

CONIC SECTION

PRACTICE SHEET

1. If the latus rectum of an ellipse is equal to one half its minor axis, what is the eccentricity of the ellipse?
 (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$
 (c) $\frac{3}{4}$ (d) $\frac{\sqrt{15}}{4}$
2. P(2,2) is a point on the parabola $y^2 = 2x$ and A is its vertex. Q is another point on the parabola such that PQ is perpendicular to AP. What is the length of PQ?
 (a) $\sqrt{2}$ (b) $2\sqrt{2}$
 (c) $4\sqrt{2}$ (d) $6\sqrt{2}$
3. The focal distance of a point on the parabola $y^2 = 12x$ is 4. What is the abscissa of the point?
 (a) 1 (b) -1
 (c) $2\sqrt{3}$ (d) -2
4. If (2, 0) is the vertex and the y - axis is the directrix of a parabola, then where is its focus?
 (a) (0, 0) (b) (-2, 0)
 (c) (4, 0) (d) (-4, 0)
5. Which one of the following points lies outside the ellipse $(x^2/a^2) + (y^2/b^2) = 1$?
 (a) (a, 0) (b) (0, b)
 (c) (-a, 0) (d) (a, b)
6. What is the equation of the parabola, whose vertex and focus are on the x-axis at distance a and b from the origin respectively? ($b > a > 0$)
 (a) $y^2 = 8(b-a)(x-a)$
 (b) $y^2 = 4(b+a)(x-a)$
 (c) $y^2 = 4(b-a)(x-a)$
 (d) $y^2 = 4(b-a)(x+a)$
7. If the eccentricity and length of latus rectum of a hyperbola are $\frac{\sqrt{13}}{3}$ and $\frac{10}{3}$ units respectively, then what is the length of the transverse axis?
 (a) $\frac{7}{2}$ unit (b) 12 unit
 (c) $\frac{15}{2}$ unit (d) $\frac{15}{4}$ unit
8. In how many points do the ellipse $\frac{x^2}{4} + \frac{y^2}{8} = 1$ and the circle $x^2 + y^2 = 9$ intersect?
 (a) One (b) Two
 (c) Four (d) None of the above
9. If the foci of the conics $\frac{x^2}{a^2} + \frac{y^2}{7} = 1$ and $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ were to coincide, then what is the value of a?
 (a) 2 (b) 3
 (c) 4 (d) 16
10. Which one of the following is correct? The eccentricity of the conic $\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1, (\lambda \geq 0)$
 (a) Increases with increase in λ
 (b) Decreases with increase in λ
 (c) Does not change with λ
 (d) None of the above
11. Consider the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($b > a$). Then, which one of the following is correct?
 (a) Real foci do not exist
 (b) Foci are $(\pm ae, 0)$
 (c) Foci are $(\pm be, 0)$
 (d) Foci are $(0, \pm be)$
12. Consider the parabolas $S_1 \equiv y^2 - 4ax = 0$ and $S_2 \equiv y^2 - 4bx = 0$. S_2 will contain S_1 , if
 (a) $a > b > 0$
 (b) $b > a > 0$
 (c) $a > 0, b < 0$ but $|b| > a$
 (d) $a < 0, b > 0$ but $b > |a|$
13. Equation of the hyperbola with eccentricity $3/2$ and foci at $(\pm 2, 0)$ is $5x^2 - 4y^2 = k^2$. What is the value of k?
 (a) $4/3$ (b) $3/4$
 (c) $(4/3)\sqrt{5}$ (d) $(3/4)\sqrt{5}$
14. What is the eccentricity of an ellipse, if its latusrectum is equal to one - half of its minor axis?
 (a) $1/4$ (b) $1/2$
 (c) $\sqrt{3}/4$ (d) $\sqrt{3}/2$
15. What does an equation of the first degree containing one arbitrary parameter passing through a fixed point represent?
 (a) Circle (b) Straight Line
 (c) Parabola (d) Ellipse
16. The ellipse $\frac{x^2}{169} + \frac{y^2}{25} = 1$ has the same eccentricity as the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. What is the ratio of a to b?
 (a) $\frac{5}{13}$ (b) $\frac{13}{5}$
 (c) $\frac{7}{8}$ (d) $\frac{8}{7}$
17. The curve $y^2 = -4ax$ where, ($a > 0$) lies in.
 (a) First and fourth quadrants
 (b) First and second quadrants
 (c) Second and third quadrants
 (d) Third and fourth quadrants
18. What is the sum of focal radii of any point on an ellipse equal to?
 (a) Length of latus rectum
 (b) Length of major axis
 (c) Length of minor axis
 (d) Length of semi latus rectum
19. What is the locus of points, the difference of whose distances from two points being constant?
 (a) Pair of straight lines (b) An ellipse
 (c) A hyperbola (d) A parabola
20. Let E be the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and C be the circle $x^2 + y^2 = 9$. If P = (1,2), then which one of the following is correct?
 (a) Q lies inside C but outside E
 (b) Q lies outside both C and E
 (c) P lies inside both C and E
 (d) P lies inside C but outside E

21. What are the equations of the directrices of the ellipse $25x^2 + 16y^2 = 400$?
- (a) $3x \pm 25 = 0$ (b) $3y \pm 25 = 0$
 (c) $x \pm 15 = 0$ (d) $y \pm 25 = 0$

22. A circle is drawn with the two foci of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the end of diameter. What is the equation of the circle?
- (a) $x^2 + y^2 = a^2 + b^2$

- (b) $x^2 + y^2 = a^2 - b^2$
 (b) $x^2 + y^2 = 2(a^2 + b^2)$
 (d) $x^2 + y^2 = 2(a^2 - b^2)$

23. If (4, 0) and (-4, 0) are the foci of ellipse and the semi-minor axis is 3, then the ellipse passes through which one of the following points?

- (a) (2, 0) (b) (0, 5)
 (c) (0, 0) (d) (5, 0)

ANSWER KEY

1.	b	2.	d	3.	a	4.	c	5.	d	6.	d	7.	c	8.	d	9.	c	10.	b
11.	d	12.	b	13.	c	14.	d	15.	b	16.	b	17.	c	18.	b	19.	c	20.	d
21.	b	22.	b	23.	d														

Solutions

Sol.1. (b)

Length of latus rectum of an ellipse is $\frac{2b^2}{a}$ where b is semi minor axis and a is semi - major

axis. As given, $\frac{2b^2}{a} = b$

$$\Rightarrow 2b = a \Rightarrow \frac{b}{a} = \frac{1}{2}$$

We know that eccentricity

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{1}{4}} = \sqrt{\frac{5}{4}}$$

Sol.2. (d)

Equation of parabola is $y^2 = 2x$, so vertex lies at origin

So, co - ordinates of vertex are A(0, 0)

Let (x_1, y_1) be the co - ordinates of the point Q

$$\therefore y_1^2 = 2x_1 \quad \dots(i)$$

$$\text{And slope of } PQ = \frac{y_1 - 2}{x_1 - 2}$$

[co - ordinates of P is (2, 2) as given]

$$\text{Also, slope of } AP = \frac{2 - 0}{2 - 0} = 1$$

Since, PQ and AP are perpendicular to each other, hence slope of AP \times Slope of PQ = -1

$$\text{So, } 1 \times \left(\frac{y_1 - 2}{x_1 - 2} \right)$$

$$\Rightarrow y_1 - 2 = -x_1 + 2$$

$$\Rightarrow x_1 + y_1 = 4 \Rightarrow x_1 = 4 - y_1$$

Putting value of x_1 in equation (i)

$$y_1^2 = 8 - 2y_1 \text{ or } y_1^2 + 2y_1 - 8 = 0$$

$$\Rightarrow y_1 = -4 \text{ and } 2$$

Hence, co - ordinates of point Q are (8, -4)

So, required length

$$PQ = \sqrt{(8-2)^2 + (-4-2)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

Sol.3. (a)

Focal distance of a point (x_1, y_1) on the parabola is $y^2 = 4ax$ is equal to its distance from directrix $x+a=0$ is $x_1 + a$.

