JEE Main
4th September 2020 : Shift - 1
Memory Based Question

PART : MATHEMATICS

1. In a group 63 % people read news paper A while 76% people read news paper B. If x% people read both A and B then x may be

(1) 37% (2) 68% (3) 29% (4) 55%

Ans. **(4)**

Sol. $n(A) = 63\%$

$n(B) = 76\%$

$n(A \cap B) = x\%$

Let $n(\cup) = 100$

$n(A) = 63,\ n(B) = 76,\ n(A \cap B) = x$

$n(A \cup B) = n(A) + n(B) - n(A \cap B) \le 100$

$63 + 76 - x \le 100$

$x \ge 39$

but $n(A \cap B) \le n(A)$ $\therefore$ $39 \le x \le 63$

2. If $f(x) = \int \frac{\sqrt{x}}{(1+x)^2}\,dx$, then find the value of $f(3) - f(1)$

(1) $\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$ (2) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

(3) $\frac{\pi}{12} + \frac{1}{3} - \frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{12} + \frac{1}{4} - \frac{\sqrt{3}}{4}$

Ans. **(2)**

Sol. $f(x) = \int \frac{\sqrt{x}}{(1+x)^2}\,dx$

Let $x = \tan^2\theta$

$dx = 2\tan\theta \sec^2\theta\, d\theta$

$f(x) = \int \frac{\tan\theta}{(1+\tan^2\theta)^2}.2\tan\theta\sec^2\theta d\theta$

$f(x) = \int \frac{\tan\theta}{\sec^4\theta}.2\tan\theta\sec^2\theta d\theta$

$f(x) = \int 2\tan^2\theta.\cos^2\theta d\theta$

$f(x) = \int 2\sin^2\theta d\theta$

$f(x) = \int (1-\cos 2\theta) d\theta$

$f(x) = \theta - \frac{\sin 2\theta}{2} + C = \theta - \frac{\tan\theta}{1+\tan^2\theta} + C$

$f(x) = \tan^{-1}\sqrt{x} - \frac{\sqrt{x}}{1+x} + C$

now $f(3) - f(1) = \tan^{-}\sqrt{3} - \frac{\sqrt{3}}{1+3} - \tan^{-1}\sqrt{1} + \frac{1}{1+1}$

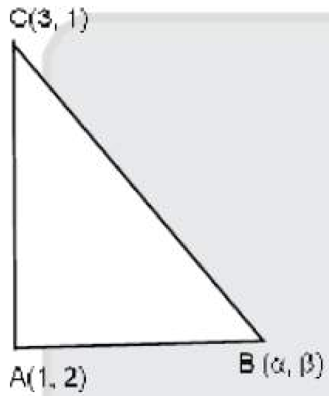
$= \frac{\pi}{3} - \frac{\pi}{4} + \frac{1}{2} - \frac{\sqrt{3}}{4} = \frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

3. Let ΔABC is a right angled triangle right angled at A such that A(1, 2), C(3, 1) and area of $\Delta ABC = 5\sqrt{5}$ then abscissa of B can be

(1) $1 + 5\sqrt{2}$ (2) $1 + 2\sqrt{5}$ (3) $1 - 5\sqrt{2}$ (4) $3 + 2\sqrt{5}$

Ans. **(2)**

Sol.



$m_{AB} = \dfrac{\beta - 2}{\alpha - 1}$

$m_{AC} = \dfrac{2-1}{1-3} = -\dfrac{1}{2}$

$AB \perp AC \qquad \therefore \qquad \dfrac{\beta-2}{\alpha-1}\left(-\dfrac{1}{2}\right) = -1$

$\beta = 2\alpha - 2 + 2 \quad \Rightarrow \quad \beta = 2\alpha$

Now area of $\Delta ABC = 5\sqrt{5} = \dfrac{1}{2}$ AB.AC

$\Rightarrow \qquad \dfrac{1}{2}\sqrt{(3-1)^2 + (1-2)^2}.\sqrt{(\alpha-1)^2 + (\beta-2)^2} = 5\sqrt{5}$

