Chapter 2
Electrostatic Potential and Capacitance

Solutions (Set-1)

Very Short Answer Type Questions :

1. What are linear isotropic dielectrics?

Sol. In an external electric field, the positive and negative charges of a non- polar molecule are displaced in opposite directions. The displacement stops when the external force on the constituent charges of the molecule is balanced by the restoring force (due to internal fields in the molecule). The non-polar molecule thus develops an induced dipole moment. The dielectric is said to be polarized by the external field. We consider only the simple situation when the induced dipole moment is in the direction of the field and is proportional to the field strength. *(Substances for which this assumption is true are called* ***linear isotropic dielectrics****)*

2. Define dielectric strength.

Sol. The maximum electric field that a dielectric medium can withstand without breaking down of its insulating property is called its dielectric strength.

3. What is the fringing effect of an electric field?

Sol. Electric field in the space between the plates of a parallel plate capacitor is non-uniform at the ends and the field lines are curved. This is known as fringing effect.

4. Write the relation between electric susceptibility and dielectric constant.

Sol. In either case, whether polar or non-polar, a dielectric develops a net dipole moment in the presence of an external field. The dipole moment per unit volume is called polarization and is denoted by P. For linear isotropic dielectrics,

$P = \chi_e E$

Here χ_e is a constant, characteristic of the dielectric and is known as the electric susceptibility of the dielelctric medium.

5. A charge q is moving along a straight line perpendicular to the bisector of an electric dipole, then what is the work done by charge against electric field?

Sol. Zero

6. When we move along the direction of electric field, then does electrostatic potential increases, decreases or remains same?

Sol. Decreases

7. Write the dimensional formula for polarisation.

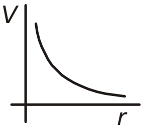
Sol. $[M^0L^{-2}T^1A^1]$

8. What is the electrostatic potential due to a charge q at its own location?

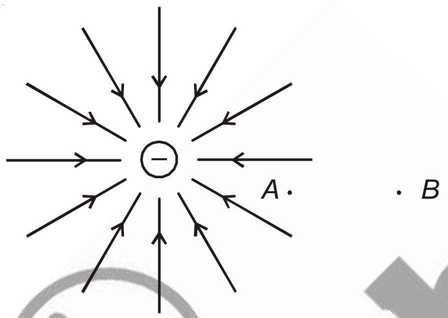
Sol. Infinite

9. Draw a graph between electrostatics potential and distance for a point charge Q.

Sol. $\because V = K\frac{Q}{r}$



10. Does the kinetic energy of a small positive charge increase or decrease in going from A to B?

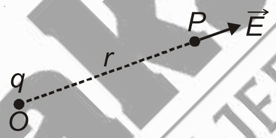


Sol. Increases

Short Answer Type Questions :

11. Derive an expression for electrostatic potential at any point due to a point charge.

Sol. Consider a point charge q placed at point O. Consider any point P in the field of the above charge. Let us calculate the potential at point P due to the charge q kept at point O.



By definition, potential at P is equal to work done against electric field in moving a unit positive charge from a large distance (∞) to point P. For that matter, a force equal and opposite to the force exerted by electric field is to be applied on the unit positive charge.

$\therefore\ \overrightarrow{F_{\text{ext}}} = -(+1C)\,\vec{E} = -\vec{E}$.

For small displacement $\overrightarrow{dr}$,work done is

$dw = \vec{F}\cdot\overrightarrow{dr} = -\vec{E}.\overrightarrow{dr}$

Now $\vec{E}$ is radially outward. Taking $\overrightarrow{dr}$ to be radially outward, the angle between $\vec{E}$ and $\overrightarrow{dr}$ becomes 0°,

So, $dw = -E\,dr = -\dfrac{q}{4\pi\varepsilon_0 r^2}dr$

Now, $w = \int dw = -\int_{\infty}^{r}\dfrac{q}{4\pi\varepsilon_0 r^2}dr$

$\Rightarrow\ w = -\dfrac{q}{4\pi\varepsilon_0}\left[-\dfrac{1}{r}\right]_{\infty}^{r}$ $\qquad \Rightarrow\ w = \dfrac{q}{4\pi\varepsilon_0 r}$ $\qquad \left(\because \int x^n dx = \dfrac{x^{n+1}}{n+1}\right)$

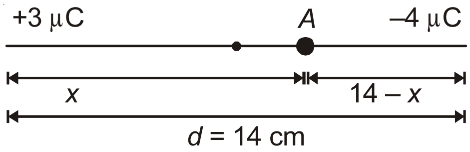
by defination is the potential at P due to the charge q

$V = \dfrac{q}{4\pi\varepsilon_0 r}$ or $V = \dfrac{Kq}{r}$, where $K = \dfrac{1}{4\pi\varepsilon_0}$

12. Two charges +3 μC and –4 μC are placed 14 cm apart. Find the point on the line joining the two charges where electric potential is zero. (Taking zero potential at infinity).

Sol. Case 1:

Let at point A, potential is zero.

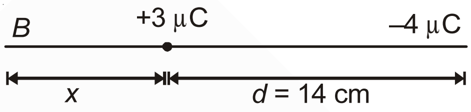


$$\frac{K\left(3\times10^{-6}\right)}{\left(\frac{x}{100}\right)}-\frac{K\left(4\times10^{-6}\right)}{\left(\frac{14-x}{100}\right)}=0$$

$$\frac{3}{x}=\frac{4}{14-x} \Rightarrow x = 6 \text{ cm}$$

Case 2:

Let potential at B is zero.



$$\frac{K\left(3\times10^{-6}\right)}{\left(\frac{x}{100}\right)}-\frac{K\left(4\times10^{-6}\right)}{\left(\frac{(14+x)}{100}\right)}=0$$

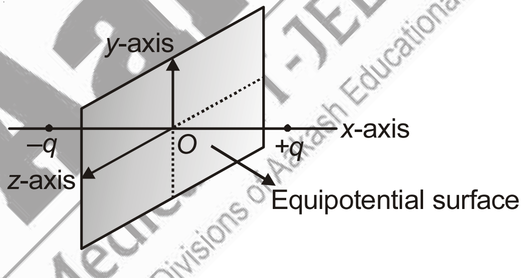
$$\frac{3}{x}=\frac{4}{14+x} \Rightarrow 42 + 3x = 4x$$

$\Rightarrow$ $x = 42$ cm

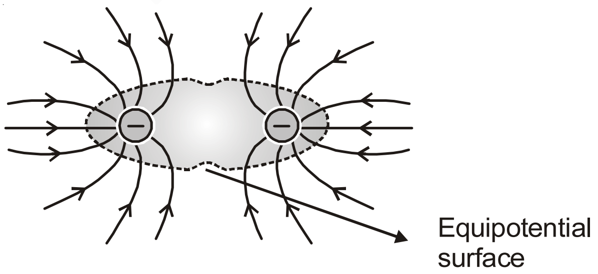
13. Draw the equipotential surface for (a) a dipole and (b) two identical negative charges placed at a certain distance apart.

Sol. Equipotential surface for

(a) a dipole



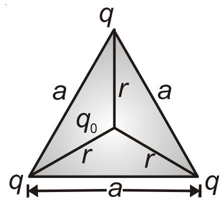
(b) Two identical negative charges placed at a certain distance apart.



14. Three identical charges (q) are arranged at the corners of an equilateral triangle of side a and a charge q_0 brought to the centre of the triangle, the three charges being held fixed at its corners. How much extra work is needed to this?

Sol. $W = 3\dfrac{Kq\,q_0}{r}$ $\left\{\because r = \dfrac{a}{\sqrt{3}}\right.$

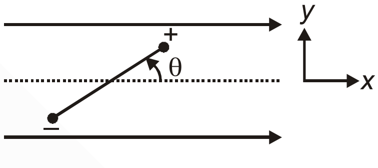
So, $W = \dfrac{3Kq\,q_0}{\left(a/\sqrt{3}\right)}$



$$\boxed{W = \dfrac{3\sqrt{3}\cdot q\cdot q_0}{4\pi\varepsilon_0\cdot a}}$$

15. Derive an expression of stored potential energy, when a dipole rotated from stable equilibrium position to any position in an external uniform electric field.

Sol. Potential energy can be associated with the orientation of an electric dipole in an electric field. Change in potential energy is related to the work done by electric field as



(a) Potential energy is U

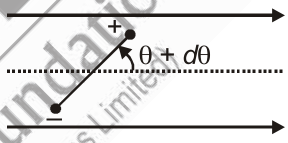
$dU = -dW_E = -\vec{\tau}\,.\,\vec{d\theta}$

$\Rightarrow\ dU = -(-pE\sin\theta\hat{k})\,.\,(d\theta\,\hat{k})$ $[\because\ \vec{d\theta} = d\theta\hat{k},\ \text{anticlockwise}]$

$\Rightarrow\ dU = pE\sin\theta\,d\theta$

$\Rightarrow\ \displaystyle\int_{U_1}^{U_2} dU = \int_{\theta_1}^{\theta_2} pE\sin\theta d\theta \Rightarrow U_2 - U_1 = -pE[\cos\theta_2 - \cos\theta_1]$

Let $\theta = 90°$ is taken as reference position or zero of potential energy.



