Chapter 5

Forces on Immersed Bodies

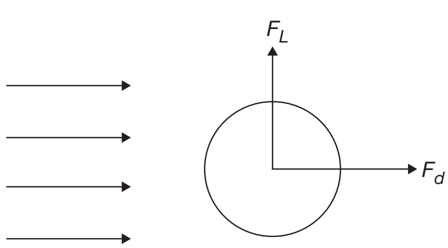
CHAPTER HIGHLIGHTS

- *Forces on immersed bodies*
- *Expressions for drag and lift*
- *General equations for lift and drag*
- *Drag force on a sphere*
- *Drag force on a cylinder*
- *Streamlined and bluff bodies*
- *Terminal velocity of a body*
- *Circulation and lift on a cylinder*
- *Stagnation points*
- *Pressure on the cylinder surface*
- *Lift on cylinder*
- *Lift coefficient for rotating cylinder*
- *Classroom practice exercise*

Forces on Immersed Bodies

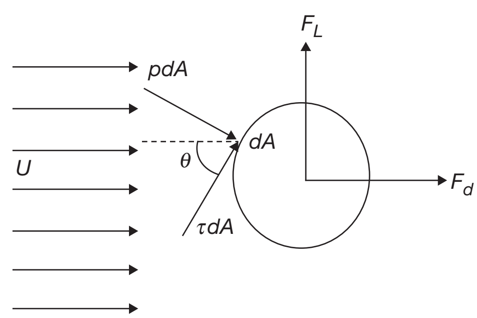
When a body moves within a fluid a force is exerted by the fluid on the body. Similarly when a body is placed in a fluid in motion then also the fluid exerts a force on the body. So whenever there is a relative motion between a fluid and a body, the fluid exerts a force on the body. The body exerts an equal and opposite force (reaction) on the fluid.

Take the case of a body placed in a fluid moving at velocity U. The moving fluid will exert a force F on the body, which can be resolved into two components—one in the direction of motion of the fluid, and other normal to it. The force in the direction of motion is called **drag force** (F_d) and the normal force is called **lift force** (F_L)



Motion of submarines, torpedoes, aeroplanes, structures such as building and bridges submerged in air or water are some of the examples when drag and lift forces are experienced.

Expressions for Drag and Lift



Consider the case of a stationary body in a stream of velocity U. On an elemental area dA on the surface of the body, let p be the static pressure and τ be the shear stress. Then forces on the elemental area are, pdA acting normal to the element and τdA acting tangential to the element. Let θ be the inclination of the element to the direction of motion of the stream: Then, drag force acting on the elemental area

$$= pdA\sin\theta + \tau dA\cos\theta$$

Lift force acting on the elemental area,

$$\tau dA\sin\theta - pdA\cos\theta$$

Now, total drag and lift forces are given by,

$$F_d = \int_A [pdA\sin\theta + \tau dA\cos\theta]$$

and $F_L = \int_A [\tau dA\sin\theta - pdA\cos\theta]$ where $\int_A$ represents integration over the entire surface of the body $\int_A p\,dA\cos\theta$ is called pressure drag or form drag $\int_A \tau\,dA\sin\theta$ is called friction drag or shear drag or skin drag or viscous drag or surface drag.

General Equations for Lift and Drag

As theoretical evaluation of the forces are not possible for any given body shapes this is done experimentally. To plan experiments properly dimensional analysis is carried out. Force exerted by a fluid on a body depends on dimension of the body, density, viscosity, elastic modulus, velocity of the fluid and the acceleration due to gravity. So the force can be expressed as

$$F = f[L, \rho, \mu, E, U, g]$$

There are seven variables in the relation involving 3 fundamental units (M, L, T). So $7 - 3 = 4$ dimensionless constants can be formed. These are

1. $\dfrac{F}{\rho U^2 L^2}$
2. $\dfrac{\rho UL}{\mu} = Re$ (Reynolds number)
3. $\dfrac{U}{\sqrt{\dfrac{E}{\rho}}} = M$ (Mach number)
4. $\dfrac{U}{\sqrt{\dfrac{g}{L}}} = Fr$ (Froude's number)

From the above, it can be written as

$$\frac{F}{\rho L^2 U^2} = f[Re, M, Fr]$$

In most of the practical cases, effect of elasticity and gravity are negligible and ignorable. With these approximation, the equation becomes

$$\frac{F}{\rho L^2 U^2} = f(Re)$$

Since, F_d and F_L are components of F and L^2 represents area (A) of the body,

$$F_d = C_d \times \frac{1}{2}\rho AU^2 \text{ and } F_L = C_L \times \frac{1}{2}\rho AU^2$$

F_d and F_L are called coefficient drag and coefficient of lift respectively and are functions of Re $\frac{1}{2}\rho U^2$ represents dynamic pressure of the undisturbed flow stream.

