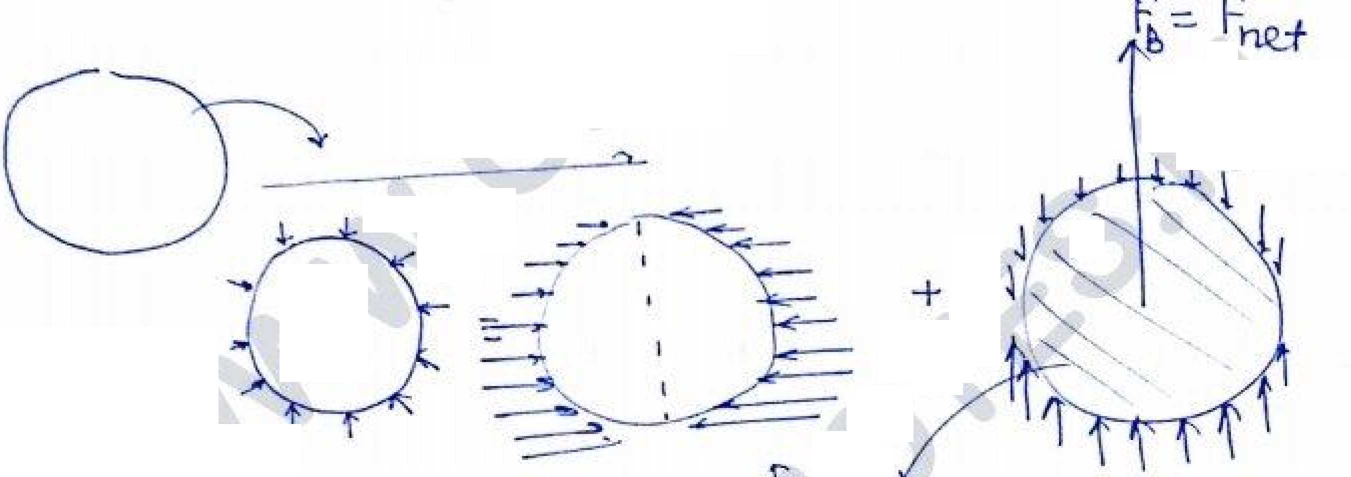
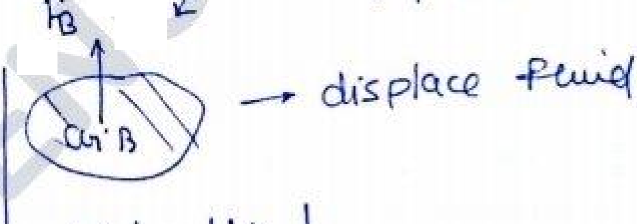
Buoyancy force and floatation:-

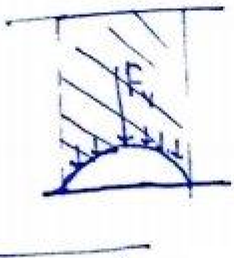
→ When a body is ~~immerse~~ immerge partially or complately in a fluid the net vertical upward hydrostatic force exerted by the fluid on the body is known as Buoyant force and it will the weight of the fluid displaced its point of application is the C.G. of displaced ~~weight~~ fluid known as centre of buoyancy.





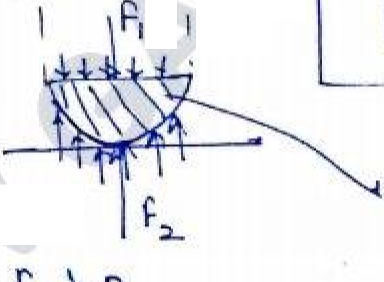
$$|F_B| = |W_{fd}|$$

*



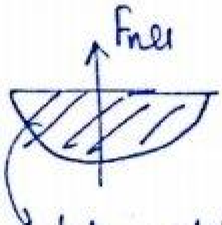
$$\boxed{F_B = 0}$$

No upward hydrostatic
No buoyancy



$F_B \neq 0$

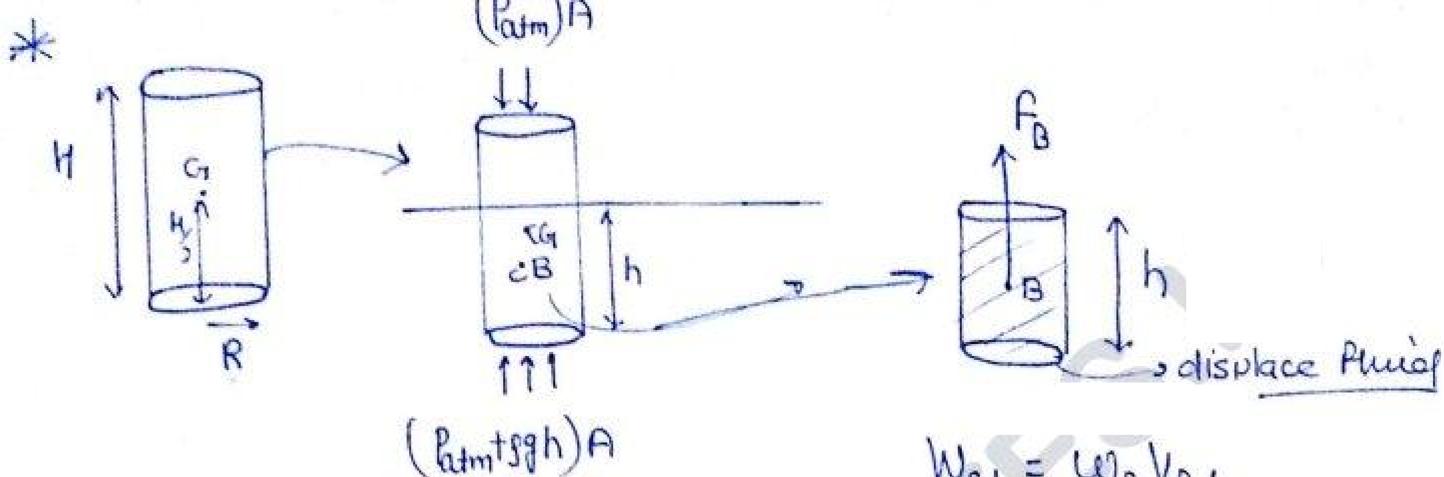
$$F_B = F_{net} = F_2 - F_1$$



$$W_{fd} = \omega_f V_{fd}$$

$$\boxed{F_B = \omega_f V_{fd}}$$

*



$$W_{fd} = \omega_f V_{fd}$$

$$F_{net} = (\rho_f g h) A$$

$$F_B = \omega_f h A \, (\uparrow)$$

$$|F_B| = \omega_f A \cdot h \, (\uparrow)$$

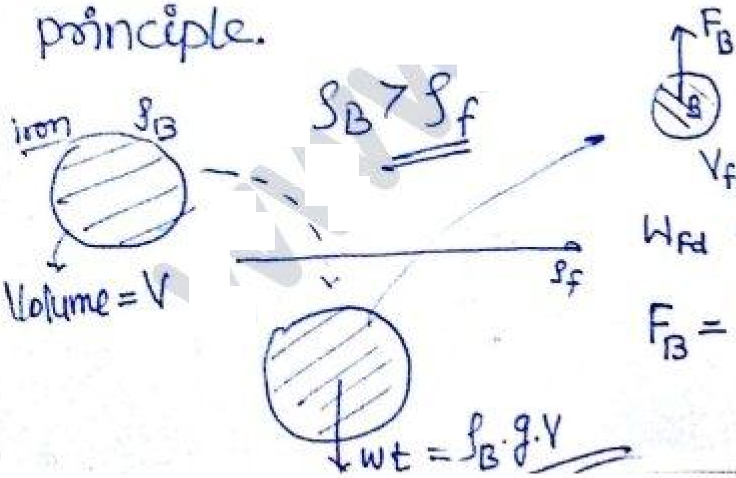
$$H > h$$

$$\boxed{\frac{H}{2} > \frac{h}{2}}$$

* Centre of gravity is above centre of buoyancy.

