UNIT 8

EXPONENTS AND POWERS

(A) Main Concepts and Results

- Exponential notation is a powerful way to express repeated multiplication of the same number. Specifically, powers of 10 express very large and very small numbers in a manner which is convenient to read, write and compare.

- For any non-zero integer a, $a^{-m} = \frac{1}{a^m}$

- Laws of exponents are

  (a) $a^m \times a^n = a^{m+n}$

  (b) $a^m \div a^n = a^{m-n}$

  (c) $(a^m)^n = a^{mn}$

  (d) $a^m \times b^m = (ab)^m$

  (e) $a^\circ = 1$, where $a \neq 0$

  (f) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

- Numbers can be expressed in expanded form by using exponents.
- Very large and very small numbers can be expressed in standard form.
- Standard form is also called **scientific notation form.**

(B) Solved Examples

In example 1 and 2, there are four options given out of which one is correct. Write the correct answer.

Example 1 : Multiplicative inverse of 2^7 is

(a) 2^{-7} (b) 7^2 (c) -2^7 (d) -2^7

Solution : The Correct answer is (a).

Example 2 : The human body has about 100 billion cells. This number can be written in exponential form as

(a) 10^{-11} (b) 10^{11} (c) 10^9 (d) 10^{-9}

Solution : The correct answer is (b).

In examples 3 to 5, fill in the blanks to make the statements true.

Example 3 : $(-4)^4 \times \left(\frac{5}{4}\right)^4 =$ _____

Solution : 5^4

Example 4 : $\left(2^{-3}\right)^2 \times \left(3^{-2}\right)^3 =$ _____

Solution : 6^{-6}

Example 5 : The distance between earth and sun is 150 million kilometres which can be written in exponential form as _______.

Solution : 1.5×10^8 km

In examples 6 and 7, state whether the statements are true (T) or false (F):

Example 6 : Very small numbers can be expressed in standard form using positive exponents.

Solution : False.

Example 7 : $(-10) \times (-10) \times (-10) \times (-10) = 10^{-4}$

Solution : False.

Example 8 : Simplify $\dfrac{(-2)^3 \times (-2)^7}{3 \times 4^6}$

Solution : $\dfrac{(-2)^3 \times (-2)^7}{3 \times 4^6} = \dfrac{(-2)^{3+7}}{3 \times \left(2^2\right)^6}$ $\{a^m \times a^n = a^{m+n}\}$

$= \dfrac{(-2)^{10}}{3 \times 2^{12}}$ $\{(a^m)^n = a^{m \times n}\}$

$= \dfrac{(-2)^{10}}{3 \times 2^{12}} = \dfrac{2^{10-12}}{3}$ $\{a^m \div a^n = a^{m-n}, (-2)^{10} = 2^{10}\}$

$= \dfrac{2^{-2}}{3} = \dfrac{1}{3 \times 2^2} = \dfrac{1}{12}$

Example 9 : Find x so that $(-5)^{x+1} \times (-5)^5 = (-5)^7$

Solution : $(-5)^{x+1} \times (-5)^5 = (-5)^7$

$(-5)^{x+1+5} = (-5)^7$ $\{a^m \times a^n = a^{m+n}\}$

$(-5)^{x+6} = (-5)^7$

On both sides, powers have the same base, so their exponents must be equal.

Therefore, $x + 6 = 7$

$x = 7 - 6 = 1$

$x = 1$

Application on Problem Solving Strategy

Example 10 :

Find x so that $(-5)^{x+1} \times (-5)^5 = (-5)^7$

Understand and Explore the Problem

- What are you trying to find?

  The value of x which satisfies the given equation.

Plan a Strategy

- You know the laws of exponents. Applying the laws of exponent in the given equation to find the value of x.

Solve

- Given $(-5)^{x+1} \times (-5)^5 = (-5)^7$

  Using the Law of exponents, $a^m \times a^n = a^{m+n}$, we get

  $(-5)^{x+1+5} = (-5)^7$

  $(-5)^{x+6} = (-5)^7$

  On both the sides, power has the same base, so their exponents must be equal,

  Therefore, $x + 6 = 7$

  $x = 7 - 6$

  So, $x = 1$

  Hence, the value of x is 1.

Revise

- Substitute the value of x in the equation and check if it satisfies the equation.

  LHS $= (-5)^{x+1} \times (-5)^5$

  $= (-5)^{1+1} \times (-5)^5$

  $= (-5)^2 \times (-5)^5$

  $= (-5)^{2+5}$

  $= (-5)^7 = \text{RHS}$

  $x = 1$ satisfies the equation. Hence our answer is correct.

Think and Discuss

(a) Try to find the value of x in the question by changing – 5 to 2. What difference do you find in the value of x? What do you infer from your answer?

(b) See if you can find the value of x if the equation is changed to

(i) $(-5)^{x+1} \times (5)^5 = (5)^7$

(ii) $(-5)^{2x} \times (5)^5 = (5)^7$

Getting Rid of Negative Exponents in Fractions

You might have to deal with fractions that have negative exponents in the numerator and denominator, like $\frac{2^{-4}}{3^{-7}}$. It's useful to be able to change them into fractions with only positive exponents because it's a simpler form. A number with negative exponent in the numerator is equivalent to the same number with positive exponent in the denominator $\Rightarrow$ $2^{-4} = \frac{2^{-4}}{1} = \frac{1}{2^4}$. A number with a negative exponent in the denominator is equivalent to the same number with positive exponent in the numerator $\Rightarrow$ $\frac{1}{3^{-7}} = \frac{3^7}{1} = 3^7$.

So. 2^{-4} gets moved from the numerator to the denominator, where it is written as 2^4. $\Rightarrow$ $\frac{2^{-4}}{3^{-7}} = \frac{3^7}{2^4}$ $\Leftarrow$ 3^{-7} moved from the denominator and becomes 3^7 in the numerator.

(C) Exercise

In questions 1 to 33, out of the four options, only one is correct. Write the correct answer.

1. In 2^n, n is known as

(a) Base (b) Constant (c) x (d) Variable

2. For a fixed base, if the exponent decreases by 1, the number becomes

(a) One-tenth of the previous number.

(b) Ten times of the previous number.

(c) Hundredth of the previous number.

(d) Hundred times of the previous number.

3. 3^{-2} can be written as

(a) 3^2 (b) $\frac{1}{3^2}$ (c) $\frac{1}{3^{-2}}$ (d) $-\frac{2}{3}$

4. The value of $\frac{1}{4^{-2}}$ is

(a) 16 (b) 8 (c) $\frac{1}{16}$ (d) $\frac{1}{8}$

5. The value of $3^5 \div 3^{-6}$ is

(a) 3^5 (b) 3^{-6} (c) 3^{11} (d) 3^{-11}

6. The value of $\left(\frac{2}{5}\right)^{-2}$ is

(a) $\frac{4}{5}$ (b) $\frac{4}{25}$ (c) $\frac{25}{4}$ (d) $\frac{5}{2}$

7. The reciprocal of $\left(\frac{2}{5}\right)^{-1}$ is

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) $-\frac{5}{2}$ (d) $-\frac{2}{5}$

8. The multiplicative inverse of 10^{-100} is

(a) 10 (b) 100 (c) 10^{100} (d) 10^{-100}

9. The value of $(-2)^{2\times3-1}$ is

(a) 32 (b) 64 (c) – 32 (d) – 64

10. The value of $\left(-\frac{2}{3}\right)^4$ is equal to

(a) $\frac{16}{81}$ (b) $\frac{81}{16}$ (c) $\frac{-16}{81}$ (d) $\frac{81}{-16}$

The table shows several powers of 10 in various forms. Note the following:

- Powers of 10 with positive integer exponents involve repeated multiplication by 10.
- Power of 10 with negative integer exponents involve repeated multiplication by $\frac{1}{10}$ (the multiplicative inverse of 10), or repeated division by 10.
- The power of 10 with an exponent of 0 equals 1.

