Arithmetic Progressions

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1. Write first four terms of the AP, when the first term a and common difference d are given as follows:
(i) a = 10, d=10
(ii) a= -2, d=0
(iii) a=4, d=-3
(iv) a=-1, d= ½
(v) a=-1.25, d=-0.25

Ans. (i) First term = a =10, d=10
Second term = a+d = 10 + 10 = 20
Third term = second term + d = 20 + 10 = 30
Fourth term = third term + d = 30 + 10 = 40
Therefore, first four terms are: 10, 20, 30, 40
(ii) First term = a = −2 , d=0
Second term = a+d = −2 + 0 = −2
Third term = second term + d = −2 + 0 = −2
Fourth term = third term + d = −2 + 0 = −2
Therefore, first four terms are:−2, −2, −2, −2
(iii) First term = a = 4, d=−3
Second term = a+d = 4 − 3= 1
Third term = second term + d = 1 − 3= −2
Fourth term = third term + d = −2 − 3= −5
Therefore, first four terms are: 4, 1, −2, −5
(iv) First term = a = −1, d= ½
Second term = a+d = −1+ ½ =−½
Third term = second term + d = −½ + ½ = 0
Fourth term = third term + d = 0 + ½ = ½
Therefore, first four terms are:−1, −½, 0, ½
(v) First term = a =−1.25, d = −0.25
Second term = a+d = −1.25 − 0.25= −1.50
Third term = second term + d = −1.50 − 0.25 = −1.75
Fourth term = third term + d = −1.75 − 0.25 = −2.00
Therefore, first four terms are: −1.25, −1.50, −1.75, −2.00

2. Find the 31st term of an AP whose 11th term is 38 and 16th term is 73.

Ans. Here a_{11}=38 and a_{16}=73
Using formula $a_n=a+(n-1)d$,to find nth term of arithmetic progression,
38=a+(11−1)(d) And 73=a+(16−1)(d)

$\Rightarrow 38=a+10d$ And $73=a+15d$
These are equations consisting of two variables.
We have, $38=a+10d$
$\Rightarrow a=38-10d$
Let us put value of a in equation ($73=a+15d$),
$73=38-10d+15d$
$\Rightarrow 35=5d$
Therefore, Common difference $=d=7$
Putting value of d in equation $38=a+10d$,
$38=a+70$
$\Rightarrow a=-32$
Therefore, common difference = d = 7 and First term = a = −32
Using formula $a_n=a+(n-1)d$, to find n^{th} term of arithmetic progression,
$a_{31}=-32+(31-1)(7)=-32+210=178$
Therefore, 31^{st}term of AP is 178.

3. If the third and the ninth terms of an AP are 4 and −8 respectively, which term of this AP is zero?

Ans. It is given that 3^{rd} and 9^{th} term of AP are 4 and −8 respectively.
It means $a_3=4$ and $a_9=-8$
Using formula $a_n=a+(n-1)d$, to find n^{th} term of arithmetic progression,
4 = a + (3 − 1)d And, −8 = a + (9 − 1)d
$\Rightarrow 4=a+2d$ And, $-8=a+8d$
These are equations in two variables.
Using equation $4=a+2d$, we can say that $a=4-2d$
Putting value of a in other equation $-8=a+8d$,
$-8=4-2d+8d$
$\Rightarrow -12=6d$
$\Rightarrow d=-2$
Putting value of d in equation $-8=a+8d$,
$-8=a+8(-2)$
$\Rightarrow -8=a-16$
$\Rightarrow a=8$
Therefore, first term $=a=8$ and Common Difference $=d=-2$
We want to know which term is equal to zero.
Using formula $a_n=a+(n-1)d$, to find n^{th} term of arithmetic progression,
$0=8+(n-1)(-2)$
$\Rightarrow 0=8-2n+2$
$\Rightarrow 0=10-2n$
$\Rightarrow 2n=10$
$\Rightarrow n=5$
Therefore, 5^{th}term is equal to 0.

4. Two AP's have the same common difference. The difference between their 100^{th}terms is 100, what is the difference between their 1000^{th}terms.

Ans. Let first term of 1^{st}AP = a
Let first term of 2^{nd}AP = a'
It is given that their common difference is same.
Let their common difference be d.
It is given that difference between their 100^{th}terms is 100.
Using formula $a_n=a+(n-1)d$, to find n^{th} term of arithmetic progression,
$a+(100-1)d-[a'+(100-1)d] = a+99d-a'-99d=100$
$\Rightarrow a-a'=100$ … (1)
We want to find difference between their 1000^{th}terms which means we want to calculate:
$a+(1000-1)d-[a'+(1000-1)d] = a+999d-a'-999d=a-a'$
Putting equation (1) in the above equation,
$a+(1000-1)d-[a'+(1000-1)d] = a+999d-a'+999d=a-a'=100$
Therefore, difference between their 1000^{th} terms would be equal to 100.

5. How many three digit numbers are divisible by 7?

Ans. We have AP starting from 105 because it is the first three digit number divisible by 7.
AP will end at 994 because it is the last three digit number divisible by 7.
Therefore, we have AP of the form 105,112,119…, 994
Let 994 is the n^{th} term of AP.
We need to find n here.
First term = a = 105, Common difference = d = 112 – 105= 7
Using formula $a_n=a+(n-1)d$, to find n^{th} term of arithmetic progression,
$994=105+(n-1)(7)$
$\Rightarrow 994=105+7n-7$
$\Rightarrow 896=7n$
$\Rightarrow n=128$
It means 994 is the 128^{th} term of AP.
Therefore, there are 128 terms in AP.

6. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?
Ans. Penalty for first day = Rs 200, Penalty for second day = Rs 250
Penalty for third day = Rs 300
It is given that penalty for each succeeding day is Rs 50 more than the preceding day.
It makes it an arithmetic progression because the difference between consecutive terms is constant.