A is the projected area on the plane perpendicular to the relative motion of the fluid for the calculations of the drag force. For the calculations of lift force, it is the projected area on a plane perpendicular to the direction of lift force.

Drag Force on a Sphere

Drag force on a sphere immersed in a fluid stream is a function of the Reynold's number $\left(Re = \dfrac{\rho UD}{\mu}\right)$

The drag force is different at different ranges of the Reynold's number

1. When $Re \leq 0.2$
   (Stoke's range)
   $F_D = 3\pi\mu UD$

   Where
   U = Velocity of moving fluid
   μ = Dynamic viscosity of the fluid
   D = Diameter of the sphere

   In this range shear drag is $\frac{2}{3}$ of the total drag and pressure drag is $\frac{1}{3}$ of his total drag.

   Expressing in the standard form,

   $$F_d = C_d\rho A\frac{U^2}{2}$$

   $$= C_d\rho \times \frac{\pi D^2}{4} \times \frac{U^2}{2} = 3\pi\mu UD$$

   From the above,

   $$C_d = \frac{24\mu}{\rho UD} = \frac{24}{Re} \text{ (Stoke's formula).}$$

2. When Re is between 0.2 and 5

   $$C_d = \frac{24}{Re}\left(1 + \frac{3}{16}Re\right) = 4.5 + \frac{24}{Re}$$

3. When Re is greater than 1 and less than 100

   $$C_d = \left(1 + \frac{3}{16}Re\right)^{\frac{1}{2}} \times \frac{24}{Re}$$

4. When Re is between 5 and 1000. In this range C_d decreases and reaches a value of 0.4.

5. When Re is between 1000 and 10^5. In this range C_d increases slightly from 0.4 to 0.5.

6. When Re is greater than 10^5.
   As Re reaches 3×10^5, C_d drops to 0.2

Drag Force on a Cylinder

When a cylinder of diameter D and length L is placed, in a fluid stream such that the length of the cylinder is at right angles to the stream direction.

1. When Re is between 1000 and 3×10^5. In this range C_d is approximately equal to 1.2.
2. When Re is between 3×10^5 and 5×10^5. In this range C_d drops from 1.2 to 0.3.

Streamlined and Bluff Bodies

When a body is placed in a flow, if the surface coincides with the streamlines, it is called a streamlined body. In this case the pressure drag will be very small as flow separation from surface takes place only at the trailing edge of the body. A body may become streamlined depending upon stream velocity and shape of the body. When the streamlines does not coincide with the surface of the body, it is called a bluff body. In this case extensive boundary layer separation and wake formation takes place resulting in very large pressure drag compared to friction drag.

Terminal Velocity of a Body

Maximum velocity attained by a falling body is called the terminal velocity. When a body is allowed to fall from rest, its velocity is increased due to acceleration due to gravity. As the velocity increases the drag force also increases. When the drag force becomes equal to the weight of the body the acceleration ceases and the body moves with a constant velocity. This velocity is called the terminal velocity. Terminal velocity of a body falling through a liquid at rest is obtained from the relation

$$W = F_d + F_b$$

$$\text{or} \quad F_d = W - F_b$$

Where
W = Weight of the body
F_d = Drag force
F_b = Buoyant force

From the above,

$$3\pi\mu DU = \frac{\pi}{6}D^3(w_s - w_f)$$

$$\text{or} \quad \frac{D^2}{18\mu}(w_s - w_f)$$

SOLVED EXAMPLES

Example 1

A sphere of 150 mm diameter is held in equilibrium by a vertical air stream of velocity 15 m/s. If density of air is 1.225 kg/m^3 and coefficient of drag is 0.43, weight of the sphere (in N) is ______.

Solution

d = 150 mm = 0.15 m
U = 15 m/s
C_d = 0.43
ρ = 1.225 kg/m^3
A = Projected area of sphere

$$= \frac{\pi d^2}{4} = \frac{\pi}{4}(0.15)^2$$

$$= 0.01767 \text{ m}^2$$

Weight of the body = Drag force

$$= \frac{1}{2}C_d \rho A U^2$$

$$= \frac{1}{2} \times 0.43 \times 1.225 \times 0.01767 \times 15^2$$

$$= 1.0471 \text{ N.}$$

Example 2

A truck with projected area of 7 m^2 travels at a speed of 65 km/h. Total resistance on the truck is 2000 N. 20% of this is due to rolling friction, 10% is due to surface friction and the rest is due to form drag. If density of air is 1.22 kg/m^3, coefficient of form drag is ______.

