CHAPTER 14

Proof of Mathematical Statements

Introduction

In ours daily lives we often try to logically judge statements and claims instead of just accepting them. For example, you must have heard or seen advertisements like "you can write faster if you use this pencil; your child will run faster if you give this tonic to her". They say that if you want to write faster then you should use that pencil and you will run faster by taking the tonic otherwise you will be left behind. Now the point is that how do we check these claims and statements? In other words, how can we check their validity?

One method is empirical observation to find out how many children came first in race after taking the tonic or height of how many children increased or writing speed of how many children increased by using that pencil. We can analyse further on the basis of the above observations. But will this method work in all situations? Can it be applied on mathematical statements?

For example, read the following statements:-

1. Multiple of a number is a multiple of all the factors of that number.
2. If a number is divisible by 8 it is also divisible by 4.
3. 0.000001 is bigger than 10^{-20}.
4. The sum of two odd numbers is always an even number.
5. The product of two numbers is bigger than the numbers.

Can we use the above described method to check such statements i.e. Is it possible to find all the multiplies and factors of every number to check statement (1) Can we will divide by 4 all the numbers till infinity that are divisible by 8 to check statement (2)?

It is clear that such statements cannot be checked by empirical method. Some general method is required so that we can find that 0.00001 is bigger or smaller thatn 10^{-20}. Similarly, we need a rule on the basis of which the sum of two odd numbers can be shown as an even number or product of numbers can be checked.

Let us find a method to check these mathematical statement.

Proving Mathematical Statements

Let us take some examples of mathematical statements.

Statemens about Numbers

Statement-1 : The sum of an odd number and an even number is always an odd number.

Proof : We can write any even number b as $b = 2k$, where k is an integer.

(By definition of even integers, since b is divisible by 2) . ..... (1)

Any odd number a can be written in the form of $a = 2k_1 +$, where k_1 is an integer.

(Adding 1 to an even number given us an odd number) ..... (2)

Now by adding (1) and (2)

$a + b = 2k_1 + 1 + 2k \quad = 2\,(k + k_1) + 1$

$= 2m + 1$ where $m = k + k_1$ and m is a integer (Why?)

Since $2m$ is an even number.

So $2m + 1$ is a odd number.

So, the sum of an even number and an odd number will always be an odd number.

You can see that here we proved the statement on the basis of definitions of even integers and odd integers.

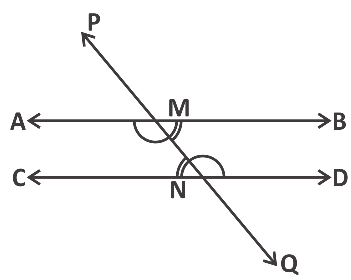
Geometric Statements

You have learnt to prove geometric statements in class-IX. For example, statements such as "The sum of interior angles of a quadrilateral is 360°" of "If a transversal intersects two parallel lines then each pair of alternate interior angles are equal."

Now, we will prove another statement and try to find out key aspects of its proof.

Statement-2 : If two parallel lines are cut by a transversal, each pair of corresponding angles are equal.

Proof : Let a **transversal PQ intersect** two parallel lines AB and CD.



Here $\angle$MND and $\angle$AMN are a pair of alternate interior angles.

And, $\angle$MNC and $\angle$BMN are another pair of alternate interior angles.

We have to see if $\angle$MND = $\angle$AMN and

$\angle$MNC = $\angle$BMN

Since $\angle$PMB and $\angle$MND are corresponding angles

$\therefore \angle PMB = \angle MND$ (By corresponding angles axiom) .....(1)

And $\angle PMB = \angle AMN$ (By vertically opposite angle theorem) .....(2)

By (1) and (2)

$\angle MND = \angle AMN$ .....(3)

Similarly,

$\angle MNC = \angle BMN$ .....(4)

So, here both the pair of alternate interior angles are equal.

Key Aspects of the Proof

Read both proofs given above carefully and list out the key aspects which are used to prove these statements.

Madhavi says that she find three key aspects of proof:-

1. We have made use of previously drawn results such as theorems, definitions or axioms to prove both the statements.
2. Each statements of proof is logically joined with previous statement.
3. Special type of symbols and signs are used to write the statements and long statements are written in brief by the use of these symbols and signs.

Do you agree with Madhavi?

Try These

1. "The sum of interior angles of a quadrilateral is 360°."

