Chapter

# 1 NUMBERS

## KEY FACTS

**1.** Numbers of the form $\frac{p}{q}, q \neq 0$, where $p$ and $q$ are integers and which can be expressed in the form of terminating or repeating decimals are called **rational numbers**.

e.g., $\frac{7}{32} = 0.21875, \frac{8}{15} = 0.5\overline{3}$ are rational numbers.

**2. Properties of operations of rational numbers**

For any rational numbers $a$, $b$, $c$.

(i) Rational numbers are closed under addition, multiplication and subtraction, i.e., $(a + b)$, $(a \times b)$ and $(a - b)$ are also rational numbers.

(ii) Rational numbers follow the **commutative law** of addition and multiplication, i.e., $a + b = b + a$ and $a \times b = b \times a$.

(iii) Rational numbers follow the **associative law** of addition and multiplication, i.e., $(a + b) + c = a + (b + c)$ and $(a \times b) \times c = a \times (b \times c)$.

(iv) **Additive identity** : 0 is the additive identity for rational numbers as $a + 0 = 0 + a = 0$.

(v) **Multiplicative identity** : 1 is the multiplicative identity for rational numbers as $a \times 1 = 1 \times a = a$.

(vi) **Additive inverse** : For every rational number '$a$', there is a rational number '$-a$' such that $a + (-a) = 0$.

(vii) **Multiplicative inverse** : For every rational number '$a$' except 0, there is a rational number $\frac{1}{a}$ such that $a \times \frac{1}{a} = 1$.

(viii) **Distributive property** : Multiplication distributes over addition in rational numbers, i.e., $a\,(b + c) = a \times b + a \times c$.

(ix) Between any two different rational numbers, there are infinitely many rational numbers. Rational numbers between any two given rational numbers $a$ and $b$ are $q_1 = \frac{1}{2}(a+b)$, $q_2 = \frac{1}{2}(q_1 + b)$, $q_3 = \frac{1}{2}(q_2 + b)$ and so on.

**3.** Numbers which when converted into decimals are expressible neither as terminating nor as repeating decimals are called **irrational numers.**

e.g., $\sqrt{2} = 1.41421356237.......$ is an irrational number.

An irrational number cannot be expressed in the form $\frac{p}{q}(q \neq 0)$ and its exact root cannot be found.

**4.** The totality of all rational and all irrational numbers is called real numbers which is denoted by $R$.

The natural numbers, whole numbers, integers non-integral rationals $\left(\frac{4}{7}, -\frac{7}{8}\right)$ etc., are all contained in real numbers.

**5. Unit's digit of any number that can be expressed as a power of any natural number between 1 and 9.**

(i) The units' digit of any number expressed as a power of 2 is any of the digits 2, 4, 8, 6 as $2^1 = \mathbf{2}$, $2^2 = \mathbf{4}$, $2^3 = \mathbf{8}$, $2^4 = \mathbf{16}$, $2^5 = \mathbf{32}$, ....... etc.

(ii) The units' digit of any number expressed as a power of 3 is any of the digits 3, 9, 7, 1 as $3^1 = \mathbf{3}$, $3^2 = \mathbf{9}$, $3^3 = \mathbf{27}$, $3^4 = \mathbf{81}$, $3^5 = \mathbf{243}$, ....... etc.

(iii) The units' digit of any number expressed as a power of 4 is 4 if the power is odd and 6 if the power is even as $4^1 = \mathbf{4}$, $4^2 = \mathbf{16}$, $4^3 = \mathbf{64}$, $4^4 = \mathbf{256}$, ......etc.

(iv) The units' digit of any number expressed as a power of 5 is always 5 as $5^1 = \mathbf{5}$, $5^2 = \mathbf{25}$, $5^3 = \mathbf{125}$, ........ etc.

(v) The units' digit of any number expressed as a power of 6 is always 6 as $6^1 = 6$, $6^2 = \mathbf{36}$, $6^3 = \mathbf{216}$, ........ etc.

(vi) The units' digit of any number expressed as a power of 7 is any of the digits 7, 9, 3, 1 as $7^1 = \mathbf{7}$, $7^2 = \mathbf{49}$, $7^3 = \mathbf{343}$, $7^4 = \mathbf{2401}$, ........ etc.

