3

Trigonometric Functions

Numerical

Q.1 Let S = {1, 2, 3, 4, 5, 6, 9}. Then the number of elements in the set T = {A ⊆ S : A ≠ ϕ and the sum of all the elements of A is not a multiple of 3} is ______________.

27th Aug Evening Shift 2021

Q.2 Let z_1 and z_2 be two complex numbers such that $\arg(z_1 - z_2) = \pi/4$ and z_1, z_2 satisfy the equation $| z - 3 | = \text{Re}(z)$. Then the imaginary part of $z_1 + z_2$ is equal to __________.

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Q.3 The probability distribution of random variable X is given by :

X	1	2	3	4	5
P(X)	K	2K	2K	3K	K

Let $p = P(1 < X < 4 \mid X < 3)$. If $5p = \lambda K$, then λ equal to __________.

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Q.4 Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ in $[0, 4\pi]$. Then $8S/\pi$ is equal to ___________.

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Q.5 The number of solutions of the equation

$|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is

18th Mar Morning Shift 2021

Q.6 The number of integral values of 'k' for which the equation 3sin x + 4 cos x = k + 1 has a solution, k∈R is ___________.

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Q.7

If $\sqrt{3}(\cos^2 x) = (\sqrt{3} - 1)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is ____________.

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Numerical Answer Key

1. **Ans. (80)**
2. **Ans. (6)**
3. **Ans. (30)**
4. **Ans. (56)**
5. **Ans. (1)**
6. **Ans. (11)**
7. **Ans. (1)**

Numerical Explanation

Ans. 1

3n type $\rightarrow$ 3, 6, 9 = P

3n − 1 type $\rightarrow$ 2, 5 = Q

3n − 2 type $\rightarrow$ 1, 4 = R

number of subset of S containing one element which are not divisible by 3 = ${}^2C_1 + {}^2C_1 = 4$

number of subset of S containing two numbers whose some is not divisible by 3

$= {}^3C_1 \times {}^2C_1 + {}^3C_1 \times {}^2C_1 + {}^2C_2 + {}^2C_2 = 14$

number of subsets containing 3 elements whose sum is not divisible by 3

$= {}^3C_2 \times {}^4C_1 + ({}^2C_2 \times {}^2C_1)2 + {}^3C_1({}^2C_2 + {}^2C_2) = 22$

number of subsets containing 4 elements whose sum is not divisible by 3

$= {}^3C_3 \times {}^4C_1 + {}^3C_2({}^2C_2 + {}^2C_2) + ({}^3C_1{}^2C_1 \times {}^2C_2)2$

$= 4 + 6 + 12 = 22$

number of subsets of S containing 5 elements whose sum is not divisible by 3.

$= {}^3C_3({}^2C_2 + {}^2C_2) + ({}^3C_2{}^2C_1 \times {}^2C_2) \times 2 = 2 + 12 = 14$

number of subsets of S containing 6 elements whose sum is not divisible by 3 = 4

$\Rightarrow$ Total subsets of Set A whose sum of digits is not divisible by 3 = 4 + 14 + 22 + 22 + 14 + 4 = 80.

Ans. 2

Let $z_1 = x_1 + iy$; $z_2 = x_2 + iy_2$

$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$

$\therefore \arg(z_1 - z_2) = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{y_1-y_2}{x_1-x_2}\right) = \frac{\pi}{4}$

$y_1 - y_2 = x_1 - x_2$ .... (1)

$|z_1 - 3| = \text{Re}(z_1) \Rightarrow (x_1 - 3)^2 + {y_1}^2 = {x_1}^2$ .... (2)

$|z_2 - 3| = \text{Re}(z_2) \Rightarrow (x_2 - 3)^2 + {y_2}^2 = {x_2}^2$ .... (3)

sub (2) & (3)

$(x_1 - 3)^2 - (x_2 - 3)^2 + {y_1}^2 - {y_2}^2 = {x_1}^2 - {x_2}^2$

$(x_1 - x_2)(x_1 + x_2 - 6) + (y_1 - y_2)(y_1 + y_2)$

$= (x_1 - x_2)(x_1 + x_2)$

$x_1 + x_2 - 6 + y_1 + y_2 = x_1 + x_2 \Rightarrow y_1 + y_2 = 6$

Ans. 3

$\sum P(X) = 1 \Rightarrow k + 2k + 3k + k = 1$

$\Rightarrow k = \frac{1}{9}$

Now, $p = P\left(\frac{kx<4}{X<3}\right) = \frac{P(X=2)}{P(X<3)} = \frac{\frac{2k}{9k}}{\frac{k}{9k}+\frac{2k}{9k}} = \frac{2}{3}$

$\Rightarrow p = \frac{2}{3}$

Now, $5p = \lambda k$

$\Rightarrow (5)\left(\frac{2}{3}\right) = \lambda(1/9)$

$\Rightarrow \lambda = 30$

Ans. 4

Given equation

$\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$

$\Rightarrow 1 - \sin^2\theta\cos^2\theta - \sin\theta\cos\theta = 0$

$\Rightarrow 2-(\sin 2\theta)^2-\sin 2\theta=0$

$\Rightarrow (\sin 2\theta)^2+(\sin 2\theta)-2=0$

$\Rightarrow (\sin 2\theta+2)(\sin 2\theta-1)=0$

$\Rightarrow \sin 2\theta=1$ or $\sin 2\theta=-2$ (Not Possible)

$\Rightarrow 2\theta=\frac{\pi}{2},\frac{5\pi}{2},\frac{9\pi}{2},\frac{13\pi}{2}$

$\Rightarrow \theta=\frac{\pi}{4},\frac{5\pi}{4},\frac{9\pi}{4},\frac{13\pi}{4}$

$\Rightarrow S=\frac{\pi}{4}+\frac{5\pi}{4}+\frac{9\pi}{4}+\frac{13\pi}{4}=7\pi$

$\Rightarrow \frac{8S}{\pi}=\frac{8\times 7\pi}{\pi}=56.00$

Ans. 5

Case I : When cot x > 0, $x\in\left[0,\frac{\pi}{2}\right]\cup\left[\pi,\frac{3\pi}{2}\right]$

$\cot x=\cot x+\frac{1}{\sin x}\Rightarrow$ not possible

Case II : When cot x < 0, $x\in\left[\frac{\pi}{2},\pi\right]\cup\left[\frac{3\pi}{2},2\pi\right]$

$-\cot x=\cot x+\frac{1}{\sin x}$

$\Rightarrow \frac{-2\cos x}{\sin x} = \frac{1}{\sin x}$

$\Rightarrow \cos x = \frac{-1}{2}$

$\Rightarrow x = \frac{2\pi}{3}, \frac{4\pi}{3}$(Rejected)

One solution.

Ans. 6

We know,

$-\sqrt{a^2+b^2} \le a\cos x + b\sin x \le \sqrt{a^2+b^2}$

$\therefore -\sqrt{3^2+4^2} \le 3\cos x + 4\sin x \le \sqrt{3^2+4^2}$

$-5 \le k+1 \le 5$

$-6 \le k \le 4$

$\therefore$ Set of integers = $-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4$ = Total 11 intergers.

Ans. 7

$\sqrt{3}(\cos^2 x) = (\sqrt{3}-1)\cos x + 1$

$\Rightarrow \sqrt{3}\cos^2 x - \sqrt{3}\cos x + \cos x - 1 = 0$

$\Rightarrow \sqrt{3}\cos x(\cos x - 1) + (\cos x - 1) = 0$

$\Rightarrow (\cos x - 1)(\sqrt{3}\cos x + 1) = 0$

$\cos x = 1$

$\Rightarrow x = 0 \; [as x \in \left[0, \frac{\pi}{2}\right]]$

and $\cos x = -\frac{1}{\sqrt{3}}$ (not possible in $x \in \left[0, \frac{\pi}{2}\right]$]

$\therefore$ Number of solution = 1

MCQ (Single Correct Answer)

Q.1. If n is the number of solutions of the equation

$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1, x \in [0, \pi]$ and S is the sum of all these solutions, then the ordered pair (n, S) is :

A (3, 13π / 9)

B (2, 2π / 3)

C (2, 8π / 9)

D (3, 5π / 3)

1st Sep Evening Shift 2021

Q.2.

The number of solutions of the equation $32^{\tan^2 x} + 32^{\sec^2 x} = 81,\ 0 \le x \le \frac{\pi}{4}$ is :

A 3

B 1

C 0

D 2

31st Aug Evening Shift 2021

Q.3. The distance of the point (1, −2, 3) from the plane x − y + z = 5 measured parallel to a line, whose direction ratios are 2, 3, −6 is :

A 3

B 5

C 2

D 1

27th Aug Morning Shift 2021

Q.4. The value of

$$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$$ is :

A $\frac{1}{4\sqrt{2}}$

B $\frac{1}{4}$

C $\frac{1}{8}$

D $\frac{1}{8\sqrt{2}}$

26th Aug Evening Shift 2021

Q.5. The sum of solutions of the equation

$\frac{\cos x}{1+\sin x} = |\tan 2x|, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ is :

A $-\frac{11\pi}{30}$

B $\frac{\pi}{10}$

C $-\frac{7\pi}{30}$

D $-\frac{\pi}{15}$

26th Aug Morning Shift 2021

Q.6. If sinθ + cosθ = ½, then 16(sin(2θ) + cos(4θ) + sin(6θ) is equal to :

A 23

B −27

C −23

D 27

27th Jul Morning Shift 2021

Q.7. The value of cot π/24 is:

A $\sqrt{2}+\sqrt{3}+2-\sqrt{6}$

B $\sqrt{2}+\sqrt{3}+2+\sqrt{6}$

C $\sqrt{2}-\sqrt{3}-2+\sqrt{6}$

D $3\sqrt{2}-\sqrt{3}-\sqrt{6}$

25th Jul Evening Shift 2021

Q.8. If tan (π/9), x, tan (7π/18) are in arithmetic progression and tan (π/9), y, tan (5π/18) are also in arithmetic progression, then |x − 2y| is equal to :

A 4

B 3

C 0

D 1

27th Jul Evening Shift 2021

Q.9. The sum of all values of x in [0, 2π], for which sin x + sin 2x + sin 3x + sin 4x = 0, is equal to :

A 8π

B 11π

C 12π

D 9π

25th Jul Morning Shift 2021

Q.10. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in R$, then the value of

$27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$ is equal to :

A 500

B 400

C 250

D 350

18th Mar Evening Shift 2021

Q.11. The number of solutions of the equation $x + 2\tan x = \pi/2$ in the interval $[0, 2\pi]$ is :

A 4

B 3

C 2

D 5

17th Mar Evening Shift 2021
Q.12.