$\Rightarrow \qquad \sqrt{(\alpha-1)^2 + (2\alpha-2)^2} = 10$

$\Rightarrow \qquad \sqrt{(\alpha-1)^2}\sqrt{5} = 10 \qquad \Rightarrow \qquad |\alpha - 1| = 2\sqrt{5} \quad \Rightarrow \qquad \alpha = 1 \pm 2\sqrt{5}$

4. Let $f(x) = |x - 2|$ and $g(x) = f(f(x))$, $x \in [0,4]$, then $\int_0^3 (g(x) - f(x))\,dx =$

(1) 1 (2) 2 (3) 3 (4) 4

Ans. **(1)**

Sol. $f(x) = |x-2| = \begin{cases} 2-x & x<2 \\ x-2 & x \ge 2 \end{cases}$

$g(x) = f(f(x)) = \begin{cases} 2-f(x) & f(x)<2 \\ f(x)-2 & f(x) \ge 2 \end{cases}$

$= \begin{cases} 2-(2-x) & 2-x<2, \quad x<2 \\ (2-x)-2 & 2-x \ge 2, \quad x<2 \\ 2-(x-2) & x-2<2, \quad x \ge 2 \\ (x-2)-2 & x-2 \ge 2, \quad x \ge 2 \end{cases}$

$= \begin{cases} x & 0<x<2 \\ -x & x \le 0 \\ 4-x & 2 \le x < 4 \\ x-4 & x \ge 4 \end{cases}$

$\int_0^3 (g(x)-f(x))dx = \int_0^2 x dx + \int_2^3 (4-x)dx - \int_0^3 |x-2| dx = 1$

5. $\sum_{r=0}^{20} {}^{50-r}C_6 =$

(1) ${}^{51}C_7 - {}^{30}C_6$ (2) ${}^{51}C_6 - {}^{30}C_6$ (3) ${}^{51}C_7 - {}^{30}C_7$ (4) ${}^{51}C_6 - {}^{30}C_7$

Ans. **(3)**

Sol. $\sum_{r=0}^{20} {}^{50-r}C_6 =$

$= {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + \ldots\ldots + {}^{30}C_6 = -{}^{30}C_7 + {}^{30}C_7 + {}^{30}C_6 + {}^{31}C_6 + {}^{32}C_6 + \ldots\ldots + {}^{50}C_6$

$= -{}^{30}C_7 + {}^{31}C_7 + {}^{31}C_6 + {}^{32}C_6 + \ldots\ldots + {}^{50}C_6$

$= -{}^{30}C_7 + {}^{32}C_7 + {}^{32}C_6 + \ldots\ldots + {}^{50}C_6$

$= -{}^{30}C_7 + {}^{51}C_7$

6. Let $x\frac{dy}{dx} - y = x^2$ (x cosx + sinx) is a differential equation. If f(π) = π then $f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) =$

(1) $\frac{\pi}{2}+2$ (2) $\frac{\pi}{2}-2$ (3) $\frac{\pi}{2}+1$ (4) $\frac{\pi}{2}-1$

Ans. (1)

Sol. Given $x\frac{dy}{dx} - y = x^2$ (xcosx + sinx)

$\Rightarrow \frac{dy}{dx} - \frac{1}{x}y = x(x\cos x + \sin x)$ $\therefore$ I.F. = $e^{\int -\frac{1}{x}dx} = e^{-\ell nx} = \frac{1}{x}$

$\therefore$ solution is $y.\frac{1}{x} = \int \frac{1}{x}.x(x\cos x + \sin x)dx + C$

$\frac{y}{x} = \int (x\cos x + \sin x)dx + C$

$\frac{y}{x} = x\sin x + C$, $\frac{\pi}{\pi} = 0 + C$ $\Rightarrow C = 1$

$y = x^2 \sin x + x$

$$\frac{dy}{dx} = x^2\cos x + 2x\sin x + 1$$

$$\frac{d^2y}{dx^2} = -x^2\sin x + 2x\cos x + 2x\cos x + 2\sin x \qquad = -x^2\sin x + 4x\cos x + 2\sin x$$

$$\therefore\ f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = \left(-\frac{\pi^2}{4} + 4.0 + 2\right) + \left(\frac{\pi^2}{4}.1 + \frac{\pi}{2}\right)$$

$$= -\frac{\pi^2}{4} + 2 + \frac{\pi^2}{4} + \frac{\pi}{2}$$

$$= \frac{\pi}{2} + 2$$

7. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be an ellipse such that LR = 10 and its eccentricity is equal to maximum value of quadratic expression $f(t) = \frac{5}{12} + t - t^2$ then $(a^2 + b^2) =$