(vii) The units' digit of any number expressed as a power of 8 is any of the digits 8, 4, 2, 6 as $8^1 = \mathbf{8}$, $8^2 = \mathbf{64}$, $8^3 = \mathbf{512}$, $8^4 = \mathbf{4096}$, ........ etc.

(viii) The units' digit of any number expressed as a power of 9 is 9 if the power is odd and 1 if the power is even as $9^1 = \mathbf{9}$, $9^2 = \mathbf{81}$, $9^3 = \mathbf{729}$, $9^4 = \mathbf{6561}$, ........ etc.

**6. Total number of factors of any number.**

The number has to be resolved into prime factors. If the prime factorisation of the given number $= 2^m \times 3^n \times 5^p \times 11^q$, then the total number of factors $= (m + 1) \times (n + 1) \times (p + 1) \times (q + 1)$.

For example, if $N = 2^4 \times 5^6 \times 7^4 \times 13^1$, then

Number of factors of $N = (4 + 1) \times (6 + 1) \times (4 + 1) \times (1 + 1) = 5 \times 7 \times 5 \times 2 = 350$.

$2^4 = 16$ $\therefore$ factors of 16 are 1, 2, 4, 8, 16, i.e., 5 in number, i.e., (4 + 1) in numbers.

Similarly $7^4 = 2401$ $\therefore$ factors of 2401 and 1, 7, 49, 343 and 2401, i.e., 5 in number and so on.

## Solved Examples

**Ex. 1.** ***If a number 573 xy is divisible by 90, then what is the value of x + y ?***

**Sol.** 573 $xy$ is divisible by 90, i.e., $(9 \times 10)$

$\Rightarrow$ 573 $xy$ is divisible by both 9 and 10.

$\Rightarrow$ $y = 0$ as a number is divisible by 10 if its ones' digit = 0.

Also, sum of digits $= 5 + 7 + 3 + x + 0 = 15 + x$

For divisibility by 9, $15 + x = 18 \Rightarrow x = 3$

$\therefore$ $x + y = 3 + 0 = \mathbf{3}$.

**Ex. 2.** ***The product of two whole numbers is 13. What is the sum of the squares of their reciprocals ?***

**Sol.** The two whole numbers whose product is 13 are 1 and 13.

$\therefore$ Required sum $= 1 + \frac{1}{13^2} = 1 + \frac{1}{169} = \mathbf{\frac{170}{169}}$.

**Ex. 3.** ***What per cent is the least rational number of the greatest rational number, if $\frac{1}{2}, \frac{2}{5}, \frac{1}{3}$ and $\frac{5}{9}$ are arranged in ascending order ?***

**Sol.** Since LCM of 2, 5, 3, 9 = 270, $\frac{1}{2} = \frac{135}{270}, \frac{2}{5} = \frac{108}{270}, \frac{1}{3} = \frac{90}{270}, \frac{5}{9} = \frac{150}{270}$

$\therefore$ Arranged in ascending order the numbers are $\frac{90}{270}, \frac{108}{270}, \frac{135}{270}, \frac{150}{270}$, i.e., $\frac{1}{3}, \frac{2}{5}, \frac{1}{2}$ and $\frac{5}{9}$.

$\therefore$ Required per cent $= \left(\frac{1}{3} \div \frac{5}{9}\right) \times 100\% = \left(\frac{1}{3} \times \frac{9}{5} \times 100\right)\% = \mathbf{60\%}$.

**Ex. 4.** ***Find the units' digit in the expression $11^1 . 12^2 . 13^3 . 14^4 . 15^5 . 16^6$ ?***

**Sol.** Units' digit in the given expression

= Units' digit of $1^1$ × Units' digit of $2^2$ × Units' digit of $3^3$ × Units' digit of $4^4$ × Units' digit of $5^5$ × Units' digit of $6^6$

= Units' digit of $(1 \times 4 \times 7 \times 6 \times 5 \times 6)$

= Units' digit of 5040 = **0.**

**Ex. 5.** ***Find the units' digit in the expression $(515)^{31} + (525)^{90}$ ?***

**Sol.** Since the units' digit of any number written as a power of 5 is always 5,

Units' digit in the expression $(515)^{31} + (525)^{90}$

= Units' digit of $5^{31}$ + Units digit of $5^{90}$

= Units' digit of $(5 + 5) = 10 =$ **0.**

**Ex. 6.** ***What is the total number of factors of the number $N = 4^{11} \times 14^5 \times 11^2$ ?***

**Sol.** $N = 4^{11} \times 14^5 \times 11^2 = (2^2)^{11} \times (2 \times 7)^5 \times 11^2 = 2^{22} \times 2^5 \times 7^5 \times 11^2 = 2^{27} \times 7^5 \times 11^2$

