt{(a-3)^2 - 4(a-4)}}{2} = a - 4, 1$

It is given that roots are non-positive

i.e. $x \le 0 \Rightarrow 2^x \le 1$

$\Rightarrow 0 < 2^x \le 1 \Rightarrow 4 < a \le 5.$

57. **(b)** The given equation can be written as $x\,(x + n) + m = 0$ If x is a non-integral rational number, then both x and $x + n$ will have the same denominator ($\ne 1$) and $x\,(x + n)$ will not be an integer.

The sum of a non-integer and an integer can never be zero.

$\therefore$ The given equation can not have non-integral rational roots.

B COMPREHENSION TYPE

1. **(c)** $\because\ \ x^2 + 2x + 2 \ne 0$ for all x, so the curve has no vertical asymptote.

Again $y = \dfrac{x+1}{x^2 + 2x + 2} = \dfrac{1}{(x+1) + \dfrac{1}{x+1}}$

Clearly $y \to 0 + 0$ if $x \to +\infty$ and $y \to 0 - 0$ if $x \to -\infty$

Thus $y = 0$ is a horizontal asymptote.

2. **(d)** $y = \dfrac{x+1}{x(x-3)}$. Points of discontinuity are $x = 0, x = 3$

Now $\lim_{x\to 0^-} y = +\infty$ and $\lim_{x\to 0^+} y = -\infty$

$\lim_{x\to 3^-} y = -\infty$ and $\lim_{x\to 3^+} y = +\infty$

Thus $x = 0$ and $x = 3$ are vertical asymptotes.

3. (a) $\frac{x^2+x+d}{x^2+2x+d} = \frac{3}{2}$ has equal roots $\Rightarrow d = 4$

[We will find the same value of d, if $\frac{x^2+x+d}{x^2+2x+d} = \frac{5}{6}$ has equal roots]

4. (b) If curves intersect, then the y coordinate and therefore the parameters are equal at the point of intersection. Hence the curves intersect where

$at^2 + bt + c_1 = -at^2 - bt - c_2$

$\Rightarrow 2at^2 + 2bt + c_1 + c_2 = 0$ …(1)

The equation has distinct roots if

$4b^2 - 4 . 2a(c_1 + c_2) > 0 \Rightarrow b^2 > 2a(c_1 + c_2)$

5. (a) For the curve S_1, we have $\frac{dy}{dx} = \frac{a}{2at+b}$

For the curve S_2, we have $\frac{dy}{dx} = -\frac{a}{2at+b}$

The curves intersect orthogonally if, at the point of intersection,

$$\frac{a}{2at+b} \times -\frac{a}{2at+b} = -1$$

$$\Rightarrow 4a^2t^2 + 4abt + b^2 - a^2 = 0$$

$$\Rightarrow (2at+b-a)(2at+b+a) = 0$$

$$\Rightarrow t = \frac{a-b}{2a} \text{ or } -\frac{a+b}{2a}$$

If the curve intersect then $t = \frac{a-b}{2a}$ and $t = -\frac{a+b}{2a}$ must satisfy equation (1)

So, for $t = \frac{a-b}{2a}$, from (1) we get

$$\frac{2a(a-b)^2}{4a^2} + \frac{2b(a-b)}{2a} + c_1 + c_2 = 0$$

$$\Rightarrow a^2 - b^2 + 2a(c_1+c_2) = 0$$

Similarly for $t = -\frac{a+b}{2}$

we get $a^2 - b^2 + 2a(c_1 + c_2) = 0$

$\therefore$ The curves intersect orthogonally if $b^2 > 2a(c_1 + c_2)$ and $a^2 - b^2 + 2a(c_1 + c_2) = 0$

6. (c) $\frac{dy}{dx}$ for S_1, S_2 exist if $t \neq -\frac{b}{2a}$

But the equation (1) has equal roots if $b^2 = 2a(c_1 + c_2)$, which are given by $t_1 = t_2 = -\frac{2b}{2.2a} = -\frac{b}{2a}$

So, the curves touch when $t = -\frac{b}{2a}$ and the tangent at point of contact is parallel to y-axis.

At point of contact $t = -\frac{b}{2a}$, so, $y = at = -\frac{b}{2}$

So, the line $y = -\frac{b}{2}$ is the common normal.