(b) Potential energy is $U + dU$

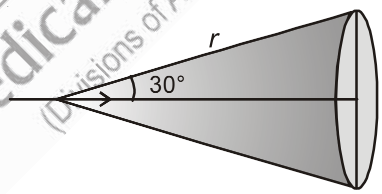
$\therefore\ \theta_1 = 90° \Rightarrow U_1 = 0$

$\Rightarrow\ U_2 - 0 = -pE[\cos\theta_2 - \cos 90°]$

$\Rightarrow\ U = -pE\cos\theta$

16. An electric dipole of dipole moment 10 μCm is placed along the axis of a right angled cone such that the vertex of cone is at the centre of dipole. If the semivertex angle of cone is 30° and slant height of curved surface is 20 cm, then find the electrostatic potential at any point of circumference of base of cone.

Sol. $\boxed{V = \dfrac{K.P.\cos\theta}{r^2 - a^2\cos^2\theta}}$



$\theta = 30°$, $P = 10 \times 10^{-6}$ cm

$r = 20$ cm

For a short dipole $a \simeq 0$

$$V = \frac{9\times10^9\times10\times10^{-6}}{\left(\dfrac{20}{100}\right)^2\times100}\times\frac{\sqrt{3}}{2} = \frac{90\times10^3\times\sqrt{3}}{2\times20\times20}\times\frac{100}{100}\times100$$

$$\boxed{V = \frac{9\sqrt{3}}{8}\times10^4 \text{ volts}}$$

17. "Inside a conductor, electrostatic field is zero", why?

Sol. Due to equal and opposite induced electric field inside the conductor, the net electrostatic field inside conductor become zero.

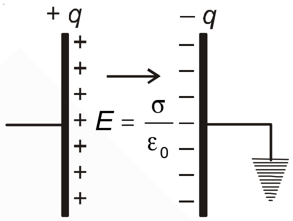
18. What is electrostatic shielding? Explain.

Sol. Electrostatic shielding/screening is the phenomenon of protecting a certain region of space from external electric field.

19. Derive an expression for capacitance of a parallel plate capacitor.

Sol. The arrangement consists of two thin conducting plates, each of area *A* and separated by *d* distance. When charge *q* is given to first plate, a charge – *q* is induced on the inner face of other plate and positive on the outer face of plate. As this face is connected to earth, a net negative charge is left on this plate. Thus, the arrangement is equivalent to two thin sheets of charge.

The electric field between the plates is



$E = \frac{\sigma}{\varepsilon_0}$ where σ is the charge density.

As $E = \frac{V}{d}$;

V is the potential difference between the plates.

$V = Ed = \frac{\sigma d}{\varepsilon_0}$

$\Rightarrow \; V = \frac{qd}{A\varepsilon_0}$ as $\sigma = \frac{q}{A}$

$\Rightarrow \; C = \frac{q}{V} = \frac{q}{\left(\frac{qd}{A\varepsilon_0}\right)}$, $C = \frac{\varepsilon_0 A}{d}$

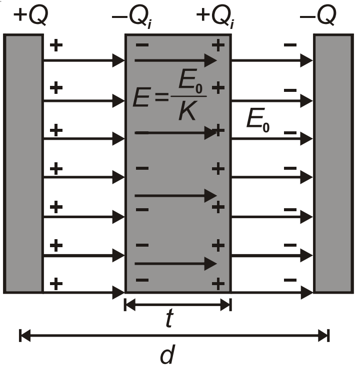
20. When a dielectric medium (slab) is inserted between the plates of a capacitor, then how is the capacitance changed? Explain.

Sol. Capacitance becomes *k* times of original capacitance where "*k*" is dielectric constant of dielectric

Effect of Dielectric Slab (Inserted along the length of plates)

When a dielectric slab is placed, between the plates of capacitor its polarisation takes place. Thus a charge $-Q_i$ appears on its left face and $+Q_i$ appears on its right face, as shown in diagram.

Potential difference between the plates can be calculated as,



$E_o = \frac{\sigma}{\varepsilon_0}$

$E = E_0 - E_p$

$E_p = \frac{\sigma_i}{\varepsilon_0}$

$\sigma_i = \frac{Q_i}{A}$

$V = E_0(d - t) + Et = E_0\left(d - t + \frac{t}{K}\right)$

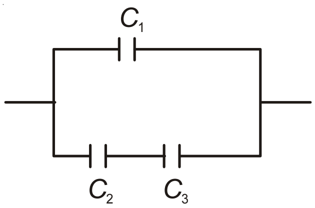
$\therefore \; V = \frac{\sigma}{\varepsilon_0}\left(d - t + \frac{t}{K}\right) = \frac{Q}{A\varepsilon_0}\left(d - t + \frac{t}{K}\right)$

As, $C = \frac{Q}{V}$ $\Rightarrow$ $C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$

Note : *When t = d,* $C = \frac{K\varepsilon_0 A}{d}$

21. A slab of material of dielectric constant *k* has the area $\left(\frac{1}{4}\right)^{th}$ as that of the plates of a parallel plate capacitor and has thickness $\frac{3}{4}$ *d* where *d* is the separation of plates. How is the capacitance changed when the slab is inserted between the plates?

Sol. $C_1 = \dfrac{\left(\dfrac{3A}{4}\right)\varepsilon_0}{d}$



$C_2 = \dfrac{\left(\dfrac{A}{4}\right)\varepsilon_0 \cdot K}{\dfrac{3d}{4}}$, $C_3 = \dfrac{\left(\dfrac{A}{4}\right)\varepsilon_0}{\dfrac{d}{4}}$

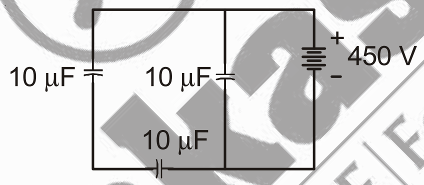
$$C_{23} = \frac{C_2 \cdot C_3}{C_2 + C_3} = \frac{\dfrac{A\varepsilon_0 \cdot K}{3d} \cdot \dfrac{A\varepsilon_0}{d}}{\dfrac{A\varepsilon_0 \cdot K}{3d} + \dfrac{A\varepsilon_0}{d}} = \frac{K \cdot A\varepsilon_0 \times 3}{3d(K+3)}$$

$$C_{23} = \frac{A\varepsilon_0}{d} \cdot \frac{K}{K+3}$$

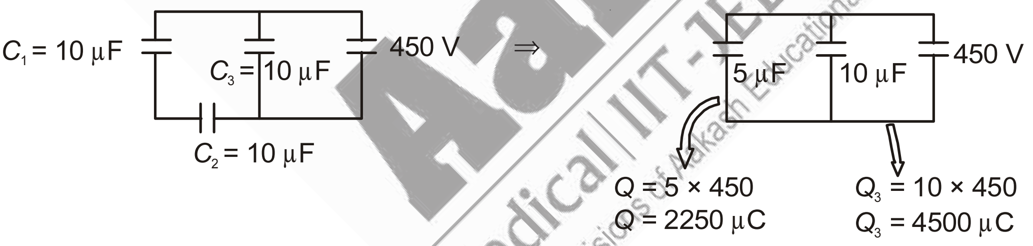
$$C_{eq} = C_1 + C_{23} = \frac{3A\varepsilon_0}{4d} + \frac{A\varepsilon_0 \cdot K}{d(K+3)} = \frac{A\varepsilon_0}{d}\left[\frac{3}{4} + \frac{K}{K+3}\right] = \frac{A\varepsilon_0}{d}\left[\frac{3K+9+4K}{4K+12}\right]$$

$$\boxed{C_{eq} = \frac{A\varepsilon_0}{d}\left(\frac{7K+9}{4K+12}\right)}$$

22. A network of three 10 μF capacitors is connected to a 450 V supply, as shown in fig. Determine the charge on each capacitor.



Sol.

