ALL INDIA TEST SERIES

FULL TEST - 15

JEE (Main)

Time Allotted: 3 Hours **Maximum Marks: 300**

General Instructions:

- The test consists of total 90 questions.
- Each subject (PCM) has 30 questions.
- This question paper contains **Three Parts.**
- **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics.
- Each part has only two sections: **Section-A and Section-B**.
- **Section – A** : Attempt all questions.
- **Section – B :** Do any five questions out of 10 Questions.

  Section-A (01 – 20, 31 – 50, 61 – 80) *contains 60 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

  Section-B (21 – 30, 51 – 60, 81 – 90) *contains 30 Numerical answer type questions with answer XXXXX.XX and each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

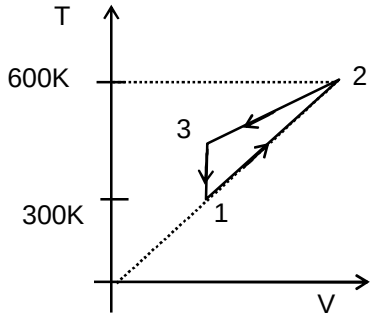
Physics

PART – A

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

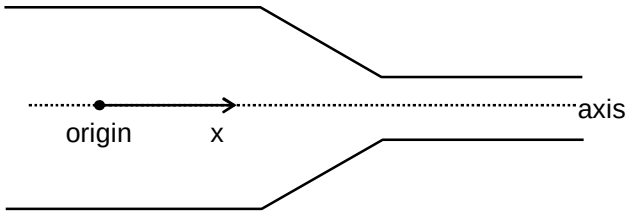
1. Two moles of an ideal gas is taken through a cyclic process 1-2-3-1 as shown in the T-V diagram. If the heat rejected in the cyclic process is 300 J, then work done by the gas in the process 2-3 is (Assume R= 8.3 J/mol K)

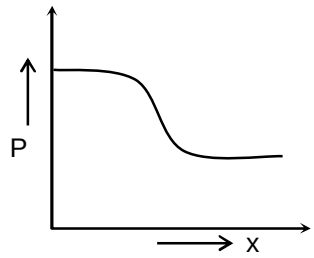


(A) −4980 J
(B) +4980 J
(C) +5280 J
(D) −5280 J

2. A sphere is released on a smooth inclined plane from the top. When it moves down its angular momentum is:
(A) conserved about every point
(B) conserved about the point of contact only
(C) conserved about the centre of the sphere only
(D) Conserved about any point on a line parallel to the inclined plane and passing through the centre of the ball.

3. An ideal liquid in streamline motion flows through a frictionless duct with varying cross-section as shown in figure. Pressure P at points along the x axis of duct is most likely to be represented by :



(A)


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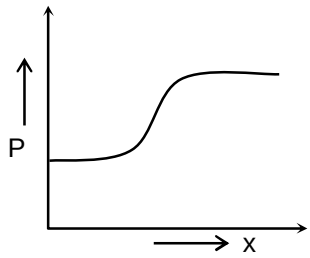

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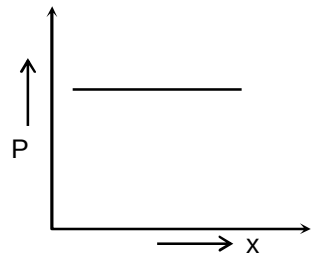

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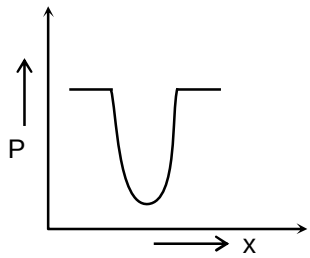
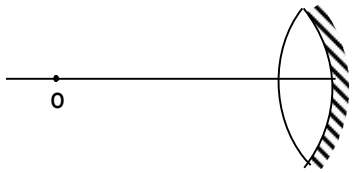
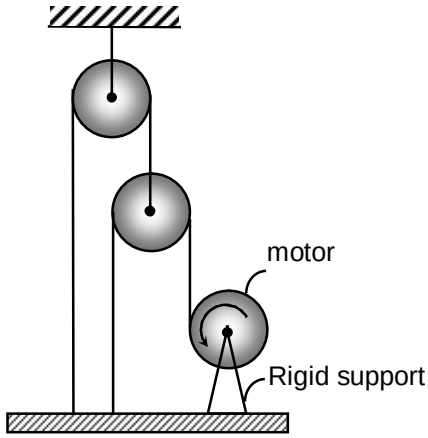


4. One face of a biconvex lens of radius $R_1 = R_2 = 30$ cm and $\mu = 1.5$ is silvered as shown in the figure. The image of a point object placed at a distance 60 cm from the lens on its optic axis is formed at a distance



(A) 60/7 cm behind the silver face
(B) 60/9 cm behind the silvered face
(C) 60/7 cm in front of silvered face
(D) none of these

5. In the arrangement shown, motor (fixed to the platform below the motor using rigid supports) is driving so that rope is winding (tangential speed at which rope is winding at motor pulley i.e. wrt motor) at a rate of v m/s, then the upward speed of platform will be



(A) $\frac{v}{3}$
(B) $\frac{v}{4}$
(C) $\frac{v}{7}$
(D) None of these

6. In an AC circuit, when applied voltage is $V = V_0 \sin(\omega t - \pi/4)$ then resulting current is $I = I_0 \sin(\omega t + \pi/4)$, then average power developed in the circuit is
(A) $V_0 I_0 \sqrt{2}$
(B) $V_0 I_0$
(C) zero
(D) None of the above

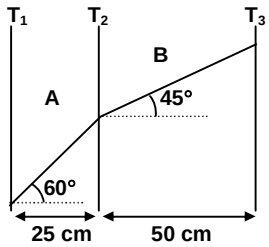
7. Two semicircular rings having same mass m and radius r are joined as shown in the figure. The moment of inertia of the system about an axis passing through the centre of mass of the system and perpendicular to the plane of the figure is

(A) $2\,mr^2$
(B) $4\,mr^2$
(C) mr^2
(D) $\frac{3}{2}mr^2$

8. A particle is moving along a straight line such that its position depends on time as $x = 2 - at + bt^2$, where a = 3 m/s, b = 1 m/s^2. Then the distance covered by the particle during first 3 seconds from starting of the motion is
(A) 2 m
(B) 5 m
(C) 7 m
(D) 4.5 m

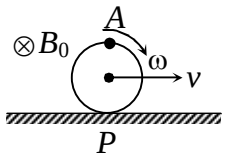
9. A metallic surface is irradiated with a light of variable wavelength. When the wavelength of light is more than 5000 Å, no photoelectrons are emitted from the surface. With an unknown wavelength, a stopping potential of 3 V is necessary to eliminate the photo current. The unknown wavelength is
(A) 2260Å
(B) 4133 Å
(C) 3126 Å
(D) 2679 Å

10. Two conductors A and B each of cross section area 5 cm^2 are connected in series. Variation of temperature (in °C) along the length (in cm) is as shown in the figure. If thermal conductivity of A is 200 J/m-sec-°C and the heat current in the conductors is 0.173 J/s. Find the thermal conductivity of B. (Take $\sqrt{3} = 1.732$)



(A) 346.40 J/m-sec-°C
(B) 173.20 J/m-sec-°C
(C) 400 J/m-sec-°C
(D) 100 J/m-sec-°C

11. A ring of radius R is rolling without slipping on a horizontal surface as shown in the figure. A uniform magnetic field B_0 exists perpendicular to the plane of motion. The potential difference between the points A and P is

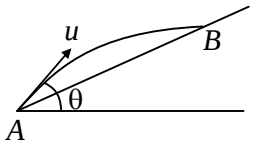


(A) B_0Rv
(B) $2B_0Rv$
(C) $\frac{B_0R^2\omega}{2}$
(D) $\frac{1}{3}B_0R^2\omega$

12. A uniform rope of length *l* and mass *M* hangs vertically from a rigid support. A block of mass *m* is attached to the free end of the rope. A transverse wave pulse of wavelength λ is produced at the lower end of the rope. The wavelength of the pulse, when it reaches the top of the rope, is

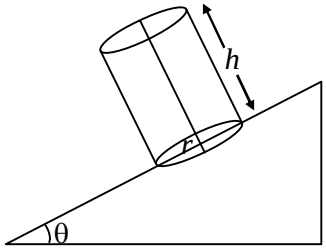
(A) $\lambda\sqrt{\frac{M-m}{m}}$
(B) $\lambda\left(\frac{m}{M-m}\right)$
(C) $\lambda\sqrt{\frac{m}{M+m}}$
(D) $\lambda\sqrt{\frac{M+m}{m}}$

13. A particle is projected from point *A* with a projection speed $u = 20$ m/s at an angle of projection $\theta = 30^o$ from horizontal as shown in the figure. It strikes an inclined plane at point *B* after one second of the projection. The separation *AB* is (g = 10 m/s^2)



(A) $4\sqrt{13}$ m
(B) $5\sqrt{13}$ m
(C) $4\sqrt{10}$ m
(D) $5\sqrt{10}$ m

14 A cylinder of height 20cm rests on an inclined plane. As the angle of inclination increases, the cylinder topples at an angle of 45º. The radius of the cylinder is



(A) 10 cm
(B) 20 cm
(C) 5 cm
(D) 15 cm

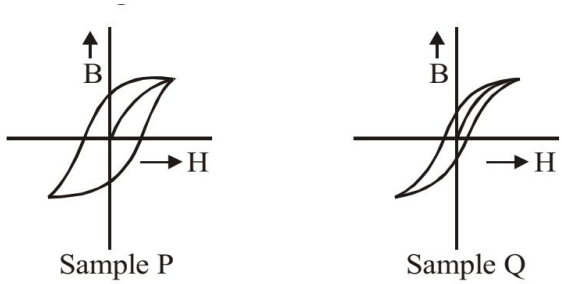
15. In Young's double slit experiment, two wavelengths of light are used simultaneously where $\lambda_2 = 2\lambda_1$. In the fringe pattern observed on the screen

(A) maxima of wavelength λ_2 can coincide with minima of wavelength λ_1
(B) fringe width of λ_2 will be double that of fringe width of λ_1
(C) n^{th} order maxima of λ_1 will coincide of $2n^{th}$ order of maxima of λ_2
(D) n^{th} order minima of λ_2 will coincide with $2n^{th}$ order of minima of λ_1

