4

SECOND DEGREE EQUATIONS

Important Ideas

- The general form of a second degree equation is

  $ax^2 + bx + c = 0,\ a \neq 0$

- To complete the second degree polynomial $x^2 + ax$, as a perfect square, add $(a/2)^2$ on both sides of the equation.

- The solutions of $ax^2 + bx + c = 0$ $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- If $b^2 - 4ac$ is
  i) a positive number, the equation has two different solutions.
  ii) zero, solutions are equal.
  iii) a negative number, the equation has no solutions.

- $(a \pm b)^2 = a^2 \pm 2ab + b^2$

- $(x + a)(x + b) = x^2 + (a+b)x + ab$

- $(x + a)(x - a) = x^2 - a^2$

PART : A (Score : 1)
(Make algebraic equations)

1. The square of a number in 25
2. If 9 is added to the square of a number gives 25.
3. If a number is added with the square of that number gives 8.
4. What number is added to $x^2 - 18x$ to get a perfect square?
5. When 89 is subtracted from a number, gives 56. What is the number?

PART : B (Score : 2)

6. The square of a number is 25
   a) What are the numbers?
   b) If the number is x, frame the equation.
7. The length of a rectangle is 2cm more than its breadth. Its area is 56 sq:cm.
   a) If x is the breadth, find its length.
   b) Write the equation representing the area?
8. The product of two consecutive even numbers is 128.
   a) If one of them is x, find the other?
   b) Frame the equation.
9. The sum of a number and its reciprocal is 12/5
   a) If the number is x, find its reciprocal.
   b) Form an equation.
10. If the length of a rectangle is 4cm more than its breadth.
    a) If the breadth is x, write the length.
    b) If the area of the rectangle is 84 sq:cm. Write an equation for finding its area?
11. Worksheet

Algebraic Expression	No. to be added to get a perfect square	Write as perfect square
1. $x^2 + 4x$	$(4/2)^2 = 2^2 = 4$	$x^2 + 4x + 4 = (x+2)^2$
2. $x^2 - 4x$		
3. $x^2 + 8x$		
4. $x^2 - 8x$		
5. $x^2 + 10x$		
6. $x^2 - 10x$		
7. $x^2 + x$		
8. $x^2 - x$		

PART : C (Score : 3)

12. If 4 is added to a number and taking its square. It is 64.
    a) If the number is taken as 'x' frame an equation.
    b) find the numbers?
13. a) What is the common difference of the sequence $(x - 1)$, x, $(x + 1)$, ...
    b) If $x - 1$ is an even number, which is the next even number?
    c) Prove that the product of two consecutive even numbers added to one gives a perfect square?
14. The length of a rectangle is 6cm more than its breadth. Its area is 1216 sq:cm.
    a) If the breadth is 'x', write its length?
    b) find its breadth & length?
15. When all sides of a square were reduced by 4cms, the area becomes 400 sq:cm.
    a) Take the length of a side of the original square as 'x', formulate an equation?
    b) What is the length of a side of the original square?
    c) What is the length of a side of the reduced square?
16. If 'x' is a natural number,
    a) Which number is added to $x^2 + 8x$ to get a perfect square
    b) If $x^2 + ax + 25$ is a perfect square, find a?
    c) If $x^2 + ax + b$ is a perfect square, prove that $a^2 = 4b$
17. The product of two consecutive multiples of 7 is 588.
    a) Write the statement in algebraic form?
    b) What are the numbers?
18. If the solutions of the equation $x^2 - kx + 16 = 0$ are equal, find the value of k?
19. If the sides of a square is reduced by 1, the area becomes 100 sq:cm.
    a) If the sides of the first square before reducing is 'x' form an equation?
    b) find the sides of the original square?
20. Consider an arithmetic sequence with common difference 2.
    a) If the first term is x, find the second term?
    b) Can the product of any two consecutive terms the sequence be -5? Justify your answer?
21. The difference of two positive numbers is 6. Their product is 216.
    a) If one number is 'x', find the other number?
    b) Find the numbers?