Principle of Floatation [Archimedes principle]

When body is submerged in fluid in equilibrium the net verticle hydrostatic force (buoyant force) becomes equal to weight of the body the body is said to in floating. It is known as archimedes principle.



$$V_{fd} = V$$

$$W_{fd} = \omega_f V_{fd}$$

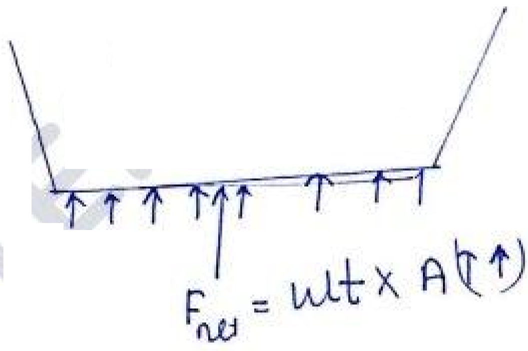
$$F_B = \omega_f \cdot V$$

$\rho_B > \rho$

$[\rho_B g V] > [\rho_f \cdot g V]$

$\boxed{Wt > \rho_f \cdot g \cdot V_{fd}}$ $\boxed{Wt > F_B}$ body will sink

Floating ☆ $\boxed{Wt = F_B}$

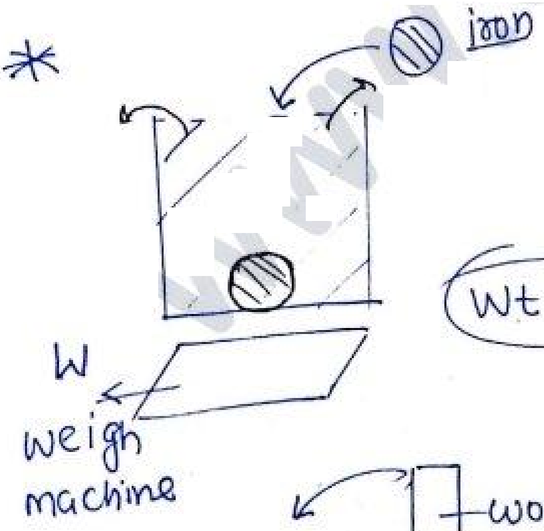


For $Wt = F_B$

displace volume increase

Area ↑ V_{fd} ↑

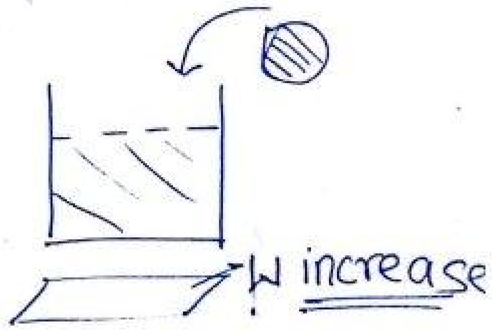
*



Reading of weighing balance will increase

$Wt > F_B$

*



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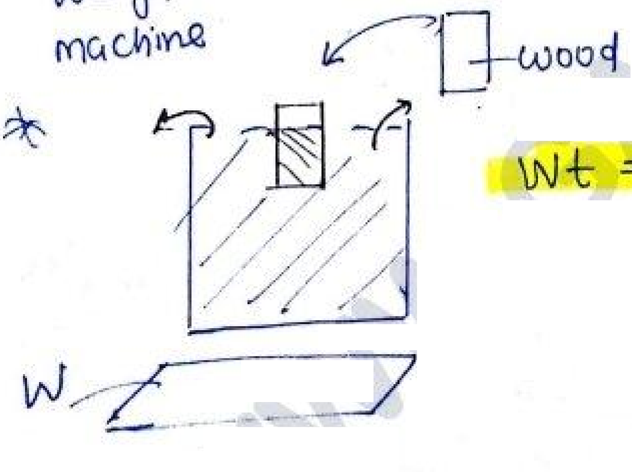
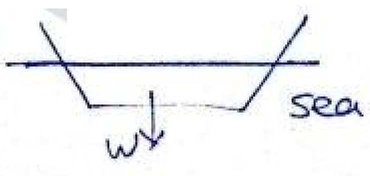
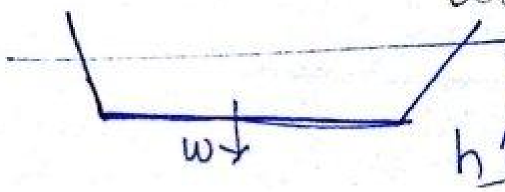


$Wt = F_B$

Reading Remain Constant.