For $y^2 = 12x$; comparing with $y^2 = 4ax$.

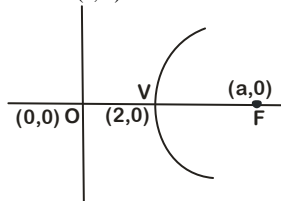
$$\text{So, } x_1 + 3 = 4$$

$$\Rightarrow x_1 = 1$$

Sol.4. (c)

Vertex is (2, 0). Since y - axis is the directrix of a parabola. Equation directrix is $x = 0$. So, axis of parabola is x - axis.

Let the focus be (a, 0)



Distance of the vertex of a parabola from directrix = its distance from focus

$$\text{So, } OV = VF \Rightarrow (2-0)^2 = (a-2)^2$$

$$\Rightarrow a^2 = 4a \Rightarrow a = 4$$

$$\Rightarrow \text{Focus is } (4, 0)$$

Sol.5. (d)

$$\text{The equation of ellipse is } \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$$

The point for which $\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 > 0$ is outside ellipse.

Since, at (a, 0), $1 + 0 - 1 = 0$. It lies on the ellipse.

$$\text{At } (0, b), 0 + 1 - 1 = 0$$

It lies on the ellipse.

$$\text{At } (-a, 0), 1 + 0 - 1 = 0$$

It lies on the ellipse.

$$\text{At } (a, b), 1 + 1 > 0$$

So, the point (a, b) lies outside the ellipse.

Sol.6. (d)

The parabola's vertex and focus lie on x - axis at points (a, 0) and (b, 0). Vertex and focus lie on the x - axis hence, the axis of parabola is x - axis. Equation of parabola Vertex whose is a point (x_1, y_1) then is $(y - y_1)^2 = 4k(x - x_1)$

So, $y_1 = 0$ and $x_1 = a$ and k = distance between focus and vertex = (b - a) so the equation is $(y - 0)^2 = 4(b - a)(x - a)$ i.e., $y^2 = 4(b - a)(x - a)$

Sol.7. (c)

Length of latus rectum of a hyperbola is $\frac{2b^2}{a}$

where a is the half of the distance between two vertex of the hyperbola.

$$\text{Latus rectum} = \frac{2b^2}{a} = \frac{10}{3}$$

$$\text{or, } b^2 = \frac{5a}{3} \quad \dots(1)$$

In case of hyperbola, $b^2 = a^2(e^2 - 1)$

Putting values of b^2 from equation (1) and

$$e = \frac{\sqrt{13}}{3} \text{ in equation (2),}$$

$$\frac{5a}{3} = a^2 \left(\frac{13}{9} - 1 \right)$$

$$\text{or, } \frac{5a}{3} = \frac{4a^2}{9}$$

$$\Rightarrow 4a^2 - 15a = 0 \text{ or } a(4 - 15a) = 0$$

$$a \neq 0, \text{ hence, } a = \frac{15}{4}$$

Length of transverse axis

$$2a = 2 \times \frac{15}{4} = \frac{15}{2}$$

Sol.8. (d)

The given equation of circle is : $x^2 + y^2 = 9$ and

$$\text{ellipse is : } \frac{x^2}{4} + \frac{y^2}{8} = 1$$

From equation (1) and (2) we get

$$\frac{x^2}{4} + \frac{9 - x^2}{8} = 1$$

$$\Rightarrow 2x^2 + 9 - x^2 = 8 \Rightarrow x^2 = -1$$

$$\Rightarrow x \text{ is not real}$$

Hence, circle and ellipse do not intersect.

Sol.9. (c)

$$\text{The equation of ellipse is given as: } \frac{x^2}{a^2} + \frac{y^2}{7} = 1$$

$$\text{Eccentricity is given by: } e = \sqrt{1 - \frac{7}{a^2}}$$

Therefore, foci of ellipse are $(\pm ae, 0)$ i.e., $(\pm a\sqrt{1-\frac{7}{a^2}})$

Now, the equation of given hyperbola is

$$\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25} \Rightarrow \frac{x^2}{\frac{144}{25}} - \frac{y^2}{\frac{81}{25}} = 1$$

$$\text{So, } a = \frac{12}{5} \text{ and } b = \frac{9}{5}$$

$$\therefore e = \sqrt{1 + \frac{81/25}{144/25}} = \sqrt{\frac{144+81}{144}} = \sqrt{\frac{225}{144}} = \frac{15}{12}$$

$$\therefore \text{Foci of hyperbola are } \left(\pm \frac{12}{5} \cdot \frac{15}{12}\right) \text{ i.e., } (\pm 3, 0)$$

Since these foci coincide

$$\Rightarrow 3 = a\sqrt{1 - \frac{7}{a^2}}$$

$$\Rightarrow 3/a = \sqrt{1 - \frac{7}{a^2}}$$

$$\Rightarrow 9/a^2 = 1 - 7/a^2$$

$$\Rightarrow 16/a^2 = 1 \Rightarrow a = 4$$

Sol.10. (b)

Equation of the given conic is an equation of ellipse

$$\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} \quad (x \geq 0)$$

$$\Rightarrow A^2 = a^2 + \lambda \text{ and } B^2 = b^2 + \lambda$$

$$\text{Eccentricity, } e = \sqrt{1 - \frac{B^2}{A^2}} = \sqrt{1 + \frac{b^2 + \lambda}{a^2 + \lambda}}$$

$$= \sqrt{\frac{a^2 + \lambda - b^2 - \lambda}{a^2 + \lambda}} = \sqrt{\frac{a^2 - b^2}{a^2 + \lambda}}$$

λ is in the denominator so, when λ increases, the eccentricity decreases.