PART : D (Score : 4)

22. The steps done by a student to find the solutions of an equation $x^2 + 10x + 16 = 0$ is given below. Fill in the blanks.

    $x^2 + 10x + 16 = 0$

    $x^2 + 10x =$ .....

    $(x +$ .....$)^2 = 9$

    $x =$ ..... or $x =$ .....

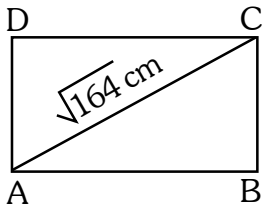
23. The algebraic expression of the sum of terms of an arithmetic sequence is $n^2 + 8n$. The sum of continuous terms starting from first of this sequence is found to be 240.
    a) Write a second degree equation based on this statement.
    b) The sum of how many terms in 240?

24. The hypotenuse of a right angled triangle is 25 cm and the difference of the other two sides is 5cm.
    a) If the smallest side is 'x', find the other side?
    b) find the lengths of two sides?

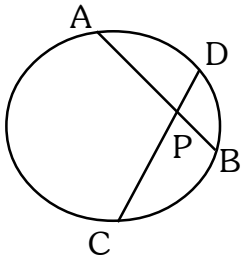
25. Meera was copying a second degree equation to solve it, the coefficient of x was wrongly written as 12 instead of -12, The answers found were -5 and -7.
    a) Write the wrongly written equation?
    b) Write the original equation?
    c) What are the solutions of the original equation?

26. The length of a rod is 56cm. It is bend into a rectangle.
    a) What is the sum of length and breadth of the rectangle?
    b) If the length of diagonal of the rectangle is 20cms. Find the length and breadth of this rectangle?

27. The perimeter of a rectangle in the figure is 36cm. AC = $\sqrt{164}$ cm
    a) Write AB + BC
    b) If AB = x, find BC?
    c) Find the length of AB.



28. In the figure two chords AB and CD intersect at P. The length of PC is 2cm more than PD. AB = 11cm, PB = 3cm.
    a) PA x PB = ...
    b) If PD = x, PC = ...
    c) Find the length of PC?



29. The perimeter of a rectangle is 48cm and its area is 135 sq:cm.
    a) If the length of one side of the rectangle is taken as $12 + x$, find the length of other side?
    b) Frame an equation to calculate the area of the rectangle?
    c) Find the length of sides of the rectangle?

PART : E (Score : 5)

30. The product of a number and another number 8 more than it is 105.
    a) If the smallest number is 'x' find the next number?
    b) On the basis of these facts, write the product.
    c) Which is the least number to be added to make the product a perfect square?
    d) What are the numbers?
31. A rope of length 40 meters is cut into two pieces and two squares are made on the floor with them. The sum of the areas enclosed is 58 sq:meters.
    a) If the length of one piece is taken as x, what is the length of the other piece?
    b) What are the length of the side of the square?
    c) Write the given facts about area as an algebraic equation.
    d) What is the length of each piece?
32. Consider two adjacent even numbers.
    a) If one of them is x, find the other number?
    b) If the product of these numbers is 143, form an equation.
    c) Find the numbers?
33. The product of two adjacent even number is 255,
    a) If the odd number between them is x, find the numbers?
    b) Frame an equation.
    c) Find the numbers?
34. An 8 meter long wire is to be bent into a rectangle. The length of the diagonal is 2 meters.
    a) Find the perimeter of the rectangle?
    b) length + breadth = ...
    c) If the length is taken as 'x', find the breadth?
    d) Can a rectangle be made from these measures? Justify your answer?
35. A box is to be made by cutting off small squares from each corner of a square of thick paper, and bending upwards. The height of the box is to be 5cm and volume 1/2 litre.
    a) What should be the length of the side of the square sheet cut off?
    b) Find the length of a side of the square sheet?

36. The length of a rectangle is 10cm more than its breadth and its area is 200 cm². Find the length and breadth.

breadth = x

length = x +.....