Ans. **126**

Sol. $LR = \frac{2b^2}{a} = 10 \quad \Rightarrow \quad b^2 = 5a$

$f(t) = \frac{5}{12} - (t^2 - t + \frac{1}{4} - \frac{1}{4}) = \frac{5}{12} + \frac{1}{4} - \left(t - \frac{1}{2}\right)^2$

$= \frac{2}{3} - \left(t - \frac{1}{2}\right)^2$

maxi $f(t) = \frac{2}{3} = e$

$b^2 = a^2(1 - e^2)$

$5a = a^2\left(1 - \frac{4}{9}\right) \Rightarrow \quad 5 = \frac{5}{9}a \quad \Rightarrow \quad a^2 = 81,\ b^2 = 45$

$a^2 + b^2 = 126$

8. If α, β are roots of $x^2 - 3x + p = 0$ and γ, δ are roots of $x^2 - 6x + q = 0$ and $\alpha, \beta, \gamma, \delta$ are in increasing geometric progression then value of $\frac{2q+p}{2q-p}$ is equal to

(1) $\frac{7}{9}$ (2) $-\frac{7}{9}$ (3) $\frac{9}{7}$ (4) $-\frac{9}{7}$

Ans. **(3)**

Sol. $\alpha = a,\ \beta = ar,\ \gamma = ar^2,\ \delta = ar^3$

$\alpha + \beta = 3 \Rightarrow a + ar = 3$ ....(1)

$\gamma + \delta = 6 \Rightarrow ar^2 + ar^3 = 6$ ....(2)

By (1) and (2) $\Rightarrow \frac{ar^2(1+r)}{a(1+r)} = \frac{6}{3} \Rightarrow r^2 = 2$

$\Rightarrow r = \sqrt{2}$

$\therefore \quad \frac{2q+p}{2q-p} = \frac{9}{7}$

9. The mean and variance of 5, 7, 12 , 10, 15, 14, a, b are 10 and 13.5 respectively then value of |a –b| =

(1) 5 (2) 6 (3) 7 (4) 8

Ans. **(3)**

Sol. $\frac{5+7+12+10+15+14+a+b}{8}=10$

$\Rightarrow 63 + a + b = 80 \Rightarrow a + b = 17$ ....(1)

$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

$\Rightarrow 13.5 = \frac{25+49+144+100+225+196+a^2+b^2}{8} - 100$

$908 = a^2 + b^2 + 739$

$a^2 + b^2 = 169$

$(a + b)^2 - 2ab = 169$

$289 - 169 = 2ab \Rightarrow ab = 60$

$\therefore |a - b|^2 = (a + b)^2 - 4ab = 289 - 240 = 49$

$\therefore |a - b| = 7$

10. If $1 + (1 - 2^2 . 1) + (1 - 4^2 . 3) + (1 - 6^2 . 5) + \ldots\ldots + (1 - 20^2 . 19) = \alpha - 220\beta$ then (α, β) =

(1) (11, 97) (2) (11, 103) (3) (10, 97) (4) (10, 103)

Ans. **(2)**

Sol. $S = 1 + \sum_{r=1}^{10} 1 - (2r)^2 (2r - 1) = 1 + 10 - \sum_{r=1}^{10} (8r^3 - 4r^2) = 11 - \left[8\left(\frac{10\times11}{2}\right)^2 - 4.\left(\frac{10\times11\times21}{6}\right)\right]$

$= 11 - [2(110)^2 - 140 \times 11]$

$= 11 - 22(1100 - 70)$

$= 11 - 220(110 - 7)$

$\therefore \quad \alpha - 220\beta$

$= 11 - 220(103)$

$\therefore \quad \alpha = 11, \beta = 103$

$(\alpha, \beta) = (11, 103)$

11. For equation $[x]^2 + 2[x + 2] - 7 = 0$, $x \in R$ number of solution of equation is/are

(1) four integer solution (2) Infinite solution

(3) No solution (4) two solution

Ans. **(2)**

Sol. $[x]^2 + 2[x + 2] - 7 = 0$

$[x]^2 + 2([x] + 2) - 7 = 0$

Let $[x] = t$

$t^2 + 2t - 3 = 0$

$t = 1, -3$

$[x] = -3, 1$

$x \in [-3, -2) \cup [1, 2)$

Hence infinite solution.