Solution

Projected area, $A = 7\text{ m}^2$
Speed of truck, U = 65 km/h

$$= \frac{65 \times 10^3}{3600} \text{ m/s}$$

$$= 18.056 \text{ m/s}$$

Density of air, ρ = 1.22 kg/m^3
Total resistance = 2000 N
Resistance due to form or

$$\text{Pressure drag} = \frac{(100 - 20 - 10)}{100} \times 2000$$

$$= 1400 \text{ N}$$

$$= C_D \times \frac{\rho A U^2}{2}$$

Where, C_D = Coefficient of form drag

$$\therefore C_D \times 1.22 \times \frac{7 \times (18.056)^2}{2} = 1400$$

$$\Rightarrow C_D = 1.0057.$$

Example 3

A flat plate, 1.6 m × 1.6 m in size moves at a velocity of 60 km/h in stagnant air of density 1.15 kg/m^3. If coefficient of drag and coefficient of lift are 0.15 and 0.75 respectively, determine the following:

(i) Lift force
(ii) Drag force
(iii) Resultant force
(iv) Power required to keep the plate in motion.

Solution

Area of plate $A = 1.6 \times 1.6$
$= 2.56\ \text{m}^2$

Velocity of plate $U = 60$ km/h

$$= \frac{60 \times 10^3}{3600} = \frac{60 \times 5}{18} = 16.67\ \text{m/s}$$

Density of air $\rho = 1.15$ kg/m^3

$C_D = 0.15$
$C_L = 0.75$

(i) Lift force, $F_L = C_L \times \dfrac{\rho A U^2}{2}$

$$= 0.75 \times 1.15 \times 2.56 \times \frac{(16.67)^2}{2}$$
$$= 306.79\ \text{N}$$

(ii) Drag force, $F_D = C_D \times \dfrac{\rho A U^2}{2}$

$$= F_L \times \frac{C_D}{C_L}$$
$$= 306.79 \times \frac{0.15}{0.75}$$
$$= 61.36\ \text{N}$$

(iii) Resultant force $F_R = \sqrt{F_D^2 + F_L^2}$

$$= \sqrt{(61.36)^2 + (306.79)^2}$$
$$= 312.87\ \text{N}$$

(iv) Power required $= F_D \times U$

$= 61.36 \times 16.67$ N-m/s or W
$= 1022.87$ W
$= 1.023$ kW.

Example 4

Coefficient of drag due to boundary layer over or thin flat plate 5 m long and 0.3 m wide is given by

$$C_D = \frac{0.455}{(\log_{10} Re_L)^{2.58}}$$

where, Re_L = Reynolds number at length L

The plate is kept immersed in a stream of water having free stream velocity of 3.5 m/s, parallel to the flow. If kinematic viscosity of water is 1×10^{-6} m^2/s, total drag force (in N) on the plate is ______.

Solution

Length of plate, $L = 5$ m
Width, $B = 0.3$ m
Kinematic viscosity, $\nu = 1 \times 10^{-6}$ m^2/s
Free stream velocity, $U = 3.5$ m/s
Reynolds number

$$Re_L = \frac{\rho U L}{\mu} = \frac{UL}{\nu} = \frac{3.5 \times 5}{1 \times 10^{-6}} = 1.75 \times 10^7$$

$$\therefore C_D = \frac{0.455}{(\log_{10} Re_L)^{2.58}}$$
$$= \frac{0.455}{\left[\log_{10} 1.75 \times 10^7\right]^{2.58}}$$
$$= \frac{0.455}{165.423}$$
$$= 2.75 \times 10^{-3}$$

Total drag force (on two sides)

$$= 2 \times C_D \times \frac{\rho A U^2}{2}$$
$$= 2 \times 2.75 \times 10^{-3} \times 1000 \times (5 \times 0.3) \times \frac{3.5^2}{2}$$
$$= 50.53\ \text{N}.$$

Example 5

A cable of 16 mm diameter stretches between 2 towers 400 m apart. If wind blows transverse to the cable at a velocity of 90 km/h, force (in N) exerted on the cable is ______. (Take unit weight of air = 12.5 N/m^3 and $C_D = 1.2$)

Solution

$C_D = 1.2$

$$\rho = \frac{w}{g} = \frac{12.5}{9.81}\ \text{kg/m}^3$$

Projected area $A = dL$

$$= \frac{16}{1000} \times 400 = 6.4\text{m}^2$$

$U = 90$ km/h

$$= \frac{90 \times 10^3}{3600} = 25\ \text{m/s}$$

Force exerted (drag force),

$$F_D = C_D \rho A \frac{U^2}{2}$$

$$= 1.2 \times \frac{12.5}{9.81} \times 6.4 \times \frac{(25)^2}{2}$$

$$= 3058.1 \text{ N}$$

Example 6

A car of weight 28 kN is moving on a horizontal road at a speed of 65 km/h. Coefficient of friction at the tyres is 0.015. Maximum cross-sectional area of the car is 2.7 m^2. If density of air is 1.2 kg/m^3 and coefficient of drag is 0.6, power (in kW) required by the car is ________.