We want to know how much money the contractor has to pay as penalty, if he has delayed the work by 30 days.
So, we have an AP of the form 200, 250, 300, 350 … 30 terms
First term = a = 200, Common difference = d = 50, n = 30

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP, we get

$S_n = \frac{30}{2}[400+(30-1)50]$

$\Rightarrow S_n = 15(400+29 \times 50)$

$\Rightarrow S_n = 15(400+1450) = 27750$

Therefore, penalty for 30 days is Rs. 27750.

7. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g, a section of Class I will plant 1 tree, a section of class II will plant two trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Ans. There are three sections of each class and it is given that the number of trees planted by any class is equal to class number.
The number of trees planted by class I = number of sections × 1 = 3 × 1 = 3
The number of trees planted by class II = number of sections × 2 = 3 × 2 = 6
The number of trees planted by class III = number of sections × 3 = 3 × 3 = 9
Therefore, we have sequence of the form 3, 6, 9 … 12 terms
To find total number of trees planted by all the students, we need to find sum of the sequence 3, 6, 9, 12 … 12 terms.
First term = a = 3, Common difference = d= 6 – 3= 3 and n = 12

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP, we get

$S_{12} = \frac{12}{2}[6+(12-1)3] = 6(6+33) = 6 \times 39 = 234$

8. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.

Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (see figure). Calculate the total volume of concrete required to build the terrace.

Ans. Volume of concrete required to build the first step, second step, third step, ……. (in m^2) are

$\frac{1}{4} \times \frac{1}{2} \times 50, \left(2 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50, \left(3 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50, \ldots$

$\Rightarrow \frac{50}{8}, 2\times\frac{50}{8}, 3\times\frac{50}{8}, \ldots\ldots$

$\therefore$ Total volume of concrete required = $\frac{50}{8}+2\times\frac{50}{8}+3\times\frac{50}{8}+\ldots\ldots$

$= \frac{50}{8}[1+2+3+\ldots\ldots]$

$= \frac{50}{8}\times\frac{15}{2}\left[2\times1+(15-1)\times1\right] \ [\because n=15]$

$= \frac{50}{8}\times\frac{15}{2}\times16$

= 750 m^3

9. For what value of n are the n^{th} term of the following two AP's are same 13, 19, 25,…. and 69, 68, 67 …

Ans. n^{th} term of 13, 19, 25 ,…… = nth term of 69, 68, 67,……

$13+(n-1)\times 6 = 69 + (n-1)(-1)$

Therefore, n = 9

10. Check whether 301 is a term of the list of numbers 5, 11, 17, 32,……..?

Ans. $d = 11-5 = 6$

$a = 5$

$a_n = a+(n-1)d$

$\Rightarrow 301 = 5+(n-1)d$

$\Rightarrow n = 151$

So, 301 is not a term of the given list.

11. Determine the AP whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12.

Ans.

$a_3 = 16$

$\Rightarrow a+2d = 16 \ldots(i)$

$a_7 = a_5 + 12$

$\Rightarrow a+6d = a+4d+12$

$\Rightarrow 2d = 12$

$\Rightarrow d = 6$

Put the valve of d in eq. (i)

$a+2\times6=16$

$\Rightarrow a=16-12$

$\Rightarrow a=4$

$4,10,16...$

12. Find the sum of AP in $-5+(-8)+(-11)+.....+(-230)$

Ans. $a=-5$

$d=-8-(5)$

$=-8+5=-3$

$a_n=-230$

$a_n=a+(n-1)d$

$\Rightarrow -230=-5+(n-1)(-3)$

$\Rightarrow -230=-5-3n+3$

$\Rightarrow -230+2=-3n$

$\Rightarrow n=76$

$S_n=\frac{n}{2}[2a+(n-1)d]$

$S_{76}=\frac{76}{2}[2\times(-5)+(76-1)(-3)]$

$=38[-10+75\times(-3)]$

$=38[-10-225]$

$=38\times(-235)$

$=-8930$

13. In an AP, $a_n=4,d=2,S_n=-14$ **find** n **and a.**

Ans. $a_n=a+(n-1)d$

$\Rightarrow 4 = a + (n-1).(2)$

$\Rightarrow a + 2n = 6...(i)$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow -14 = \frac{n}{2}(a + 4)$

$\Rightarrow -28 = n[6 - 2n + 4)$ $[\because a = 6 - 2n]$

$\Rightarrow n^2 - 5n - 14 = 0$

$\Rightarrow n = 7, \quad n = -2$

$a = -8$

14. Find $a_{30} - a_{20}$ for the AP in -9, -14, -19, -24…

Ans. $a = -9$

$d = -14 - (-9) = -14 + 9 = -5$

$a_{30} - a_{20} = (a + 29d) - (a + 19d)$

$= 10d = 10 \times (-5) = -50$

15. Find the sum to n term of the AP in 5, 2, -1, -4, -7…….

Ans. $a = 5, d = 2 - 5 = -3$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{n}{2}[2 \times 5 + (n-1)(-3)]$

$= \frac{n}{2}[10 - 3n + 3]$

$= \frac{n}{2}[13 - 3n]$

16. Find the sum of first 24 terms of the list of no. whose nth term is given by $a_n = 3 + 2n$

Ans. $a_n = 3 + 2n$

Put $n = 1, 2, 3, ...$

$a_1 = 5, \quad a_2 = 7, \quad a_3 = 9...$

$a = 5,\ \ d = 7 - 5 = 2$

$$S_{24} = \frac{24}{2}\left[2 \times 5 + (24 - 1) \times 2\right]$$

$$= 12\left[10 + 46\right] = 672$$