CHAPTER

5

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. Value of $\left(\frac{2i}{1+i}\right)^2$ is

(a) i (b) $2i$
(c) $1-i$ (d) $1-2i$

2. If $\left(\frac{1-i}{1+i}\right)^{100} = a+ib$ then

(a) $a=2, b=-1$ (b) $a=1, b=0$
(c) $a=0, b=1$ (d) $a=-1, b=2$

3. $1 + i^2 + i^4 + i^6 + ... + i^{2n}$ is

(a) positive (b) negative
(c) 0 (d) cannot be determined

4. If $(x+iy)(2-3i)=4+i$, then

(a) $x=-14/13, y=5/13$ (b) $x=5/13, y=14/13$
(c) $x=14/13, y=5/13$ (d) $x=5/13, y=-14/13$

5. If $4x+i(3x-y)=3+i(-6)$, where x and y are real numbers, then the values of x and y are

(a) $x=\frac{3}{5}$ and $y=\frac{33}{4}$ (b) $x=\frac{3}{4}$ and $y=\frac{22}{3}$
(c) $x=\frac{3}{4}$ and $y=\frac{33}{4}$ (d) $x=\frac{3}{4}$ and $y=\frac{33}{5}$

6. If $z=x-i\,y$ and $z^{\frac{1}{3}}=p+iq$, then $\left(\frac{x}{p}+\frac{y}{q}\right)\Big/(p^2+q^2)$ is equal to

(a) –2 (b) –1 (c) 2 (d) 1

7. The polar form of the complex number $(i^{25})^3$ is

(a) $\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$ (b) $\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}$
(c) $\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$ (d) $\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}$

8. If $z_1=\sqrt{3}+i\sqrt{3}$ and $z_2=\sqrt{3}+i$, then in which quadrant $\left(\frac{z_1}{z_2}\right)$ lies?

(a) I (b) II (c) III (d) IV

9. The solutions of the quadratic equation $ax^2+bx+c=0$, where $a, b, c \in R$, $a \neq 0$, $b^2-4ac<0$, are given by $x=$?

(a) $\frac{b\pm\sqrt{4ac-b^2i}}{2a}$ (b) $\frac{-b\pm\sqrt{4ac+b^2i}}{2a}$
(c) $\frac{-b\pm\sqrt{4ac-b^2i}}{2a}$ (d) $\frac{-b\pm\sqrt{4ab-c^2i}}{2a}$

10. If $x^2+x+1=0$, then what is the value of x ?

(a) $\frac{1+\sqrt{3}i}{2}$ (b) $\frac{-1\pm\sqrt{3}i}{2}$
(c) $\frac{-1\pm\sqrt{3}i}{3}$ (d) $\frac{-1\pm\sqrt{2}i}{2}$

11. The solution of $\sqrt{3x^2-2}=2x-1$ are :

(a) (2,4) (b) (1,4) (c) (3,4) (d) (1,3)

12. If α, β are roots of the equation $x^2-5x+6=0$, then the equation whose roots are $\alpha+3$ and $\beta+3$ is

(a) $2x^2-11x+30=0$ (b) $-x^2+11x=0$
(c) $x^2-11x+30=0$ (d) $2x^2-5x+30=0$

13. Value of k such that equations $2x^2+kx-5=0$ and $x^2-3x-4=0$ have one common root, is

(a) –1, –2 (b) $-3, -\frac{27}{4}$
(c) $3, \frac{4}{27}$ (d) –2, –3

14. If $a<b<c<d$, then the nature of roots of $(x-a)(x-c)+2(x-b)(x-d)=0$ is

(a) real and equal (b) complex
(c) real and unequal (d) None of these

15. For the equation $\frac{1}{x+a}-\frac{1}{x+b}=\frac{1}{x+c}$, if the product of roots is zero, then sum of roots is

(a) $-\frac{2bc}{b+c}$ (b) $\frac{2ca}{c+a}$
(c) $\frac{bc}{c+a}$ (d) $\frac{-bc}{b+c}$

16. Product of real roots of the equation $t^2x^2+|x|+9=0$

(a) is always positive (b) is always negative
(c) does not exist (d) None of these

17. If p and q are the roots of the equation $x^2+px+q=0$, then
(a) $p=1, q=-2$ (b) $p=0, q=1$
(c) $p=-2, q=0$ (d) $p=-2, q=1$

18. The roots of the given equation $(p-q)x^2+(q-r)x+(r-p)=0$ are :
(a) $\frac{p-q}{r-p}, 1$ (b) $\frac{q-r}{p-q}, 1$
(c) $\frac{r-p}{p-q}, 1$ (d) None of these

19. If α, β are the roots of $ax^2+bx+c=0$, then $\alpha\beta^2+\alpha^2\beta+\alpha\beta$ equals
(a) $\frac{c(a-b)}{a^2}$ (b) 0
(c) $\frac{-bc}{a^2}$ (d) abc

20. The roots of equation $x-\frac{2}{x-1}=1-\frac{2}{x-1}$ is
(a) one (b) two
(c) infinite (d) None of these

21. If $z_1=3+i$ and $z_2=i-1$, then
(a) $|z_1+z_2|>|z_1|+|z_2|$ (b) $|z_1+z_2|<|z_1|-|z_2|$
(c) $|z_1+z_2|\le|z_1|+|z_2|$ (d) $|z_1+z_2|<|z_1|+|z_2|$

22. Let z be any complex number such that $|z|=4$ and $\arg(z)=\frac{5\pi}{6}$, then value of z is
(a) $-2\sqrt{3}-2i$ (b) $2\sqrt{3}-i$
(c) $\sqrt{2}+3i$ (d) $-2\sqrt{3}+2i$

23. If $z=\frac{3-i}{2+i}+\frac{3+i}{2-i}$, then value of arg (zi) is
(a) 0 (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

24. If the complex numbers z_1, z_2, z_3 represents the vertices of an equilateral triangle such that $|z_1|=|z_2|=|z_3|$, then value of $z_1+z_2+z_3$ is
(a) 0 (b) 1 (c) 2 (d) $\frac{3}{2}$

25. $(1+i)^8+(1-i)^8$ equal to
(a) 2^8 (b) 2^5 (c) $2^4\cos\frac{\pi}{4}$ (d) $2^8\cos\frac{\pi}{8}$

26. The conjugate of the complex number $\frac{2+5i}{4-3i}$ is equal to :
(a) $\frac{7-26i}{25}$ (b) $\frac{-7-26i}{25}$
(c) $\frac{-7+26i}{25}$ (d) $\frac{7+26i}{25}$

27. If $z=1+i$, then the multiplicative inverse of z^2 is (where, $i=\sqrt{-1}$)
(a) 2i (b) $1-i$
(c) $-\frac{i}{2}$ (d) $\frac{i}{2}$

28. $\left(\frac{1}{1-2i}+\frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$ is equal to :
(a) $\frac{1}{2}+\frac{9}{2}i$ (b) $\frac{1}{2}-\frac{9}{2}i$
(c) $\frac{1}{4}-\frac{9}{4}i$ (d) $\frac{1}{4}+\frac{9}{4}i$

29. The complex number $\left(\frac{1+2i}{1-i}\right)$ lies in:
(a) I quadrant (b) II quadrant
(c) III quadrant (d) IV quadrant

30. Amplitude of $\frac{1+\sqrt{3}i}{\sqrt{3}+1}$ is :
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

31. The value of $(1+i)^4\left(1+\frac{1}{i}\right)^4$ is
(a) 12 (b) 2
(c) 8 (d) 16

32. Evaluate: $(1+i)^6+(1-i)^3$.
(a) $-2-10i$ (b) $2-10i$
(c) $-2+10i$ (d) $2+10i$

33. If $(x+iy)^{\frac{1}{3}}=a+ib$, where $x, y, a, b \in R$, then $\frac{x}{a}-\frac{y}{b}=$
(a) a^2-b^2 (b) $-2(a^2+b^2)$
(c) $2(a^2-b^2)$ (d) a^2+b^2

34. The value of $\frac{i^{4n+1}-i^{4n-1}}{2}$. is
(a) i (b) 2i (c) $-i$ (d) $-2i$

35. $\sqrt{-3}\,\sqrt{-6}$ is equal to
(a) $3\sqrt{2}$ (b) $-3\sqrt{2}$ (c) $3\sqrt{2}i$ (d) $-3\sqrt{2}i$

36. If $z(2-i)=(3+i)$, then z^{20} is equal to
(a) 2^{10} (b) -2^{10}
(c) 2^{20} (d) -2^{20}

37. The real part of $\frac{(1+i)^2}{(3-i)}$ is
(a) $\frac{1}{3}$ (b) $\frac{1}{5}$
(c) $-\frac{1}{3}$ (d) None of these

38. The multiplicative inverse of $\frac{3+4i}{4-5i}$ is
(a) $\frac{8}{25}-\frac{31}{25}i$ (b) $-\frac{8}{25}-\frac{31}{25}i$
(c) $-\frac{8}{25}+\frac{31}{25}i$ (d) None of these

39. What is the conjugate of $\dfrac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}}$?

(a) $-3i$ (b) $3i$ (c) $\frac{3}{2}i$ (d) $-\frac{3}{2}i$

40. If $z = \dfrac{7-i}{3-4i}$, then $|z|^{14} =$

(a) 2^7 (b) $2^7 i$ (c) -2^7 (d) $-2^7 i$

41. Represent $z = 1 + i\sqrt{3}$ in the polar form.

(a) $\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$ (b) $\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$

(c) $2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ (d) $4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

42. The modulus of $\dfrac{(1+i\sqrt{3})(2+2i)}{(\sqrt{3}-i)}$ is

(a) 2 (b) 4 (c) $3\sqrt{2}$ (d) $2\sqrt{2}$

43. The argument of the complex number $\left(\frac{i}{2} - \frac{2}{i}\right)$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{12}$ (d) $\frac{\pi}{2}$

44. The square root of $(7 - 24i)$ is

(a) $\pm(3-5i)$ (b) $\pm(3+4i)$

(c) $\pm(3-4i)$ (d) $\pm(4-3i)$

45. Solve $\sqrt{5}x^2 + x + \sqrt{5} = 0$.

(a) $\pm\frac{\sqrt{19}}{5}i$ (b) $\pm\frac{\sqrt{19}i}{2}$

(c) $\dfrac{-1\pm\sqrt{19}i}{2\sqrt{5}}$ (d) $\dfrac{-1\pm\sqrt{19}i}{\sqrt{5}}$

46. If α and β are the roots of the equation $x^2 + 2x + 4 = 0$, then $\dfrac{1}{\alpha^3} + \dfrac{1}{\beta^3}$ is equal to

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 32 (d) $\frac{1}{4}$

47. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then the value of $\dfrac{1}{a\alpha+b} + \dfrac{1}{a\beta+b}$ equals

(a) $\frac{ac}{b}$ (b) 1 (c) $\frac{ab}{c}$ (d) $\frac{b}{ac}$

48. If $1 - i$, is a root of the equation $x^2 + ax + b = 0$, where $a, b \in R$, then the values of a and b are,

(a) 2, 2 (b) −2, 2 (c) −2, −2 (d) 1, 2

49. Which of the following is correct for any two complex numbers z_1 and z_2?

(a) $|z_1 z_2| = |z_1||z_2|$

(b) $\arg(z_1 z_2) = \arg(z_1) \cdot \arg(z_2)$

(c) $|z_1 + z_2| = |z_1| + |z_2|$

(d) $|z_1 + z_2| \geq |z_1| - |z_2|$

50. A number $z = a + ib$, where a and b are real numbers, is called

(a) complex number (b) real number

(c) natural number (d) integer

51. If $ax^2 + bx + c = 0$ is a quadratic equation, then equation has no real roots, if

(a) $D > 0$ (b) $D = 0$

(c) $D < 0$ (d) None of these

52. If $z = a + ib$, then real and imaginary part of z are

(a) $Re(z) = a, Im(z) = b$ (b) $Re(z) = b, Im(z) = a$

(c) $Re(z) = a, Im(z) = ib$ (d) None of these

53. Which of the following options defined 'imaginary number'?

(a) Square root of any number

(b) Square root of positive number

(c) Square root of negative number

(d) Cube root of number

54. If $x = \sqrt{-16}$, then

(a) $x = 4i$ (b) $x = 4$

(c) $x = -4$ (d) All of these

55. If $z_1 = 6 + 3i$ and $z_2 = 2 - i$, then $\dfrac{z_1}{z_2}$ is equal to

(a) $\frac{1}{5}(9+12i)$ (b) $9+12i$

(c) $3+2i$ (d) $\frac{1}{5}(12+9i)$

56. The value of $(1 + i)^5 \times (1 - i)^5$ is

(a) −8 (b) 8i (c) 8 (d) 32

57. If $z_1 = 2 - i$ and $z_2 = 1 + i$, then value of $\left|\dfrac{z_1+z_2+1}{z_1-z_2+1}\right|$ is

(a) 2 (b) 2i (c) $\sqrt{2}$ (d) $\sqrt{2}i$

58. If $\dfrac{(1+i)^3}{(1-i)^3} - \dfrac{(1-i)^3}{(1+i)^3} = x + iy$

(a) $x = 0, y = -2$ (b) $x = -2, y = 0$

(c) $x = 1, y = 1$ (d) $x = -1, y = 1$

59. Additive inverse of $1 - i$ is

(a) $0 + 0i$ (b) $-1 - i$

(c) $-1 + i$ (d) None of these

60. If z is a complex number such that $z^2 = (\bar{z})^2$, then

(a) z is purely real

(b) z is purely imaginary

(c) either z is purely real or purely imaginary

(d) None of these

61. If $|z| = 1$, $(z \neq -1)$ and $z = x + iy$, then $\left(\frac{z-1}{z+1}\right)$ is
(a) purely real (b) purely imaginary
(c) zero (d) undefined

62. If $\bar{z}$ be the conjugate of the complex number z, then which of the following relations is false?
(a) $|z| = |\bar{z}|$ (b) $z \cdot \bar{z} = |\bar{z}|^2$
(c) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$ (d) $\arg z = \arg \bar{z}$

63. If $\sqrt{a+ib} = x + iy$, then possible value of $\sqrt{a-ib}$ is
(a) $x^2 + y^2$ (b) $\sqrt{x^2+y^2}$
(c) $x + iy$ (d) $x - iy$

64. A value of k for which the quadratic equation $x^2 - 2x(1 + 3k) + 7(2k + 3) = 0$ has equal roots is
(a) 1 (b) 2 (c) 3 (d) 4

65. The roots of the equation $3^{2x} - 10.3^x + 9 = 0$ are
(a) 1, 2 (b) 0, 2 (c) 0, 1 (d) 1, 3

66. If $x^2 + y^2 = 25$, $xy = 12$, then x =
(a) $\{3, 4\}$ (b) $\{3, -3\}$
(c) $\{3, 4, -3, -4\}$ (d) $\{-3, -3\}$

67. If the roots of the equations $px^2 + 2qx + r = 0$ and $qx^2 - 2\left(\sqrt{pr}\right)x + q = 0$ be real, then
(a) $p = q$ (b) $q^2 = pr$
(c) $p^2 = qr$ (d) $r^2 = pq$

68. If $a > 0$, $b > 0$, $c > 0$, then both the roots of the equation $ax^2 + bx + c = 0$.
(a) Are real and negative (b) Have negative real parts
(c) Are rational numbers (d) None of these

69. If a and b are the odd integers, then the roots of the equation $2ax^2 + (2a + b)x + b = 0$, $a \neq 0$, will be
(a) rational (b) irrational
(c) non-real (d) equal

70. If $2 + i\sqrt{3}$ is a root of the equation $x^2 + px + q = 0$, where p and q are real, then (p, q) =
(a) (–4, 7) (b) (4, –7) (c) (4, 7) (d) (–4, –7)

71. If the sum of the roots of the equation $x^2 + px + q = 0$ is equal to the sum of their squares, then
(a) $p^2 - q^2 = 0$ (b) $p^2 + q^2 = 2q$
(c) $p^2 + p = 2q$ (d) None of these