Solution

$C_D = 0.012$

$\rho = 1.2$ kg/m^3

$A = 2.7$ m^2

Weight, $W = 28$ kN

$\mu = 0.015$

$$U = 65 \text{ km/h} = 65 \times \frac{1000}{3600} = 18.06 \text{ m/s}$$

Force on the car,

F = Drag force + Frictional force at the tyres

$$= C_D\, \rho \text{A} \frac{U^2}{2} + \mu W$$

$$= 0.6 \times 1.2 \times 2.7 \times \frac{(18.06)^2}{2} + 0.015 \times 28 \times 10^3$$

$$= 317.03 + 420$$

$$= 737.03 \text{ N}$$

Power required,

$P = F \times U$

$= 737.03 \times 18.06$ W

$= 13310.76$

$= 13.31$ kW.

Example 7

A sign board of 2.5 m × 1 m is mounted on top of a 4 m high pole. If wind blows at a speed of 45 km/h against the board, bending moment (in Nm) at the base of the pole is ______. [Take, $C_D = 1.85$ and density of air = 1.25 kg/m^3]

Solution

$C_D = 1.85$

$\rho = 1.25$ kg/m^3

$A = 2.5 \times 1$

$= 2.5$ m^2

$U = 45$ km/h

$$= 45 \times \frac{5}{18} \text{ m/s}$$

Height of pole, $h = 4$ m

Drag force,

$$F_D = C_D \times \frac{\rho A U^2}{2}$$

$$= 1.85 \times 1.25 \times 2.5 \times \frac{(12.5)^2}{2}$$

$= 451.66$ N

Bending moment at the base of pole

$= F_D \times h$

$= 451.66 \times 4$

$= 1806 .64$ N/m

Example 8

An aluminium ball of 2 mm diameter falls in an oil of specific gravity 0.9 and reaches the terminal velocity of 0.5 cm/s. Assuming that Reynolds number is in Stoke's range determine the value of dynamic viscosity of the oil. (Specific gravity of aluminium = 2.8)

Solution

When the Reynold's number is in the Stoke's range,

Viscous resistance = $3\pi\mu dv$

Net down ward force

= Viscous resistance

$$\text{That is, } (w_s - w)\frac{4}{3}\neq r^3$$

$$= 3\pi\mu dv$$

$$= 6\pi\mu rv$$

$$\Rightarrow \mu = \frac{2(w_s - w)\, r^2}{9v}$$

$$= \frac{2 \times 9810(2.8 - 0.9) \times (0.001)^2}{9 \times 0.5 \times 10^{-2}}$$

$= 0.8284$ Ns/m^2.

Example 9

A kite weighing 10 N and having area 1 m^2 makes an angle 8° to the horizontal when flying in a wind of 36 km/h. If pull on the string attached to the kite is 49 N and it is inclined to the horizontal at 45°. determine the lift and drag coefficients, [density of air is 1.2 kg/m^3]

Solution

Weight of the kite = 10 N

Area of the kite = 1 m^2

Angle with horizontal = 8°

Angle made by string with horizontal = 45°

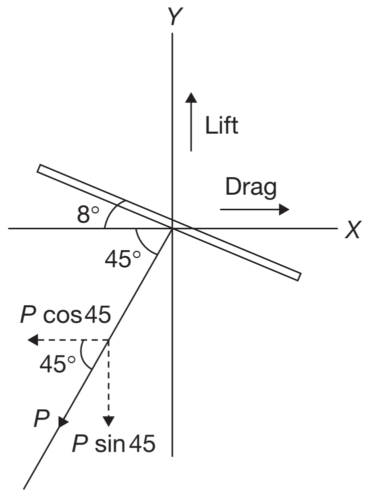
Pull on the string = 49 N

Density of air $\rho = 1.2$ kg/m^3

$U = 36$ km/h

$$= 36 \times \frac{5}{18}$$

$= 10$ m/s.