If for $x \in \left(0, \frac{\pi}{2}\right)$, $\log_{10}\sin x + \log_{10}\cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$, $n > 0$, then the value of n is equal to :

A 16

B 9

C 12

D 20

16th Mar Morning Shift 2021

Q.13. The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval $[0, \pi]$ is equal to :

A 2

B 3

C 4

D 8

16th Mar Morning Shift 2021

Q.14. If 0 < x, y < π and cosx + cosy − cos(x + y) = 3/2, then sinx + cosy is equal to :

A $\frac{1+\sqrt{3}}{2}$

B $\frac{1}{2}$

C $\frac{\sqrt{3}}{2}$

D $\frac{1-\sqrt{3}}{2}$

25th Feb Evening Shift 2021

Q.15. All possible values of θ ∈ [0, 2π] for which sin 2θ + tan 2θ > 0 lie in :

A $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

B $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

C $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

D $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

25th Feb Morning Slot 2021

Q.16.

If $e^{(\cos^2 x+\cos^4 x+\cos^6 x+\ldots\infty)\log_e 2}$ satisfies the equation t^2 - 9t + 8 = 0, then the value of $\frac{2\sin x}{\sin x+\sqrt{3}\cos x}\left(0<x<\frac{\pi}{2}\right)$ is :

A $\sqrt{3}$

B $\frac{3}{2}$

C $2\sqrt{3}$

D $\frac{1}{2}$

24th Feb Morning Slot 2021

MCQ Answer Key

1. **Ans.** (A)
2. **Ans.** (B)
3. **Ans.** (D)
4. **Ans.** (C)
5. **Ans.** (A)
6. **Ans.** (C)
7. **Ans.** (B)
8. **Ans.** (C)
9. **Ans.** (D)
10. **Ans.** (C)
11. **Ans.** (B)
12. **Ans.** (C)
13. **Ans.** (C)
14. **Ans.** (A)
15. **Ans.** (D)
16. **Ans.** (D)

MCQ Explanation

Ans 1.

$$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1$$

$$2\cos x\left(4\left(\sin^2\frac{\pi}{4}-\sin^2 x\right)-1\right)=1$$

$$2\cos x\left(4\left(\frac{1}{2}-\sin^2 x\right)-1\right)=1$$

$$2\cos x(2-4\sin^2 x-1)=1$$

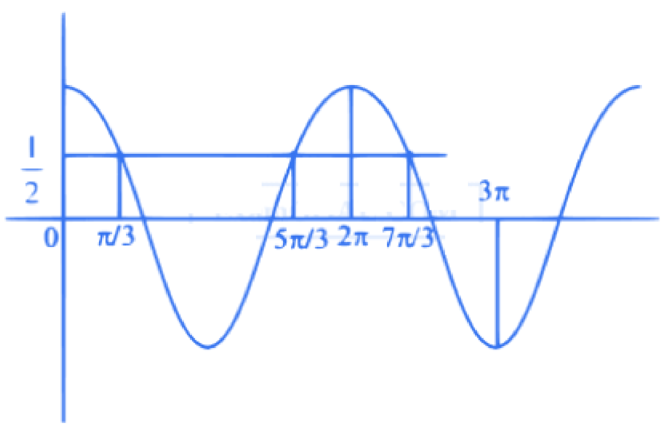
$$2\cos x(1-4\sin^2 x)=1$$

$$2\cos x(4\cos^2 x-3)=1$$

$$4\cos^3 x-3\cos x=\frac{1}{2}$$

$$\cos 3x=\frac{1}{2}$$

$$x\in[0,\pi]\therefore 3x\in[0,3\pi]$$



Ans 2.

$(32)^{\tan^2 x} + (32)^{\sec^2 x} = 81$

$\Rightarrow (32)^{\tan^2 x} + (32)^{1+\tan^2 x} = 81$

$\Rightarrow (32)^{\tan^2 x} = \frac{81}{33}$

taking log of base 32 both side,

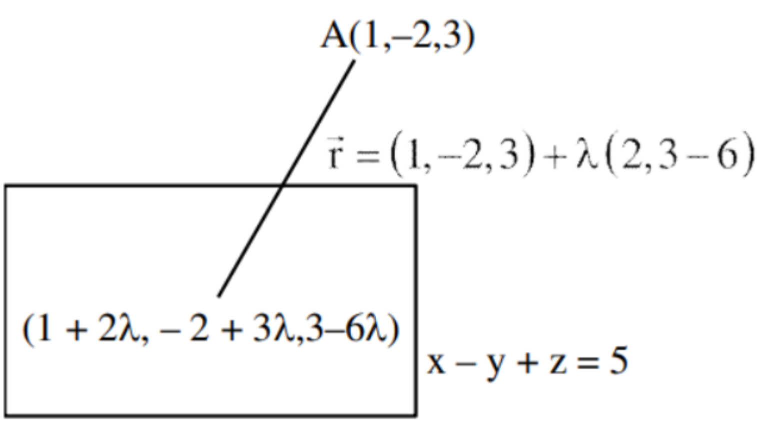
$\Rightarrow$ tan^{2}x = $\log_{32}\left(\frac{81}{33}\right)$

$\Rightarrow$ tan x = $\sqrt{\log_{32}\left(\frac{81}{33}\right)}$

As value of $\sqrt{\log_{32}\left(\frac{81}{33}\right)}$ belongs to (0, 1).

In interval $0 \le x \le \frac{\pi}{4}$ only one solution.

Ans 3.



$$(1+2\lambda)+2-3\lambda+3-6\lambda=5$$

$$\Rightarrow 6-7\lambda=5 \Rightarrow \lambda=\frac{1}{7}$$

so, $P=\left(\frac{9}{7},-\frac{11}{7},\frac{15}{7}\right)$

$$AP=\sqrt{\left(1-\frac{9}{7}\right)^2+\left(-2+\frac{11}{7}\right)^2+\left(3-\frac{15}{7}\right)^2}$$

$$AP=\sqrt{\left(\frac{4}{49}\right)+\frac{9}{49}+\frac{36}{49}}=1$$

Ans 4.

$$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$$

$$2\sin^2\frac{\pi}{8}\sin^2\frac{2\pi}{8}\sin^2\frac{3\pi}{8}$$

$$\sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}$$

$$\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}$$

$$\frac{1}{4}\sin^2\left(\frac{\pi}{4}\right)=\frac{1}{8}$$

Ans 5.

$$\frac{\cos x}{1+\sin x}=|\tan 2x|$$

$$\Rightarrow \frac{\cos^2 x/2-\sin^2 x/2}{(\cos x/2+\sin x/2)^2}=|\tan 2x|$$

$$\Rightarrow \frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}} = |\tan 2x|$$

$$\Rightarrow \frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}} = |\tan 2x|$$

$$\Rightarrow \frac{\tan\frac{\pi}{4}-\tan\frac{x}{2}}{\tan\frac{\pi}{4}+\tan\frac{x}{2}} = |\tan 2x|$$

$$\Rightarrow \tan^2\left(\frac{\pi}{4}-\frac{\pi}{2}\right) = \tan^2 2x$$

$$\Rightarrow 2x = n\pi \pm \left(\frac{\pi}{4}-\frac{\pi}{2}\right)$$

$$\Rightarrow x = \frac{-3\pi}{10}, \frac{-\pi}{6}, \frac{\pi}{10}$$

or sum $= \frac{-11\pi}{6}$.

Ans 6.

$$\sin\theta + \cos\theta = \frac{1}{2}$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{1}{4}$$

$$\sin 2\theta = -\frac{3}{4}$$

Now :

$$\cos 4\theta = 1 - 2\sin^2 2\theta$$

$$= 1 - 2\left(-\frac{3}{4}\right)^2$$

$$= 1 - 2 \times \frac{9}{16} = -\frac{1}{8}$$

$$\sin 6\theta = 3\sin 2\theta - 4\sin^3 2\theta$$

$$= (3 - 4\sin^2 2\theta).\sin 2\theta$$

$$= \left[3 - 4\left(\frac{9}{16}\right)\right].\left(-\frac{3}{4}\right)$$

$$\Rightarrow \left[\frac{3}{4}\right] \times \left(-\frac{3}{4}\right) = -\frac{9}{16}$$

$$16[\sin 2\theta + \cos 4\theta + \sin 6\theta]$$

= $16\left(-\frac{3}{4} - \frac{1}{8} - \frac{9}{16}\right) = -23$

Ans 7.

$$\cot\theta = \frac{1+\cos 2\theta}{\sin 2\theta} = \frac{1+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$$

$$\theta = \frac{\pi}{24}$$

$$\Rightarrow \cot\left(\frac{\pi}{24}\right) = \frac{1+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$$

$$= \frac{\left(2\sqrt{2}+\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)} \times \frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}$$

$$= \frac{2\sqrt{6}+2\sqrt{2}+3+\sqrt{3}+\sqrt{3}+1}{2}$$

$$= \sqrt{6} + \sqrt{2} + \sqrt{3} + 2$$

Ans 8.

$$x = \frac{1}{2}\left(\tan\frac{\pi}{9} + \tan\frac{7\pi}{18}\right)$$

and $2y = \tan\frac{\pi}{9} + \tan\frac{5\pi}{18}$

so, $x - 2y = \frac{1}{2}\left(\tan\frac{\pi}{9} + \tan\frac{7\pi}{18}\right) - \left(\tan\frac{\pi}{9} + \tan\frac{5\pi}{18}\right)$

$$\Rightarrow |x - 2y| = \left|\frac{\cot\frac{\pi}{9} - \tan\frac{\pi}{9}}{2} - \tan\frac{5\pi}{18}\right| = \left|\cot\frac{2\pi}{9} - \cot\frac{2\pi}{9}\right| = 0$$

$$\left(as\ \tan\frac{5\pi}{18} = \cot\frac{2\pi}{9}; \tan\frac{7\pi}{18} = \cot\frac{\pi}{9}\right)$$

Ans 9.

$$(\sin x + \sin 4x) + (\sin 2x + \sin 3x) = 0$$

$$\Rightarrow 2\sin\frac{5x}{2}\left\{\cos\frac{3x}{2} + \cos\frac{x}{2}\right\} = 0$$

$$\Rightarrow 2\sin\frac{5x}{2}\left\{2\cos x\cos\frac{x}{2}\right\} = 0$$

$2\sin\frac{5x}{2}=0 \Rightarrow \frac{5x}{2}=0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$

$\Rightarrow x=0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi$

$\cos\frac{x}{2}=0 \Rightarrow \frac{x}{2}=\frac{\pi}{2} \Rightarrow x=\pi$

$\cos x=0 \Rightarrow x=\frac{\pi}{2}, \frac{3\pi}{2}$

So, sum $=6\pi+\pi+2\pi=9\pi$

Ans 10.