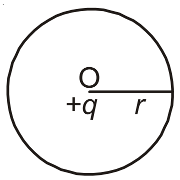


$Q_1 = Q_2 = 2250\ \mu C$

$Q_3 = 4500\ \mu C$

23. Prove that for an isolated spherical conductor the capacitance is directly proportional to the radius of conductor.

Sol. Capacity of an isolated spherical conductor



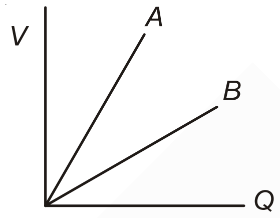
Consider a sphere with centre *O* and radius *r*, which is supplied with a charge = *+q*. This charge is distributed uniformly over the outer surface of the sphere. Thus, the potential at every point on the surface is same and is given by

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

As $C = \frac{q}{V}$

$C = 4\pi\varepsilon_0 r$

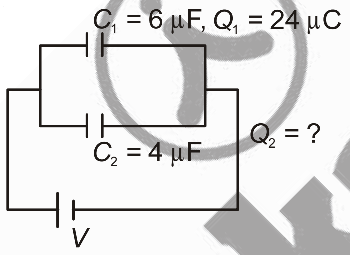
24. Graphs for two capacitors of capacitances C_1 and C_2 are shown in figure. The area of plates for both capacitors are same but separation between plates is double for C_1 to that of C_2. Which of the graph corresponds to C_1 and C_2 and why?



Sol. *A* for C_1 and *B* for C_2

25. Two capacitors of capacitance 6 μF and 4 μF are connected in parallel with a battery. The charge on 6μF capacitor is 24 μC, then find the charge on another capacitor and voltage of battery.

Sol. $\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$

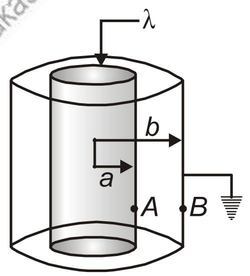


$$\frac{24}{6} = \frac{Q_2}{4}$$

$$\boxed{Q_2 = 16\ \mu C}$$

26. A long charged conducting cylinder of linear charge density λ is surrounded by a hollow co-axial conducting cylinder. The outer surface of hollow cylinder is earthed. Find the potential difference between cylinders, if radius of inner cylinder is a and outer hollow cylinder is *b*.

Sol. $\int_B^A dv = -\int_b^a \vec{E}\cdot\vec{dr}$



$$V_A - V_B = -\int_b^a \frac{\lambda}{2\pi\varepsilon_0 r} dr$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}\left[\log_e r\right]_b^a$$

$$= \frac{-\lambda}{2\pi\varepsilon_0}\left(\log_e a - \log_e b\right)$$

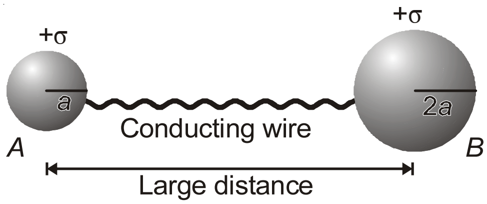
$$\Rightarrow \quad V_A - V_B = \frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{b}{a}\right)$$

∴ Cylinder *B* is earthed so $V_B = 0$ then,

$$\boxed{V_A - 0 = \frac{\lambda}{2\pi\varepsilon_0}\log_e\left(\frac{b}{a}\right)}$$

27. Two conducting sphere of radius a and 2a having same charge density $+\sigma$ are placed far distance away. When they are connected by a conducting wire then due to charge re-distribution their potential change. Find the common potential of each sphere.

Sol. Now, $V_A = V_B$



$$\frac{K.q_A}{a} = \frac{K.q_B}{2a}$$

$$q_A = \frac{q_B}{2}$$

Also, $q_A + q_B = \left(\sigma 4\pi a^2 + \sigma 4\pi (2a)^2\right)$

$$\frac{q_B}{2} + q_B = \sigma 4\pi a^2 5$$

$$q_B = \frac{40}{3}\pi a^2 \cdot \sigma$$

$$\therefore \quad V = \frac{Kq}{r} = \frac{K\cdot\sigma\cdot 4\pi r^2}{r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\sigma 4\pi r^2}{r}$$

$$V = \frac{\sigma r}{\varepsilon_0}$$

Common potential $V = \dfrac{K\cdot q_B}{2a}$

$$= \frac{1}{4\pi\varepsilon_0 \times 2a} \times \frac{40\pi a^2\cdot\sigma}{3}$$

$$\boxed{V = \frac{5\sigma\cdot a}{3\varepsilon_0}}$$

28. Determine the electrostatic potential energy of a system consisting of two charges 5 μC and –3 μC placed in an external electric field $E = \dfrac{2\times 10^5}{r^2}$ V/m at (–3 cm, 0, 0) and (3 cm, 0, 0) respectively, where r is distance from origin. What would the electrostatic energy of the configuration be?

Sol. Given electric field is due to a unit positive point charge q placed at origin.

$$\frac{K\cdot q}{r^2} = \frac{2\times 10^5}{r^2} \Rightarrow \boxed{q = \frac{200}{9}\,\mu\text{C}}$$

$$\text{Energy} = \frac{K\left(\frac{200}{9}\times 10^{-6}\right)\left(5\times 10^{-6}\right)}{3\times 10^{-2}} - \frac{K\left(\frac{200}{9}\times 10^{-6}\right)\left(3\times 10^{-6}\right)}{3\times 10^{-2}} - \frac{K\left(5\times 10^{-6}\right)\left(3\times 10^{-6}\right)}{6\times 10^{-2}}$$

$$= 9\times 10^9\left(\frac{200}{9}\times\frac{5}{3} - \frac{200}{9} - \frac{5}{2}\right)\times\frac{10^{-12}}{10^{-2}}$$

$$= 9\left(\frac{200}{9}\times\left(\frac{5-3}{3}\right) - \frac{5}{2}\right)\frac{1}{10} = \frac{9}{10}\left(\frac{200}{9}\times\frac{2}{3} - \frac{5}{2}\right)$$

$$= \frac{40}{3} - \frac{9}{4} = \frac{160-27}{12} \Rightarrow \boxed{\text{Energy} = \frac{133}{12}\text{J}}$$

29. Define the polar and non-polar dielectric materials with examples.

Sol. In general the dielectric can be classified into Polar and Non-polar dielectrics. The molecules of a substance may be polar or non-polar. In a non-polar molecule, the centres of positive and negative charges coincide. The molecule thus has no permanent (or intrinsic) dipole moment. Examples of non-polar molecules are oxygen (O_2) and hydrogen (H_2) molecules which, because of their symmetry, have no dipole moment. On the other hand, a polar molecule is one in which the centres of positive and negative charges are separated (even when there is no external field). Such molecules have a permanent dipole moment. An ionic molecule such as HCl or a molecule of water (H_2O) are examples of polar molecules.

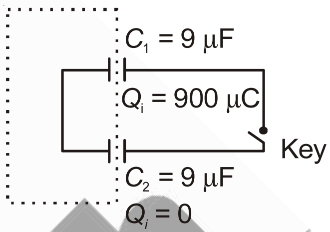
30. A 9 μF capacitor is charged by 100 V battery. The capacitor is disconnected from battery and connected with another identical uncharged capacitor. What is the electrostatic energy stored by the system?

Sol. For isolated system

Net initial charge = Net final charge

900 + 0 = 9 V + 9 V

V = 50 volts



$$U = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2} \times 18\mu F \times 50 \times 50$$

$= 9 \times 2500 \times 10^{-6} = 22400 \times 10^{-6}$ J

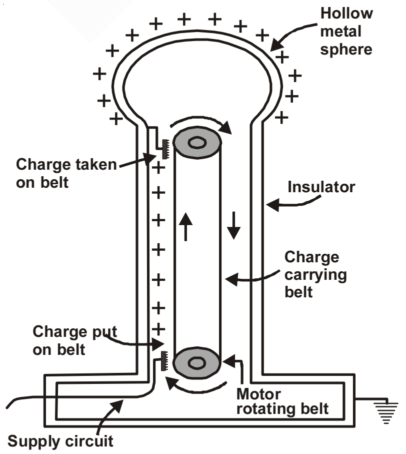
$= 2.2 \times 10^{-2}$ J

Long Answer Type Questions :

31. Discuss briefly the principle, construction and working of Van-de-Graff electrostatic generator.

Sol. Van de Graaff Generator

If we place a charge anywhere in a conductor, the charge will move to the outside surface, and the field inside the conductor will be zero. Robert Van de Graaff took advantage of this concept in 1931 to build an accelerator, an apparatus, that produces highly energetic charged particles. Such particles are useful for microscopic probes of matter and as cancer treatments. Van de Graaff used a device similar in concept to the apperatus shown schematically in figure. An insulated belt (or chain) continuously brings charge to the inside of a hollow conductor, which then moves to the outside surface of the conductor. The electric potential on the spherical conducting surface increases as charge flows to its surface $V = \dfrac{q}{4\pi\varepsilon_0 R}$. An ion source produces charged atoms whose sign is such as to be repelled from the region of high potential and thus accelerated. Such devices are called Van de Graaff accelerators of Van de Graaff generators, and the beams they or other accelerators produce play an important role in modern techonology. for example, such beems are used to make microcircuits.