Drag force, $F_D = P\cos 45 = 49\cos 45$
$= 34.65$ N

$$= C_D\,\rho A\frac{U^2}{2}$$

$$\therefore\ 34.65 = C_D \times 1.2 \times 1 \times \frac{10^2}{2}$$

$$\Rightarrow C_D = 0.577$$

Lift force F_L = Component of P in the vertical direction + Weight of the kite

$= 49\sin 45 + 10 = 44.65$ N

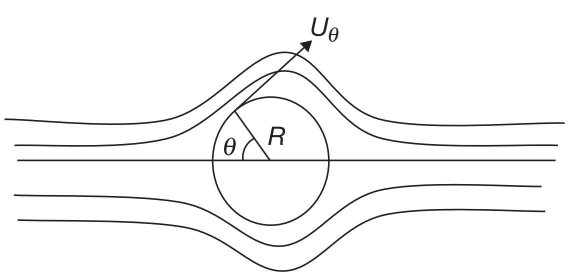
$$\therefore\ 44.65 = C_L \times 1.2 \times 1 \times \frac{10^2}{2}$$

$$= 60\ C_L$$

$$\Rightarrow C_L = 0.744.$$

Circulation and Lift on a Cylinder

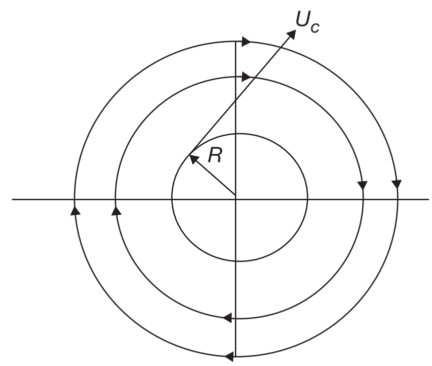
Stationary Cylinder



When an ideal fluid is flowing over a stationary cylinder of radius R with a uniform velocity the flow pattern will be symmetrical. Velocity u at any point on the surface is given by,

$$u_\theta = 2U\sin\theta$$

Constant Circulation Imparted to Cylinder

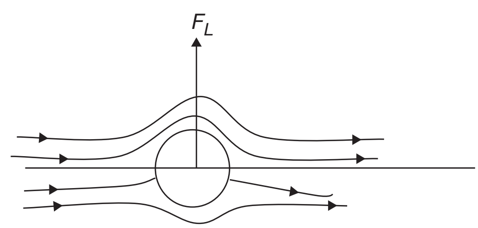


Peripheral velocity U_c is given by,

$$U_c = \frac{\Gamma}{2\pi R}$$

Where, Γ = circulation

Composite flow pattern: When the above two flow patterns are superimposed the flow patterns will be as shown below.



Velocity $u = u_\theta + u_c$

$$= 2U\sin\theta + \frac{\Gamma}{2\pi R}$$

Velocity will be more in the upper position. Velocity will be less and pressure will be more in the lower portion. The flow is symmetrical about the vertical axis. So there is no drag force acting.

Stagnation Points

Stagnation points are points on the surface of the cylinder where velocity is zero.

$$\therefore\ 2U\sin\theta + \frac{\Gamma}{2\pi R} = 0$$

$$\text{or } \sin\theta = \frac{\Gamma}{4\pi UR} = 0$$

Pressure on the Cylinder Surface

Pressure at a point on the cylinder is obtained by applying Bernoulli's equation between an unaffected point in the upstream and any point on the surface. Thus,

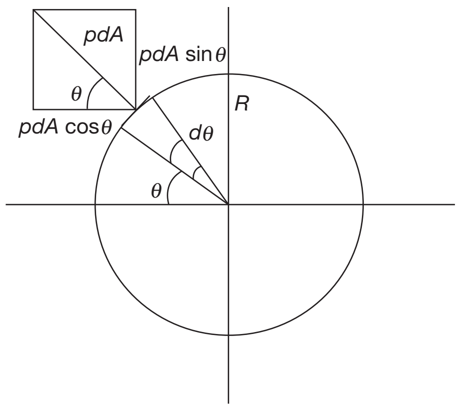
$$p_0 + \frac{1}{2}\rho U^2 = p + \frac{1}{2}\rho U^2$$

Substituting the value of u,

$$p_0 + \frac{1}{2}\rho U^2 = p + \frac{1}{2}\rho\left(2U\sin\theta + \frac{\Gamma}{2\pi R}\right)^2$$

$$\therefore\; p = p_0 + \frac{1}{2}\rho U^2\left[1 - \left(2\sin\theta + \frac{\Gamma}{2\pi UR}\right)^2\right]$$

Lift on Cylinder



For a cylinder rotating in a uniform flow field, considering an elemental length $Rd\theta$ along circumference.

$$dA = Rd\theta L$$

Where, L = Length of the cylinder

Force on dA

$$dF = p\,Da = pRd\theta L$$

Resolving the force in the horizontal and vertical directions

$$dF_D = pRd\theta L\cos\theta$$

and $dF_L = -pRd\theta L\sin\theta$

By integrating the above we get total drag F_D and total lift F_L.