12. Integration: $\int \frac{x^2}{(x\sin x + \cos x)^2} dx$ is equal to

(1) $\frac{\sin x + x\cos x}{x\sin x + \cos x} + C$ (2) $\frac{\sin x + x\cos x}{x\sin x - \cos x} + C$

(3) $\frac{x\cos x - \sin x}{\cos x - x\sin x} + C$ (4) $\frac{\sin x - x\cos x}{x\sin x + \cos x} + C$

Ans. **(4)**

Sol. $\int \frac{x^2 \quad dx}{(x\sin x + \cos x)^2} = \int \frac{x}{\cos x} \cdot \frac{x\cos x}{(x\sin x + \cos x)^2} dx$

$= \frac{x}{\cos x}\int \frac{x\cos x}{(x\sin x + \cos x)^2} dx - \int \left[\frac{d}{dx}(x\sec x)\int \frac{x\cos x}{(x\sin x + \cos x)^2} dx\right] dx$

$= \frac{x}{\cos x}\left(-\frac{1}{x\sin x + \cos x}\right) + \int \sec^2 x \quad dx$

$\left(\because \int \frac{x\cos x}{(x\sin x + \cos x)^2} dx = \frac{-1}{x\sin x + \cos x}\right.$ and (और),

$\left.\frac{d}{dx}(x\sec x) = \sec x + x\sec x\tan x = \sec x\left(1 + \frac{x\sin x}{\cos x}\right) = \sec^2 x(x\sin x + \cos x)\right)$

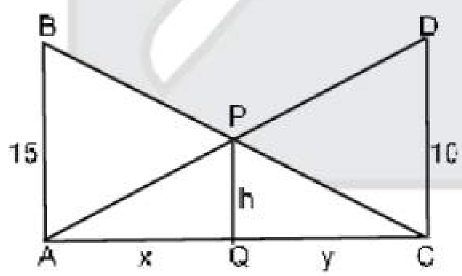
$= \frac{-x}{\cos x(x\sin x + \cos x)} + \frac{\sin x}{\cos x} + C$

$= \frac{-x + x\sin^2 x + \sin x\cos x}{\cos x(x\sin x + \cos x)} = \frac{\sin x - x\cos x}{x\sin x + \cos x} + C$

13. Two poles AB and CD of height 15 m and 10 m respectively. A and C are on level ground. Point of intersection of AD and BC is P then height of P is

Ans. 6

Sol.



$\Delta AQP \sim \Delta ACD \quad \Rightarrow \quad \frac{x}{h} = \frac{x+y}{10} \quad .....(1)$

$\because \quad \Delta CQP \sim \Delta CAB \Rightarrow \quad \frac{y}{h} = \frac{x+y}{15} \quad .....(2)$

$(1) + (2) \rightarrow \frac{x+y}{h} = (x+y)\left(\frac{1}{10} + \frac{1}{15}\right) \quad \Rightarrow \quad h = 6$

14. Consider two statements

S_1 : ~p → (~q ↔ ~p) is a tautology

S_2 : $(\sim q \wedge p) \rightarrow q$ is a fallacy then

(1) Statement I is true, statement II is false

(2) Statement I is false, statement II is true

(3) Both true

(4) Both false

Ans. **(4)**

Sol. I : $\sim p \rightarrow (\sim q \leftrightarrow \sim p)$

p	q	$\sim p$	$\sim q$	$\sim q \leftrightarrow \sim p$	(I) $\sim p \rightarrow (\sim q \leftrightarrow \sim p)$	$p \wedge \sim q$	(II) $(\sim q \wedge p) \rightarrow q$
T	T	F	F	T	T	F	T
T	F	F	T	F	T	T	F
F	T	T	F	F	F	F	T
F	F	T	T	T	T	F	T

both are false

15. If u = $\frac{2z+i}{z-ki}$ where z = x + iy and k > 0

Curve Re (u) + Im(u) = 1 cuts y-axis at two point P and Q such that PQ = 5 then value of k is