[The common tangent can be found by putting $t = -\frac{b}{2a}$ in the expression for x]

9. (a) The equations are

$x^2 - 2p_1x + 1 = 0$ ...(1)

$x^2 - 4p_2x + 2 = 0$ ...(2)

$x^2 - 6p_3x + 3 = 0$ ...(3)

Let α and β be roots of equation (1), β and γ be roots of equation (2) and γ and α be roots of equation (3), then

$\alpha + \beta = 2p_1$ and $\alpha\beta = 1$

$\beta + \gamma = 4p_2$ and $\beta\gamma = 2$

$\gamma + \alpha = 6p_3$ and $\gamma\alpha = 3$

From above

$$\alpha\beta^2\gamma = 2 \Rightarrow \beta^2 = \frac{2}{3};$$

$$\alpha^2\beta\gamma = 3 \Rightarrow \alpha^2 = \frac{3}{2} \text{ and } \alpha\beta\gamma^2 = 6 \Rightarrow y^2 = 6$$

Clearly α, β and γ all three must have same signs, so

$$\alpha = \sqrt{\frac{3}{2}}, \beta = \sqrt{\frac{2}{3}} \text{ and } \gamma = \sqrt{6}$$

or $\alpha = -\sqrt{\frac{3}{2}}, \beta = -\sqrt{\frac{2}{3}}$ and $\gamma = -\sqrt{6}$

10. (a) $f\left(\frac{a^2-3}{a} + x\right) = f\left(\frac{a^2-3}{a} - x\right)$

$\Rightarrow f(x)$ is symmetrical about $x = \frac{a^2-3}{a}$

So, the vertex of the parabola is $\left(\frac{a^2-3}{a}, -\frac{(a^2+3)^2}{a}\right)$

Clearly, discriminant of quadratic $= (a^2+3)^2 > 0$

Equation of the parabola is

$$\left[y + \frac{(a^2+3)^2}{a}\right] = a\left[x - \left(\frac{a^2-3}{a}\right)\right]^2$$

11. **(d)** We have $f(0) = -12a$

$f(1) = (1-2a)(a+6)$

$f(0)\,f(1) < 0 \Rightarrow a \in (-\infty, -6) \cup \left(0, \frac{1}{2}\right)$

12. **(c)** $f(a\lambda) < 0 \Rightarrow a(\lambda - 2)\left(a^2\lambda + 6\right) < 0$

$\Rightarrow -\frac{6}{a^2} < \lambda < 2 \ (\because a > 0)$

C REASONING TYPE

1. **(a)** $f(x) = x^2 + x + 1 \ \Delta = 1 - 4 = -3 < 0$

$\Rightarrow f(x)$ is positive for all $x \in R$ (since $a > 0$)

If $\Delta < 0$ then $ax^2 + bx + c$, a will have same sign for all $x \in R$

Both statement-1 and statement-2 are true and statement-2 is correct explanation of statement-1.

2. **(d)** $x \in (2, 3) \ \ x > 2$ and $x < 3$

$(x-2)(x-3) < 0$

$x^2 - 5x + 6 < 0$

If $\alpha < x < \beta$ and $(\alpha < \beta)$ then $ax^2 + bx + c$ and a will have opposite sign. Statement-1is false statement-2 is true.

3. **(a)** $a \sin x + 1 - 2\sin^2 x = 2a - 7$

$2\sin^2 x - a\sin x + 2a - 8 = 0$

$\sin x = \frac{a \pm \sqrt{a^2 - 16a + 64}}{4} = \frac{a \pm (a-8)}{4} = \frac{a-4}{2}$,

$2 - 1 \le \sin x \le 1 \Rightarrow -2 \le a - 4 \le 2$

$2 \le a \le 6$

So both statement-1 and statement-2 are true and statement-2 is correct reason of statement-1.

4. **(a)** In a triangle sum of two sides greater than the other $\Rightarrow a^2 + 2a + 2a + 3 > a^2 + 3a + 8 \Rightarrow a > 5$. (for positive a, $a^2 + 3a + 8$ is the greatest side)

5. **(c)** Let $f(x) = (x^2 - 4x + 3) + \lambda\,(x^2 - 6x + 8)$

$= (x-1)(x-3) + \lambda\,(x-2)(x-4)$

If $\lambda = 0$ then $f(x) = 0$ has roots 1 and 3

Now $f(1) = 3\lambda, f(2) = -1, f(3) = -\lambda, f(4) = 3$

If $\lambda > 0$ then $f(x) = 0$ has a root in $(1, 2)$ and other in $(3, 4)$.

If $\lambda < 0$ then $f(x) = 0$ has a root in $(2, 3)$ and other in $(-\infty, 1)$ or $(4, \infty)$

Clearly, second statement may not be always true, for example if $f(x) = (x-1)(x-2) + \lambda\,(x-3)(x-4)$ then nature of roots will depend on values of λ.

6. **(a)** The equation can be rewritten as

$(x+1)\,[x\,(a+2) + a] = 0$

For both roots integer $\frac{a}{a+2}$ must be integer

$\Rightarrow \ a = 0, -1, -3, -4$

7. **(d)** If $q^2 - 4pr < 0$, then $px^2 - qx + r \ne 0$ for any x implies that the curve has no vertical asymptote but it can have other asymptotes.

D MULTIPLE CORRECT CHOICE TYPE

1. **(b,c,d)** If $f(x) = Ax^2 - |G|\,x - H$, then $f(0) = -H < 0$ and $f(-1) = A + |G| - H > 0$. So, $f(x) = 0$ has one root in $(-1, 0)$ hence the equation has a negative fraction root. Also, $f(2) = 4A - 2|G| - H = 2\,(A - |G|) + (A - H) + A > 0$. So, $f(x) = 0$ has one root in $(0, 2)$, hence the equation has a positive root, which cannot exceed 2.