$$\text{Total } F_L = -\int_0^{2\pi} pRL\sin\theta\,d\theta$$

$$\text{But } p = p_0 + \frac{1}{2}\rho U^2\left[1 - \left(2\sin\theta + \frac{\Gamma}{2\pi uR}\right)^2\right]$$

$$\therefore\; F_L = -RL\left[\int_0^{2\pi} p_0\sin\theta\,d\theta - \int_0^{2\pi}\frac{1}{2}\rho U^2\sin\theta\,d\theta\right.$$

$$-\frac{1}{2}\rho u^2\left\{\int_0^{2\pi} 4\sin^3\theta\,d\theta + \int_0^{2\pi}\frac{2\Gamma}{\pi UR}\sin^2\theta\,d\theta\right.$$

$$\left.\left.+\int_0^{2\pi}\frac{\Gamma^2}{4\pi^2U^2R^2}\sin\theta\,d\theta\right\}\right]$$

$$\int_0^{2\pi}\sin^n\theta\,d\theta = 0 \text{ for odd values of } n.$$

So the above expression reduces to,

$$F_L = RL\left(\frac{1}{2}\rho U^2 \times \frac{2\Gamma}{\pi UR}\right)\int_0^{2\pi}\sin^2\theta\,d\theta$$

$$= \frac{\rho LU\Gamma}{\pi}\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta$$

$$= \frac{\rho LU\Gamma}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi} = \rho UL\Gamma$$

This equation is known as Kutta–Joukowski equation.

This equation is applicable to other bodies of any shape, including airfoil.

Lift Coefficient for Rotating Cylinder

$$F_L = C_L\frac{\rho AU^2}{2} \quad \text{Or} \quad C_L = \frac{2F_L}{\rho U^2 A}$$

But $F_L = \rho LU\Gamma$

$$\therefore\; C_L = \frac{2\rho LU\Gamma}{\rho U^2 A} = \frac{2\rho LU\Gamma}{\rho U^2 2RL}\quad\left[\therefore A = 2RL = \frac{\Gamma}{UR}\right]$$

But $\Gamma = 2\pi RU_c$

$$\therefore\; C_L = \frac{2\pi RU_c}{UR} = \frac{2\pi U_c}{U}$$

$\therefore$ C_L depends on the ratio $\frac{u_c}{U}$.

The flow pattern over a rotating cylinder in a uniform flow field is symmetrical about the vertical axis of the cylinder and there is no drag on the cylinder.

The phenomenon of production of lift force on a rotating cylinder in a moving mass of fluid is called **Magnus effect** (named after the German physicist HG Magnus, who did the first experimental investigations in this).

Example 10

A polystyrene spherical particle of density 1048 kg/m^3 falls in air, obeying Stoke's law. If density and kinematic viscosity of air are 1.2 kg/m^3 and 1.5 × 10^{-5} m^2/s respectively, determine largest diameter and terminal velocity of the particle. (Assume Stoke's law is valid upto Reynolds number = 1)

Solution

$\rho_s = 1048$ kg/m^3

$\rho_a = 1.2$ kg/m^3

$\nu_a = 1.5 \times 10^{-5}$ m^2/s

$Re_{\max} = 1$

$$= \frac{UD}{\nu_a} \quad \text{or} \quad U = \frac{\nu_a}{D}$$

Terminal velocity,

$$U = \frac{D^2}{18\mu}(w_s - w_f)$$

$$\therefore \frac{v_a}{D} = \frac{D^2}{18\mu}(w_s - w_f)$$

$$\frac{1.5\times10^{-5}}{D} = \frac{D^2 g(1048-1.2)}{18\times1.8\times10^{-5}}[\because \mu = \rho v]$$

$$= 1.2 \times 1.5 \times 10^{-5} = 1.8 \times 10^{-5}$$

$$\Rightarrow D^3 = \frac{1.5\times18\times1.8\times10^{-10}}{9.81\times1046.8}$$

$$= 4.7326 \times 10^{-3}$$

$$\Rightarrow D = 7.7929 \times 10^{-5}\text{m} = 0.0779 \text{ mm}$$

Terminal velocity,

$$U = \frac{v_a}{D} = \frac{1.5\times10^{-5}}{7.7929\times10^{-5}} = 0.1925 \text{ m/s.}$$

Example 11

An airplane weighing 118 kN has a wing span of 12 m and a wing area of 20 sqm. If it flies at a velocity of 360 km/h at a steady level in still air, determine:

(i) Coefficient of lift.

(ii) Total drag on the wing (Take $C_d = 0.06$).