72. If a root of the equations $x^2 + px + q = 0$ and $x^2 + \alpha x + \beta = 0$ is common, then its value will be (where $p \neq \alpha$ and $q \neq \beta$)
(a) $\frac{q-\beta}{\alpha-p}$ (b) $\frac{p\beta-\alpha q}{q-\beta}$
(c) $\frac{q-\beta}{\alpha-p}$ or $\frac{p\beta-\alpha q}{q-\beta}$ (d) None of these

73. If $x^2 + ax + 10 = 0$ and $x^2 + bx - 10 = 0$ have a common root, then $a^2 - b^2$ is equal to
(a) 10 (b) 20 (c) 30 (d) 40

74. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then
(a) $a < 2$ (b) $2 \leq a \leq 3$
(c) $3 < a \leq 4$ (d) $a > 4$

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

75. **Statement - I :** Roots of quadratic equation $x^2 + 3x + 5 = 0$ is $x = \frac{-3 \pm i\sqrt{11}}{2}$.
Statement - II : If $x^2 - x + 2 = 0$ is a quadratic equation, then its roots are $\frac{1 \pm i\sqrt{7}}{2}$.
(a) Statement I is correct (b) Statement II is correct
(c) Both are correct (d) Both are incorrect

76. **Statement - I :** Let z_1 and z_2 be two complex numbers such that $\overline{z_1} + i\,\overline{z_2} = 0$ and $\arg(z_1 \cdot z_2) = \pi$, then $\arg(z_1)$ is $\frac{3\pi}{4}$.
Statement - II : $\arg(z_1 \cdot z_2) = \arg z_1 + \arg z_2$.
(a) Statement I is correct (b) Statement II is correct
(c) Both are correct (d) Neither I nor II is correct

77. Which of the following are correct?
I. Modulus of $\frac{1+i}{1-i}$ is 1.
II. Argument of $\frac{1+i}{1-i}$ is $\frac{\pi}{2}$.
III. Modulus of $\frac{1}{1+i}$ is $\sqrt{2}$.
IV. Argument of $\frac{1}{1+i}$ is $\frac{\pi}{4}$.
(a) I and II are correct (b) III and IV are correct
(c) I, II and III are correct (d) All are correct

78. **Statement - I :** If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$, then $(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$.
Statement II : If $z = x + iy$, then $|z| = \sqrt{x^2 + y^2}$.
(a) Statement I is correct (b) Statement II is correct
(c) Both are correct (d) Neither I nor II is correct

79. Consider the following statements
I. Additive inverse of $(1 - i)$ is equal to $-1 + i$.
II. If z_1 and z_2 are two complex numbers, then $z_1 - z_2$ represents a complex number which is sum of z_1 and additive inverse of z_2.

III. Simplest form of $\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$ is $1 + 2\sqrt{2}$ i.

Choose the correct option.
(a) Only I and II are correct.
(b) Only II and III are correct.
(c) I, II and III are correct.
(d) I, II and III are incorrect.

80. Consider the following statements.
I. Representation of $z = x + iy$ in terms of r and θ is called polar form of the complex number.
II. $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

Choose the correct option.
(a) Only I is incorrect.
(b) Only II is correct.
(c) Both I and II are incorrect.
(d) Both I and II are correct.

81. Consider the following statements.
I. Let z_1 and z_2 be two complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$ then $\arg(z_1) - \arg(z_2) = 0$
II. Roots of quadratic equation $x^2 + 3x + 5 = 0$ is $x = \frac{-3 \pm i\sqrt{11}}{2}$

Choose the correct option.
(a) Only I is correct. (b) Only II is correct.
(c) Both are correct. (d) Neither I nor II is correct.

82. Consider the following statements.
I. The value of $x^3 + 7x^2 - x + 16$, when $x = 1 + 2i$ is $-17 + 24i$.
II. If $iz^3 + z^2 - z + i = 0$ then $|z| = 1$

Choose the correct option.
(a) Only I is correct. (b) Only II is correct.
(c) Both are correct. (d) Both are incorrect.

83. Consider the following statements.
I. If z, z_1, z_2 be three complex numbers then $z\bar{z} = |z|^2$
II. The modulus of a complex number $z = a + ib$ is defined as $|z| = \sqrt{a^2 + b^2}$.
III. Multiplicative inverse of $z = 3 - 2i$ is $\frac{3}{13} + \frac{2}{13}i$

Choose the correct option.
(a) Only I and II are correct.
(b) Only II and III are correct.
(c) Only I and III are correct.
(d) All I, II and III are correct.

84. Consider the following statements.
I. Modulus of $\frac{1+i}{1-i}$ is 1.
II. Argument of $\frac{1+i}{1-i}$ is $\frac{\pi}{2}$.

Choose the correct option.
(a) Only I is correct. (b) Only II is correct.
(c) Both are correct. (d) Both are incorrect.

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

85.

Column - I (Complex Nos.)	**Column - II (Multiplicative inverse)**
(A) $4 - 3i$	(1) $\frac{\sqrt{5}}{14} - i\frac{3}{14}$
(B) $\sqrt{5} + 3i$	(2) $\frac{4}{25} + i\frac{3}{25}$
(C) $-i$	(3) $0 + i$

Codes:

	A	B	C
(a)	1	2	3
(b)	2	1	3
(c)	1	3	2
(d)	2	3	1

86. Simplify the complex numbers given in column-I and match with column-II.

Column - I	**Column - II**
(A) $(1-i)^4$	(1) $-\left(\frac{22}{3} + i\frac{107}{27}\right)$
(B) $\left(\frac{1}{3} + 3i\right)^3$	(2) $-4 + 0i$
(C) $\left(-2 - \frac{1}{3}i\right)^3$	(3) $-\frac{242}{27} - 26i$

Codes:

	A	B	C
(a)	1	2	3
(b)	2	1	3
(c)	3	1	2
(d)	2	3	1

87.

Column - I	**Column -II**
(A) i^{-1}	(1) -1
(B) i^{-2}	(2) $-i$
(C) i^{-3}	(3) i
(D) i^4	(4) 1

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	2	1	3	4
(c)	2	3	4	1
(d)	1	4	3	2

88.

Column - I (Complex Number)	Column - II (a + ib form)
(A) $(1-i)-(-1+6i)$	(1) $-\frac{21}{5}-\frac{21}{10}i$
(B) $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$	(2) -4
(C) $\left(\frac{1}{3}+3i\right)^3$	(3) $-\frac{22}{3}-\frac{107}{27}i$
(D) $(1-i)^4$	(4) $2-7i$
(E) $\left(-2-\frac{1}{3}i\right)^3$	(5) $\frac{-242}{27}-26i$

Codes:

	A	B	C	D	E
(a)	5	4	3	2	1
(b)	4	1	5	2	3
(c)	4	2	5	1	3
(d)	3	1	2	5	4

89.

Column – I (Complex Number)	Column - II (Multiplicative Inverse)
(A) $4-3i$	(1) $\frac{\sqrt{5}}{14}-\frac{3}{14}i$
(B) $\sqrt{5}+3i$	(2) $\frac{1}{49}-\frac{4\sqrt{3}\,i}{49}$
(C) $-i$	(3) $0+i.1$
(D) $\left(2+\sqrt{3}\,i\right)^2$	(4) $\frac{4}{25}+i\frac{3}{25}$

Codes:

	A	B	C	D
(a)	3	2	1	4
(b)	4	3	1	2
(c)	2	1	3	4
(d)	4	1	3	2

90.

Column - I (Quadratic Equation)	Column - II (Roots)
(A) $2x^2+x+1=0$	(1) $\frac{1\pm\sqrt{7}\,i}{2}$
(B) $x^2+3x+9=0$	(2) $\frac{-1\pm\sqrt{7}\,i}{4}$
(C) $-x^2+x-2=0$	(3) $\frac{-3\pm\sqrt{11}\,i}{2}$
(D) $x^2+3x+5=0$	(4) $\frac{-3\pm\left(3\sqrt{3}\right)i}{2}$

Codes:

	A	B	C	D
(a)	3	1	4	2
(b)	3	4	1	2
(c)	2	4	1	3
(d)	2	1	4	3

91.

Column - I (Complex Number)	Column - II (Polar form)
(A) $(1-i)$	(1) $\sqrt{2}\left[\cos\left(\frac{-3\pi}{4}\right)+i\sin\left(\frac{-3\pi}{4}\right)\right]$
(B) $(-1+i)$	(2) $2\left[\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right]$
(C) $(-1-i)$	(3) $\sqrt{2}\left[\cos\left(\frac{-\pi}{4}\right)+i\sin\left(\frac{-\pi}{4}\right)\right]$
(D) $\left(\sqrt{3}+i\right)$	(4) $\sqrt{2}\left[\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right]$

Codes:

	A	B	C	D
(a)	3	4	1	2
(b)	3	1	4	2
(c)	2	4	1	3
(d)	2	1	4	3

92.

Column - I	Column - II
(A) If $z = x + iy$, then modulus z is	(1) $a + i\,0$
(B) The modulus of complex number $x + iy$ is	(2) $0 + bi$
(C) Complex numbers which lie on x-axis are in the form of	(3) $\sqrt{x^2+y^2}$
(D) Complex numbers which lie on y-axis are in the form of	(4) distance of the point from the origin

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	4	3	1	2
(c)	4	1	3	2
(d)	2	3	1	4

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

93. $i^{57}+\frac{1}{i^{25}}$, when simplified has the value

(a) 0 (b) $2i$
(c) $-2i$ (d) 2

94. If $z = 2 - 3i$, then the value of $z^2 - 4z + 13$ is

(a) 1 (b) –1 (c) 0 (d) None of these

95. If $\frac{c+i}{c-i} = a + ib$, where a, b, c are real, then $a^2 + b^2$ is equal to:

(a) 7 (b) 1 (c) c^2 (d) $-c^2$

96. If $x + iy = \frac{a + ib}{a - ib}$, then $x^2 + y^2 =$

(a) 1 (b) 2 (c) 0 (d) 4

97. If $z = x + iy$, $z^{\frac{1}{3}} = a - ib$ and $\frac{x}{a} - \frac{y}{b} = k(a^2 - b^2)$, then value of k equals

(a) 2 (b) 4 (c) 6 (d) 1

98. $2x^2 - (p + 1)x + (p - 1) = 0$. If $\alpha - \beta = \alpha\beta$, then what is the value of p?

(a) 1 (b) 2 (c) 3 (d) –2

99. If $z_1 = 2 + 3i$ and $z_2 = 3 + 2i$, then $z_1 + z_2$ equals to $a + ai$. Value of 'a' is equal to

(a) 3 (b) 4 (c) 5 (d) 2

100. If $z_1 = 2 + 3i$ and $z_2 = 3 - 2i$, then $z_1 - z_2$ equals to $-1 + bi$. The value of 'b' is

(a) 1 (b) 2 (c) 3 (d) 5

101. If $z = 5i\left(\frac{-3}{5}i\right)$, then z is equal to $3 + bi$. The value of 'b' is

(a) 1 (b) 2 (c) 0 (d) 3

102. If $z_1 = 6 + 3i$ and $z_2 = 2 - i$, then $\frac{z_1}{z_2}$ is equal to $\frac{1}{a}(9 + 12i)$. The value of 'a' is

(a) 1 (b) 2 (c) 4 (d) 5

103. Value of $i^{4k} + i^{4k+1} + i^{4k+2} + i^{4k+3}$ is

(a) 0 (b) 1 (c) 2 (d) 3

104. If $z = i^9 + i^{19}$, then z is equal to $a + ai$. The value of 'a' is

(a) 0 (b) 1 (c) 2 (d) 3

105. If $z = i^{-39}$, then simplest form of z is equal to $a + i$. The value of 'a' is

(a) 0 (b) 1 (c) 2 (d) 3

106. If $(1 - i)^n = 2^n$, then the value of n is

(a) 1 (b) 2
(c) 0 (d) None of these

107. The value of $(1 + i)^5 (1 - i)^5$ is 2^n. 'n' is equal to

(a) 2 (b) 3 (c) 4 (d) 5

108. The value of $(1 + i)^8 + (1 - i)^8$ is 2^n. Value of n is

(a) 2 (b) 3 (c) 4 (d) 5

109. Roots of $x^2 + 2 = 0$ are $\pm\sqrt{n}\, i$. The value of n is

(a) 1 (b) 2 (c) 3 (d) 4

110. If $z_1 = \sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$ and $z_2 = \sqrt{3}\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$, then $|z_1 z_2|$ is equal to $\sqrt{m}$. Value of m is

(a) 6 (b) 3 (c) 2 (d) 5

111. The modulus of $\sqrt{2i} - \sqrt{-2i}$ is:

(a) 2 (b) $\sqrt{2}$ (c) 0 (d) $2\sqrt{2}$

112. If $z = x + iy$, $z^{1/3} = a - ib$, then $\frac{x}{a} - \frac{y}{b} = k(a^2 - b^2)$ where k is equal to

(a) 1 (b) 2 (c) 3 (d) 4

113. If the equations $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k(2x^2 - 1) + px + 4x^2 + 2 = 0$ have both roots common, then the value of $(2r - p)$ is :

(a) 0 (b) 1/2
(c) 1 (d) None of these

114. $\arg \bar{z} + \arg z; z \neq 0$ is equal to :

(a) $\frac{\pi}{4}$ (b) π (c) 0 (d) $\frac{\pi}{2}$

115. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1 - \arg z_2$ is equal to

(a) $\frac{\pi}{2}$ (b) $-\pi$ (c) 0 (d) $\frac{-\pi}{2}$

116. If $z = 2 - 3i$, then value of $z^2 - 4z + 13$ is

(a) 0 (b) 1 (c) 2 (d) 3

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reason is correct.

117. Assertion : Let f(x) be a quadratic expression such that $f(0) + f(1) = 0$. If –2 is one of the root of $f(x) = 0$, then other root is $\frac{3}{5}$.

Reason : If α and β are the zeroes of $f(x) = ax^2 + bx + c$, then sum of zeroes $= -\frac{b}{a}$, product of zeroes $= \frac{c}{a}$.

118. Assertion : If $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$, then $\frac{z_1}{z_2}$ is purely imaginary.

Reason : If z is purely imaginary, then $z + \bar{z} = 0$.

119. Assertion : The greatest integral value of λ for which $(2\lambda - 1)x^2 - 4x + (2\lambda - 1) = 0$ has real roots, is 2.

Reason : For real roots of $ax^2 + bx + c = 0$, $D \geq 0$.

120. Assertion : Consider z_1 and z_2 are two complex numbers such that $|z_1| = |z_2| + |z_1 - z_2|$, then $\mathrm{Im}\left(\frac{z_1}{z_2}\right) = 0$.

Reason : $\arg(z) = 0 \Rightarrow z$ is purely real.

121. Assertion : If P and Q are the points in the plane XOY representing the complex numbers z_1 and z_2 respectively, then distance $|PQ| = |z_2 - z_1|$.

Reason : Locus of the point P(z) satisfying $|z - (2 + 3i)| = 4$ is a straight line.