2. **(a,b,c,d)** $a > b > c$ ...(1)

and given equation is

$$(a + b - 2c)\,x^2 + (b + c - 2a)\,x + (c + a - 2b) = 0 \quad ...(2)$$

$\because$ Equation (2) has a root in the interval $(-1, \ 0)$

$\therefore \ f(-1)\,f(0) < 0$

$\Rightarrow \ (2a - b - c)(c + a - 2b) < 0$ ...(3)

From (1), $a > b \ \Rightarrow a - b > 0$ and

$a > c \ \Rightarrow a - c > 0 \ \therefore \ 2a - b - c > 0$ ...(4)

From (3) and (4), $c + a - 2b < 0$ or $c + a < 2b$. Option (a) is correct. Again, the sum of coefficients of the equation = 0, that is one root is 1and the other root is $\frac{c + a - 2b}{a + b - 2c}$, which is a rational number as a, b, c are rational. Hence, both the roots of the equation are rational .

$\Rightarrow$ (b) is correct. Further, the discriminate of equation $ax^2 + 2bx + c = 0$ is $D = 4b^2 - 4ac$.

As deduced earlier, $c + a < 2b$

$\Rightarrow 4b^2 > (c + a)^2$

$\Rightarrow 4b^2 > c^2 + a^2 + 2ac$

$\Rightarrow 4b^2 - 4ac > c^2 + a^2 - 2ac$

$= (c-a)^2 \Rightarrow 4b^2 - 4ac > 0 \Rightarrow D > 0$. Also, each of a,b,c are positive.

$\therefore$ The equation $ax^2 + 2bx + c = 0$ has real and negative roots. So (c) is also correct. Similarly (d) is also correct.

3. **(b,c)** Let $f(x) = (x-a)(x-c) + \lambda(x-b)(x-d)$. Now $f(a) = \lambda(a-b)(a-d)$ and $f(c) = \lambda(c-b)(c-d)$

$\therefore f(a)f(c) = \lambda^2(a-b)(c-d)(c-b)(c-d) < 0$

[Since $a < b < c < d$]

Hence, $f(x) = 0$ has one real root between a and c.

Again $f(b)f(d) = (b-a)(b-c)(d-a)(d-c) < 0$.

Hence $f(x) = 0$ has one real root between b and d.

4. **(b,c)** The given equation is,

$$\pi^e(x-\pi)(x-\pi-e) + e^\pi(x-e)(x-\pi-e) + (\pi^\pi + e^e)(x-e)(x-\pi) = 0$$

Let

$$f(x) = \pi^e(x-\pi)(x-\pi-e) + e^\pi(x-e)(x-\pi-e) + (\pi^\pi + e^e)(x-e)(x-\pi)$$

Then $f(e) = \pi^e(e-\pi)(-\pi) > 0$ [$\because e < \pi$]

and $f(\pi) = e^\pi(\pi - e)(-e) < 0$

$\therefore$ Equation $f(x) = 0$ has a real root in (e, π).

Again $f(\pi+e) = (\pi^\pi + e^e)(\pi)(e) > 0$.

$\therefore$ Equation $f(x) = 0$ has a real root in $(\pi, e+\pi)$.

$\therefore f(x) = 0$ has a real roots in (e, π)

and other in $(\pi, \pi + e)$

Also, $\pi - e < e$

$\therefore$ Equation $f(x) = 0$ has two real roots in $(\pi - e, \pi + e)$.

5. **(b,c)** If $x \ge a$, then $|x-a| = x - a$, and the equation becomes $x^2 - 2ax - a^2 = 0 \Rightarrow x = a(1 \pm \sqrt{2})$

$\because a < 0$; $\therefore a(1+\sqrt{2}) < a$,

hence $x = a(1+\sqrt{2})$ cannot be the solution.

$\therefore x = a(1-\sqrt{2}) > a$ is the solution, which is positive.

Again if $x < a$, then $|x-a| = -(x-a)$

and the equation becomes

$x^2 + 2ax - 5a^2 = 0 \Rightarrow x = (-1 \pm \sqrt{6})a$

Again as $a < 0$; $\therefore a(-1-\sqrt{6}) > 0 > a$,

hence $x = a(-1-\sqrt{6})$ cannot be the solution.

$\therefore x = a(-1+\sqrt{6}) < a$ is the solution, which is negative.

6. **(a,b,c,d)** The given equation is

$(1+\lambda)x^2 - (6+4\lambda)x + 8 + 3\lambda = 0$

Its discriminant

$D = (6+4\lambda)^2 - 4(1+\lambda)(8+3\lambda) = 4(\lambda^2 + \lambda + 1)$

$$= 4\left[\left(\lambda + \frac{1}{2}\right)^2 + \frac{3}{4}\right] > 0, \ \forall \lambda \in \mathbf{R}$$

$\therefore$ Roots are real and unequal for all λ.