Sol.11. (d)

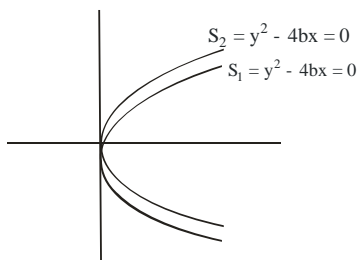
$$\text{Given equation } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Since $b > a$

$$\therefore \text{Foci} = (0, \pm be)$$

Sol.12. (b)

If a and $b > 0$, then graphic representation would be as follows:



S_2 will contain S_1 ,

If latusrectum of $S_2 >$ latusrectum of S_1

$$\Rightarrow 4b > 4a$$

$$\therefore b > a > 0$$

Sol.13. (c)

Given equation of hyperbola

$$5x^2 - 4y^2 = k^2$$

$$\Rightarrow \frac{x^2}{\frac{k^2}{5}} - \frac{y^2}{\frac{k^2}{4}} = 1$$

$$\therefore a = \frac{k}{\sqrt{5}} \text{ and } b = \frac{k}{2}$$

The eccentricity $3/2$ and foci at $(\pm 2, 0)$ of

$$5x^2 - 4y^2 = k^2$$

$$\text{Then, } e = \frac{3}{2} \text{ and } \pm ae = 2$$

$$\Rightarrow \frac{k}{\sqrt{5}} \cdot \frac{3}{2} = 2 \Rightarrow k = \frac{4}{3}\sqrt{5}$$

Sol.14. (d)

$$\text{Since, Latusrectum of an ellipse} = \frac{2b^2}{a}$$

and minor axis = $2b$

$$\therefore b = \frac{2b^2}{a} \Rightarrow a = 2b$$

$$\text{Also, } e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 - \frac{b^2}{4b^2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

Sol.15. (b)

From the given information, we have an equation of the first degree which contains one arbitrary parameter. Therefore the required equation represents a straight line.

Sol.16. (b)

$$e_1 = \sqrt{1 - \frac{25}{169}} = \frac{12}{13} \Rightarrow e_2 = \sqrt{1 - \frac{b^2}{a^2}}$$

$$\therefore e_1 = e_2$$

$$\therefore \frac{12}{13} = \sqrt{1 - \frac{b^2}{a^2}} \Rightarrow \frac{a}{b} = \frac{13}{5}$$

Sol.17. (c)

Left hand parabola always lie in second and third parabola.

Sol.18. (b)

Equal to length of major axis.

Sol.19. (c)

We know that the locus of the difference whose distances from two points being constant, is a hyperbola.

Sol.20. (d)

For a point $p(1, 2)$

$$4(1)^2 + 9(2)^2 - 36 = 40 - 36 > 0$$

$$\text{and } 1^2 + 2^2 - 9 = 5 - 9 < 0$$

\therefore point p lies outside of E and inside of C .

Sol.21. (b)

$$25x^2 + 16y^2 = 400$$

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$

$$e = 3/5$$

so directrix is parallel to y axis.

equation of directrix $y = \pm b/e$

$$y = \pm 25/3$$

Sol.22. (b)

\therefore Foci of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are $(ae, 0)$ and

$(-ae, 0)$ equation of circle with centre $(0, 0)$ and radius ae is $x^2 + y^2 = (ae)^2$ [where, $(ae)^2 = a^2 - b^2$]

$$\therefore x^2 + y^2 = a^2 - b^2$$

Sol.23. (d)

$$2ae = 8 \Rightarrow ae = 4$$

We know that

$$e = \sqrt{1 - \frac{b^2}{a^2}} \Rightarrow \left(\frac{4}{a}\right)^2 = \left(1 - \frac{9}{a^2}\right)$$

$$\Rightarrow \frac{16}{a^2} = \frac{a^2 - 9}{a^2} \Rightarrow a^2 = 25 \Rightarrow a = 5$$

$$\text{Thus, the equation of the ellipse is } \frac{x^2}{25} + \frac{y^2}{9} = 1$$

Which is satisfied by $(5, 0)$. Hence, the ellipse passes through $(5, 0)$.

NDA PYQ

1. What are the points of intersection of the curve $4x^2 - 9y^2 = 1$ with its conjugate axis?
 (a) $\left(\frac{1}{2}, 0\right)$ and $\left(-\frac{1}{2}, 0\right)$
 (b) $(0, 2)$ and $(0, -2)$
 (c) $(0, 3)$ and $(0, -3)$
 (d) No such points exist
[NDA (I) - 2011]
2. What is the sum of the focal distances of a point of an ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
 (a) a
 (b) b
 (c) 2a
 (d) 2b
[NDA-2011(1)]
3. What is the focal distance of any point $P(x_1, y_1)$ on the parabola $y^2 = 4ax$?
 (a) $x_1 + y_1$
 (b) $x_1 y_1$
 (c) ax_1
 (d) $a + x_1$
[NDA-2011(2)]
4. What is the eccentricity of the conic $4x^2 + 9y^2 = 144$?
 (a) $\frac{\sqrt{5}}{3}$
 (b) $\frac{\sqrt{5}}{4}$
 (c) $\frac{3}{\sqrt{5}}$
 (d) $\frac{2}{3}$
[NDA (I) - 2012]
5. If the latus rectum of an ellipse is equal to half of the minor axis, then what is its eccentricity?
 (a) $2/\sqrt{3}$
 (b) $1/\sqrt{3}$
 (c) $\sqrt{3}/2$
 (d) $1/\sqrt{2}$
[NDA (I)-2012]
6. The sum of the focal distances of a point on the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is:
 (a) 4 units
 (b) 6 units
 (c) 8 units
 (d) 10 units
[NDA (II) - 2012]
7. The eccentricity e of an ellipse satisfies the condition.
 (a) $e < 0$
 (b) $0 < e < 1$
 (c) $e = 1$
 (d) $e > 1$
[NDA (II) - 2012]
8. The difference of focal distances of any point on a hyperbola is equal to:
 (a) Latus rectum
 (b) Semi-transverse axis
 (c) Transverse axis
 (d) Semi-latus rectum
[NDA (I) - 2013]
9. The equation of the ellipse whose vertices are at $(\pm 5, 0)$ and foci at $(\pm 4, 0)$ is:
 (a) $\frac{x^2}{25} + \frac{y^2}{9} = 1$
 (b) $\frac{x^2}{9} + \frac{y^2}{25} = 1$
 (c) $\frac{x^2}{16} + \frac{y^2}{25} = 1$
 (d) $\frac{x^2}{25} + \frac{y^2}{16} = 1$
[NDA-2013(1)]
10. The foci of the hyperbola $4x^2 - 9y^2 - 1 = 0$ are:
 (a) $(\pm\sqrt{13}, 0)$
 (b) $\left(\pm\frac{\sqrt{13}}{6}, 0\right)$
 (c) $\left(0, \pm\frac{\sqrt{13}}{6}\right)$
 (d) None of these
[NDA (II) - 2013]
11. The axis of the parabola $y^2 + 2x = 0$ is:
 (a) $x = 0$
 (b) $y = 0$
 (c) $x = 2$
 (d) $y = 2$
[NDA (II) - 2013]
12. The length of latus rectum of the ellipse $4x^2 + 9y^2 = 36$ is:
 (a) $4/3$
 (b) $8/3$
 (c) 6
 (d) 12
[NDA-2013(2)]
13. What is the equation of parabola whose vertex is at $(0, 0)$ and focus is at $(0, -2)$?
 (a) $y^2 + 8x = 0$
 (b) $y^2 - 8x = 0$
 (c) $x^2 + 8y = 0$
 (d) $x^2 - 8y = 0$
[NDA (I) - 2014]
14. What is the sum of the major and minor axes of the ellipse whose eccentricity is $4/5$ and length of latus rectum is 14.4 units?
 (a) 32 units
 (b) 48 units
 (c) 64 units
 (d) None of these
[NDA (I) - 2014]
15. What is the length of the latus rectum of an ellipse $25x^2 + 16y^2 = 400$?
 (a) $25/2$
 (b) $25/4$
 (c) $16/5$
 (d) $32/5$
[NDA (II) - 2014]
16. The point on the parabola $y^2 = 4ax$ nearest to the focus has its abscissa.
 (a) $x = 0$
 (b) $x = a$
 (c) $x = a/2$
 (d) $x = 2a$
[NDA (I) - 2015]
17. The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through the point $(3\sqrt{5}, 1)$ and the length of its latus rectum is $4/3$ units. The length of the conjugate axis is:
 (a) 2 units
 (b) 3 units
 (c) 4 units
 (d) 5 units
[NDA (I) - 2015]
18. Consider any point P on the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ in the first quadrant. Let r and s represent its distances from $(4, 0)$ and $(-4, 0)$ respectively, then $(r + s)$ is equal to:
 (a) 10 unit
 (b) 9 unit
 (c) 8 unit
 (d) 6 unit
[NDA (II) - 2015]
19. The eccentricity of the hyperbola $16x^2 - 9y^2 = 1$ is?
 (a) $3/5$
 (b) $5/3$
 (c) $4/5$
 (d) $5/4$
[NDA (II) - 2015]
20. The equation of the hyperbola having latus rectum and eccentricity 8 and $\frac{3}{\sqrt{5}}$ respectively?
 (a) $\frac{x^2}{25} - \frac{y^2}{20} = 1$
 (b) $\frac{x^2}{40} - \frac{y^2}{20} = 1$
 (c) $\frac{x^2}{40} - \frac{y^2}{30} = 1$
 (d) $\frac{x^2}{30} - \frac{y^2}{25} = 1$