32. (i) When a capacitor of capacitance C charged upto a potential V, then derive an expression for energy stored in the capacitor.

(ii) If electric field between plates of capacitor is E then derive the expression of energy density.

Sol. (i) Energy stored in a capacitor during the charging of a capacitor, work has to be done to add charge to the capacitor against its potential. This work is stored in the capacitor as electrical energy.

Suppose during the charging of capacitor its potential at any instant is given by

$V = \frac{q}{C}$ small amount of work done in adding a charge dq is given by

$$dW = \frac{q}{C}\,dq$$

Total work done in giving a charge Q to the condenser is

$$W = \int_0^Q \frac{q}{C}dq$$

$$\therefore W = \left[\frac{q^2}{2C}\right]_0^Q$$

$$\therefore W = \frac{Q^2}{2C}$$

$$\therefore U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

(ii) Energy density in parallel plate capacitor

The volume of a parallel plate capacitor is Ad.

$\therefore$ Energy density $u = \dfrac{U}{Ad} = \dfrac{\frac{1}{2}CV^2}{Ad}$ where $C = \varepsilon_0\dfrac{A}{d}$, $V = Ed$

$$\therefore u = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)\left(\frac{E^2d^2}{Ad}\right) = \frac{1}{2}\varepsilon_0 E^2$$

33. Derive an expression for electrostatic potential due to an electric dipole at any point $p(r,\theta)$.

Sol. Potential due to an electric dipole

In the previous chapter on electric charges and field, we have already calculated the electric field due to an electric dipole and seen that for an ideal (short) dipole, the electric field varies inversely as r^3, we now determine the potential due to an electric dipole.

AB be an electric dipole of length $2a$ and let P be any point where $OP = r$.

Let θ be the angle between r and the dipole axis.

$AB = 2a$, $AO = OB = a$, $OP = r$

In Δ OAC, $\cos\theta = \dfrac{OC}{OA} = \dfrac{OC}{a}$

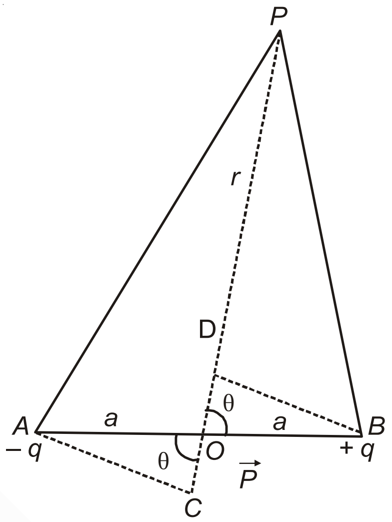
$\therefore OC = a\cos\theta$

Also $OD = a\cos\theta$

If $r >> a$, $PA \approx PC = OP + OC = r + a \cos\theta$

$PB \approx PD = OP - OD = r - a\cos\theta$

V is the potential due to electric dipole,



$$V = \left(\frac{1}{4\pi\varepsilon_0}\right)\left[\frac{q}{PB} - \frac{q}{PA}\right]$$

$$V = \left(\frac{1}{4\pi\varepsilon_0}\right) q\left[\frac{1}{(r - a\cos\theta)} - \frac{1}{(r + a\cos\theta)}\right]$$

$$V = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2a\,q\cos\theta}{(r^2 - a^2\cos^2\theta)} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{p\cos\theta}{(r^2 - a^2\cos^2\theta)}$$

where P is dipole moment.

Special Cases

1. When the point P lies on the axial line of the dipole on the side of positive charge, $\theta = 0$; $\cos\theta = 1$

   $$\therefore\ V = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{p}{(r^2 - a^2)}$$

2. When P lies on other side, $\theta = 180$; $\cos\theta = -1$.

   $$\therefore\ V = -\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{p}{(r^2 - a^2)}$$

3. When the point P lies on equatorial line, $\theta = 90$; $\cos\theta = 0$.

   $\therefore\ V = 0$

4. In general $V = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{\vec{p}\cdot\vec{r}}{r^3}$, for a short dipole, at a point whose position vector with respect to dipole is $\vec{r}$.

34. Three capacitors of capacitance C_1, C_2 and C_3 are connected (i) in series and (ii) in parallel. Find the energy stored in each combination when they are connected to the same battery of emf V one by one.

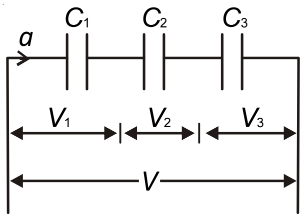
Sol. Capacitors in series

Consider three capacitors connected in series. Consider the shown arrangement of three capacitors in series. Let a V potential is applied to the circuit and + Q be the charge, which will became on all the capacitors. Let V_1, V_2, V_3 be the individual potentials of the capacitors and their respective capacitance is C_1, C_2, C_3.

Therefore, $Q = C_1V_1 = C_2V_2 = C_3V_3$

But $V = V_1 + V_2 + V_3 = \dfrac{Q}{C}$

where C is the effective capacitance .



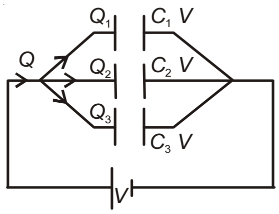
$$\therefore\ \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\therefore\ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Capacitors in parallel

Consider three capacitors connected in parallel. A voltage V is applied across the combination. Q_1, Q_2, Q_3 be the charge on each capacitor and C_1, C_2, C_3 be their respective capacitance.

$Q_1 = C_1 V$, $Q_2 = C_2 V$, $Q_3 = C_3 V$



As $Q = CV$

where C is the net capacitance of the combination,

$\therefore\ Q = Q_1 + Q_2 + Q_3$

$\therefore\ CV = C_1 V + C_2 V + C_3 V$

$\therefore\ C = C_1 + C_2 + C_3$

Energy stored :-

In series combination

$$U = \frac{1}{2}\left(\frac{Q^2}{C_1} + \frac{Q^2}{C_2} + \frac{Q^2}{C_3}\right)$$

$$\boxed{U = \frac{Q^2}{2}\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)}$$

In parallel combination

$$U = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \frac{1}{2}C_3V^2$$

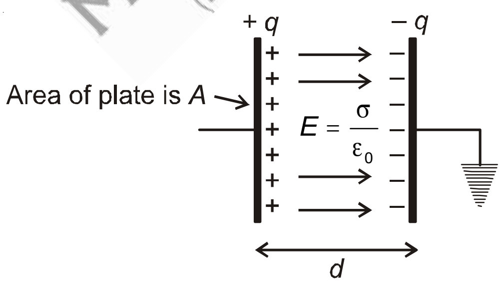
$$\boxed{U = \frac{1}{2}(C_1 + C_2 + C_3)V^2}$$

35. What is a parallel plate capacitor? Explain the principle of a capacitor. Derive an expression for the capacitance of a parallel plate capacitor.

Sol. Parallel plate capacitor

The arrangement consists of two thin conducting plates, each of area A and separated by d distance. When charge q is given to first plate, a charge $-q$ is induced on the inner face of other plate and positive on the outer face of plate. As this face is connected to earth, a net negative charge is left on this plate. Thus, the arrangement is equivalent to two thin sheets of charge.

The electric field between the plates is



$E = \dfrac{\sigma}{\varepsilon_0}$ where σ is the charge density.

As $E = \dfrac{V}{d}$;

V is the potential difference between the plates.

$$V = Ed = \frac{\sigma d}{\varepsilon_0}$$

$$\Rightarrow V = \frac{qd}{A\varepsilon_0} \text{ as } \sigma = \frac{q}{A}$$

$$\Rightarrow C = \frac{q}{V} = \frac{q}{\left(\dfrac{qd}{A\varepsilon_0}\right)} \qquad C = \frac{\varepsilon_0 A}{d}$$

36. (i) What is the dimensional formula of capacitance?

(ii) Why does the electric field inside a dielectric decrease when it is placed in an external electric field?

(iii) A parallel plate capacitor has a capacitance of 3 μF in air and 24 μF when dielectric medium is introduced. What is dielectric constant of the medium?

Sol. (i) $[M^{-1} L^{-2} A^{2} T^{4}]$

(ii) Due to induced electric field in opposite directions

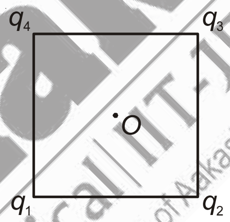
(iii) $C_0 = 3\ \mu F$

$C = 24\ \mu F = C_0 K$

$$\Rightarrow K = \frac{24}{3}$$

$$\Rightarrow K = 8$$

37. (i) In the given figure, there are four point charges placed at the vertex of a square of side 7 cm. If $q_1 = +18\ \mu C$, $q_2 = -24\ \mu C$, $q_3 = +35\ \mu C$ and $q_4 = +16\ \mu C$, then find the electric potential at the centre O of the square, assume the potential to be zero at infinity.