*





it will sink

bcom

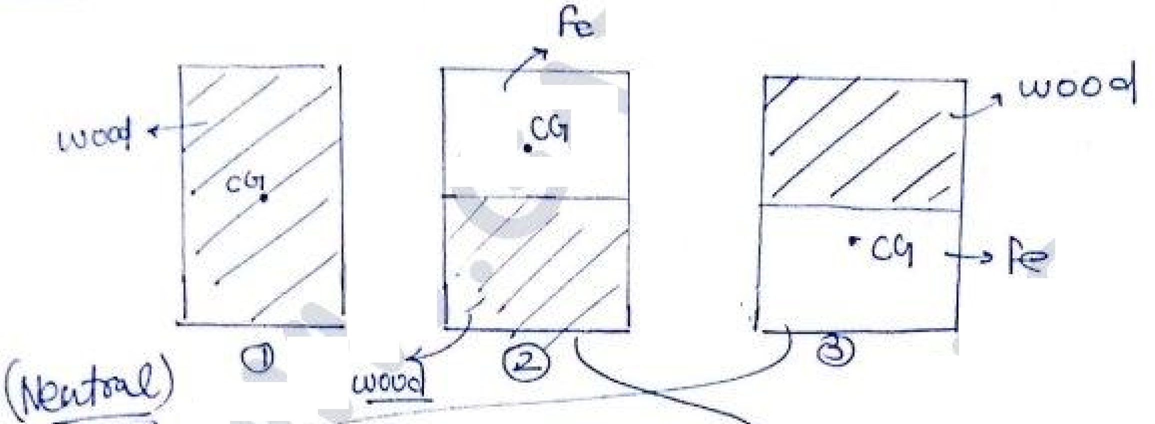
Rived $\rho_R < \rho_S$

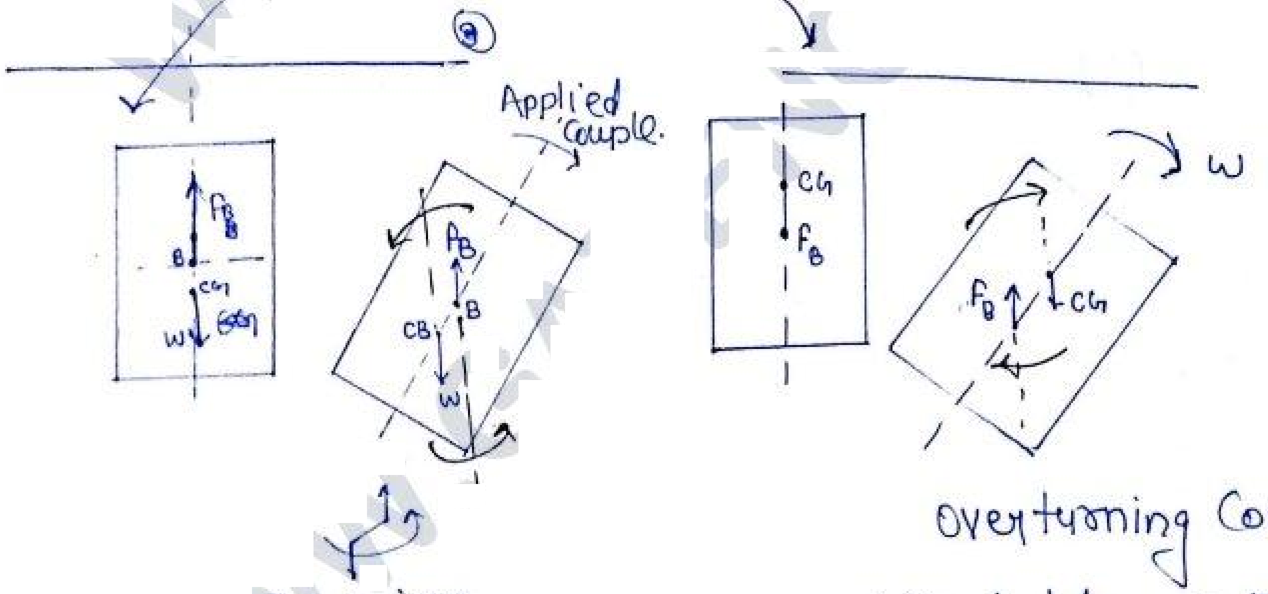
h ↑

Equilibrium Conditions:-

stable eq^m (Restoring) | unstable eq^n (overturning) | Neutral eq^n

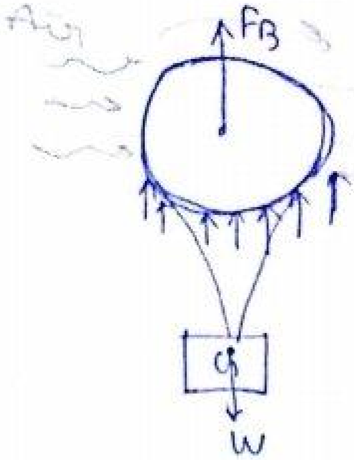
(1) For completely submerged body ⇒





Restoring Couple
⇒ Stable eq^m
(B above C.G)

Overturning Couple
⇒ Unstable eq^m
(B below C.G)

* 

C.G. below the B
Stable

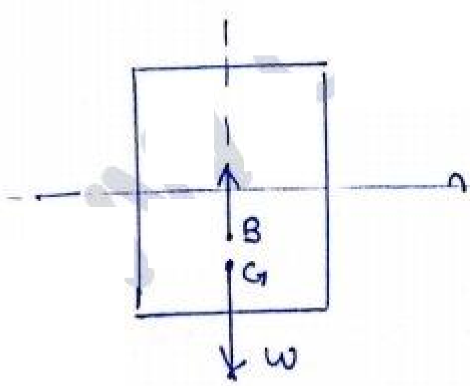
① A body is said to in stable equilibrium when centre of buoyancy (B) is above the centre of gravity (G).

② In unstable eq^m when centre of buoyancy (B) is below centre of gravity (G)

③ In neutral eq^n when centre of buoyancy (B) coinside C.G. (G)