   Prove this statement and find out all three key aspects of the proof.

Think and Discuss

Read the proof of the two statements given above, which we have proved, and discuss the following questions in the class:-

1. Which definitions, theorems and axioms were used?
2. List out those signs, symbols which are used while proving the statements.

Understanding Proofs

Let us try to understand how the all three key aspects given above help to read, understand and write proofs.

1. **Use of "Definitions, previously drawn theorems and axioms".**

If a statement is given to you to prove, how will you start?

Obviously for this you need all those known information on the basis of which the statement can be proved. This information can be postulates, definitions and proved statements. Therefore, to prove a statement first you need to think which information you have, so that you can use the information in appropriate situations.

In the chapter on similarity we used AA criteria to prove SAS nad SSS theorem of congruency in two triangles. Similarly to prove that $\sqrt{2}$ is an irrational number, we used definition of rational numbers and to prove that adjacent sides of a parallelogram are equal we used the theorem "pair of alternate angles".

Try These

Find out the definitions which we used to prove the statements given above.

2. Deductive Reasoning

It is important to think while proving a statement that what is the basis of writing one statement after a previous statement. To prove mathematical statements, we need information of previously known definitions and postulates and proved theorems.

1. Write the deductive statement of the following statements:-

"l and m are parallel lines"

We know that there is no common point in parallel lines by the definition of parallel lines (Reasoning).

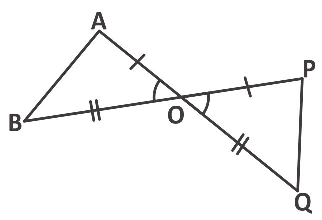
Therefore, we can say that if l and m are any two parallel lines then there will not be any common point on them. (Deductive statement)

Let us consider another example.

2. If $a + 5 = b$ and $c = b$

then $a + 5 = c$. so, $a = c - 5$

3. In both the examples given above on the basis of definitions and postulates a statement is deduced from the previous statement. Similarly we deduce a statement from the previous statement on basis of proved statements and theorems.



Let us take an example:-

In ΔAOB and ΔPOQ, $\angle AOB = \angle POQ$,

$OA = OP$ and $OB = OQ$

On the basis of SAS congruency theorem we can say that

$\Delta AOB \cong \Delta POQ$

4. Deductive reasoning helps in reaching from general truth statements to specific truth statements. For example, if we prove that product of two odd numbers is an odd number once then if we know the given numbers are odd, then we can say that the product of two odd numbers is an odd number without multiplying them.

7428391 × 607349 will an odd number because 7428391 and 607349 are odd numbers.

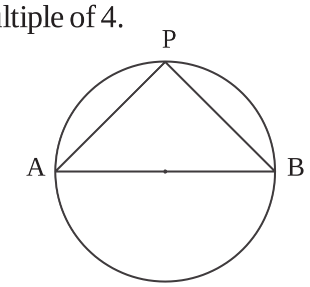
Try These

1. Prove that - the sum of any two successive odd numbers is a multiple of 4.
2. Write the deduced statement on the basis of given statements:-

   Statement-a : Square is a rectangle

   Statement-b : Rectangle is a parallelogram.

   Statement-c : Chord AB makes $\angle APB$ on perimeter of circle.

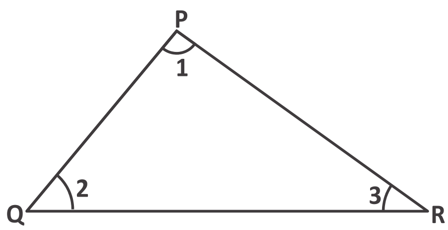


3. If ABCE and PQRS are two rectangles then what we can deduce about their angles, sides and diagonals? What we can say about their congruency and similarity.

Can we prove mathematical statements by measuring and cutting

While learning mathematics, we often accept some facts at a general level by measuring or considering some specific examples. The sum of the interior angles of a triangle is 180°. To show this we either measure the angles or cut the corner of triangle and put them together to show that sum of interior angles is 180°. But it is not the proof of this satement. By showing it we can't generalise this statement for all the triangles.

We know that the sum of interior angles of a triangle is 180°. This is a theorem which is a generalised statement applicable on all triangles. Suppose you measure the angles of a triangle and the sum of angles is 180° even then we cannot say this is true some any other triangle. We cannot repeat the same process for all triangles. Further, if sum of the angles is more or less than 180° then we say that it is not measured properly. We say this because we know that the sum of angles of a triangle on a plane surface is 180° and this can be generalised also so that it is true for all triangles.