$15\sin^2\alpha + 10(1 - \sin^2\alpha)^2 = 6$

$\Rightarrow 25\sin^2\alpha - 20\sin^2\alpha + 4 = 0$

$\Rightarrow 25\sin^2\alpha - 10\sin^2\alpha - 10\sin^2\alpha + 4 = 0$

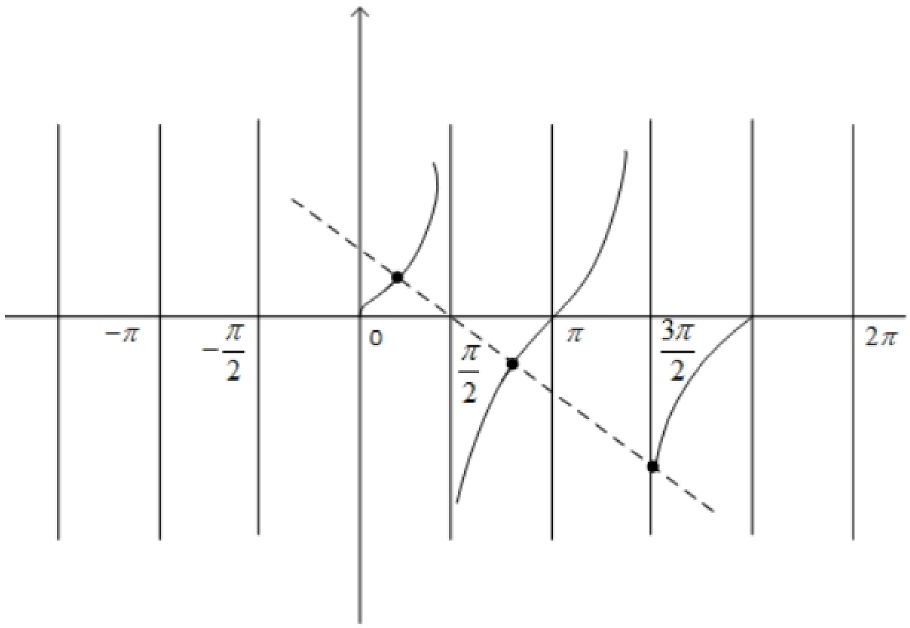
$\Rightarrow (5\sin^2\alpha - 2)^2 = 0 \Rightarrow \sin^2\alpha = \frac{2}{5}$

$\therefore \cos^2\alpha = \frac{3}{5}$

$\therefore 27\sec^2\alpha + 8\text{cosec}^6\alpha = 27\left(\frac{5}{3}\right)^3 + 8\left(\frac{5}{2}\right)^3$

$= 125 + 125 = 250$

Ans 11.



$x + 2\tan x = \frac{\pi}{2}$ in [0, 2π]

$2\tan x = \frac{\pi}{2} - x$

$2\tan x = \frac{\pi}{2} - x$

$\tan x = \frac{\pi}{4} - \frac{x}{2}$

$y = \tan x$ and $y = \frac{-x}{2} + \frac{\pi}{4}$

3 intersection points on the graph.

$\therefore$ 3 solutions.

Ans 12.

$$\log_{10}(\sin x) + \log_{10}(\cos x) = -1$$

$\sin x \cos x = \frac{1}{10}$ .... (1)

and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$

$$\Rightarrow \sin x + \cos x = \left(\frac{n}{10}\right)^{\frac{1}{2}}$$

$\Rightarrow \sin^2 x + \cos^2 x + 2 \sin x \cos x = \frac{n}{10}$ (squaring)

$\Rightarrow 1 + 2\left(\frac{1}{10}\right) = \frac{n}{10}$ (using equation (1))

$$\Rightarrow \frac{n}{10} = \frac{12}{10}$$

$$\Rightarrow n = 12$$

Ans 13.

$$(81)^{\sin^2 x} + (81)^{1-\sin^2 x} = 30$$

$$(81)^{\sin^2 x} + \frac{81}{(81)^{\sin^2 x}} = 30$$

Let $(81)^{\sin^2 x} = t$

$t + \frac{81}{t} = 30 \Rightarrow t^2 + 81 = 30t$

$t^2 - 30t + 81 = 0$

$t^2 - 27t - 3t + 81 = 0$

$(t - 3)(t - 27) = 0$

$t = 3, 27$

$(81)^{\sin^2 x} = 3, 3^3$

$3^{4\sin^2 x} = 3^1, 3^3$

$4\sin^2 x = 1, 3$

$\sin^2 x = \frac{1}{4}, \frac{3}{4}$

in [0, π] sin x > 0

$\sin x = \frac{1}{2}, \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}$

Number of solution = 4

Ans 14.

$$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)-\left[2\cos^2\left(\frac{x+y}{2}\right)-1\right]=\frac{3}{2}$$

$$2\cos\left(\frac{x+y}{2}\right)\left[\cos\left(\frac{x-y}{2}\right)-\cos\left(\frac{x+y}{2}\right)\right]=\frac{1}{2}$$

$$2\cos\left(\frac{x+y}{2}\right)\left[2\sin\left(\frac{x}{2}\right).\sin\left(\frac{y}{2}\right)\right]=\frac{1}{2}$$

$$\cos\left(\frac{x+y}{2}\right).\sin\left(\frac{x}{2}\right).\sin\left(\frac{y}{2}\right)=\frac{1}{8}$$

Possible when $\frac{x}{2}=30^\circ$ & $\frac{y}{2}=30^\circ$

$$x=y=60^\circ$$

$$\sin x+\cos y=\frac{\sqrt{3}}{2}+\frac{1}{2}=\frac{\sqrt{3}+1}{2}$$

Ans 15.

$$\sin 2\theta+\tan 2\theta>0$$

$$\Rightarrow \sin 2\theta+\frac{\sin 2\theta}{\cos 2\theta}>0$$

$$\Rightarrow \sin 2\theta\frac{(\cos 2\theta+1)}{\cos 2\theta}>0\Rightarrow \tan 2\theta(2\cos^2\theta)>0$$

Note : $\cos 2\theta\neq 0$

$$\Rightarrow 1-2\sin^2\theta\neq\theta\Rightarrow \sin\theta\neq\pm\frac{1}{\sqrt{2}}$$

Now, $\tan 2\theta(1+\cos 2\theta) > 0$

$\Rightarrow \tan 2\theta > 0$ (as $\cos 2\theta + 1 > 0$)

$\Rightarrow 2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$

$\Rightarrow \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

Ans 16.

$e^{(\cos^2 x+\cos^4 x+\ldots\ldots\ldots\ldots\infty)\ln 2} = 2^{\cos^2 x+\cos^4 x+\ldots\ldots\ldots\ldots\infty}$

$= 2^{\frac{\cos^2 x}{1-\cos^2 x}}$

$= 2^{\cot^2 x}$

Given, $t^2 - 9t + 8 = 0 \Rightarrow t = 1, 8$

$\Rightarrow 2^{\cot^2 x} = 1, 8 \Rightarrow cot^2 x = 0, 3$

$0 < x < \frac{\pi}{2} \Rightarrow \cot x = \sqrt{3}$

$\therefore \frac{2\sin x}{\sin x+\sqrt{3}\cos x} = \frac{2}{1+\sqrt{3}\cot x} = \frac{2}{4} = \frac{1}{2}$

TOPIC **Circular System, Trigonometric Ratios, Domain and Range of Trigonometric Functions, Trigonometric Ratios of Allied Angles**

1. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ the expression

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals: **[Jan. 9, 2019 (I)]**

(a) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

(b) $13 - 4\cos^6\theta$

(c) $13 - 4\cos^2\theta + 6\cos^4\theta$

(d) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

2. Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ where $x \in R$ and $k \geq 1$.

Then $f_4(x) - f_6(x)$ equals **[2014]**

(a) $\frac{1}{4}$ (b) $\frac{1}{12}$

(c) $\frac{1}{6}$ (d) $\frac{1}{3}$

3. If $2\cos\theta + \sin\theta = 1$ $\left(\theta \neq \frac{\pi}{2}\right)$,

then $7\cos\theta + 6\sin\theta$ is equal to: **[Online April 11, 2014]**

(a) $\frac{1}{2}$ (b) 2

(c) $\frac{11}{2}$ (d) $\frac{46}{5}$

4. The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$

can be written as : **[2013]**

(a) sinA cosA + 1 (b) secA cosecA + 1

(c) tanA + cotA (d) secA + cosecA

5. The value of $\cos 255° + \sin 195°$ is **[Online May 26, 2012]**

(a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (b) $\frac{\sqrt{3}-1}{\sqrt{2}}$

(c) $-\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)$ (d) $\frac{\sqrt{3}+1}{\sqrt{2}}$

6. Let $f(x) = \sin x$, $g(x) = x$.

Statement 1: $f(x) \leq g(x)$ for x in $(0, \infty)$

Statement 2: $f(x) \leq 1$ for x in $(0, \infty)$ but $g(x) \to \infty$ as $x \to \infty$. **[Online May 7, 2012]**

(a) Statement 1 is true, Statement 2 is false.

(b) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

(c) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.

(d) Statement 1 is false, Statement 2 is true.

7. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is **[2006]**

(a) $\frac{3}{2}x^2$ (b) $\sqrt{\frac{x^3}{8}}$

(c) $\frac{1}{2}x^2$ (d) πx^2

TOPIC 2 Trigonometric Identities, Conditional Trigonometric Identities, Greatest and Least Value of Trigonometric Expressions

8. The value of $\cos^3\left(\frac{\pi}{8}\right).\cos\left(\frac{3\pi}{8}\right)+\sin^3\left(\frac{\pi}{8}\right).\sin\left(\frac{3\pi}{8}\right)$ is **[Jan. 9, 2020 (I)]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

9. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}}=\frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}}=\frac{1}{\sqrt{10}}$,

$\alpha,\beta\in\left(0,\frac{\pi}{2}\right)$, then $\tan(\alpha+2\beta)$ is equal to _____.

[Jan. 8, 2020 (II)]

10. If $L=\sin^2\left(\frac{\pi}{16}\right)-\sin^2\left(\frac{\pi}{8}\right)$ and

$M=\cos^2\left(\frac{\pi}{16}\right)-\sin^2\left(\frac{\pi}{8}\right)$, then : **[Sep. 05, 2020 (II)]**

(a) $L=-\frac{1}{2\sqrt{2}}+\frac{1}{2}\cos\frac{\pi}{8}$ (b) $L=\frac{1}{4\sqrt{2}}-\frac{1}{4}\cos\frac{\pi}{8}$

(c) $M=\frac{1}{4\sqrt{2}}+\frac{1}{4}\cos\frac{\pi}{8}$ (d) $M=\frac{1}{2\sqrt{2}}+\frac{1}{2}\cos\frac{\pi}{8}$

11. The set of all possible values of θ in the interval $(0, \pi)$ for which the points $(1, 2)$ and $(\sin\theta, \cos\theta)$ lie on the same side of the line $x + y = 1$ is : **[Sep. 02, 2020 (II)]**

(a) $\left(0,\frac{\pi}{2}\right)$ (b) $\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$

(c) $\left(0,\frac{3\pi}{4}\right)$ (d) $\left(0,\frac{\pi}{4}\right)$

12. The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is: **[April 12, 2019 (II)]**

(a) $15(3+\sqrt{3})$ (b) $15(5-\sqrt{3})$

(c) $15(3-\sqrt{3})$ (d) $15(1+\sqrt{3})$

13. The value of **[April 9, 2019 (II)]**

$\cos^2 10°-\cos 10°\cos 50°+\cos^2 50°$ is :

(a) $\frac{3}{4}+\cos 20°$ (b) 3/4

(c) $\frac{3}{2}(1+\cos 20°)$ (d) 3/2

14. Two poles standing on a horizontal ground are of heights 5m and 10m respectively. The line joining their tops makes an angle of 15° with the ground. Then the distance (in m) between the poles, is: **[April. 09, 2019 (II)]**

(a) $5(2+\sqrt{3})$ (b) $5(\sqrt{3}+1)$

(c) $\frac{5}{2}(2+\sqrt{3})$ (d) $10(\sqrt{3}-1)$

15. The value of sin 10° sin 30° sin 50° sin 70° is:

[April. 09, 2019 (II)]

(a) $\frac{1}{16}$ (b) $\frac{1}{32}$

(c) $\frac{1}{18}$ (d) $\frac{1}{36}$

16. If $\cos(\alpha+\beta)=\frac{3}{5}$, $\sin(\alpha-\beta)=\frac{5}{13}$ and $0<\alpha,\beta<\frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to : **[April 8, 2019 (I)]**

(a) $\frac{63}{52}$ (b) $\frac{63}{16}$

(c) $\frac{21}{16}$ (d) $\frac{33}{52}$

17. If $\sin^4\alpha+4\cos^4\beta+2=4\sqrt{2}\sin\alpha\cos\beta$; $\alpha,\beta\in[0,\pi]$, then $\cos(\alpha+\beta)-\cos(\alpha-\beta)$ is equal to : **[Jan. 12, 2019 (II)]**

(a) 0 (b) -1

(c) $\sqrt{2}$ (d) $-\sqrt{2}$

18. Let $f_k(x)=\frac{1}{k}\left(\sin^k x+\cos^k x\right)$ for k = 1, 2, 3, ... Then for all $x\in R$, the value of $f_4(x)-f_6(x)$ is equal to :

[Jan. 11, 2019 (I)]

(a) $\frac{1}{12}$ (b) $\frac{1}{4}$

(c) $\frac{-1}{12}$ (d) $\frac{5}{12}$

19. The value of **[Jan. 10, 2019 (II)]**

$\cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\cdot\ldots\cdot\cos\frac{\pi}{2^{10}}\cdot\sin\frac{\pi}{2^{10}}$

(a) $\frac{1}{512}$ (b) $\frac{1}{1024}$

(c) $\frac{1}{256}$ (d) $\frac{1}{2}$

20. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is : **[2017]**

(a) $-\frac{7}{9}$ (b) $-\frac{3}{5}$

(c) $\frac{1}{3}$ (d) $\frac{2}{9}$

21. If m and M are the minimum and the maximum values of $4+\frac{1}{2}\sin^2 2x - 2\cos^4 x, x \in R$, then M – m is equal to : **[Online April 9, 2016]**

(a) $\frac{9}{4}$ (b) $\frac{15}{4}$

(c) $\frac{7}{4}$ (d) $\frac{1}{4}$

22. If $\cos\alpha + \cos\beta = \frac{3}{2}$ and $\sin\alpha + \sin\beta$ $\frac{1}{2}$ and θ is the the arithmetic mean of α and β, then $\sin 2\theta + \cos 2\theta$ is equal to : **[Online April 11, 2015]**

(a) $\frac{3}{5}$ (b) $\frac{7}{5}$

(c) $\frac{4}{5}$ (d) $\frac{8}{5}$

23. If $\operatorname{cosec}\theta = \frac{p+q}{p-q}$ $(p \neq q \neq 0)$, then $\left|\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right|$ is equal to: **[Online April 9, 2014]**

(a) $\sqrt{\frac{p}{q}}$ (b) $\sqrt{\frac{q}{p}}$

(c) $\sqrt{pq}$ (d) pq

24. If $A = \sin^2 x + \cos^4 x$, then for all real x : **[2011]**

(a) $\frac{13}{16} \le A \le 1$ (b) $1 \le A \le 2$

(c) $\frac{3}{4} \le A \le \frac{13}{16}$ (d) $\frac{3}{4} \le A \le 1$

25. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$ **[2010]**

(a) $\frac{56}{33}$ (b) $\frac{19}{12}$

(c) $\frac{20}{7}$ (d) $\frac{25}{16}$

26. Let **A** and **B** denote the statements

A : $\cos\alpha + \cos\beta + \cos\gamma = 0$

B : $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$, then : **[2009]**

(a) **A** is false and **B** is true
(b) both **A** and **B** are true
(c) both **A** and **B** are false
(d) **A** is true and **B** is false

27. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p+q)$ is **[2007]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{2}$ (d) 2.

28. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is **[2006]**

(a) $\frac{(1-\sqrt{7})}{4}$ (b) $\frac{(4-\sqrt{7})}{3}$

(c) $-\frac{(4+\sqrt{7})}{3}$ (d) $\frac{(1+\sqrt{7})}{4}$

29. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$ then the difference between the maximum and minimum values of u^2 is given by **[2004]**

(a) $(a-b)^2$ (b) $2\sqrt{a^2+b^2}$

(c) $(a+b)^2$ (d) $2(a^2+b^2)$

30. Let α, β be such that $\pi < \alpha - \beta < 3\pi$.

If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$ **[2004]**

(a) $\frac{-6}{65}$ (b) $\frac{3}{\sqrt{130}}$

(c) $\frac{6}{65}$ (d) $-\frac{3}{\sqrt{130}}$

31. The function $f(x) = \log\left(x+\sqrt{x^2+1}\right)$, is **[2003]**

(a) neither an even nor an odd function
(b) an even function
(c) an odd function
(d) a periodic function.

32. The period of $\sin^2\theta$ is **[2002]**

(a) π^2 (b) π
(c) 2π (d) $\pi/2$

33. Which one is not periodic? **[2002]**

(a) $|\sin 3x| + \sin^2 x$ (b) $\cos\sqrt{x} + \cos^2 x$

(c) $\cos 4x + \tan^2 x$ (d) $\cos 2x + \sin x$

TOPIC 3 Solutions of Trigonometric Equations

34. If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in the interval : **[Sep. 02, 2020 (II)]**

(a) $\left(-\frac{5}{4}, -1\right)$ (b) $\left[-1, -\frac{1}{2}\right]$

(c) $\left(-\frac{1}{2}, -\frac{1}{4}\right]$ (d) $\left[-\frac{3}{2}, -\frac{5}{4}\right]$

35. The number of distinct solutions of the equation, $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$ in the interval $[0, 2\pi]$, is _____. **[Jan. 9, 2020 (I)]**

36. The number of solutions of the equation $1 + \sin^4 x = \cos^2 3x$, $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is : **[April 12, 2019 (I)]**

(a) 3 (b) 5

(c) 7 (d) 4

37. Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to : **[April 12, 2019 (II)]**

(a) R (b) [1, 4]

(c) [3,7] (d) [2, 6]

38. If $[x]$ denotes the greatest integer $\leq x$, then the system of linear equations

$[\sin\theta]\,x + [-\cos\theta]\,y = 0$

$[\cot\theta]\,x + y = 0$ **[April 12, 2019 (II)]**

(a) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

(b) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

(c) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

(d) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

39. Let $S = \{\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is: **[April 9, 2019 (I)]**

(a) $\frac{13\pi}{6}$ (b) $\frac{5\pi}{3}$

(c) 2π (d) π

40. If $0 \leq x < \frac{\pi}{2}$, then the number of values of x for which $\sin x - \sin 2x + \sin 3x = 0$, is: **[Jan. 09, 2019 (II)]**

(a) 3 (b) 1

(c) 4 (d) 2

41. The number of solutions of $\sin 3x = \cos 2x$, in the interval $\left(\frac{\pi}{2}, \pi\right)$ is **[Online April 15, 2018]**

(a) 3 (b) 4

(c) 2 (d) 1

42. If sum of all the solutions of the equation $8\cos x \cdot \left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right) - \frac{1}{2}\right) - 1$ in $[0, \pi]$ is $k\pi$, then k is equal to : **[2018]**

(a) $\frac{13}{9}$ (b) $\frac{8}{9}$

(c) $\frac{20}{9}$ (d) $\frac{2}{3}$

43. If $0 \leq x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is: **[2016]**

(a) 7 (b) 9

(c) 3 (d) 5

44. The number of $x \in [0, 2\pi]$ for which

$$\left|\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x}\right| = 1 \text{ is}$$

[Online April 9, 2016]

(a) 2 (b) 6

(c) 4 (d) 8

45. The number of values of α in $[0, 2\pi]$ for which $2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha = 2$, is: **[Online April 9, 2014]**

(a) 6 (b) 4

(c) 3 (d) 1

46. Let $A = \{\theta : \sin(\theta) = \tan(\theta)\}$ and $B = \{\theta : \cos(\theta) = 1\}$ be two sets. Then : **[Online April 25, 2013]**

(a) $A = B$

(b) $A \not\subset B$

(c) $B \not\subset A$

(d) $A \subset B$ and $B - A \neq \phi$

47. The number of solutions of the equation $\sin 2x - 2\cos x + 4\sin x = 4$ in the interval $[0, 5\pi]$ is : **[Online April 23, 2013]**

(a) 3 (b) 5
(c) 4 (d) 6

48. **Statement-1:** The number of common solutions of the trigonometric equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is two.