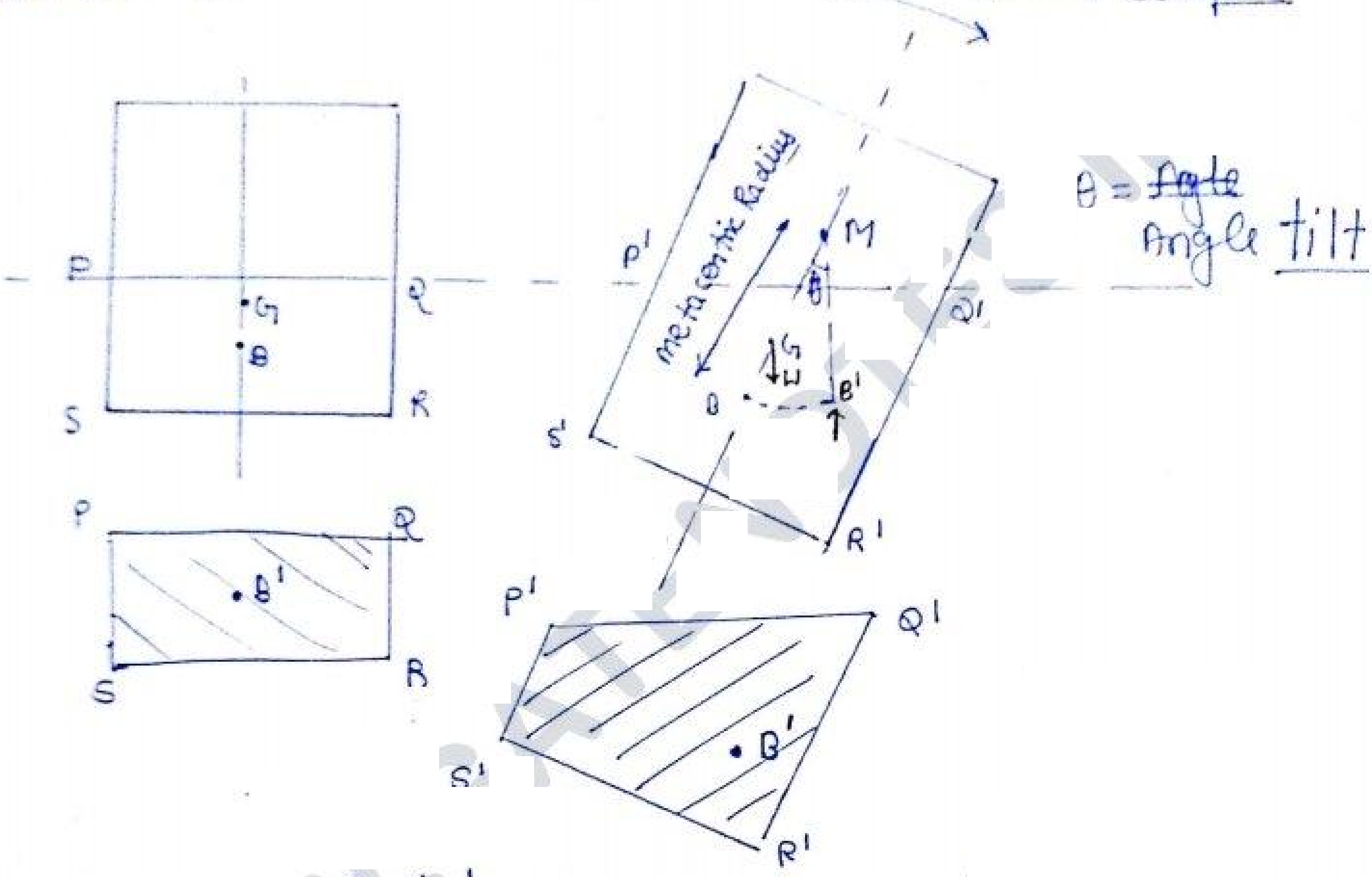
(2) For partially submerged body:-

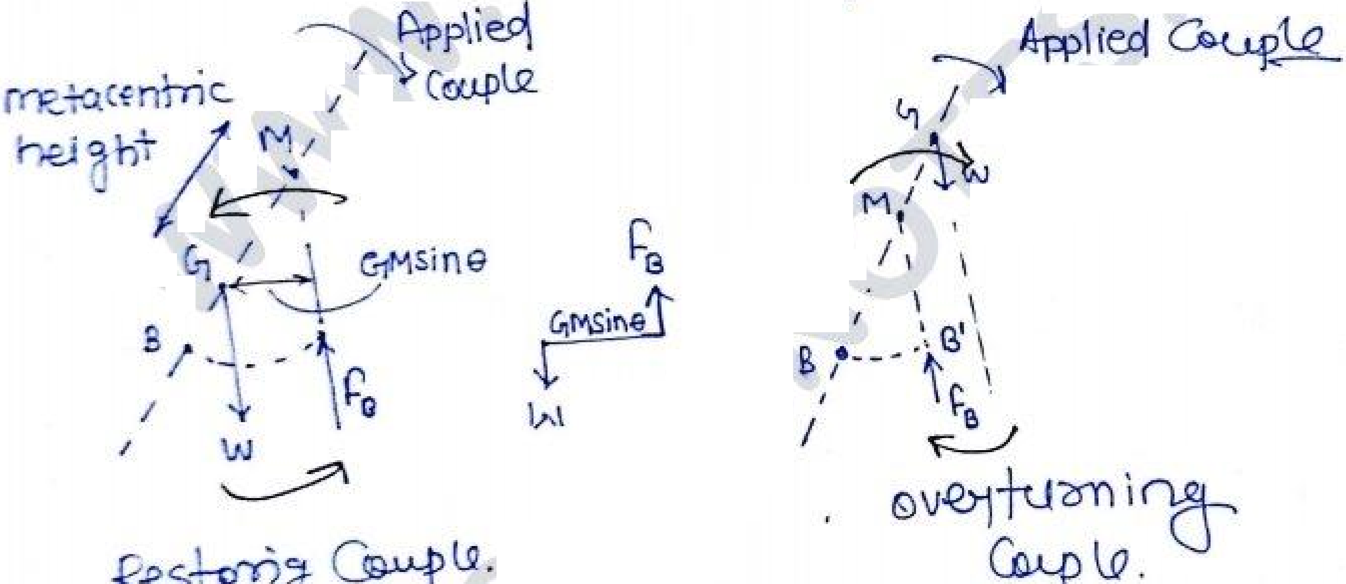
* when $B > G$ (B is above G)



Always stable eq^m

* When (B is below G)

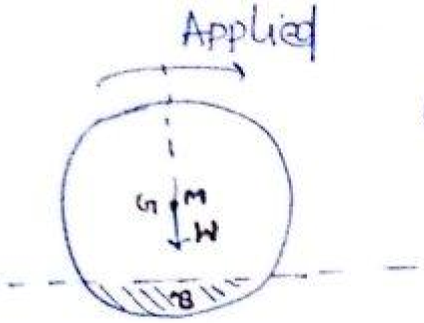




$$\text{Restoring Couple} = W(GM\sin\theta)$$

- +ve → S.E. $\quad GM > 0$
- −ve → U.S.E $\quad GM < 0$
- 0 → N.E. $\quad GM = 0$

*



$GM = 0$

N.E.

* Meta Centre is the point of intersection of new line of action of buoyant force and centroidal axis.
* The distance b/w the metacentre and centre gravity is known as metacentric ~~len~~ height.
* Meta centre may lie inside or outside the body C.G. & centre of buoyancy always lies inside the body.

For partially submerge bodyes

a) a body is said to be in stable eq^m when M above G

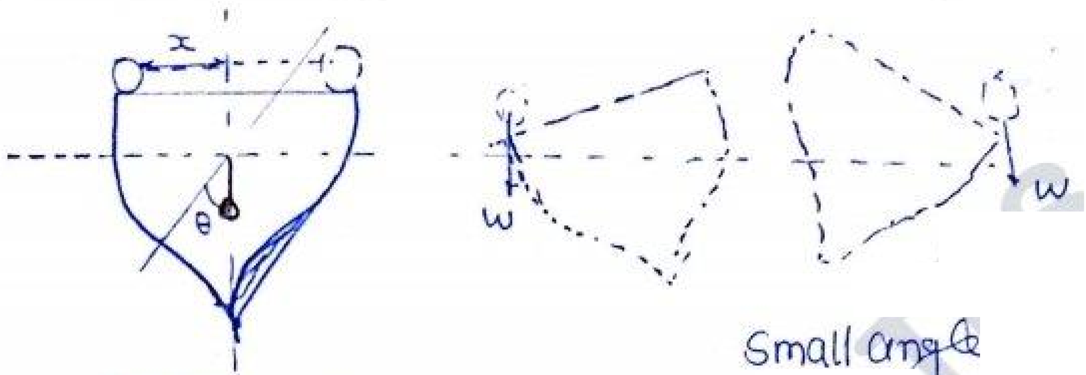
b) In U.S.E. , M below G

c) N.E. when M coinsides G.

Calculation of Metacentric height:-

GM
- exprementa l
- Analytical

Experimental:-



Applied couple $= w(2x)$

Small angle

$\sin\theta = \theta$

$\theta = \tan\theta$

Rotating couple $= W\, GM \sin\theta$

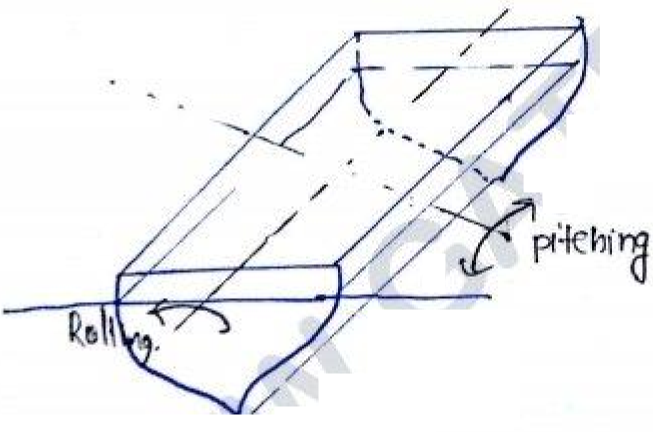
$$W\, GM \sin\theta = w \cdot 2x \Rightarrow GM = \frac{w(2x)}{W \tan\theta}$$

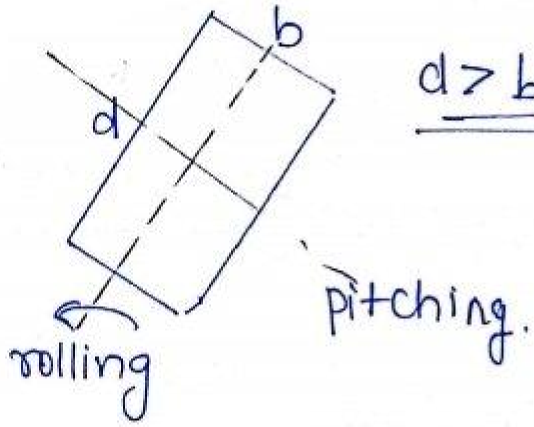
Analytical:-

metacentric Radius

$$\boxed{BM = \frac{I}{\gamma_{fd}}}$$

least $\boxed{\text{MoI}}$ of the water line area about tilting axis.