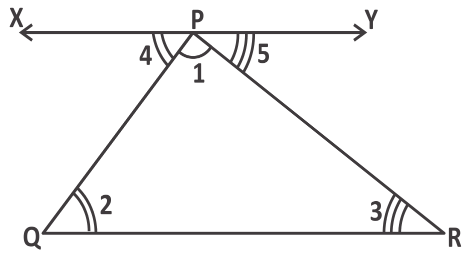


In mathematics, to prove a statement deductive reasoning is used so that any statement can be checked and established true in a generalised form. For example imagine a triangle, shape and measurement of whose angles is unknown. The conclusion for this triangle will be applicable for all triangles.

Theorem-1 : The sum of interior angles of a triangle is 180°

Proof : A triangle PQR is given whose angle are $\angle 1$, $\angle 2$ and $\angle 3$.

To Prove : $\angle 1 + \angle 2 + \angle 3 = 180°$



Construction : Draw a line XPY parallel to QR and passing through point P so that we can use the property of parallel lines.

In figure,

$\angle 4 + \angle 1 + \angle 5 = 180°$ (XPY is a line) .....(1)

$\angle 4 = \angle 2$ and $\angle 5 = \angle 3$ (Pair of alternate angles) .....(2)

On putting the value of $\angle 4$ and $\angle 5$ in (1)

$\angle 2 + \angle 1 + \angle 3 = 180°$ .....(3)

So $\angle 1 + \angle 2 + \angle 3 = 180°$ .....(4)

We can say this is true for any triangle, for different values of $\angle 1$ and $\angle 2$ and different length of PQ and QR, etc.

Try These

Prove that

1. The exterior angle of a triangle is equal to the sum of two interior opposite angles.
2. If two angles of a triangle are equal then its two opposite sides are also equal.
3. The diagonal of a parallelogram divides it into two congruent triangles.
4. If in two right triangles, the hypotenuse and one side of the first triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

3. Writing exact, brief and clear mathematical statements by using mathematical language.

Read this property of natural number

$n_1 + n_2 = n_2 + n_1 \qquad \forall\, n_1, n_2 \in N$ (1)

Can you tell which property of natural numbers is described?

This statement tells about the commutative property of sum of natural numbers. In other words we can say that even if we change the order of two natural numbers, their sum will remain the same. The same is written in brief with the help of letters, symbols or signs in the above statement. The two natural numbers are denoted as n_1 and n_2. And there are two new symbols $\forall$ and $\in$.

This small statement tells us that the sum of two natural numbers does not depend on what is added to what. It means that we can replace the value of n_1 and n_2 by any value of natural numbers and we get $n_1 + n_2 = n_2 + n_1$, for every value of natural number.

Similarly we can also write law of commutative property for multiplication.

Now read the definition of rational numbers which is written with the help of letters and symbols.

$$Q = \frac{p}{q} \qquad \text{Where } p, q \in I \ \&\ q \neq 0$$

In words, "can write any rational number in the form of $\frac{p}{q}$ where p, q are integers and the value of q is not 0.

Try These

Write in words the following mathematical statements:-

(i) $a^m \times a^n = a^{m+n} \quad a, m, n \in N$

(ii) $p(x + y) = px + py \quad \forall p, x, y \in R$

(ii) Write all properties of natural numbers in symbols.

Symbols and Mathematical Statements in Mathematics

Definitions, properties and rules are written in brief by the use of mathematical language. If we will not use mathematical signs, symbols while proving and writing mathematical statements will have to write more and more. Mathematical language use signs to write a statements in accurate form. So using mathematical language is both necessary and helpful while proving theorems.

Generally many signs are used in mathematics, here are some signs and their meanings.

S.No.	Sign	Meaning
1.	$=$	Is equal to
2.	$<$	Less than
3.	$>$	Greater than
4.	$\therefore$	Therefore
5.	$\because$	Since
6.	$\neq$	Is not equal to
7.	$\forall$	For all
8.	$\in$	Belongs to
9.	$\notin$	does not belong to
10.	$\sim$	Is similar to
11.	$\cong$	Is congruent to
12.	$\Rightarrow$	Implies
13.	$\parallel$	Is parallel to

Example-1. Write the following literal (word) statements as mathematical statements:-

(A) Commutative property is not satisfied while doing subtraction of integers.