(1) 1 (2) 2 (3) 3 (4) 4

Ans. **(2)**

Sol. $u = \frac{2(x+iy)+i}{(x+iy)-ki} = \frac{2x+(2y+1)i}{x+(y-k)i} \times \frac{x-(y-k)i}{x-(y-k)i}$

Real part of u = Re(u) = $\frac{2x^2+(2y+1)(y-k)}{x^2+(y-k)^2}$

Imaginary part of u = Im(u) = $\frac{x(2y+1)-2x(y-k)}{x^2+(y-k)^2}$

Now Re(u) + Im(u) = 1

$$\frac{2x^2+(2y+1)(y-k)+x(2y+1)-2x(y-k)}{x^2+(y-k)^2}=1$$

for y-axis put x = 0 ⇒ $\frac{(2y+1)(y-k)}{(y-k)^2}=1$

⇒ (2y + 1) (y – k) = $(y-k)^2$

⇒ (y – k) (y + (1 + k) = 0

y = k, –(1 + k)

Now point P(0,k), Q (0,– (1 + k))

PQ = |2k + 1| = 5

2k + 1 = ± 5

2k = 4, – 6

k = 2 – 3

hence k = 2 (k > 0)

16. Probability of hitting a target is $\frac{1}{10}$ then find the minimum number of trials so that probability of at least one success is greater than $\frac{1}{4}$ is

Ans. **(3)**

Sol. $p = \frac{1}{10},\ q = \frac{9}{10}$

p (not hitting in n trials) = $\left(\frac{9}{10}\right)^n$

$\therefore$ p (at least one hit) = $1 - \left(\frac{9}{10}\right)^n \geq \frac{1}{4}$

$\Rightarrow \left(\frac{9}{10}\right)^n \leq \frac{3}{4}$

$(.9)^n \leq .75$

$n = 3 \Rightarrow 0.729 \leq .75$ which is true

17. Let A = $\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$, $0 < \theta < \frac{\pi}{24}$ and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then which statement is false

(1) $a^2 - b^2 = \frac{1}{2}$ (2) $a^2 + b^2 \in (0,1)$ (3) $a^2 - d^2 = 0$ (4) $a^2 - c^2 = 1$

Ans. **(1)**

Sol. $A^2 = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos^2\theta - \sin^2\theta & 2i\sin\theta\cos\theta \\ 2i\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} = \begin{bmatrix} \cos2\theta & i\sin2\theta \\ i\sin2\theta & \cos2\theta \end{bmatrix}$

$A^3 = \begin{bmatrix} \cos2\theta & i\sin2\theta \\ i\sin2\theta & \cos2\theta \end{bmatrix}\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos3\theta & i\sin3\theta \\ i\sin3\theta & \cos3\theta \end{bmatrix}$

$\therefore$ $A^5 = \begin{bmatrix} \cos5\theta & i\sin5\theta \\ i\sin5\theta & \cos5\theta \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$a = \cos5\theta$, $b = i\sin5\theta$, $c = i\sin5\theta$, $d = \cos5\theta$

$a = d$, $b = c$

(1) $a^2 - b^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

(2) $a^2 + b^2 = \cos^2 5\theta - \sin^2 5\theta = \cos10\theta \in (0,1)$ as $0 < \theta < \frac{\pi}{24}$

(3) $a^2 - d^2 = 0$

(4) $a^2 - c^2 = a^2 - b^2 = 1$

18. If $\left(a - \sqrt{2}b\cos x\right)\left(a + \sqrt{2}b\cos y\right) = a^2 - b^2$ then value of $\frac{dy}{dx}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is -

(1) $\frac{a-b}{a+b}$ (2) $\frac{a+b}{a-b}$ (3) $\frac{2a+b}{a-b}$ (4) $\frac{2a-b}{a-b}$