7. **(a,b,d)** We first assume that the discriminant is a perfect square. That is $b^2 - 4ac = d^2$, d is an odd integer as a, b, c are odd. Let $a = 2k+1$, $b = 2m+1$, $c = 2n+1$, $d = 2p+1$. Then

$(2m+1)^2 - 4(2k+1)(2n+1) = (2p+1)^2$

$\Rightarrow (2m+1)^2 - (2p+1)^2 = 4(2k+1)(2n+1)$

$\Rightarrow (2m-2p)(2m+2p+2) = 4(2k+1)(2n+1)$

$\Rightarrow (m-p)(m+p+1) = (2k+1)(2n+1)$

Now for any integral value of m and p, the LHS of above is always even but RHS is always odd, which is a contradiction. Hence, the discriminant cannot be perfect square. So the roots cannot be rational and hence cannot be integer also. Clearly discriminant cannot be zero.

8. **(b,d)** The roots are non-real complex,

$\Rightarrow$ discriminant, $b^2 - 4ac < 0$

Hence $f(x) = ax^2 + bx + c$ must have the same sign for all real x. Now $f(-1) = a - b + c < 0$ (given condition)

Thus $f(x) < 0$ for all $x \in \mathbf{R}$. In particular $f(-2) < 0$.

That is, $4a - 2b + c < 0 \Rightarrow 4a + c < 2b$

Also, $f\left(-\frac{1}{2}\right) < 0 \Rightarrow a - 2b + 4c < 0$

9. **(a,b,c)** The discriminant

$D = 4(a-1)^2 - 4(a+5) = 4(a+1)(a-4)$

The roots of the equation are real, if

$D \ge 0 \Rightarrow (a+1)(a-4) \ge 0 \Rightarrow a \le -1$ or $a \ge 4$

$\Rightarrow a \in (-\infty, -1] \cup [4, \infty)$.

The roots are positive if $D \ge 0$, $a + 5 > 0$ and

$a - 1 < 0$ [$\because$ coefficient of x^2 is $1 > 0$]

$\Rightarrow (a+1)(a-4) \ge 0$, $a + 5 > 0$ and

$a - 1 < 0 \Rightarrow a \le -1$ or $a \ge 4$ and $a > -5$ and

$a < 1 \Rightarrow a \in (-5, -1]$

The roots are of opposite sign, if

$a + 5 < 0 \Rightarrow a < -5$

The roots are negative, if $D \geq 0$, $a+5>0$ and $a-1>0$

$\Rightarrow a \leq -1$ or $a \geq 4$, $a>-5$ and

$a>1 \Rightarrow a \geq 4 \Rightarrow a \in [4, \infty)$

10. (a,d) Discriminant $D = 4p^2 - 4(p^2-1) = 4 > 0$

$\because$ Roots of the equation are real and distinct

Now both the roots are less than 4 if

$D \geq 0$, $f(4) > 0$ and $4 > -\frac{-2p}{2}$

$\Rightarrow 16 - 8p + p^2 - 1 > 0$ and

$4 > p \Rightarrow (p-3)(p-5) > 0$ and $p < 4$

$\Rightarrow p < 3$ or $p > 5$ and $p < 4 \Rightarrow p \in (-\infty, 3)$

Again both the roots are greater than

-2 if $D \geq 0$, $f(-2) > 0$ and $-2 < -\frac{-2p}{2}$

$\Rightarrow (4 + 4p + p^2 + 1) > 0$ and

$3 < p \Rightarrow (p+3)(p+1) > 0$ and $p > -3$

$\Rightarrow p < -3$ or $p > -1$ and $p > -3 \Rightarrow p \in (-1, \infty)$

Further exactly one root lies in the interval $(-2, 4)$

if $D > 0$ and $f(-2)f(4) < 0$

$\Rightarrow (p+3)(p+1)(p-3)(p-5) < 0$

$\Rightarrow p \in (-3, -1) \cup (3, 5)$

Finally, 1 lies between the roots if $D > 0$ and $f(1) < 0$

$\Rightarrow 1 - 2p + p^2 - 1 < 0 \Rightarrow p(p-2) < 0$

$\Rightarrow 0 < p < 2 \Rightarrow p \in (0, 2)$

Alternatively :

$x^2 - 2px + p^2 - 1 = 0 \Rightarrow (x-p)^2 = 1$

$\therefore x = p \pm 1$

Both the roots are less than 4 if $p+1 < 4$ and $p-1 < 4 \Rightarrow p < 3$

Both the roots are greater than -2 if $p+1 > -2$ and $p-1 > -2 \Rightarrow p > -1$

Exactly one root lies in $(-2, 4)$ if $-2 < p+1 < 4$ or $-2 < p-1 < 4$ but not both