Power	**Repeated multiplication**	**Standard form**
10^3	$1 \times 10 \times 10 \times 10$	1000
10^2	$1 \times 10 \times 10$	100
10^1	1×10	10
10^0	1	1
10^{-1}	$1 \times \frac{1}{10}$	0.1
10^{-2}	$1 \times \frac{1}{10} \times \frac{1}{10}$	0.01
10^{-3}	$1 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$	0.001

11. The multiplicative inverse of $\left(-\frac{5}{9}\right)^{-99}$ is

(a) $\left(-\frac{5}{9}\right)^{99}$ (b) $\left(\frac{5}{9}\right)^{99}$ (c) $\left(\frac{9}{-5}\right)^{99}$ (d) $\left(\frac{9}{5}\right)^{99}$

12. If x be any non-zero integer and m, n be negative integers, then $x^m \times x^n$ is equal to

(a) x^m (b) x^{m+n} (c) x^n (d) x^{m-n}

13. If y be any non-zero integer, then y^0 is equal to

(a) 1 (b) 0 (c) – 1 (c) Not defined

14. If x be any non-zero integer, then x^{-1} is equal to

(a) x (b) $\frac{1}{x}$ (c) $-x$ (c) $\frac{-1}{x}$

15. If x be any integer different from zero and m be any positive integer, then x^{-m} is equal to

(a) x^m (b) $-x^m$ (c) $\frac{1}{x^m}$ (d) $\frac{-1}{x^m}$

16. If x be any integer different from zero and m, n be any integers, then $(x^m)^n$ is equal to

(a) x^{m+n} (b) x^{mn} (c) $x^{\frac{m}{n}}$ (d) x^{m-n}

17. Which of the following is equal to $\left(-\frac{3}{4}\right)^{-3}$?

(a) $\left(\frac{3}{4}\right)^{-3}$ (b) $-\left(\frac{3}{4}\right)^{-3}$ (c) $\left(\frac{4}{3}\right)^{3}$ (d) $\left(-\frac{4}{3}\right)^{3}$

18. $\left(-\frac{5}{7}\right)^{-5}$ is equal to

(a) $\left(\frac{5}{7}\right)^{-5}$ (b) $\left(\frac{5}{7}\right)^{5}$ (c) $\left(\frac{7}{5}\right)^{5}$ (d) $-\frac{7}{5}^{5}$

19. $\left(\frac{-7}{5}\right)^{-1}$ is equal to

(a) $\frac{5}{7}$ (b) $-\frac{5}{7}$ (c) $\frac{7}{5}$ (d) $\frac{-7}{5}$

20. $(-9)^3 \div (-9)^8$ is equal to

(a) $(9)^5$ (b) $(9)^{-5}$ (c) $(-9)^5$ (d) $(-9)^{-5}$

21. For a non-zero integer x, $x^7 \div x^{12}$ is equal to

(a) x^5 (b) x^{19} (c) x^{-5} (d) x^{-19}

22. For a non-zero integer x, $(x^4)^{-3}$ is equal to

(a) x^{12} (b) x^{-12} (c) x^{64} (d) x^{-64}

23. The value of $(7^{-1} - 8^{-1})^{-1} - (3^{-1} - 4^{-1})^{-1}$ is

(a) 44 (b) 56 (c) 68 (d) 12

24. The standard form for 0.000064 is

(a) 64×10^4 (b) 64×10^{-4} (c) 6.4×10^5 (d) 6.4×10^{-5}

25. The standard form for 234000000 is

(a) 2.34×10^8 (b) 0.234×10^9 (c) 2.34×10^{-8} (d) 0.234×10^{-9}

26. The usual form for 2.03×10^{-5}

(a) 0.203 (b) 0.00203 (c) 203000 (d) 0.0000203

Explore

Use a pattern to raise 10 to a zero or negative power

Step 1: Copy the table and complete the next two rows by evaluating 10^2 and 10^1.

Step 2: Look at the rows you have completed. How does the standard form changes each time the exponent decreases by 1?

Step 3: Use the pattern you identified in Step 2 to complete the remaining rows in the table.

Power	Standard
10^3	1000
10^2	?
10^1	?
10^0	?
10^{-1}	?
10^{-2}	?
10^{-3}	?

27. $\left(\frac{1}{10}\right)^0$ is equal to

(a) 0 (b) $\frac{1}{10}$ (c) 1 (d) 10

28. $\left(\frac{3}{4}\right)^5 \div \left(\frac{5}{3}\right)^5$ is equal to

(a) $\left(\frac{3}{4} \div \frac{5}{3}\right)^5$ (b) $\left(\frac{3}{4} \div \frac{5}{3}\right)^1$ (c) $\left(\frac{3}{4} \div \frac{5}{3}\right)^0$ (d) $\left(\frac{3}{4} \div \frac{5}{3}\right)^{10}$

29. For any two non-zero rational numbers x and y, $x^4 \div y^4$ is equal to

(a) $(x \div y)^0$ (b) $(x \div y)^1$ (c) $(x \div y)^4$ (d) $(x \div y)^8$

30. For a non-zero rational number p, $p^{13} \div p^8$ is equal to

(a) p^5 (b) p^{21} (c) p^{-5} (d) p^{-19}

31. For a non-zero rational number z, $(z^{-2})^3$ is equal to

(a) z^6 (b) z^{-6} (c) z^1 (d) z^4

32. Cube of $-\frac{1}{2}$ is

(a) $\frac{1}{8}$ (b) $\frac{1}{16}$ (c) $-\frac{1}{8}$ (d) $-\frac{1}{16}$

33. Which of the following is not the reciprocal of $\left(\frac{2}{3}\right)^4$?

(a) $\left(\frac{3}{2}\right)^4$ (b) $\left(\frac{3}{2}\right)^{-4}$ (c) $\frac{2}{3}^{-4}$ (d) $\frac{3^4}{2^4}$

In questions 34 to 65, fill in the blanks to make the statements true.

34. The multiplicative inverse of 10^{10} is ___________.

35. $a^3 \times a^{-10}$ = __________.

Draw Conclusions

Use your observations to complete this exercise.

Write the power of 10 in standard form.

1. 10^4 2. 10^9 3. 10^{-4} 4. 10^{-6}

36. $5^0 =$ ________.

37. $5^5 \times 5^{-5} =$ ________.

38. The value of $\left(\frac{1}{2^3}\right)^2$ is equal to ________.

39. The expression for 8^{-2} as a power with the base 2 is ________.

40. Very small numbers can be expressed in standard form by using ________ exponents.

41. Very large numbers can be expressed in standard form by using ________ exponents.

42. By multiplying $(10)^5$ by $(10)^{-10}$ we get ________.

43. $\left[\left(\frac{2}{13}\right)^{-6} \div \left(\frac{2}{13}\right)^{3}\right]^3 \times \left(\frac{2}{13}\right)^{-9} =$ ________

44. Find the value $[4^{-1} + 3^{-1} + 6^{-2}]^{-1}$.

45. $[2^{-1} + 3^{-1} + 4^{-1}]^0 =$ ______

46. The standard form of $\left(\frac{1}{100000000}\right)$ is ______.

47. The standard form of 12340000 is ______.

48. The usual form of 3.41×10^6 is _______.

49. The usual form of 2.39461×10^6 is _______.

50. If $36 = 6 \times 6 = 6^2$, then $\frac{1}{36}$ expressed as a power with the base 6 is _______.

Key Concept

Scientific Notation

A number is written in scientific notation if it has the form $c \times 10^n$ where $c > 1$, $c < 10$, and n is an integer.