16. A galvanometer gives full scale reading of 50 mA, when a potential difference across its terminals is 0.15 V. It can be used as an ammeter of range 0-100 A by connecting a shunt resistance of magnitude

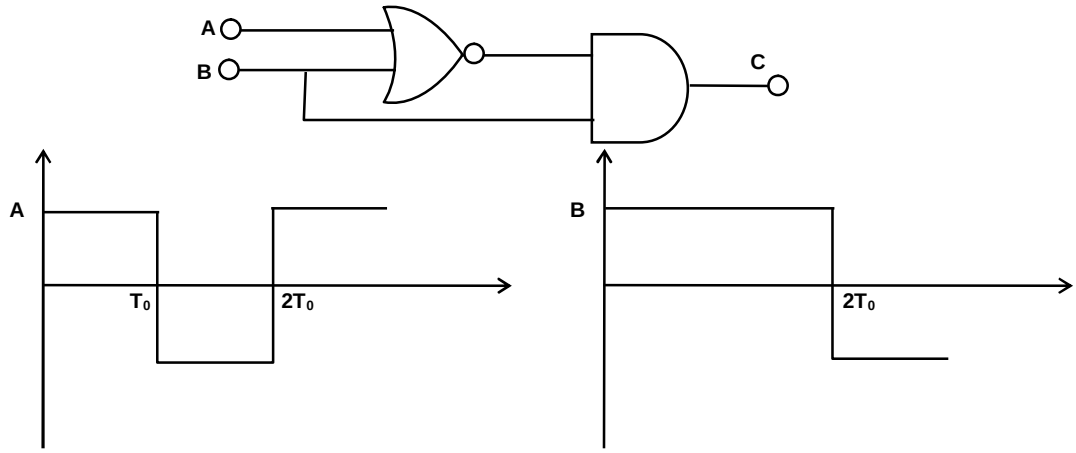
(A) 0.00075 Ω
(B) 0.00045 Ω
(C) 0.0015 Ω
(D) 3 Ω

17. If the B-H curves of two samples of P and Q of iron are as shown below, then which one of the following statements is CORRECT ?

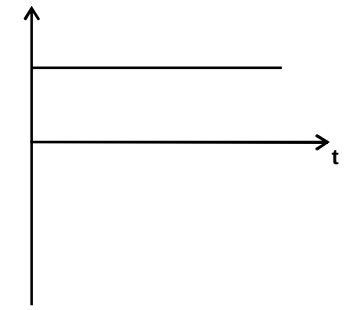


(A) Both P and Q are suitable for making permanent magnet
(B) P is suitable for making permanent magnet and Q is suitable for making electromagnet
(C) P is suitable for making electromagnet and Q is suitable for permanent magnet
(D) Both P and Q are suitable for making electromagnets

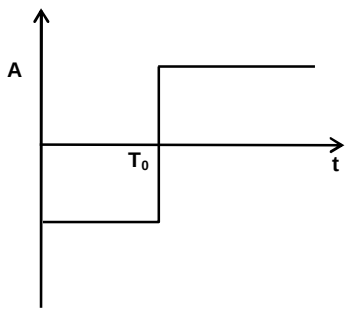
18. Correct output for given logic circuit and inputs is:



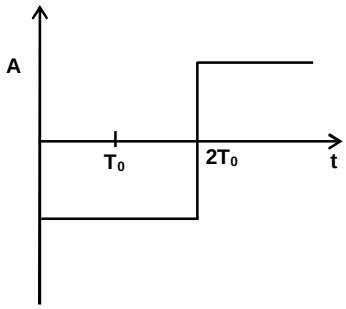
(A)



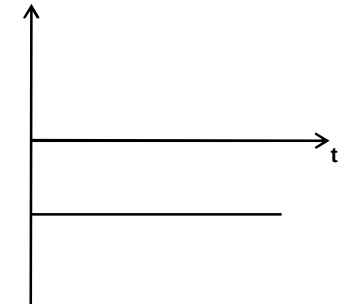
(B)



(C)

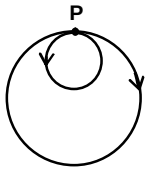


(D)

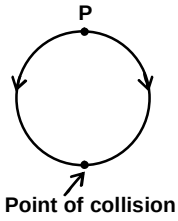


19. A neutral particle at rest in a uniform magnetic field, decays into two charged particles of different masses at point P as shown in the figure. The energy released goes to their kinetic energy and particles move in the plane of the paper. Magnetic field is into the plane of paper. Select the diagram which describes path followed by the particles most appropriately.

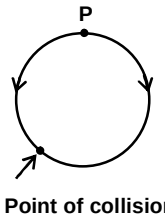
(A)



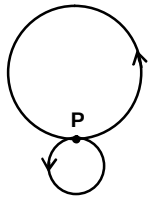
(B)



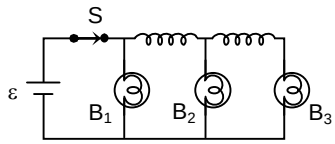
(C)



(D)



20. Three identical bulbs and two coils are connected to dc source as shown in the figure. Resistance of coils is negligible. After some time switch S is opened. Immediately after the switch S is opened, brightness of three bulbs will be in the order of :



(A) $B_1 > B_2 > B_3$

(B) $B_1 > B_2 = B_3$

(C) $B_1 < B_2 < B_3$

(D) $B_1 = B_2 < B_3$

SECTION – B
(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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21. $y(x,t) = \dfrac{0.8}{\left[(8x+10t)^2+5\right]}$ represents a travelling pulse where x and y are in meter and t is in second. The maximum particle displacement in meter is

22. When the screw end of screw gauge is touching the stud, the zero of circular scale lies 12 divisions below the reference line. If screw gauge measures diameter of a wire as 5 mm then actual diameter of the wire (in mm) is [Given least count of screw gauge = 0.01mm]

23. Two tuning forks with natural frequencies 332 Hz each move relative to a stationary detector. One fork moves away from the detector while the other moves towards the detector at the same speed. The detector observes beats of frequency 5 Hz. The speed (in m/s) of the tuning fork is (velocity of sound in air is 332 m/s):

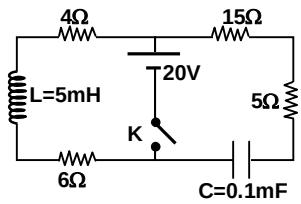
24. An unpolarized light passes through three polarizing sheets whose polarizing directions make angles of 30°, 60° and 30° with the y-axis in the same sense. The fraction of initial intensity $\left(\dfrac{I}{I_0}\right)$ transmitted by the system is

25. The average density of Earth's crust 10 km beneath the surface is 2.5 gm/cm^3. The speed of longitudinal seismic waves at that depth is 5.4 km/s. The bulk modulus of Earth's crust considering its behavior as fluid at that depth (in GPa) is

26. A liquid cools from 50ºC to 45ºC in 5 minutes and from 45ºC to 41.5ºC in the next 5 minutes. The temperature of the surrounding in ºC is (Assume newton's law of cooling is applicable)

27. A household refrigerator with a coefficient of performance 1.2 removes heat from the refrigerated space at the rate of 60kJ/min. What would be the cost (in Rs) of running this refrigerator for one day (assuming each day it is used for 8 hours and cost of one electrical unit is Rs 6, also 1 unit of electricity is 1kW-hour)

28. In the circuit shown, the key (K) is closed at t = 0, the current (in ampere) through the key K at the instant t = $10^{-3}\ell n2$ sec is (Take 1/√2= 0.71)



29. In a potentiometer experiment, two cells connected in series get balanced at 25 cm length on the potentiometer wire. Now the connections of terminals of the cell of lower emf are reversed then the balancing length is obtained at 17 cm on the potentiometer wire. The ratio of (higher to lower) emf's of two cells will be

30. Two thin prisms of dispersive power ω and ω' are combined such that they neither produce average deviation nor do they produce dispersion. If the angle of the first prism is 2° and the refractive index is 1.5 for yellow light, 1.49 for red light and 1.51 for violet light. Then the value of the dispersive power ω' for the second prism is

Chemistry

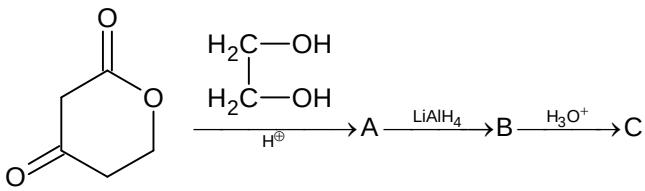
PART – B

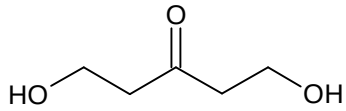
SECTION – A
(One Options Correct Type)

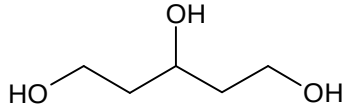
This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

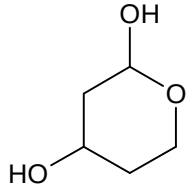
31. 4 gram of radioactive isotope X-148 were collected on 1st March 2021 and kept in a sealed tube. On 1st November 2021, it was found only 0.5 gm of it remained. Half-life period of this isotope.
(A) 30 days
(B) 81.67 days
(C) 122.5 days
(D) 100 days

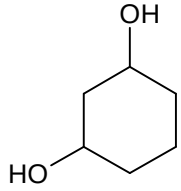
32. Identify correct statements:
(A) In the manufacture of tyre rubber, 80% of sulphur is used as cross linking agent.
(B) PHBV undergoes bacterial degration in environment.
(C) Clean water would have BOD value more then 17 ppm.
(D) Increase in the green house gases is decreasing the temperature of the earth's atmosphere.

33. 

Product (C) will be
(A) 

(B) 

(C) 

(D) 

34. $X(OH)_3$ has following equilibrium in H_2O

$X(OH)_3(s) \rightleftharpoons X^{+3}(aq) + 3OH^-(aq), K_{sp}$

$X(OH)_3(s) + OH^-(aq) \rightleftharpoons \left[X(OH)_4\right]^-, K_C$

At the minimum point of solubility of $X(OH)_3(s)$, which of the following relationship is correct?