Statement-2: The number of solutions of the equation, $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, \pi]$ is two. **[Online April 22, 2013]**

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1.
(b) Statement-1 is true; Statement-2 is true; Statement-2 is **not** a correct explanation for statement-1.
(c) Statement-1 is false; Statement-2 is true.
(d) Statement-1 is true; Statement-2 is false.

49. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has : **[2012]**

(a) infinite number of real roots
(b) no real roots
(c) exactly one real root
(d) exactly four real roots

50. The possible values of $\theta \in (0, \pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are **[2011RS]**

(a) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(b) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$

(c) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(d) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$

51. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is **[2006]**

(a) 4 (b) 6
(c) 1 (d) 2

52. The number of solution of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi)$ is **[2002]**

(a) 2 (b) 3
(c) 0 (d) 1

Hints & Solutions

1. **(b)** $3(\sin\theta-\cos\theta)^4+6(\sin\theta+\cos\theta)^2+4\sin^6\theta$

$=3(1-2\sin\theta\cos\theta)^2+6(1+2\sin\theta\cos\theta)+4\sin^6\theta$

$=3(1+4\sin^2\theta\cos^2\theta-4\sin\theta\cos\theta)+6$

$\qquad -12\sin\theta\cos\theta+4\sin^6\theta$

$=9+12\sin^2\theta\cos^2\theta+4\sin^6\theta$

$=9+12\cos^2\theta(1-\cos^2\theta)+4(1-\cos^2\theta)^3$

$=9+12\cos^2\theta-12\cos^4\theta+4(1-\cos^6\theta-3\cos^2\theta+3\cos^4\theta)$

$=9+4-4\cos^6\theta$

$=13-4\cos^6\theta$

2. **(b)** Let $f_k(x)=\frac{1}{k}(\sin^k x+\cos^k x)$

Consider $f_4(x)-f_6(x)=\frac{1}{4}(\sin^4 x+\cos^4 x)$

$\qquad -\frac{1}{6}(\sin^6 x+\cos^6 x)$

$=\frac{1}{4}[1-2\sin^2 x\cos^2 x]\ -\frac{1}{6}[1-3\sin^2 x\cos^2 x]$

$=\frac{1}{4}-\frac{1}{6}=\frac{1}{12}$

3. **(d)** Given $2\cos\theta+\sin\theta=1$

Squaring both sides, we get

$(2\cos\theta+\sin\theta)^2=1^2$

$\Rightarrow 4\cos^2\theta+\sin^2\theta+4\sin\theta\cos\theta=1$

$\Rightarrow 3\cos^2\theta+(\cos^2\theta+\sin^2\theta)+4\sin\theta\cos\theta=1$

$\Rightarrow 3\cos^2\theta+\cancel{1}+4\sin\theta\cos\theta=\cancel{1}$

$\Rightarrow 3\cos^2\theta+4\sin\theta\cos\theta=0$

$\Rightarrow \cos\theta(3\cos\theta+4\sin\theta)=0$

$\Rightarrow 3\cos\theta+4\sin\theta=0 \Rightarrow 3\cos\theta=-4\sin\theta$

$\Rightarrow \frac{-3}{4}=\tan\theta=\sqrt{\sec^2\theta-1}=\frac{-3}{4}$

$\left(\because \tan\theta=\sqrt{\sec^2\theta-1}\right)$

$\Rightarrow \sec^2\theta-1=\left(\frac{-3}{4}\right)^2=\frac{9}{16}$

$\Rightarrow \sec^2\theta=\frac{9}{16}+1=\frac{25}{16}\Rightarrow \sec\theta=\frac{5}{4}$

or $\boxed{\cos\theta=\frac{4}{5}}$ ...(1)

Now, $\sin^2\theta+\cos^2\theta=1\Rightarrow \sin^2\theta+\left(\frac{4}{5}\right)^2=1$

$\sin^2\theta+\frac{4}{5}=1 \Rightarrow \sin^2\theta=1-\frac{16}{25}=\frac{9}{25}$

$\sin\theta=\pm\frac{3}{5}$ ...(2)

Taking $\left(\sin\theta=+\frac{3}{5}\right)$ because $\left(\sin\theta=-\frac{3}{5}\right)$ cannot satisfy the given equation.

Therefore; $7\cos\theta+6\sin\theta$

$=7\times\frac{4}{5}+6\times\frac{3}{5}=\frac{28}{5}+\frac{18}{5}=\frac{46}{5}$

4. **(b)** Given expression can be written as

$\frac{\sin A}{\cos A}\times\frac{\sin A}{\sin A-\cos A}+\frac{\cos A}{\sin A}\times\frac{\cos A}{\cos A-\sin A}$

$\left(\because \tan A=\frac{\sin A}{\cos A} \text{ and } \cot A=\frac{\cos A}{\sin A}\right)$

$=\frac{1}{\sin A-\cos A}\left\{\frac{\sin^3 A-\cos^3 A}{\cos A\sin A}\right\}$

$\because a^3-b^3=(a-b)(a^2+ab+b^2)$

$=\frac{\sin^2 A+\sin A\cos A+\cos^2 A}{\sin A\cos A}$

$=1+\sec A\operatorname{cosec} A$

5. **(c)** Consider $\cos 255°+\sin 195°$

$=\cos(270°-15°)+\sin(180°+15°)$

$=-\sin 15°-\sin 15°$

$=-2\sin 15°=-2\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)=-\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)$

6. **(c)** Let $f(x)=\sin x$ and $g(x)=x$

Statement-1: $f(x)\le g(x)\ \forall x\in(0,\infty)$

i.e., $\sin x\le x\ \forall x\in(0,\infty)$

which is true

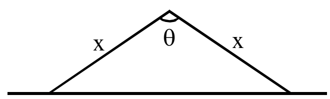
Statement-2: $f(x)\le 1\ \forall\ x\in(0,\infty)$

i.e., $\sin x\le 1\ \forall\ x\in(0,\infty)$

It is true and

$g(x)=x\to\infty$ as $x\to\infty$ also true.

7. **(c)** Area $= \frac{1}{2}x^2 \sin\theta$



Maximum value of $\sin\theta$ is 1 at $\theta = \frac{\pi}{2}$

$A_{max} = \frac{1}{2}x^2$

8. **(b)** $\cos^3\frac{\pi}{8}\left[4\cos^3\frac{\pi}{8} - 3\cos\frac{\pi}{8}\right]$

$+\sin^3\frac{\pi}{8}\left[3\sin\frac{\pi}{8} - 4\sin^3\frac{\pi}{8}\right]$

$= 4\cos^6\frac{\pi}{8} - 4\sin^6\frac{\pi}{8} - 3\cos^4\frac{\pi}{8} + 3\sin^4\frac{\pi}{8}$

$= 4\left[\left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right)\right]$

$\left[\left(\sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8} + \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right)\right]$

$-3\left[\left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right)\left(\cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8}\right)\right]$

$= \cos\frac{\pi}{4}\left[4\left(1 - \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right) - 3\right]$

$= \frac{1}{\sqrt{2}}\left[1 - \frac{1}{2}\right] = \frac{1}{2\sqrt{2}}$

9. **(1)** $\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos^2\beta}{2}} = \frac{1}{10}$

$\Rightarrow \frac{\sqrt{2}\sin\beta}{\sqrt{2}} = \frac{1}{\sqrt{10}}$

$\therefore \tan\alpha = \frac{1}{7}$ and $\sin\beta = \frac{1}{\sqrt{10}}$

$\tan\beta = \frac{1}{3}$

$\therefore \tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2.\frac{1}{3}}{1-\frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{3}{4}$

$\tan(\alpha+2\beta) = \frac{\tan\alpha + \tan 2\beta}{1 - \tan\alpha\tan 2\beta}$

$= \frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}.\frac{3}{4}} = \frac{\frac{4+21}{28}}{\frac{25}{28}} = 1$

10. **(d)** $L + M = 1 - 2\sin^2\frac{\pi}{8} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ ...(i)

and $L - M = -\cos\frac{\pi}{8}$ ...(ii)

From equations (i) and (ii),

$L = \frac{1}{2}\left(\frac{1}{\sqrt{2}} - \cos\frac{\pi}{8}\right) = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\frac{\pi}{8}$ and

$M = \frac{1}{2}\left(\frac{1}{\sqrt{2}} + \cos\frac{\pi}{8}\right) = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

11. **(a)** Let $f(x, y) = x + y - 1$

Given $(1, 2)$ and $(\sin\theta, \cos\theta)$ are lies on same side.

$\therefore f(1, 2)\cdot f(\sin\theta, \cos\theta) > 0$

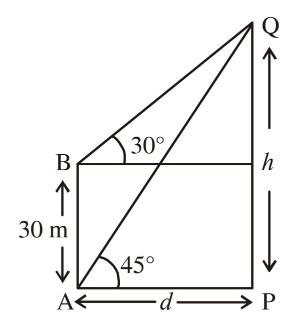
$\Rightarrow 2[\sin\theta + \cos\theta - 1] > 0$

$\Rightarrow \sin\theta + \cos\theta > 1 \Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$

$\Rightarrow \theta + \frac{\pi}{4} \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$

12. **(a)** Let the height of the tower be h and distance of the foot of the tower from the point A is d.