Standard form	Product form	Scientific notation
325,000	$3.25 \times 100{,}000$	3.25×10^5
0.0005	5×0.0001	5×10^{-4}

51. By multiplying $\left(\frac{5}{3}\right)^4$ by _______ we get 5^4.

52. $3^5 \div 3^{-6}$ can be simplified as __________.

53. The value of 3×10^{-7} is equal to ________.

54. To add the numbers given in standard form, we first convert them into numbers with __ exponents.

55. The standard form for 32,50,00,00,000 is __________.

56. The standard form for 0.000000008 is __________.

57. The usual form for 2.3×10^{-10} is ____________.

58. On dividing 8^5 by _________ we get 8.

59. On multiplying _________ by 2^{-5} we get 2^5.

60. The value of $[3^{-1} \times 4^{-1}]^2$ is _________.

61. The value of $[2^{-1} \times 3^{-1}]^{-1}$ is _________.

62. By solving $(6^0 - 7^0) \times (6^0 + 7^0)$ we get ________.

63. The expression for 3^5 with a negative exponent is _________.

64. The value for $(-7)^6 \div 7^6$ is _________.

65. The value of $[1^{-2} + 2^{-2} + 3^{-2}] \times 6^2$ is ________ .

In questions 66 to 90, state whether the given statements are true (T) or false (F).

66. The multiplicative inverse of $(-4)^{-2}$ is $(4)^{-2}$.

67. The multiplicative inverse of $\left(\frac{3}{2}\right)^2$ is not equal to $\left(\frac{2}{3}\right)^{-2}$.

68. $10^{-2} = \frac{1}{100}$

69. $24.58 = 2 \times 10 + 4 \times 1 + 5 \times 10 + 8 \times 100$

70. $329.25 = 3 \times 10^2 + 2 \times 10^1 + 9 \times 10^0 + 2 \times 10^{-1} + 5 \times 10^{-2}$

71. $(-5)^{-2} \times (-5)^{-3} = (-5)^{-6}$

72. $(-4)^{-4} \times (4)^{-1} = (4)^5$

73. $\left(\frac{2}{3}\right)^{-2} \times \left(\frac{2}{3}\right)^{-5} = \left(\frac{2}{3}\right)^{10}$

74. $5^0 = 5$

75. $(-2)^0 = 2$

76. $\left(-\frac{8}{2}\right)^0 = 0$

77. $(-6)^0 = -1$

78. $(-7)^{-4} \times (-7)^2 = (-7)^{-2}$

79. The value of $\frac{1}{4^{-2}}$ is equal to 16.

80. The expression for 4^{-3} as a power with the base 2 is 2^6.

81. $a^p \times b^q = (ab)^{pq}$

82. $\frac{x^m}{y^m} = \left(\frac{y}{x}\right)^{-m}$

83. $a^m = \frac{1}{a^{-m}}$

84. The expontential form for $(-2)^4 \times \left(\frac{5}{2}\right)^4$ is 5^4.

85. The standard form for 0.000037 is 3.7×10^{-5}.

Key Concept

Definition of Zero and Negative Exponents

Let a be a non-zero number, and let n be an integer.

Words	Algebra	Example
a to the zero power is 1.	$a^0 = 1$	$5^0 = 1$
a^{-n} is the reciprocal of a^n.	$a^{-n} = \frac{1}{a^n}$	$2^{-3} = \frac{1}{2^3}$
a^n is the reciprocal of a^{-n}.	$a^n = \frac{1}{a^{-n}}$	$2 = \frac{1}{2^{-1}}$

86. The standard form for 203000 is 2.03×10^5

87. The usual form for 2×10^{-2} is not equal to 0.02.

88. The value of 5^{-2} is equal to 25.

89. Large numbers can be expressed in the standard form by using positive exponents.

90. $a^m \times b^m = (ab)^m$

91. Solve the following:

(i) 100^{-10} (ii) $2^{-2} \times 2^{-3}$ (iii) $\left(\frac{1}{2}\right)^{-2} \div \left(\frac{1}{2}\right)^{-3}$

92. Express $3^{-5} \times 3^{-4}$ as a power of 3 with positive exponent.

93. Express 16^{-2} as a power with the base 2.

94. Express $\frac{27}{64}$ and $\frac{-27}{64}$ as powers of a rational number.

95. Express $\frac{16}{81}$ and $\frac{-16}{81}$ as powers of a rational number.

96. Express as a power of a rational number with negative exponent.

(a) $\left(\left(\frac{-3}{2}\right)^{-2}\right)^{-3}$ (b) $(2^5 \div 2^8) \times 2^{-7}$

97. Find the product of the cube of (–2) and the square of (+4).

98. Simplify:

(i) $\left(\frac{1}{4}\right)^{-2} + \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2}$

(ii) $\left(\left(\frac{-2}{3}\right)^{-2}\right)^{3} \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times \frac{1}{6}$

(iii) $\frac{49 \times z^{-3}}{7^{-3} \times 10 \times z^{-5}} (z \neq 0)$

(iv) $(2^5 \div 2^8) \times 2^{-7}$

99. Find the value of x so that

(i) $\left(\frac{5}{3}\right)^{-2} \times \left(\frac{5}{3}\right)^{-14} = \left(\frac{5}{3}\right)^{8x}$

(ii) $(-2)^3 \times (-2)^{-6} = (-2)^{2x-1}$

(iii) $(2^{-1} + 4^{-1} + 6^{-1} + 8^{-1})^x = 1$

100. Divide 293 by 10,00,000 and express the result in standard form.

101. Find the value of x^{-3} if $x = (100)^{1-4} \div (100)^0$.

102. By what number should we multiply $(-29)^0$ so that the product becomes $(+29)^0$.

103. By what number should $(-15)^{-1}$ be divided so that quotient may be equal to $(-15)^{-1}$?

Plan Strategy :

- Identify too much/too little information.

  When you read a problem, you must decide if the problem has too much or too little information. If the problem has too much information, you must decide what information to use to solve the problem. If the problem has too little information, then you should determine what additional information you need to solve the problem.

- Read the problems below and decide if there is too much or too little information in each problem. If there is too much information, tell what information you would use to solve the problem. If there is too little information, tell what additional information you would need to solve the problem.

- On Monday, 20 students took an examination. There were 10 students who scored above 85 and 10 students who scored below 85. What was the average score?

- Aayesha is practising for a marathon. She ran for 50 minutes on Monday, 70 minutes on Wednesday, and 45 minutes on Friday. On Tuesday and Thursday, she lifted weights at the gym for 45 minutes each day. She swam for 45 minutes over the weekend. What was the average amount of time per day Aayesha spent running last week?

104. Find the multiplicative inverse of $(-7)^{-2} \div (90)^{-1}$.

105. If $5^{3x-1} \div 25 = 125$, find the value of x.

106. Write 39,00,00,000 in the standard form.

107. Write 0.000005678 in the standard form.

108. Express the product of 3.2×10^6 and 4.1×10^{-1} in the standard form.

109. Express $\dfrac{1.5\times10^{6}}{2.5\times10^{-4}}$ in the standard form.

110. Some migratory birds travel as much as 15,000 km to escape the extreme climatic conditions at home. Write the distance in metres using scientific notation.

111. Pluto is 59,1,30,00, 000 m from the sun. Express this in the standard form.

112. Special balances can weigh something as 0.00000001 gram. Express this number in the standard form.

113. A sugar factory has annual sales of 3 billion 720 million kilograms of sugar. Express this number in the standard form.