(A) $\left[OH^-\right] = \sqrt{\left(\frac{K_{sp}}{K_c}\right)^{1/4}}$

(B) $\left[OH^-\right] = \left(\frac{3K_{sp}}{K_c}\right)^{1/4}$

(C) $\left[OH^-\right] = \left(\frac{3K_{sp}}{K_c}\right)^{1/2}$

(D) $\left[OH^-\right] = \left(\frac{K_{sp}}{K_c}\right)^{1/4}$

35. Out of $I_3^+, I_3^-, NO_2^+, N_3^-, SO_2$ linear species are

(A) SO_2, I_3^+, N_3^+

(B) I_3^-, SO_2, N_3^-

(C) I_3^+, I_3^-

(D) NO_2^+, I_3^-, N_3^-

36. Which of the following is not a sulphide ore?

(A) Copper pyrite

(B) Galena

(C) Limonite

(D) Copper glance

37. For the reaction

$3A + 5B \longrightarrow 4C \qquad \Delta H = -600$ kJ

If 9 mole of A are mixed with 20 mole of B, enthalpy change for the reaction will be

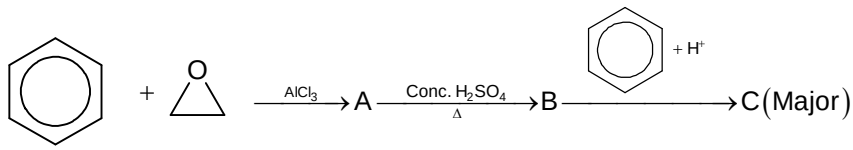
(A) -600 kJ

(B) -1800 kJ

(C) -2400 kJ

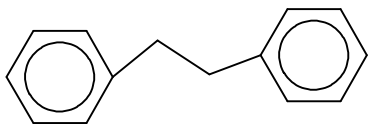
(D) -1200 kJ

38.

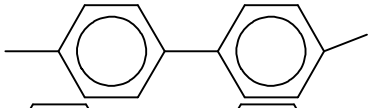


Compound C will be:

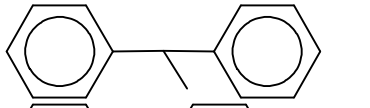
(A)



(B)



(C)



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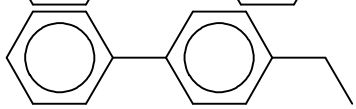


39. Hollow sphere of A made body centred cubic unit cell. Solid spheres B are present in all hollow spheres A in such way that both centres are common. Radius of B is $\frac{1}{4}$th of radius of A. What is the ratio of total volume of B unoccupied by A in unit cell with volume of unit cell.

(A) $\frac{7\sqrt{3}}{512}$

(B) $\frac{63\sqrt{3}}{256}$

(C) $\frac{7\sqrt{3}}{256}$

(D) $\frac{63\sqrt{3}}{512}$

40. If each orbital can hold maximum number of five electrons, the number of elements in 4^{th} period of periodic table (long form) will be

(A) 18 elements
(B) 30 elements
(C) 45 elements
(D) 32 elements

41. Consider the products of following reaction

(i) $Fe + HNO_3\,(\text{very dil.}) \longrightarrow$

(ii) $Mn + HNO_3\,(\text{very dil.}) \longrightarrow$

(iii) $Sn + HNO_3\,(\text{very dil.}) \longrightarrow$

(iv) $Zn + HNO_3\,(\text{conc.}) \longrightarrow$

(v) $Zn + HNO_3\,(\text{very dilute}) \longrightarrow$

(vi) $Pb + HNO_3\,(\text{dil.}) \longrightarrow$

In how many reactions, NH_4NO_3 form as one of the product?

(A) 2
(B) 3
(C) 4
(D) 5

42. Find out the incorrectly matched pairs

	Molecule	shape
(A)	NH_4^+	Tetrahedral
(B)	BrF_3	T – shape
(C)	BrF_5	Trigonal bipyramidal
(D)	SF_4	See – saw

43. Compound 'X' made of two most occurring elements in the earth crust and used in construction of building. When compound 'X' reacts with carbon (C), forms a poisonous gas 'Y', which is stable diatomic molecule. Compounds X and Y are

(A) CaO, CO_2

(B) SiO_2, N_2

(C) SiO_2, CO_2

(D) SiO_2, CO

44. Arrange the following in decreasing order of polarity in bonds
NH_3, AsH_3, PH_3 and SbH_3

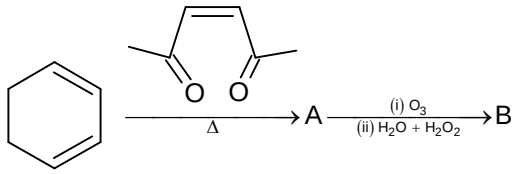
(A) $NH_3 > PH_3 > AsH_3 > SbH_3$

(B) $SbH_3 > AsH_3 > PH_3 > NH_3$

(C) $NH_3 > AsH_3 > PH_3 > SbH_3$

(D) $NH_3 > SbH_3 > AsH_3 > PH_3$

45.



Number of oxygen atom in compound (B) will be

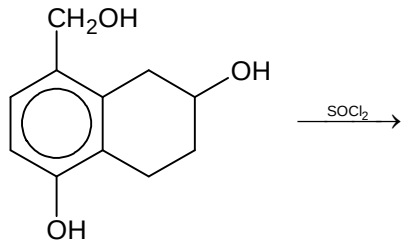
(A) 6

(B) 4

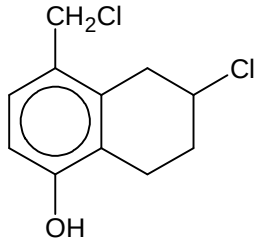
(C) 2

(D) 3

46.



(A)



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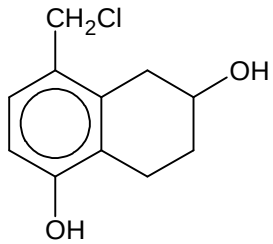
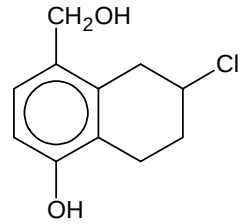
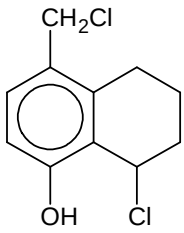
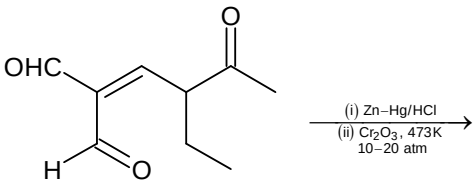
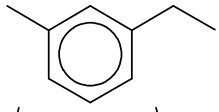
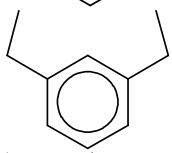


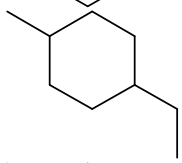
(C) 

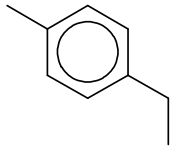
(D) 

47. 

(A) 

(B) 

(C) 

(D) 

48. Statement – I : Acid rain water pH is less than 5.6.
Statement – II : Normal rain water pH is 5.6.
(A) Statement – I is true, Statement – II is false.
(B) Statement – I and Statement – II both are false.
(C) Statement – I is false, Statement – II is true.
(D) Statement – I and Statement – II both are correct.

49. Identify the incorrect reaction.
(A) $S + NaOH \longrightarrow Na_2S + Na_2S_2O_3 + H_2O$
(B) $P_4 + NaOH \longrightarrow NaH_2PO_2 + PH_3$
(C) $C + NaOH \longrightarrow Na_2CO_3 + H_2$
(D) $Cl_2 + \underset{\text{(Hot \& conc.)}}{NaOH} \longrightarrow NaCl + NaClO_3 + H_2O$

50. Xanthopretic tests are used for the identification of
(A) Ketoses
(B) Proteins
(C) Aldoses
(D) Amines

SECTION – B
(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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51. [(No. of S – O – S bond in γ form of SO_3) + (no. of P – O – P bonds in P_4O_{10})] is x. Find x/4.

52. 0.1 molal aqueous solution of $CoCl_3.xNH_3$ shows $\Delta T_f = 0.558^oC\left(K_f \text{ for } H_2O = 1.86 \text{ K molality}^{-1}\right)$. Assuming degree of dissociation unity and coordination number of Co is six. The ratio of coordination number of cobalt with x will be

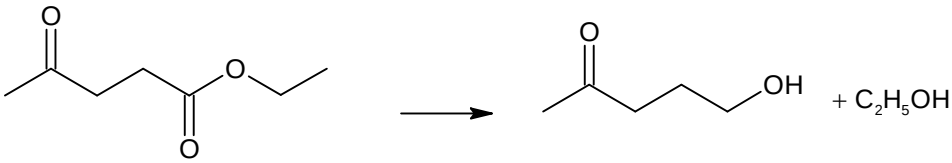
53. 1 litre of a hard water sample contains 2 mg of $CaCl_2$ and 3 mg of $MgCl_2$. What will be the total hardness in terms of parts of $CaCO_3$ per 10^6 parts of water by mass?

54. Volume strength of 12 N H_2O_2 will be

55. Consider the following compounds in liquid form $N_2, H_2O_2, HF, O_2, NH_3, CCl_4, C_6H_6, CHCl_3, C_6H_5Cl$ when charged comb is brought near their following stream as per diagram, then find b/a?
a – number of compounds show deflection
b – number of compounds do not show deflection

56. Total volume of gases evolved by passing 0.965 A current for 5 hour through an aqueous solution of CH_3COONa at 25^oC and 1 atm. $\left(R = 0.0821 \text{ atm lit } K^{-1} \text{ mole}^{-1}\right)$
(Answer should be up to two decimal places)

57. x = number of reagents required for following transformation
y = number of reagents which are not required for following transformation



Reagents $\Rightarrow$ $LiAlH_4$, Acetone, H_3O^+, Ethylene glycol with H^+, DMF
Find the x/y?

58. Consider the following statements

(a) Solvay process cannot be used to prepare K_2CO_3 due to precipitation of $KHCO_3$.