(ii) An electric field $\vec{E} = \left(2\hat{i} + 3\hat{j}\right)$ N/C exists in the space. If potential at the origin is taken to be 10 volt, then find the potential at (2 m, 1 m).

Sol. (i) $V_0 = \dfrac{Kq_1}{r} + \dfrac{Kq_2}{r} + \dfrac{Kq_3}{r} + \dfrac{Kq_4}{r}$

$$= \frac{1}{4\pi\varepsilon_0 r}\left[+18 - 24 + 35 + 16\right] \times 10^{-6}$$

$$= \frac{9 \times 10^9}{\left(7/\sqrt{2}\right) \times 10^{-2}} \times 45 \times 10^{-6} \text{ volt}$$

$$\Rightarrow V_0 = \frac{9 \times 45 \times 10^5}{\left(7/\sqrt{2}\right)} \text{volt}$$

(ii) $\vec{E} = 2\hat{i} + 3\hat{j}$

$$\int dV = \int -\vec{E} \cdot \vec{dr}$$

$$\int_{10}^{V} dV = -\int_{(0,0)}^{(2,1)} \left(2\hat{i} + 3\hat{j}\right) \cdot \left(dx\,\hat{i} + dy\,\hat{j}\right)$$

$$V - 10 = -\int_{(0,0)}^{(2,1)} 2dx + 3dy$$

$$V - 10 = -\left(2x + 3y\right)_{(0,0)}^{(2,1)}$$

$V = 10 - (2 \times 2 + 3 \times 1) = 10 - 7$

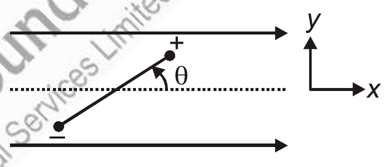
$V = 3$ volt

38. (i) Derive an expression for potential energy of an electric dipole in an uniform electric field.

(ii) Three identical charges q are at the vertices of an equilateral triangle of side L. How much work is done in bringing them closer to an equilateral triangle of side $\frac{L}{2}$?

Sol. (i) Potential Energy of a dipole in an electric field

Potential energy can be associated with the orientation of an electric dipole in an electric field. Change in potential energy is related to the work done by electric field as

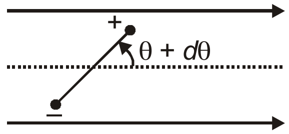


(a) Potential energy is U

$dU = -dW_E = -\vec{\tau} \cdot \vec{d\theta}$

$\Rightarrow$ $dU = -(-pE\sin\theta\hat{k}) \cdot (d\theta\,\hat{k})$ [$\because$ $\vec{d\theta} = d\theta\hat{k}$, anticlockwise]

$\Rightarrow$ $dU = pE \sin\theta\, d\theta$



(b) Potential energy is $U + dU$

$\Rightarrow$ $\int_{U_1}^{U_2} dU = \int_{\theta_1}^{\theta_2} pE\sin\theta d\theta \Rightarrow U_2 - U_1 = -pE[\cos\theta_2 - \cos\theta_1]$

Let $\theta = 90°$ is taken as reference position or zero of potential energy.

$\therefore$ $\theta_1 = 90° \Rightarrow U_1 = 0$

$\Rightarrow$ $U_2 - 0 = -pE[\cos\theta_2 - \cos 90°]$

$\Rightarrow$ $U = -pE\cos\theta$

(ii) $W_{ext} = U_f - U_i$

$$W = \frac{3q^2}{4\pi\varepsilon_0}\left(\frac{1}{\left(L/2\right)} - \frac{1}{L}\right) = \frac{3q^2}{4\pi\varepsilon_0 L}$$

39. (i) "At the surface of a charged conductor electrostatics field must be normal to the surface at every point." Explain it.

(ii) An electrical technician requires a capacitance of 4 μF in a circuit across a potential difference of 500 V. A large number of 1 μF capacitors are available to him each of which can withstand a potential difference of not more than 150 V. Suggest a possible arrangement that requires the minimum number of capacitors.

Sol. (i) A charge conductor behaves as an equipotential surface. The potential at each point on the surface of it is same. So, work done to move a unit positive over the surface from one point to another point is zero.

As, $W = q\,\Delta V = 0$

Also, $W = 0 = q\int \vec{E}\cdot\vec{ds}$

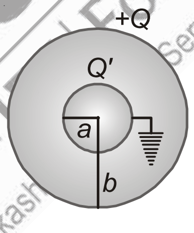
Hence, $\vec{E} \perp \vec{ds}$

So, elelctric field lines must be normal to the surface at every point.

(ii) The circuit consists of 16 rows with each row containing 4 capacitors each of value 1 μF. This circuit is connected to a 500 volts battery.

40. A spherical capacitor consists of two concentric spherical conductors of radius *a* and *b* respectively ($b > a$). Then inner sphere is earthed and outer sphere is given a positive charge of *Q*, then find the capacitance of the system.

Sol. $V_{inner} = 0$ (as grounded)



$$\frac{KQ'}{a} + \frac{KQ}{b} = 0$$

$$\boxed{Q' = -\frac{Qa}{b}}$$

$$\Delta V = \frac{K}{b}\left(-Q\frac{a}{b}\right) + \frac{KQ}{b} = \frac{KQ}{b}\left(\frac{b-a}{b}\right)$$

$$C = \frac{Q}{\Delta V} \Rightarrow C = \frac{Qb^2}{KQ(b-a)}$$

$$\Rightarrow \boxed{C = \frac{4\pi\varepsilon_0 \cdot b^2}{b-a}}$$

41. (i) Derive the relation between electric field intensity and electrostatic potential.

(ii) In a parallel plate capacitor with a dielectric medium of dielectric constant 8, each plate has an area of $2\times10^{-3}\ \text{m}^2$ and distance between the plates is 2 mm. Calculate the capacitance of capacitor, if this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

Sol. (i) Relation between Electric Field and Potential

As we know that $\Delta V = \frac{-W_E}{q_0}$

$\Rightarrow \quad V_f - V_i = \frac{-W_E}{q_0}$

Now, $W_E = \int \vec{F} \cdot \overrightarrow{dr}$, where $\vec{F}$ is the force due to electric field.

$\Rightarrow \quad V_f - V_i = \frac{-\int \vec{F} \cdot \overrightarrow{dr}}{q_0}$

or, $V_f - V_i = -\int_{r_i}^{r_f} \vec{E} \cdot \overrightarrow{dr}$ $\qquad \left[\text{If } V_\infty = 0, \text{ then } V = -\int_{\infty}^{r} E dr\right]$

The above result can also be expressed in differential form as,

$dV = -\vec{E} \cdot \overrightarrow{dr} \left[\text{as } dV = \frac{-dW_E}{q_0}\right]$

The negative sign in the expression $dV = -\vec{E} \cdot \overrightarrow{dr}$ signifies that as one moves in the direction of electric field, potential decreases.

In Cartesian form, $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

$\overrightarrow{dr} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$\Rightarrow \quad dV = -E_x dx - E_y dy - E_z dz$

$\Rightarrow \quad E_x = \frac{-\partial V}{\partial x}; E_y = \frac{-\partial V}{\partial y}; E_z = \frac{-\partial V}{\partial z}$

(ii) $K = 8$, $A = 2 \times 10^{-3}$ m^2, $d = 2 \times 10^{-3}$ mm

$V = 100$ volts

$C = \frac{A\varepsilon_0 K}{d}$

$= \frac{2 \times 10^{-3} \times 8.85 \times 10^{-12} \times 8}{2 \times 10^{-3}}$

$= 70.8 \times 10^{-12}$ F

$Q = CV = 70.8 \times 10^{-12} \times 100$

$Q = 70.8 \times 10^{-8}$ C.

42. (i) What is capacitance?

(ii) Two charges $q_1 = +4$ μC and $q_2 = -4$ μC are placed at a distance 90 cm. Estimate the potential energy of system in eV taking the potential energy as zero when charges q_1 and q_2 are at infinite separation.

Sol. (i) Any conducting object that carries a charge is characterized by an electric potential that is constant everywhere on and within that object. If two such conductors have a potential difference between them then, as any potential difference is able to accelerate charges, the system effectively stores energy. Such a device that can maintain a potential difference, storing energy by storing charge is called capacitor.

(ii) $U = \dfrac{K \cdot q_1 q_2}{r} = \dfrac{9\times10^{9}\times4\times10^{-6}\times-4\times10^{-6}}{90}\times 100$

$U = 160 \times 10^{-2}$ J $= 1.6$ J

$\because$ 1 eV $= 1.6 \times 10^{-19}$ J

so, $U = 10^{19}$ eV

43. Derive an expression for force on each plate of parallel plate capacitor if magnitude of charge on each plate is Q, area of each plate is A and separation between plates is d.