∴ Total number of prime factors of

$N = (27 + 1) \times (5 + 1) \times (2 + 1) = 28 \times 6 \times 3 =$ **504.**

**Ex. 7.** ***The numbers 1, 3, 5, ........, 25 are multiplied together. What is the number of zeros at the right end of the product ?***

**Sol.** Since all the numbers to be multiplied are odd number, the digit at the units' place will be 5. Hence the number of zeros at the right end of the product is zero.

**Ex. 8.** ***In a division sum, the remainder is 6 and the divisor is 5 times the quotient and is obtained by adding 2 to thrice of remainder. Find the dividend.***

**Sol.** Divisor = $3 \times 6 + 2 = 20$ (∵ Remainder = 6)

$$\text{Quotient} = \frac{20}{5} = 4$$

∴ Dividend = Divisor × Quotient + Remainder

= $20 \times 4 + 6 =$ **86.**

**Ex. 9.** ***A three digit number 3a5 is added to another 3-digit number 933 to give a 4-digit number 12b8, which is divisible by 11. Then, find the value of a + b ?***

**Sol.** Given, $a + 3 = b$ ... $(i)$

$$\begin{array}{rccc} & 3 & a & 5 \\ + & 9 & 3 & 3 \\ \hline & 12 & b & 8 \\ \hline \end{array}$$

For $12b8$ to be divisible by 11,

$(8 + 2) - (b + 1) = 0 \Rightarrow 10 - b - 1 = 0 \Rightarrow 9 - b = 0 \Rightarrow b = 9$

∴ From $(i)$ $a = 6$

∴ $a + b = 6 + 9 =$ **15.**

**Ex. 10.** ***If $\frac{1}{891} = 0.00112233445566778899$........ Then what is the value of $\frac{198}{891}$ ?***

**Sol.** $\frac{198}{891} = \frac{1}{891} \times 198 = \frac{2}{9} = 0.2222$.........

**Ex. 11.** ***What is the value of $\frac{1}{5} \div \frac{1}{5} \div \frac{1}{5} \div \frac{1}{5} \div \frac{1}{5} \div 5 \div 5 \div 5$?***

**Sol.** $\frac{1}{5} \div \frac{1}{5} \div \frac{1}{5} \div \frac{1}{5} \div \frac{1}{5} \div 5 \div 5 \div 5$

$= \frac{1}{5} \div \frac{1}{5} \div \frac{1}{5} \div \frac{1}{5} \div \frac{1}{5} \div 5 \div 1 = \frac{1}{5} \div \frac{1}{5} \div \frac{1}{5} \div \frac{1}{5} \div \frac{1}{5} \div 5$

$= \frac{1}{5} \div \frac{1}{5} \div \frac{1}{5} \div \frac{1}{5} \div \frac{1}{25} = \frac{1}{5} \div \frac{1}{5} \div \frac{1}{5} \div \frac{1}{5} \times 25 = \frac{1}{5} \div \frac{1}{5} \div \frac{1}{5} \div 5$

$= \frac{1}{5} \div \frac{1}{5} \div \left(\frac{1}{5} \times \frac{1}{5}\right) = \frac{1}{5} \div \frac{1}{5} \div \frac{1}{25} = \frac{1}{5} \div \frac{1}{5} \times 25 = \frac{1}{5} \div 5 = \mathbf{\frac{1}{25}}$.

## Question Bank–1

**1.** If the remainder obtained by subtracting a number from its own square is 4 times the number, what is the number ?

(a) 4 (b) 3
(c) 6 (d) 5

**2.** The difference between the squares of two consecutive odd integers is always divisible by

(a) 8 (b) 7
(c) 6 (d) 3

**3.** A number when divided by 296 leaves 75 as remainder. If the same number is divided by 37, the remainder obtained is

(a) 2 (b) 1
(c) 11 (d) 8

**4.** A number when divided by 5 leaves 3 as remainder. If the square of the same number is divided by 5, the remainder obtained is

