Mathematics

Chapterwise Practise Problems (CPP) for JEE (Main & Advanced)

Chapter - Inverse Trigonometry Function

Level-1

SECTION - A

Straight Objective Type

This section contains multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

1. The number of solution(s) of the equation $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$ is

   (A) 0 (B) 1

   (C) 2 (D) 3

2. The sum of all the solutions of the equation $\tan^{-1}\left(\frac{2x}{x^2-1}\right)+\cot^{-1}\left(\frac{x^2-1}{2x}\right)=\frac{2\pi}{3}$ is

   (A) $\frac{2}{\sqrt{3}}$ (B) $-\frac{2}{\sqrt{3}}$

   (C) $\frac{-1}{\sqrt{3}}$ (D) $\frac{-4}{\sqrt{3}}$

3. Let $y = f(x) = \sin^{-1}([x] - 1)$, where [.] represents greatest integral function. It should be noted that $\sin^{-1} x$ is defined for $-1 \le x \le 1$. The domain of function is

   (A) (1, 3] (B) [0, 3)

   (C) (2, 3) (D) (–1, 3)

4. If $\sum_{k=1}^{100} \sin^{-1} x_k = 50\,\pi, k \in N$, then $\sum_{k=1}^{100} k x_k^3$ equals

   (A) 5050 (B) 10^3

   (C) 10^6 (D) $(5050)^2$

5. The solution of $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ is/are

   (A) –1 (B) $\frac{1}{2}$

   (C) $\frac{1}{6}$ (D) $\frac{1}{6}$ and –1

6. The value of x which satisfies equation $2\tan^{-1} 2x = \sin^{-1}\frac{4x}{1+4x^2}$ is

   (A) $\left[\frac{1}{2},\infty\right)$ (B) $\left(-\infty,-\frac{1}{2}\right]$

   (C) $[-1,1]$ (D) $\left[-\frac{1}{2},\frac{1}{2}\right]$

7. The sum $\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{4n}{n^4-2n^2+2}\right)$ is equal to

   (A) $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{3}$ (B) $4\tan^{-1}1$

   (C) $\frac{\pi}{2}$ (D) $\sec^{-1}\left(-\sqrt{2}\right)$

SECTION - B

Multiple Correct Answer Type

This section contains multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

8. $\tan\left[\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right]$ does not equal to

   (A) $\frac{3+\sqrt{5}}{2}$ (B) $\frac{\sqrt{5}+3}{4}$

   (C) $\frac{3-\sqrt{5}}{2}$ (D) $\frac{\sqrt{5}-3}{4}$

9. The value of $\sin^{-1}(\cos(\sin^{-1}x)) + \cos^{-1}(\sin(\cos^{-1}x))$ is not equal to

   (A) π (B) $\frac{\pi}{2}$

   (C) $\frac{\pi}{4}$ (D) 0

10. If the equations

$\sin^{-1} x + \cos^{-1} y = \frac{3\pi}{2}$ ...(i)

$2\sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$ ...(ii)

$3\sin^{-1} x = \sin^{-1}(3x - 4x^3)$ ...(iii)

hold simultaneously, then the number of values of y is not equal to

(A) 2 (B) 1

(C) 0 (D) 3

11. If $\tan^{-1}(x) + \tan^{-1}(1-x) = \cot^{-1}\left(\frac{5}{6}\right)$ then x may be

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{1}{4}$ (D) $\frac{3}{4}$

12. Identify the pair(s) of functions which are identical?

(A) $f(x) = \tan(\text{arc}\cos x)$, $f(x) = \frac{\sqrt{1-x^2}}{x}$

(B) $f(x) = \tan(\text{arc}\cot x)$, $f(x) = \frac{1}{x}$

(C) $f(x) = \sin(\text{arc}\tan x)$, $f(x) = \frac{x}{\sqrt{1+x^2}}$

(D) $f(x) = \cos(\text{arc}\tan x)$, $f(x) = \sin(\text{arc}\cot x)$

13. If $\cos(2\sin^{-1}x) = \frac{1}{9}$, then x can attain value(s)

(A) $-\frac{2}{3}$ (B) $\frac{2}{3}$

(C) $-\frac{3}{2}$ (D) $\frac{3}{2}$

14. If $\alpha = \tan^{-1}(\tan 4)$, $\beta = \tan^{-1}(\tan(-6))$, $\gamma = \sin^{-1}(\sin 10)$, then

(A) $\alpha < \beta < \gamma$ (B) $\alpha > \beta > \gamma$

(C) $\alpha, \frac{\beta}{2}, \gamma$ are in AP (D) $\beta - \alpha = \gamma$

15. The value of λ for which the equation

$\lambda x^2 + \sin^{-1}(x^2 - 2x + 2) + \cos^{-1}(x^2 - 2x + 2) = 0$ has not a real solution is