(b) Hemihydrate of calcium sulphate is known as gypsum.

(c) Phenol can give electrophilic aromatic substitution reactions in absence of Lewis acid.

(d) Salicyclic acid can give white/yellow solution with Br_2 / CS_2 .

(e) Phenol gives paraquinol with $S_2O_8^{-2} / OH^-$ as a major product.

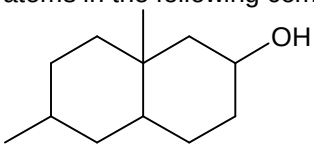
(f) None alkaline metal hydride are ionic in nature.

Find a/b. If a = number of correct statements.

b = number of incorrect statements.

59. Radioactive mixture of species A and B emitting 13000 $\beta-$particle per min initially. A was emitting 1250 $\beta-$particle per min after 100 min. If half-lives of A and B are 200 min and 6250 hours then what was the ratio of initial activities of B and A in the mixture.

60. What will be ratio of total number of chiral carbon atoms to the total number of achiral carbon atoms in the following compounds



SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

61. Let $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$ and matrix B is such that $AB^TA = B$ and $BA^TB = I$ where I is unit matrix of order 2, then matrix B^2 is

(A) $\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 2 \\ 2 & -5 \end{bmatrix}$

(D) $\begin{bmatrix} 29 & 12 \\ 12 & 5 \end{bmatrix}$

62. A curve is passing through (0, 2) such that geometric mean of sub-tangent and subnormal at any of its point is equal to the ratio of length of normal to the length of tangent at the same point, then the equation of curve is

(A) $y = 2(e^x - x)$

(B) $y = 2(x - e^x)$

(C) $y = 2e^x$

(D) $y = 2e^{2x}$

63. Let a, b, c be sides, m_a, m_b, m_c are the length of altitudes and d_a, d_b, d_c are the distances from the vertices to the orthocentre in an acute angle triangle, then $m_ad_a + m_bd_b + m_cd_c$ is equal to

(A) $a^2 + b^2 + c^2$

(B) $\frac{a^2 + b^2 + c^2}{2}$

(C) $\frac{a^2 + b^2 + c^2}{3}$

(D) $\frac{a^2 + b^2 + c^2}{4}$

64. Let equations $x^2 + 2x + a = 0$ and $ax^2 + x + 2 = 0$, $a \in R$ have atleast one common root, then product of all the possible values of a is

(A) 1

(B) 3

(C) 0

(D) –3

65. If y = y(x) is solution of differential equation $(1-x^2)\frac{dx}{dy}+xy=x$ such that y(1) = 1 and g(x) = $(y-1)^2$, then $\lim\limits_{x\to1}\frac{g(x)}{(x-1)^2}$ is equal to

(A) 4
(B) 3
(C) 2
(D) –1

66. Let ABC be a triangle with sides a, b, c and corresponding angles A, B, C respectively. If angle A = 3B, then $(a^2-b^2)(a-b)$ is equal to

(A) a^2b
(B) ab^2
(C) bc^2
(D) b^2c

67. Let $L=\lim\limits_{x\to0}\frac{3\lambda x+(\lambda-2)\sin x}{\left(\sin^{-1}(x)\right)^3}$. If L is finite, then

(A) $L=\frac{1}{3}$
(B) $L=\frac{1}{2}$
(C) $\lambda=\frac{1}{2}$
(D) $\lambda = 1$

68. If $x \geq 1$ and $\int\sin\left(\cot^{-1}\left(\sqrt{x^2-1}\right)\right)dx=f(x)+k$ where f(1) = 0, then the value of $f(e^2)$ is equal to

(A) 1
(B) $\frac{1}{2}$
(C) –2
(D) 2

69. If x and y are independent variable and $f(x)=\left(\int_0^x e^{x-y}f'(y)dy\right)-(x^2-x+1)e^x$, where f(x) is a differentiable function, then f(x) is

(A) $(x^2 – x + 1)e^x$
(B) $(2x + 1)e^x$
(C) $(2x – 1)e^x$
(D) $(x^2 – x + 1)e^x$

70. If $y=\cos^{-1}\left(\cos(|2x|-f(x))\right)$, where f(x) = 2 sgn(x), then $\left.\frac{dy}{dx}\right|_{x=-\frac{5\pi}{8}}$ is

(A) –2
(B) 0
(C) 2
(D) 3

71. If $\tan^2 2° + \tan^2 4° + \tan^2 6° + ... + \tan^2 88° = a$, then the value of $\sum_{\theta=1°}^{89°} \frac{\sin^4\theta + \cos^4\theta}{\sin^2\theta\cos^2\theta}$ is

(A) 2(89 + 4a)
(B) 2(89 – 2a)
(C) 2(89 + 2a)
(D) 2(89 – 4a)

72. If two events A and B are such that P(A) = 0.4, P(B) = 0.5 and $P(A \cap B') = 0.30$, then $P\left(\frac{B}{A \cup B'}\right)$ is

(A) $\frac{2}{3}$
(B) $\frac{2}{5}$
(C) $\frac{1}{6}$
(D) $\frac{1}{5}$

73. Let A = {1, 2, 3}, B = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} and $f : A \to B$ is one-one function. If N is the total number of such possible function and P is the probability of selection of a function out of there N function such that $f(x) > x^2$, then P is equal to

(A) $\frac{63}{880}$
(B) $\frac{63}{440}$
(C) $\frac{72}{440}$
(D) $\frac{32}{440}$

74. For distinct x, y and z, if k denotes number of dissimilar term in simplified expansion of $(x + y + z)^{18} + (x + y – z)^{18}$, then the value of k is
(A) 101
(B) 100
(C) 51
(D) 50

75. If the variable line y = px + q is tangent to an ellipse $3x^2 + 4y^2 = 6$, then the locus of point P(p, q) is

(A) $\frac{x^2}{3} - \frac{y^2}{3/2} = 1$
(B) $\frac{x^2}{3} - \frac{y^2}{3/2} = -1$
(C) $\frac{x^2}{3/4} - \frac{y^2}{3/2} = 1$
(D) $\frac{x^2}{3/4} - \frac{y^2}{3/2} = -1$

76. If $\vec{a}$ and $\vec{c}$ are unit vector and $\vec{a} = 2(\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a})$, then

(A) $\left[\bar{a}\ \bar{b}\ \bar{c}\right] = \frac{1}{4}$

(B) $\bar{a} \cdot \bar{b} = \frac{1}{2}$

(C) $\bar{a} \cdot \bar{c} = \frac{1}{2}$

(D) $\bar{a} \cdot \bar{b} = 2\left[\bar{a}\ \bar{b}\ \bar{c}\right]$

77. Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{3} = 1$ of eccentricity e is confocal with the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$, then

(A) number of common tangent of hyperbola and ellipse is 4
(B) number of common tangent of hyperbola and ellipse is 2
(C) number of common tangent of hyperbola and ellipse is 0
(D) number of common tangent of hyperbola and ellipse is 1

78. Two tangents are drawn to parabola $y^2 = 8x$ from the point P(2, 5), which touches the parabola at A and B, then the acute angle between tangent PA and PB is

(A) $\tan^{-1}\left(\frac{3}{5}\right)$

(B) $\sin^{-1}\left(\frac{3}{5}\right)$

(C) $\cos^{-1}\left(\frac{3}{5}\right)$

(D) $\cot^{-1}\left(\frac{3}{5}\right)$

79. If the straight line ax + may + 1 = 0, bx + (m + 1)by + 1 = 0 and cx + (m + 2)cy + 1 = 0 (where a, b, c and m are non-zero) are concurrent, then a, b, c are in

(A) A.P. for $m > 0$
(B) A.P. for $m < 0$
(C) G.P. for $m \in R$
(D) H.P. for $m \in R$

80. Which of the following is logically equivalent to $((p \wedge \sim p) \vee (q \vee \sim q)) \vee (p \rightarrow q)$ is

(A) $p \rightarrow \sim p$
(B) $q \leftrightarrow \sim q$
(C) $(p \rightarrow q) \vee (q \rightarrow p)$
(D) $(p \rightarrow q) \leftrightarrow (q \rightarrow p)$

SECTION – B
(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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81. Let $f : A \rightarrow \left[-\frac{1}{2}, 0\right]$ is such that $f^3(x) - 3xf(x) + x^3 + 1 = 0$ and f(x) is an invertible function, then number of solution(s) of the equation $f(f(x)) = f^{-1}(x)$ is

82. If $\sin(x - y)$, $\cos x$, $\sin(x + y)$ are in A.P. where $x, y \in \left[0, \frac{\pi}{2}\right]$ and minimum value of $x + y = \frac{a\pi}{b}$, then $a + b$ is equal to (where a and b are coprime)

83. If N is number of pairs of two divisors of 75600 such that the product of these divisors is 60480, then the value of N is

84. If $\left|\frac{z}{z-1}\right| = \frac{1}{2}$, then the minimum value of $|3z + 1 - 6i|$ is equal to

85. If $5a + 12b + 13c = 52$ and least value of $a^2 + b^2 + c^2 = k$, then $[k]$ is equal to
(Where [.] denotes greatest integer function and $a, b, c \in R$)

86. If the line $\frac{x-2}{a} = \frac{y-b}{1} = \frac{z-4}{4}$ meets the circle $x^2 + y^2 - 8x + 2y - 1 = 0$ at one point, then the maximum value of $(a + b)$ is

87. Let $A_0, A_1, A_2, A_3, A_4, A_5$ be the vertices of a regular hexagon inscribed in a circle of radius unity. If p, q, r denotes the length of segment A_0A_2, A_0A_4, A_2A_4, then the value of $(p + q)r$ is equal to

88. A tangent to parabola $y^2 = 16x$ meets the hyperbola $xy = c^2$ at two points A and B. The locus of midpoint of AB is another parabola whose length of latus rectum is

89. For pair of lines $ax^2 + by^2 + 2hxy = 0$ if the slope of one of the angle bisector is 3, then $\left|\frac{a-b}{h}\right|$ is equal to

90. Let a, b, c be positive real numbers such that $a^2 + b^2 + c^2 = 3$, then if $\frac{1}{1+ab} + \frac{1}{1+bc} + \frac{1}{1+ca} \geq k$, then k is equal to

ALL INDIA TEST SERIES

FULL TEST - 15

JEE (Main)

ANSWERS, HINTS & SOLUTIONS

Physics

PART – A

SECTION – A

1. D

Sol. $W_{1-2} + W_{2-3} + W_{3-1} = -300$ ($\Delta U = 0$ in a cyclic process).