By the diagram,



$\tan 45° = \frac{h}{d} = 1$

$h = d$ ...(i)

$\tan 30° = \frac{h-30}{d}$

$\sqrt{3}(h-30) = d$ ...(ii)

Put the value of h from (i) to (ii),

$\sqrt{3}d = d + 30\sqrt{3}$

$d = \frac{30\sqrt{3}}{\sqrt{3}-1} = 15\sqrt{3}\left(\sqrt{3}+1\right) = 15\left(3+\sqrt{3}\right)$

13. **(b)** $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$

$$= \left(\frac{1+\cos 20°}{2}\right) + \left(\frac{1+\cos 100°}{2}\right) - \frac{1}{2}(2\cos 10° \cos 50°)$$

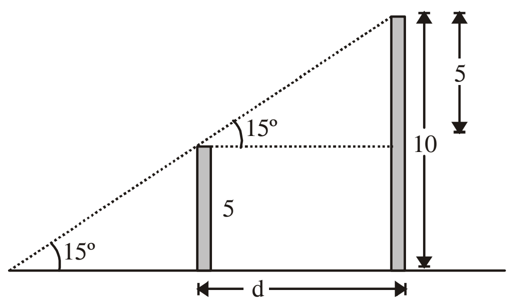
$$= 1 + \frac{1}{2}(\cos 20° + \cos 100°) - \frac{1}{2}[\cos 60° + \cos 40°]$$

$$= \left(1 - \frac{1}{4}\right) + \frac{1}{2}[\cos 20° + \cos 100° - \cos 40°]$$

$$= \frac{3}{4} + \frac{1}{2}[2\cos 60° \times \cos 40° - \cos 40°]$$

$$= \frac{3}{4}$$

14. **(a)**



By the diagram,

$$\tan 15° = \frac{5}{d} \Rightarrow d = \frac{5}{\tan 15°} = \frac{5\left(\sqrt{3}+1\right)}{\sqrt{3}-1}$$

$$= \frac{5\left(4+2\sqrt{3}\right)}{2} = 5\left(2+\sqrt{3}\right)$$

15. **(a)** $\because \sin(60° + A).\sin(60° - A) \sin A = \frac{1}{4} \sin 3A$

$\therefore \sin 10° \sin 50° \sin 70° = \sin 10° \sin(60° - 10°)$

$\sin(60° + 10°) = \frac{1}{4} \sin 30°$

$\Rightarrow \sin 10° \sin 30° \sin 50° \sin 70° = \frac{1}{4} \sin^2 30 = \frac{1}{16}$

16. **(b)** $\because \alpha + \beta$ and $\alpha - \beta$ both are acute angles.

$\cos(\alpha + \beta) = \frac{3}{5}$, then $\sin(\alpha + \beta) = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$

$\tan(\alpha + \beta) = \frac{4}{3}$

And $\sin(\alpha - \beta) = \frac{5}{13}$, then

$$\cos(\alpha - \beta) = \sqrt{1 - \left(\frac{5}{13}\right)^2} = \frac{12}{13}$$

$\Rightarrow \tan(\alpha - \beta) = \frac{5}{12}$

Now, $\tan 2\alpha = \tan((\alpha + \beta) + (\alpha - \beta))$

$$= \frac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta).\tan(\alpha-\beta)} = \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3}.\frac{5}{12}} = \frac{63}{16}$$

17. **(d)** $\because$ The given equation is

$\sin^4 \alpha + 4\cos^4 \beta + 2 = 4\sqrt{2} \sin \alpha \cdot \cos \beta$, $\alpha, \beta \in [0, \pi]$

Then, by A.M., G.M. ineqality;

A.M. $\geq$ G.M.

$$\frac{\sin^4 \alpha + 4\cos^4 \beta + 1 + 1}{4} \geq \left(\sin^4 \alpha \cdot 4\cos^4 \beta \cdot 1 \cdot 1\right)^{\frac{1}{4}}$$

$\sin^4\alpha + 4\cos^4\beta + 1 + 1 \geq 4\sqrt{2} \sin \alpha \cdot |\cos \beta|$

Inequality still holds when $\cos\beta < 0$ but L.H.S. is positive than $\cos\beta > 0$, then

L.H.S. = R.H.S

$\therefore \sin^4 \alpha = 1$ and $\cos^4 \beta = \frac{1}{4}$

$\Rightarrow \alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{4}$

$\therefore \cos(\alpha + \beta) - \cos(\alpha - \beta)$

$$= \cos\left(\frac{\pi}{2} + \beta\right) - \cos\left(\frac{\pi}{2} - \beta\right)$$

$$= -\sin\beta - \sin\beta = -2\sin\frac{\pi}{4} = -\sqrt{2}$$

18. **(a)** $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$

$$f_4(x) = \frac{1}{4}[\sin^4 x + \cos^4 x]$$

$$= \frac{1}{4}\left[(\sin^2 x + \cos^2 x)^2 - \frac{(\sin 2x)^2}{2}\right]$$

$$= \frac{1}{4}\left[1 - \frac{(\sin 2x)^2}{2}\right]$$

$$f_6(x) = \frac{1}{6}[\sin^6 x + \cos^6 x]$$

$$= \frac{1}{6}\left[(\sin^2 x + (\cos^2 x) - \frac{3}{4}(\sin^2 x)^2\right]$$

$=\frac{1}{6}\left[1-\frac{3}{4}(\sin 2x)^2\right]$

Now $f_4(x)-f_{(6)}(x)=\frac{1}{4}-\frac{1}{6}-\frac{(\sin 2x)^2}{8}+\frac{1}{8}(\sin 2x)^2$

$=\frac{1}{12}$

19. **(a)** $A=\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}...\cos\frac{\pi}{2^{10}}.\sin\frac{\pi}{2^{10}}$

$=\frac{1}{2}\left(\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}...\cos\frac{\pi}{2^9}\sin\frac{\pi}{2^9}\right)$

$=\frac{1}{2^8}\left(\cos\frac{\pi}{2^2}.\sin\frac{\pi}{2^2}\right)=\frac{1}{2^9}\sin\frac{\pi}{2}$

$=\frac{1}{512}$

20. **(a)** We have

$5\tan^2 x-5\cos^2 x=2(2\cos^2 x-1)+9$

$\Rightarrow 5\tan^2 x-5\cos^2 x=4\cos^2 x-2+9$

$\Rightarrow 5\tan^2 x=9\cos^2 x+7$

$\Rightarrow 5(\sec^2 x-1)=9\cos^2 x+7$

Let $\cos^2 x=t$

$\Rightarrow \frac{5}{t}-9t-12=0$

$\Rightarrow 9t^2+12t-5=0$

$\Rightarrow 9t^2+15t-3t-5=0$

$\Rightarrow (3t-1)(3t+5)=0$

$\Rightarrow t=\frac{1}{3}$ as $t\neq-\frac{5}{3}$.

$\cos 2x=2\cos^2 x-1=2\left(\frac{1}{3}\right)-1=-\frac{1}{3}$

$\cos 4x=2\cos^2 2x-1=2\left(-\frac{1}{3}\right)^2-1=-\frac{7}{9}$

21. **(b)** $4+\frac{1}{2}\sin^2 2x-2\cos^4 x$

$4+2(1-\cos^2 x)\cos^2 x-2\cos^4 x$

$-4\left\{\cos^4 x-\frac{\cos^2 x}{2}-1+\frac{1}{16}-\frac{1}{16}\right\}$

$-4\left\{\left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16}\right\}$

$0\leq\cos^2 x\leq 1$

$-\frac{1}{4}\leq\cos^2 x-\frac{1}{4}\leq\frac{3}{4}$

$0\leq\left(\cos^2 x-\frac{1}{4}\right)^2\leq\frac{9}{16}$

$-\frac{17}{16}\leq\left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16}\leq\frac{9}{16}-\frac{17}{16}$

$\frac{17}{4}\geq-4\left\{\left(\cos^2 x-\frac{1}{4}\right)^2-\frac{17}{16}\right\}\geq\frac{1}{2}$

$M=\frac{17}{4}$

$m=\frac{1}{2}$

$M-m=\frac{17}{4}-\frac{2}{4}=\frac{15}{4}$

22. **(b)** Let $\cos\alpha+\cos\beta=\frac{3}{2}$

$\Rightarrow 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=\frac{3}{2}$ ...(i)

and $\sin\alpha+\sin\beta=\frac{1}{2}$

$\Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=\frac{1}{2}$ ...(ii)

On dividing (ii) by (i), we get

$\tan\left(\frac{\alpha+\beta}{2}\right)=\frac{1}{3}$

Given : $\theta=\frac{\alpha+\beta}{2}\Rightarrow 2\theta=\alpha+\beta$

Consider $\sin 2\theta+\cos 2\theta=\sin(\alpha+\beta)+\cos(\alpha+\beta)$

$=\frac{\frac{2}{3}}{1+\frac{1}{9}}+\frac{1-\frac{1}{9}}{1+\frac{1}{9}}=\frac{6}{10}+\frac{8}{10}=\frac{7}{5}$

23. **(b)** $\text{cosec}\theta=\frac{p+q}{p-q}$, $\sin\theta=\frac{p-q}{p+q}$

$\cos\theta=\pm\sqrt{1-\sin^2\theta}=\sqrt{1-\left(\frac{p-q}{p+q}\right)^2}=\frac{2\sqrt{pq}}{(p+q)}$

$\left|\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right|=\left|\frac{\cot\frac{\pi}{4}\cot\frac{\theta}{2}-1}{\cot\frac{\pi}{4}+\cot\frac{\theta}{2}}\right|=\left|\frac{\cot\frac{\theta}{2}-1}{\cot\frac{\theta}{2}+1}\right|$

$=\left|\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right|$

On rationalizing denominator, we get

$$\left|\left(\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right)\left(\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right)\right|$$

$$=\left|\frac{\cos\theta}{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right|$$

$$=\left|\frac{\cos\theta}{1+\sin\theta}\right|=\left|\frac{2\sqrt{pq}/(p+q)}{1+\frac{(p-q)}{p+q}}\right|=\frac{\sqrt{pq}}{p}=\sqrt{\frac{q}{p}}$$