(B) The square of an integer is greater than or equal to that number.

Solution : (A) $a - b \neq b - a \quad \forall\ a, b \in \mathrm{I}$

To write this mathematical statement we use two variables a and b and the signs $\neq$, $\forall$, $\in$.

(B) $x^2 \geq x \quad \forall x \in N$

Here x denotes a natural number.

Try These

Write mathematical statements for the following:-

(i) On multiplying an integer by 1 we will get the same integer.

(ii) The sum of two sides of a triangle is greater than the third side.

(iii) The sum of two fractional numbers is also a fractional number.

Exercise-1

1. State whether the following mathematical statements are true or false. Justify your answers:-
   1. The sum of interior angles of a quadrilateral is 350°.
   2. $x^2 \geq 0$ for a real number x.
   3. The sum of two even numbers is always an even number.
   4. All prime numbers are odd.
   5. $3n + 1 > 4$, where n is a natural number.
   6. $x^2 > 0$, where x is a real number.
   7. $(a + b) + c = a + (b + c) \quad \forall\ a, b, c \in N$
   8. $(p - q) + r = p - (q + r) \quad \forall\ p, q, r \in Q$
   9. $(x + y) - z = x + (y - z) \quad \forall\ x, y, z \in R$

2. Some axiom, theorem and definitions are given in the following table. Read carefully-

1.	Whole is greater than part (Axiom)
2.	If all three sides of a triangle are different in measure, then that triangle is called a scalene triangle. (Definition)
3.	If n is an odd integer then it can be expressed as, $n = 2k + 1$, where k is any integer (Definition).
4.	Two triangles are congruent if the three sides of one traingle are equal to the corresponding three sides of the other triangle (Theorem)
5.	If two things are equal to the same third thing then they also equal one another. (Axiom)

Write possible conclusions on the basis of the above statements for the following statement.

(i) In triangle RST and XYZ, RS = XY, ST = YZ and TR = ZX.

(We can deduce this conclusion on the basis of statement –4 that $\Delta RST \cong \Delta XYZ$)

(ii) $\frac{AB}{2} = AC$

(iii) $l = \frac{k+5}{2}$ and $2m = k + 5$, where k, l and $m \in R$

(iv) In ΔDEF, $DE \neq EF \neq FD$

(v) 141 is an odd integer.

3. If n_1 and n_2 are two even integers and k_1 and k_2 are any two integers then,

(i) Write n_1 and n_2 in the form of k_1 and k_2 respectively by using definition of even integers.

(ii) Write the product $n_1 n_2$ in the form of k_1 and k_2.

(iii) Write $n_1 + n_2$ in the form of k_1 and k_2.

(iv) Is $n_1 \times n_2$ even number or odd number? Why?

(v) Is $n_1 + n_2$ even number or odd number? Why?

4. If $ax^2 + bx + c = 0$ is a quadratic equation where $a, b, c \in R$ and $a \neq 0$ then which of the following equations is a quadratic equation. Give reasons.

(i) $ax^2 - bx + c = 0$ (ii) $bx + c = 0$ (iii) $ax^2 + c = 0$

(iv) $ax^2 = 0$ (v) $bx = 0$

5. Definition of rational number (Q) is given below:-

$$Q = \frac{p}{q} \text{ where } \forall\, p, q \in I \text{ and } q \neq 0$$

(i) Write the definition of rational number in words.

(ii) Is $\frac{6}{0}$ a rational number?

(iii) Is $\frac{81}{1}$ a rational number? Give reason on the basis of given definition.

(iv) If $\frac{b+9}{a-5}$ is a rational number, where $a, b \in N$ (Natural numbers), then which value of a is not valid here? Why?

(v) If $\frac{p^2+7}{q^2-25}$ is a rational expression then here, 5 and –5 cannot be values of q (Use definition).

Methods of Proving Mathematical Statements

So, generally we prove mathematical statements by deductive reasoning. Let us consider some more examples:-

"If ΔABC is an equilateral triangle then it is also an isosceles triangle."

If a triangle is equilateral then its three sides are equal. It mean that if all its sides are equal then its any two sides will also be equal. So, it would also be isosceles.

Let us now try to express these facts into symbolical form.

A : ΔABC is a triangle.

B : ΔABC is an isosceles triangle.