122. Assertion : The equation $ix^2 - 3ix + 2i = 0$ has non-real roots.

Reason : If a, b, c are real and $b^2 - 4ac \geq 0$, then the roots of the equation $ax^2 + bx + c = 0$ are real and if $b^2 - 4ac < 0$, then roots of $ax^2 + bx + c = 0$ are non-real.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

123. If $|z-4|<|z-2|$, its solution is given by
(a) $Re(z)>0$ (b) $Re(z)<0$
(c) $Re(z)>3$ (d) $Re(z)>2$

124. The equation whose roots are twice the roots of the equation, $x^2-3x+3=0$ is:
(a) $4x^2+6x+3=0$ (b) $2x^2-3x+3=0$
(c) $x^2-3x+6=0$ (d) $x^2-6x+12=0$

125. The roots of the equation $4^x-3\ .\ 2^{x+3}+128=0$ are
(a) 4 and 5 (b) 3 and 4
(c) 2 and 3 (d) 1 and 2

126. If one root of the equation $x^2+px+12=0$ is 4, while the equation $x^2+px+q=0$ has equal roots , then the value of 'q' is
(a) 4 (b) 12
(c) 3 (d) $\frac{49}{4}$

127. For the equation $3x^2+px+3=0, p>0$, if one of the root is square of the other, then p is equal to
(a) 1/3 (b) 1
(c) 3 (d) 2/3

128. Value of $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}-1$ is
(a) –2 (b) 0 (c) –1 (d) 1

129. Modulus of $z=\frac{(1+i\sqrt{3})(\cos\theta+i\sin\theta)}{2(1-i)(\cos\theta-i\sin\theta)}$ is
(a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

130. The modulus and amplitude of $\frac{1+2i}{1-(1-i)^2}$ are
(a) $\sqrt{2}$ and $\frac{\pi}{6}$ (b) 1 and 0
(c) 1 and $\frac{\pi}{3}$ (d) 1 and $\frac{\pi}{4}$

131. If $|z^2-1|=|z|^2+1$, then z lies on
(a) imaginary axis (b) real axis
(c) origin (d) None of these

132. If $z=2+i$, then $(z-1)(\bar{z}-5)+(\bar{z}-1)(z-5)$ is equal to
(a) 2 (b) 7
(c) –1 (d) –4

133. If $z=r(\cos\theta+i\sin\theta)$, then the value of $\frac{z}{\bar{z}}+\frac{\bar{z}}{z}$ is
(a) $\cos 2\theta$ (b) $2\cos 2\theta$
(c) $2\cos\theta$ (d) $2\sin\theta$

134. The square root of i is
(a) $\pm\frac{1}{\sqrt{2}}(-1+i)$ (b) $\pm\frac{1}{\sqrt{2}}(1+i)$
(c) $\pm\frac{1}{\sqrt{2}}(1-i)$ (d) None of these

135. The number of real roots of $\left(x+\frac{1}{x}\right)^3+\left(x+\frac{1}{x}\right)=0$ is
(a) 0 (b) 2
(c) 4 (d) 6

136. If the roots of the equation $\frac{a}{x-a}+\frac{b}{x-b}=1$ are equal in magnitude and opposite in sign, then
(a) $a=b$ (b) $a+b=1$
(c) $a-b=1$ (d) $a+b=0$

137. Find the value of a such that the sum of the squares of the roots of the equation $x^2-(a-2)x-(a+1)=0$ is least.
(a) 4 (b) 2
(c) 1 (d) 3

138. If α, β are the roots of the equation $(x-a)(x-b)=5$, then the roots of the equation $(x-\alpha)(x-\beta)+5=0$ are
(a) a, 5 (b) b, 5
(c) a, α (d) a, b

139. The complex number z which satisfies the condition $\left|\frac{i+z}{i-z}\right|=1$ lies on
(a) circle $x^2+y^2=1$ (b) the x-axis
(c) the y-axis (d) the line $x+y=1$

140. The value of $(z+3)(\bar{z}+3)$ is equivalent to
(a) $|z+3|^2$ (b) $|z-3|$
(c) z^2+3 (d) None of these

141. $|z_1+z_2|=|z_1|+|z_2|$ is possible, if
(a) $z_2=\bar{z}_1$ (b) $z_2=\frac{1}{z_1}$
(c) $\arg(z_1)=\arg(z_2)$ (d) $|z_1|=|z_2|$

142. $\sin x+i\cos 2x$ and $\cos x-i\sin 2x$ are conjugate to each other for
(a) $x=n\pi$ (b) $x=\left(n+\frac{1}{2}\right)\frac{\pi}{2}$
(c) $x=0$ (d) No value of x

143. The modulus of the complex number z such that $|z+3-i|=1$ and $\arg(z)=\pi$ is equal to
(a) 3 (b) 2
(c) 9 (d) 4

144. If $Z=\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$, then polar form of Z is
(a) $\sqrt{2}\left(\cos\frac{5\pi}{12}-i\sin\frac{5\pi}{12}\right)$ (b) $\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$
(c) $\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ (d) $\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)$

145. $(x - iy)(3 + 5i)$ is the conjugate of $(-6 - 24i)$, then x and y are
(a) $x = 3, y = -3$ (b) $x = -3, y = 3$
(c) $x = -3, y = -3$ (d) $x = 3, y = 3$

146. If z is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then
(a) $|z| = 0$ (b) $|z| = 1$
(c) $|z| > 1$ (d) $|z| < 1$

147. The amplitude of $\sin\frac{\pi}{5} + i\left(1 - \cos\frac{\pi}{5}\right)$ is
(a) $\frac{\pi}{5}$ (b) $\frac{2\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $\frac{\pi}{15}$

148. If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, then $(x^2 + y^2)^2 =$
(a) $\frac{a^2+b^2}{c^2+d^2}$ (b) $\frac{a+b}{c+d}$
(c) $\frac{c^2+d^2}{a^2+b^2}$ (d) $\left(\frac{a^2+b^2}{c^2+d^2}\right)^2$

149. If the equation $(m - n)x^2 + (n - l)x + l - m = 0$ has equal roots, then l, m and n satisfy
(a) $2l = m + n$ (b) $2m = n + l$
(c) $m = n + l$ (d) $l = m + n$

150. If the product of the roots of the equation $(a + 1)x^2 + (2a + 3)x + (3a + 4) = 0$ be 2, then the sum of roots is
(a) 1 (b) –1
(c) 2 (d) –2

151. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then $\frac{\alpha}{a\beta+b} + \frac{\beta}{a\alpha+b} =$
(a) $\frac{2}{a}$ (b) $\frac{2}{b}$ (c) $\frac{2}{c}$ (d) $-\frac{2}{a}$

152. If one root of $ax^2 + bx + c = 0$ be square of the other, then the value of $b^3 + ac^2 + a^2c$ is
(a) 3abc (b) –3abc
(c) 0 (d) None of these

153. If α, β are the roots of $(x - a)(x - b) = c$, $c \neq 0$, then the roots of $(x - \alpha)(x - \beta) + c = 0$ shall be
(a) a, c (b) b, c
(c) a, b (d) a + c, b + c

154. If the roots of the equation $ax^2 + bx + c = 0$ are α, β, then the value of $\alpha\beta^2 + \alpha^2\beta + \alpha\beta$ will be
(a) $\frac{c(a-b)}{a^2}$ (b) 0
(c) $-\frac{bc}{a^2}$ (d) None of these

155. If α, β be the roots of the equation $2x^2 - 35x + 2 = 0$, then the value of $(2\alpha - 35)^3 \cdot (2\beta - 35)^3$ is equal to
(a) 1 (b) 64
(c) 8 (d) None of these

156. If the sum of the roots of the equation $x^2 + px + q = 0$ is three times their difference, then which one of the following is true?
(a) $9p^2 = 2q$ (b) $2q^2 = 9p$
(c) $2p^2 = 9q$ (d) $9q^2 = 2p$

157. If the ratio of the roots of $x^2 + bx + c = 0$ and $x^2 + qx + r = 0$ be the same, then
(a) $r^2c = b^2q$ (b) $r^2b = c^2q$
(c) $rb^2 = cq^2$ (d) $rc^2 = bq^2$

158. If the roots of the equation $x^2 - 5x + 16 = 0$ are α, β and the roots of equation $x^2 + px + q = 0$ are $\alpha^2 + \beta^2, \frac{\alpha\beta}{2}$, then
(a) $p = 1, q = -56$ (b) $p = -1, q = -56$
(c) $p = 1, q = 56$ (d) $p = -1, q = 56$

159. If A.M. of the roots of a quadratic equation is $\frac{8}{5}$ and A.M. of their reciprocals is $\frac{8}{7}$, then the equation is
(a) $5x^2 - 16x + 7 = 0$ (b) $7x^2 - 16x + 5 = 0$
(c) $7x^2 - 16x + 8 = 0$ (d) $3x^2 - 12x + 7 = 0$

160. If the roots of $4x^2 + 5k = (5k + 1)x$ differ by unity, then the negative value of k is
(a) –3 (b) –5
(c) $-\frac{1}{5}$ (d) $-\frac{3}{5}$

161. Sum of all real roots of the equation $|x-2|^2 + |x-2| - 2 = 0$ is
(a) 2 (b) 4 (c) 5 (d) 6

162. If $|z+4| \leq 3$, then the maximum value of $|z+1|$ is
(a) 6 (b) 0 (c) 4 (d) 10

163. Value of $\frac{(\cos\theta + i\sin\theta)^4}{(\cos\theta - i\sin\theta)^3}$ is
(a) $\cos 5\theta + i\sin 5\theta$ (b) $\cos 7\theta + i\sin 7\theta$
(c) $\cos 4\theta + i\sin 4\theta$ (d) $\cos\theta + i\sin\theta$

164. The value of $2 + \cfrac{1}{2 + \cfrac{1}{2 + \ldots\ldots\ldots\ldots \infty}}$ is
(a) $1 - \sqrt{2}$ (b) $1 + \sqrt{2}$
(c) $1 \pm \sqrt{2}$ (d) None of these

165. If $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots\ldots \text{ to } \infty}}}$, then
(a) x is an irrational number
(b) $2 < x < 3$
(c) $x = 3$
(d) None of these

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(b)** $\left(\frac{2i}{1+i}\right)^2 = \frac{4i}{1+i^2+2i} = \frac{-4}{1-1+2i} = \frac{-4}{2i}$

$= \frac{-2}{i} = 2i \left(\because \frac{1}{i} = -i\right)$

2. **(b)** $\frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = \frac{(1-i)^2}{1-i^2} = \frac{1+i^2-2i}{2} = -i$

$\therefore\ (-i)^{100} = (i)^{100} = (i^4)^{25} = 1$

$\Rightarrow 1 = a + ib$

$\Rightarrow a = 1, b = 0$

3. **(d)** Given expression $= 1 + i^2 + i^4 + ... + i^{2n}$
$= 1 - 1 + 1 - 1 + ... + (-1)^n$, which cannot be determined unless n is known.

4. **(b)** $x + iy = \frac{4+i}{2-3i} = \frac{(4+i)(2+3i)}{13} = \frac{5+14i}{13}$

$\therefore\ x = 5/13,\ y = 14/13$

5. **(c)** We have, $4x + i(3x - y) = 3 + i(-6)$
Now, equating the real and the imaginary parts of above equation, we get
$4x = 3$ and $3x - y = -6$

$\Rightarrow\ x = \frac{3}{4}$ and $3 \times \frac{3}{4} - y = -6$

or $\frac{9}{4} + 6 = y\ \Rightarrow\ \frac{9+24}{4} = y$

$\therefore\ \ y = \frac{33}{4}$

hence, $x = \frac{3}{4}$ and $y = \frac{33}{4}$

6. **(a)** $z^{\frac{1}{3}} = p + iq$

$\Rightarrow z = p^3 + (iq)^3 + 3p(iq)(p + iq)$

$\Rightarrow x - iy = p^3 - 3pq^2 + i(3p^2q - q^3)$

$\therefore x = p^3 - 3pq^2 \Rightarrow \frac{x}{p} = p^2 - 3q^2$

$y = q^3 - 3p^2q \Rightarrow \frac{y}{q} = q^2 - 3p^2$

$\therefore \frac{x}{p} + \frac{y}{q} = -2p^2 - 2q^2$

$\therefore \left(\frac{x}{p} + \frac{y}{q}\right) \Big/ (p^2 + q^2) = -2$

7. **(b)** $z = (i^{25})^3 = (i)^{75} = i^{4 \times 18 + 3} = (i^4)^{18}(i)^3$
$= i^3 = -i = 0 - i$
Polar form of $z = r(\cos\theta + i\sin\theta)$

$= 1\left\{\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right\}$

$= \cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$

8. **(a)** $\frac{z_1}{z_2} = \frac{\sqrt{3}+i\sqrt{3}}{\sqrt{3}+i} = \left(\frac{3+\sqrt{3}}{4}\right) + \left(\frac{3-\sqrt{3}}{4}\right)i$

which is represented by a point in first quadrant.

9. **(c)** Quadratic equation

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ or $x = \frac{-b \pm \sqrt{4ac - b^2}i}{2a}$

10. **(b)** $b^2 - 4ac = 1^2 - 4 \times 1 \times 1$
$[\because\ a = 1, b = 1, c = 1]$
$b^2 - 4ac = 1 - 4 = -3$
$\therefore$ the solutions are given by

$x = \frac{-1 \pm \sqrt{-3}}{2 \times 1} = \frac{-1 \pm \sqrt{3}i}{2}$

11. **(d)** Given $\sqrt{3x^2 - 2} = 2x - 1$
squaring both the sides
$\Rightarrow\ 3x^2 - 2 = 4x^2 + 1 - 4x \Rightarrow\ x^2 - 4x + 3 = 0$
$\Rightarrow\ (x - 3)(x - 1) = 0 \Rightarrow\ x = 1, 3.$

12. **(c)** Let $\alpha + 3 = x$
$\therefore\ \alpha = x - 3$ (replace x by $x - 3$)
So the required equation
$(x - 3)^2 - 5(x - 3) + 6 = 0$
$\Rightarrow x^2 - 6x + 9 - 5x + 15 + 6 = 0$
$\Rightarrow x^2 - 11x + 30 = 0$

13. **(b)** Let α be the common root
$\therefore\ \ 2\alpha^2 + k\alpha - 5 = 0$
$\alpha^2 - 3\alpha - 4 = 0$
Solving both equations

$\frac{\alpha^2}{-4k - 15} = \frac{\alpha}{-5 + 8} = \frac{1}{-6 - k}$

$\Rightarrow\ \alpha^2 = \frac{4k + 15}{k + 6}$ and $\alpha = \frac{-3}{k + 6}$

$\Rightarrow \left(\frac{-3}{k+6}\right)^2 = \frac{4k+15}{k+6}$

$\Rightarrow (4k+15)(k+6) = 9$

$\Rightarrow 4k^2 + 39k + 81 = 0$

$\Rightarrow k = -3$ or $k = -27/4$

14. (c) Here, $3x^2 - (a+c+2b+2d)x + (ac+2bd) = 0$

$\therefore D = (a+c+2b+2d)^2 - 12(ac+2bd)$

$= [(a+2d)-(c+2b)]^2 + 4(a+2d)(c+2b) - 12(ac+2bd)$

$= [(a+2d)-(c+2b)]^2 + 8(c-b)(d-a) > 0.$

Hence roots are real and unequal.

15. (a) $\frac{1}{x+a} - \frac{1}{x+b} = \frac{1}{x+c}$

$\frac{b-a}{x^2+(b+a)x+ab} = \frac{1}{x+c}$

or $x^2 + (a+b)x + ab = (b-a)x + (b-a)c$

or $x^2 + 2ax + ab + ca - bc = 0$

Since product of the roots = 0

$ab + ca - bc = 0$

$a = \frac{bc}{b+c}.$

Thus, sum of roots $= -2a = \frac{-2bc}{b+c}$

16. (a) Product of real roots $= \frac{9}{t^2} > 0,\ \forall\ t \in R$

$\therefore$ Product of real roots is always positive.

17. (a) $p + q = -p$ and $pq = q \Rightarrow q(p-1) = 0$

$\Rightarrow q = 0$ or $p = 1$.

If $q = 0$, then $p = 0$. i.e. $p = q$

$\therefore p = 1$ and $q = -2$.

18. (c) Given equation is

$(p-q)x^2 + (q-r)x + (r-p) = 0$

By using formula for finding the roots

viz : $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$, we get

$$x = \frac{(r-q) \pm \sqrt{(q-r)^2 - 4(r-p)(p-q)}}{2(p-q)}$$

$$\Rightarrow x = \frac{(r-q) \pm (q+r-2p)}{2(p-q)} = \frac{r-p}{p-q}, 1$$

19. (a) Given. $ax^2 + bx + c = 0$ and α, β are roots of given equation

$\therefore \alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$ ...... (i)

Now, $\alpha\beta^2 + \alpha^2\beta + \alpha\beta = \alpha\beta(\beta + \alpha) + \alpha\beta$

$= \frac{c}{a} \cdot \left(-\frac{b}{a}\right) + \frac{c}{a}$ [Using equation (i)]

$= -\frac{cb}{a^2} + \frac{c}{a}$

$= \frac{-cb+ac}{a^2} = \frac{c(a-b)}{a^2}$

20. (b) Consider the given equation

$x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$

By taking L.C.M, we get

$\frac{x(x-1)-2}{x-1} = \frac{x-1-2}{x-1}$

$\Rightarrow x(x-1) - 2 = x - 3$

$\Rightarrow x^2 - x - 2 = x - 3$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x-1)^2 = 0$

$\Rightarrow x = 1, 1$

Thus, the given equation has two roots.