114. The number of red blood cells per cubic millimetre of blood is approximately 5.5 million. If the average body contains 5 litres of blood, what is the total number of red cells in the body? Write the standard form. (1 litre = 1,00,000 mm^3)

115. Express each of the following in standard form:

(a) The mass of a proton in gram is

$$\frac{1673}{1000000000000000000000000000}$$

(b) A Helium atom has a diameter of 0.000000022 cm.

(c) Mass of a molecule of hydrogen gas is about

0.00000000000000000000334 tons.

(d) Human body has 1 trillon of cells which vary in shapes and sizes.

(e) Express 56 km in m.

(f) Express 5 tons in g.

(g) Express 2 years in seconds.

(h) Express 5 hectares in cm^2 (1 hectare = 10000 m^2)

116. Find x so that $\left(\frac{2}{9}\right)^3 \times \left(\frac{2}{9}\right)^{-6} = \left(\frac{2}{9}\right)^{2x-1}$

117. By what number should $\left(\frac{-3}{2}\right)^{-3}$ be divided so that the quotient may be $\left(\frac{4}{27}\right)^{-2}$?

In questions 118 and 119, find the value of *n*.

118. $\frac{6^n}{6^{-2}} = 6^3$

119. $\frac{2^n \times 2^6}{2^{-3}} = 2^{18}$

120. $\frac{125 \times x^{-3}}{5^{-3} \times 25 \times x^{-6}}$

121. $\frac{16 \times 10^2 \times 64}{2^4 \times 4^2}$

122. If $\frac{5^m \times 5^3 \times 5^{-2}}{5^{-5}} = 5^{12}$, find m.

123. A new born bear weighs 4 kg. How many kilograms might a five year old bear weigh if its weight increases by the power of 2 in 5 years?

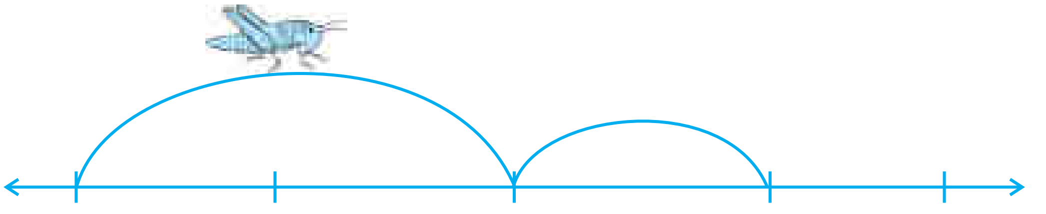
124. The cells of a bacteria double in every 30 minutes. A scientist begins with a single cell. How many cells will be there after

(a) 12 hours (b) 24 hours

125. Planet A is at a distance of 9.35×10^6 km from Earth and planet B is 6.27×10^7 km from Earth. Which planet is nearer to Earth?

126. The cells of a bacteria double itself every hour. How many cells will there be after 8 hours, if initially we start with 1 cell. Express the answer in powers.

127. An insect is on the 0 point of a number line, hopping towards 1. She covers half the distance from her current location to 1 with each hop. So, she will be at $\frac{1}{2}$ after one hop, $\frac{3}{4}$ after two hops, and so on.



(a) Make a table showing the insect's location for the first 10 hops.

(b) Where will the insect be after n hops?

(c) Will the insect ever get to 1? Explain.

128. Predicting the ones digit, copy and complete this table and answer the questions that follow.

Powers Table

x	1^x	2^x	3^x	4^x	5^x	6^x	7^x	8^x	9^x	10^x
1	1	2								
2	1	4								
3	1	8								
4	1	16								
5	1	32								
6	1	64								
7	1	128								
8	1	256								
Ones Digits of the Powers	1	2,4,8,6								

(a) Describe patterns you see in the ones digits of the powers.

(b) Predict the ones digit in the following:

1. 4^{12} 2. 9^{20} 3. 3^{17} 4. 5^{100} 5. 10^{500}

(c) Predict the ones digit in the following:

1. 31^{10} 2. 12^{10} 3. 17^{21} 4. 29^{10}

129. **Astronomy** The table shows the mass of the planets, the sun and the moon in our solar system.

Celestial Body	Mass (kg)	Mass (kg) Standard Notation
Sun	1,990,000,000,000,000,000,000,000,000,000	1.99×10^{30}
Mercury	330,000,000,000,000,000,000,000	
Venus	4,870,000,000,000,000,000,000,000	
Earth	5,970,000,000,000,000,000,000,000	
Mars	642,000,000,000,000,000,000,000,000	
Jupiter	1,900,000,000,000,000,000,000,000,000	
Saturn	568,000,000,000,000,000,000,000,000	
Uranus	86,800,000,000,000,000,000,000,000	
Neptune	102,000,000,000,000,000,000,000,000	
Pluto	12,700,000,000,000,000,000,000	
Moon	73,500,000,000,000,000,000,000	

(a) Write the mass of each planet and the Moon in scientific notation.

(b) Order the planets and the moon by mass, from least to greatest.

(c) Which planet has about the same mass as earth?

130. **Investigating Solar System** The table shows the average distance from each planet in our solar system to the sun.

Planet	Distance from Sun (km)	Distance from Sun (km) Standard Notation
Earth	149,600,000	1.496×10^{8}
Jupiter	778,300,000	
Mars	227,900,000	
Mercury	57,900,000	
Neptune	4,497,000,000	
Pluto	5,900,000,000	
Saturn	1,427,000,000	
Uranus	2,870,000,000	
Venus	108,200,000	

(a) Complete the table by expressing the distance from each planet to the Sun in scientific notation.

(b) Order the planets from closest to the sun to farthest from the sun.

131. This table shows the mass of one atom for five chemical elements. Use it to answer the question given.

Element	Mass of atom (kg)
Titanium	7.95×10^{-26}
Lead	3.44×10^{-25}

Silver	1.79×10^{-25}
Lithium	1.15×10^{-26}
Hydrogen	1.674×10^{-27}

(a) Which is the heaviest element?

(b) Which element is lighter, Silver or Titanium?

(c) List all five elements in order from lightest to heaviest.

132. The planet Uranus is approximately 2,896,819,200,000 metres away from the Sun. What is this distance in standard form?

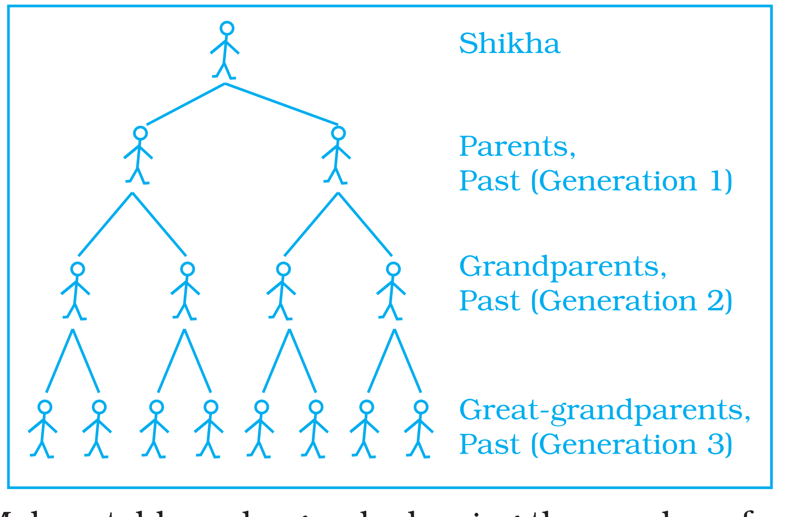
133. An inch is approximately equal to 0.02543 metres. Write this distance in standard form.