(a) 9 (b) 4
(c) 1 (d) 3

**5.** When $N$ is divided by 4, the remainder is 3. What is the remainder when $2N$ is divided by 4.

(a) 2 (b) 3
(c) 4 (d) 8

**6.** The units' digit in the expression $(11^1 + 12^2 + 13^2 + 14^4 + 15^5 + 16^6)$ is

(a) 1 (b) 9
(c) 7 (d) 0

**7.** The digit in the units' place of

$[(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - (16)^4 + 259]$ is

(a) 1 (b) 4
(c) 5 (d) 6

**8.** The number of digits in $(48^4 \times 5^{12})$ is

(a) 18 (b) 16
(c) 14 (d) 12

**9.** Which one of the following is a rational number ?

(a) $(\sqrt{2})^2$ (b) $2\sqrt{2}$
(c) $2 + \sqrt{2}$ (d) $\frac{\sqrt{2}}{2}$

**10.** If $x$ is a rational number and $y$ is an irrational number, then

(a) Both $x + y$ and $xy$ are necessarily irrational.
(b) Both $x + y$ and $xy$ are necessarily rational.
(c) $xy$ is necessarily irrational, but $x + y$ can be either rational or irrational.
(d) $x + y$ is necessarily irrational, but $xy$ can be either rational or irrational.

**11.** Consider the following statements :

A number $a_1\, a_2\, a_3\, a_4\, a_5\, a_6$ is divisible by 11 if

**1.** $(a_1 + a_3 + a_5) - (a_2 + a_4 + a_6) = 0$
**2.** $(a_1 + a_3 + a_5) - (a_2 + a_4 + a_6)$ is divisible by 11

Which of these statements is/are correct ?

(a) 1 alone (b) 2 alone
(c) Both 1 and 2 (d) Neither 1 nor 2

**12.** The number 111,111,111,111 is divisible by

(a) 9 and 11 (b) 5 and 11
(c) 3 and 9 (d) 3 and 11

**13.** If $1^2 + 2^2 + 3^2 + ........ + 512^2 = m$, then $2^2 + 4^2 + 6^2 + ........ + 1024^2$ is equal to

(a) $3m$ (b) $4m$
(c) $m^2$ (d) $m^3$

**14.** If $m$ and $n$ are positive integers, then the digit in the units' place of $5^n + 6^m$ is always

(a) 1 (b) 5
(c) 6 (d) $n + m$

**15.** What number should replaced $M$ in this multiplication problem ?

$$\begin{array}{r} 3M4 \\ \times\ 4 \\ \hline 1216 \\ \hline \end{array}$$

(a) 0 (b) 5
(c) 7 (d) 8

**16.** $m$ and $n$ are integers and $\sqrt{mn} = 10$. Which of the following cannot be a value of $m + n$ ?

(a) 25 (b) 52
(c) 101 (d) 50

**17.** If a six digit number $93p25q$ is divisible by 88, then the values of $p$ and $q$ are respectively

(a) 2 and 8 (b) 8 and 2
(c) 8 and 6 (d) 6 and 8

**18.** What is the 25th digit to the right of the decimal point in the decimal of $\frac{6}{11}$ ?

(a) 5 (b) 3
(c) 4 (d) 6

**19.** A number when divided by the sum of 555 and 445 gives two times their difference as quotient and 30 as remainder. The number is

(a) 220030 (b) 22030
(c) 1220 (d) 1250

**20.** There are four prime numbers written is ascending order. The product of the first three is 385 and that of last three is 1001. Find the first number.

(a) 5 (b) 7
(c) 11 (d) 17

**21.** What is the highest power of 5 that divides
$90 \times 80 \times 70 \times 60 \times 50 \times 40 \times 30 \times 20 \times 10$

(a) 10 (b) 12
(c) 14 (d) 15

**22.** Five digit numbers are formed such that either all the digits are even or all the digits are odd. If no digit is allowed to be repeated in one number find the difference between the maximum possible number with odd digits and the minimum possible number with even digits

(a) 77063 (b) 79999
(c) 72841 (d) 86420

**23.** Which one of the following numbers will completely divide $(3^{25} + 3^{26} + 3^{27} + 3^{28})$ ?

(a) 11 (b) 16
(c) 25 (d) 30

**24.** A number when divided successively by 4 and 5 leaves remainders 1 and 4 respectively. When it is successively divided by 5 and 4, then the respective remainders will be

(a) 1, 2 (b) 2, 3
(c) 3, 2 (d) 4, 1

**25.** If 2.5252525........ $= \frac{p}{q}$ (in the lowest form) then what is the value of $\frac{q}{p}$ ?

(a) 0.4 (b) 0.42525
(c) 0.0396 (d) 0.396

**26.** What is the sum of two numbers whose difference is 45, and the quotient of the greater number by the lesser number is 4 ?