21. (d) $z_1 + z_2 = 2 + 2i$

$\Rightarrow |z_1 + z_2| = \sqrt{4+4} = \sqrt{8}$

Now $|z_1| = \sqrt{10}$, $|z_2| = \sqrt{2}$.

It is clear that, $|z_1 + z_2| < |z_1| + |z_2|$

22. (d) Let $z = r(\cos\theta + i\sin\theta)$. Then $r = 4$, $\theta = \frac{5\pi}{6}$

$\therefore z = 4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

$= 4\left(-\frac{\sqrt{3}}{2} + \frac{i}{2}\right) = -2\sqrt{3} + 2i$

23. (d) $z = \frac{3-i}{2+i} + \frac{3+i}{2-i} = \frac{(3-i)(2-i)+(3+i)(2+i)}{(2+i)(2-i)}$

$\Rightarrow z = 2 \Rightarrow (iz) = 2i$, which is the positive imaginary quantity

$\therefore \arg(iz) = \frac{\pi}{2}$

24. (a) Let the complex number z_1, z_2, z_3 denote the vertices A, B, C of an equilateral triangle ABC. Then, if O be the origin we have $OA = z_1$, $OB = z_2$, $OC = z_3$,

Therefore $|z_1| = |z_2| = |z_3| \Rightarrow OA = OB = OC$

i.e. O is the circumcentre of ΔABC

Hence $z_1 + z_2 + z_3 = 0$.

25. (b) $(1+i)^8 + (1-i)^8$

$= \{(1+i)^2\}^4 + \{(1-i)^2\}^4$

$= \{1+2i+i^2\}^4 + \{1-2i+i^2\}^4$

$= (1+2i-1)^4 + (1-2i-1)^4$

$= 2^4 . i^4 + (-2)^4 . i^4$

$= 2^4 + 2^4$ [Since $i^4 = 1$]

$= 2 \times 2^4$

$= 2^5$

26. (c) Let $z=\frac{2+5i}{4-3i}$ Rationalize,

$=\frac{2+5i}{4-3i}\times\frac{4+3i}{4+3i}$

$=\frac{8+26i-15}{(4)^2-(3i)^2}=\frac{8+26i-15}{16+(9)}$ $(\because i^2=-1)$

$=\frac{-7+26i}{16+9}=\frac{-7+26i}{25}$

27. (c) Let $z=1+i$

then $z^2=(1+i)^2$

$=1^2+i^2+2.1.i$

$=1+i^2+2i$

$=1-1+2i$ $(\because i^2=-1)$

$=2i$

Now, $2i\times-\frac{i}{2}\Rightarrow -i^2=1$

Hence, $-\frac{i}{2}$ is multiplicative inverse of z^2.

28. (d) Let $z=\left(\frac{1}{1-2i}+\frac{2}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$

$=\left[\frac{1+i+3-6i}{(1-2i)(1+i)}\right]\left[\frac{3+4i}{2-4i}\right]$

$=\left[\frac{4-5i}{3-i}\right]\left[\frac{3+4i}{2-4i}\right]=\left[\frac{32+i}{2-14i}\right]$

$=\frac{32+i}{2-14i}\times\frac{2+14i}{2+14i}=\frac{64+448i+2i-14}{4+196}$

$=\frac{50+450i}{200}=\frac{1}{4}+\frac{9}{4}i$

29. (b) Let $z=\frac{1+2i}{1-i}$ be the given complex number.

$\Rightarrow z=\frac{1+2i}{1-i}\times\frac{1+i}{1+i}=\frac{1+i+2i+2i^2}{1-i^2}$

$=\frac{-1+3i}{2}=\frac{-1}{2}+\frac{3}{2}i$

$\Rightarrow (x,y)=\left(\frac{-1}{2},\frac{3}{2}\right)$ which lies in IInd quadrant.

30. (c) Let $r(\cos\theta+i\sin\theta)=\frac{1+i\sqrt{3}}{\sqrt{3}+1}=\frac{1}{\sqrt{3}+1}+i\frac{\sqrt{3}}{\sqrt{3}+1}$

$\Rightarrow r\cos\theta=\frac{1}{\sqrt{3}+1};\ r\sin\theta=\frac{\sqrt{3}}{\sqrt{3}+1}$

$\Rightarrow \tan\theta=\sqrt{3}\Rightarrow\theta=\frac{\pi}{3}.$

31. (d) $(1+i)^4\times\left(1+\frac{1}{i}\right)^4=(1+i)^4\times(1-i)^4$

$=(1-i^2)^4=(1+1)^4=2^4=16.$

32. (a) $(1+i)^6=\{(1+i)^2\}^3=(1+i^2+2i)^3=(1-1+2i)^3$

$=8i^3=-8i$ and

$(1-i)^3=1-i^3-3i+3i^2$

$=1+i-3i-3=-2-2i$

$\therefore$ $(1+i)^6+(1-i)^3=-8i-2-2i=-2-10i$

33. (b) $(x+iy)^{\frac{1}{3}}=a+ib$

$\Rightarrow x+iy=(a+ib)^3$

$\Rightarrow x+iy=a^3-ib^3+i3a^2b-3ab^2$

$=a^3-3ab^2+i(3a^2b-b^3)$

$\Rightarrow x=a^3-3ab^2$ and $y=3a^2b-b^3$

So, $\frac{x}{a}-\frac{y}{b}=a^2-3b^2-3a^2+b^2$

$=-2a^2-2b^2=-2(a^2+b^2)$

34. (a) $\frac{i^{4n+1}-i^{4n-1}}{2}=\frac{i^{4n}i-i^{4n}i^{-1}}{2}$

$=\frac{i-\frac{1}{i}}{2}=\frac{i^2-1}{2i}=\frac{-2}{2i}=i$

35. (b) $\sqrt{-3}=i\sqrt{3},\ \sqrt{-6}=i\sqrt{6}$

So, $\sqrt{(-3)}\sqrt{(-6)}=i^2\,3\sqrt{2}=-3\sqrt{2}$

36. (b) We have, $z(2-i)=(3+i)$

$\Rightarrow z=\left(\frac{3+i}{2-i}\right)\times\left(\frac{2+i}{2+i}\right)=\frac{5+5i}{5}$

$\Rightarrow z=1+i$

$\Rightarrow z^2=2i\Rightarrow z^{20}=-2^{10}$

37. (d) $(1+i)^2=1+i^2+2i=2i$

$\therefore$ $\frac{(1+i)^2}{3-i}=\frac{2i(3+i)}{3^2-i^2}=\frac{6i-2}{10}=\frac{-1+3i}{5}$

$\therefore$ Real part $=\frac{-1}{5}$.

38. (b) Let $z=\frac{3+4i}{4-5i}\times\frac{4+5i}{4+5i}=-\frac{8}{41}+\frac{31}{41}i$

Then, $\bar{z}=-\frac{8}{41}-\frac{31}{41}i$

and $|z|=\sqrt{\left(-\frac{8}{41}\right)^2+\left(\frac{31}{41}\right)^2}=\frac{5}{\sqrt{41}}$

$\therefore$ Multiplicative inverse of z

$=\frac{\bar{z}}{|z|^2}=\frac{-\frac{8}{41}-\frac{31}{41}i}{\frac{25}{41}}=-\frac{8}{25}-\frac{31}{25}i$

39. (c) Let $z = \dfrac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}} \times \dfrac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}+\sqrt{5-12i}}$

$= \dfrac{5+12i+5-12i+2\sqrt{25+144}}{5+12i-5+12i}$

$= \dfrac{3}{2i} = \dfrac{3i}{-2} = 0 - \dfrac{3}{2}i$

Therefore, the conjugate of $z = 0 + \dfrac{3}{2}i$

40. (a) $z = \dfrac{7-i}{3-4i} = \dfrac{7-i}{3-4i} \times \dfrac{3+4i}{3+4i} = \dfrac{21+4+i(28-3)}{25}$

$= 1 + i$

$\therefore \ |z| = |1+i| = \sqrt{2}$

$\therefore \ |z|^{14} = \left(\sqrt{2}\right)^{14} = \left[\left(\sqrt{2}\right)^2\right]^7 = 2^7$

41. (c) Let $1 = r\cos\theta$, $\sqrt{3} = r\sin\theta$

By squaring and adding, we get

$r^2(\cos^2\theta + \sin^2\theta) = 4$

i.e, $r = \sqrt{4} = 2$

Therefore, $\cos\theta = \dfrac{1}{2}$, $\sin\theta = \dfrac{\sqrt{3}}{2}$, which gives $\theta = \dfrac{\pi}{3}$

Therefore, required polar form is

$z = 2\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)$.

42. (d) $\dfrac{\left(1+i\sqrt{3}\right)(2+2i)}{\sqrt{3}-i} = \dfrac{2+2\sqrt{3}i+2i-2\sqrt{3}}{\sqrt{3}-i}$

$= \dfrac{\left(2-2\sqrt{3}\right)+\left(2\sqrt{3}+2\right)i}{\sqrt{3}-i} \times \dfrac{\sqrt{3}+i}{\sqrt{3}+i}$

$= \dfrac{2\sqrt{3}-6+2i-2\sqrt{3}i+6i+2\sqrt{3}i-2\sqrt{3}-2}{3+1}$

$= \dfrac{8i-8}{4} = -2+2i$

$\therefore$ Modulus $= \sqrt{(-2)^2+(2)^2} = 2\sqrt{2}$.

43. (d) Since $\left(\dfrac{i}{2} - \dfrac{2}{i}\right) = \dfrac{i}{2} - \dfrac{2i}{i^2} = \dfrac{i}{2} + 2i = \dfrac{5}{2}i$

So, argument is $\tan^{-1}\left(\dfrac{b}{a}\right) = \tan^{-1}\left(\dfrac{\frac{5}{2}}{0}\right) = \dfrac{\pi}{2}$.

44. (d) Let $z = 7 - 24i$

$= 7 - 2\cdot4\cdot3i = 16 - 9 - 2\cdot4\cdot3i$

$= (4)^2 + (-3i)^2 - 2\cdot4\cdot3i$

$= (4-3i)^2$

$\therefore \ \sqrt{7-24i} = \pm(4-3i)$

45. (c) Here, $b^2 - 4ac = 1^2 - 4 \times \sqrt{5} \times \sqrt{5} = 1 - 20 = -19$

Therefore, the solutions are

$\dfrac{-1\pm\sqrt{-19}}{2\sqrt{5}} = \dfrac{-1\pm\sqrt{19}i}{2\sqrt{5}}$

46. (d) Given equation is $x^2 + 2x + 4 = 0$

Since α, β are roots of this equation

$\therefore \ \alpha + \beta = -2$ and $\alpha\beta = 4$

Now, $\dfrac{1}{\alpha^3} + \dfrac{1}{\beta^3} = \dfrac{\alpha^3+\beta^3}{(\alpha\beta)^3}$

$= \dfrac{(\alpha+\beta)\left(\alpha^2+\beta^2-\alpha\beta\right)}{(\alpha\beta)^3} = \dfrac{(-2)\left((\alpha+\beta)^2-3\alpha\beta\right)}{4\times4\times4}$

$= \dfrac{-2(4-12)}{4\times4\times4} = \dfrac{(-2)\times(-8)}{4\times4\times4} = \dfrac{1}{4}$

47. (d) Since α, β are roots of the equation $ax^2 + bx + c = 0$

$\therefore \ \alpha + \beta = \dfrac{-b}{a}, \ \alpha\beta = \dfrac{c}{a}$ ... (i)

Now, $\dfrac{1}{a\alpha+b} + \dfrac{1}{a\beta+b} = \dfrac{a(\alpha+\beta)+2b}{(a\alpha+b)(a\beta+b)}$

$= \dfrac{a(\alpha+\beta)+2b}{a^2\alpha\beta + ab(\alpha+\beta) + b^2}$

$= \dfrac{a\left(-\dfrac{b}{a}\right)+2b}{a^2\cdot\dfrac{c}{a} + ab\left(-\dfrac{b}{a}\right)+b^2} = \dfrac{b}{ac}$. [using (i)]

48. (b) Since complex roots always occur in conjugate pair.

$\therefore$ Other conjugate root is $1 + i$.

Sum of roots $= \dfrac{-a}{1} = (1-i) + (1+i) \Rightarrow a = -2$

Product of roots $= \dfrac{b}{1} = (1-i)(1+i) \Rightarrow b = 2$

49. (a) (a) $|z_1 z_2| = |z_1||z_2|$

(b) $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$

(c) $|z_1 + z_2| \neq |z_1| + |z_2|$

(d) $|z_1 + z_2| \le |z_1| - |z_2|$

50. (a) A number $z = a + ib$ where $a, b \in R$ is called complex number.

51. (c) For a quadratic equation $ax^2 + bx + c = 0$

$x = \dfrac{-b\pm\sqrt{D}}{2a}$

$x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

For real roots $D \ge 0$. If roots are not real, then $D < 0$.

52. (a) Here, $z = a + ib$, then real part of z, i.e., $Re(z) = a$ and imaginary part of z, i.e., $Im(z) = b$.

53. (c) Square root of negative number is imaginary in general $(a)^{\frac{1}{2n}}$, where $a < 0$ and $n \in N$ gives imaginary number.

54. (a) Here, $x = \sqrt{-16}$

$$x = \sqrt{-1 \times 16}$$

$$= \sqrt{-1} \times \sqrt{4 \times 4} = 4i$$

55. (a) Let $z_1 = 6 + 3i$ and $z_2 = 2 - i$

Then, $\dfrac{z_1}{z_2} = (6+3i)\dfrac{1}{2-i} = \dfrac{(6+3i)(2+i)}{(2-i)(2+i)}$

$$= (6+3i)\left(\frac{2}{2^2+(-1)^2} + i\frac{1}{2^2+(-1)^2}\right)$$

$$= (6+3i)\left(\frac{2}{5} + i\frac{1}{5}\right)$$

$$= (6+3i)\frac{(2+i)}{5}$$

$$= \frac{1}{5}[12 - 3 + i(6 + 6)]$$

$$= \frac{1}{5}(9 + 12i)$$

56. (d) $(1 + i)^5 (1 - i)^5 = (1 - i^2)^5$
$= 2^5 = 32$

57. (c) $\left|\dfrac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right| = \left|\dfrac{2 - i + 1 + i + 1}{2 - i - (1+i) + 1}\right|$

$[\because z_1 = 2 - i \text{ and } z_2 = 1 + i]$

$$= \left|\frac{4}{2 - i - 1 - i + 1}\right| = \left|\frac{4}{2-2i}\right| = \left|\frac{2}{1-i}\right| = \frac{2}{|1-i|}$$

$$\left[\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$$

$$= \frac{2}{\sqrt{(1)^2 + (-1)^2}} \qquad \left[\because |z| = \sqrt{a^2+b^2}\right]$$

$$= \frac{2}{\sqrt{2}} = \sqrt{2}.$$

58. (a) $\dfrac{(1+i)^3}{(1-i)^3} - \dfrac{(1-i)^3}{(1+i)^3} = x + iy$

$$\Rightarrow \frac{(1 + i^2 + 2i)^3 - (1 + i^2 - 2i)^3}{(1 - i^2)^3} = x + iy$$

$$\Rightarrow \frac{8i^3 + 8i^3}{2^3} = x + iy$$

$\Rightarrow 2i^3 = x + iy \Rightarrow -2i = x + iy$

$\Rightarrow x = 0, y = -2$

59. (c) If $z = x + iy$ is the additive inverse of $1 - i$, then
$(x + iy) + (1 - i) = 0$
$\Rightarrow x + 1 = 0, y - 1 = 0$
$\Rightarrow x = -1, y = 1$
$\therefore$ The additive inverse of $1 - i$ is $z = -1 + i$
Trick: Since $(1 - i) + (-1 + i) = 0$.