Now, if statement A is right then statement B is also right.

So, we expressed it in the following way.

$A \Rightarrow B$ (we read it as "if A then B" or 'A implies B').

Here $\Rightarrow$ this symbol stand for implies.

Try These

1. Write some more statements of this type which we can prove by deductive reasoning.
2. Does $A \Rightarrow B$ also show that $B \Rightarrow A$? Give reason.

Use of Implies

Let us consider some examples.

Statement-1 : If $x^2 = 4$ then $x = 2, -2$

$A : x^2 = 4$

$B : x = \pm 2$

We know that if $x^2 = 4$, then values of x will be 2 and –2. So $A \Rightarrow B$

Statement-2 : If m is a multiple of 9 then m is also a multiple of 3.

A : m is a multiple of 9

B : m is a multiple of 3

We know that if a number is multiple of 9 then it will be also multiple of 3. So $A \Rightarrow B$.

Try These

Find out appropriate logical relation in the following statements and use sign ($\Rightarrow$) to express them.

1. P : Quadrilateral ABCD is a rectangle.

   Q : Quadrilateral ABCD is a square.

2. A : Point P_1, lies on line l and m.

   B : Lines l and m are not parallel.

Proving Some More Statements

Now we prove and test those statements which we can't prove directly by deductive reasoning.

Statement-3 : Square of an odd number is an odd number.

Proof : Let n be an odd number then

$$n = 2k + 1$$

(By the definition of odd numbers we know this)

On squaring both sides,

$n^2 = (2k + 1)^2$

$(2k + 1)^2 = 4k^2 + 4k + 1$

$=2(2k^2+2k)+1$ (Since, $2k^2 + 2k$ is also an integer, so say $b = 2k^2 + k$)

$=2b + 1$

So $n^2 = 2b + 1$

Since $2b$ is an even number therefore $2b + 1$ will be an odd number.

Clearly, square of an odd number is an odd number.

Statement-4 : Prove that $\sqrt{2}$ is irrational.

To prove this statement first of all we will assume that $\sqrt{2}$ is rational. By the use of definition of rational numbers we will reach the conclusion that $\sqrt{2}$ is not a rational number. By this, $\sqrt{2}$ will be proved as irrational number.

Let us consider the proof of this statement.

Proof : Let us take the adverse of statement as true, that is, $\sqrt{2}$ is a rational number. Then according to definition of rational number,

$$\sqrt{2}=\frac{a}{b} \text{ where } a,b\in I,\ b\neq 0 \quad ....(1)$$

a and b are also prime.

On squaring both side in (1)

$$\left(\sqrt{2}\right)^2=\left(\frac{a}{b}\right)^2$$

$$2b^2=a^2 \quad ....(2)$$

$$2m=a^2 \text{ where } m=b^2,\ m\in I$$

$2m=(2n)^2$ Where $a=2n, n\in I$ (If a^2 is an even number then a which is an integer will also be even)

Now, on putting value of a in equation (2).

$2b^2=(2n)^2 \Rightarrow b^2=2n^2$ (Suppose that $n^2=p$ is any integer)

$\therefore b^2=2p$ where $p\in I$

(So, b^2 is an even number therefore b which is an integer will also be an even number)

$\Rightarrow b=2q$ where $q\in I$

It means, a and b have at least one common factor, therefore a and b are not coprime. But this contradicts the fact that a and b are coprime. So this proves that $\sqrt{2}$ is an irrational number.

So, we conclude that $\sqrt{2}$ is irrational.

This method of verification is called proof by contradiction. In this method we assume that the contradictory statement of the given statement is true and then we logially try to prove the assumption wrong. As a result, the actual statement is proven true. Thus, this method is often used to prove statements. As you may have understood, we assume the negation as true.

Negating a statement is called negation. For this we use a special symbol. Negation of statement P is denoted by ~P (it means tilde P).

Lets consider some examples:-

1. P : x and y both are integers.

   ~P : both x and y are not integers.

2. B : Line segement AB is perpendicular on line segment PT.

   ~B : Line segment AB, is not perpendicular on line segment PT.

Try These

State the negations for the following statements.

1. C : A tangent intersects a circle at one point.
2. D : Arithmetic mean is greater than geometric mean.
3. R : $b^2 - a^2$ is a negative number.