24. **(d)** $A=\sin^2 x+\cos^4 x$

$=\sin^2 x+\cos^2 x(1-\sin^2 x)$

$=\sin^2 x+\cos^2 x-\frac{1}{4}(2\sin x.\cos x)^2$

$=1-\frac{1}{4}\sin^2(2x)$

$\because -1\le \sin 2x\le 1$

$\Rightarrow 0\le \sin^2(2x)\le 1$

$\Rightarrow 0\ge -\frac{1}{4}\sin^2(2x)\ge -\frac{1}{4}$

$\Rightarrow 1\ge 1-\frac{1}{4}\sin^2(2x)\ge 1-\frac{1}{4}$

$\Rightarrow 1\ge A\ge \frac{3}{4}$

25. **(a)** $\cos(\alpha+\beta)=\frac{4}{5}\Rightarrow \tan(\alpha+\beta)=\frac{3}{4}$

$\sin(\alpha-\beta)=\frac{5}{13}\Rightarrow \tan(\alpha-\beta)=\frac{5}{12}$

$\tan 2\alpha=\tan[(\alpha+\beta)+(\alpha-\beta)]$

$$=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}=\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}.\frac{5}{12}}=\frac{56}{33}$$

26. **(b)** Given that

$\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)=-\frac{3}{2}$

$\Rightarrow\ 2[\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)]+3=0$

$\Rightarrow\ 2[\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)]+\sin^2\alpha+\cos^2\alpha+\sin^2\beta+\cos^2\beta+\sin^2\gamma+\cos^2\alpha=0$

$\Rightarrow\ [\sin^2\alpha+\sin^2\beta+\sin^2\gamma+2\sin\alpha\sin\beta+2\sin\beta\sin\gamma+2\sin\gamma\sin\alpha]+[\cos^2\alpha+\cos^2\beta+\cos^2\gamma+2\cos\alpha\cos\beta+2\cos\beta\cos\gamma+2\cos\gamma\cos\alpha]=0$

$[\because \cos(A-B)=\cos A.\cos B+\sin A.\sin B]$

$\Rightarrow\ [\sin\alpha+\sin\beta+\sin\gamma]^2+(\cos\alpha+\cos\beta+\cos\gamma)^2=0$

$\Rightarrow\ \sin\alpha+\sin\beta+\sin\gamma=0$ and $\cos\alpha+\cos\beta+\cos\gamma=0$

$\therefore$ A and B both are true.

27. **(c)** Given that $p^2+q^2=1$

$\therefore\ p=\cos\theta$ and $q=\sin\theta$ satisfy the given equation

Then $p+q=\cos\theta+\sin\theta$

We know that

$-\sqrt{a^2+b^2}\le a\cos\theta+b\sin\theta\le\sqrt{a^2+b^2}$

$\therefore -\sqrt{2}\le\cos\theta+\sin\theta\le\sqrt{2}$

Hence max. value of $p+q$ is $\sqrt{2}$

28. **(c)** $\cos x+\sin x=\frac{1}{2}\Rightarrow 1+\sin 2x=\frac{1}{4}$

$\Rightarrow \sin 2x=-\frac{3}{4}$,

$\therefore \pi<2x<2\pi$

$\Rightarrow \frac{\pi}{2}<x\le\pi$ ...(i)

$\frac{2\tan x}{1+\tan^2 x}=-\frac{3}{4}$

$\Rightarrow\ 3\tan^2 x+8\tan x+3=0$

$\therefore\ \tan x=\frac{-8\pm\sqrt{64-36}}{6}=-\frac{-4\pm\sqrt{7}}{3}$

for $\frac{\pi}{2}<x<\pi$, $\tan x<0$

$\therefore\ \tan x=\frac{-4-\sqrt{7}}{3}$

29. **(a)** $u^2=a^2+b^2+2\sqrt{\begin{matrix}(a^4+b^4)\cos^2\theta\sin^2\theta\\ +a^2b^2(\cos^4\theta+\sin^4\theta)\end{matrix}}$ ...(1)

Now, $(a^4+b^4)\cos^2\theta\sin^2\theta+a^2b^2(\cos^4\theta+\sin^4\theta)$

$=(a^4+b^4)\cos^2\theta\sin^2\theta+a^2b^2(1-2\cos^2\theta\sin^2\theta)$

$=(a^4+b^4-2a^2b^2)\cos^2\theta\sin^2\theta+a^2b^2$

$=(a^2-b^2)^2.\frac{\sin^2 2\theta}{4}+a^2b^2$ ...(2)

$\because 0\le\sin^2 2\theta\le 1$

$\Rightarrow 0\le(a^2-b^2)^2\frac{\sin^2 2\theta}{4}\le\frac{(a^2-b^2)^2}{4}$

$\Rightarrow a^2b^2\le(a^2-b^2)^2\frac{\sin^2 2\theta}{4}+a^2b^2$

$\le(a^2-b^2)^2.\frac{1}{4}+a^2b^2$ ....(3)

From (1)

$$a^2+b^2+2\sqrt{a^2b^2} \le u^2 \le a^2+b^2+\frac{2}{2}\sqrt{(a^2+b^2)^2}$$

$$(a+b)^2 \le u^2 \le 2(a^2+b^2)$$

$\therefore$ Max. value – Min. value

$$= 2(a^2+b^2)-(a+b^2) = (a-b)^2$$

30. **(d)** $\pi < \alpha - \beta < 3\pi$

$$\Rightarrow \frac{\pi}{2} < \frac{\alpha-\beta}{2} < \frac{3\pi}{2} \Rightarrow \cos\frac{\alpha-\beta}{2} < 0 \quad ....(1)$$

$$\sin\alpha + \sin\beta = -\frac{21}{65}$$

$$\Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -\frac{21}{65} \quad ....(2)$$

$$\cos\alpha + \cos\beta = -\frac{27}{65}$$

$$\Rightarrow 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -\frac{27}{65} \quad ....(3)$$

Squaring and adding (2) and (3), we get

$$4\cos^2\frac{\alpha-\beta}{2} = \frac{(21)^2+(27)^2}{(65)^2} = \frac{1170}{65\times 65}$$

$$\therefore \cos^2\frac{\alpha-\beta}{2} = \frac{9}{130} \Rightarrow \cos\frac{\alpha-\beta}{2} = -\frac{3}{\sqrt{130}} \quad \text{[from (1)]}$$

31. **(c)** Given $f(x) = \log(x+\sqrt{x^2+1})$

$$f(-x) = \log\left\{-x+\sqrt{x^2+1}\right\} = \log\left\{\frac{x^2-x^2+1}{x+\sqrt{x^2+1}}\right\}$$

$$= -\log(x+\sqrt{x^2+1}) = -f(x)$$

$\Rightarrow f(x)$ is an odd function.

32. **(b)** We know that $\sin^2\theta = \frac{1-\cos 2\theta}{2}$;

Since period of $\cos 2\theta = \frac{2\pi}{2} = \pi$

Hence period of $\sin^2\theta$ is also π.

33. **(b)** we know that $\cos\sqrt{x}$ is non periodic

$\therefore$ $\cos\sqrt{x}+\cos^2 x$ can not be periodic.

34. **(b)** $\sin^4\theta + \cos^4\theta = -\lambda$

$$\Rightarrow (\sin^2\theta+\cos^2\theta)^2 - 2\sin^2\theta\cdot\cos^2\theta = -\lambda$$

$$\Rightarrow 1-2\sin^2\theta\cos^2\theta = -\lambda$$

$$\Rightarrow \lambda = \frac{(\sin 2\theta)^2}{2} - 1$$

$\Rightarrow$ as $\sin^2 2\theta \in [0, 1]$

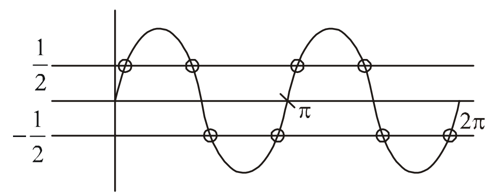
$$\Rightarrow \lambda \in \left[-1, \frac{-1}{2}\right]$$

35. **(8)** $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$

$\Rightarrow$ $\log_{1/2}|\sin x\cos x| = 2$

$$\Rightarrow \quad |\sin x\cos x| = \frac{1}{4}$$

$$\Rightarrow \quad \sin 2x = \pm\frac{1}{2}$$



Hence, total number of solutions = 8.

36. **(c)** Consider equation, $1+\sin^4 x = \cos^2 3x$

L.H.S. $= 1+\sin^4 x$ and R.H.S. $= \cos^2 3x$

$\because$ L.H.S. ≥ 1 and R.H.S. $\le 1 \Rightarrow$ L.H.S. = R.H.S. = 1

$\sin^4 x = 0$, and $\cos^2 3x = 1$

$\Rightarrow$ $\sin x = 0$ and $(4\cos^2 x - 3)^2\cos^2 x = 1$

$\Rightarrow$ $\sin x = 0$ and $\cos^2 x = 1 \Rightarrow x = 0, \pm\pi, \pm 2\pi$

Hence, total number of solutions is 5.

37. **(d)** Given equation is, $\cos 2x + \alpha\sin x = 2\alpha - 7$

$1-2\sin^2 x + \alpha\sin x = 2\alpha - 7$

$2\sin^2 x - \alpha\sin x + (2\alpha - 8) = 0$

$$\Rightarrow \quad \sin x = \frac{\alpha\pm\sqrt{\alpha^2-8(2\alpha+8)}}{4}$$

$$\Rightarrow \quad \sin x = \frac{\alpha\pm(\alpha-8)}{4} \Rightarrow \sin x = \frac{\alpha-4}{4}$$

[$\sin x = 2$ (rejected)]

$\because$ equation has solution, then $\frac{\alpha-4}{4} \in [-1, 1]$

$\Rightarrow$ $\alpha \in [2, 6]$

38. **(a)** According to the question, there are two cases.

Case 1 : $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

In this interval, $[\sin\theta] = 0$, $[-\cos\theta] = 0$ and $[\cot\theta] = -1$

Then the system of equations will be ;

$0\cdot x + 0\cdot y = 0$ and $-x+y = 0$

Which have infinitely many solutions.

Case 2 : $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

In this interval, $[\sin\theta] = -1$ and $[-\cos\theta] = 0$, Then the system of equations will be ;

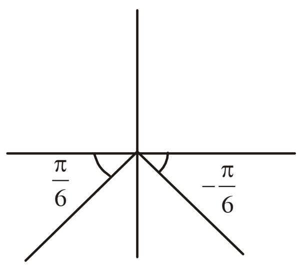
$-x+0\cdot y=0$ and $[\cot\theta]x+y=0$

Clearly, $x=0$ and $y=0$ which has unique solution.