21. What is the eccentricity of rectangular hyperbola?
(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) $\sqrt{5}$ (d) $\sqrt{6}$ [NDA (II) - 2016]
22. If the ellipse $9x^2 + 16y^2 = 144$ intercepts the line $3x + 4y = 12$, then what is the length of the chord so formed?
(a) 5units (b) 6units
(c) 8units (d) 10units [NDA (II) - 2016]
- Direction (for next two):** Consider the following for the next two items that follow:
Consider the parabola $y = x^2 + 7x + 2$ and the straight line $y = 3x - 3$.
23. What are the coordinates of the point on the parabola which is closest to the straight line?
(a) (0, 2) (b) (-2, -8)
(c) (-7, 2) (d) (1, 10) [NDA (II) - 2016]
24. What is the shortest distance from the above point on the parabola to the line?
(a) $\frac{\sqrt{10}}{2}$ (b) $\frac{\sqrt{10}}{5}$
(c) $\frac{1}{\sqrt{10}}$ (d) $\frac{\sqrt{5}}{4}$ [NDA (II) - 2016]
25. What is the equation of the ellipse having foci $(\pm 2, 0)$ and the eccentricity $1/4$?
(a) $\frac{x^2}{64} + \frac{y^2}{60} = 1$ (b) $\frac{x^2}{60} + \frac{y^2}{64} = 1$
(c) $\frac{x^2}{20} + \frac{y^2}{24} = 1$ (d) $\frac{x^2}{24} + \frac{y^2}{20} = 1$ [NDA (I) - 2017]
26. The position of the point (1,2) relative to the ellipse $2x^2 + 7y^2 = 20$ is:
(a) Outside the ellipse
(b) Inside the ellipse but not at the focus
(c) On the ellipse
(d) At the focus [NDA (II) - 2017]
27. The equation of the ellipse whose centre is at origin, major axis is along x-axis with eccentricity $\frac{3}{4}$ and latus rectum 4 units is:
(a) $\frac{x^2}{1024} + \frac{7y^2}{64} = 1$ (b) $\frac{49x^2}{1024} + \frac{7y^2}{64} = 1$
(c) $\frac{7x^2}{1024} + \frac{49y^2}{64} = 1$ (d) $\frac{x^2}{1024} + \frac{y^2}{64} = 1$ [NDA (II) - 2017]
28. Geometrically $\text{Re}(z^2 - i) = 2$, where $i = \sqrt{-1}$ and Re is the real part, represents:
(a) Circle (b) Ellipse
(c) Rectangular hyperbola (d) Parabola [NDA (II) - 2017]
29. A man running round a racecourse notes that the sum of the distance of two flag-posts from him is always 10 m and the distance between the flag posts is 8m. The area of the path he encloses is:
(a) $18\pi\text{sq m}$ (b) $15\pi\text{sq m}$
(c) $12\pi\text{sq m}$ (d) $8\pi\text{sq m}$
30. What is the equation of the ellipse whose vertices are $(\pm 5, 0)$ and foci are at $(\pm 4, 0)$?
(a) $\frac{x^2}{25} + \frac{y^2}{9} = 1$ (b) $\frac{x^2}{16} + \frac{y^2}{9} = 1$
(c) $\frac{x^2}{25} + \frac{y^2}{16} = 1$ (d) $\frac{x^2}{9} + \frac{y^2}{25} = 1$ [NDA (II) - 2017]
31. Equation $2x^2 - 3y^2 - 6 = 0$ represents
(a) A circle (b) A parabola
(c) An ellipse (d) A hyperbola [NDA - (I) 2018]
32. Two parabolas $y^2 = 4ax$ and $x^2 = 4ay$ intersect at
(a) at two points located on the line $y = x$
(b) only at origin
(c) At three points out of them one lies on line $y + x = 0$
(d) only at $(4a, 4a)$ [NDA - (I) 2019]
33. The sum of the focal distances of a point on an ellipse is constant and equal to:
(a) length of minor axis
(b) length of major axis
(c) length of latus rectum
(d) sum of lengths of minor axis and major axis [NDA - (I) 2019]
34. If the angle between the lines joining the end points of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with one of its foci is $\frac{\pi}{2}$ then what is the eccentricity of the ellipse?
(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$
(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2\sqrt{2}}$ [NDA - (II) 2019]
35. Let $P(x, y)$ be any point on ellipse $25x^2 + 16y^2 = 400$. If $Q(0, 3)$ and $R(0, -3)$ are two points, then what is $(PQ + PR)$ equal to ?
(a) 12 (b) 10
(c) 8 (d) 6 [NDA 2020]
36. In the parabola, $y^2 = x$, what is the length of the chord passing through the vertex and inclined to the x-axis at an angle θ ?
(a) $\sin\theta \cdot \sec^2\theta$ (b) $\cos\theta \cdot \csc^2\theta$
(c) $\cot\theta \cdot \sec^2\theta$ (d) $2\tan\theta \cdot \csc^2\theta$ [NDA 2020]
37. If any point on a hyperbola is $(3\tan\theta, 2\sec\theta)$, then what is the eccentricity of the hyperbola?
(a) $\frac{3}{2}$ (b) $\frac{5}{2}$
(c) $\frac{\sqrt{11}}{2}$ (d) $\frac{\sqrt{13}}{2}$ [NDA (I) 2021]
38. Consider the following with regard to eccentricity (e) of a conic section:
1. $e = 0$ for circle 2. $e = 1$ for parabola
3. $e < 1$ for ellipse
Which of the above are correct?
(a) 1 and 2 only (b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

[NDA-(I) 2021]

Consider the following for next (02) question.

The two ends of latus rectum of a parabola are $(-2, 4)$ and $(-2, -4)$.