Area = x (x +10)

x (x +.....) = 200

x^2 + ... = 200

x^2 + 10x + ... = 200 + ...

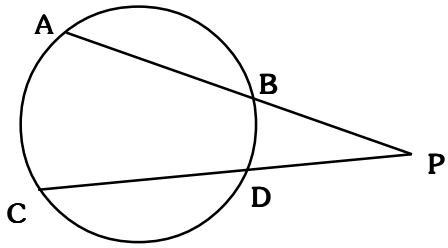
(x + ...)² = ...

x + ... = ...

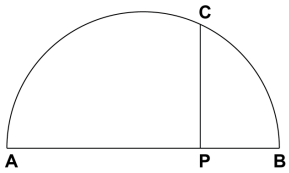
x = ...

∴ Breadth = ...cm, Length = ...cm

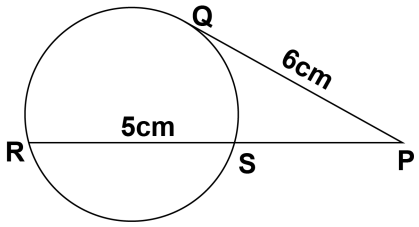
37. In figure the chords AB and CD intersect at P. AP = 12cm, AB = 7cm, CD = 4cm.
    a) Find the length of PB?
    b) PC x PD = ...
    c) If DP = x, find PC?
    d) Find the length of PC?



38. In figure AB is the diameter of the semicircle. AB is perpendicular to PC. Also AP = BP+6. PC = $\sqrt{16}$ cm
    a) Write the relation between the lengths PA, PB and PC?
    b) If PB = x, write an equation connecting the lengths PA, PB and PC.
    c) Find the length of PB.
    d) What is the radius of the semicircle?



39. In figure PQ is the tangent of the circle. PQ = 6cm, RS = 5cm.
    a) PS x PR = ...
    b) If PS = x, find PR?
    c) Find the length of PS?
    d) Find the length of PR?



40. Given picture is the dates in a calendar. A, B, C, D denotes the dates.
    a) If A = x, write B, C, D.
    b) If A x C = 84 form a second degree equation?
    c) Find the number corresponding to A?
    d) Write the numbers in the boxes B, C and D.

A	B
C	D

Answers : PART : A
(Make algebraic equations)

1. $x^2 = 25$
2. $x^2 + 9 = 25$
3. $x^2 + x = 8$
4. $18/2 = 9$
5. $x - 89 = 56$
   $x = 56 + 89 = 145$

PART : B

6. a) The numbers are 5 and -5
   b) $x^2 = 25$
7. a) Take the breadth as x, length $= x + 2$
   b) $x(x + 2) = 56$ or $x^2 + 2x - 56 = 0$
8. a) $x + 2$
   b) $x(x + 2) = 128$ or $x^2 + 2x - 128 = 0$
9. a) $1/x$
   b) $x + 1/x = 12/5$
10. a) length $= x + 4$
    b) $x(x + 4) = 84$ or $x^2 + 4x - 84 = 0$
11. Worksheet

Algebraic Expression	No. to be added to get a perfect square	Write as perfect square
1. $x^2 + 4x$	$(4/2)^2 = 2^2 = 4$	$x^2 + 4x + 4 = (x + 2)^2$
2. $x^2 - 4x$	$(4/2)^2 = 2^2 = 4$	$x^2 - 4x + 4 = (x - 2)^2$
3. $x^2 + 8x$	$(8/2)^2 = 4^2 = 16$	$x^2 + 8x + 16 = (x + 4)^2$
4. $x^2 - 8x$	$(8/2)^2 = 4^2 = 16$	$x^2 - 8x + 16 = (x - 4)^2$
5. $x^2 + 10x$	$(10/2)^2 = 5^2 = 25$	$x^2 + 10x + 25 = (x + 5)^2$
6. $x^2 - 10x$	$(10/2)^2 = 5^2 = 25$	$x^2 - 10x + 25 = (x - 5)^2$
7. $x^2 + x$	$(1/2)^2 = 1/4$	$x^2 + x + 1/4 = (x + 1/2)^2$
8. $x^2 - x$	$(1/2)^2 = 1/4$	$x^2 - x + 1/4 = (x - 1/2)^2$