134. The volume of the Earth is approximately 7.67×10^{-7} times the volume of the Sun. Express this figure in usual form.

135. An electron's mass is approximately $9.1093826 \times 10^{-31}$ kilograms. What is this mass in grams?

136. At the end of the 20th century, the world population was approximately 6.1×10^{9} people. Express this population in usual form. How would you say this number in words?

137. While studying her family's history. Shikha discovers records of ancestors 12 generations back. She wonders how many ancestors she has had in the past 12 generations. She starts to make a diagram to help her figure this out. The diagram soon becomes very complex.



(a) Make a table and a graph showing the number of ancestors in each of the 12 generations.

(b) Write an equation for the number of ancestors in a given generation *n*.

138. About 230 billion litres of water flows through a river each day. How many litres of water flows through that river in a week? How many litres of water flows through the river in an year? Write your answer in standard notation.

139. A half-life is the amount of time that it takes for a radioactive substance to decay to one half of its original quantity.

Suppose radioactive decay causes 300 grams of a substance to decrease to 300×2^{-3} grams after 3 half-lives. Evaluate 300×2^{-3} to determine how many grams of the substance are left.

Explain why the expression 300×2^{-n} can be used to find the amount of the substance that remains after n half-lives.

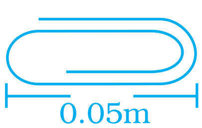
140. Consider a quantity of a radioactive substance. The fraction of this quantity that remains after t half-lives can be found by using the expression 3^{-t}.

(a) What fraction of substance remains after 7 half-lives?

(b) After how many half-lives will the fraction be $\frac{1}{243}$ of the original?

141. One Fermi is equal to 10^{-15} metre. The radius of a proton is 1.3 Fermis. Write the radius of a proton in metres in standard form.

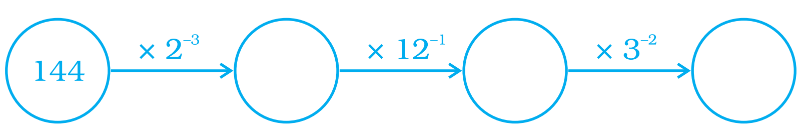
142. The paper clip below has the indicated length. What is the length in standard form.



143. Use the properties of exponents to verify that each statement is true.

(a) $\frac{1}{4}(2^n) = 2^{n-2}$ (b) $4^{n-1} = \frac{1}{4}(4)^n$ (c) $25(5^{n-2}) = 5^n$

144. Fill in the blanks



145. There are 864,00 seconds in a day. How many days long is a second? Express your answer in scientific notation.

146. The given table shows the crop production of a State in the year 2008 and 2009. Observe the table given below and answer the given questions.

Crop	2008 Harvest (Hectare)	Increase/Decrease (Hectare) in 2009
Bajra	1.4×10^3	– 100
Jowar	1.7×10^6	– 440,000
Rice	3.7×10^3	– 100
Wheat	5.1×10^5	+ 190,000

(a) For which crop(s) did the production decrease?

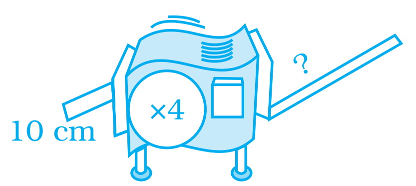
(b) Write the production of all the crops in 2009 in their standard form.

(c) Assuming the same decrease in rice production each year as in 2009, how many acres will be harvested in 2015? Write in standard form.

147. **Stretching Machine**

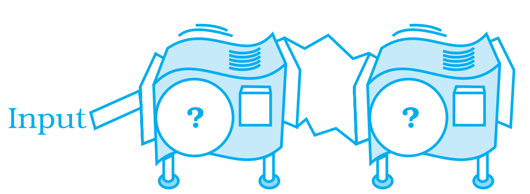
Suppose you have a stretching machine which could stretch almost anything. For example, if you put a 5 metre stick into a (× 4) stretching machine (as shown below), you get a 20 metre stick.

Now if you put 10 cm carrot into a (× 4) machine, how long will it be when it comes out?

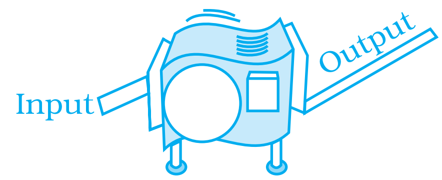


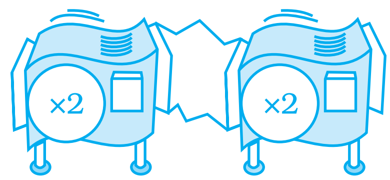
148. Two machines can be hooked together. When something is sent through this hook up, the output from the first machine becomes the input for the second.

(a) Which two machines hooked together do the same work a ($\times 10^2$) machine does? Is there more than one arrangement of two machines that will work?



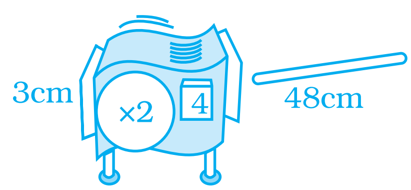
(b) Which stretching machine does the same work as two (× 2) machines hooked together?





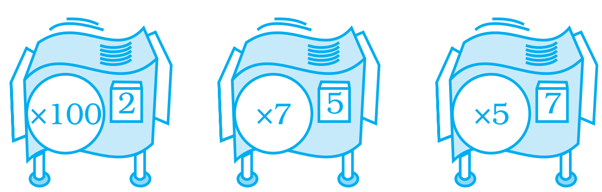
149. Repeater Machine

Similarly, repeater machine is a hypothetical machine which automatically enlarges items several times. For example, sending a piece of wire through a ($\times 2^4$) machine is the same as putting it through a (× 2) machine four times. So, if you send a 3 cm piece of wire through a ($\times 2^4$) machine, its length becomes $3 \times 2 \times 2 \times 2 \times 2 =$ 48 cm. It can also be written that a base (2) machine is being applied 4 times.



What will be the new length of a 4 cm strip inserted in the machine?

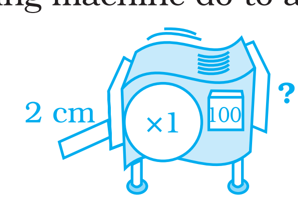
150. For the following repeater machines, how many times the base machine is applied and how much the total stretch is?



(a) (b) (c)

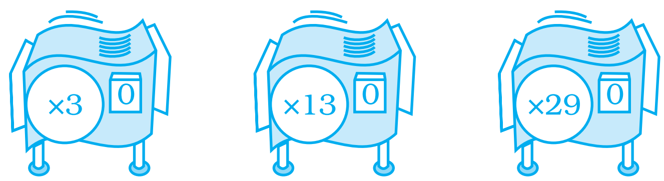
151. Find three repeater machines that will do the same work as a (×64) machine. Draw them, or describe them using exponents.

