

बहुविकल्पीय प्रश्न (Multiple choice question)

1. $\int \frac{dx}{1+x^2} = \dots\dots\dots$

- a) $\sin^{-1}x + c$ b) $\tan^{-1}x + c$
 c) $\log(1+x^2) + c$ d) $\cot^{-1}x + c$

2. $\int \frac{2x}{1+x^2} = \dots\dots\dots$

- a) $\sin^{-1}x + c$ b) $\tan^{-1}x + c$
 c) $\log(1+x^2) + c$ d) $\cot^{-1}x + c$

3. $\int x^n dx = \dots\dots\dots$

- a) $x^{n+1} + c$ b) $x^{n-1} + c$
 c) $\frac{x^n}{n+1} + c$ d) $\frac{x^{n+1}}{n+1} + c$

4. $\int e^{2x} dx = \dots\dots\dots$

- a) $2e^x + c$ b) $e^{-x} + c$
 c) $\frac{e^{2x}}{2} + c$ d) $\frac{e^{-2x}}{2} + c$

5. $\int \frac{1}{x} dx = \dots\dots\dots$

- a) $-\frac{1}{x^2} + c$ b) 1
 c) $\log x + c$ d) None of these

6. $\int \sqrt{x} dx = \dots\dots\dots$

- a) $x^{1/2} + c$ b) $x^{3/2} + c$
 c) $\frac{2}{3}x^{3/2} + c$ d) None of these

7. $\int (3x+5)dx = \dots\dots\dots$

- a) $3x^2 + 5x + c$ b) $3x^3 + 5x + c$
 c) $\frac{3x^2}{5} + c$ d) $\frac{3x^2}{2} + 5x + c$

ANSWER

- (1) b (2) c (3) d
 (4) c (5) c (6) c
 (7) d

रिक्त स्थानों को भरें (Fill in the blanks)

1. $\int (x^{2/3} + 1)dx = \dots\dots\dots$

2. $\int (4e^{3x} + 1)dx = \dots\dots\dots$

3. $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx = \dots\dots\dots$

4. $\int (x+2)dx = \dots\dots\dots$

5. $\int e^x [f(x) + f'(x)]dx = \dots\dots\dots$

6. $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx = \dots\dots\dots$

7. $\int \sqrt{ax+b} dx = \dots\dots\dots$

ANSWER

1. $\frac{3}{5}x^{5/3} + x + c$

2. $\frac{4}{3}e^{3x} + x + c$

3. $\frac{2}{3}x^{3/2} + 2x^{1/2} + c$

4. $\frac{x^2}{2} + 2x + c$

5. $e^x \cdot f(x) + c$

6. $\tan x - x + c$

7. $\frac{2(ax+b)^{3/2}}{3a}$

लघु उत्तरीय प्रश्न (Short Answer type question)

1. Evaluate $\int \frac{x+1}{x-1} dx$

Ans :-

$$\begin{aligned}\text{We have } I &= \int \frac{x+1}{x-1} dx \\ &= \int \frac{x-1+2}{x-1} dx \\ &= \int \left(\frac{x-1}{x-1} + \frac{2}{x-1} \right) dx \\ &= \int \left(1 + \frac{2}{x-1} \right) dx \\ &= \int 1 dx + \int \frac{2}{x-1} dx \\ &= x + 2 \log|x-1| + c\end{aligned}$$

2. Evaluate $\int (4x^3 + 3x^2 + 2x + 4) dx$

Ans :-

$$\begin{aligned}\text{We have } I &= \int (4x^3 + 3x^2 + 2x + 4) dx \\ &= \int 4x^3 dx + \int 3x^2 dx + \int 2x dx + \int 4 dx \\ &= 4 \cdot \frac{x^4}{4} + 3 \cdot \frac{x^3}{3} + 2 \cdot \frac{x^2}{2} + 4x + c \\ &= x^4 + x^3 + x^2 + 4x + c\end{aligned}$$