60. (c) Let $z = x + iy$, then its conjugate $\bar{z} = x - iy$

Given that $z^2 = (\bar{z})^2$

$\Rightarrow x^2 - y^2 + 2ixy = x^2 - y^2 - 2ixy$
$\Rightarrow 4ixy = 0$
If $x \neq 0$, then $y = 0$ and if $y \neq 0$, then $x = 0$.

61. (b) $z = x + iy$
$\Rightarrow |z|^2 = x^2 + y^2 = 1$ ... (i)

Now, $\left(\dfrac{z-1}{z+1}\right) = \dfrac{(x-1)+iy}{(x+1)+iy} \times \dfrac{(x+1)-iy}{(x+1)-iy}$

$$= \frac{(x^2+y^2-1)+2iy}{(x+1)^2+y^2} = \frac{2iy}{(x+1)^2+y^2}$$

[By equation (i)]

Hence, $\left(\dfrac{z-1}{z+1}\right)$ is purely imaginary.

62. (d) Let $z = x + iy$, $\bar{z} = x - iy$

Since $\arg(z) = \theta = \tan^{-1}\dfrac{y}{x}$

$\arg(\bar{z}) = \theta = \tan^{-1}\left(\dfrac{-y}{x}\right)$

Thus, $\arg(z) \neq \arg(\bar{z})$.

63. (d) $\sqrt{a + ib} = x + yi$

$\Rightarrow \left(\sqrt{a+ib}\right)^2 = (x + yi)^2$

$\Rightarrow a = x^2 - y^2$, $b = 2xy$ and hence

$\sqrt{a - ib} = \sqrt{x^2 - y^2 - 2xyi} = \sqrt{(x - yi)^2} = x - iy$

Note: In the question, it should have been given that $a, b, x, y \in R$.

64. (b) Given equation is $x^2 - 2x(1 + 3k) + 7(2k + 3) = 0$
Since, it has equal roots.
$\therefore$ Discriminant $D = 0$
$\Rightarrow b^2 - 4ac = 0 \Rightarrow 4(1 + 3k)^2 - 4 \times 7(2k + 3) = 0$
$\Rightarrow 1 + 9k^2 + 6k - 14k - 21 = 0$
$\Rightarrow 9k^2 - 8k - 20 = 0$
$\Rightarrow 9k^2 - 18k + 10k - 20 = 0$
$\Rightarrow 9k(k - 2) + 10(k - 2) = 0$

$\Rightarrow k = \dfrac{-10}{9}, 2$

Only $k = 2$ satisfy given equation.

65. (b) Given equation is $3^{2x} - 10.3^x + 9 = 0$ can be written as $(3^x)^2 - 10(3^x) + 9 = 0$

Let $a = 3^x$, then it reduces to the equation

$a^2 - 10a + 9 = 0 \Rightarrow (a-9)(a-1) = 0$

$\Rightarrow a = 9, 1$

Now, $a = 3^x$

$\Rightarrow 9 = 3^x \Rightarrow 3^2 = 3^x \Rightarrow x = 2$

and $1 = 3^x \Rightarrow 3^0 = 3^x \Rightarrow x = 0$

Hence, roots are 0, 2.

66. (c) $x^2 + y^2 = 25$ and $xy = 12$

$\Rightarrow x^2 + \left(\frac{12}{x}\right)^2 = 25$

$\Rightarrow x^4 + 144 - 25x^2 = 0$

$\Rightarrow (x^2 - 16)(x^2 - 9) = 0$

$\Rightarrow x^2 = 16$ and $x^2 = 9$

$\Rightarrow x = \pm 4$ and $x = \pm 3$.

67. (b) Equations $px^2 + 2qx + r = 0$ and

$qx^2 - 2\left(\sqrt{pr}\right)x + q = 0$ have real roots, then

from first

$4q^2 - 4pr \geq 0 \Rightarrow q^2 - pr \geq 0$

$\Rightarrow q^2 \geq pr$ ... (i)

and from second

$4(pr) - 4q^2 \geq 0$ (for real root)

$\Rightarrow pr \geq q^2$ ... (ii)

From (i) and (ii), we get result

$q^2 = pr$.

68. (b) The roots of the equations are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(i) Let $b^2 - 4ac > 0$, $b > 0$

Now, if $a > 0$, $c > 0$, $b^2 - 4ac < b^2$

$\Rightarrow$ the roots are negative.

(ii) Let $b^2 - 4ac < 0$, then the roots are given by

$$x = \frac{-b \pm i\sqrt{(4ac - b^2)}}{2a}, \quad \left(i = \sqrt{-1}\right)$$

which are imaginary and have negative real part.

$[\because b > 0]$

$\therefore$ In each case, the roots have negative real part.

69. (a) Given equation $2ax^2 + (2a + b)x + b = 0$, $(a \neq 0)$

Now, its discriminant $D = B^2 - 4AC$

$= (2a + b)^2 - 4.2ab = (2a - b)^2$

Hence, D is a perfect square. So, given equation has rational roots.

70. (a) Since $2 + i\sqrt{3}$ is a root, therefore, $2 - i\sqrt{3}$ will be other root. Now sum of the roots $= 4 = -p$ and product of roots $= 7 = q$. Hence $(p, q) = (-4, 7)$.

71. (c) Let the roots be α and β

$\Rightarrow \alpha + \beta = -p$, $\alpha\beta = q$

Given, $\alpha + \beta = \alpha^2 + \beta^2$

But $\alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow -p = (-p)^2 - 2q$

$\Rightarrow p^2 - 2q = -p \Rightarrow p^2 + p = 2q$

72. (c) Let the common root be y.

Then, $y^2 + py + q = 0$ and $y^2 + \alpha y + \beta = 0$

On solving by cross multiplication, we have

$$\frac{y^2}{p\beta - q\alpha} = \frac{y}{q - \beta} = \frac{1}{\alpha - p}$$

$$\therefore \quad y = \frac{q - \beta}{\alpha - p} \text{ and } \frac{y^2}{y} = y = \frac{p\beta - q\alpha}{q - \beta}.$$

73. (d) Let α be a common root, then

$\alpha^2 + a\alpha + 10 = 0$ ... (i)

and $\alpha^2 + b\alpha - 10 = 0$ ... (ii)

From (i) – (ii),

$$(a - b)\alpha + 20 = 0 \Rightarrow \alpha = -\frac{20}{a - b}$$

Substituting the value of α in (i), we get

$$\left(-\frac{20}{a-b}\right)^2 + a\left(-\frac{20}{a-b}\right) + 10 = 0$$

$\Rightarrow 400 - 20a(a - b) + 10(a - b)^2 = 0$

$\Rightarrow 40 - 2a^2 + 2ab + a^2 + b^2 - 2ab = 0$

$\Rightarrow a^2 - b^2 = 40$.

74. (a) Given equation is $x^2 - 2ax + a^2 + a - 3 = 0$

If roots are real, then $D \geq 0$

$\Rightarrow 4a^2 - 4(a^2 + a - 3) \geq 0$

$\Rightarrow -a + 3 \geq 0$

$\Rightarrow a - 3 \leq 0 \Rightarrow a \leq 3$

As roots are less than 3, hence $f(3) > 0$.

$9 - 6a + a^2 + a - 3 > 0$

$\Rightarrow a^2 - 5a + 6 > 0$

$\Rightarrow (a - 2)(a - 3) > 0 \Rightarrow$ either $a < 2$ or $a > 3$

Hence, $a < 2$ satisfy all.

STATEMENT TYPE QUESTIONS

75. (c) I. Given $x^2 + 3x + 5 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get

$a = 1$, $b = 3$, $c = 5$

Now, $D = b^2 - 4ac$

$= (3)^2 - 4 \times 1 \times 5 = 9 - 20 = -11 < 0$

$$\Rightarrow x = \frac{-3 \pm \sqrt{-11}}{2 \times 1}$$

$$\therefore x = \frac{-3 \pm i\sqrt{11}}{2} \quad \left[\because \sqrt{-1} = i\right]$$

II. Given $x^2 - x + 2 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we get

a = 1, b = –1, c = 2
Now, $D = b^2 - 4ac$
$= (-1)^2 - 4 \times 1 \times 2 = 1 - 8 = -7 < 0$

$\Rightarrow x = \dfrac{-(-1) \pm \sqrt{-7}}{2 \times 1}$

$= \dfrac{1 \pm i\sqrt{7}}{2}$ $\quad [\because \sqrt{-1} = i]$

76. **(c)** Given that, $\overline{z_1} + i\,\overline{z_2} = 0$

$\Rightarrow z_1 = iz_2$, i.e. $z_2 = -iz_1$
Thus, $\arg(z_1 z_2) = \arg z_1 + \arg(-iz_1) = \pi$
$\because \arg(z_1 z_2) = \arg z_1 + \arg z_2$
$\Rightarrow \arg(-iz_1^{\,2}) = \pi$
$\Rightarrow \arg(-i) + 2\arg(z_1) = \pi$
$\Rightarrow \dfrac{-\pi}{2} + 2\arg(z_1) = \pi$
$\Rightarrow \arg(z_1) = \dfrac{3\pi}{4}$

77. **(a)** I. We have,

$\dfrac{1+i}{1-i} = \dfrac{1+i}{1-i} \times \dfrac{1+i}{1+i} = \dfrac{1-1+2i}{1+1} = i = 0 + i$

Now, let us put $0 = r\cos\theta$, $1 = r\sin\theta$
Squaring and adding,
$r^2 = 1$, i.e, $r = 1$
So, $\cos\theta = 0$, $\sin\theta = 1$

Therefore, $\theta = \dfrac{\pi}{2}$

Hence, the modulus of $\dfrac{1+i}{1-i}$ is 1 and the argument is $\dfrac{\pi}{2}$.

II. We have :

$\dfrac{1}{1+i} = \dfrac{1-i}{(1+i)(1-i)} = \dfrac{1-i}{1+1} = \dfrac{1}{2} - \dfrac{i}{2}$

Let $\dfrac{1}{2} = r\cos\theta$, $-\dfrac{1}{2} = r\sin\theta$
Proceeding as

$r = \dfrac{1}{\sqrt{2}}$; $\cos\theta = \dfrac{1}{\sqrt{2}}$, $\sin\theta = \dfrac{-1}{\sqrt{2}}$

Therefore, $\theta = \dfrac{-\pi}{4}$
[$\because \cos\theta > 0$ and $\sin\theta < 0$ is in IV quadrant]

Hence, the modulus of $\dfrac{1}{1+i}$ is $\dfrac{1}{\sqrt{2}}$ and the argument is $-\dfrac{\pi}{4}$.

78. **(c)** $(a + ib)(c + id)(e + if)(g + ih) = A + iB$
Taking modulus on both sides, we get
$|(a + ib)(c + id)(e + if)(g + ih)| = |A + iB|$

$\Rightarrow |a + ib|\,|c + id|\,|e + if|\,|g + ih| = |A + iB|$
$[\because |z_1 z_2 \ldots z_n| = |z_1||z_2||z_3|\ldots|z_n|]$

$\Rightarrow \sqrt{a^2+b^2}\sqrt{c^2+d^2}\sqrt{e^2+f^2}\sqrt{g^2+h^2} = \sqrt{A^2+B^2}$
$\left[\because \text{If } z = a + ib, \text{ then } |z| = \sqrt{a^2+b^2}\right]$

Squaring on both sides, we get
$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$

79. **(c)** I. Additive inverse of $(1-i) = -(1-i) = -1 + i$

II. Since, difference of two complex numbers is also a complex number and $z_1 - z_2$ can be written as $(z_1) + (-z_2)$ which is sum of z_1 and additive inverse of z_2.

III. $\dfrac{5+\sqrt{2}i}{1-\sqrt{2}i} \times \dfrac{1+\sqrt{2}i}{1+\sqrt{2}i} = \dfrac{5+5\sqrt{2}i+\sqrt{2}i-2}{1+2}$

$= \dfrac{3+6\sqrt{2}i}{3} = 1+2\sqrt{2}i$

80. **(d)** By definition, both the statements are correct.

81. **(c)** II. $x^2 + 3x + 5 = 0$

$x = \dfrac{-3 \pm \sqrt{(3)^2 - 4(5)}}{2}$

$= \dfrac{-3 \pm \sqrt{9-20}}{2}$

$= \dfrac{-3 \pm \sqrt{11}i}{2}$

82. **(c)** I. $x = 1 + 2i$
$\Rightarrow (x-1) = 2i$
$\Rightarrow (x-1)^2 = (2i)^2 \Rightarrow x^2 - 2x + 5 = 0$
Consider
$x^3 + 7x^2 - x + 16 = x(x^2 - 2x + 5) + 9(x^2 - 2x + 5) + (12x - 29)$
$= x(0) + 9(0) + 12x - 29$
$= -17 + 24i$
II. $iz^3 + z^2 - z + i = 0$
$z^3 - iz^2 + iz + 1 = 0$ (Dividing both side by i)
$\Rightarrow (z-i)(z^2+i) = 0$
$\Rightarrow z = i$ or $z^2 = -i$
Now, $z = i \Rightarrow |z| = |i| = 1$
$z^2 = -i \Rightarrow |z^2| = |-i| = 1$
$\Rightarrow |z|^2 = 1$
$\Rightarrow |z| = 1$

83. **(d)** I. $z\bar{z} = (a + ib)(a - ib)$
$= a^2 - (ib)^2 = a^2 + b^2$
$= |z|^2$

III. $z^{-1} = \frac{3}{(3)^2+(-2)^2} + \frac{i(2)}{3^2+(-2)^2}$

$= \frac{3}{13} + \frac{2}{13}i$

84. (c) $\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{(1+i)^2}{1-i^2}$

$= \frac{1+i^2+2i}{2} = \frac{2i}{2} = i \equiv 0+i$

$\left|\frac{1+i}{1-i}\right| = |i| = 1$

Now, $r\cos\theta = 0, r\sin\theta = 1$

$r^2 = 1 \Rightarrow r = 1$

$\therefore \cos\theta = 0$ and $\sin\theta = 1$

$\Rightarrow \theta = \frac{\pi}{2}$

MATCHING TYPE QUESTIONS

85. (b) A. Let $z = 4 - 3i$

Then, its multiplicative inverse is

$\frac{1}{z} = \frac{1}{4-3i} = \frac{1}{4-3i} \times \frac{4+3i}{4+3i} = \frac{4+3i}{16-9i^2}$

[use $(a - b)(a + b) = a^2 - b^2$]

$= \frac{4+3i}{16+9}$ [$\because i^2 = -1$]

$= \frac{4+3i}{25} = \frac{4}{25} + \frac{3i}{25}$

B. Let $z = \sqrt{5} + 3i$

Then, its multiplicative inverse is

$\frac{1}{z} = \frac{1}{\sqrt{5}+3i} = \frac{1}{\sqrt{5}+3i} \times \frac{\sqrt{5}-3i}{\sqrt{5}-3i}$

$= \frac{\sqrt{5}-3i}{5-9i^2}$ [use $(a + b)(a - b) = a^2 - b^2$]

$= \frac{\sqrt{5}-3i}{5+9} = \frac{\sqrt{5}-3i}{14}$ [$\because i^2 = -1$]

$= \frac{\sqrt{5}}{14} - \frac{3i}{14}$

C. Let $z = -i$

Then, its multiplicative inverse is

$\frac{1}{z} = -\frac{1}{i} = -\frac{1}{i} \times \frac{i}{i} = \frac{-i}{i^2} = \frac{-i}{-1} = i$ [$\because i^2 = -1$]

$= 0 + i.$

86. (d) A. $(1 - i)^4 = [(1 - i)^2]^2$

$= (1 + i^2 - 2i)^2$ [use $(a - b)^2 = a^2 + b^2 - 2ab$]

$= (1 - 1 - 2i)^2$ [$\because i^2 = -1$]

$= (-2i)^2 = (-2)^2\, i^2$

$= 4(-1) = -4 + 0i$

B. $\left(\frac{1}{3}+3i\right)^3 = \left(\frac{1}{3}\right)^3 + (3i)^3 + 3 \times \frac{1}{3} \times 3i\left(\frac{1}{3}+3i\right)$

$= \frac{1}{27} + 27i^3 + 3i\left(\frac{1}{3}+3i\right)$

$= \frac{1}{27} - 27i + 3i \times \frac{1}{3} + 3i \times 3i$ [$\because i^3 = -i$]