152. What will the following machine do to a 2 cm long piece of chalk?



153. In a repeater machine with 0 as an exponent, the base machine is applied 0 times.

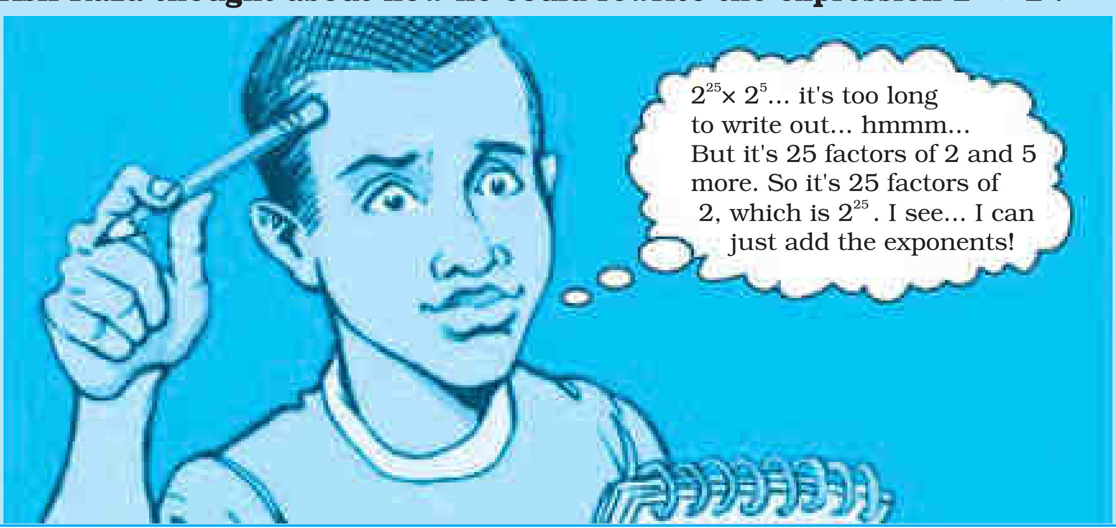
(a) What do these machines do to a piece of chalk?



(b) What do you think the value of 6^0 is?

You have seen that a hookup of repeater machines with the same base can be replaced by a single repeater machine. Similarly, when you multiply exponential expressions with the same base, you can replace them with a single expression.

Asif Raza thought about how he could rewrite the expression $2^{20} \times 2^5$.



Asif Raza's idea is one of the *product laws of exponents*, which can be expressed like this:

Multiplying Expressions with the Same Base

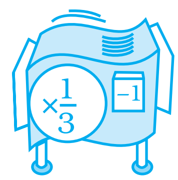
$$a^b \times a^c = a^{b+c}$$

Actually, this law can be used with more than two expressions. As long as the bases are the same, to find the product you can add the exponents and use the same base. For example:

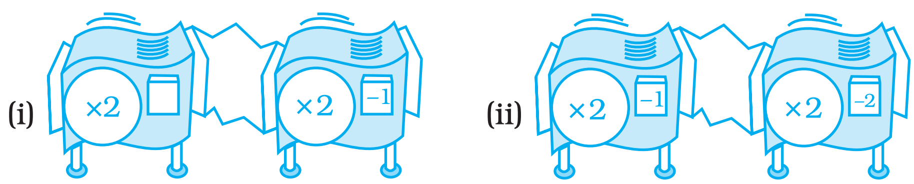
$$3^2 \times 3^3 \times 3^{10} = 3^{2+3+10}$$
$$= 3^{15}$$

154. Shrinking Machine

In a shrinking machine, a piece of stick is compressed to reduce its length. If 9 cm long sandwich is put into the shrinking machine below, how many cm long will it be when it emerges?

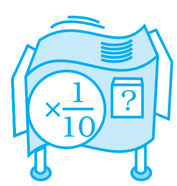


155. What happens when 1 cm worms are sent through these hook-ups?



156. Sanchay put a 1cm stick of gum through a (1×3^{-2}) machine. How long was the stick when it came out?

157. Ajay had a 1cm piece of gum. He put it through repeater machine given below and it came out $\frac{1}{100,000}$ cm long. What is the missing value?



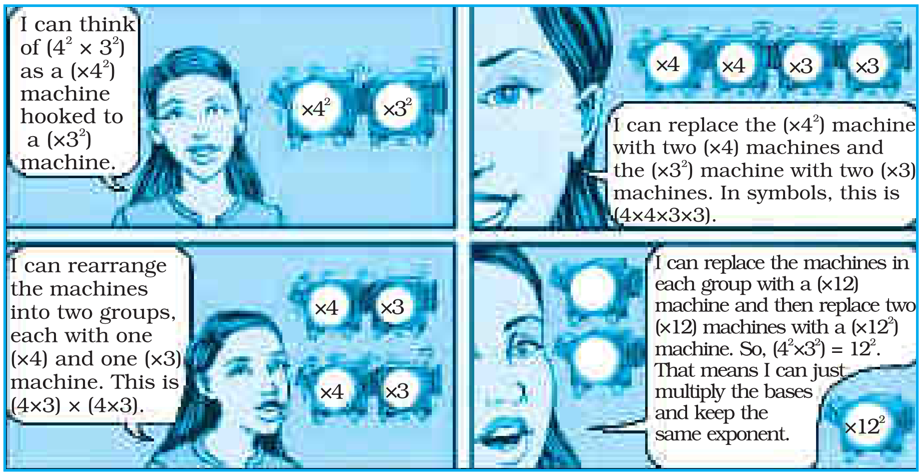
158. Find a single machine that will do the same job as the given hook-up.

(a) a $(\times 2^3)$ machine followed by $(\times 2^{-2})$ machine.

(b) a $(\times 2^4)$ machine followed by $\left(\times\left(\frac{1}{2}\right)^2\right)$ machine.

(c) a $(\times 5^{99})$ machine followed by a (5^{-100}) machine.

Maya multiplied ($4^2 \times 3^2$) by thinking about stretching machines.



Use Maya's idea to multiply $5^3 \times 2^3$

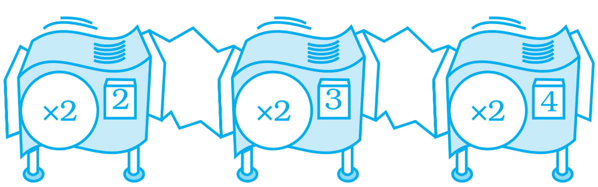
Maya's idea is another *product law of exponents*.

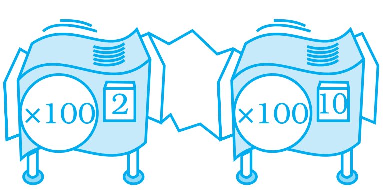
Multiplying Expressions with the Same Exponents

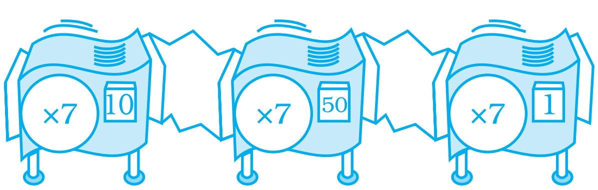
$$a^c \times b^c = (a \times b)^c$$

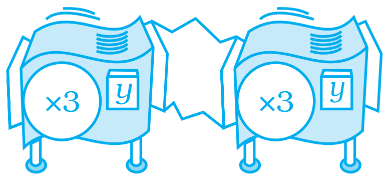
You can use this law with more than two expressions. If the exponents are the same, multiply the expressions by multiplying the bases and using the same exponent. For example, $2^8 \times 3^8 \times 7^8 = (2 \times 3 \times 7)^8 = 42^8$

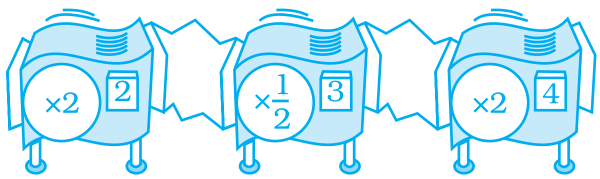
159. Find a single repeater machine that will do the same work as each hook-up.