(A) $-\frac{\pi}{3}$ (B) $-\frac{\pi}{2}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

16. Let $f(x) = e^{\cos^{-1}\sin\left(x+\frac{\pi}{3}\right)}$, then

(A) $f\left(\frac{8\pi}{9}\right) = e^{\frac{5\pi}{18}}$ (B) $f\left(\frac{8\pi}{9}\right) = e^{\frac{13\pi}{18}}$

(C) $f\left(-\frac{7\pi}{4}\right) = e^{\frac{\pi}{12}}$ (D) $f\left(-\frac{7\pi}{4}\right) = e^{\frac{11\pi}{12}}$

17. Let $f(x) = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ then which of the following is correct?

(A) f(x) is increasing in its domain

(B) f(x) has neither a maxima nor minima

(C) Range of f(x) is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \{0\}$

(D) f(x) has two asymptotes

SECTION - C

Linked Comprehension Type

This section contains paragraph. Based upon this paragraph, 2 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for Question No. 18

One of the most widely used method of summation is "method of difference". It consists in writting the general terms as a difference of two terms in such a way that when we add all the terms of the series, the term cancel in pairs and we are left with one or two terms.

18. $\sin^{-1}\frac{1}{\sqrt{2}} + \sin^{-1}\frac{\sqrt{2}-1}{\sqrt{6}} + \sin^{-1}\frac{\sqrt{3}-\sqrt{2}}{\sqrt{12}} +$ ..... upto ∞ equals

(A) $-\frac{\pi}{2}$ (B) $\frac{\pi}{2}$

(C) $-\frac{\pi}{4}$ (D) $\frac{\pi}{4}$

Paragraph for Question Nos. 19 and 20

Let us consider $\cot^{-1} y = \frac{\pi}{2} - 4\tan^{-1} x$

19. In the above expression, *y* is defined if *x* belongs to

(A) $(-\infty, \infty)$ (B) $(-1, 1)$

(C) $\left(1-\sqrt{2}, \sqrt{2}-1\right)$ (D) $\left[1-\sqrt{2}, \sqrt{2}-1\right]$

20. *y* as algebraic function of *x*, is given by

(A) $\frac{4x(1-x^2)}{x^4-6x^2+1}$ (B) $\frac{x(1-x^2)}{x^4-x^2-6}$

(C) $\frac{4x(1-x^2)}{x^2-6x+1}$ (D) $\frac{x(1-x^2)}{x^2-x-6}$

SECTION-D

Single-Match Type

This section contains Single match questions. Each question contains statements given in two columns which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p,q,r,s. Four options A,B,C and D are given below. Out of which, only one shows the right matching

21. Match the following

Column I	**Column II**
(A) $\cos^{-1}\lambda+\cos^{-1}\mu+\cos^{-1}\nu = 3\pi$, then $\lambda\mu+\mu\nu+\nu\lambda$ is	(p) 2n
(B) $\sum_{i=1}^{10}\cos^{-1} x_i = 0$, then $\sum_{i=1}^{10} x_i$ is,	(q) $\sin^{-1}x-\pi/6$
(C) $\sum_{i=1}^{2n}\sin^{-1} x_i = n\pi$, then $\sum_{i=1}^{2n} x_i$ is	(r) 10
(D) $f(x) = \sin^{-1}\left\{\frac{\sqrt{3}}{2}x - \frac{1}{2}\sqrt{1-x^2}\right\}$, $-1/2 \le x \le 1$, is	(s) 3

A	**B**	**C**	**D**
(A) q	s	r	p
(B) p	q	r	s
(C) s	r	p	q
(D) q	r	p	s

22. Match the following

Column I	**Column II**
(A) $\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3} =$	(p) $\pi/6$
(B) $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} =$	(q) $\pi/2$
(C) If $A = \tan^{-1}\left(\frac{x\sqrt{3}}{2\lambda - x}\right)$, $B = \tan^{-1}\left(\frac{2x-\lambda}{\lambda\sqrt{3}}\right)$ then A–B =	(r) $\pi/4$
(D) $\tan^{-1}\frac{1}{7} + 2\tan^{-1}\frac{1}{3} =$	(s) π

A	**B**	**C**	**D**
(A) q	s	r	p
(B) p	q	r	s
(C) s	r	p	q
(D) q	s	p	r

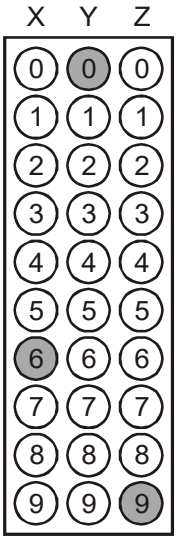
23. Match the following :