Testing of Statements : Sometimes, it is not to easy to find logic while deducing statements and not easy to prove them. Sometimes statements are not correct and have to be proved false. How do we go ahead to prove a mathematical statement as false?

Statement-5 : All prime numbers are odd.

We find that it is difficult to find a logical connection in this statement because there is no fixed pattern to finding prime number. It is clear that it is not possible to examine truth criteria for infinite prime numbers. But if we find any prime number which is not odd, then this statements proved false. There is a counter example. 2 is one prime number which is not odd. So given statement is false.

Find counter examples for following statement.

Statement-6 : $\forall x \in R$ if x^2 is a rational number then x is also a rational number. In this statement, if $x^2 = 2$ which is a rational number then we will get $x = \sqrt{2}$ which is not a rational number. So, only one example has proved this statement false. We can find more examples for this.

Notice that a statement can be disproved by just one counter example. Because in mathematics, a statement is generally accepted if and only if it is valid in every situation. Therefore, if a statement is disproved in a situation, then it is false. This is said to bedisproving by counter example.

Try These

Find one or more counter examples for the statements and disprove them.

a. Product of any two positive rational numbers is greater than both the rational numbers.

b. All similar figures are also congruent.

Example-2. Prove that $2k + 7$ is an odd integer where k is any integer.

Solution : If n is an odd integer, then we can express $n = 2k + 1$ where k is any integer.

We have to prove that $n = 2k + 7$ is an odd integer $\forall\ k \in I$

$n = 2k + 7$

$= 2k + 6 + 1$

$= 2(k + 3) + 1$ ----------(1)

Let $k + 3 = m \quad \forall\ m \in I$ ----------(2)

By (1) and (2) $n = 2m + 1$

$n = 2m + 1$ is an odd integer $\forall\ m \in I$ (According to definition of odd numbers)

Clearly $2k + 7$ is an odd integer.

Exercise - 2

1. Use mathematical symbols to write the following statements

   A. Integers are closed under multiplication.

   B. The subtraction of rational numbers is not commutative.

2. Give the answer on the basis of instructions for the mathematical statements.

   A. Statements : $n^3 \geq n \quad \forall\ n \in Q$

      (i) Write this statement in words.

      (ii) Is this statement true?

      (iii) For which numbers is the statement true.

      (iv) Can we take $n = \frac{\sqrt{2}}{7}$ in this statement?

   B. $x^2 = 1 \Rightarrow x = 1$

      (i) Write this statement in the form of 'if-then'.

      (ii) Is this statement true?

      (iii) Is statement true if when $x \notin N$

3. State whether the following statements are true or false. Give the reason for your answers:-

   (i) All polygons are pentagons.

   (ii) All hexagons are polygons.

   (iii) All even numbers are not divisible by 2.

   (iv) Some real numbers are irrational.

   (v) All real numbers are not rational.

4. It is given that ABCD is a parallelogram and $\angle B = 80°$, then what can you say about other angles of the parallelogram?

5. Prove that $4m + 9$ is an odd integer where m is an integer.

6. Write negation for following statements by using appropriate symbols:-

   A. M : $\sqrt{7}$ is an irrational number.

   B. A : $6 + 3 = 9$

   C. D : Some rational numbers are integers.

   D. P : Triangle PQR is equilateral.

7. Find logical connections in the following statements and write them by using ($\Rightarrow$):-

   A. A : All interior angles of a triangle ΔABC are equal.

   B : Triangle ΔABC is an equilateral triangle.

   B. T : P $(a) = 0$

   S : $(x - a)$ is a factor of the polynomial (x)

   C. P : x and y are two odd numbers.

   Q : $x + y$ is an even number.

Contrapositive

Some statements are difficult to prove. In given form, consider the following statements:-

A_1 : If two triangles are not similar then they are also not congruent.

This statement can also be written as:-

A_2 : If two triangles are similar then they are also congruent.

A_3 : If two traingles are not congruent, they are also not similar.

A_4 : If two traingles are congruent then they are also similar.

Now explain, which are comparable statements in the four statements given above? Obviously statements A_1 and A_4 are comparable statements, because these two statements logically express the same fact and this is their contrapositive form.

Although statements A_1 and A_4 logically express same fact but in statement A_4 it is easy to find logic and use it as compared to statement A_1. Statement A_4 is contrapositive form of statement A_1. So, to prove some statements we have to convert them into contrapositive form.