(a) 100 (b) 90
(c) 80 (d) 75

**27.** The number of factors of a number $N = 2^3 \times 3^2 \times 5^3$ is

(a) 18 (b) 45
(c) 48 (d) 9

**28.** $P = 441 \times 484 \times 529 \times 576 \times 625$. The total number of factors are

(a) 607 (b) 5706
(c) 1024 (d) 6075

**29.** If $N = 12^3 \times 3^4 \times 5^2$, then the total number of even factors of $N$ is

(a) 25 (b) 121
(c) 144 (d) 84

**30.** The units' digit of the sum $1 + 9 + 9^2 + \ldots\ldots\ldots 9^{1006}$ is

(a) 2 (b) 1
(c) 9 (d) 0

## Answers

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** (d) | **2.** (a) | **3.** (b) | **4.** (b) | **5.** (a) | **6.** (b) | **7.** (b) | **8.** (b) | **9.** (a) | **10.** (d) |
| **11.** (c) | **12.** (d) | **13.** (b) | **14.** (a) | **15.** (a) | **16.** (d) | **17.** (c) | **18.** (a) | **19.** (a) | **20.** (a) |
| **21.** (a) | **22.** (a) | **23.** (d) | **24.** (b) | **25.** (d) | **26.** (d) | **27.** (c) | **28.** (d) | **29.** (c) | **30.** (d) |

## Hints and Solutions

**1.** (d) Let the number be $x$. Then,

$x^2 - x = 4x \Rightarrow x^2 = 5x \Rightarrow x = 5$

**2.** (a) Let the two consecutive odd integers be $(2x + 1)$ and $(2x + 3)$.

$\therefore$ $(2x+3)^2 - (2x+1)^2 = 4x^2 + 12x + 9 - 4x^2 - 4x - 1$

$= 8x + 8 = 8(x + 1)$

Hence the difference is divisible by 8.

**3.** (b) Given number $= 296n + 75$

$= 37 \times 8 \times n + (37 \times 2 + 1)$

$= 37(8n + 2) + 1$

$\therefore$ Required remainder = 1

**4.** (b) Let the given number $= 5n + 3$

$\therefore$ Square of the number

$= (5n + 3)^2$

$= 25n^2 + 30n + 9$

$= 5 \times 5n^2 + 5 \times 6n + 5 + 4$

$= 5\,(5n^2 + 6n + 1) + 4$

$\therefore$ Required remainder = 4.

**5.** (a) $N = 4Q + 3$, where $Q$ is the quotient.

$\therefore$ $2N = 8Q + 6 = 4 \times 2Q + 4 + 2 = 4\,(2Q + 1) + 2$

$\Rightarrow$ Required remainder = 2.

**6.** (b) Units' digit of the given expression

= Units' digit of $1^1$ + Units' digit of $2^2$ + Units' digit of $3^3$ + Units' digit of $4^4$ + Units' digit of $5^5$ + Units' digit of $6^6$

= Units' digit of $(1 + 4 + 7 + 6 + 5 + 6)$

= Units' digit of 29 = **9**.

[**Note:** $3^3 = 27$, $4^4 = 256$; units' digit for any power of 5 and 6 = 5 and 6 respectively].

**7.** (b) Units' place digit

= Units' digit of $1^{98}$ + Units' digit of $1^{29}$ – Units' digit of $6^{100}$ + Units' digit of $5^{35}$ – Units' digit of $6^4$ + 9

$= 1 + 1 - 6 + 5 - 6 + 9 =$ **4**

[**Note:** The units' digit of any positive integral power of 1 is always 1, of any positive integral power of 5 is always 5 and of any positive integral power of 6 is always 6.]

**8.** (b) $48^4 \times 5^{12} = (3 \times 16)^4 \times 5^{12} = (3 \times 2^4)^4 \times 5^{12}$

$= 3^4 \times 2^{16} \times 5^{12} = 3^4 \times 2^4 \times 2^{12} \times 5^{12}$

$= (3 \times 2)^4 \times (2 \times 5)^{12} = 6^4 \times 10^{12}$

$= 1296 \times 10^{12}$

$\therefore$ Number of digits = 4 + 12 = 16.