Column - I	**Column - II**
(A) The number of solution of $\frac{x}{2} + \frac{\sin x}{\cos x} = \frac{\pi}{4}$ in $[-\pi, \pi]$	(p) 1
(B) The number of solution of equation $\sin^{-1}\left(\lvert x^2-1\rvert\right) + \cos^{-1}\left(\lvert 2x^2-5\rvert\right) = \frac{\pi}{2}$	(q) 0
(C) The number of solution of $x^4 - 2x^2\sin^2\left(\frac{\pi}{2}x\right) + 1 = 0$	(r) 3
(D) The number of solution of $x^2 + 2x + 2\sec^2 x\pi + \tan^2 x\pi = 0$	(s) 2

SECTION-E

Integer Answer Type

This section contains Integer type questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y and Z(say) are 6, 0 and 9, respectively, then the correct darkening of bubbles will look like the following :



24. Number of solution(s) of the equation $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ is

25. Number of solution(s) of equation $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \ldots\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \ldots\right) = \frac{\pi}{2}$ is

26. If $\cot^{-1}\left(\frac{n}{\pi}\right) > \cos^{-1}\cos\left(\frac{13\pi}{6}\right)$, $n \in N$, then the maximum value of n is

27. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$ and $f(1) = 2$, $f(h + k) = f(h).\ f(k)\ \forall\ h, k \in N$, then $x^{f(2008)} + y^{f(2009)} + z^{f(2010)} - \frac{x+y+z}{x^{f(2008)} + y^{f(2009)} + z^{f(2010)}}$ equals

28. If $f(x) = x^{13} - x^{11} + x^9 + x^3 + x + 2$ and $f(\sin^{-1}(\sin 8)) = \lambda$, where λ is constant and $f(\tan^{-1}(\tan 8)) = K - \lambda$ then K is equal to

29. The least value of n for which $(n-2)x^2+8x+n+4 > \sin^{-1}(\sin 12) + \cos^{-1}(\cos 12)$, $\forall x \in R$, where $n \in N$, is ____

30. If x_1, x_2, x_3, x_4 are the roots of the equation $x^4 - x^3 \sin 2\beta + x^2 \cos 2\beta - x \cos\beta - \sin\beta = 0$ such that $\tan^{-1}x_1 + \tan^{-1}x_2 + \tan^{-1}x_3 + \tan^{-1}x_4 = n\pi + \frac{\pi}{\lambda} + \mu\beta$. Then $|\lambda\mu|$ is _____

31. Number of solutions of the equation $\sin^{-1}\left(\left|\log_6^2(\cos x) - 1\right|\right) + \cos^{-1}\left(\left|3\log_6^2(\cos x) - 7\right|\right) = \frac{\pi}{2}$, if $x \in [0, 4\pi]$ is

32. The number of solution of the equation $\cos^{-1}(1 - x) + m\cos^{-1}x = \frac{n\pi}{2}$, where $m > 0$, $n \leq 0$ is

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SECTION - A

Straight Objective Type

This section contains multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

1. If $\cos^{-1} x_1 - \cos^{-1} x_2$

$+\cos^{-1}(x_1x_2 + \sqrt{1-x_1^2}\sqrt{1-x_2^2}) = 0,$

then which of the following is true?

(A) $x_1 + x_2 \le 1$ (B) $x_1 + x_2 \ge -1$

(C) $x_1 \le x_2$ (D) $x_1 \ge x_2$

2. If $\sin^{-1}\left(\sin\left(\frac{2x^2+4}{1+x^2}\right)\right) < \pi - 3$, then

(A) $|x| > 1$ (B) $|x| < 1$

(C) $|x| = 1$ (D) $|x| \le 1$

3. Number of solutions of $[\sin^{-1}x] + [\cos^{-1}x] = 4$ is

(where [.] denotes the greatest integer function)

(A) 1 (B) 0

(C) 2 (D) 3

4. The value of $\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{1}{2r^2}\right)$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$

(C) π (D) 2π

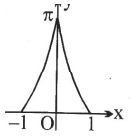
5. The range of the function $f(x) = \sec^{-1}(x) + \tan^{-1}(x)$, is

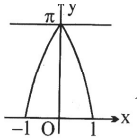
(A) $(0, \pi)$ (B) $\left(\frac{-\pi}{2}, \frac{3\pi}{2}\right)$

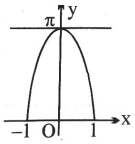
(C) $\left(0, \frac{3\pi}{4}\right)$ (D) none

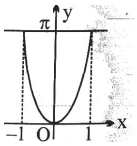
6. Which one of the following can best represent the graph of the function,

$f(x) = \cos^{-1}(2x^2 - 1)$?