39. What is the maximum number of parabolas that can be drawn through these two points as end points of latus rectum?

(a) Only one
(c) Four

(b) Two
(d) Infinite

[NDA-(II) 2021]

40. Consider the following statements in respect of such parabolas:

1. One of the parabolas passes through the origin $(0, 0)$
2. The focus of one of the parabolas lies at $(-2, 0)$

Which of the above statements is/are correct?

(a) 1 only
(c) Both 1 and 2

(b) 2 only
(d) Neither 1 nor 2

[NDA-(II) 2021]

41. What is the equation of the parabola with focus $(-3, 0)$ and direction $x - 3 = 0$?

(a) $y^2 = 3x$
(c) $y^2 = 12x$

(b) $x^2 = 12y$
(d) $y^2 = -12x$

[NDA-(I) 2022]

42. What is the distance between the foci of the ellipse $x^2 + 2y^2 = 1$?

(a) 1
(c) 2

(b) $\sqrt{2}$
(d) $2\sqrt{2}$

[NDA-(I) 2022]

43. The centre of an ellipse is at $(0, 0)$, major axis is on the y -axis. If the ellipse passes through $(3, 2)$ and $(1, 6)$, then what is its eccentricity?

(a) $\frac{\sqrt{3}}{2}$
(c) $\frac{\sqrt{5}}{2}$

(b) $\sqrt{3}$
(d) $\sqrt{5}$

[NDA 2022 (II)]

44. An equilateral triangle is inscribed in a parabola $x^2 = \sqrt{3}y$ where one vertex of the triangle is at the vertex of the parabola. If p is the length of side of the triangle and q is the length of the latus rectum, then which one of the following is correct?

(a) $p = q$
(c) $p = 2\sqrt{3}q$

(b) $p = \sqrt{3}q$
(d) $2\sqrt{3}p = q$

[NDA 2022 (II)]

Consider the following for the next two (02) items that follow:

$P(x, y)$ is any point on the ellipse $x^2 + 4y^2 = 1$. Let E, F be the foci of the ellipse.

45. What is $PE + PF$ equal to?

(a) 1
(c) 3

(b) 2
(d) 4

[NDA - 2023 (1)]

46. Consider the following points:

1. $\left(\frac{\sqrt{3}}{2}, 0\right)$

2. $\left(\frac{\sqrt{3}}{2}, \frac{1}{4}\right)$

3. $\left(\frac{\sqrt{3}}{2}, -\frac{1}{4}\right)$

Which of the above points lie on latus rectum of ellipse

(a) 1 and 2 only

(c) 1 and 3 only

(b) 2 and 3 only

(d) 1, 2 and 3

[NDA - 2023 (1)]

47. What is the equation of directrix of parabola $y^2 = 4bx$, where $b < 0$ and $b^2 + b - 2 = 0$?

(a) $x+1=0$
(c) $x-1=0$

(b) $x-2=0$
(d) $x+2=0$

[NDA-2023 (2)]

48. Consider the following in respect of the equation

$$\frac{x^2}{24-k} + \frac{y^2}{k-16} = 2$$

1. The equation represents an ellipse if $k=19$

2. The equation represents a hyperbola if $k=12$

3. The equation represents a circle if $k=20$

How many of the statements given above are correct?

(a) only one
(c) all three

(b) only two
(d) none

[NDA-2023 (2)]

49. Consider the following statements in respect of hyperbola

$$\frac{x^2}{\cos^2 \theta} - \frac{y^2}{\sin^2 \theta} = 1.$$

1. The two foci are independent of θ

2. The eccentricity is $\sec \theta$

3. The distance between the two foci is 2 units.

How many of the statements given above are correct?

(a) only one
(c) all three

(b) only two
(d) none

[NDA-2023 (2)]

50. In the parabola $y^2 = 8x$, the focal distance of a point P lying on it is 8 units. Which of the following statements is/are correct?

1. The coordinates of P can be $(6, 4\sqrt{3})$.

2. The perpendicular distance of P from the directrix of parabola is 8 units

Select the correct answer using the code given below:

(a) 1 only
(c) Both 1 and 2

(b) 2 only
(d) Neither 1 nor 2

[NDA-2024 (1)]

51. What is the eccentricity of the ellipse if the angle between the straight lines joining the foci to an extremity of the minor axis is 90° ?

(a) $\frac{1}{3}$
(c) $\frac{1}{\sqrt{3}}$

(b) $\frac{1}{2}$
(d) $\frac{1}{\sqrt{2}}$

[NDA-2024 (1)]

52. The foci of the ellipse $4x^2 + 9y^2 = 1$ are Q and R . If $P(x, y)$ is any point on the ellipse, then what is $PQ + PR$ equal to?

(a) 2
(c) $2/3$

(b) 1
(d) $1/3$

[NDA-2024 (2)]

53. Consider the points $P(4k, 4k)$ and $Q(4k, -4k)$ lying on the parabola $y^2 = 4kx$. If the vertex is A , then what is $\angle PAQ$ equal to?

(a) 60°
(c) 120°

(b) 90°
(d) 135°

[NDA-2024 (2)]

ANSWER KEY

1.	d	2.	c	3.	d	4.	a	5.	c	6.	b	7.	b	8.	c	9.	a	10.	b
11.	b	12.	b	13.	c	14.	c	15.	d	16.	a	17.	c	18.	a	19.	b	20.	a
21.	a	22.	a	23.	b	24.	c	25.	a	26.	a	27.	b	28.	c	29.	b	30.	a
31.	d	32.	a	33.	b	34.	b	35.	b	36.	b	37.	d	38.	a	39.	b	40.	c
41.	d	42.	b	43.	a	44.	c	45.	b	46.	d	47.	b	48.	c	49.	c	50.	c
51.	d	52.	b	53.	b														

Solutions

Sol. 1. (d)

A hyperbola never meet/intersect conjugate axis in real points.

Sol. 2. (c)

Sum of focal distances is always equal to length of major axis = 2a

Sol. 3. (d)

Focal distance is always equal to distance of that point from directrix i.e. $a + x_1$

Sol. 4. (a)

The given equation of conic is

$$4x^2 + 9y^2 = 144 \Rightarrow \frac{x^2}{36} + \frac{y^2}{16} = 1$$

Which represent an ellipse, here $a > b$

$$\therefore a^2 = 36 \Rightarrow a = 6$$

$$b^2 = 16 \Rightarrow b = 4$$

Now, eccentricity, $b^2 = a^2(1 - e^2)$

$$\Rightarrow 16 = 36(1 - e^2) \Rightarrow \frac{4}{9} = -e^2$$

$$\Rightarrow e^2 = 1 - \frac{4}{9} = \frac{5}{9} \Rightarrow e = \frac{\sqrt{5}}{3}$$

Sol. 5. (b)

Since, the sum of foci distances of a point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to 2b.

When $b > a$

$$\therefore a^2 = 4, b^2 = 9 \Rightarrow a = 2, b = 3$$

\therefore Sum of the focal distances = $2 \times 3 = 6$ units

Sol. 6. (b)

The eccentricity of ellipse lies between 0 and 1

Sol. 7. (b)

For ellipse $0 < e < 1$

Sol. 8. (c)

By definition of hyperbola,

A hyperbola is the set of points in a plane, where distances from two fixed points in the plane have a constant difference i.e., transverse axis. The two fixed points are the foci of the hyperbola.

Sol. 9.