$= \frac{1}{27} - 27i + i + 9i^2$

$= \frac{1}{27} - 27i + i - 9$ [$\because i^2 = -1$]

$= \left(\frac{1}{27} - \frac{9}{1}\right) - i(27 - 1)$

$= \left(\frac{1-243}{27}\right) - 26i = -\frac{242}{27} - 26i$

C. $(-1)^3\left(2+\frac{1}{3}i\right)^3$

$= -\left[(2)^3 + \left(\frac{1}{3}i\right)^3 + 3 \times 2 \times \frac{1}{3}i\left(2+\frac{1}{3}i\right)\right]$

$= -\left[8 + \frac{1}{27}i^3 + 2i\left(2+\frac{1}{3}i\right)\right]$

$= -\left[8 - \frac{1}{27}i + 4i + \frac{2}{3}i^2\right]$ [$\because i^3 = -i$]

$= -\left[8 - \frac{1}{27}i + 4i - \frac{2}{3}\right]$ [$\because i^2 = -1$]

$= -\left[\left(\frac{8}{1} - \frac{2}{3}\right) + i\left(\frac{4}{1} - \frac{1}{27}\right)\right]$

$= -\left[\left(\frac{24-2}{3}\right) + i\left(\frac{108-1}{27}\right)\right]$

$= -\left[\frac{22}{3} + i\frac{107}{27}\right]$

87. (b) We know that,

$i = \sqrt{-1},\ i^2 = -1$

$\Rightarrow i^{-1} = \frac{1}{i} \times \frac{i}{i} = \frac{i}{-1} = -i$

$\Rightarrow i^{-2} = \frac{1}{i^2} = \frac{1}{-1} = -1$

$\Rightarrow\ i^{-3} = \dfrac{1}{i^3} = \left\{\dfrac{i}{i^3 \times i}\right\}$

[multiplying numerator and denominator by i]

$\Rightarrow\ \dfrac{i}{i^4} = i$

$\Rightarrow\ i^{-4} = \dfrac{1}{i^4} = \dfrac{1}{1} = 1$

88. **(b)** (A) $(1-i)-(-1+i.6) = (1-i)+(1-6i)$
$= 1+1-i-6i$
$= 2-7i = (a+ib)$,
where $a = 2, b = -7$

(B) $\left(\dfrac{1}{5}+i.\dfrac{2}{5}\right)-\left(4+i.\dfrac{5}{2}\right) = \left(\dfrac{1}{5}+\dfrac{2}{5}i\right)+\left(-4-\dfrac{5}{2}i\right)$

$= \dfrac{1}{5}-4+\dfrac{2}{5}i-\dfrac{5}{2}i = -\dfrac{19}{5}-\left(-\dfrac{2}{5}+\dfrac{5}{2}\right)i$

$= -\dfrac{21}{5}-\dfrac{21}{10}i$

(C) $\left(\dfrac{1}{3}+3i\right)^3 = \left(\dfrac{1}{3}\right)^3+3\left(\dfrac{1}{3}\right)^2(3i)+3.\left(\dfrac{1}{3}\right)(3i)^2+(3i)^3$

$= \dfrac{1}{27}+i+9(-1)+27i^3$

$= \dfrac{1}{27}+i+9(-1)+27i.(i^2)$

$= \dfrac{1}{27}+i+9(-1)+27i.(-1)$

$= \dfrac{1}{27}+i-9-27i = -\dfrac{242}{27}-26i$

(D) $(1-i)^4 = [(1-i)^2]^2$
$= (1+i^2-2i)^2 = (1-1-2i)^2$
$= (-2i)^2 = 4i^2 = 4(-1) = -4$

(E) $\left(-2-\dfrac{1}{3}i\right)^3$

$= (-2)^3-3(-2)^2.\left(\dfrac{1}{3}i\right)+3(-2)\left(-\dfrac{1}{3}i\right)^2-\left(\dfrac{1}{3}i\right)^3$

$= -8-4i-6\times\dfrac{1}{9}(i^2)-\dfrac{1}{27}i^3$

$= -8-4i-\dfrac{2}{3}(-1)-\dfrac{1}{27}i.i^2$

$= -8-4i+\dfrac{2}{3}-\dfrac{1}{27}i.(-1)$

$= -8-4i+\dfrac{2}{3}+\dfrac{1}{27}i$

$= -\dfrac{22}{3}-\dfrac{107}{27}i$

89. **(d)** (A) We have multiplicative inverse of $4-3i$

$= \dfrac{1}{4-3i}\times\dfrac{4+3i}{4+3i}$

$= \dfrac{4+3i}{4^2-9i^2} = \dfrac{4+3i}{16+9} = \dfrac{4+3i}{25} = \dfrac{4}{25}+i\dfrac{3}{25}$

(B) We have multiplicative inverse of $\sqrt{5}+3i$

$= \dfrac{1}{\sqrt{5}+3i}\times\dfrac{\sqrt{5}-3i}{\sqrt{5}-3i}$ (multiply by conjugate)

$= \dfrac{\sqrt{5}-3i}{5-9i^2} = \dfrac{\sqrt{5}-3i}{5+9} = \dfrac{\sqrt{5}-3i}{14} = \dfrac{\sqrt{5}}{14}-\dfrac{3}{14}i$

$[\because\ (a+ib)(a-ib) = a^2+b^2]$

(C) We have multiplicative inverse of $-i = \dfrac{1}{-i}$.

Multiply by conjugate

$= \dfrac{1}{-i}\times\dfrac{i}{i} = \dfrac{-i}{i^2} = \dfrac{-i}{-1} = i = 0+i\,.\,1$

(D) $z = (2+\sqrt{3}\,i)^2 = 4+3i^2+4\sqrt{3}\,i$

$= 1+4\sqrt{3}\,i$

$\therefore\ \dfrac{1}{z} = \dfrac{1}{4+\sqrt{3}\,i} = \dfrac{1-4\sqrt{3}\,i}{1+48}$ (On rationalizing)

90. **(c)** (A) $2x^2+x+1 = 0$. Comparing with $ax^2+bx+c = 0$
$a = 2, b = 1, c = 1$
$b^2-4ac = 1^2-4.2.1 = 1-8 = -7$

$\therefore\ x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a} = \dfrac{-1\pm\sqrt{-7}}{2.2}$

$= \dfrac{-1\pm\sqrt{7}i}{4}$

(B) $x^2+3x+9 = 0 \therefore a = 1, b = 3, c = 9$
$b^2-4ac = 3^2-4.1.9 = 9-36 = -27$

$\therefore\ x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a} = \dfrac{-3\pm\sqrt{-27}}{2\times1}$

$= \dfrac{-3\pm(3\sqrt{3})i}{2}$

(C) $-x^2+x-2 = 0$ or $x^2-x+2 = 0$
$a = 1, b = -1, c = 2$

Hence, $x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \dfrac{-(-1)\pm\sqrt{(-1)^2-4\times1\times2}}{2\times1}$

$= \dfrac{1\pm\sqrt{1-8}}{2}$

$= \dfrac{1\pm\sqrt{-7}}{2} = \dfrac{1\pm\sqrt{7}i}{2}$

(D) $x^2+3x+5=0$

$a=1, b=3, c=5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-3\pm\sqrt{(3)^2-4\times1\times5}}{2\times1}$$

$$x=\frac{-3\pm\sqrt{9-20}}{2}$$

$$x=\frac{-3\pm\sqrt{-11}}{2}$$

$$x=\frac{-3\pm\sqrt{11}i}{2}$$

91. **(a)**

(A) We have $1-i=r(\cos\theta+i\sin\theta)$

$\Rightarrow$ $r\cos\theta=1, r\sin\theta=-1$

By squaring and adding, we get

$r^2(\cos^2\theta+\sin^2\theta)=1^2+(-1)^2$

$\Rightarrow r^2 . 1=1+1 \Rightarrow r^2=2$

$\therefore$ $r=\sqrt{2}$, By dividing $\frac{r\sin\theta}{r\cos\theta}=\frac{-1}{1}=-1$

$\Rightarrow$ $\tan\theta=-1$ i.e., θ lies in fourth quadrant.

$\Rightarrow$ $\theta=-45$

$\Rightarrow$ $\theta=-\frac{\pi}{4}$

$\therefore$ Polar form of $1-i$

$$=\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)$$

(B) We have $-1+i=r(\cos\theta+i\sin\theta)$

$\Rightarrow$ $r\cos\theta=-1$ and $r\sin\theta=1$

By squaring and adding, we get

$r^2(\cos^2\theta+\sin^2\theta)=(-1)^2+1^2 \Rightarrow r^2. 1=1+1$

$\therefore$ $r^2=2$ $\therefore$ $r=\sqrt{2}$

By dividing, $\frac{r\sin\theta}{r\cos\theta}=\frac{1}{-1}=-1 \Rightarrow \tan\theta=-1$

$\therefore$ θ lies in second quadrant ;

$\theta=180°-45°=135°$ i.e $\theta=\frac{3\pi}{4}$

$\therefore$ Polar form of $-1+i$

$$=\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$$

(C) we have $-1-i=r(\cos\theta+i\sin\theta)$

$\Rightarrow$ $r\cos\theta=-1$ and $r\sin\theta=-1$

By squaring and adding, we get

$r^2(\cos^2\theta+\sin^2\theta)=(-1)^2+(-1)^2$

$\Rightarrow$ $r^2 . 1=1+1$

$\Rightarrow$ $r^2=2$

$\therefore$ $r=\sqrt{2}$

By dividing $\frac{r\sin\theta}{r\cos\theta}=\frac{-1}{-1}=1 \Rightarrow \tan\theta=1$

$\therefore$ θ lies in IIIrd quadrant.

$\theta=-180°+45°=-135°$ or $\theta=-\frac{3\pi}{4}$

$\therefore$ Polar form of $-1-i$

$$=\sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right)+i\sin\left(-\frac{3\pi}{4}\right)\right)$$

(D) $r=\sqrt{3}+i=r(\cos\theta+i\sin\theta)$

$\therefore$ $r\cos\theta=\sqrt{3}$, $r\sin\theta=1$

Squaring and adding $r^2=3+1=4, r=2$

Also $\tan\theta=\frac{1}{\sqrt{3}}$, $\sin\theta$ and $\cos\theta$ both are positive.

$\therefore$ θ lies in the I quadrant

$\therefore$ $\theta=30°=\frac{\pi}{6}$

$\therefore$ Polar form of z is $2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

92. **(b)** $z=x+iy \Rightarrow |z|=\sqrt{x^2+y^2}$

INTEGER TYPE QUESTIONS

93. **(a)** $i^{57}+\frac{1}{i^{25}}=(i^4)^{14}.i+\frac{1}{(i^4)^6.i}$

$=i+\frac{1}{i}$ $(\because i^4=1)$

$=i-i$ $\left(\because \frac{1}{i}=-i\right)$

$=0$

94. **(c)** $z=2-3i \Rightarrow z-2=-3i$

Squaring, we get

$z^2-4z+4=-9 \Rightarrow z^2-4z+13=0$

95. **(b)** Given: $\frac{c+i}{c-i}=a+ib$

Then, $a+ib=\frac{c+i}{c-i}\times\frac{c+i}{c+i}=\frac{c^2-1+2ic}{c^2+1}$

$\Rightarrow$ $a=\frac{c^2-1}{c^2+1}$ and $b=\frac{2c}{c^2+1}$

$\Rightarrow$ $(a^2+b^2)=\frac{(c^2-1)^2+4c^2}{(c^2+1)^2}$

$$=\frac{c^4+1-2c^2+4c^2}{(c^2+1)^2}=\frac{(c^2+1)^2}{(c^2+1)^2}=1.$$

96. (a) We have,

$$x + iy = \frac{a+ib}{a-ib} \quad ...(i)$$

Its conjugate,

$$x - iy = \frac{a-ib}{a+ib} \quad ...(ii)$$

Multiply (i) and (ii),

$$(x+iy)(x-iy) = \frac{a+ib}{a-ib} \times \frac{a-ib}{a+ib}$$

$x^2 + y^2 = 1.$

97. (b) $(x+iy)^{\frac{1}{3}} = a - ib$

$x + iy = (a - ib)^3 = (a^3 - 3ab^2) + i(b^3 - 3a^2 b)$

$\Rightarrow x = a^3 - 3ab^2,\ y = b^3 - 3a^2 b$

$\Rightarrow \frac{x}{a} = a^2 - 3b^2,\ \frac{y}{b} = b^2 - 3a^2$

$\therefore \frac{x}{a} - \frac{y}{b} = a^2 - 3b^2 - b^2 + 3a^2$

$\frac{x}{a} - \frac{y}{b} = 4(a^2 - b^2) = k(a^2 - b^2)$

$\therefore$ k = 4.

98. (b) $2x^2 - (p+1)x + (p-1) = 0$

Given $\alpha - \beta = \alpha\beta \Rightarrow (\alpha+\beta)^2 - 4\alpha\beta = \alpha^2\beta^2$

$\Rightarrow \frac{(p-1)^2}{4} = \frac{(p+1)^2}{4} - \frac{4(p-1)}{2}$

$\Rightarrow 2(p - 1) = p \Rightarrow p = 2.$

99. (c) $z_1 = 2 + 3i, z_2 = 3 + 2i$

$z_1 + z_2 = (2 + 3i) + (3 + 2i) = 5 + 5i$

Hence, a = 5

100. (d) $z_1 = 2 + 3i$ and $z_2 = 3 - 2i$

$z_1 - z_2 = (2 + 3i) - (3 - 2i) = -1 + 5i \equiv -1 + bi$

Hence, b = 5.

101. (c) $z = 5i\left(\frac{-3}{5}i\right) = -3i^2 = -3(-1) = 3 \equiv 3 + 0i$

Hence, b = 0

102. (d) $\frac{z_1}{z_2} = \frac{6+3i}{2-i} = \frac{6+3i}{2-i} \times \frac{2+i}{2+i}$

$= \frac{12+6i+6i-3}{4-i^2} = \frac{9+12i}{5}$

$= \frac{1}{5}(9+12i) \equiv \frac{1}{a}(9+12i)$

Hence, a = 5.

103. (a) Consider

$i^{4k} + i^{4k+1} + i^{4k+2} + i^{4k+3}$

$= (i^4)^k + (i^4)^k . i + (i^4)^k . i^2 + (i^4)^k . i^3$

$= 1 + i + i^2 + i^3 = 1 + i - 1 - i = 0$

104. (a) $z = i^9 + i^{19} = (i^4)^2 . i + (i^4)^4 . i^3$

$= i + i^3 \qquad (\because i^4 = 1)$

$= i - i = 0 \qquad (\because i^3 = 1)$

$\equiv 0 + 0\,i$

Hence, a = 0

105. (a) $z = i^{-39} = \frac{1}{i^{39}} = \frac{1}{(i^4)^9} \cdot \frac{1}{i^3}$

$= 1 \cdot \frac{1}{i^3} \qquad (\because i^4 = 1)$

$= \frac{1}{i^2 \cdot i} = \frac{-1}{i} \qquad (\because i^2 = -1)$

$= i \qquad (\because \frac{1}{i} = -i)$

$\equiv 0 + i$

Hence, value of a = 0.