3. Integrate $\int e^{3x-5} dx$

Ans :-

$$\begin{aligned}\text{We have } I &= \int e^{3x-5} dx \\ \text{Put } 3x-5 &= t \\ \therefore 3dx &= dt \\ \therefore dx &= \frac{1}{3} dt \\ \therefore I &= \frac{1}{3} \int e^t dt = \frac{1}{3} e^t + c = \frac{1}{3} e^{3x-5} + c\end{aligned}$$

4. Evaluate $\int \frac{x^4 + 3x^2 + 1}{x^3} dx$

Ans :-

$$\begin{aligned}\text{We have } I &= \int \frac{x^4 + 3x^2 + 1}{x^3} dx \\ &= \int \left(\frac{x^4}{x^3} + \frac{3x^2}{x^3} + \frac{1}{x^3} \right) dx \\ &= \int \left(x + \frac{3}{x} + \frac{1}{x^3} \right) dx \\ &= \int x dx + 3 \int \frac{1}{x} dx + \int \frac{1}{x^3} dx\end{aligned}$$

$$\begin{aligned}&= \frac{x^2}{2} + 3 \log|x| + \left(\frac{-1}{2x^2} \right) + c \\ &= \frac{x^2}{2} + 3 \log|x| - \frac{1}{2x^2} + c\end{aligned}$$

5. Evaluate $\int 4x^2 \sqrt{x^3 + 3} dx$

Ans :-

$$\begin{aligned}\text{We have } I &= \int 4x^2 \sqrt{x^3 + 3} dx \\ \text{Put } x^3 + 3 &= t \quad \therefore 3x^2 dx = dt \\ \therefore x^2 dx &= \frac{1}{3} dt \\ \therefore I &= \frac{4}{3} \int \sqrt{t} dt = \frac{4}{3} \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c \\ &= \frac{4}{3} \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{4}{3} \times \frac{2}{3} t^{\frac{3}{2}} + c \\ &= \frac{8}{9} (x^3 + 3)^{\frac{3}{2}} + c\end{aligned}$$

6. Evaluate $\int \left(\frac{1+x \log x}{x} \right) e^x dx$

Ans :-

$$\begin{aligned}\text{We have } I &= \int \left(\frac{1+x \log x}{x} \right) e^x dx \\ &= \int e^x \left(\log x + \frac{1}{x} \right) dx \dots\dots\dots \text{eqn(1)}\end{aligned}$$

$$\text{Let } f(x) = \log x \quad \therefore f'(x) = \frac{1}{x}$$

$\therefore$ From eqn(1)

$$\begin{aligned}I &= \int e^x [f(x) + f'(x)] dx \\ &= e^x f(x) + c \\ &= e^x \log x + c\end{aligned}$$

7. Integrate $\frac{2x}{1+x^2}$

Ans :-

$$\begin{aligned}\text{We have } I &= \int \frac{2x}{1+x^2} dx \\ \text{Put } 1+x^2 &= t \quad \therefore 2x dx = dt \\ \therefore I &= \int \frac{dt}{t} = \log|t| + c \\ &= \log|1+x^2| + c\end{aligned}$$

8. Evaluate $\int \frac{(x^3+8)(x-1)}{x^2-2x+4} dx$

Ans :-

$$\begin{aligned}\text{We have } I &= \int \frac{(x^3+8)(x-1)}{x^2-2x+4} dx \\ &= \int \frac{(x^3+2^3)(x-1)}{x^2-2x+4} dx \\ &= \int \frac{(x+2)(x^2-2x+4)(x-1)}{(x^2-2x+4)} dx \\ &= \int (x+2)(x-1) dx \\ &= \int (x^2+x-2) dx \\ &= \frac{x^3}{3} + \frac{x^2}{2} - 2x + c\end{aligned}$$

9. If $\frac{dy}{dx} = 5x^2 + 2$, then find the value of y.

Ans :-

$$\begin{aligned}\text{We have } \frac{dy}{dx} &= 5x^2 + 2 \\ \therefore dy &= (5x^2 + 2) dx \\ \text{Integrating both sides, we get} \\ \int dy &= \int (5x^2 + 2) dx \\ y &= 5 \cdot \frac{x^3}{3} + 2x + c \\ \therefore y &= \frac{5}{3}x^3 + 2x + c\end{aligned}$$