Answers : PART : C

12. a) $(x + 4)$

    b) $(x + 4)^2 = 64$

    $(x + 4) = \pm 8$

    $x = 8 - 4$ or $x = -8 - 4$

    $x = 4$ or $x = -12$

13. a) Common difference $= 1$

    b) $(x + 1)$

    c) $(x - 1)(x + 1) + 1 = x^2 - 1 + 1 = x^2$

14. a) length $= x + 6$

    b) $x(x + 6) = 1216$

    $x^2 + 6x = 1216$

    $x^2 + 6x + 9 = 1216 + 9$

    $(x + 3)^2 = 35^2$

    $x + 3 = \pm 35$

    $x = 35 - 3$ or $x = -35 - 3$

    $x = 32$

    breadth $= 32$ $\therefore$ length $= 32 + 6 = 38$ cm

15. a) length of a side of the large square $= x$

    $(x - 4)^2 = 400$

    b) $x - 4 = \pm 20$

    $x = 20 + 4$ or $x = -20 + 4$

    $x = 24$

    $\therefore$ length of a side of the large square $= 24$ cm

    c) length of a side of the second square $= 24 - 4 = 20$ cm

16. a) $(8/2)^2 = 4^2 = 16$

    b) 10 or -10

    c) $(a/2)^2 = b$

    $a^2/4 = b$

    $a^2 = 4b$

17. a) If we take the consecutive multiples of 7 are x and $x + 7$, then

    $x(x + 7) = 588$

    b) $x^2 + 7x = 588$

    $x^2 + 7x - 588 = 0$

$(x - 21)(x + 28) = 0$

$x - 21 = 0$ or $x + 28 = 0$

$x = 21$ or $x = -28$ (or Take any other method)

18. $x^2 - kx + 16 = 0$

$a = 1, b = -k, c = 16$

If the solutions are equal, $b^2 - 4ac = 0$

$(-k)^2 - 4 \times 1 \times 16 = 0$

$k^2 - 64 = 0$

$k^2 = 64$

$k = \pm 8$

19. a) If the length of the first square is taken as 'x', then

$(x - 1)^2 = 100$

b) $x - 1 = \sqrt{100}$

$x - 1 = \pm 10$

$x = 10 + 1$ or $x = -10 + 1$

$x = 11$

$\therefore$ A side of the first square = 11 cm

20. a) $x + 2$

b) $x(x + 2) = -5$

$x^2 + 2x + 5 = 0$

$b^2 - 4ac = (2)^2 - 4 \times 1 \times 5$

$= 4 - 20 = -16$

$b^2 - 4ac < 0$ $\therefore$ The product should not be - 5

21. a) $x + 6$

b) $x(x + 6) = 216$

$x^2 + 6x = 216$

$x^2 + 6x + 9 = 216 + 9$

$(x + 3)^2 = 15^2$

$x + 3 = 15$

$x = 15 - 3 = 12$

numbers are 12 and 18

PART : D

22. $x^2 + 10x + 16 = 0$

$x^2 + 10x = -16$

$x^2 + 10x + 5^2 = -16 + 25$

$(x + 5)^2 = 9$

$x + 5 = \pm 3$

$x = 3 - 5$ or $x = -3 - 5$

$x = -2$ or $x = -8$

23. a) $n^2 + 8n = 240$

b) $n^2 + 8n + 16 = 240 + 16$

$(n + 4)^2 = 256$

$n + 4 = 16$

$n = 16 - 4 = 12$

If we added 12 terms to get a sum 240

24. a) $x + 5$

b) $x^2 + (x + 5)^2 = 25^2$

$x^2 + x^2 + 10x + 25 = 625$

$2x^2 + 10x - 600 = 0$

$x^2 + 5x - 300 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25 + 1200}}{2}$$