Sol. 10. (b)

Given equation of hyperbola is

$$4x^2 - 9y^2 = 1 \Rightarrow \frac{x^2}{(1/4)} - \frac{y^2}{(1/9)} = 1$$

Here: $a^2 = 1/4$ and $b^2 = 1/9$

\therefore Foci of the hyperbola = $(\pm ae, 0)$

$$= \left(\pm a \frac{\sqrt{a^2 + b^2}}{a}, 0 \right) = \left(\pm \sqrt{a^2 + b^2}, 0 \right)$$

$$= \left(\pm \sqrt{\frac{1}{4} + \frac{1}{9}}, 0 \right) = \left(\pm \frac{\sqrt{13}}{6}, 0 \right)$$

Sol. 11. (b)

Given equation of parabola is:

$$y^2 + 2x = 0 \Rightarrow y^2 = -2x$$

Which is of the form $y^2 = -4ax$

So, axis of the parabola is $y = 0$

Sol. 12. (b)

length of latus rectum of the ellipse $4x^2 + 9y^2 = 36$

$$x^2/9 + y^2/4 = 0$$

$$a = 3, b = 2$$

$$LR = 8/3$$

Sol. 13. (c)

Given vertex of the parabola = (0, 0)

and focus of the parabola = (0, -2)

Let P be any point on the parabola, then equation directrix is $y - 2 = 0$

\therefore Equation of parabola is

$$\Rightarrow \frac{PS = PM}{\sqrt{(x-0)^2 + (y+2)^2}} = \frac{|y-2|}{\sqrt{1}}$$

$$\Rightarrow \sqrt{x^2 + (y+2)^2} = |y-2|^2$$

$$\Rightarrow x^2 + y^2 + 4 + 4y = y^2 + 4 - 4y$$

$x^2 = -8y$, which is the required equation of parabola.

Sol. 14. (c)

We know that, length of major axes of an ellipse = 2a and length of minor axes of an ellipse = 2b

Given that,

eccentricity of an ellipse = $4/5 \dots (i)$

and length of latus rectum of an ellipse = 14.4 units

$$\Rightarrow \frac{2b^2}{a} = 14.4 \Rightarrow \frac{b^2}{a} = 7.2$$

$$\Rightarrow b^2 = 7.2a$$

Since,

$$b^2 = a^2(1 - e^2)$$

$$\Rightarrow 7.2a = a^2 \left[1 - \left(\frac{4}{5} \right)^2 \right]$$

[from eqs.(i) and (ii)]

$$\Rightarrow 7.2a = a^2 \left(1 - \frac{16}{25} \right)$$

$$\Rightarrow 7.2a = a^2 = \frac{9}{25}$$

$$\Rightarrow 9a^2 - 7.2 \times 25a = 0$$

$$\Rightarrow 9a^2 - 36 \times 5a = 0$$

$$\Rightarrow 9a(a - 20) = 0$$

$$\Rightarrow a = 20 \quad (\because a \neq 0)$$

Putting the value of a in Eq. (ii), we get

$$b^2 = 7.2 \times 20$$

$$\Rightarrow b^2 = 72 \times 2 = 144 \Rightarrow b^2 = (12)^2$$

Hence, the sum of the major and minor axes

$$= 2a + 2b$$

$$= 2(a+b) = 2(20+12)$$

$$= 2 \times 32 = 64 \text{ units}$$

Sol. 15. (d)

Equation of ellipse is $25x^2 + 16y^2 = 400$

$$\Rightarrow \frac{x^2}{16} + \frac{y^2}{25} = 1$$

Here, $a^2 = 16$ and $b^2 = 25$

\therefore length of latus rectum

$$= \frac{2a^2}{b} = \frac{2 \times 16}{5} = \frac{32}{5}$$

Sol. 16. (a)

\therefore Required abscissa is $x = 0$

Sol. 17. (c)

Since, hyperbola passes through $(3\sqrt{5}, 1)$

$$\therefore 2 - \frac{(3\sqrt{5})^2}{a^2} - \frac{1}{b^2} = 1$$

$$\Rightarrow \frac{45}{a^2} - \frac{1}{b^2} = 1$$

$$\Rightarrow 45b^2 - a^2 = a^2 b^2 \dots (i)$$

$$\text{Also, } \frac{2b^2}{a} = \frac{4}{3}$$

$$\Rightarrow 6b^2 = 4a$$

$$\Rightarrow a = \frac{6b^2}{4} \dots (ii)$$

On putting the value from Eq. (ii) in Eq. (i), we get

$$45b^2 - \left(\frac{6b^2}{4} \right)^2 = \left(\frac{6b^2}{4} \right)^2 b^2$$

$$\Rightarrow 45b^2 - \frac{36b^2}{16} = \frac{36b^4 \cdot b^2}{16}$$

$$\Rightarrow 45b^2 = \frac{36b^2}{16} [b^2 + 1]$$

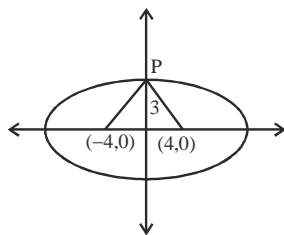
$$\Rightarrow 45 \times 16 = 36b^2 (b^2 + 1)$$

$$\Rightarrow b^4 + b^2 = 20$$

$$\therefore b^2 = 2$$

$\therefore 2b = 4 = \text{length of conjugate axis}$

Sol. 18. (a)



The sum of the distance of any point P from the foci of an ellipse

$$= 2\sqrt{(ae)^2 + b^2}$$

$$= 2\sqrt{16+9}$$

$$\therefore r + s = 10 \text{ cm}$$

Sol. 19. (b)

Equation of Hyperbola: $16x^2 - 9y^2 = 1$

$$\Rightarrow a^2 = \frac{1}{16} \text{ and } b^2 = \frac{1}{9}$$

$$\therefore e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$$

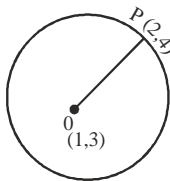
Sol. 20. (a)

Given, length of latus rectum

$$= \frac{2b^2}{a} = 8 \quad \dots(i)$$

$$\text{and eccentricity} = \sqrt{1 + \left(\frac{b}{a}\right)^2} = \frac{3}{\sqrt{5}}$$

$$\Rightarrow 1 + \left(\frac{b}{a}\right)^2 = \frac{9}{5}$$



$$\Rightarrow \frac{b^2}{a^2} = \frac{4}{5} \quad \dots(ii)$$

On solving equations (i) and (ii), we get

$$a^2 = 25 \text{ and } b^2 = 20$$

\therefore Equation of hyperbola will be:

$$\frac{x^2}{25} - \frac{y^2}{20} = 1$$

Sol. 21. (a)

We know that,

Equation of rectangular hyperbola

$$x^2 - y^2 = p^2 \Rightarrow \frac{x^2}{p^2} - \frac{y^2}{p^2} = 1$$

Here, length of transverse axis and conjugate axis are equal.

$$\therefore \text{Eccentricity} = \sqrt{1 + \left(\frac{b}{a}\right)^2} = \sqrt{1 + \left(\frac{p}{p}\right)^2}$$

$$= \sqrt{1+1} = \sqrt{2}$$

Sol. 22. (a)

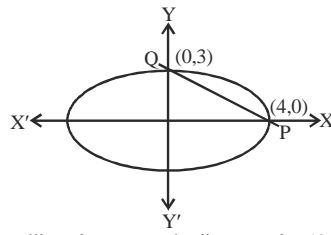
Given,

$$9x^2 + 16y^2 = 144$$

$$\Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 1 \text{ and } 3x + 4y = 12$$

$$\Rightarrow \frac{x}{4} + \frac{y}{3} = 1 \quad \dots(ii)$$

From figure,



Given ellipse intercepts the line at point (4,0) and (0,3).