$d > b$

Rolling — $I_{rolling} = \frac{db^3}{12}$

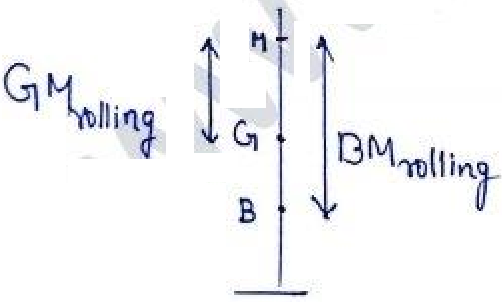
Pitching — $I_{pitching} = \frac{bd^3}{12}$

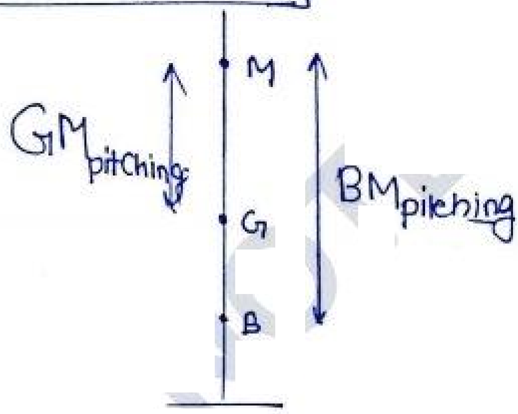
$$b < d$$

$$I_{rolling} < I_{pitching}$$

$$\frac{I_{rolling}}{V_{fd}} < \frac{I_{pitching}}{V_{fd}}$$

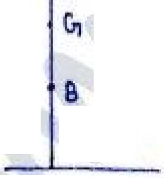
$$* \boxed{BM_{Rolling} < BM_{pitching}}$$





$$* \boxed{GM_{rolling} < GM_{pitching}}$$

Note

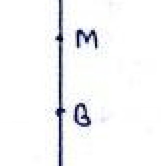


S.E.

$BM > BG$

$GM > 0$

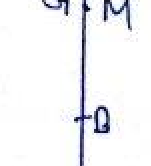
$$\boxed{GM = BM - BG}$$



U.S.E.

$GM < 0$

$$\boxed{BM < BG}$$



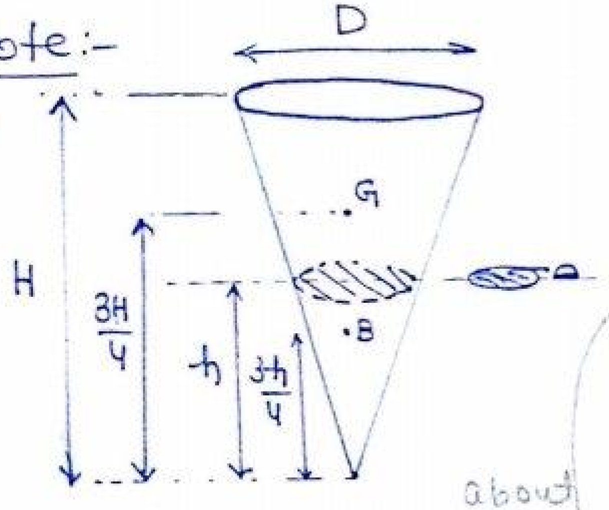
N.E.

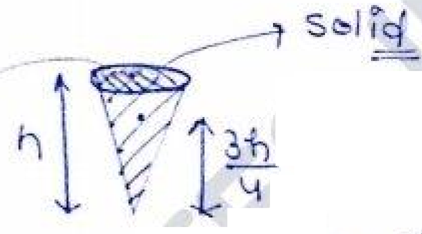
$GM = 0$

$BG = BM$

Note:-

Cone



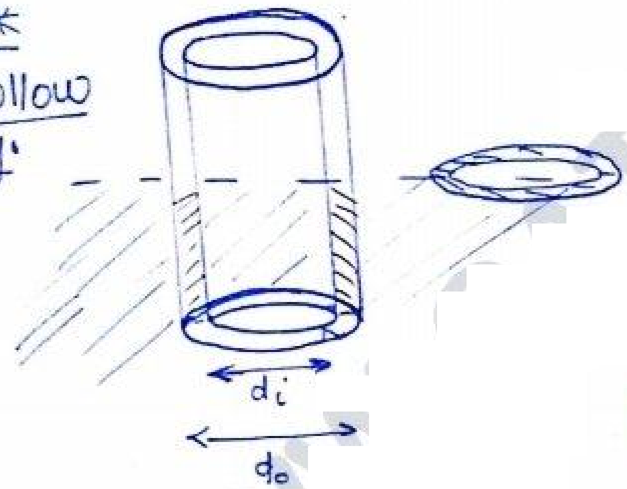


$$V_{fd} = \frac{1}{3}\pi\left(\frac{d}{2}\right)^2 h$$

about this surface

$$I = \frac{\pi}{64} d^2$$

*
Hollow Cy.



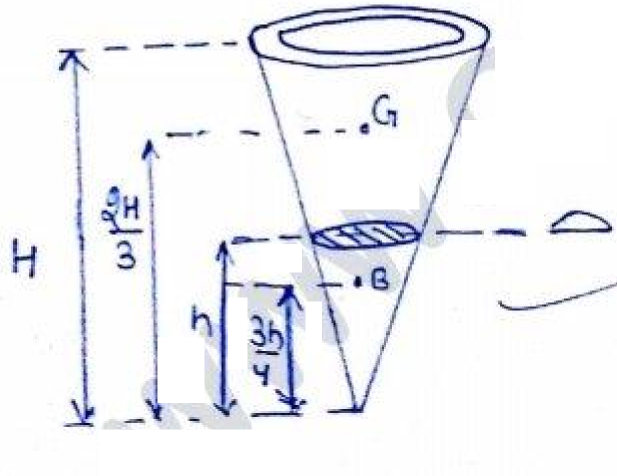
$$I = \frac{\pi}{64}\left[d_o^4 - d_i^4\right]$$

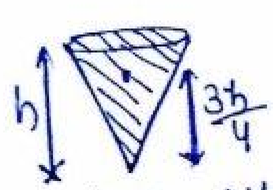
$$BM = \frac{I}{V_{fd}} \qquad V_{fd} = \frac{\pi(d_o^2 - d_i^2)}{4} \times h$$