$$x = \frac{-5 \pm 35}{2}$$

$$x = \frac{-5 + 35}{2} \quad \text{or} \quad x = \frac{-5 - 35}{2}$$

$$x = \frac{30}{2} \quad \text{or} \quad x = \frac{40}{2}$$

$x = 15$ $\therefore$ Smallest side = 15 cm Largest side = 15 + 5 = 20 cm

25. a) Solutions of the wrong equations are -5 and -7

$\therefore$ Equation $(x + 5)(x + 7) = 0$ or $x^2 + 12x + 35 = 0$

b) $x^2 - 12x + 35 = 0$

c) $x^2 - 12x = -35$

$x^2 - 12x + 36 = -35 + 36$

$(x - 6)^2 = 1$

$x - 6 = \pm 1$

$x = 1 + 6$ or $x = -1 + 6$

$x = 7$ or $x = 5$

26. a) length + breadth = 56/2 = 28 cm

b) If the length = x, breadth = $28 - x$, diagonal = 20 cm

$x^2 + (28 - x)^2 = 20^2$

$x^2 + 784 - 56x + x^2 = 400$

$2x^2 - 56x + 384 = 0$

$x^2 - 28x + 192 = 0$

$(x - 16)(x - 12) = 0$

$x - 16 = 0$ or $x - 12 = 0$

$x = 16$ or $x = 12$

$\therefore$ Length = 16cm, Breadth = 12cm

27. a) AB + BC = 36/2 = 18 cm

b) If AB = x, BC = $18 - x$

c) $x^2 + (18 - x)^2 = (\sqrt{164})^2$

$x^2 + 324 - 36x + x^2 = 164$

$2x^2 - 36x + 160 = 0$

$x^2 - 18x + 80 = 0$

$(x - 10)(x - 8) = 0$

$x - 10 = 0$ or $x - 8 = 0$

$x = 10$ or $x = 8$

$\therefore$ AB = 10 cm

28. a) PA x PB = PC x PD

b) If PD = x, PC = $x + 2$

c) PA x PB = PC x PD

$8 \times 3 = x(x + 2)$

$x^2 + 2x = 24$

$x^2 + 2x - 24 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 + 96}}{2}$$

$$x = \frac{-2 \pm 10}{2}$$

$$x = \frac{-2 + 10}{2} \quad \text{or} \quad x = \frac{-2 - 10}{2}$$

$$x = \frac{8}{2} \quad \text{or} \quad x = \frac{-12}{2}$$

$x = 4$

$\therefore$ Length of PC $= 4 + 2 = 6$cm

29. a) breadth $= 24 - (12 + x) = 12 - x$

b) $(12 + x)(12 - x) = 135$

$12^2 - x^2 = 135$

$144 - x^2 = 135$

$x^2 = 9$

$x = \pm 3$

$x = 3$

Length $= 12 + 3 = 15$cm ; Breadth $= 12 - 3 = 9$cm

PART : E

30. a) If the smallest number $= x$, another number : $x + 8$

b) $x(x + 8) = 105$

c) $x^2 + 8x = 105$

To get the product as a perfect square, add $(8/2)^2 = 16$

d) $x^2 + 8x + 16 = 105 + 16$

$(x + 4)^2 = 11^2$

$x + 4 = \pm 11$

$x + 4 = 11$ or $x + 4 = -11$

$x = 11 - 4$ or $x = -11 - 4$

$x = 7$

The smallest number $= 7$; Another number $= 7 + 8 = 15$

31. a) Length of other piece = $40 - x$

b) x, $10 - x$

c) $x^2 + (10 - x)^2 = 58$

d) $x^2 - 100 - 20x + x^2 = 58$

$2x^2 - 20x + 42 = 0$