\therefore length of chord

$$PQ = \sqrt{(4-0)^2 + (0-3)^2}$$

$$= 5 \text{ unit}$$

Solutions (for next two):

$$\text{Given, } y = x^2 + 7x + 2 = \left(x + \frac{7}{2}\right)^2 - \frac{41}{4}$$

Let the co-ordinates of the point on this parabola be

$$\left(P - \frac{7}{2}, P^2 - \frac{41}{4}\right)$$

The equation on the line is $y = 3x - 3 \Rightarrow y - 3x + 3 = 0$

Distance of the point from the line

$$z = \frac{\left(P^2 - \frac{41}{4}\right) - 3\left(P - \frac{7}{2}\right) + 3}{\sqrt{10}}$$

$$\therefore \frac{dz}{dt} = \frac{1}{\sqrt{10}}(2P - 3) = 0$$

$$\Rightarrow P = 3/2$$

Sol. 23. (b)

Co-ordinates of the point on the parabola which is closest to the straight line

$$= \left(P - \frac{7}{2}, P^2 - \frac{41}{4}\right) = \left(\frac{3}{2} - \frac{7}{2}, \left(\frac{3}{2}\right)^2 - \frac{41}{4}\right)$$

$$= (-2, -8) \quad \left[\text{For } P = \frac{3}{2}\right]$$

Sol. 24. (c)

Shortest distance from the point to parabola

$$= \frac{P^2 - \frac{41}{4} - 3\left(P - \frac{7}{2}\right) + 3}{\sqrt{10}} = \frac{\left(\frac{3}{2}\right)^2 - \frac{41}{4} - 3\left(\frac{3}{2} - \frac{7}{2}\right) + 3}{\sqrt{10}}$$

$$= \frac{\frac{9}{4} - \frac{41}{4} + 6 + 3}{\sqrt{10}} = \frac{-8 + 6 + 3}{\sqrt{10}} = \frac{1}{\sqrt{10}} \text{ unit}$$

Sol. 25. (a)

The foci of the ellipse is $(\pm 2, 0)$

i.e. $\pm ae = \pm 2$

and $e = 1/4$

$$\Rightarrow a = 2 \times 4 = 4$$

$$\text{and } b^2 = a^2(1 - e^2) = 64$$

$$\left(1 - \frac{1}{16}\right) = 60$$

$$\therefore \text{the required equation is } \frac{x^2}{64} + \frac{y^2}{60} = 1$$

Sol. 26. (a)

Equation of ellipse: $2x^2 + 7y^2 = 20$

Putting $x = 1$ and $y = 2$ on LHS, we get

$$2(1)^2 + 7(2)^2 = 2 + 28 = 30 > 20$$

Sol. 27. (b)

Latus rectum = 4

$$\text{or } \frac{2b^2}{a} = 4$$

$$\text{or } b^2 = 2a$$

$$\therefore e = \frac{3}{4} \quad \dots(i)$$

$$\Rightarrow \sqrt{1 - \frac{b^2}{a^2}} = \frac{3}{4} \Rightarrow \frac{a^2 - b^2}{a^2} = \frac{9}{16}$$

$$\Rightarrow \frac{a^2 - 2a}{a^2} = \frac{9}{16} \Rightarrow \frac{a - 2}{a} = \frac{9}{16}$$

$$\Rightarrow 7a = 32 \Rightarrow a = \frac{32}{7}$$

$$\therefore b^2 = 2 \times \frac{32}{7} = \frac{64}{7}$$

Now, required equation of ellipse is given as:

$$\frac{x^2}{\left(\frac{32}{7}\right)^2} + \frac{y^2}{\left(\frac{64}{7}\right)} = 1$$

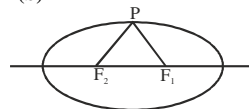
$$\text{or } \frac{49x^2}{1024} + \frac{7y^2}{64} = 1$$

Sol. 28. (c)

Re $(z^2 - i) = 2$ or Re $[(x+iy)^2 - i] = 2$ {where, $z = x + iy$ } or Re $(x^2 - y^2 + 2ixy - i) = 2$ or $x^2 - y^2 = 2$

$$\text{or } \frac{x^2}{2} - \frac{y^2}{2} = 1$$

Sol. 29. (b)



$$PF_1 + PF_2 = 10$$

$$\Rightarrow 2a = 10 \Rightarrow a = 5$$

$$F_1F_2 = 8 \Rightarrow 2c = 8 \Rightarrow c = 4$$

$$a^2 = b^2 + c^2 \Rightarrow b^2 = 3^2 \Rightarrow b = 3$$

$$\text{Area} = \pi ab = \pi \times 5 \times 3 = 15\pi \text{ sq. m}$$

Sol. 30. (a)

$$c = 4, a = 5, b^2 = 25 - 16 = 9$$

$$\therefore \text{Equation of ellipse is } \frac{x^2}{25} + \frac{y^2}{9} = 1$$

Sol. 31. (d)

$$\frac{x^2}{3} - \frac{y^2}{2} = 1$$

this equation represents a hyperbola

Sol. 32. (a)

$$x^2 = 4ay \text{ and } y^2 = 4ax.$$

by solving these equations solutions are

$(0,0)$ and $(4a,4a)$ these points are lie on $y = x$ line

Sol. 33. (b)

Sum of focal distances is always equal length of major axis.

Sol. 34. (b)

If F is focus of ellipse and F(ae, 0)

A is one end point of minor axis on positive y axis A(0,b)

B is second end point of minor axis on negative y axis B(0,-b)

AF and BF lines are given perpendicular.

So slope of AF \times slope of BF = -1

$$\left(\frac{b-0}{0-ae}\right)\left(\frac{-b-0}{0-ae}\right) = -1$$

$$\Rightarrow \left(\frac{-b^2}{a^2e^2}\right) = -1$$

$$\Rightarrow e^2 = \frac{b^2}{a^2} \text{ eccentricity}$$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - e^2}$$

$$\Rightarrow e^2 = 1 - e^2$$

$$\Rightarrow 2e^2 = 1$$

$$\Rightarrow e = \frac{1}{\sqrt{2}}$$

Sol. 35. (b)

Given ellipse $25x^2 + 16y^2 = 400$

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$

$a = 4$, $b = 5$ end points of major axes are (0,5) and (0,-5) coordinates of foci are Q(0,3) and R(0,-3)