Solid Cy./closed from bottom $\quad I = \frac{\pi}{64} d_o^4$

Imp
* **Hollow Cone**

C.G. of hollow cone $= \frac{2H}{3}$ from Apex

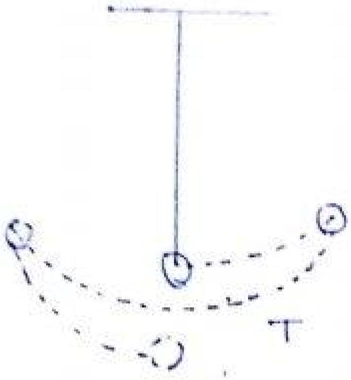




displaced Vol. (Solid

$$V_{fd} = \frac{1}{3}\pi\left(\frac{d}{2}\right)^2 \cdot h$$

Time period of oscillation :-



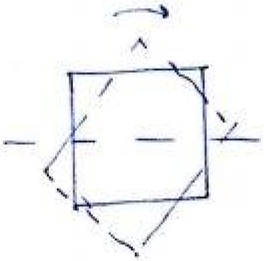
$$a \propto -x$$

$$F = -ma$$

$$T = -I\alpha = -I\frac{d^2\theta}{dt^2}$$

$\downarrow$

Restoring Couple.



$$W(GM)\sin\theta = -I\frac{d^2\theta}{dt}$$

$$mg(GM)\sin\theta = -mk^2\frac{d^2\theta}{dt}$$

$$\frac{d^2\theta}{dt^2} + \left[\frac{g(GM)}{k^2}\right]\theta = 0$$

$$\omega^2 = \frac{g(GM)}{k^2}$$

$$T = \frac{2\pi}{\omega}$$

$$\boxed{T = 2\pi\sqrt{\frac{k^2}{g(GM)}}}$$

$$\downarrow T \propto \frac{1}{\sqrt{GM}\uparrow}$$

k = radius of Gyration

GM = Metacentric height.

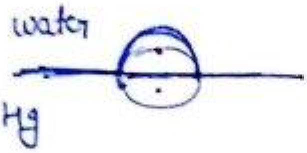
GM $\uparrow$ $\rightarrow$ Stability $\uparrow$ $\rightarrow$ T $\downarrow$ $\rightarrow$ Comfort $\downarrow$

war ship GM 1.3M — 1.8M

Cargo ship GM 0.5M — 1M

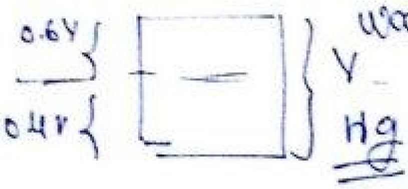
Que! A metalic body floats at the interface of Hg & water in such a way that 40% of it's volume is submersed in Hg and 60% in water find the density of body.

Sol^n



$\cancel{(-\rho g) \times 0.4}$

$\cancel{(\rho_w - \rho_b) g \times 0.60 V = (\rho_b - \rho_{Hg}) g \times 0.40 V}$



water $\cancel{(\rho_b - 1000) \times 0.3 = (13600 - \rho_b) \times 2}$

$\cancel{3\rho_b - 3000 = 27,200 - 2\rho_B}$

$Wt_B = W_{fd}$ floating

$w_B V = w_f W_{fd} =$

$\rho_B g V = \rho_w g (0.6V) + \rho_{Hg} g (0.4V)$

$\rho_B = 1000 \times 0.6 + 13.6 \times 1000 \times 0.4$

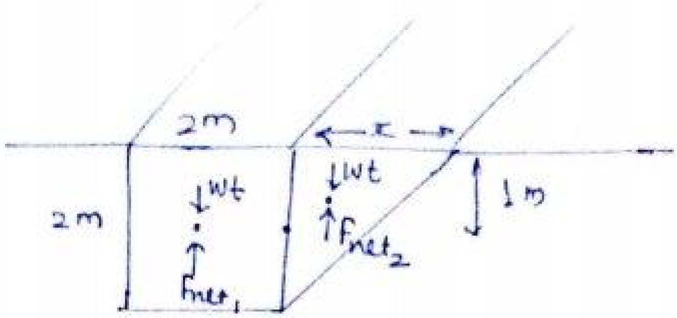
$\rho_B = 6040 \ kg/m^3.$

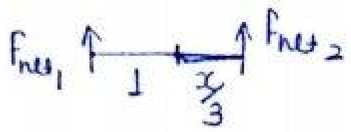
Q. 35 $Wt = (2 \times 2 \times 2) \times 1 = \left(\frac{1}{2} x \times x \times 2 \right) \times 2 \times \times \frac{x}{3}$

$12 = x^2$

$x = 2\sqrt{3}$

or





$$F_{net_1} \times 1 = F_{net_2} \times \frac{x}{3}$$

$$(F_{B_1} - W_1) \times 1 = (F_{B_2} - W_2) \times \frac{x}{3}$$

$$\rho_f V_1 - \rho_B V_1 = (\rho_f V_2 - \rho_B V_2)\frac{x}{3}$$

$$V_1 = V_2 \times \frac{x}{3}$$

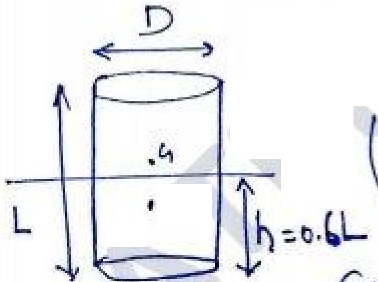
$$2 \times 2 \times B = \frac{1}{2} \times x \times 2 \times \frac{x}{3}$$

$$x = 2\sqrt{3}\ \underline{\underline{m}}$$

Q.33 $S_g = 0.6$

$\rho_w = 1000\ kg/m^3$

$\rho_s = 600\ kg/m^3$



$GM = 0$

$$W_t = W_{fd}$$

$$\omega_B (A \cdot L) = \omega_w (A \cdot h) \Rightarrow \frac{\omega_B}{\omega_w} = \frac{h}{L}$$

$$\frac{\omega_B}{\omega_w} L = h$$

$$S_B L = h$$

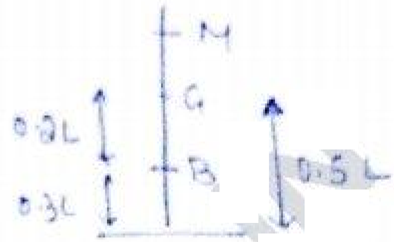
$$\boxed{h = 0.6 \cdot L}$$

Start with here

$$\hookrightarrow GM = BM - BG$$

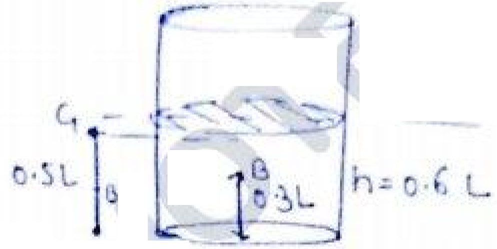
$$\underline{GM = 0} \quad (\text{Given})$$

$$0 = BM - BG$$



Find

$$\hookrightarrow BM = \frac{I}{V_{fg}}$$



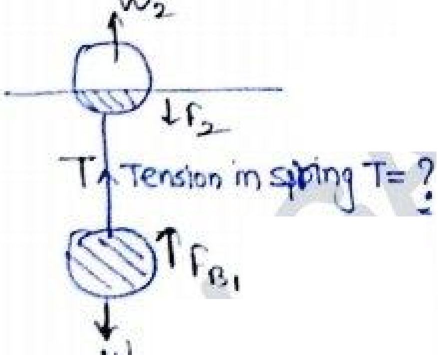
$$BM = \frac{\frac{\pi}{64} D^4}{\frac{\pi D^2}{4} \times (0.6 L)} = \frac{D^2}{9.6 L}$$

then equat $\Rightarrow$

$$\frac{D^2}{9.6 L} = 0.2 L$$

$$\frac{L^2}{D^2} = \frac{25}{16 \times 3} \Rightarrow \frac{L}{D} = \frac{5}{4\sqrt{3}}$$

*



4.w.