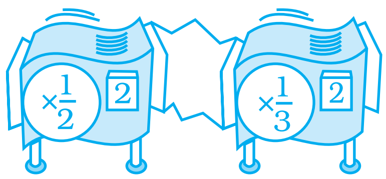
(a) 

(b) 

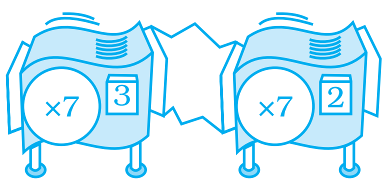
(c) 

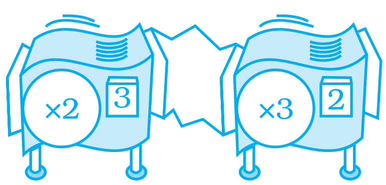
(d) 

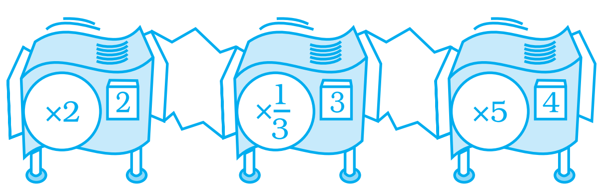
(e) 

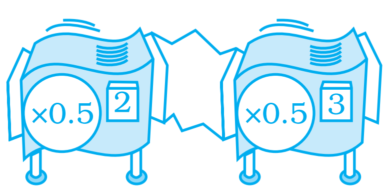
(f) 

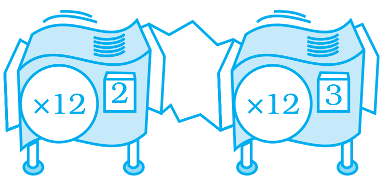
160. For each hook-up, determine whether there is a single repeater machine that will do the same work. If so, describe or draw it.

(a) 

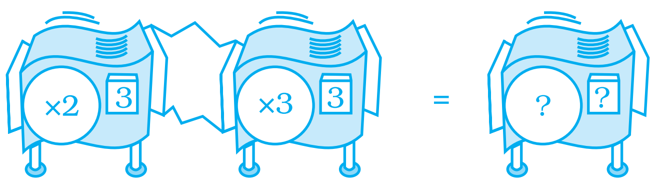
(b) 

(c) 

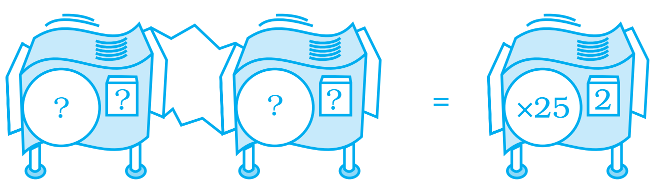
(d) 

(e) 

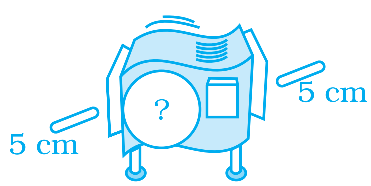
161. Shikha has an order from a golf course designer to put palm trees through a (× 2^3) machine and then through a (× 3^3) machine. She thinks she can do the job with a single repeater machine. What single repeater machine should she use?

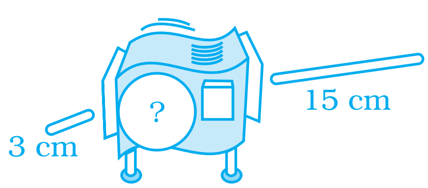


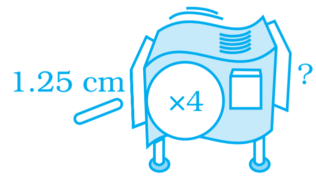
162. Neha needs to stretch some sticks to 25^2 times their original lengths, but her (×25) machine is broken. Find a hook-up of two repeater machines that will do the same work as a (×25^2) machine. To get started, think about the hookup you could use to replace the (×25) machine.

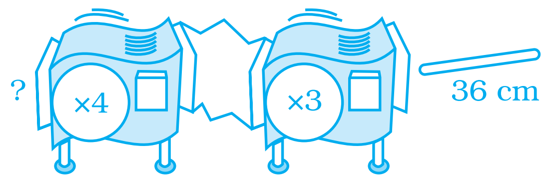


163. Supply the missing information for each diagram.

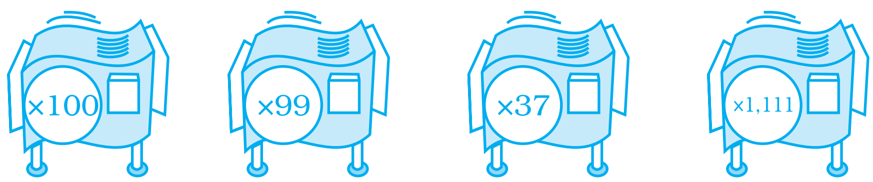
(a) 

(b) 

(c) 

(d) 

164. If possible, find a hook-up of prime base number machine that will do the same work as the given stretching machine. Do not use (× 1) machines.



(a) (b) (c) (d)

165. Find two repeater machines that will do the same work as a (× 81) machine.

166, Find a repeater machine that will do the same work as a $(\times \frac{1}{8})$ machine.

167. Find three machines that can be replaced with hook-ups of (× 5) machines.

168. The left column of the chart lists the lengths of input pieces of ribbon. Stretching machines are listed across the top. The other entries are the outputs for sending the input ribbon from that row through the machine from that column. Copy and complete the chart.

Input Length	**Machine**			
	× 2			
	1	5		
3				15
	14		7	

169. The left column of the chart lists the lengths of input chains of gold. Repeater machines are listed across the top. The other entries are the outputs you get when you send the input chain from that row through the repeater machine from that column. Copy and complete the chart.

Input Length	Repeater Machine		
	$\times 2^3$		
	40		125
2			
		162	

170. Long back in ancient times, a farmer saved the life of a king's daughter. The king decided to reward the farmer with whatever he wished. The farmer, who was a chess champion, made an unusal request:

"I would like you to place 1 rupee on the first square of my chessboard, 2 rupees on the second square, 4 on the third square, 8 on the fourth square, and so on, until you have covered all 64 squares. Each square should have twice as many rupees as the previous square." The king thought this to be too less and asked the farmer to think of some better reward, but the farmer didn't agree.

How much money has the farmer earned?

[Hint: The following table may help you. What is the first square on which the king will place at least Rs 10 lakh?]

Position of Square on chess board	Amount (in Rs)
1st square	1
2nd square	2
3rd square	4

171. The diameter of the Sun is 1.4×10^9 m and the diameter of the Earth is 1.2756×10^7 m. Compare their diameters by division.

172. Mass of Mars is 6.42×10^{29} kg and mass of the Sun is 1.99×10^{30} kg. What is the total mass?

173. The distance between the Sun and the Earth is 1.496×10^8 km and distance between the Earth and the Moon is 3.84×10^8 m. During solar eclipse the Moon comes in between the Earth and the Sun. What is distance between the Moon and the Sun at that particular time?

174. A particular star is at a distance of about 8.1×10^{13} km from the Earth. Assuring that light travels at 3×10^8 m per second, find how long does light takes from that star to reach the Earth.