$W_{3-1} = 0$ $W_{1-2} = P(V_2 - V_1) = nR(T_2 - T_1) = 600R = 4980$ J

Therefore $W_{2-3} = -5280$ J

2. D

Sol. As the inclined plane is smooth, the sphere can never roll rather it will just slip down.
Hence, the angular momentum remains conserved about any point on a line parallel to the inclined plane and passing through the centre of the ball.

3. A

Sol. From equation

Av = constant, $v \propto \frac{1}{\text{Area of cross section}}$

$P + \frac{1}{2}\rho v^2 = \text{constant}$

If velocity increases then pressure decreases

4. C

Sol. $\frac{1}{f_\ell} = (1.5 - 1)\left[\frac{1}{30} - \frac{1}{-30}\right]$

$\Rightarrow f_\ell = 30$ cm

$P = P_1 + P_2 + P_3 = \frac{1}{30} + \frac{1}{30} + \frac{1}{15} = \frac{4}{30} = -\frac{1}{F_m}$

$\therefore F_m = -7.5$ cm

$\frac{1}{v} + \frac{1}{-60} = -\frac{2}{15} \Rightarrow \frac{1}{v} = \frac{1}{60} - \frac{2}{15} \Rightarrow v = -\frac{60}{7}$ cm

5. B

Sol. $3Tv_0 + T(v_0 - v) = 0$

$\Rightarrow 4v_0 = v$

$v_0 = \frac{v}{4}$

6. C

Sol. $P_{av} = V_{rms} I_{rms} \cos\phi$

Since, $\phi = \frac{\pi}{2}$

$\Rightarrow P_{av} = 0$.

7. B

Sol. For 1 ring,

$I_{CM} = mr^2 - m(2r/\pi)^2$;

$I = I_{CM} + m(r^2 + (2r/\pi)^2) = 2mr^2$

COM of system will be point of intersection of the two half rings.

Hence, the moment of inertia of the system is $I = 2mr^2 + 2mr^2 = 4mr^2$

8. D

Sol. $v = 2t - 3$; $v = 0$ at $t = 1.5$ s; $x(t = 1.5) = -0.25$ m; $x(t = 3) = 2$m ; $x(t = 0) = 2$m

Therefore distance travelled in 3 sec = 2(2 + 0.25) = 4.5 m

9. A

Sol. Work function, ϕ = 12400/5000= 2.48 eV

hc/λ – Work function = KE max = 3eV

hc/λ = 5.48 eV $\Rightarrow$ λ = 2260 Å

10. A

Sol. $K_A A \frac{dT_A}{dx} = K_B A \frac{dT_B}{dx}$

$200 \times \tan 60° = K_B \tan 45°$

$K_B = 200\sqrt{3} = 200 \times 1.732 = 346.40$ J/m-sec-°C

11. B

Sol. Potential difference, $V_A - V_P = \int (\vec{v} \times \vec{B}).d\vec{l} = 2B_0Rv$

12. D

Sol. Using $v = \sqrt{\frac{T}{\mu}}$, μ = mass per unit length of the rope

If v_t is velocity at top and v_b is velocity at bottom then

$$\frac{\lambda_t}{\lambda_b}=\frac{v_t}{v_b}=\sqrt{\frac{T_t}{T_b}}=\sqrt{\frac{(M+m)g}{mg}}=\sqrt{\frac{M+m}{m}}$$

$$\lambda_t=\lambda\sqrt{\frac{M+m}{m}}$$

13. B

Sol. Time of flight = $\frac{2u\sin\theta}{g}=\frac{2\times 20\times\frac{1}{2}}{10}=2\text{sec}$

After 1 sec, the projectile is at maximum height.

Maximum height $(H)=\frac{u^2\sin^2\theta}{2g}=\frac{20^2\times\left(\frac{1}{2}\right)^2}{2\times 10}=5\text{ m}$

Range $(R)=\frac{u^2\sin 2\theta}{g}=\frac{20^2\times\frac{\sqrt{3}}{2}}{10}=20\sqrt{3}\text{ m}$

$$\overline{AB}=\sqrt{H^2+\left(\frac{R}{2}\right)^2}=\sqrt{5^2+(10\sqrt{3})^2}=\sqrt{325}=5\sqrt{13}\text{ m}$$

14. A

Sol. The cylinder begins to topple when the vertical line through CG passes beyond the base of cylinder.

$$\tan\theta=\frac{r}{\left(\frac{h}{2}\right)}=1$$

or $r=\frac{h}{2}=10\text{cm}$

15. B

Sol. $\beta_2/\beta_1=\lambda_2/\lambda_1=2$

Fringe width of λ_2 = 2* Fringe width of λ_1

$2n^{th}$ order maxima of λ_1 coincide with n^{th} order maxima of λ_2

16. C

Sol. $G=\frac{0.15}{0.05}=3\Omega$

$$S=\frac{i_gG}{(I-i_g)}=\frac{0.05\times 3}{(100-0.05)}=0.0015\ \Omega$$

17. B

Sol. For permanent magnet we prefer a material with high retentivity (so as to make a stronger magnet) and high coercivity (so that magnetization can not be wiped out easily). For electromagnet we prefer high saturated magnetism low coercivity and least possible area of hysteresis loop so that electromagnet develops high magnetization, is easily demagnetized and energy loss in a magnetization cycle is least. Therefore, P is suitable for making permanent magnet and Q for making electromagnet.

18. D

Sol. $y=\overline{A+B}\cdot B$

19. C

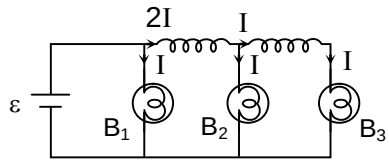
Sol. Momentum and charges are equal and opposite, so sense of the motions are opposite.

$\therefore R = \dfrac{mv}{qB}$

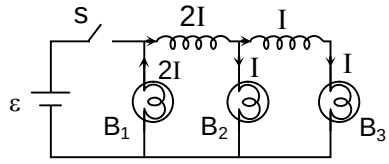
$\Rightarrow$ Their radii are equal but angular velocities will be different.

20. B

Sol. Currents before opening the switch 'S'



Currents just after opening the switch 'S'



SECTION – B

21. 00000.16

Sol. Given $y(x,t) = \dfrac{0.8}{[(8x+10t)^2+5]}$

y will be maximum when denominator is minimum.

So, $\left[(8x+10t)^2+5\right]$ must be minimum. It will be minimum when x = 0 and t = 0

So, $y_{max} = \dfrac{0.8}{0+5}$

or, y_{max} = 0.16 m

22. 00004.88

Sol. When circular scale lies below reference line error is called positive and when lies above reference line error is called negative

error = least count × no. of division which differs from reference line

or error = + 0.01×12 = 0.12 mm

Actual diameter = Reading – error

or Actual diameter = 5 mm – [+ 0.12 mm]

or Actual diameter = 4.88 mm

23. 00002.50

Sol. Given $f_1 - f_2 = 5$

or $\left(\dfrac{v}{v-v_s}\right)f - \left(\dfrac{v}{v+v_s}\right)f = 5$ or $\left[\dfrac{1}{(1-v_s/v)} - \dfrac{1}{(1+v_s/v)}\right]f = 5$

or $\left[\left(1-\dfrac{v_s}{v}\right)^{-1} - \left(1+\dfrac{v_s}{v}\right)^{-1}\right]f = 5$ or $\left[\left(1+\dfrac{v_s}{v}\right) - \left(1-\dfrac{v_s}{v}\right)\right]f = 5$

or $\dfrac{2v_s f}{v} = 5$ or Speed of tuning fork $v_s = \dfrac{5v}{2f}$

Substituting the values, we get $v_s = \dfrac{5\times 332}{2\times 332} = 2.5 \text{ m/s}$

24. 00000.28

Sol. $I = (I_o/2)\cos^2 30^o \times \cos^2 30^o$

$= \frac{I_0}{2} \times \frac{3}{4} \times \frac{3}{4} = \frac{9I_0}{32}$

25. 00072.90

Sol. $v = \sqrt{\frac{B}{\rho}}$

$\Rightarrow B = \rho v^2 = 2.5 \times 10^3 \times (5.4 \times 10^3)^2 = 72.90 \times 10^9$ N/m^2 = 72.90 (GPa)

26. 00033.33

Sol. From Newton's law of cooling

$$\left(\frac{\theta_1 - \theta_2}{t}\right) = k\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$

In the first case, $\left(\frac{50-45}{5}\right) = k\left[\frac{50+45}{2} - \theta_0\right]$ …(i)

In the second case, $\left(\frac{45-41.5}{5}\right) = k\left[\frac{45+41.5}{2} - \theta_0\right]$ …(ii)

From (i) and (ii)

$\theta_0 = 33.33°C$

27. 00040.00

Sol. COP = $T_2/(T_1-T_2)$=1.2 ; T_1/T_2 = 11/6 ; Q_1 = 110 kJ/ min ; W = 50 kJ/ min

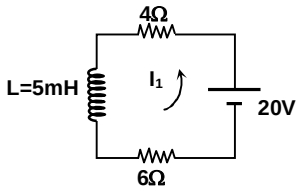
W = (5/6) kJ/ sec; $\Rightarrow$ 5/6 units per hour

Total cost = (5/6) × 6 × 8 = 40 Rs per day

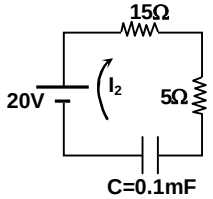
28. 00002.21

Sol. $I_1 = \frac{20}{10}\left(1 - e^{-\frac{t}{5\times10^{-4}}}\right)$

$= \frac{3}{2} = 1.5$ A



$I_2 = \frac{20}{20} e^{-\frac{t}{2\times10^{-3}}}$ = 1/√2 A= 0.71A



From superposition

$I = I_1 + I_2 = 1.50 + 0.71$

= 2.21 A

29. 00005.25

Sol. $\frac{E_1 + E_2}{E_1 - E_2} = \frac{25}{17} \quad \Rightarrow \quad \frac{E_1}{E_2} = \frac{21}{4} = 5.25$.