39. **(c)** $2\cos^2\theta+3\sin\theta=0$

$(2\sin\theta+1)(\sin\theta-2)=0$

$\Rightarrow \sin\theta=-\dfrac{1}{2}$ or $\sin\theta=2\rightarrow$ Not possibe



The required sum of all solutions in $[-2\pi, 2\pi]$ is

$$=\left(\pi+\frac{\pi}{6}\right)+\left(2\pi-\frac{\pi}{6}\right)+\left(-\frac{\pi}{6}\right)+\left(-\pi+\frac{\pi}{6}\right)=2\pi$$

40. **(d)** $\sin x-\sin 2x+\sin 3x=0$

$\Rightarrow \sin x-2\sin x.\cos x+3\sin x-4\sin^3 x=0$

$\Rightarrow 4\sin x-4\sin^3 x-2\sin x.\cos x=0$

$\Rightarrow 2\sin x(1-\sin^2 x)-\sin x.\cos x=0$

$\Rightarrow 2\sin x.\cos^2 x-\sin x.\cos x=0$

$\Rightarrow \sin x.\cos x(2\cos x-1)=0$

$\therefore \sin x=0, \cos x=0, \cos x=\dfrac{1}{2}$

$\therefore x=0,\dfrac{\pi}{3}\quad \because x\in\left[0,\dfrac{\pi}{2}\right)$

41. **(d)** $\sin 3x=\cos 2x$

$\Rightarrow 3\sin x-4\sin^3 x=1-2\sin^2 x$

$\Rightarrow 4\sin^3 x-2\sin^2 x-3\sin x+1=0$

$\Rightarrow \sin x=1, \dfrac{-2\pm2\sqrt{5}}{8}$

In the interval $\left(\dfrac{\pi}{2},\pi\right)$, $\sin x=\dfrac{-2+2\sqrt{5}}{8}$

So, there is only one solution.

42. **(a)** $\because 8\cos x\left(\cos^2\dfrac{\pi}{6}-\sin^2 x-\dfrac{1}{2}\right)=1$

$\Rightarrow 8\cos x\left(\dfrac{3}{4}-\dfrac{1}{2}-\sin^2 x\right)=1$

$\Rightarrow 8\cos x\left(\dfrac{1}{4}-(1-\cos^2 x)\right)=1$

$\Rightarrow 8\cos x\left(\dfrac{1}{4}-1+\cos^2 x\right)=1$

$\Rightarrow 8\cos x\left(\cos^2 x-\dfrac{3}{4}\right)=1$

$\Rightarrow 8\left(\dfrac{4\cos^3 x-3\cos x}{4}\right)=1$

$\Rightarrow 2(4\cos^3 x-3\cos x)=1$

$\Rightarrow 2\cos 3x=1\Rightarrow \cos 3x=\dfrac{1}{2}$

$\therefore 3x=2n\pi\pm\dfrac{\pi}{3}, n\in I$

$\Rightarrow x=\dfrac{2n\pi}{3}\pm\dfrac{\pi}{9}$

In $x\in[0,\pi]: x=\dfrac{\pi}{9},\dfrac{2\pi}{3}+\dfrac{\pi}{9},\dfrac{2\pi}{3}-\dfrac{\pi}{9}$, only

Sum of all the solutions of the equation

$$=\left(\frac{1}{9}+\frac{2}{3}+\frac{1}{9}+\frac{2}{3}-\frac{1}{9}\right)\pi=\frac{13}{9}\pi$$

43. **(a)** $\cos x+\cos 2x+\cos 3x+\cos 4x=0$

$\Rightarrow 2\cos 2x\cos x+2\cos 3x\cos x=0$

$\Rightarrow 2\cos x\left(2\cos\dfrac{5x}{2}\cos\dfrac{x}{2}\right)=0$

$\cos x=0,\ \cos\dfrac{5x}{2}=0,\ \cos\dfrac{x}{2}=0$

$x=\pi,\dfrac{\pi}{2},\dfrac{3\pi}{2},\dfrac{\pi}{5},\dfrac{3\pi}{5},\dfrac{7\pi}{5},\dfrac{9\pi}{5}$

44. **(d)** $\left|\sqrt{2\sin^4 x+18\cos^2 x}-\sqrt{2\cos^4 x+18\sin^2 x}\right|=1$

$\sqrt{2\sin^4 x+18\cos^2 x}-\sqrt{2\cos^4 x+18\sin^2 x}=\pm1$

$\sqrt{2\sin^4 x+18\cos^2 x}=\pm1+\sqrt{2\cos^4 x+18\sin^2 x}$

by squaring both the sides we will get 8 solutions

45. **(c)** $2\sin^3\alpha-7\sin^2\alpha+7\sin\alpha-2=0$

$\Rightarrow 2\sin^2\alpha(\sin\alpha-1)-5\sin\alpha(\sin\alpha-1)+2(\sin\alpha-1)=0$

$\Rightarrow (\sin\alpha-1)(2\sin^2\alpha-5\sin\alpha+2)=0$

$\Rightarrow \sin\alpha-1=0$ or $2\sin^2\alpha-5\sin\alpha+2=0$

$\sin\alpha=1$ or $\sin\alpha=\dfrac{5\pm\sqrt{25-16}}{4}=\dfrac{5\pm3}{4}$

$\alpha=\dfrac{\pi}{2}$ or $\sin\alpha=\dfrac{1}{2},2$

Now, $\sin\alpha\neq2$

for, $\sin \alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{3}, \frac{2\pi}{3}$

There are three values of α between $[0, 2\pi]$

46. **(b)** Let $A = \{\theta : \sin\theta = \tan\theta\}$

and $B = \{\theta : \cos\theta = 1\}$

Now, $A = \left\{\theta : \sin\theta = \frac{\sin\theta}{\cos\theta}\right\}$

$= \{\theta : \sin\theta(\cos\theta - 1) = 0\}$

$= \{\theta = 0, \pi, 2\pi, 3\pi, ...\}$

For $B : \cos\theta = 1 \Rightarrow \theta = \pi, 2\pi, 4\pi, ...$

This shows that A is not contained in B. i.e. $A \not\subset B$. but $B \subset A$.

47. **(a)** $\sin 2x - 2\cos x + 4\sin x = 4$

$\Rightarrow 2\sin x \,.\, \cos x - 2\cos x + 4\sin x - 4 = 0$

$\Rightarrow (\sin x - 1)(\cos x - 2) = 0$

$\because \cos x - 2 \neq 0, \therefore \sin x = 1$

$\therefore x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$

48. **(b)** $2\sin^2\theta - \cos 2\theta = 0$

$\Rightarrow 2\sin^2\theta - (1 - 2\sin^2\theta) = 0$

$\Rightarrow 2\sin^2\theta - 1 + 2\sin^2\theta = 0$

$\Rightarrow 4\sin^2\theta = 1 \Rightarrow \sin\theta = \pm\frac{1}{2}$

$\therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \theta \in [0, 2\pi]$

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Now $2\cos^2\theta - 3\sin\theta = 0$

$\Rightarrow 2(1 - \sin^2\theta) - 3\sin\theta = 0$

$\Rightarrow -2\sin^2\theta - 3\sin\theta + 2 = 0$

$\Rightarrow -2\sin^2\theta - 4\sin\theta + \sin\theta + 2 = 0$

$\Rightarrow 2\sin^2\theta - \sin\theta + 4\sin\theta - 2 = 0$

$\Rightarrow \sin\theta(2\sin\theta - 1) + 2(2\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = \frac{1}{2}, -2$

But $\sin\theta = -2$, is not possible

$\therefore \sin\theta = \frac{1}{2}, -2 \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Hence, there are two common solution, there each of the statement-1 and 2 are true but statement-2 is not a correct explanation for statement-1.

49. **(b)** Given equation is $e^{\sin x} - e^{-\sin x} - 4 = 0$

Put $e^{\sin x} = t$ in the given equation, we get

$t^2 - 4t - 1 = 0$

$\Rightarrow t = \frac{4 \pm \sqrt{16+4}}{2} = \frac{4 \pm \sqrt{20}}{2}$

$= \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$

$\Rightarrow e^{\sin x} = 2 \pm \sqrt{5}$ $(\because t = e^{\sin x})$

$\Rightarrow e^{\sin x} = 2 - \sqrt{5}$ and $e^{\sin x} = 2 + \sqrt{5}$

$\Rightarrow e^{\sin x} = 2 - \sqrt{5} < 0$

and $\sin x = \ln(2 + \sqrt{5}) > 1$

So, rejected.

Hence, given equation has no solution.

$\therefore$ The equation has no real roots.

50. **(d)** $\sin 4\theta + 2\sin 4\theta \cos 3\theta = 0$

$\sin 4\theta(1 + 2\cos 3\theta) = 0$

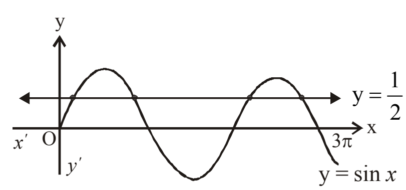
$\sin 4\theta = 0$ or $\cos 3\theta = -\frac{1}{2}$

$4\theta = n\pi; n \in I$

or $3\theta = 2n\pi \pm \frac{2\pi}{3}, n \in I$

$\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ or $\theta = \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{4\pi}{9}$ $[\because \theta, \in (0, \pi)]$

51. **(a)**



$2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow (\sin x + 3)(2\sin x - 1) = 0$

$\Rightarrow \sin x = \frac{1}{2}$ and $\sin x \neq -3$

$\therefore$ In $[0, 3\pi]$, x has 4 values.

52. **(b)** $\because \tan x + \sec x = 2\cos x;$

$\Rightarrow \sin x + 1 = 2\cos^2 x$

$\Rightarrow \sin x + 1 = 2(1 - \sin^2 x);$

$\Rightarrow 2\sin^2 x + \sin x - 1 = 0;$

$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0$

$\Rightarrow \sin x = \frac{1}{2}, -1.;$

$\Rightarrow x = 30°, 150°, 270°.$

Number of solution = 3