Example-3. Write contrapositive form for the following statements:-

If a number is divisible by 25 then it is also divisible by 5.

Solution : If a number is not divisible by 5 then it is also not divisible by 25.

Example-4. If $x^2 - 6x + 5$ is even, then x is odd. Where $\forall\ x \in z$

Solution : Let us express this statement in contrapositive form.

If x is not odd, then $x^2 - 6x + 5$ is not even. $\forall\ x \in z$

Now x is not odd $\Rightarrow x$ is even

$x = 2k,\ k \in z$ (According to definition of even integers)

$x^2 - 6x + 5 \quad \Rightarrow (2k)^2 - 6(2k)^2 - 6(2k) + 5$

$\Rightarrow 4k^2 - 12k + 4 + 1 \quad \Rightarrow 2\ (2k^2 - 6k + 2) + 1$

$\Rightarrow 2b + 1$ where $b = 2k^2 - 6k + 2$ and b is an integer $(b \in z)$

According to definition of odd integers we can say that $2b + 1$ is an odd integer.

It means, if x is not odd, then $x^2 = 6x + 5$ is not even

Exercise - 3

1. Prove that the sum of interior angles of *a* polygon with *n* sides whose all angles are equal is $n\left[180-\dfrac{360}{n}\right]^{\circ}$ where $n \geq 3$.
2. Prove that the sum of n term of an A.P. is $3p^2 + 4p$, if the n^{th} term of that AP is $6n + 1$.
3. Prove that the sum of three successive even integers is always a multiple of 6.
4. Prove that 8 is a factor of $(2n+3)^2 - (2n-3)^2$, where n is a natural number.
5. Prove that if the sum of squares of two successive whole numbers is divided by 4, then remainder always comes as 1.

What We Have Learnt

1. For proving statements:-
   (i) We use previously proved theorems, definitions and axioms.
   (ii) Each statement of proof logically connects with the earlier statements.
   (iii) During the writing of statements we use specific symbols so that large sentences can be written in brief.
2. We can write brief, clear and exact mathematical statements by using mathematical language.

   $\forall, \in, \cong$ etc. are some symbols of mathematical language.
3. Methods of proving mathematical statements are:-
   (i) Deductive - reasoning.
   (ii) By counter example.

Answer Sheet

Exercise - 1

1.	(i)	False	(ii)	True	(iii)	True
	(iv)	Flase	(v)	False	(vi)	False
	(vii)	True	(viii)	False	(ix)	True.

2. (i) $\Delta RST \cong \Delta XYZ$ (By Statement-4)
   (ii) AB > AC (By statement-1)
   (iii) $l = m$ (By statement-5)
   (iv) ΔDEF is a scalene triangle (By statement-2)
   (v) $141 = 2 \times 70 + 1$ (By statement-3)
3. (i) $n_1 = 2k_{1,}\ n_{2} = 2k_2$
   (ii) $n_1 \times n_2 = (k_1 \times k_2)$
   (iii) $n_1 + n_2 = 2\ (k_1 + k_2)$
   (iv) $n_1 + n_2 = 2\ (k_1 \times k_2) = 2k$, according to definition of even numbers.
   (v) Even number.
4. In all (i), (ii), (iii) $a \neq 0$
5. (ii) No (iii) Yes (iv) $a = 5$ is not valid
   (v) $q \in I - \{5, -5\}$

Exercise - 2

1. (i) $a.b = c \ \forall\ a, b, c \in I$ (ii) $p - q \neq q - p\ \forall\ p, q \in Q$
2. A. (i) Cube of a rational number is greater than that number
   (ii) No
   (iii) True for $n \in N$
   (iv) No
   (v) No, cube of a negative number is smaller than that number.
   B. (i) If square of a number is 1, then the value of that number will be 1.
   (ii) Yes
   (iii) No
3. (i) False (ii) True (iii) False (iv) True (iv) False
   (vi) False
4. $\angle A = 100°$, $\angle C = 100°$, $\angle D = 80°$.
6. (i) ~M : $\sqrt{7}$ is not an irrational number
   (ii) ~ A : $6 + 3 \neq 9$
   (iii) ~D : Some rational numbers are not whole numbers.
   (iv) ~P : Triangle PQR is not equilateral.
7. (i) $A \Rightarrow B, B \Rightarrow A$ (ii) $S \Rightarrow T, T \Rightarrow S$ (iii) $P \Rightarrow Q$