30. 00000.04

Sol. For dispersion without average deviation

$(n_y - 1)A = (n'_y - 1)A'$ …(i)

For no dispersion

$A(n_v - n_R) = (n'_v - n'_R)A'$ …(ii)

$\Rightarrow (n_y - 1)A\omega = (n'_y - 1)A'\omega'$

$\Rightarrow \omega = \omega' \quad [\because (n_y - 1)A = (n'_y - 1)A']$

$\Rightarrow \omega' = \omega = \frac{n_v - n_R}{n_y - 1} = \frac{1.51 - 1.49}{1.5 - 1} = \frac{0.02}{0.5} = .04$

SECTION – A

31. B

Sol. Time (t) = March + April + May + June + July + August + September + October
= 31 + 30 + 31 + 30 + 31 + 31 + 30 + 31
= 245 days

$$\lambda = \frac{2.303}{245}\log_{10}\left(\frac{4}{0.5}\right) = \frac{2.303}{245}\log_{10}\left(2^3\right)$$

$$\lambda = 3\times\frac{2.303}{245}\log_{10}2 = \frac{3\times0.693}{245}$$

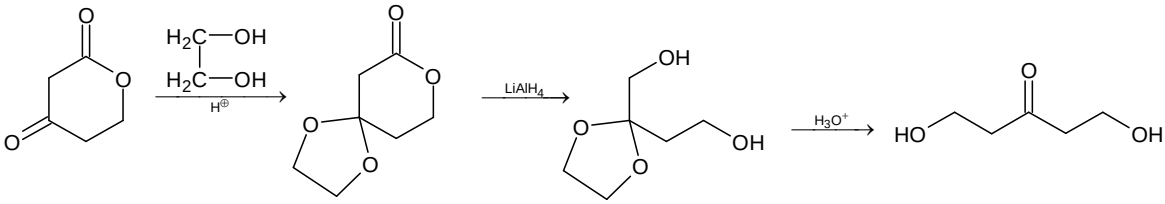
$$t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{\left(3\times\frac{0.693}{245}\right)} = \frac{245}{3} = 81.67\,\text{days}$$

32. D

Sol. Fact

33. A

Sol.



34. B

Sol. Solubility (S) $= \left[X^{+3}\right] + \left[X(OH)_4^-\right]$

$$S = \frac{K_{sp}}{\left[OH^-\right]^3} + K_c\left[OH^-\right]$$

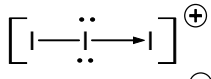
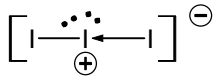
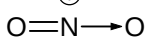
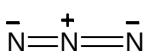
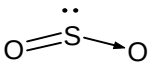
Differentiate above with respect to $\left[OH^-\right]$, for minimum solubility

$$\frac{dS}{d\left[OH^-\right]} = \frac{-3K_{sp}}{\left[OH^-\right]^4} + K_c = 0$$

$$\left[OH^-\right] = \left(\frac{3K_{sp}}{K_c}\right)^{1/4}$$

35. D

Sol.

I_3^+		v-shape
I_3^-		Linear
NO_2^+		Linear
N_3^-		Linear
SO_2		v-shape

36. C

Sol. (A) Copper pyrite $\rightarrow CuFeS_2$

(B) Galena $\rightarrow PbS$

(C) Limonite $\rightarrow Fe_2O_3.3H_2O$

(D) Copper glance $\rightarrow Cu_2S$

37. B

Sol.

	$3A$	$+\quad 5B$	$\longrightarrow$	$4C$	$\Delta H = -600$ kJ
$t = 0$	9	20		0	
t	$9-9=0$	$20-\frac{5}{3}\times 9$			

= 5 mole

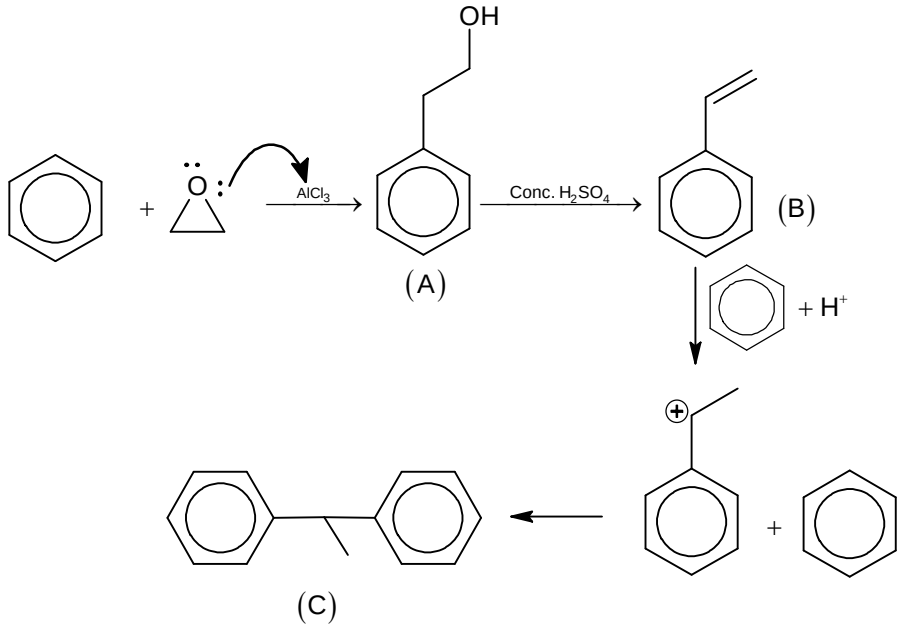
A will be the limiting reagent.

Final change in enthalpy $= -\frac{600}{3}\times 9$

= - 1800 kJ

38. C

Sol.



39. D

Sol. In BCC number of atoms of $A = 1+\frac{1}{8}\times 8 = 2$

B is inside A so volume unoccupied by A in unit cell $= 2\left(\frac{4}{3}\pi r_A^3 - \frac{4}{3}\pi r_B^3\right)$

$4r_A = a\sqrt{3}$ (By diagonal)

$$\text{Ratio} = \frac{2\times\frac{4}{3}\pi\left(r_A^3 - r_B^3\right)}{a^3} = \frac{2\times\frac{4}{3}\pi\left(r_A^3 - \left(\frac{1}{4}r_A\right)^3\right)}{\left(\frac{4r_A}{\sqrt{3}}\right)^3}$$

$$= \left(\frac{\frac{8\pi}{3} \times \frac{63}{64} r_A^3}{64 r_A^3} \times 3\sqrt{3} \right) = \frac{63\sqrt{3}}{512}$$

40. C

Sol. In 4^{th} period s, p, d will present.
So number of orbitals 1 + 3 + 5 = 9
In each orbital maximum number of electron = 5
Hence, total number of elements in 4^{th} period = 5 × 9 = 45 elements.

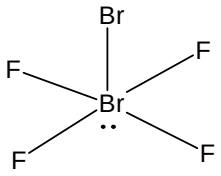
41. B

Sol. In i, iii and v reaction NH_4NO_3 is also formed with metal nitrate.

42. C

Sol.

BrF_5



sp^3d^2

square pyramidal

43. D

Sol. $\underset{(X)}{SiO_2} + 2C \longrightarrow Si + \underset{\text{(Poisonous gas stable)}}{2CO}$

44. D

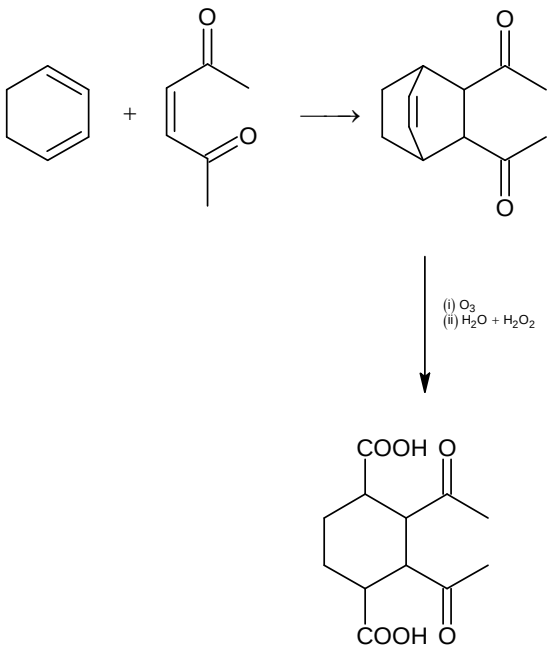
Sol. Electronegativities are H – 2.1, P – 2.1, N – 3 , As – 2.0, Sb – 1.9
Polarity $\propto$ difference in electronegativity order is

$NH_3 > SbH_3 > AsH_3 > PH_3$

Difference in electronegativity	0.9	0.2	0.1	0

45. A

Sol.

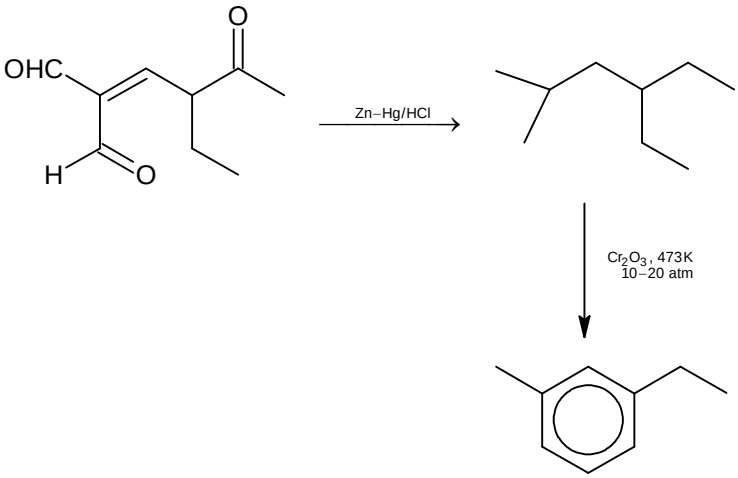


46. A

Sol. Benzyl and alkyl position is reactive toward nucleophilic substitution reaction. Phenol will not react with $SOCl_2$.