$\Rightarrow p \in (-3, -1) \cup (3, 5)$

One root is less than 1 and other greater than 1 if $p+1 < 1 < p-1$ or $p-1 < 1 < p+1 \Rightarrow 0 < p < 2$

NOTE : *The alternate method is easier than the general method, so if the roots of quadratic in terms of parameter come out to be free of radical the alternative method is better.*

11. (a,c) We have $a\alpha^2 + b\alpha + c = 0$ ...(1)

and $-a\beta^2 + b\beta + c = 0$ ...(2)

Now, Let $f(x) = \frac{a}{2}x^2 + bx + c$

Then

$f(\alpha) = \frac{a}{2}\alpha^2 + b\alpha + c = \frac{a}{2}\alpha^2 - a\alpha^2 = -\frac{a}{2}\alpha^2$

[from (1)]

and $f(\beta) = \frac{a}{2}\beta^2 + b\beta + c = \frac{a}{2}\beta^2 + a\beta^2 = \frac{3}{2}a\beta^2$

[from (2)]

$\therefore f(\alpha)f(\beta) = -\frac{a}{2}\alpha^2 . \frac{3}{2}a\beta^2 = -\frac{3}{4}a^2\alpha^2\beta^2 < 0$

$\therefore$ $f(x) = 0$ has a real root between α and β.

12. (b,c) Let $\frac{x^2 + ax + 3}{x^2 + x + a} = y$

$\Rightarrow x^2(1-y) - x(y-a) + 3 - ay = 0$

$\because$ $x \in \mathbf{R} \Rightarrow (y-a)^2 - 4(1-y)(3-ay) \geq 0$

$\Rightarrow (1-4a)y^2 + (2a+12)y + a^2 - 12 \geq 0$ ...(1)

(1) is true for all $y \in \mathbf{R}$, if $1 - 4a > 0$ and $D \leq 0$.

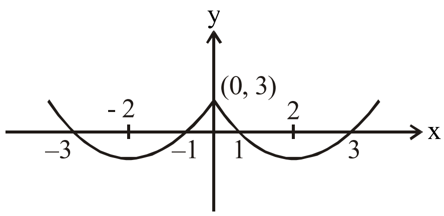
$\Rightarrow$ $a < \frac{1}{4}$ and $4(a+6)^2 - 4(a^2-12)(1-4a) \leq 0$

$\Rightarrow$ $a < \frac{1}{4}$ and $4a^3 - 36a + 48 \leq 0 \Rightarrow a < \frac{1}{4}$

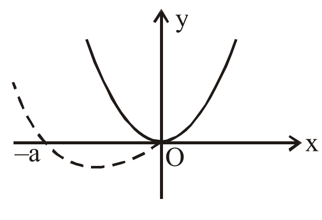
and $4a^3 \leq 36a - 48$ $\Rightarrow$ $4a^3 < 36\left(\frac{1}{4}\right) - 48$

$\Rightarrow$ $4a^3 + 39 < 0$ $\left[\because a < \frac{1}{4}\right]$

13. (a, d) $x^2 - 4|x| + 3 = |k-1|$ has exactly four distinct roots if $|k-1| < 3 \Rightarrow -2 < k < 4$



14. (a, d)



Clearly one root of $x^2 + ax + b = 0$ will be zero and other root will be negative. So, $b = 0$ and $a > 0$

15. (a, b) The vertex $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ lies in the third quadrant and $f(0) < 0$

$\therefore -\frac{b}{2a} < 0,\ -\frac{D}{4a} < 0$ and $c < 0$. Also $a > 0$

$\therefore a > 0, \quad b > 0$ and $c < 0$

16. (a, b) Clearly x can not be an odd integer, otherwise $x^2 + 2px + 2q \neq 0$

If x is an even integer then $x^2 + 2px$ is a multiple of 4 but $2q$ is not so $x^2 + 2px + 2q \neq 0$

$\therefore$ Equation has no integral roots. Again $x^2 + 2px + 2q = 0 \Rightarrow (x+p)^2 = p^2 - 2q \Rightarrow x$ cannot be fractional either.

ALTERNATIVELY :

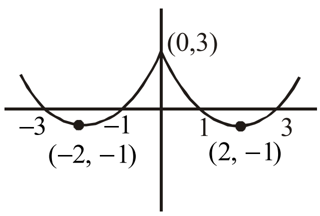
Discriminant $D = 4p^2 - 8q$. If D is a perfect square then $4p^2 - 8q = (2k)^2$ for some $k \in I$

$\therefore\ p^2 - k^2 = 2q$, which is not possible as LHS is a multiple of 8 or an odd integer.

17. (a,b,d) $x^2 - 4|x| + 3 = k - 1$

Clearly for four distinct roots

$-1 < k - 1 < 3 \ \Rightarrow 0 < k < 4$



18. (b,d) The equation $ax^2 + bx + c = 0$ must have roots of opposite sign

$\Rightarrow$ a and c are of opposite sign. If $x = 0$ is one of the roots of $ax^2 + bx + c = 0$ then later equation will have three roots.