10. Evaluate $\int \frac{x^2+1}{x-1} dx$

Ans :-

$$\begin{aligned}\text{We have } I &= \int \frac{x^2+1}{x-1} dx = \int \frac{x^2-1+2}{x-1} dx \\ &= \int \left(\frac{x^2-1}{x-1} + \frac{2}{x-1} \right) dx \\ &= \int \left[(x+1) + \frac{2}{x-1} \right] dx \\ &= \frac{x^2}{2} + x + 2\log(x-1) + c\end{aligned}$$

11. Integrate $\int \left(6e^x - \frac{3}{x^2} \right) dx$

Ans :-

$$\begin{aligned}\text{We have } I &= \int \left(6e^x - \frac{3}{x^2} \right) dx \\ &= \int 6e^x dx - \int \frac{3}{x^2} dx\end{aligned}$$

$$\begin{aligned}&= 6 \int e^x dx - 3 \int \frac{1}{x^2} dx \\ &= 6e^x - 3 \left(-\frac{1}{x} \right) + c \\ &= 6e^x + \frac{3}{x} + c\end{aligned}$$

12. Evaluate $\int (4x^3 - 3x^2 + 2x + 1) dx$

Ans :-

$$\begin{aligned}\text{We have } I &= \int (4x^3 - 3x^2 + 2x + 1) dx \\ &= \int 4x^3 dx - \int 3x^2 dx + \int 2x dx + \int 1 dx \\ &= 4 \cdot \frac{x^4}{4} - 3 \cdot \frac{x^3}{3} + 2 \cdot \frac{x^2}{2} + x + c \\ &= x^4 - x^3 + x^2 + x + c\end{aligned}$$

13. Evaluate $\int (ax^3 + bx^2 + cx + d) dx$

Ans :-

$$\begin{aligned}\text{We have } I &= \int (ax^3 + bx^2 + cx + d) dx \\ &= \int ax^3 dx + \int bx^2 dx + \int cxdx + \int ddx \\ &= a \cdot \frac{x^4}{4} + b \cdot \frac{x^3}{3} + c \cdot \frac{x^2}{2} + dx + k \quad \left(\begin{array}{l} \text{where} \\ k = \text{constant} \end{array} \right) \\ \therefore I &= \frac{ax^4}{4} + \frac{bx^3}{3} + \frac{cx^2}{2} + dx + k\end{aligned}$$

14. Evaluate $I = \int \frac{(x-3)^2}{\sqrt{x}} dx$

Ans :-

$$\begin{aligned}\text{We have } I &= \int \frac{(x-3)^2}{\sqrt{x}} dx \\ &= \int \frac{x^2 - 6x + 9}{\sqrt{x}} dx = \int (x^{3/2} - 6x^{1/2} + 9x^{-1/2}) dx \\ &= \frac{x^{3/2+1}}{\frac{3}{2}+1} - 6 \cdot \frac{x^{1/2+1}}{\frac{1}{2}+1} + 9 \cdot \frac{x^{-1/2+1}}{-\frac{1}{2}+1} + c \\ &= \frac{x^{5/2}}{5/2} - 6 \cdot \frac{x^{3/2}}{3/2} + 9 \cdot \frac{x^{1/2}}{1/2} + c \\ &= \frac{2}{5}x^{5/2} - 6 \times \frac{2}{3}x^{3/2} + 9 \times 2x^{1/2} + c \\ &= \frac{2}{5}x^{5/2} - 4x^{3/2} + 18x^{1/2} + c\end{aligned}$$

15. Evaluate $\int x\sqrt{x^2+5} dx$

Ans :-

$$\text{We have } I = \int x\sqrt{x^2+5} dx$$

$$\text{Put } x^2+5 = t \quad \therefore 2x dx = dt$$

$$\therefore x dx = \frac{1}{2} dt$$

$$\therefore I = \frac{1}{2} \int \sqrt{t} dt = \frac{1}{2} \cdot \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$= \frac{1}{2} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{1}{2} \times \frac{2}{3} (x^2+5)^{\frac{3}{2}} + c$$

$$= \frac{1}{3} (x^2+5)^{\frac{3}{2}} + c$$