(iii) Power required to keep the air plane at this velocity.

(iv) Theoretical value of boundary circulations around the wing (Take density of air = 1.25 kg/m^3).

Solution

$W = 118$ kN

$= 118 \times 10^3$ N

$L = 12$ m

$A = 20$ m^2

$U = 360$ km/h

$$= 360 \times \frac{1000}{3600} = 100 \text{ m/s}$$

$\rho = 1.25$ kg/m^3

(i) At steady level

$$W = F_L = C_L \rho A \frac{U^2}{2}$$

i.e., $118 \times 10^3 = C_L \times 1.25 \times 20 \times \frac{(100)^2}{2}$ N

$\Rightarrow C_L = 0.944$

(ii) $F_D = C_d \dfrac{\rho A U^2}{2}$

$$= 0.06 \times 1.25 \times 20 \times \frac{(100)^2}{2} = 7500 \text{ N}$$

(iii) Power required

$= F_d \times U = 7500 \times 100$

$= 750000$ W $= 750$ kW

(iv) $F_L = W = \rho U \Gamma L$

Where Γ = Circulation

$\therefore 118 \times 10^3 = 1.25 \times 100 \times \Gamma \times 12$

$\Rightarrow \Gamma = 78.67$ m^2/s

Exercises

1. A smooth flat plate is in a parallel flow stream. The ratio of the drag over the upstream half of the plate to that over the entire plate is ______. Assume the average skin friction coefficient is proportional to $(Re_x)^{-1/2}$.

2. The following items consists of two statements one labeled as 'Assertion (A)' and the other as 'Reason (R)'. You have to examine these two statements carefully and select the answers from given options

Assertion (A): If a cylindrical body is placed in a fluid stream and is rotated, then a lift force is produced on the body.

Reason (R): Rotation of the cylinder disturbs the symmetrical pattern which in turn alters the pressure distribution on the body.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true and R is not correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

3. On an immersed body in a flowing fluid, the lift force is

(A) due to buoyant force.

(B) always in the opposite direction to gravity.

(C) due to wake phenomenon.

(D) the dynamic fluid force component normal to approach velocity.

4. Which one of the following statements relating to vortex flow is INCORRECT?

(A) In the formation of a free vortex, stream-lines are axisymmetric and irrotational.

(B) In a forced vortex, work transfer between the fluid and the surroundings takes place and flow is rotational.

(C) In a free vortex, radial motion towards the core takes place due to variation of depth of water in the whirlpool or due to the difference of pressure resulting from higher velocity near the core.

(D) In a free vortex, there is no variation of energy from streamline and irrotationality is not deviated from near the core.

5. Quetion consists of two statements, one labeled as the 'Statements I' and the other as 'Statements II'. You have to examine these two statements carefully and select the answer from given options.

Statement I: A tennis ball with a rough surface can be made to curve more markedly than the one with a smooth surface.

Statement II: Circulation is increased in the case of a rough ball.

(A) Both Statement I and Statement II are individually true and Statement II is the correct explanation of Statement I.
(B) Both Statement I and Statement II are individually true and Statement II is NOT the correct explanation of Statement I.
(C) Statement I is true but Statement I is false.
(D) Statement I is false but Statement II is true.

6. An aircraft is flying in level flight at a speed of 200 km/h through air (density, $\rho = 1.2$ kg/m^3, and viscosity $\mu = 1.6 \times 10^{-5}$ N-s/m^2). The lift coefficient at this speed is 0.4 and the drag coefficient is 0.0065. The mass of the aircraft is 800 kg. The effective lift area of the aircraft is

(A) 21.2 m^2 (B) 10.6 m^2
(C) 2.2 m^2 (D) 1.1 m^2

7. Which one of the following gives the lift force F_L produced on a rotating circular cylinder per unit length in a uniform flow?

(A) $F_L = \dfrac{\mu\Gamma}{\rho}$ (B) $F_L = \rho u \Gamma$

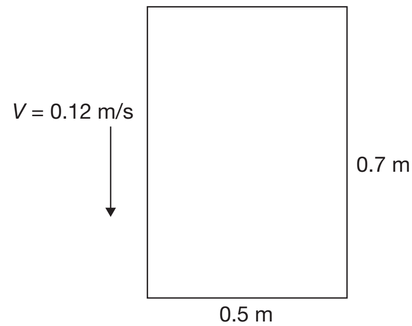
(C) $F_L = \dfrac{\rho\Gamma}{u}$ (D) $F_L = \dfrac{\rho u}{\Gamma}$

Where
U = Free stream velocity
G = Circulation
ρ = Density of the fluid

8. A thin flat plate 0.5 m × 0.7 m in size settles in a large tank of water with a terminal velocity of 0.12 m/s. The coefficients of drag,

$C_D = \dfrac{1.328}{\sqrt{R_L}}$ for a laminar boundary layer and $C_D = \dfrac{0.072}{(R_L)^{1/5}}$ for a turbulent boundary layer, where R_L, is the plate Reynolds number.