Ans. **(2)**

Sol. $\left(a-\sqrt{2}b\cos x\right)\left(-\sqrt{2}b\sin y\right)\frac{dy}{dx}+\sqrt{2}b\sin x\left(a+\sqrt{2}b\cos y\right)=0$

$$\frac{dy}{dx}=\frac{\sqrt{2}b\sin x(a+\sqrt{2}b\cos y)}{\sqrt{2}b\sin y\left(a-\sqrt{2}b\cos x\right)}=\frac{\sin x\left(a+\sqrt{2}b\cos y\right)}{\sin y\left(a-\sqrt{2}b\cos x\right)}$$

$$\left.\frac{dy}{dx}\right|_{\left(\frac{\pi}{4},\frac{\pi}{4}\right)}=\frac{a+b}{a-b}$$

19. $f(x + y) = f(x) + f(y) + xy^2 + x^2y$ and $\lim_{x\to 0}\frac{f(x)}{x}=1$, then find value of $f'(3)$

Ans. 10

Sol. $f(x + y) = f(x) + f(y) + xy^2 + x^2y$

$f'(x + y) = f'(x) + 0 + y^2 + 2xy$

put $y = -x$

$f'(0) = f'(x) + x^2 - 2x^2$

$1 = f'(x) - x^2$

$f'(x) = 1 + x^2$

$f'(3) = 10$

20. If f is twice differentiable function for $x \in R$ such that $f(2) = 5$, $f'(2) = 8$ and $f'(x) \ge 1$, $f''(x) \ge 4$ then

(1) $f(5) + f'(5) \le 26$
(2) $f(5) + f'(5) \ge 28$
(3) $f(5) + f'(5) \le 28$
(4) none of these

Ans. **(2)**

Sol. Given $f'(x) \ge 1 \Rightarrow \int_2^5 f'(x)dx \ge \int_2^5 1.dx$

$\Rightarrow \left(f(x)\right)_2^5 \ge \left(x\right)_2^5 \quad \Rightarrow f(5) - f(2) \ge 3 \quad \Rightarrow f(5) \ge 8 \quad ......(1)$

Now $f''(x) \ge 4 \Rightarrow \int_2^5 f''(x)dx \ge \int_2^5 4dx$

$= \left(f'(x)\right)_2^5 \ge \left(4x\right)_2^5$

$\Rightarrow f'(5) - f'(2) \ge 12$

$\Rightarrow f'(5) \ge 20 \quad .... (2)$

$(1) + (2) \Rightarrow f(5) + f'(5) \ge 28$

21. If $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$, then $\frac{a_7}{a_{13}} =$

Ans. 8

Sol. General term $\frac{10!}{r_1!r_2!r_3!}(2x^2)^{r_1}.(3x)^{r_2}(4)^{r_3}$

$$a_7=\frac{10!.2^3.3.4^6}{3!\ 1!\ 6!}+\frac{10!2^2.3^3.4^5}{2!3!5!}+\frac{10!.2.3^5.4^4}{1!.5!.4!}+\frac{10!.3^7.4^3}{7!3!}$$

$$a_{13}=\frac{10!.2^6.3.4^3}{6!\ 1!\ 3!}+\frac{10!.2^5.3^3.4^2}{5!3!\ 2!}+\frac{10!2^4 3^5.4^1}{4!\ 5!\ 1!}+\frac{10!.2^3.3^7.4^0}{3!\ 7!}$$

$$\frac{a_7}{a_{13}}=2^3=8$$

22. If from a point (3,3) on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ a normal is drawn which cuts x axis at (9,0) then value of (a^2, e^2) is

(1) $\left(\frac{9}{2},3\right)$ (2) $\left(\frac{9}{2},1\right)$ (3) (9,3) (4) (3,9)

Ans. **(1)**

Sol. Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

P(3,3) lies on hyperbola then $\frac{1}{a^2}-\frac{1}{b^2}=\frac{1}{9}$ ....(1)

Normal at (3,3) is

$\frac{a^2x}{3}+\frac{b^2y}{3}=a^2+b^2$

pass through (9,0)

$3a^2 = a^2 + b^2 = 2a^2 = b^2$

then $\frac{1}{a^2}-\frac{1}{2a^2}=\frac{1}{9}$

$2a^2 = 9 \Rightarrow a^2 = \frac{9}{2}$ and $b^2 = 9$

$e^2 = 1+\frac{b^2}{a^2} = 1 + 2 = 3$