Que A hollow cylinder of length 1m and ~~to~~ have internal & external dia 0.4m & 0.6 m respectively and both ends are open the weight of the cylinder is 700N anaylys wheather the cylinder would be stable while floating in water with its axis verticle.

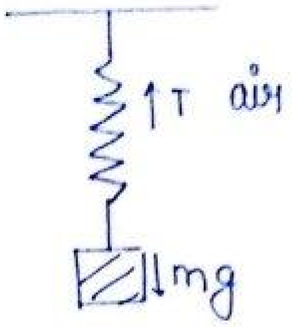
W = 700 N

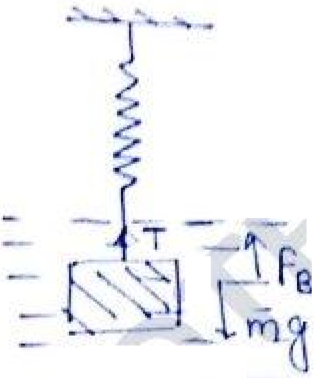
$$\cancel{wt} = F_B$$

$$7\cancel{00} = \rho_w \times g \left(\frac{\pi}{4}\left(d_o^2 - d_i^2\right)\right)$$

$$F_B = 157.079 \, \underline{\underline{N}}$$

Real and Apparent weight Concept:-

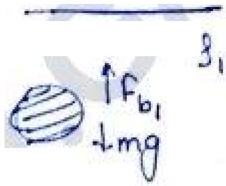




Real weight $T = mg$

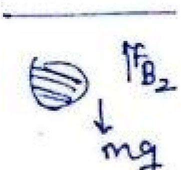
$$\boxed{T = mg - F_B}$$

Q.39 ρ_B -



$\rho_1 = 800$

$$30 = mg - \rho_1 g V \quad (1)$$



$\rho_1 = 1200N$

$$15 = mg - \rho_2 g V \quad (2)$$

$$15 = (\rho_2 - \rho_1) \times 10 \times V$$

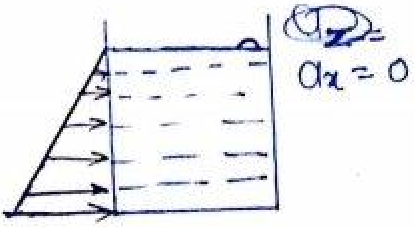
$$15 = (1200 - 800) \times 10 \times V$$

$$V = 3.82 \text{ lit}$$

Fluid under Relative motion:-

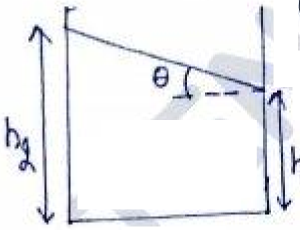
Example :- fuel tanks of car, aeroplane, gasoline tanker, water tanker etc.

- Horizontal motion:-

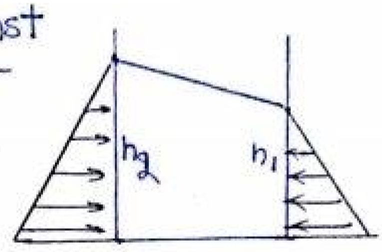


$P = \rho g h$

$a_x = \text{Const}$

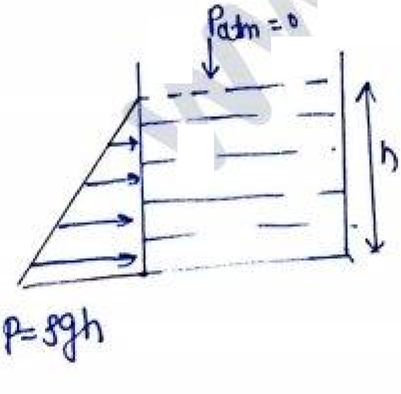


$$\boxed{\tan\theta = \frac{a_x}{g}}$$



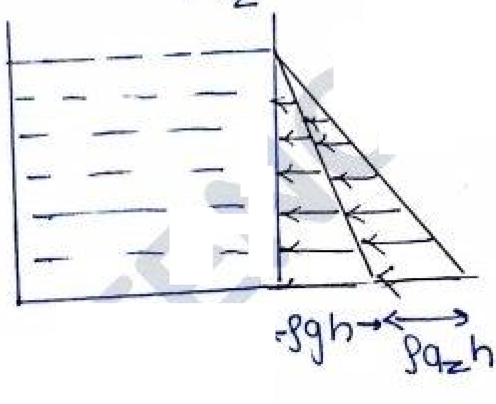
$\rho g h_2$ $\rho g h_1$

- Vertical Motion



$P = \rho g h$

$\uparrow a_z = \text{Const}$



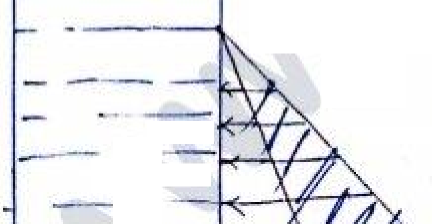
$\rho g h \rightarrow \leftrightarrow \rho a_z h$

$P = \rho g h + \rho a_z h$

$$\boxed{P = \rho(g + a_z)h}$$

&

$\downarrow a_z = \text{Const}$



$\rho g h$

$P = \rho g h - \rho a_z h$

$$\boxed{P = \rho(g - a_z)h}$$

★ Free Fall

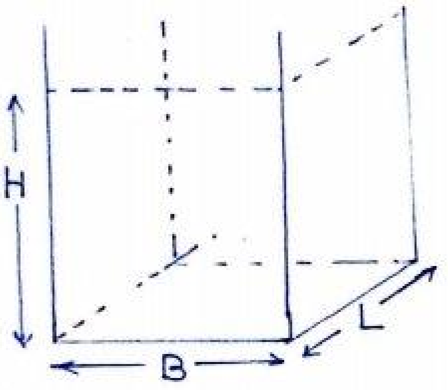
$a_z = g$

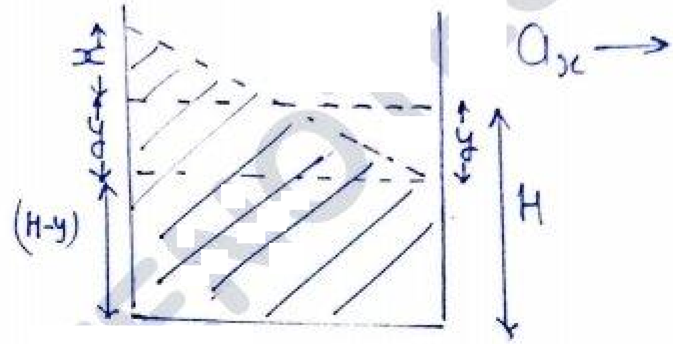
$\boxed{P = 0}$ gauge

$\boxed{P = P_{atm}}$

Absolute

Ques Find the rise and fall of the liquid in the container if the container is in constant horizontal motion and no liquid split out.