19. (b,d) Given, $x^2 - 2x + \sin^2\alpha \ \Rightarrow \sin^2\alpha = 2x - x^2$

We know that $-1 \le \sin\alpha \le 1$

$\therefore\ 0 \le \sin^2\alpha \le 1$ or $0 \le 2x - x^2 \le 1$

If $2x - x^2 \ge 0$ then $0 \le x \le 2$

If $2x - x^2 \le 1$ then $(x-1)^2 \ge 0$, which is true

$\therefore\ 0 \le x \le 2$

20. (a,b,c,d) We have $a > 0$ and $b^2 - 4ac < 0$. Obviously, $f(-2) > 0 \Rightarrow 4a - 2b + c > 0$

Now consider,

$g(x) = f(x) + f'(x) + f''(x)$

$\quad = ax^2 + (2a+b)x + (2a+b+c)$

Discriminant of $g(x)$

$= (2a+b)^2 - 4a(2a+b+c) = b^2 - 4ac - 4a^2 < 0$

$\therefore\ g(x) > 0\ \forall\, x \in R \Rightarrow g(-2) > 0, g(2) > 0$ and $g(0) > 0$

21. (a,b,c,d) Let $f(x) = ax^2 + bx + c$

Its discriminant $D = b^2 - 4ac < 0$

And $f(-1) = a - b + c > 0$

So, $f(x) > 0$ for all $x \in R$

So, $a > 0$

$f(0) > 0 \Rightarrow c > 0$

$f(1) > 0 \Rightarrow a + b + c > 0$

$f(-2) > 0 \Rightarrow 4a - 2b + c > 0$

22. (a,b,d) If $D_1 = b^2 - 4ac < 0, D_2 = b^2 - 4ac < 0$, as the root is non-real

$\Rightarrow$ Both roots will be common.

$\Rightarrow \frac{a}{c} = \frac{b}{b} = \frac{c}{a} = 1 \Rightarrow a = c$

Now, $b^2 - 4ac < 0$

$\Rightarrow b^2 - 4a^2 (\text{or } 4c^2) < 0 \Rightarrow |b| < 2|a| \,(or\, 2|c|)$.

23. (a,b,c,d) Let $f(x) = cx^2 + bx - 2a$

$\frac{b+c}{2} > a \Rightarrow b + c - 2a > 0 \Rightarrow f(1) > 0$

Now $f(x) = 0$ has no real root.

Therefore, $f(x) > 0$ for all x or $f(x) < 0$

But $f(1) > 0$. Therefore f (x) > 0 for all x

$\Rightarrow \quad f(0) > 0 \Rightarrow a < 0$

Clearly, $c > 0$.

$\Rightarrow \quad ac < 0$. Also $f(-1) > 0 \Rightarrow c - b > 2a$

Similarly, $f(1/2) > 0 \Rightarrow \frac{c+2b}{8} > a$.

24. (a,b,c) Let $f(x) = ax^2 + bx + c$

– 3 and 3 are lying between the roots of $f(x) = 0$

$\Rightarrow \quad f(-3) > 0$ and $f(3) > 0$

$\Rightarrow \quad 9a + 3|b| + c > 0$

Also, 0, – 2, 2 are lying between the roots.

therefore $c > 0$ and $4a + 2|b| + c > 0$

25. (b, d) Let β be the other root

$\Rightarrow \quad \alpha + \beta = -\frac{1}{2} \Rightarrow \beta = -\frac{1}{2} - \alpha$

and $4\alpha^2 + 2\alpha - 1 = 0$

$\Rightarrow 4\alpha^2 = -2\alpha + 1 \Rightarrow 4\alpha^3 = -2\alpha^2 + \alpha$

$\Rightarrow \quad 4\alpha^3 - 3\alpha = -\frac{1}{2} - \alpha = \beta$

26. (a, b, c) $f(0) = c > 0, f(-2) = 4a - 2b + c < 0$.

$\Rightarrow \quad f(0).f(-2) < 0$

$\Rightarrow$ One root lie in the interval (– 2, 0).

$\Rightarrow$ Roots are real.

Product of roots $= \frac{c}{a} > 0$ if $a > 0$.

$\Rightarrow$ Both the roots are negative

$\Rightarrow$ Sum of root < 0

$\Rightarrow -\frac{b}{a} < 0 \Rightarrow \frac{b}{a} > 0 \Rightarrow$ if $a > 0$ then $b > 0$

27. (a, c, d) Let $f(x) = (2x-a)(3x-b) + (3x-b)(4x-c) + (4x-c)(2x-a) = 0$

$f\left(\frac{a}{2}\right) = \left(\frac{3a}{2} - b\right)(2a-c) > 0$ since $\frac{a}{2} < \frac{b}{3} < \frac{c}{4}$

$f\left(\frac{b}{3}\right) = \left(\frac{4b}{3} - c\right)\left(\frac{2b}{3} - a\right) < 0$

since $f\left(\frac{a}{2}\right) f\left(\frac{b}{3}\right) < 0$

$\Rightarrow$ one real root lies between $\left(\frac{a}{2}, \frac{b}{3}\right)$

$\Rightarrow$ both roots are real

Also, $f\left(\frac{c}{4}\right) > 0 \Rightarrow f\left(\frac{b}{3}\right) f\left(\frac{c}{4}\right) < 0$

$\Rightarrow$ other root lies between $\left(\frac{b}{3}, \frac{c}{4}\right)$.