175. By what number should $(-15)^{-1}$ be divided so that the quotient may be equal to $(-5)^{-1}$?

176. By what number should $(-8)^{-3}$ be multiplied so that that the product may be equal to $(-6)^{-3}$?

177. Find *x*.

(1) $-\frac{1}{7}^{\,-5} \div -\frac{1}{7}^{\,-7} = (-7)^x$

(2) $\frac{2}{5}^{\,2x+6} \times \frac{2}{5}^{\,3} = \frac{2}{5}^{\,x+2}$

(3) $2^x + 2^x + 2^x = 192$

(4) $\frac{-6}{7}^{\,x-7} = 1$

(5) $2^{3x} = 8^{2x+1}$

(6) $5^x + 5^{x-1} = 750$

178. If $a = -1$, $b = 2$, then find the value of the following:

(1) $a^b + b^a$ (2) $a^b - b^a$ (3) $a^b \times b^2$ (4) $a^b \div b^a$

179. Express each of the following in exponential form:

(1) $\frac{-1296}{14641}$ (2) $\frac{-125}{343}$ (3) $\frac{400}{3969}$ (4) $\frac{-625}{10000}$

180. Simplify:

(1) $\frac{1}{2}^{\,2} - \frac{1}{4}^{\,3\;\;-1} = \times 2^{-3}$

(2) $\frac{4}{3}^{-2} - \frac{3}{4}^{2 \ (-2)}$

(3) $\left(\frac{4}{13}\right)^{4} \times \left(\frac{13}{7}\right)^{2} \times \left(\frac{7}{4}\right)^{3}$

(4) $\left(\frac{1}{5}\right)^{45} \times \left(\frac{1}{5}\right)^{-60} - \left(\frac{1}{5}\right)^{+28} \times \left(\frac{1}{5}\right)^{-43}$

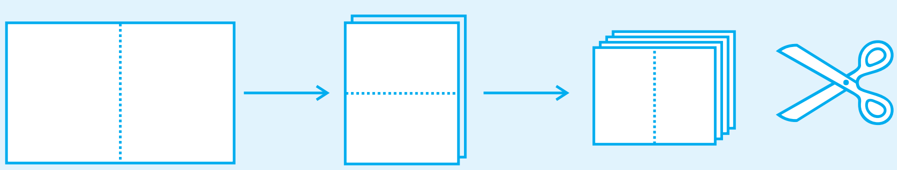
(5) $\frac{(9)^{3} \times 27 \times t^{4}}{(3)^{-2} \times (3)^{4} \times t^{2}}$

(6) $\frac{\left(3^{-2}\right)^{2} \times \left(5^{2}\right)^{-3} \times \left(t^{-3}\right)^{2}}{\left(3^{-2}\right)^{5} \times \left(5^{3}\right)^{-2} \times \left(t^{-4}\right)^{3}}$

(D) ACTIVITIES

Activity 1

To make ballot papers for the upcoming school elections, cut a sheet of paper in half. Stack the two pieces and cut them in half again. Stack the resulting four pieces and cut them in half again. Repeat this process, creating smaller sized ballot papers.



Count the ballots and record the result in the table below.

Number of Cuts	Number of Ballot papers
1	2
2	4
3	8
4	16

Now, answer the following:

(a) Can you predict the number of ballot papers after *n* number of cuts?

(b) Suppose you could make 40 cuts. How many ballot papers would you have?

(c) How many cuts would it take to make enough ballot papers for 512 students at school?

(d) Suppose you start with a sheet of paper having area 324 cm^2. Copy and complete the table to show the area of each ballot paper after each of the first 10 cuts.

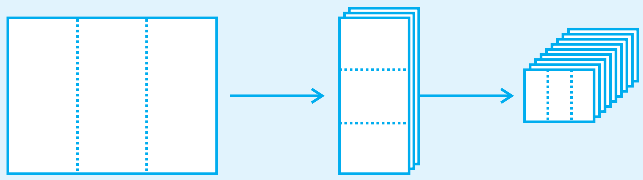
Number of Cuts	**Area (cm^2)**
0	324
1	162
2	81
3	4
5	
6	
7	
8	
9	
10	

Observe the table and write an expression which gives the area of ballot papers after *n* number of cuts on sheet of given area A.

(e) If you want 512 ballot papers each having area 16 cm^2, then what will be the area of starting paper?

Activity 2

1. To meke ballot papers cut a sheet of paper into three. Stack the three pieces and cut the stack into three. Stack all the pieces and cut the stack into three again.



(a) Complete the table to show the number of ballot papers after five such steps:

Number of Steps	Number of Ballot Paper
1	3
2	
3	
4	
5	

(b) Suppose you continue this process. How many ballot papers would you have after 15 steps? How many would you have after n cuts?

(c) How many steps would it take to make at least one lakh ballot paper?

Crossword

Across

(1) In $x^m \times x^n = x^p$, p is the _________ of m and n.

(1) –7 is the _______ of 2 exponents –5 and –2.

(2) Very large numbers like 6,250,000,000 can be conveniently written using _______.

(3) The value of a^n if $n = 0$.

(4) Very small numbers can be expressed in standard form using _____ exponents.

Down

(5) The value of 3^{-2}.

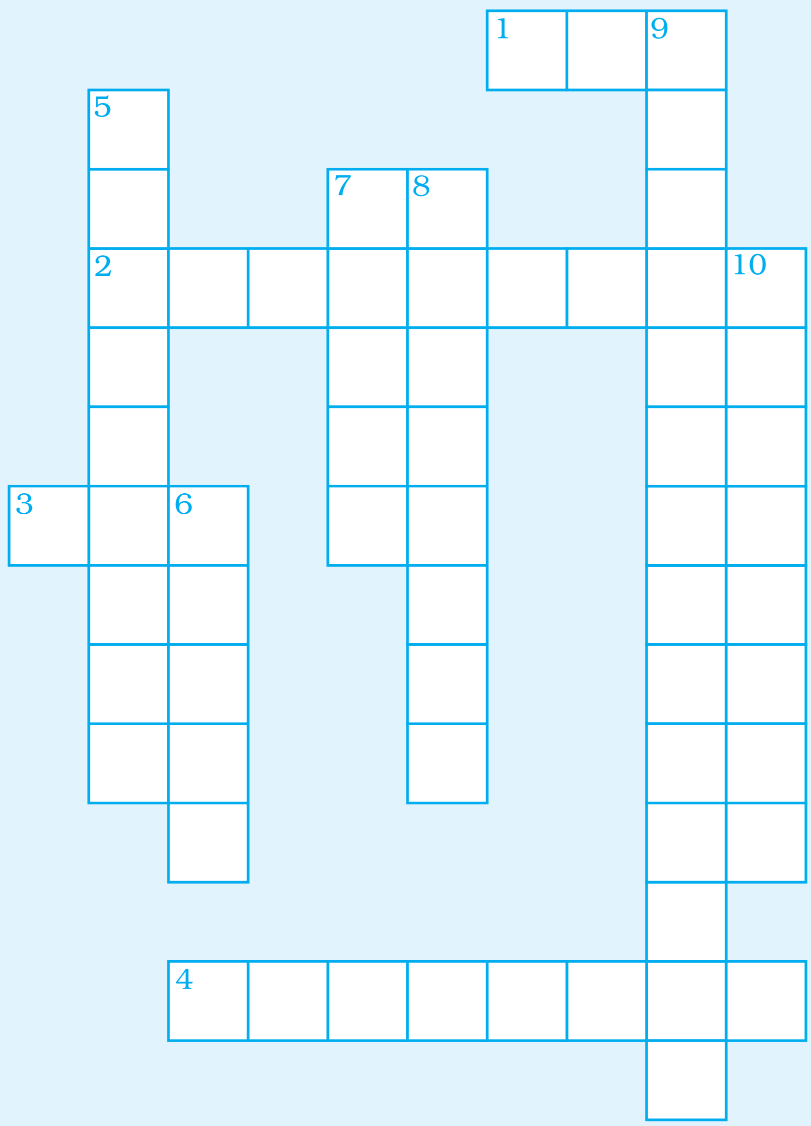
(6) The value of $\frac{1}{2^{-3}}$.

(7) 5^7 is read as 5 raised to the ______ of 7.

(8) As the exponent decreases by 1, the value becomes ______ of the previous value.

(9) a^{-m} is the ______ inverse of a^m.

(10) 1.24×10^{-4} is known as the ________ form of 0.000124.



Rough Work

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