$x^2 - 10x + 21 = 0$

$(x - 7)(x - 3) = 0$

$x - 7 = 0$ or $x - 3 = 0$

$x = 7$ or $x = 3$

Length of scales are $7 \times 4 = 28$ metre and $3 \times 4 = 12$ metre

32. a) $x + 2$

b) $x(x + 2) = 143$

$x^2 + 2x = 143$

c) $x^2 + 2x + 1 = 143 + 1$

$(x + 1)^2 = 144$

$x + 1 = \pm 12$

$x = 12 - 1$ or $x = -12 - 1$

$x = 11$

Numbers are 11 and $11 + 2 = 13$

33. a) $(x - 1)$; $(x + 1)$

b) $(x - 1)(x + 1) = 255$

$x^2 - 1 = 255$

$x^2 = 256$

$x = 16$

Numbers are $(16 - 1) = 15$ and $(16 + 1) = 17$

34. a) Perimetre = 8 m

b) Length + Breadth = 4 m

c) Breadth = $4 - x$

d) $x^2 + (4 - x)^2 = 2^2$

$x^2 + 16 - 8x + x^2 = 4$

$2x^2 - 8x + 12 = 0$

$x^2 - 4x + 6 = 0$

$b^2 - 4ac = (-4)^2 - 4 \times 1 \times 6$

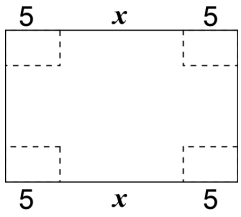
$= 16 - 24 = -8$

$b^2 - 4ac < 0$, there is no solutions

So we couldn't construct the rectangle

35. a) 5 cm

b) One side of the rectangle $= x + 5 + 5 = x + 10$



Volume = 1/2 litre

$x^2 \times 5 = 500$

$x^2 = 500/5 = 100$

$x = 10$ $\therefore$ length of one side $= 10 + 5 + 5 = 20$ cm

36. Breadth = x

Length $= x + 10$

Area = $x(x + 10)$

$x(x + 10) = 200$

$x^2 + 10x = 200$

$x^2 + 10x + 25 = 200 + 25 = 225$

$(x + 5)^2 = 15^2$

$x + 5 = \pm 15$

$x = 15 - 5$ or $x = -15 - 5$

$x = 10$

$\therefore$ Breadth $= 10$cm ; length $= 10 + 10 = 20$cm

37. a) PB = $12 - 7 = 5$cm

b) PA x PB = PC x PD

c) PC $= x + 4$

d) $12 \times 5 = (x + 4)x$

$x^2 + 4x = 60$

$x^2 + 4x + 4 = 64$

$(x + 2)^2 = 8^2$

$x + 2 = \pm 8$

$x = 8 - 2$ or $x = -8 - 2$

$x = 6$

$\therefore$ PC $= 6 + 4 = 10$cm

38. a) PA x PB $= PC^2$

b) $(x + 6)x = (\sqrt{16})^2$

$x^2 + 6x = 16$

c) $x^2 + 6x + 9 = 16 + 9$

$(x + 3)^2 = 25 = 5^2$

$x + 3 = \pm 5$

$x = 5 - 3$ or $x = -5 - 3$

$x = 2$

PB = 2cm

d) AB = AP + PB = 8 + 2 = 10cm

Radius = 10/2 = 5cm

39. a) PS x PR = PQ^2

b) PR = $x + 5$

c) $x(x + 5) = 6^2$

$x^2 + 5x - 36 = 0$

$(x + 9)(x - 4) = 0$

$x + 9 = 0$ or $x - 4 = 0$

$x = -9$ or $x = 4$

$\therefore$ PS = 4cm

d) PR = 5 + 4 = 9cm

40. a) A = x B = $x + 1$

C = $x + 7$ D = $x + 8$

b) $x(x + 8) = 84$

$x^2 + 8x = 84$

$x^2 + 8x + 16 = 84 + 16$

$(x + 4)^2 = 100$

$x + 4 = \pm 10$

$x = 10 - 4$ or $x = -10 - 4$

$x = 6$

c) A = 6

d) B = 7, C = 13, D = 14