28. (a, c) Let $f(x) = x^2 - (k-1)x + k(k+4)$

$f(1) < 0$ or $f(k+2) < 0$

$\Rightarrow 1 - (k-1) + k(k+4) < 0$ or

$(k+2)^2 - (k-1)(k+2) + k(k+4) < 0$

$k^2 + 3k + 2 < 0$ or $k^2 + 7x + 6 < 0$

$\Rightarrow k \in (-2, -1)$ or $k \in (-6, -1)$

29. (b, c, d) Let $f(x) = ax^2 + 8x - (20 + 4a)$

$f(2) = 4a + 16 - (20 + 4a) = -4 < 0$

$f(0) < 0 \Rightarrow (20 + 4a) > 0 \Rightarrow a > -5$ ...(1)

If $f(x) < 0\ \forall x \in [0, 2] \Rightarrow a > 0$ and $D > 0$ ...(2)

$D > 0 \Rightarrow 64 + 4a(20 + 4a) > 0$

$\Rightarrow 8a^2 + 5a + 4 > 0$

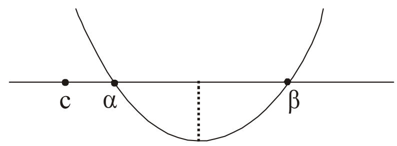
or $a \in (-\infty, -4) \cup (-1, \infty)$ ...(3)

From (1), (2) and (3) we get $a \in (0, \infty)$

Also if $a = 0$, $f(x) = 8x - 20 < 0 \Rightarrow x < \frac{5}{2}$

$\therefore a \in [0, \infty)$

30. (a, b, c) Discriminant > 0 $\Rightarrow a^4 > 4b^2$



$c < -\frac{a^2}{2}$ and $f(c) > 0$,

where $f(x) = x^2 + a^2x + b^2 \Rightarrow c^2 + a^2c + b^2 > 0$

31. (a, b) Let α be the common root

$\Rightarrow a\alpha^2 + 2b\alpha + c = 0$ and $a\alpha^2 + 2ap^2\alpha + a = 0$

$\Rightarrow 2(ap^2 - b)\alpha = (c - a) \Rightarrow \alpha = \frac{c-a}{2(ap^2 - b)}$

and since a, b, c are A.P. Let d be the common difference

$\Rightarrow \alpha = \left(\frac{d}{ap^2 - b}\right)$

$\Rightarrow \left(\frac{d}{ap^2 - b}\right)^2 + 2p^2\left(\frac{d}{ap^2 - b}\right) + 1 = 0$

$\Rightarrow d^2 + 2p^2(ap^2 - b)d + (ap^2 - b)^2 = 0$

$\Rightarrow p^2 = \frac{b^2 + d^2}{b^2 - d^2} \Rightarrow \alpha = -\frac{c}{a}$.

Let the other root be

$\beta \Rightarrow \alpha\beta = 1 \Rightarrow \beta = -\frac{a}{c}$.

E MATRIX-MATCH TYPE

1. A - t; B - s; C - p; D - q

(A) $7x^2 - 8x + 9 < 0 \Rightarrow b^2 - 4ac = 64 - 4 \times 7 \times 9 < 0 \Rightarrow x \in \phi$

(B) $2x^2 - 4x + 5 > 0 \Rightarrow b^2 - 4ac = 16 - 4 \times 10 < 0 \Rightarrow x \in R$

(C) $x^2 - 4x + 4 = (x-2)^2 > 0 \Rightarrow x \in R - \{2\}$

(D) $(x+1)(x-6) < 0 \Rightarrow x \in (-1, 6)$

2. A - r, t; B - p, r; C - s; D - r, t

(A) $\because b$ is AM of a and c. $\Rightarrow b > \sqrt{ac} \Rightarrow b^2 - ac > 0$

$\Rightarrow$ The equation has real and distinct roots. Also, a, b, c are positive so the roots are negative.

(B) $f(0) = -a^2 - 4 < 0 \Rightarrow f(x) = x^2 - (a+1)x - a^2 - 4 = 0$ has roots of opposite sign.

(C) $\because b$ is HM of a and $c \Rightarrow b < \sqrt{ac} \Rightarrow b^2 - ac < 0$

$\Rightarrow$ The equation has imaginary roots.

(D) Discriminant of the equation is

$D = (b^2 + a^2 - c^2)^2 - 4a^2b^2$

$= \{(a+b)^2 - c^2\}\{(a-b)^2 - c^2\} > 0\ (\because |a \pm b| < c)$

$\therefore$ roots of equation are real and distinct. Also the product of roots > 0 $\Rightarrow$ roots have same sign.