(PQ+PR)=sum of focal distances = length of major axes = 10 unit

Sol. 36. (b)

chord inclined with x axis at angle θ

and passing through vertex (origin)

then equation of chord will be $y = \tan\theta \cdot x$

it will cut the parabola at A(0,0) and B($\cot^2\theta, \cot\theta$)

$$\text{length of chord} = \sqrt{\cot^4\theta + \cot^2\theta}$$

$$= \cot\theta \sqrt{\cot^2\theta + 1}$$

$$= \cot\theta \sec\theta$$

$$= \cos\theta \cdot \sec^2\theta$$

Sol. 37. (d)

any point on a hyperbola is (3tan θ , 2 sec θ),

$$x = 3\tan\theta \Rightarrow \tan\theta = x/3$$

$$y = 2\sec\theta \Rightarrow \sec\theta = y/2$$

$$\sec^2\theta - \tan^2\theta = 1$$

$$\frac{y^2}{4} - \frac{x^2}{9} = 1$$

$$e = \sqrt{1 + \frac{9}{4}} = \sqrt{\frac{13}{4}} = \frac{\sqrt{13}}{2}$$

Sol. 38. (a)

for ellipse $0 < e < 1$

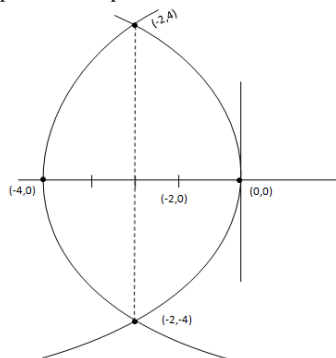
Sol. 39. (b)

End point of LR are (-2, 4) and (-2, -4)

Length of LR = 8 unit

Focal length = 2 unit

Two parabola are possible



Sol. 40. (c)

From above graph statements are correct.

Sol. 41. (d)

focus (-3,0)

director $x-3=0$

parabola $y^2 = -12x$

Sol. 42.

$$\frac{x^2}{1} + \frac{y^2}{1/2} = 1$$

$$e = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

distance between foci

$$\Rightarrow 2ae = 2 \times 1 \times \frac{1}{\sqrt{2}} = \sqrt{2}$$

Sol. 43. (a)

equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where ($b > a$)

ellipse passes through (3, 2) and (1, 6)

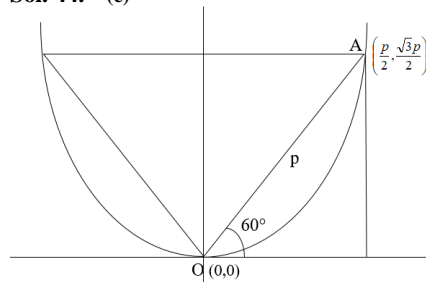
$$\frac{9}{a^2} + \frac{4}{b^2} = 1 \quad \dots\dots(i)$$

$$\frac{1}{a^2} + \frac{16}{b^2} = 1 \quad \dots\dots(ii)$$

by both equations $a^2 = 35$ and $b^2 = 140$

$$e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{35}{140}} = \frac{\sqrt{3}}{2}$$

Sol. 44. (c)



side of triangle is p, so its horizontal component is $p\cos 60^\circ$ and vertical component is $p\sin 60^\circ$.

so coordinates of A is $\left(\frac{p}{2}, \frac{\sqrt{3}p}{2}\right)$

this point lie on parabola so this will satisfy equation of parabola $x^2 = \sqrt{3}y$

$$\frac{p^2}{4} = \sqrt{3} \frac{\sqrt{3}p}{2}$$

$$p = 6$$

$$\text{and letus rectum} = q = \sqrt{3}$$

$$\text{i.e. } p/q = 6/\sqrt{3}$$

$$p = 2\sqrt{3}q$$

Sol. 45. (b)

ellipse $x^2 + 4y^2 = 1$

$$a = 1 \text{ and } b = \frac{1}{2}$$

sum of focal distances is equal to length of major axis $= 2a = 2$

Sol. 46. (d)

ellipse $x^2 + 4y^2 = 1$

$$a = 1 \text{ and } b = \frac{1}{2}$$

$$e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

$$\text{focii } (\pm ae, 0) = \left(\pm \frac{\sqrt{3}}{2}, 0\right)$$

$$\text{and L.R. equation } x = \frac{\sqrt{3}}{2}$$

All given points are lie on this line

Sol. 47. (b)

$$b^2 + b - 2 = 0$$

$$b = -2 \text{ given } (b < 0)$$

equation of directrix of parabola $y^2 = -8x$

equation of directrix is $x = 2$ and $x = -b$

Sol. 48. (c)

$$\frac{x^2}{24-k} + \frac{y^2}{k-16} = 2$$

If $k = 19$

$$\frac{x^2}{5} + \frac{y^2}{3} = 2$$

$$\frac{x^2}{10} + \frac{y^2}{6} = 1$$

Equation represents ellipse.

If $k = 12$

$$\frac{x^2}{12} - \frac{y^2}{4} = 2$$

Equation represents hyperbola.

If $k = 20$

$$\frac{x^2}{4} + \frac{y^2}{4} = 2$$

$$x^2 + y^2 = 8$$

equation represents circle.

All three statements are correct

Sol. 49. (c)

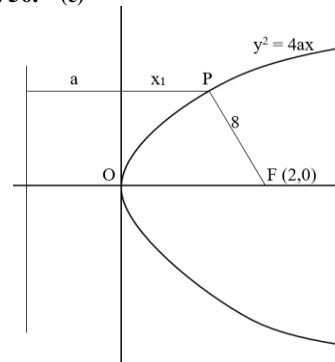
$$\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$$

$$e = \sqrt{1 + \frac{\sin^2\theta}{\cos^2\theta}} = \sec\theta$$

Focii are $ae = \cos\theta \cdot \sec\theta = 1$

Distance between (1,0) and (-1,0) is 2 units.

Sol. 50. (c)



according to above diagram

$a + x_1 = \text{focal distance (for all } y^2 = 4ax)$

$$2 + x_1 = 8 \quad (\text{here } a = 2 \text{ is focal length})$$

$$x_1 = 6$$

put $x = 6$ in equation of parabola so P is $(6, 4\sqrt{3})$

$a + x_1 = \text{perpendicular distance from directrix.}$
both statements are correct.

Sol. 51. (d)

coordinates of foci $F_1(ae, 0)$ and $F_2(-ae, 0)$

one end point of minor axis is A(0,b)

angle between AF_1 and AF_2 is 90°

we know that if two lines are perpendicular than $m_1 m_2 = -1$

$$\text{slope of } AF_1 = \frac{0-b}{ae-0} = -\frac{b}{ae}$$

$$\text{slope of } AF_2 = \frac{0-b}{-ae-0} = \frac{b}{ae}$$

$$\left(-\frac{b}{ae}\right)\left(\frac{b}{ae}\right) = -1$$

$$\frac{b^2}{a^2} = e^2$$

and eccentricity of ellipse

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - e^2}$$

$$e^2 = 1 - e^2$$

$$2e^2 = 1 \Rightarrow e = \frac{1}{\sqrt{2}}$$

Sol. 52. (b)

$$4x^2 + 9y^2 = 1$$

$$\frac{x^2}{1/4} + \frac{y^2}{1/9} = 1$$

Sum of distances of any point on ellipse from
both axes = length of major axis
here $a = 1/2$

so length of major axis = $2a = 1$

Sol. 53. (b)

$P(4k, 4k)$, $Q(4k, 4k)$, $A(0, 0)$

slope of $PA = 1$ and $PQ = -1$

here $m_1 m_2 = -1$ so both lines are perpendicular.