(A) 

(B) 

(C) 

(D) 

7. $\cot^{-1} 3 + \cot^{-1}\frac{9}{2} + \cot^{-1}\frac{33}{4} + \cot^{-1}\frac{129}{8}$ + ... upto ∞ equals

(A) $\frac{\pi}{4}$ (B) 0

(C) $\frac{-\pi}{4}$ (D) $\frac{3\pi}{4}$

SECTION - B

Multiple Correct Answer Type

This section contains multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

8. The least value of $(\sin^{-1}x)^3 + (\cos^{-1}x)^3$, $-1 \le x \le 1$, is not equal to

(A) $\frac{\pi^3}{32}$ (B) $\frac{\pi^3}{16}$

(C) $\frac{7\pi^3}{8}$ (D) $\frac{7\pi^3}{16}$

9. Let $t_k = \tan^{-1}\left(\frac{2}{k^2}\right)$ then

(A) $\sum_{k=1}^{n} t_k = \tan^{-1}\left(\frac{n^2+3n}{2+n-n^2}\right), n > 2$

(B) $\sum_{k=1}^{\infty} t_k = \frac{3\pi}{4}$

(C) $\sum_{k=1}^{n} t_k = \tan^{-1}\left(\frac{n^2}{n+2}\right), n > 2$

(D) $\sum_{k=1}^{\infty} t_k = \frac{\pi}{4}$

SECTION - C

Linked Comprehension Type

This section contains paragraph. Based upon this paragraph, 2 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for question nos. 10 and 11

Consider a real-valued function

$f(x) = \sqrt{\sin^{-1} x + 2} + \sqrt{1 - \sin^{-1} x}$

10. The domain of definition of f (x) is

(A) $[-1,1]$ (B) $[\sin 1, 1]$

(C) $[-1, \sin 1]$ (D) $[-1, 0]$

11. The range of f (x) is

(A) $\left[0,\sqrt{3}\right]$ (B) $\left[1,\sqrt{3}\right]$

(C) $\left[1,\sqrt{6}\right]$ (D) $\left[\sqrt{3},\sqrt{6}\right]$

SECTION-D

Single-Match Type

This section contains Single match questions. Each question contains statements given in two columns which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p,q,r,s. Four options A,B,C and D are given below. Out of which, only one shows the right matching

12. Column I contains functions and Column II contains their range. Match the entries of Column I with the entries of Column II.

Column - I	**Column - II**
(A) $f(x) = \sin^{-1}\left(\frac{x}{1+\lvert x\rvert}\right)$	(P) $(0, \pi)$
(B) $g(x) = \cos^{-1}\left(\frac{x}{1+\lvert x\rvert}\right)$	(Q) $\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$
(C) $h(x) = \tan^{-1}\left(\frac{x}{1+\lvert x\rvert}\right)$	(R) $\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$
(D) $k(x) = \cot^{-1}\left(\frac{x}{1+\lvert x\rvert}\right)$	(S) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

13. Let $t_1 = (\sin^{-1}x)^{\sin^{-1}x}$, $t_2 = (\sin^{-1}x)^{\cos^{-1}x}$, $t_3 = (\cos^{-1}x)^{\sin^{-1}x}$, $t_4 = (\cos^{-1}x)^{\cos^{-1}x}$ then,

Column I	**Column II**
(A) for $x \in (0, \cos 1)$	(p) $t_1 > t_2 > t_4 > t_3$
(B) for $x \in \left(\cos 1, \frac{1}{\sqrt{2}}\right)$	(q) $t_4 > t_3 > t_1 > t_2$
(C) for $x \in \left(\frac{1}{\sqrt{2}}, \sin 1\right)$	(r) $t_2 > t_1 > t_4 > t_3$
(D) for $x \in (\sin 1, 1)$	(s) $t_3 > t_4 > t_1 > t_2$

	A	**B**	**C**	**D**
(A)	q	s	r	p
(B)	p	q	r	s
(C)	q	p	s	r
(D)	q	r	p	s

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ANSWERS

LEVEL-1

1. (B)	2. (D)	3. (B)	4. (A)	5. (C)	6. (D)
7. (D)	8. (A,B,D)	9. (A,C,D)	10. (A,B,D)	11. (A,B)	12. (A,B,C,D)
13. (A,B)	14. (B,C,D)	15. (A,C,D)	16. (B,C)	17. (A,B,D)	18. (B)
19. (C)	20.(A)	21. (C)	22. (D)	23. (A-r B-s C-s D-q)	
24.(2)	25. (2)	26. (5)	27. (2)	28. (4)	29. (5)
30. (2)	31. (4)	32. (0)			

LEVEL-2

1. (D)	2. (B)	3. (B)	4. (B)	5. (A)	6. (B)
7. (A)	8. (B,C,D)	9. (A,B)	10. (C)	11. (D)	
12. (A-S B-P C-R D-Q)		13. (A)			

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