F NUMERIC/INTEGER ANSWER TYPE

1. Ans : 2

Let $y = x+\sqrt{x^2+k^2} = \dfrac{k^2}{\sqrt{x^2+k^2}-x}$

$\Rightarrow \dfrac{k^2}{y} = \sqrt{x^2+k^2}-x$

$\therefore\ 2x = y-\dfrac{k^2}{y}$ ...(1)

Let $z = 2(k-x)\left(x+\sqrt{x^2+k^2}\right)$

$= \left(2k-y+\dfrac{k^2}{y}\right)y$

$= k^2-y^2+2ky = 2k^2-(k-y)^2 \le 2k^2$

$\therefore\ \dfrac{z}{k^2} \le 2$

2. Ans : 6

The inequality is $x^2+|x-a|-3<0$ ...(1)

Let $x \ge a$, then (1) is $x^2+x-(a+3)<0$. It will have a solution if $D>0 \Rightarrow 1+4(a+3)>0 \Rightarrow a>-\dfrac{13}{4}$ and the solution is $\dfrac{-1-\sqrt{4a+13}}{2}<x<\dfrac{-1+\sqrt{4a+13}}{2}$. Clearly it contains negative values of x.

Let $x<a$, then (1) is $x^2-x+a-3<0$. It will have a solution, if $D>0 \Rightarrow 1-4(a-13)>0 \Rightarrow a<\dfrac{13}{4}$ and the solution is $\dfrac{1-\sqrt{13-4a}}{2}<x<\dfrac{1+\sqrt{13-4a}}{2}$

It will have a negative value of x, if $\dfrac{1-\sqrt{13-4a}}{2}<0$

$\Rightarrow 1-\sqrt{13-4a}<0 \Rightarrow \sqrt{13-4a}>1 \Rightarrow 13-4a>1 \Rightarrow a<3$

Combining these three value of a, we get $-\dfrac{13}{4}<a<3$.

So, the integral values of a are $-3, -2, -1, 0, 1, 2$.

3. Ans : 1210

Roots of $x^2-10cx-11d=0$ are a and $b \Rightarrow a+b=10c$ and $ab=-11d$

Similarly c and d are the roots of $x^2-10ax-11b=0 \Rightarrow c+d=10a$ and $cd=-11b$

$\Rightarrow a+b+c+d=10(a+c)$ and $abcd=121bd$

$\Rightarrow b+d=9(a+c)$ and $ac=121$

Also we have $a^2-10ac-11d=0$ & $c^2-10ac-11b=0$

$\Rightarrow a^2+c^2-20ac-11(b+d)=0$

$\Rightarrow (a+c)^2-22\times 121-99(a+c)=0 \Rightarrow a+c=121$ or -22

For $a+c=-22$ we get $a=c$

$\therefore$ rejecting this value we have $a+c=121$

$\therefore\ a+b+c+d=10(a+c)=1210$

4. Ans : 13

$y+z=5-x$ and $(y+z)x+yz=3$

$\Rightarrow yz=3-x(5-x)=3-5x+x^2$

Now, $(y-z)^2\ge 0 \Rightarrow (5-x)^2-4(3-5x+x^2)\ge 0$

$\Rightarrow 3x^2-10x-13\le 0$

$\Rightarrow (x+1)(3x-13)\le 0 \Rightarrow -1\le x\le \dfrac{13}{3}$

5. Ans : 880

a = sum of the roots = odd

$\Rightarrow$ Both roots cannot be odd

$\therefore$ one root must be 2. So $4-2a+b=0$

and $a+b=35$ (given)

On solving we get, $a=13, b=22$

So, $f(x)=x^2-13x+22$

$f(1)+f(2)+.......+f(10)$

$=(1^2+2^2+.......+10^2)-13(1+2+........+10)+22\times 10$

$=\dfrac{10\times 11\times 21}{6}-13\times\dfrac{10\times 11}{2}+220=-110$

Also, $f(10)=-8$

$\Rightarrow [f(1)+f(2)+f(3)+..........+f(10)]\,f(10)=880$

6. Ans : 2

$ax^2+bx+c=0$ has roots α and β

$\Rightarrow\ \alpha+\beta=-\dfrac{b}{a},\ \alpha\beta=\dfrac{c}{a}.$

If the roots of equation $a^5x^2+ba^2c^2x+c^5=0$ are γ and δ, then

$\gamma+\delta=-\dfrac{b}{a}\left(\dfrac{c}{a}\right)^2=(\alpha+\beta)\alpha^2\beta^2=\alpha^3\beta^2+\alpha^2\beta^3$

Clearly roots are $\alpha^3\beta^2$ and $\alpha^2\beta^3$

$\Rightarrow\ \alpha^5\beta^5=32 \Rightarrow\ \alpha\beta=2$