47. A

Sol.



48. D

Sol. Both are correct statement. Fact.

49. C

Sol. Carbon and NaOH does not react with each other.

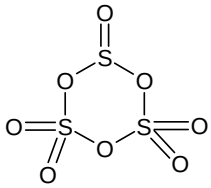
50. B

Sol. This test is used for proteins.

SECTION – B

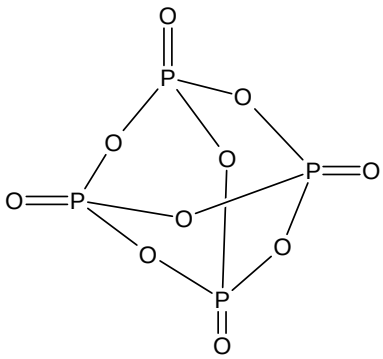
51. 00002.25

Sol. γ form of SO_3 is S_3O_9



3 S – O – S bonds

P_4O_{10}



6 P – O – P bonds

x = 3 + 6 = 9

$$\text{Ratio} = \frac{x}{4} = \frac{9}{4} = 2.25$$

52. 00001.20

Sol. $CoCl_3.xNH_3$

$$\left[CoCl_{3-y}.(NH_3)_x\right]Cl_y \rightleftharpoons \left[CoCl_{3-y}(NH_3)_x\right]^{+y}_{(aq)} + yCl^-_{(g)}$$

$$i = 1 + y$$

$$\Delta T_f = iK_f m$$

$$0.558 = (1+y)0.558 \times 0.1$$

y = 2

Hence, compound will be

$$\left[Co(NH_3)_5\,Cl\right]Cl_2$$

x = 5

$$\text{Ratio} = \frac{\text{C.N. of cobalt}}{x} = \frac{6}{5} = 1.20$$

53. 00004.95

Sol. $2\text{mg of } CaCl_2 \equiv \frac{100}{111} \times 2 \text{ mg of } CaCO_3$

$= 1.8 \text{ mg of } CaCO_3$

$2\text{mg of } MgCl_2 \equiv \frac{100}{95} \times 3 \text{ mg of } CaCO_3$

$= 3.15 \text{ mg of } CaCO_3$

One litre of hard water contains 1.8 + 3.15

= 4.95

54. 00067.20

Sol. Volume strength $= 5.6 \times$ Normality

$= 5.6 \times 12$

= 67.20

55. 00001.25

Sol. Polar molecules will show deflection near charged comb.

$a = H_2O_2, HF, NH_3, CHCl_3, C_6H_5Cl \quad (5)$

$b = O_2, CCl_4, C_6H_6, N_2 \quad (4)$

$$\frac{b}{a} = \frac{5}{4} = 1.25$$

56. 00001.76

Sol. $2CH_3COONa \xrightarrow{\text{Electrolysis}} 2NaOH + H_2 + C_2H_6 + 2CO_2$

$$CH_3COONa \xrightarrow{\text{Electrolysis}} NaOH + \frac{1}{2}H_2 + \frac{1}{2}C_2H_6 + CO_2$$

For above 1 F is required to form 2 mole gases.

Charge supplied $= i \times t = 0.965 \times 60 \times 60$ C

So moles of gases formed $= 2 \times$ Charge supplied/1F

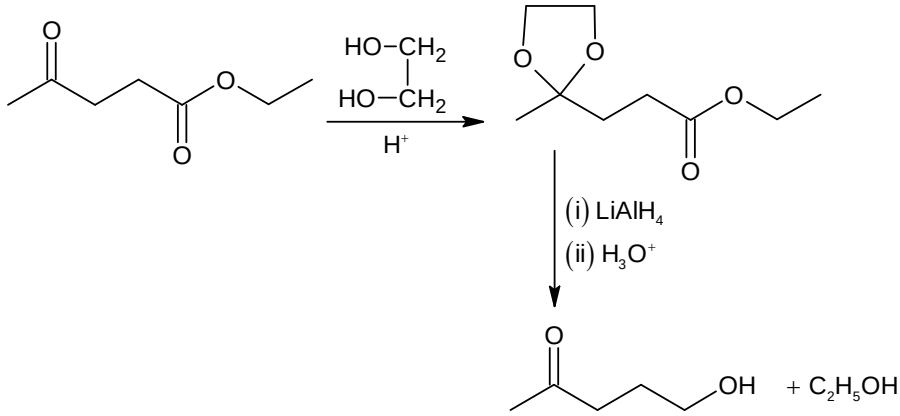
$$= \frac{2 \times 0.965 \times 60 \times 60}{1F} = 0.072$$

$$PV = nRT \Rightarrow V = \frac{nRT}{P}$$

$$V = \frac{0.072 \times 0.0821 \times 298}{1} = 1.761 \text{ lit}$$

57. 00001.50

Sol.



$$x = 3\left(LiAlH_4, \text{ Ethylene glycol, } H_3O^+\right)$$

$$y = 2(\text{Acetone, DMF})$$

$$\frac{x}{y} = \frac{3}{2} = 1.50$$

58. 00000.50

Sol. Number of correct statements = 2 (c, e)
Number of incorrect statements = 4 (a, b, d, f)

$$\frac{a}{b} = \frac{2}{4} = 0.50$$

59. 00001.60

Sol. Let A_o and B_o are initial activities of A and B respectively. After 200 min A will $1/4^{th}$ of A_o.

$t = 0 \quad A_o + B_o = 13000\ \beta - \text{particle / min} \quad \ldots$ (i)

$t = 200 \quad \frac{A_o}{4} = 12500\ \beta - \text{particle / min} \quad \ldots$ (ii)

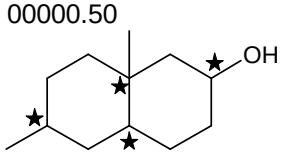
$A_o = 5000 \quad B_o = 13000 - 5000$

$= 8000$

$$\frac{B_o}{A_o} = \frac{8000}{5000} = 1.60$$

60. 00000.50

Sol.



Number of chiral carbon atoms = 4 (* marked)
Number of achiral carbon atoms = 8 (Rest are achiral C atoms)

$$\text{Ratio} = \frac{4}{8} = 0.50$$

SECTION – A

61. B

Sol. From both equation $B^2 = A^{-1}$

62. C

Sol. G.M. of sub-tangent and subnormal = $\sqrt{\frac{y}{\frac{dy}{dx}} \cdot y \cdot \frac{dy}{dx}} = |y|$, ratio of length of normal to the tangent =

$$\left| \frac{y\sqrt{1+\left(\frac{dy}{dx}\right)^2}}{y\sqrt{1+\frac{1}{\left(\frac{dy}{dx}\right)^2}}} \right| = \left|\frac{dy}{dx}\right|$$

$\Rightarrow |y| = \left|\frac{dy}{dx}\right| \Rightarrow y' = \pm y \Rightarrow y = 2e^x$ or $y = 2e^{-x}$

63. B

Sol. $m_a = c \sin B$ and $d_a = 2R \cos A$

$m_b = a \sin C$ and $d_b = 2R \cos B$

$m_c = a \sin B$ and $d_c = 2R \cos C$

$\Rightarrow m_a d_a + m_b d_b + m_c d_c$

$\Rightarrow 2R \cos A \, c \sin B + 2R \cos B \, a \sin C + 2R \cos C \, a \sin B$

$\Rightarrow bc \cos A + ac \cos B + ab \cos C$

$$\Rightarrow bc\left(\frac{b^2+c^2-a^2}{2bc}\right) + ac\left(\frac{a^2+c^2-b^2}{2ac}\right) + ab\left(\frac{a^2+b^2-c^2}{2ab}\right) \Rightarrow \frac{a^2+b^2+c^2}{2}$$

64. C

Sol. $x^2 + 2x + a = 0 \Rightarrow a = -x^2 - 2x$ and $ax^2 + x + 2 = 0 \Rightarrow (-x^2 - 2x)x^2 + (x + 2) = 0$

$\Rightarrow (x + 2)(-x^3 + 1) = 0 \Rightarrow x = -2$ or 1 or $x^2 + x + 1 = 0$ when $x = -2 \Rightarrow a = 0$

$x = 1 \Rightarrow a = -3$

$\therefore$ Product of all possible values of a is 0

65. C

Sol. $(1-x^2)\frac{dx}{dy} = x(1-y) \Rightarrow \left(\frac{1}{x} - x\right)\frac{dx}{dy} = (1-y)$

$$\Rightarrow \left(\frac{1}{x} - x\right)dx = (1-y)dy \Rightarrow \ln|x| - \frac{x^2}{2} = -\frac{(1-y)^2}{2} + c$$

$\Rightarrow \ln x^2 = x^2 - (y - 1)^2 + c$

$\because$ $y(1) = 1 \Rightarrow c = -1 \Rightarrow g(x) = (y - 1)^2 = x^2 - \ln x^2 - 1$

$$\Rightarrow \lim_{x\to 1} \frac{g(x)}{(x-1)^2} = \lim_{x\to 1} \frac{x^2 - \ln x^2 - 1}{(x-1)^2} = 2$$

66. C

Sol. $(a^2 - b^2)(a - b) = 8R^3(\sin^2 A - \sin^2 B)(\sin A - \sin B)$

$\Rightarrow 8R^3(\sin^2 (3B) - \sin^2 B)(\sin 3B - \sin B) \Rightarrow 8R^3(\sin (3B) - \sin B)^2(\sin (3B) + \sin B)$

$\Rightarrow 8R^3 \sin^2 4B \sin B \Rightarrow 8R^3 \sin^2(\pi - c) \sin B \Rightarrow 8R^3 \sin^2 C \sin B \Rightarrow bc^2$

67. C

Sol. $L = \lim\limits_{x\to 0} \dfrac{3\lambda x + (\lambda-2)\left(x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!}.....\right)}{\dfrac{(\sin^{-1}(x))^3}{x^3}\cdot x^3} = \lim\limits_{x\to 0} \dfrac{x(4\lambda-2) - \left(\dfrac{\lambda-2}{6}\right)x^3.....}{x^3}$

$\because$ limit is finite $\lambda = \dfrac{1}{2}$ and $L = \dfrac{1}{4}$

68. D

Sol. $\int \sin\left(\cot^{-1}\left(\sqrt{x^2-1}\right)\right)dx = \int \dfrac{dx}{x} = \ln x + k$