Sol. $F = qE$

$$F = q\frac{\sigma}{2\varepsilon_0}$$

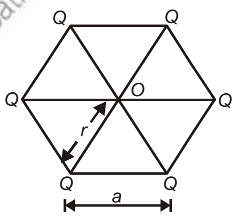
$$F = \frac{1}{2}\frac{\sigma q}{\varepsilon_0} \Rightarrow F = \frac{q^2}{2A\varepsilon_0}$$

44. (i) A regular hexagon of side a has a charge Q at each of its vertices. Determine the electrostatic potential due to this charge array at the centre of Hexagon.

(ii) The plates of a parallel plate capacitor has an area of 30 cm^2 each and separation 2 mm is charged by a battery of emf 200 V. Calculate the electrostatic energy stored in the capacitor.

Sol. (i) $V_0 = \dfrac{KQ}{r}\times 6$

$$\boxed{V_0 = \frac{6KQ}{a}}$$

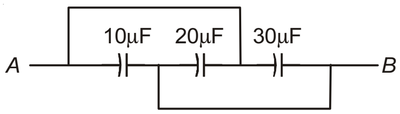


(ii) $U = \dfrac{1}{2}CV^2 = \dfrac{1}{2}\dfrac{A\varepsilon_0}{d}V^2 = \dfrac{30}{10000}\times\dfrac{8.85\times10^{-12}}{2\times2\times10^{-3}}\times200\times200$

$U = 26.55 \times 10^{-9}$ J $= 2.7 \times 10^{-8}$ J

45. (i) Two concentric spherical conductors of radius a and b ($b > a$) inner sphere has q_1 charge and outer sphere has q_2. When they are connected by a conducting wire then prove that charge on inner sphere must be zero.

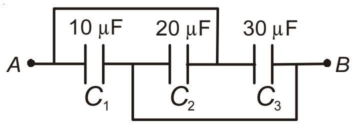
(ii) Find the equivalent capacitance between A and B.

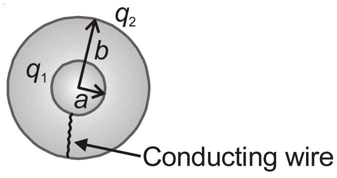


(iii) If the given combination is connected with a battery of emf 200 volt then find the charge on each capacitor.

Sol. (i) Charge on inner sphere will become zero, as all the charges flow to outer sphere through conducting wire.

(ii) $C_{eq} = 60\ \mu F$





Here C_1, C_2, C_3 are in parallel.

(iii) $Q = CV$

$Q_1 = 10 \times 200 = 2000\ \mu C$

$Q_2 = 20 \times 200 = 4000\ \mu C$

$Q_3 = 30 \times 200 = 6000\ \mu C$

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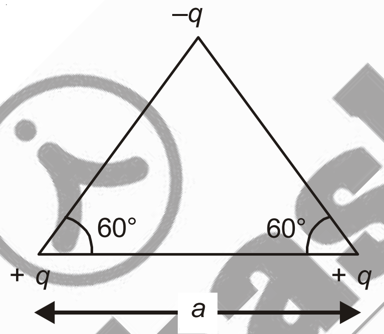
Chapter 2

Electrostatic Potential and Capacitance

Solutions (Set-2)

1. Find the electrostatic potential energy of the system shown in the figure.



(1) $\frac{Kq^2}{a}$ (2) $\frac{3Kq^2}{a}$ (3) $-\frac{Kq^2}{a}$ (4) $-\frac{2Kq^2}{a}$

Sol. Answer (3)

$$U = \frac{kq\times(-q)}{a} + \frac{k(q)(-q)}{a} + \frac{k(q)(q)}{a}$$

2. Two charges q and $-q$ are placed at (a, 0) and ($-a$, 0) on the x-axis. Another charge $2q$ is taken from (0, 0) to (0, a), then the work done to do so

(1) $\frac{q}{2\pi\varepsilon_0 a}$ (2) Zero (3) Infinite (4) $\frac{q}{4\pi\varepsilon_0 a}$

Sol. Answer (2)

$W = q(V_f - V_i) = 0$

3. In moving a unit positively charged body from point A to point B, external work done is 40 J and body acquires a kinetic energy of 20 J. The potential difference, $V_B - V_A$, is

(1) 20 V (2) –20 V (3) 40 V (4) –40 V

Sol. Answer (1)

$\Delta U = W - \Delta K = 40 - 20 = 20$ J

$$\Delta V = \frac{\Delta U}{q} = \frac{20}{1} = 20 \text{ V}$$

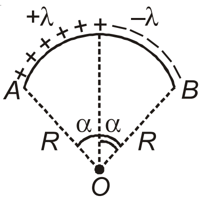
4. A charge of 5 nC is uniformly distributed on a ring of radius 4 cm. Find the potential at the axis at a distance of 3 cm from the centre.

(1) 0.09 V (2) 600 V (3) 900 V (4) 300 V

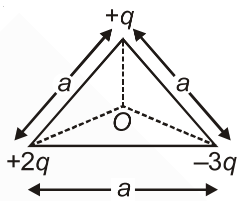
Sol. Answer (3)

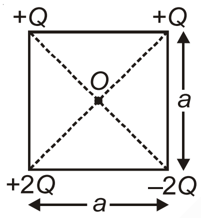
$$V = \frac{Kq}{\sqrt{r^2 + x^2}}$$

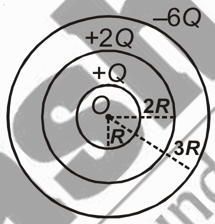
5. In which of the following case, potential at point O is not zero?

(1)


(AB is circular arc)

(2)


(3)


(4)


Sol. Answer (3)

In case 3,

$$V = \frac{kQ}{a/\sqrt{2}} + \frac{kQ}{a/\sqrt{2}} + \frac{2kQ}{a/\sqrt{2}} \frac{-2kQ}{a/\sqrt{2}} \neq 0$$

6. In any region, if electric field is define as $\vec{E} = (\hat{i} + 2\hat{j} + \hat{k})$ V/m, then the potential difference between two points $A(0, 0, 0)$ and $B(2, 3, 4)$ in that region, is

(1) 6 V (2) 12 V (3) 8 V (4) 9 V

Sol. Answer (2)

$$\Delta V = -\vec{E} \cdot \overrightarrow{\Delta r} \Rightarrow V_B - V_A = \vec{E} \cdot \left(\overrightarrow{r_A} - \overrightarrow{r_B}\right)$$

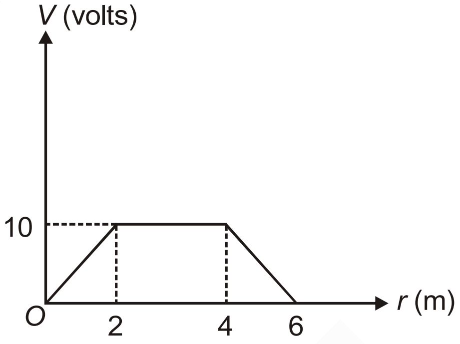
7. If the potential function is define as $V = (-3x + 4y + 12z)$ V, then magnitude of electric field $E(x, y, z)$ is

(1) 16 V/m (2) 12 V/m (3) $\sqrt{14}$ V/m (4) 13 V/m

Sol. Answer (4)

$$E_x = \frac{-\partial V}{\partial x} = 3;\ E_y = \frac{-\partial V}{\partial y} = -4;\ E_z = \frac{-\partial V}{\partial z} = -12 \Rightarrow E = 13 \text{ V/m}$$

8. From the following graph, find the value of $|\vec{E}|$ at $r = 3.5$ m

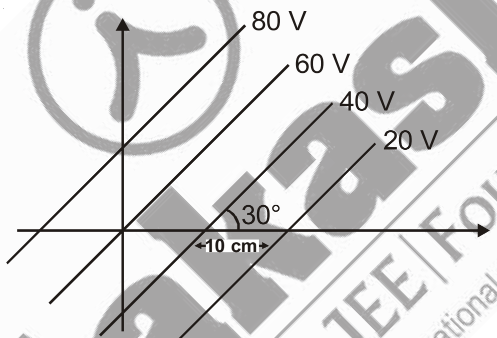


(1) 3.5 V/m (2) ∞ (3) $\frac{100}{35}$ V/m (4) Zero

Sol. Answer (4)

$E = -dV/dr$. $|E|$ = slope = 0

9. If the given figure shows equipotential surfaces, then the magnitude of electric field is

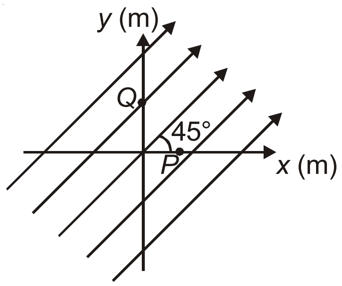


(1) 50 N/C (2) 100 N/C (3) 200 N/C (4) 400 N/C

Sol. Answer (4)

$E = \Delta V/\Delta r$, $\Delta V = 20$ V, $\Delta r = (10\,\text{cm}) \times \sin 30 = 5\,\text{cm} = 0.05$ m

10. A uniform electric field of 200 V/m is directed at 45° with x-axis as shown in figure. The co-ordinates of point P and point Q are (1, 0) and (0, 2). Find the potential difference, $V_P - V_Q$ (in volts).