Assume $\mu = 10^{-3}$ N$^-$ s/m^2 and $\rho = 1000$ kg/m^3



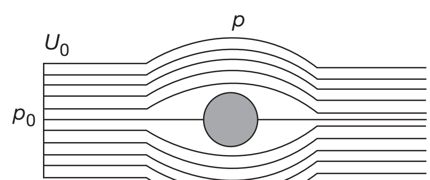
The submerged weight of the plate is

(A) 0.0115 N (B) 0.0118 N
(C) 0.0231 N (D) 0.0376 N

9. Match List I (Flow pattern past a cylinder) with List II (Pressure distribution) and select the correct answer using the codes given

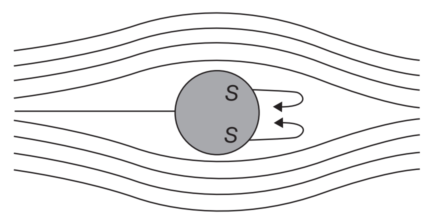
List I

a.



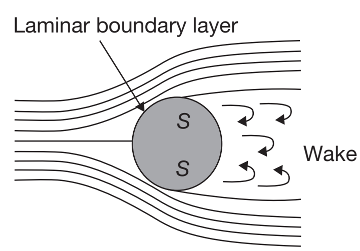
Re < 0.5

b.



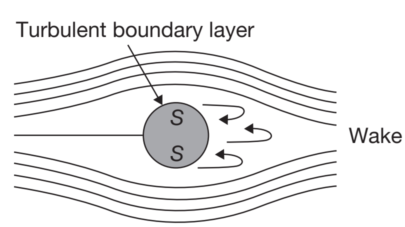
2 < *Re* < 30

c.



$10^3 \le Re \le 2 \times 10^5$

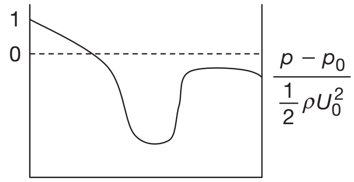
d.



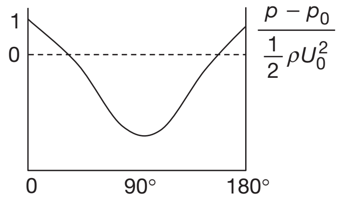
$Re > 2 \times 10^5$

List II

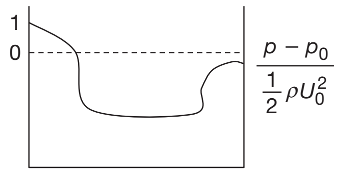
1.



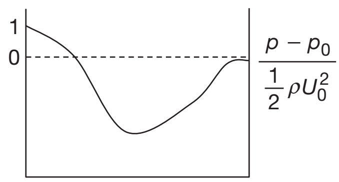
2.



3.



4.



Codes:

	a	b	c	d		a	b	c	d
(A)	2	5	1	3	(B)	4	3	5	2
(C)	2	1	5	4	(D)	4	3	1	2

10. A closed cylindrical vessel of radius R completely filled with a liquid of density ρ is rotated at an angular velocity ω about its vertical axis. The total fluid pressure force acting on the top is

(A) $\frac{\rho}{4}\omega^2\pi R^4$

(B) $\frac{\rho}{4}\omega^2\pi R^3$

(C) $\frac{\rho}{4}\omega^2\pi R^2$

(D) $\frac{\rho}{4}\omega^2\pi R$

Previous Years' Questions

Direction for questions 1 and 2:

An automobile with projected area 2.6 m^2 is running on a road with a speed of 120 km/h. The mass density and the kinematic viscosity of air are 1.2 kg/m^3 and 1.5 × 10^{-5} m^2/s, respectively. The drag coefficient is 0.30.

1. The drag force on the automobile is **[GATE, 2008]**

(A) 620 N (B) 600 N
(C) 580 N (D) 520 N

2. The metric horse power required to overcome the drag force is **[GATE, 2008]**

(A) 33.23 (B) 31.23
(C) 23.23 (D) 20.23

Answer Keys

Exercises

1. $\frac{1}{\sqrt{2}}$ **2.** A **3.** D **4.** D **5.** C **6.** B **7.** B **8.** C **9.** A **10.** A

Previous Years' Questions

1. D **2.** C