**9.** (a) $(\sqrt{2})^2 = 2$, a rational number.

**12.** (d) Sum of digits = 12, hence divisible by 3

Sum of digits at odd place – Sum of digits at even places

= 6 – 6 = 0, hence divisible by 11.

**13.** (b) $2^2 + 4^2 + 6^2 + \ldots\ldots + (1024)^2$

$= 2^2\,(1^2 + 2^2 + 3^2 + \ldots\ldots + 512^2) = 4m.$

**14.** (a) The units' digit of any positive integer power of 5 is always 5 and that of any positive integral power of 6 is always 6.

$\therefore$ Units' digit of $5^m + 6^n$ = Units' digit of (5 + 6)

= Units' digit of 11 = **1**.

**16.** (d) $\sqrt{mn} = 10 \Rightarrow mn = 100$

$\therefore$ The possible pairs of $m$ and $n$ are

$(m, n) = (1, 100), (2, 50), (4, 25), (5, 20), (10, 10)$

$\Rightarrow$ $m + n$ can be 101, 52, 29, 25, 20

So 50 cannot be a value of $m + n$.

**17.** (c) A number is divisible by 88 if it is divisible by $(8 \times 11)$, i.e., by both 8 and 11.

For a number to be divisible by 8 the last three digits should be divisible by 8, i.e., from the given options we can see that $q = 6$ as 256 is div. by 8.

$\therefore$ Sum of digits at odd places of given number $= 6 + 2 + 3 = 11$

Sum of digits at even places $= 5 + p + 9 = 14 + p$

For divisibility by 11, $14 + p - 11 = 22 \Rightarrow p = 8$.

**18.** (a) $\frac{6}{11} = 0.54545454\ldots\ldots$

Thus we can see that odd places are occupied by the digit 5 and even places by the digit 4. Therefore, the 25th digits to the right of the decimal point in $\frac{6}{11}$ is 5.

**19.** (a) Divisor = 555+ 445 = 1000

Quotient = 2 × (555 – 445) = 220

Remainder = 30

∴ Dividend = Divisor × Quotient + Remainder

= 1000 × 220 + 30 = 220030.

**20.** (a) Let the numbers be $a$, $b$, $c$, $d$. Then,

$abc = 385$ and $bcd = 1001$

$$\Rightarrow \frac{abc}{bcd} = \frac{385}{1001} = \frac{a}{d} = \frac{5}{13} \Rightarrow a = 5, d = 13$$

**21.** (a) All the numbers that are multiplied have 0 as units' digit, so all of them are divisible by 5 once. Also 50 can be divided by $5^2$. So the highest power of 5 that divides the given product = (9 + 1) = 10.

**22.** (a) Maximum possible number with odd digits = 97531 and the minimum possible number with even digits = 20468.

∴ Required difference = 97531 – 20468 = 77063

**23.** (d) $3^{25} + 3^{26} + 3^{27} + 3^{28}$

$= 3^{25}(1 + 3 + 3^2 + 3^3)$

$= 3^{25} \times (1 + 3 + 9 + 27) = 3^{25} \times 40$

$= 3^{24} \times 3 \times 4 \times 10 = 3^{24} \times 4 \times 30,$

which is divisible by 30.

**24.** (b) Let the number be $x$. Then,

∴ $y = 5 \times 1 + 4 = 9$

$\Rightarrow$ $x = 4 \times y + 1 = 4 \times 9 + 1 = 37$

| | |
|---|---|
| 4 | $x$ |
| 5 | $y$ 1 (Remainder) |
| | 1 4 (Remainder) |

∴ When 37 is successively divided by 5 and 4, we get the remainder as 2 and 3 respectively.