(1) $\frac{200}{\sqrt{2}}$ (2) $200\sqrt{2}$ (3) $\frac{-200}{\sqrt{2}}$ (4) $-200\sqrt{2}$

Sol. Answer (1)

$$\vec{E} = \frac{200}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$$

$$\overrightarrow{r_{PQ}} = \overrightarrow{r_P} - \overrightarrow{r_Q} = \hat{i} - 2\hat{j}$$

$$V_P - V_Q = -\vec{E}.\overrightarrow{r_{PQ}} = \frac{200}{\sqrt{2}}\text{volt}$$

11. There are two concentric conducting shells. The potential of outer shell is 10 V and that of inner shell is 15 V. If the outer shell is grounded, the potential of inner shell becomes/remains

(1) 25 V (2) 15 V (3) 10 V (4) 5 V

Sol. Answer (4)

Potential difference is only due to charge on inner shell. So potential difference between the two shells remains unchanged.

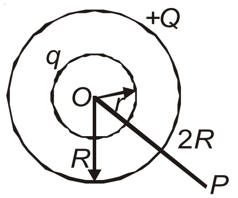
$\Rightarrow$ $V - 0 = 15 - 10$

$\Rightarrow$ $V = 5$ volt

12. There are two concentric hollow conducting spherical shells of radii r and R ($R > r$). The charge on the outer shell is Q. What charge should be given to the inner shell, so that the potential at a point P, at a distance $2R$ from the common centre, is zero?

(1) $\frac{-Qr}{R}$ (2) $\frac{-QR}{r}$ (3) $\frac{-2QR}{r}$ (4) $-Q$

Sol. Answer (4)



Since P is outside, both spheres behave like point charges located at O.

(Common centre)

For zero potential at P,

$$V = 0 = \frac{K\left(Q+q\right)}{2R}$$

$\Rightarrow$ $q = -Q$

13. The graph between the surface charge density and radius of curvature for an isolated conductor at constant potential is

(1) Straight line with positive slope (2) Parabola

(3) Rectangular hyperbola (4) Straight line negative slope

Sol. Answer (3)

$\sigma \propto \frac{1}{r}$ (Rectangular hyperbola)

14. Which of the following statements is not true regarding a conductor?

    (1) The surface of any charged conductor is an equipotential surface

    (2) When a conductor is earthed, charge always flows from conductor to earth

    (3) Electrostatic field inside the conductor is zero

    (4) Electrostatic potential through out the volume of conductor is constant

Sol. Answer (2)

Charge may flow in either direction.

15. An air filled parallel plate capacitor having circular plates of diameter *D* is given a charge Q. The magnitude of the force acting between plates is

    (1) $\dfrac{Q^2}{2\varepsilon_0}$ (2) $\dfrac{Q^2}{2\pi D^2\varepsilon_0}$ (3) $\dfrac{Q^2}{\pi D^2\varepsilon_0}$ (4) $\dfrac{2Q^2}{\pi\varepsilon_0 D^2}$

Sol. Answer (4)

$F = Q^2/2A\varepsilon_0$, $A = \pi D^2/4$

16. Air filled capacitor is charged by a battery and after charging battery is removed. A slab of dielectric material is inserted in it to fill the space completely. The electric field in the capacitor is

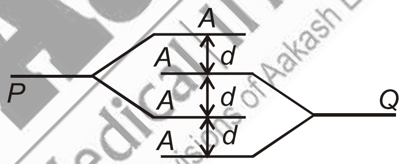
    (1) Increased (2) Decreased

    (3) Remains constant (4) First increased then decreased

Sol. Answer (2)

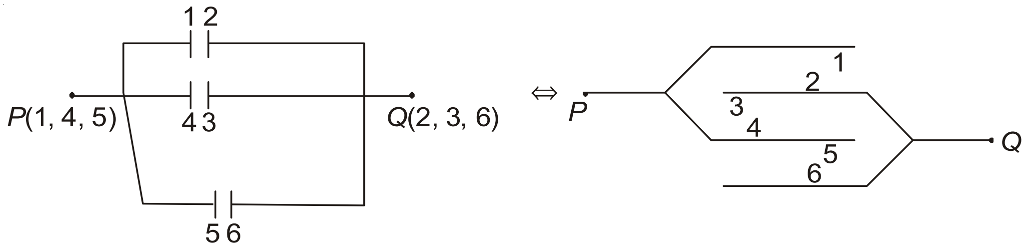
$E = V/d = \dfrac{Q}{Cd} = E \propto \dfrac{1}{C}$

17. If the area of each plate is *A* and the separation between them is *d*, then find the equivalent capacitance between *P* and *Q*



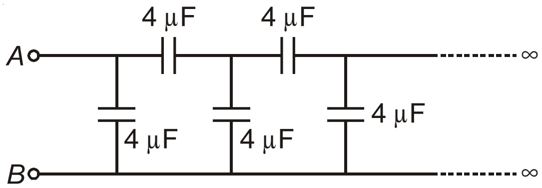
(1) $\dfrac{\varepsilon_0 A}{3d}$ (2) $\dfrac{2\varepsilon_0 A}{d}$ (3) $\dfrac{\varepsilon_0 A}{d}$ (4) $\dfrac{3\varepsilon_0 A}{d}$

Sol. Answer (4)



$C_{eq} = 3C = \dfrac{3\varepsilon_0 A}{d}$

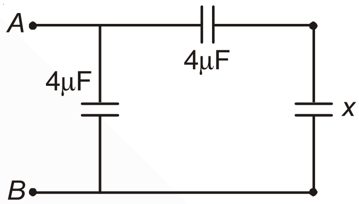
18. Find the equivalent capacitance between point *A* and *B*.



(1) $(1+\sqrt{5})\,\mu F$ (2) $2(1+2\sqrt{5})\,\mu F$ (3) $2+\sqrt{20}\ \mu F$ (4) $1+\sqrt{2}\ \mu F$

Sol. Answer (3)

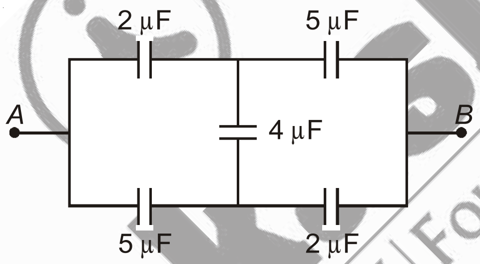
Let *x* be the equivalent capacitance between *A* and *B*.



$\Rightarrow \quad \dfrac{4x}{4+x}+4=x$

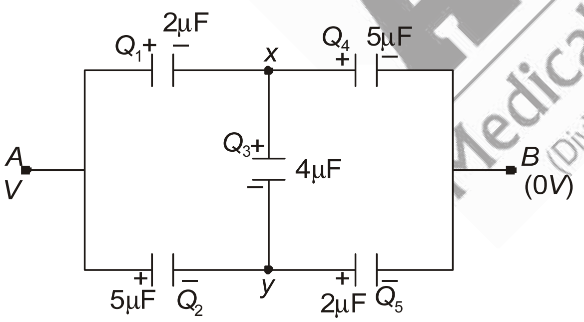
Or $x = \left(2+\sqrt{20}\right)\mu F$

19. Find the capacitance between *A* and *B*.



(1) $\dfrac{7}{12}\mu F$ (2) $4\ \mu F$ (3) $\dfrac{12}{7}\mu F$ (4) $3.2\mu F$

Sol. Answer (4)



By Nodal analysis

$Q_1 = 2(V-x)$; $Q_2 = 5(V-y)$

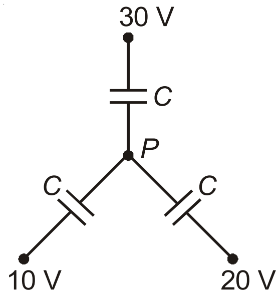
$Q_3 = (x-y)\times 4$; $Q_4 = (x-0)5$; $Q_5 = (y-0)\times 2$

Now, $Q_1 = Q_3 + Q_4$; $Q_2 + Q_3 = Q_5$

Solve for Q_1 and Q_2.