$\because$ $f(x) = \ln x \Rightarrow f(e^2) = 2$

69. C

Sol. $f(x) = \int\limits_0^x e^{x-y} f'(y)\,dy - (x^2 - x + 1)e^x$ ..... (1)

$\Rightarrow f'(x) = e^x \int\limits_0^x e^{-y} f'(y)\,dy + e^x\left(1\cdot e^{-x}\cdot f'(x)\right) - (2x-1)e^x - (x^2 - x + 1)e^x$

$\Rightarrow \int\limits_0^x e^{x-y} f'(y)\,dy = (x^2 + x)e^x$ ..... (2)

From equation (1) and (2), we get $f(x) = (2x - 1)e^x$

70. C

Sol. At $x = -\dfrac{5\pi}{8}$; $y = \cos^{-1}(\cos(-2x+2)) = 2\pi - (-2x + 2)) = 2x - 2 + 2\pi$

$\dfrac{dy}{dx} = 2$

71. A

Sol. $\dfrac{\sin^4\theta + \cos^4\theta}{\sin^2\theta\cos^2\theta} = 4\,\text{cosec}^2 2\theta - 2 = 2 + 4\cot^2 2\theta$

$\Rightarrow \sum\limits_{\theta=1^\circ}^{89^\circ} (2 + 4\cot^2 2\theta) = 2(89 + 4a)$

72. C

Sol. $P(A \cup B') = 0.60$; $P(A \cap B) = 0.10$

$\Rightarrow P\left(\dfrac{B}{A\cup B'}\right) = \dfrac{P(B\cap(A\cup B'))}{P(A\cup B')} = \dfrac{P(A\cap B)}{P(A\cup B')} = \dfrac{1}{6}$

73. B

Sol. $N = 12 \times 11 \times 10 = 1320$

$P = \dfrac{{}^3C_1\cdot{}^7C_1\cdot{}^9C_1}{1320}$

74. B

Sol. Simplified expansion will be

$2\left({}^{18}C_0(x+y)^{18}z^0 + {}^{18}C_2(x+y)^{16}z^2 + {}^{18}C_4(x+y)^{14}z^4 ..... {}^{18}C_{18}z^{18}\right)$

$\Rightarrow$ Numbers of dissimilar term are = 19 + 17 + 15 ..... 1 = 100

75. D

Sol. Equation of tangent to ellipse $y = mx \pm \sqrt{2m^2 + \frac{3}{2}}$ by comparing both equations

$y = px + q$ and $y = mx \pm \sqrt{2m^2 + \frac{3}{2}}$

$\Rightarrow q = \sqrt{2m^2 + \frac{3}{2}}$, $p = m \Rightarrow \frac{p^2}{3/4} - \frac{q^2}{3/2} = -1$

76. B

Sol. $\bar{a} = 2(\bar{b} \times \bar{c}) + (\bar{c} \times \bar{a}) \Rightarrow \bar{a} \cdot \bar{a} = 1 = 2[\bar{a}\ \bar{b}\ \bar{c}] \Rightarrow [\bar{a}\ \bar{b}\ \bar{c}] = \frac{1}{2}$

$\Rightarrow \bar{a} \cdot \bar{b} = [\bar{a}\ \bar{b}\ \bar{c}] = \frac{1}{2}$ and $\bar{a} \cdot \bar{c} = 0$

77. C

Sol. Focii of ellipse are (–2, 0), (2, 0) so value of a = 1

Equation hyperbola $\frac{x^2}{1} + \frac{y^2}{3} = 1$; equation of tangent $y = mx \pm \sqrt{m^2 - 3}$

Equation of ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$; equation of tangent $y = mx \pm \sqrt{8m^2 + 4}$ for common tangent

$m^2 - 3 = 8m^2 + 4$, $m^2 = -1$ not possible

78. B

Sol. Tangent to the parabola $y^2 = 8x$ will be $y = mx + \frac{2}{m}$ this tangent passes through (2, 5)

$\Rightarrow 5 = 2m + \frac{2}{m} \Rightarrow 2m^2 - 5m + 2 = 0 \Rightarrow m = 2, \frac{1}{2}$

Angle between tangent = $\tan\theta = \left|\frac{2 - \frac{1}{2}}{1 + 2 \times \frac{1}{2}}\right| = \frac{3}{4}$; $\theta = \tan^{-1}\left(\frac{3}{4}\right)$

79. D

Sol. The determinant of coefficient $\begin{vmatrix} a & ma & 1 \\ b & (m+1)b & 1 \\ c & (m+2)c & 1 \end{vmatrix} = 0 = \begin{vmatrix} 1 & m & \frac{1}{a} \\ 1 & m+1 & \frac{1}{b} \\ 1 & m+2 & \frac{1}{c} \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 1 & m & \frac{1}{a} \\ 0 & 1 & \frac{1}{b} - \frac{1}{a} \\ 0 & 1 & \frac{1}{c} - \frac{1}{b} \end{vmatrix} = 0 \Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$\Rightarrow$ a, b, c are in H.P.

80. C

Sol. $((p \wedge \sim p) \vee (q \vee \sim q)) \vee (p \to q)$

$(F \vee T) \vee (p \to q)$

$T \vee (p \to q) = T$ only option (C) is tautology

SECTION – B
(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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81. 00001.00

Sol. $f : A \to \left[-\frac{1}{2}, 0\right]$; $f^3(x) + x^3 + 1 = 3xf(x)$

$\Rightarrow f(x) + x + 1 = 0 \Rightarrow f(x) = -x - 1$

$\Rightarrow f(f(x)) = x$ and $f^{-1}(x) = -x - 1 \Rightarrow x = -x - 1$

$x = -\frac{1}{2}$ only one solution

82. 00005.00

Sol. $\sin(x - y) + \sin(x + y) = 2\cos x \Rightarrow \sin x \cos y = \cos x$

If $\cos x = 0 \Rightarrow \cos y = 0 \Rightarrow x + y = \pi$

If $\cos x \neq 0 \Rightarrow \tan x \cos y = 1 \Rightarrow \tan x = \sec y$ for minimum value $x = \frac{\pi}{4}$, $y = 0$

83. 00024.00

Sol. $x = 75600 = 2^4\, 3^3\, 5^2\, 7^1$

Let divisor is $2^{a_1} \cdot 3^{b_1} \cdot 5^{c_1} \cdot 7^{d_1}$ and $2^{a_2} \cdot 3^{b_2} \cdot 5^{c_2} \cdot 7^{d_2}$

Product of divisors = 60480 = $2^6 \cdot 3^3 \cdot 5^1 \cdot 7^1$

$\Rightarrow a_1 + a_2 = 6 \Rightarrow 3$ ways

$\Rightarrow b_1 + b_2 = 3 \Rightarrow 4$ ways

$\Rightarrow c_1 + c_2 = 1 \Rightarrow 2$ ways

$\Rightarrow d_1 + d_2 = 1 \Rightarrow 2$ ways

Total way $N = \frac{3 \times 4 \times 2 \times 2}{2} = 24$

84. 00004.00

Sol. Equation of circle satisfying the equation $\left|\frac{z}{z-1}\right| = \frac{1}{2}$ is $(x+1)\left(x - \frac{1}{3}\right) + y^2 = 0$; minimum distance of circle from point $\left(-\frac{1}{3}, 2\right)$ is $\frac{4}{3}$

85. 00008.00

Sol. Let (a, b, c) lies on plane 5x + 12y + 13z – 52 = 0, then distance of origin from plane is

$\sqrt{k} = \left|\frac{52}{\sqrt{25+144+169}}\right| = \left|\frac{52}{13\sqrt{2}}\right| \Rightarrow 2\sqrt{2}$

86. 00004.00

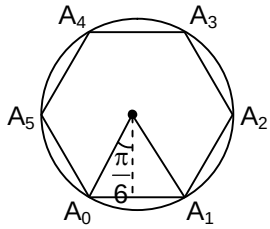
Sol. Line $\frac{x-2}{a} = \frac{y-b}{1} = \frac{z-4}{4}$ intersect the xy plane at p(2 – a, b – 1, 0)

$\because$ Point p satisfy the equation of circle

$\Rightarrow (2-a)^2 + (b-1)^2 - 8(2-a) + 2(b-1) - 1 = 0$
$\Rightarrow (a+2)^2 + b^2 = 18$
$\therefore\ a = -2 + 3\sqrt{2}\cos\theta\ ;\ b = 3\sqrt{2}\sin\theta$
$\Rightarrow a + b = -2 + 3\sqrt{2}(\cos\theta + \sin\theta)$
Maximum (a + b) = 4

87. 00006.00

Sol. $A_0A_2 = A_0A_4 = A_2A_4 = 2.1 \sin 60° = \sqrt{3}$
$\Rightarrow (p+q)r = (2\sqrt{3})\sqrt{3} = 6$



88. 00002.00

Sol. Let equation of tangent to parabola $ty = x + 4t^2$ ..... (1)

Midpoint of AB is (h, k) then AB will be $\frac{xk + yh}{2} - c^2 = hk - c^2$ ..... (2)

By comparing (1) and (2), we get
$\Rightarrow k^2 = -2h \Rightarrow y^2 = -2x$ length of latus rectum = 2

89. 00002.66
(Range 00002.66 – 00002.67)

Sol. Equation of pair of angle bisector will be $3y^2 - 8xy - 3x^2 = 0$ ..... (1)

(As slope of first bisector is 3 then slope of second is $-\frac{1}{3}$)

Equation of angle bisector of $ax^2 + by^2 + 2hxy = 0$ will be $\frac{x^2 - y^2}{a-b} = \frac{xy}{h}$ ..... (2)

By comparing both equation $\left|\frac{a-b}{h}\right| = \frac{8}{3}$

90. 00001.50

Sol. Using A.M. $\geq$ H.M. and $a^2 + b^2 + c^2 \geq ab + bc + ca$

$$\frac{1}{ab+1} + \frac{1}{bc+1} + \frac{1}{ca+1} \geq \frac{9}{3+ab+bc+ca} \geq \frac{9}{3+a^2+b^2+c^2} \geq \frac{9}{6}$$