| | |
|---|---|
| 5 | 37 |
| 4 | 7 – 2 |
| | 1 – 3 |

**25.** (d) $\frac{p}{q} = 2.525252$ ........

$$2.\overline{52} = 2\frac{52}{99} = \frac{250}{99}$$

$$\therefore \frac{q}{p} = \frac{99}{250} = 0.396.$$

**26.** (d) Let the lesser number be $x$. Then,

Greater number = $x$ + 45

Given, $\frac{x+45}{x} = 4 \Rightarrow x + 45 = 4x$

$\Rightarrow 3x = 45 \Rightarrow x = 15$

Then, required sum = $x + x + 45 = 30 + 45 = 75$

**27.** (c) Number of factors of $N\,(2^3 \times 3^2 \times 5^3)$

$= (3 + 1) \times (2 + 1) \times (3 + 1)$

$= 4 \times 3 \times 4 = 48$

**28.** (d) $P = (21)^2 \times (22)^2 \times (23)^2 \times (24)^2 \times (25)^2$

$= (3 \times 7)^2 \times (2 \times 11)^2 \times (23)^2 \times (2^3 \times 3)^2 \times (5^2)^2$

$= 3^2 \times 7^2 \times 2^2 \times 11^2 \times 23^2 \times 2^6 \times 3^2 \times 5^4$

$= 2^8 \times 3^4 \times 5^4 \times 7^2 \times 11^2 \times 23^2$

Total number of factors

$= (8+1) \times (4+1) \times (4+1) \times (2\times1) \times (2+1) \times (2+1)$

$= 9 \times 5 \times 5 \times 3 \times 3 \times 3 = 6075$

**29.** (c) $N = (2^2 \times 3)^3 \times 3^4 \times 5^2$

$= 2^6 \times 3^7 \times 5^2$

∴ Total number of factors of $N$

$= (6 + 1) \times (7 + 1) \times (2 + 1)$

$= 7 \times 8 \times 3 = 168$

Some of these are odd factors and some of these are even factors. The odd factors are formed with the combination of 3$s$ and 5$s$.

∴ Total number of odd factors

$= (7 + 1) \times (2 + 1) = 8 + 3 = 24$

∴ Number of even factors = 168 – 24 = 144.

**30.** (d) The units' digit of each term successively is 1, 9, 1, 9, ......... as the power of 9 is odd in the second term and even in the third term and so on. First term = 1

The units' digit of first two term = 0 as 1 + 9 = 10,

The units' digit of first three terms = 1 as 1 + 9 + 1 = 11

The units' digit of first four terms = 0 as 1 + 9 + 1 + 9 = 20

$\Rightarrow$ Units' digit is 0 or 1 depending upon the number of terms whether even or odd.

∴ Units' digit of the sum of 1006 terms, i.e., even number of terms = 0.

## Self Assessment Sheet–1

**1.** $16^5 + 2^{15}$ is divisible by
(a) 31 (b) 13
(c) 27 (d) 33

**2.** Find the number of divisors of 10800.
(a) 57 (b) 60
(c) 72 (d) none of these

**3.** The units digit of the expression $125^{813} \times 553^{3703} \times 4532^{828}$ is
(a) 4 (b) 2
(c) 0 (d) 5

**4.** If $M = 2^2 \times 3^5$, $N = 2^3 \times 3^4$, then the number of factors of $N$ that are common with factors of $M$ is
(a) 8 (b) 5
(c) 18 (d) 15

**5.** Find the greatest number by which the expression $7^{2n} - 3^{2n}$ is always exactly divisible.
(a) 4 (b) 10
(c) 20 (d) 40

**6.** Given that $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + 10^2 = 385$, then the value of $(2^2 + 4^2 + 6^2 + \ldots\ldots\ldots + 20^2)$ is equal to
(a) 770 (b) 1540
(c) 1155 (d) $(385)^2$

**7.** There is one number which is formed by writing one digit 6 times. Such number is always divisible by:
(*e.g.*, 0.111111, 0.444444 etc.)
(a) 7 (b) 11
(c) 13 (d) all of these

**8.** The number of integers $x$ for which the number $\sqrt{x^2 + x + 1}$ is rational is:
(a) infinite (b) one
(c) two (d) three

**9.** A number divided by 296 leaves 75 as remainder. If the same number is divided by 37, the remainder obtained is
(a) 2 (b) 1
(c) 11 (d) 8

**10.** How many prime factors are there in the expression $(12)^{43} \times (34)^{48} \times (2)^{57}$?
(a) 282 (b) 237
(c) 142 (d) 61

## Answers

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** (d) | **2.** (b) | **3.** (c) | **4.** (d) | **5.** (d) | **6.** (b) | **7.** (d) | **8.** (c) | **9.** (b) | **10.** (a) |