$$V_i = B \times H \times L$$

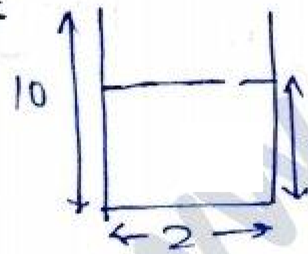
$$V_f = \left[\frac{1}{2}(x+y)B + (H-y)\cdot B\right]L$$

No spillage. $V_i = V_f$

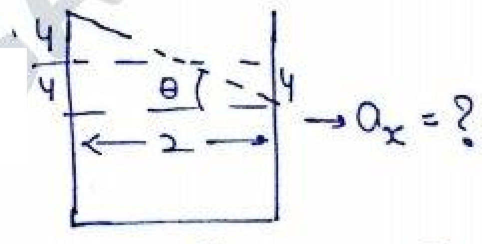
$$B \times H \times L = \frac{1}{2}xBL + \frac{1}{2}yBL + HBL - HBL$$

$$\boxed{x = y}$$

WB Pg20
Q.52



$\Rightarrow$



$$\tan\theta = \frac{a_x}{g} = \frac{8}{2} = 4$$

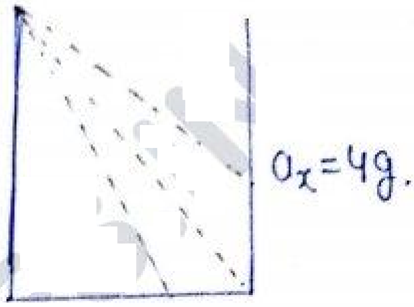
$$a_x = 4g \ m/s$$

Q.53

(ii) $V_i = 6 \times 2 \times 3$

$V_i = 36 \ m^3$

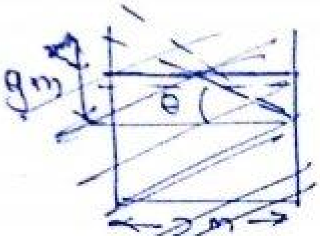
~~tanθ~~ =



(i) $a_x = 4.5g \ m/s^2$

$\tan\theta = \frac{a_x}{g} = \frac{4.5g}{g} = \frac{x}{2}$

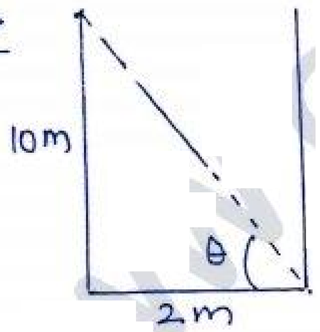
$x = 9$



we directly can't tell actual diagram of $4.5g \ m/s$

⇒

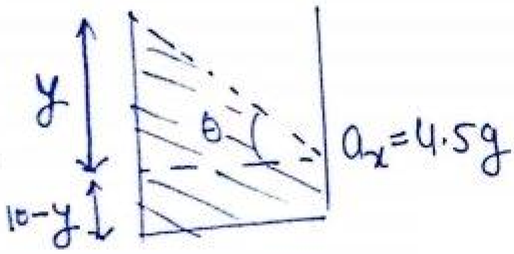
1st step



$\tan\theta = \frac{a_x}{g} = \frac{10}{2}$

$a_x = 5g$

So 4.5g above
6g below



$\tan\theta = \frac{a_x}{g} = \frac{4.5g}{g} = 4.5$

$\frac{y}{2} = 4.5$

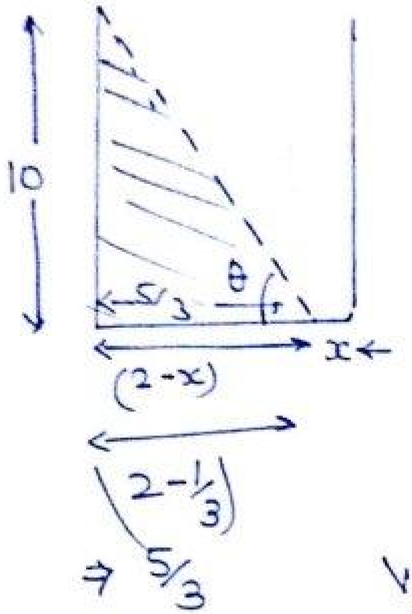
$y = 9 \ m$

$V_f = \left[\frac{1}{2} \times 2 \times 9 + 2 \times 1\right] \times 3$

$V_f = 33 \ m^3$

Spilled Vol $= 36 - 33 = 3 \ m^3$

$a_x = 6g$



$$\tan\theta = \frac{6g}{g} = \frac{10}{2-x}$$

$$12 - 6x = 10$$

$$6x = 2$$

$$x = \frac{1}{3}$$

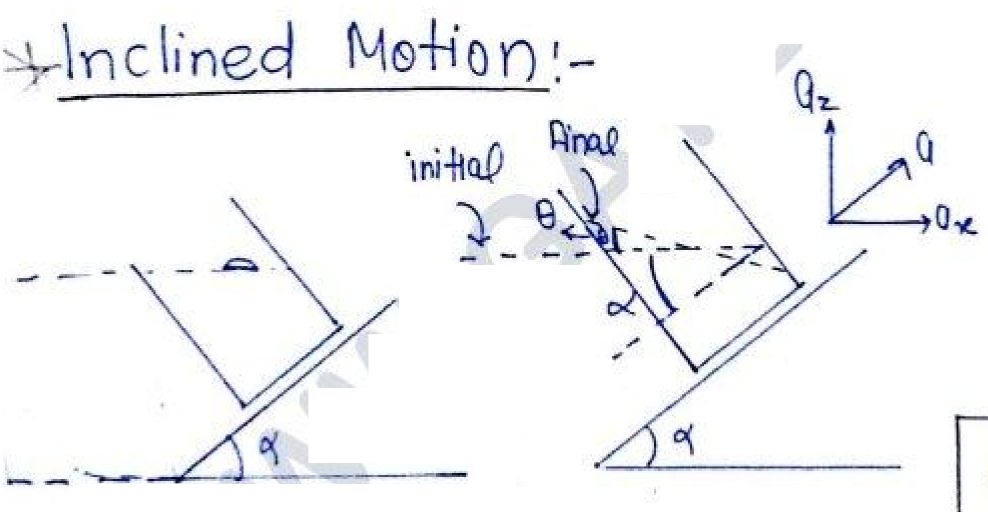
$$V_f = \frac{1}{2} \times \frac{5}{3} \times \cancel{10}^{5} \times 3$$

$$V_f = 25 \text{ m}^3$$

$$\text{Spilled Vol.} = 36 - 25$$

$$\underline{\Delta V} = 11 \text{ m}^3$$

*Inclined Motion:-



$$\tan\theta = \frac{a_x}{g_{eff.}}$$

$$\boxed{\tan\theta = \frac{a_x}{g + a_z}}$$ upward

$$\boxed{\tan\theta = \frac{a_x}{g - a_z}}$$ downward