106. (c) $(1 - i)^n = 2^n$

Take modulus, both the side $|(1 - i)^n| = |2^n|$

$|1 - i|^n = |2|^n$

$\Rightarrow \left[\sqrt{1^2 + (-1)^2}\right]^n = 2^n$

$\Rightarrow \left(\sqrt{2}\right)^n = 2^n \Rightarrow 2^{\frac{n}{2}} = 2^n$

$\Rightarrow \frac{n}{2} = n \quad \Rightarrow n = 0$

107. (d) $(1 + i)^5 (1 - i)^5 = [(1 + i)(1 - i)]^5$

$= (1 - i^2)^5 = [1 - (-1)]^5$

$= 2^5$

108. (d) $(1 + i)^8 + (1 - i)^8 = [\{(1 + i)^2\}^4 + \{(1 - i)^2\}^4]$

$= [(1 + i^2 + 2i)^4 + (1 + i^2 - 2i)^4]$

$= [(2i)^4 + (-2i)^4] = 16i^4 + 16i^4$

$= 32i^4 = 32 = 2^5$

109. (b) $x^2 + 2 = 0 \Rightarrow x^2 = -2$

$\Rightarrow x = \pm\sqrt{-2} = \pm\sqrt{2}\, i$

110. (a) $z_1 = \sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$

$= \sqrt{2}\left[\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right] = 1 + i$

$|z_1| = \sqrt{2}$

and $z_2 = \sqrt{3}\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$

$= \sqrt{3}\left[\frac{1}{2}+i\frac{\sqrt{3}}{2}\right]$

$|z_2| = \sqrt{\frac{3}{4}+\frac{9}{4}} = \sqrt{3}$

$|z_1 z_2| = |z_1||z_2| = \sqrt{2}\cdot\sqrt{3} = \sqrt{6}$

111. **(a)** As we know, if $z = a + ib$, then

$|z| = \sqrt{a^2+b^2}$

Let $z = \sqrt{2i} - \sqrt{-2i}$

$= \sqrt{2i} - i\sqrt{2i}\ \left(\because \sqrt{-1} = i\right)$

$= \sqrt{2i}(1-i)$

Now, $|z| = |\sqrt{2}\sqrt{i}(1-i)|$

$= \sqrt{2}|\sqrt{i}||1-i| = \sqrt{2}\times 1\times\sqrt{1^2+(-1)^2}$

$= \sqrt{2}\times\sqrt{2} = 2$

112. **(d)** $z^{1/3} = a - ib \Rightarrow z = (a-ib)^3$

$\therefore\ x + iy = a^3 + ib^3 - 3ia^2b - 3ab^2$. Then

$x = a^3 - 3ab^2 \Rightarrow \frac{x}{a} = a^2 - 3b^2$

$y = b^3 - 3a^2b \Rightarrow \frac{y}{b} = b^2 - 3a^2$

So, $\frac{x}{a} - \frac{y}{b} = 4(a^2 - b^2)$

113. **(a)** Given equations are

$k(6x^2+3) + rx + 2x^2 - 1 = 0$ and

$6k(2x^2-1) + px + 4x^2 + 2 = 0$

$\Rightarrow (6k+2)x^2 + rx + 3k - 1 = 0$ ...(i)

$\Rightarrow (12k+4)x^2 + px - 6k + 2 = 0$ ...(ii)

Let α and β be the roots of both equations (i) and (ii).

$\therefore\ \alpha+\beta = \frac{-r}{6k+2}$ (from (i))

and $\alpha+\beta = \frac{-p}{12k+4}$ (from (ii))

$\therefore\ \frac{-r}{2(1+3k)} = \frac{-p}{4(1+3k)} \Rightarrow \frac{-r}{2} = \frac{-p}{4}$

$\Rightarrow -2r = -p \Rightarrow 2r - p = 0.$

114. **(c)** Let $z = r(\cos\theta + i\sin\theta)$

Then $r = |z|$ and $\theta = \arg(z)$

Now $z = r(\cos\theta + i\sin\theta)$

$\Rightarrow \bar{z} = r(\cos\theta - i\sin\theta)$

$= r[\cos(-\theta) + i\sin(-\theta)]$

$\therefore \arg(\bar{z}) = -\theta$

$\Rightarrow \arg(\bar{z}) = -\arg(z)$

$\Rightarrow \arg(\bar{z}) + \arg(z) = 0.$

115. **(c)** $|z_1+z_2| = |z_1| + |z_2|$

$\Rightarrow$ z_1 and z_2 are collinear and are to the same side of origin; hence $\arg z_1 - \arg z_2 = 0$.

116. **(a)** We have, $z = 2 - 3i$

$\Rightarrow z - 2 = -3i \Rightarrow (z-2)^2 = (-3i)^2$

$\Rightarrow z^2 - 4z + 4 = 9i^2 \Rightarrow z^2 - 4z + 13 = 0$

ASSERTION - REASON TYPE QUESTIONS

117. **(a)** Since $x = -2$ is a root of $f(x)$.

$\therefore$ $f(x) = (x+2)(ax+b)$

But $f(0) + f(1) = 0$

$\therefore$ $2b + 3a + 3b = 0$

$\Rightarrow$ $-\frac{b}{a} = \frac{3}{5}$.

118. **(b)** We have,

$|z_1+z_2|^2 = |z_1|^2 + |z_2|^2$

$\Rightarrow$ $|z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\theta_1-\theta_2) = |z_1|^2 + |z_2|^2$

where $\theta_1 = \arg(z_1)$, $\theta_2 = \arg(z_2)$

$\Rightarrow$ $\cos(\theta_1-\theta_2) = 0$

$\Rightarrow$ $\theta_1 - \theta_2 = \frac{\pi}{2}$

$\Rightarrow$ $\arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2}$

$\Rightarrow$ $\text{Re}\left(\frac{z_1}{z_2}\right) = 0$

$\therefore$ $\frac{z_1}{z_2}$ is purely imaginary.

If z is purely imaginary, then $z + \bar{z} = 0$.

119. **(d)** For real roots, $D \ge 0$

$\Rightarrow (-4)^2 - 4(2\lambda-1)(2\lambda-1) \ge 0 \Rightarrow (2\lambda-1)^2 \le 4$

$\Rightarrow -2 \le 2\lambda - 1 \le 2$

$\Rightarrow -\frac{1}{2} \le \lambda \le \frac{3}{2}$

$\therefore$ Integral values of λ are 0 and 1

Hence, greatest integral value of $\lambda = 1$.

120. **(a)** We have, $\arg(z) = 0$

$\Rightarrow$ z is purely real

$\therefore$ Reason is true.

Also, $|z_1| = |z_2| + |z_1 - z_2|$

$\Rightarrow |z_1 - z_2|^2 = (|z_1| - |z_2|)^2$

$\Rightarrow |z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos(\theta_1-\theta_2)$

$= |z_1|^2 + |z_2|^2 - 2|z_1||z_2|$

$\Rightarrow \cos(\theta_1 - \theta_2) = 1 \quad \Rightarrow \theta_1 - \theta_2 = 0$

$\Rightarrow \arg(z_1) - \arg(z_2) = 0$

$\Rightarrow \arg\left(\frac{z_1}{z_2}\right) = 0$

$\Rightarrow \frac{z_1}{z_2}$ is purely real.

$\Rightarrow \text{Im}\left(\frac{z_1}{z_2}\right) = 0$

121. (c) Assertion is a standard result.
$|z - (2 + 3i)| = 4$
$\Rightarrow$ Distance of P(z) from the point (2, 3) is equal to 4.
$\Rightarrow$ Locus of P is a circle with centre at (2, 3) and radius 4.

122. (d) We have,
$ix^2 - 3ix + 2i = 0$
or $i(x^2 - 3x + 2) = 0$
$\Rightarrow x^2 - 3x + 2 = 0 \qquad [\because i \neq 0]$
$\Rightarrow x = 1, 2$, which are real.

CRITICALTHINKING TYPE QUESTIONS

123. (c) Given $|z-4| < |z-2|$ Let $z = x + iy$
$\Rightarrow |(x-4)+iy)| < |(x-2)+iy|$
$\Rightarrow (x-4)^2 + y^2 < (x-2)^2 + y^2$
$\Rightarrow x^2 - 8x + 16 < x^2 - 4x + 4 \Rightarrow 12 < 4x$
$\Rightarrow x > 3 \Rightarrow \text{Re}(z) > 3$

124. (d) The given equation is $x^2 - 3x + 3 = 0$
Let a, b be the roots of the given equation then,
$a + b = 3, ab = 3$
We know, $(a - b)^2 = (a + b)^2 - 4ab = 9 - 12$
$\Rightarrow a - b = \sqrt{3}i$
So, $a = \frac{3+\sqrt{3}i}{2}$ and $b = \frac{3-\sqrt{3}i}{2}$
If A and B are the roots of the new equation which are double of the founded roots then
$A = 3 + \sqrt{3}i$ and $B = 3 - \sqrt{3}i$
So, $A + B = 6$ and $AB = 9 + 3 = 12$
Thus the new equation is
$x^2 - 6x + 12 = 0$

125. (b) We have, $4^x - 3 \,.\, 2^{x+3} + 128 = 0$
$\Rightarrow 2^{2x} - 3 \,.\, 2^x \,.\, 2^3 + 128 = 0$
$\Rightarrow 2^{2x} - 24 \,.\, 2^x + 128 = 0$
$\Rightarrow y^2 - 24y + 128 = 0$ where $2^x = y$
$\Rightarrow (y - 16)(y - 8) = 0 \Rightarrow y = 16, 8$
$\Rightarrow 2^x = 16$ or $2^x = 8 \Rightarrow x = 4$ or 3

126. (d) 4 is a root of $x^2 + px + 12 = 0$
$\Rightarrow 16 + 4p + 12 = 0 \Rightarrow p = -7$

Now, the equation $x^2 + px + q = 0$
has equal roots.

$\therefore p^2 - 4q = 0 \Rightarrow q = \frac{p^2}{4} = \frac{49}{4}$

127. (c) Let α, α^2 be the roots of $3x^2 + px + 3$.
$\therefore \quad \alpha + \alpha^2 = -p/3$ and $\alpha^3 = 1$
$\Rightarrow \quad (\alpha - 1)(\alpha^2 + \alpha + 1) = 0$
$\Rightarrow \quad \alpha = 1$ or $\alpha^2 + \alpha = -1$
If $\alpha = 1, p = -6$ which is not possible as $p > 0$
If $\alpha^2 + \alpha = -1 \Rightarrow -p/3 = -1 \Rightarrow p = 3$.

128. (a) Given expression

$$= \frac{i^{10}\left(i^{582} + i^{580} + i^{578} + i^{576} + i^{574}\right)}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} - 1$$

$= i^{10} - 1 = (i^2)^5 - 1 = (-1)^5 - 1$
$= -1 - 1 = -2$

129. (c) $|z| = \frac{|1+i\sqrt{3}|\,|\cos\theta + i\sin\theta|}{2|1-i|\,|\cos\theta - i\sin\theta|} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

130. (b) Suppose, $z = \frac{1+2i}{1-(1-i)^2}$

$= \frac{1+2i}{1-(1^2 + i^2 - 2i)} \qquad [\text{using } (a-b)^2]$

$= \frac{1+2i}{1+2i} \qquad (\because i^2 = -1)$

$= 1 = 1 + 0\,.\,i$

$|z| = \sqrt{(\text{Real part})^2 + (\text{Img. Part})^2}$

and amp $(z) = \tan^{-1}\left[\frac{\text{Img. part}}{\text{Real part}}\right]$

$\therefore \quad |z| = 1$ and amp $(z) = \tan^{-1}\left(\frac{0}{1}\right) = 0$

131. (a) Let $z = x + iy$,
$\because \quad |z^2 - 1| = |z|^2 + 1$
$\Rightarrow \quad |x^2 - y^2 - 1 + i2xy| = |x + iy|^2 + 1$
$\Rightarrow \quad (x^2 - y^2 - 1)^2 + 4x^2y^2 = (x^2 + y^2 + 1)^2$
$\Rightarrow \quad 4x^2 = 0 \Rightarrow x = 0$
Hence, z lies on y-axis or imaginary axis.

132. (d) $(z-1)(\bar{z}-5) + (\bar{z}-1)(z-5)$

$= 2\text{Re}\left[(z-1)(\bar{z}-5)\right]$

$\left[\because z_1\bar{z}_2 + z_2\bar{z}_1 = 2\text{Re}(z_1 z_2)\right]$

$= 2\text{ Re}[(1 + i)(-3 - i)] = 2(-2) = -4$

[Given $z = 2 + i$]

133. (b) Given, $z = r(\cos\theta + i\sin\theta)$;

$\bar{z} = r(\cos\theta - i\sin\theta)$

$$\therefore \quad \frac{z}{\bar{z}} = \frac{r(\cos\theta + i\sin\theta)}{r(\cos\theta - i\sin\theta)}$$

$= (\cos\theta + i\sin\theta)(\cos\theta - i\sin\theta)^{-1}$

$= (\cos\theta + i\sin\theta)(\cos\theta + i\sin\theta)$

$= (\cos\theta + i\sin\theta)^2 = \cos 2\theta + i\sin 2\theta$

$$\therefore \quad \frac{\bar{z}}{z} = (\cos\theta - i\sin\theta)(\cos\theta + i\sin\theta)^{-1}$$

$= (\cos\theta - i\sin\theta)(\cos\theta - i\sin\theta)$

$= (\cos\theta - i\sin\theta)^2 = (\cos 2\theta - i\sin 2\theta)$

$$\therefore \quad \frac{z}{\bar{z}} + \frac{\bar{z}}{z} = \cos 2\theta + i\sin 2\theta + \cos 2\theta - i\sin 2\theta$$

$= 2\cos 2\theta$

134. (b) Let $z = i = \frac{1}{2} + \frac{1}{2}i^2 + 2\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}i$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}i\right)^2 + 2\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}i$$

$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)^2$$

$$\therefore \quad \sqrt{i} = \frac{\pm 1}{\sqrt{2}}(1+i).$$

135. (a) We have, $\left(x + \frac{1}{x}\right)^3 + \left(x + \frac{1}{x}\right) = 0$

$$\Rightarrow \left(x + \frac{1}{x}\right)\left[\left(x + \frac{1}{x}\right)^2 + 1\right] = 0$$

$\Rightarrow$ either $x + \frac{1}{x} = 0$

$\Rightarrow x^2 = -1 \Rightarrow x = \pm i$

or $\left(x + \frac{1}{x}\right)^2 + 1 = 0$

$$\Rightarrow x^2 + \frac{1}{x^2} + 3 = 0$$

$\Rightarrow x^4 + 3x^2 + 1 = 0$

$$\Rightarrow x^2 = \frac{-3 \pm \sqrt{9-4}}{2} = \frac{-3 \pm \sqrt{5}}{2} < 0$$

$\therefore$ There is no real root.

136. (d) Given equation is $\frac{a}{x-a} + \frac{b}{x-b} = 1$

$\Rightarrow a(x - b) + b(x - a) = (x - a)(x - b)$

$\Rightarrow x^2 - x(a + b) + ab = ax - ab + bx - ab$

$\Rightarrow x^2 - 2x(a + b) + 3ab = 0$

So, sum of roots $= \alpha + (-\alpha) = 2(a + b)$

or $a + b = 0$.

137. (c) Let α, β be the roots of the equation.

$\therefore \alpha + \beta = a - 2$ and $\alpha\beta = -(a + 1)$

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (a - 2)^2 + 2(a + 1)$

$= (a - 1)^2 + 5$

$\therefore \alpha^2 + \beta^2$ will be minimum if $(a - 1)^2 = 0$, i.e. $a = 1$.

138. (d) Since α, β are roots of the equation

$(x - a)(x - b) = 5$ or $x^2 - (a + b)x + (ab - 5) = 0$

$$\left.\begin{aligned}\therefore\ \alpha + \beta = a + b \ \text{ or } \ a + b = \alpha + \beta \\ \text{and } \alpha\beta = ab - 5 \ \text{ or } \ ab = \alpha\beta + 5\end{aligned}\right\} \quad \text{... (i)}$$

Taking another equation

$(x - \alpha)(x - \beta) + 5 = 0$

or $x^2 - (\alpha + \beta)x + (\alpha\beta + 5) = 0$

or $x^2 - (a + b)x + ab = 0$ [using (i)]

$\therefore$ Its roots are a, b.

139. (b) Given, $\left|\frac{i+z}{i-z}\right| = 1$

Let $z = x + iy$

$$\therefore \quad \left|\frac{i + x + iy}{i - (x + iy)}\right| = 1$$

$$\Rightarrow \left|\frac{x + i(1+y)}{-x + i(1-y)}\right| = 1$$

$$\Rightarrow \sqrt{x^2 + (1+y)^2} = \sqrt{(-x)^2 + (1-y)^2}$$

$\Rightarrow x^2 + 1 + y^2 + 2y = x^2 + 1 + y^2 - 2y$

$\Rightarrow 4y = 0 \Rightarrow y = 0$

Hence, z lies on x-axis.

140. (a) $(z+3)(\bar{z}+3) = z\bar{z} + 3z + 3\bar{z} + (3)^2$

$$= |z|^2 + 3\left(\frac{z+\bar{z}}{2}\right) \times 2 + 3^2 \qquad \left[\because |z|^2 = z\bar{z}\right]$$

$= |z|^2 + 2 \times 3 \times (\text{Re}(z)) + 3^2$

$= |z|^2 + 2\text{Re}(3z) + (3)^2 = |z + 3|^2$

141. (c) Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$,

$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$\because |z_1 + z_2| = |z_1| + |z_2|$ [given]

$\Rightarrow |(r_1\cos\theta_1 + r_2\cos\theta_2) + i(r_1\sin\theta_1 + r_2\sin\theta_2)|$

$= r_1 + r_2$

$$\Rightarrow \sqrt{r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1 - \theta_2)} = r_1 + r_2$$

$\Rightarrow r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1 - \theta_2) = r_1^2 + r_2^2 + 2r_1r_2$

$\Rightarrow \cos(\theta_1 - \theta_2) = 1$

$\Rightarrow \theta_1 - \theta_2 = 0 \Rightarrow \theta_1 = \theta_2$

$\Rightarrow \arg(z_1) = \arg(z_2)$.

142. (d) Let $z = \sin x + i\cos 2x$

According to the given condition,

$\bar{z} = \cos x - i\sin 2x$

$\therefore$ sin x – i cos 2x = cos x – i sin 2x

$\Rightarrow$ (sin x – cos x) + i(sin 2x – cos 2x) = 0

On equating real and imaginary parts, we get

sin x – cos x = 0, sin 2x – cos 2x = 0

$\Rightarrow$ tan x = 1 and tan 2x = 1

$\Rightarrow x = \frac{\pi}{4}$ and $2x = \frac{\pi}{4}$

$\Rightarrow x = \frac{\pi}{4}$ and $x = \frac{\pi}{8}$

which is not possible.

143. (a) Let z = x + iy

$\therefore |z + 3 - i| = |(x + 3) + i(y - 1)| = 1$

$\Rightarrow \sqrt{(x+3)^2 + (y-1)^2} = 1$ ... (i)

$\because$ arg z = π

$\Rightarrow \tan^{-1}\frac{y}{x} = \pi$

$\Rightarrow \frac{y}{x} = \tan \pi = 0$

$\Rightarrow$ y = 0 ... (ii)

From equations (i) and (ii), we get

x = –3, y = 0

$\therefore$ z = –3

$\Rightarrow |z| = |-3| = 3$

144. (b) Given that :

$$Z = \frac{1-i}{\frac{1}{2} + \frac{\sqrt{3}}{2}i}$$

$$= \frac{2(i-1)}{1+i\sqrt{(3)}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$$

$$= \frac{2\left(i+\sqrt{3}-1+i\sqrt{3}\right)}{1+3}$$

$$= \frac{\sqrt{3}-1}{2} + \frac{\sqrt{3}+1}{2}i$$

Now, put $\frac{\sqrt{3}-1}{2} = r\cos\theta$, $\frac{\sqrt{3}+1}{2} = r\sin\theta$

Squaring and adding, we obtain

$$r^2 = \left(\frac{\sqrt{3}-1}{2}\right)^2 + \left(\frac{\sqrt{3}+1}{2}\right)^2$$

$$= \frac{2\left(\sqrt{3}\right)^2+1}{4} = \frac{2\times 4}{4} = 2$$

Hence, $r = \sqrt{2}$ which gives :

$$\cos\theta = \frac{\sqrt{3}-1}{2\sqrt{2}}, \ \sin\theta = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Therefore, $\theta = \frac{\pi}{4} + \frac{\pi}{6} = \frac{5\pi}{12}$

Hence, the polar form is $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$.

145. (a) (x – iy) (3 + 5i)

$= 3x + 5xi - 3yi - 5yi^2$

$= 3x + (5x - 3y)i + 5y$ $[\because i^2 = -1]$

$= (3x + 5y) + (5x - 3y)i$ ... (i)

Given, (x – iy) (3 + 5i) = –6 + 24i

$\left[\begin{array}{l}\text{using equation (i), and } z = (a+ib) \\ \Rightarrow \bar{z} = (a-ib)\end{array}\right]$

On comparing the real and imaginary parts of both sides, we get

3x + 5y = –6 and 5x – 3y = 24

Solving the above equations by substitution or elimination method, we get

x = 3, y = –3

146. (b) Let z = x + iy, then $\frac{z-1}{z+1} = \frac{x-1+iy}{x+1+iy}$

$$= \frac{(x-1+iy)}{(x+1+iy)} \times \frac{(x+1-iy)}{(x+1-iy)}$$

$$= \frac{\left[\left(x^2-1\right) - i(x-1)y + i(x+1)y + y^2\right]}{\left[(x+1)^2 + y^2\right]}$$

For purely imaginary, real (z) = 0

$\Rightarrow x^2 + y^2 = 1, |z| = 1.$

147. (c) $\sin\frac{\pi}{5} + i\left(1 - \cos\frac{\pi}{5}\right) = 2\sin\frac{\pi}{10}\cos\frac{\pi}{10} + i\,2\sin^2\frac{\pi}{10}$

$$= 2\sin\frac{\pi}{10}\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)$$

For amplitude, $\tan\theta = \frac{\sin\frac{\pi}{10}}{\cos\frac{\pi}{10}} = \tan\frac{\pi}{10} \Rightarrow \theta = \frac{\pi}{10}$.

148. (a) $x + iy = \sqrt{\frac{a+ib}{c+id}} \Rightarrow x - iy = \sqrt{\frac{a-ib}{c-id}}$

Also, $x^2 + y^2 = (x + iy)(x - iy) = \sqrt{\frac{a^2+b^2}{c^2+d^2}}$

$\Rightarrow (x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$.

149. (b) As sum of coefficients is zero, hence one root is 1 and other root is $\frac{l-m}{m-n}$.

Since roots are equal,

$\therefore \frac{l-m}{m-n} = 1 \Rightarrow 2m = n + l.$

150. (b) It is given that

$$\alpha\beta = 2 \Rightarrow \frac{3a+4}{a+1} = 2$$

$$\Rightarrow 3a + 4 = 2a + 2 \Rightarrow a = -2$$

Also, $\alpha + \beta = -\frac{2a+3}{a+1}$

Putting this value of a, we get sum of roots

$$= -\frac{2a+3}{a+1} = -\frac{-4+3}{-2+1} = -1.$$

151. (d) $\alpha + \beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$ and $\alpha^2 + \beta^2 = \frac{(b^2 - 2ac)}{a^2}$

Now, $\frac{\alpha}{a\beta + b} + \frac{\beta}{a\alpha + b} = \frac{\alpha(a\alpha + b) + \beta(a\beta + b)}{(a\beta + b)(a\alpha + b)}$

$$= \frac{a(\alpha^2 + \beta^2) + b(\alpha + \beta)}{\alpha\beta a^2 + ab(\alpha + \beta) + b^2} = \frac{a\frac{(b^2 - 2ac)}{a^2} + b\left(-\frac{b}{a}\right)}{\left(\frac{c}{a}\right)a^2 + ab\left(-\frac{b}{a}\right) + b^2}$$

$$= \frac{b^2 - 2ac - b^2}{a^2c - ab^2 + ab^2} = \frac{-2ac}{a^2c} = -\frac{2}{a}.$$

152. (a) Let α, α^2 be the two roots. Then,

$$\alpha + \alpha^2 = -\frac{b}{a} \qquad \text{... (i)}$$

$$\text{and } \alpha \cdot \alpha^2 = \frac{c}{a} \qquad \text{... (ii)}$$

On cubing both sides of (i),

$$\alpha^3 + \alpha^6 + 3\alpha\alpha^2(\alpha + \alpha^2) = -\frac{b^3}{a^3}$$

$$\Rightarrow \frac{c}{a} + \frac{c^2}{a^2} + 3\frac{c}{a}\left(-\frac{b}{a}\right) = -\frac{b^3}{a^3} \qquad \text{[by (i) and (ii)]}$$

$$\Rightarrow b^3 + ac^2 + a^2c = 3abc.$$

153. (c) Given $(x - a)(x - b) = c$

$\therefore$ $\alpha + \beta = a + b$ and $\alpha\beta = ab - c$

Now, given equation $(x - \alpha)(x - \beta) + c = 0$

$\Rightarrow$ $x^2 - (\alpha + \beta)x + \alpha\beta + c = 0$

If its roots be p and q, then

$p + q = (\alpha + \beta) = a + b$

$pq = \alpha\beta + c = ab - c + c = ab$

So, it can be given by $x^2 - (a + b)x + ab = 0$

So, its roots will be a and b.

154. (a) $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now, $\alpha\beta^2 + \alpha^2\beta + \alpha\beta = \alpha\beta(\beta + \alpha) + \alpha\beta$

$$= \alpha\beta(1 + \alpha + \beta) = \frac{c}{a}\left\{1 + \left(-\frac{b}{a}\right)\right\} = \frac{c(a-b)}{a^2}.$$

155. (b) Since α, β are the roots of the equation $2x^2 - 35x + 2 = 0$

Also, $\alpha\beta = 1$

$$\therefore \quad 2\alpha^2 - 35\alpha = -2 \text{ or } 2\alpha - 35 = \frac{-2}{\alpha}$$

$$2\beta^2 - 35\beta = -2 \text{ or } 2\beta - 35 = \frac{-2}{\beta}$$

Now, $(2\alpha - 35)^3 (2\beta - 35)^3 = \left(\frac{-2}{\alpha}\right)^3 \left(\frac{-2}{\beta}\right)^3$

$$= \frac{8 \cdot 8}{\alpha^3\beta^3} = \frac{64}{1} = 64.$$

156. (c) Let α, β are roots of $x^2 + px + q = 0$

So, $\alpha + \beta = -p$ and $\alpha\beta = q$

Given that $(\alpha + \beta) = 3(\alpha - \beta) = -p$

$$\Rightarrow \alpha - \beta = \frac{-p}{3}$$

Now, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$$\Rightarrow \frac{p^2}{9} = p^2 - 4q \text{ or } 2p^2 = 9q.$$

157. (c) Let α, β be the roots of $x^2 + bx + c = 0$ and α', β' be the roots of $x^2 + qx + r = 0$.

Then, $\alpha + \beta = -b$, $\alpha\beta = c$, $\alpha' + \beta' = -q$, $\alpha'\beta' = r$

It is given that $\frac{\alpha}{\beta} = \frac{\alpha'}{\beta'} \Rightarrow \frac{\alpha + \beta}{\alpha - \beta} = \frac{\alpha' + \beta'}{\alpha' - \beta'}$

$$\Rightarrow \frac{(\alpha + \beta)^2}{(\alpha - \beta)^2} = \frac{(\alpha' + \beta')^2}{(\alpha' - \beta')^2} \Rightarrow \frac{b^2}{b^2 - 4c} = \frac{q^2}{q^2 - 4r}$$

$$\Rightarrow b^2 r = q^2 c.$$

158. (b) Since roots of the equation $x^2 - 5x + 16 = 0$ are α, β.

$$\Rightarrow \alpha + \beta = 5 \text{ and } \alpha\beta = 16 \text{ and } \alpha^2 + \beta^2 + \frac{\alpha\beta}{2} = -p$$

$$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta + \frac{\alpha\beta}{2} = -p$$

$$\Rightarrow 25 - 32 + 8 = -p$$

$$\Rightarrow p = -1 \text{ and } (\alpha^2 + \beta^2)\left(\frac{\alpha\beta}{2}\right) = q$$

$$\Rightarrow [(\alpha + \beta)^2 - 2\alpha\beta]\left[\frac{\alpha\beta}{2}\right] = q$$

$$\Rightarrow q = [25 - 32]\frac{16}{2} = -56$$

So, $p = -1$, $q = -56$.

159. (a) Let the roots are α and β.

$\Rightarrow \dfrac{\alpha+\beta}{2} = \dfrac{8}{5}$

$\Rightarrow \alpha + \beta = \dfrac{16}{5}$ ... (i)

and $\dfrac{\frac{1}{\alpha}+\frac{1}{\beta}}{2} = \dfrac{8}{7} \Rightarrow \dfrac{\alpha+\beta}{2\alpha\beta} = \dfrac{8}{7} \Rightarrow \dfrac{\left(\frac{16}{5}\right)}{2\left(\frac{8}{7}\right)} = \alpha\beta$

$\Rightarrow \alpha\beta = \dfrac{7}{5}$ ... (ii)

$\therefore$ Equation is $x^2 - \left(\dfrac{16}{5}\right)x + \dfrac{7}{5} = 0$

$\Rightarrow 5x^2 - 16x + 7 = 0$

160. (c) $4x^2 + 5k = (5k + 1)x$

$\Rightarrow 4x^2 - (5k + 1)x + 5k = 0$; $(\alpha - \beta) = 1$

$\therefore \alpha + \beta = \dfrac{(5k+1)}{4}$ and $\alpha\beta = \dfrac{5k}{4}$

Now, $\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$\Rightarrow \alpha - \beta = \sqrt{\dfrac{(5k+1)^2}{16} - \dfrac{4\cdot 5k}{4}} = 1$

$\therefore 25k^2 - 70k - 15 = 0$

$\Rightarrow (5k + 1)(k - 3) = 0 \Rightarrow k = -\dfrac{1}{5}, 3.$

161. (b) **Case I :** $x - 2 > 0$, Putting $x - 2 = y$, $y > 0$

$x > 2$

$\therefore y^2 + y - 2 = 0 \Rightarrow y = -2, 1$

$\Rightarrow x = 0, 3$

But $0 < 2$, Hence $x = 3$ is the real root.

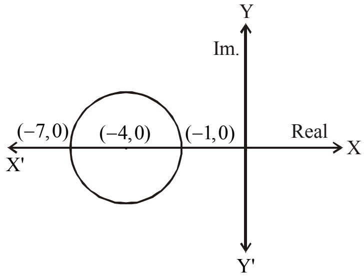
Case II : $x - 2 < 0 \Rightarrow x < 2, y < 0$

$y^2 - y - 2 = 0 \Rightarrow y = 2, -1 \Rightarrow x = 4, x = 1$

Since $4 \not< 2$, only $x = 1$ is the real root.

Hence the sum of the real roots $= 3 + 1 = 4$

162. (a) z lies on or inside the circle with centre (– 4, 0) and radius 3 units.



From the Argand diagram maximum value of $|z+1|$ is 6.

163. (b) $\dfrac{(\cos\theta + i\sin\theta)^4}{(\cos\theta - i\sin\theta)^3} = (\cos\theta + i\sin\theta)^4 (\cos\theta - i\sin\theta)^{-3}$

$= (\cos 4\theta + i\sin 4\theta)\{\cos(-\theta) + i\sin(-\theta)\}^{-3}$

$= (\cos 4\theta + i\sin 4\theta)\{\cos(-3)(-\theta) + i\sin(-3)(-\theta)\}$

$= (\cos 4\theta + i\sin 4\theta)\{\cos 3\theta + i\sin 3\theta\}$

$= \cos 4\theta\cos 3\theta - \sin 4\theta\sin 3\theta$

$+ i(\sin 4\theta\cos 3\theta + \sin 3\theta\cos 4\theta)$

$= \cos(4\theta + 3\theta) + i\sin(4\theta + 3\theta) = \cos 7\theta + i\sin 7\theta$

164. (b) Let $x = 2 + \dfrac{1}{2 + \dfrac{1}{2 + ...\infty}}$

$\Rightarrow x = 2 + \dfrac{1}{x}$ [On simplification]

$\Rightarrow x = 1 \pm \sqrt{2}$

But the value of the given expression cannot be negative or less than 2, therefore $1 + \sqrt{2}$ is required answer.

165. (c) $x = \sqrt{6+x} \Rightarrow x^2 = 6 + x$

$\Rightarrow x^2 - x - 6 = 0 \Rightarrow (x - 3)(x + 2) = 0 \Rightarrow x = 3, -2$

$x = -2$ will be rejected as $x > 0$. Hence, $x = 3$ is the solution.