Now, $C_{eq} = \dfrac{Q_1+Q_2}{V}$

20. Find the potential of point P.

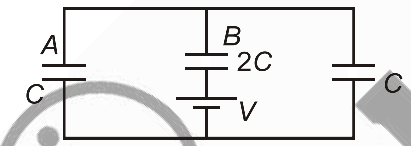


(1) 15 V (2) 60 V (3) 10 V (4) 20 V

Sol. Answer (4)

$$V = \frac{C\times30+C\times20+C\times10}{C+C+C} = 20\text{ V}$$

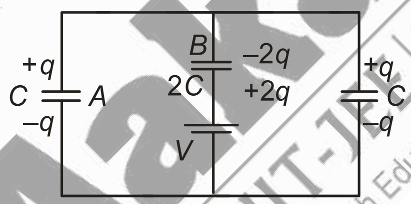
21. In the circuit shown, the charges on the capacitors A and B are respectively



(1) CV, CV (2) $\frac{CV}{2}$, $\frac{CV}{2}$ (3) CV, $\frac{CV}{2}$ (4) $\frac{CV}{2}$, CV

Sol. Answer (4)

Applying Kirchhoff's loop law,



$$V = \frac{2q}{2C} + \frac{q}{C}$$

$$\Rightarrow \quad q = \frac{CV}{2}$$

$$\therefore \quad 2q = CV$$

22. The ratio of energy stored by the series combination of two identical capacitors to their parallel combination, when connected to same supply voltage, is

(1) 1 (2) $\frac{1}{2}$ (3) 4 (4) $\frac{1}{4}$

Sol. Answer (4)

Series :

$$C_{eq} = \frac{C}{2}$$

$$E = \frac{1}{2}\cdot\frac{C}{2}.V^2$$

Parallel :

C_{eq} . = $2C$

$$E' = \frac{1}{2}.2C.V^2 \Rightarrow \frac{E}{E'} = \frac{1}{4}$$

23. A capacitor is charged until it stores an energy of 1 J. A second uncharged capacitor is connected to it, so that charge distributes equally. The final energy stored in the second capacitor is

(1) 1 J (2) 0.5 J (3) 0.25 J (4) 0.125 J

Sol. Answer (3)

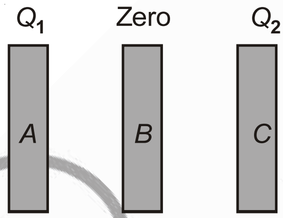
$$\Delta U = \frac{1}{2}.\frac{C^2}{C+C}.V^2 = \frac{1}{2}CV^2$$

$$\because \ \frac{1}{2}CV^2 = 1.0\,\text{J}$$

$$\Rightarrow \ \Delta U = 0.5\,\text{J}$$

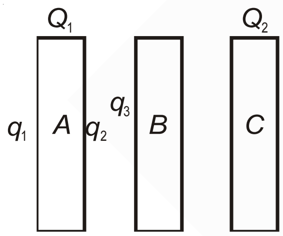
Remaining 0.5 J distributes equally between two capacitors. That is, each capacitor has energy = 0.25 J

24. An arrangement of three large metallic parallel plates with respective charges marked in figure is shown. What is the charge appearing on the left face of plate *B*?



(1) Zero (2) $\frac{Q_2 - Q_1}{2}$ (3) $-Q_1$ (4) $+Q_2$

Sol. Answer (2)



As electric field intensity inside any metal plate is zero.

$$q_1 = \frac{Q_1 + Q_2}{2}$$

$$q_2 = Q_1 - q_1 = \frac{Q_1 - Q_2}{2}$$

$$q_3 = -q_2 = \frac{Q_2 - Q_1}{2}$$

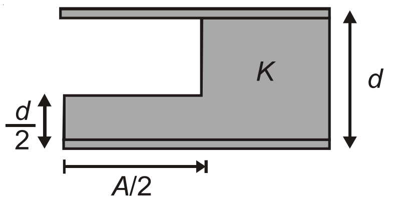
25. An air filled parallel plate capacitor of capacitance 50 μF is connected to a battery of 100 V. A slab of dielectric constant 4 is inserted in it to fill the space completely. Find the extra charge flown through the battery till it attains the steady state.

(1) 2.5 mC (2) 5 mC (3) 20 mC (4) 15 mC

Sol. Answer (4)

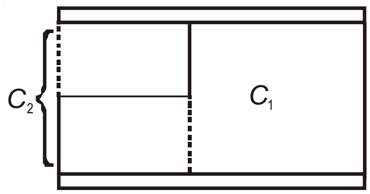
$Q = CV$, $Q' = C'V \Rightarrow \Delta Q = C'V - CV = (200 - 50) \times 100 = 15000$ mC = 15 mC

26. A parallel plate capacitor, with plate area A and plate separation d, is filled with a dielectric slab as shown. What is the capacitance of the arrangement?



(1) $\frac{K(K+3)}{2(K+1)}\frac{\varepsilon_0 A}{d}$ (2) $\frac{3}{4}\frac{K\varepsilon_0 A}{d}$ (3) $\frac{4}{3}\frac{K\varepsilon_0 A}{d}$ (4) $\left(\frac{K+1}{2}\right)\frac{\varepsilon_0 A}{d}$

Sol. Answer (1)



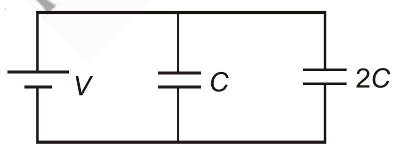
The system can be visualized as shown in diagram

$$\Rightarrow \quad C_1 = \frac{\varepsilon_0 K \frac{A}{2}}{d} = \frac{\varepsilon_0 KA}{2d}$$

$$C_2 = \frac{\varepsilon_0 \frac{A}{2}}{\frac{d}{2}+\frac{d}{2K}} = \frac{\varepsilon_0 AK}{d(K+1)}$$

$$C = C_1 + C_2 = \frac{\varepsilon_0 KA}{d}\left[\frac{1}{2}+\frac{1}{K+1}\right] = \frac{\varepsilon_0 AK}{d}\frac{(K+3)}{2(K+1)}$$

27. In the arrangement shown, the battery is disconnected and a dielectric slab of dielectric constant K is inserted between the plates of capacitor with capacitance C, so as to completely fill the space. What is the final potential difference across capacitor with capacitance $2C$?



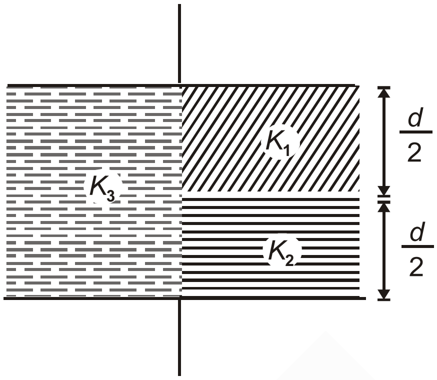
(1) Zero (2) V (3) $\frac{3V}{2+K}$ (4) $\frac{V}{K+2}$

Sol. Answer (3)

$Q_1 + Q_2 = CV + 2CV = 3CV$

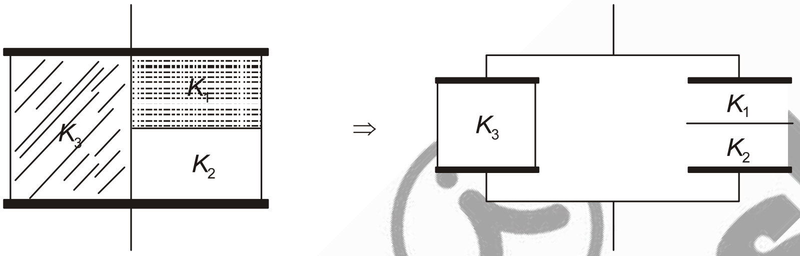
$$V = \frac{Q_1+Q_2}{2C+kC} = \frac{3CV}{C(2+k)} = \frac{3V}{2+K}$$

28. Air filled capacitor of capacitance 2 μF is filled with three dielectric material of dielectric constants $K_1 = 4$, $K_2 = 4$ and $K_3 = 6$ as shown in the figure. The new capacitance of the capacitor is



(1) 50 μF (2) 2 μF (3) 20 μF (4) 10 μF

Sol. Answer (4)



$$C_{eq} = \frac{k_3\varepsilon_0 \frac{A}{2}}{d/2} + \frac{\varepsilon_0 A}{\frac{d}{2k_1}+\frac{d}{2k_2}}, \text{ where } \frac{\varepsilon_0 A}{d} = 2\,\mu\text{F}$$

29. Positive and negative point charges of equal magnitude are kept at $\left(0, 0, \frac{a}{2}\right)$ and $\left(0, 0, \frac{-a}{2}\right)$, respectively. The work done by the electric field when another positive point charge is moved from (– a, 0, 0) to (0, a, 0) is

(1) Positive

(2) Negative

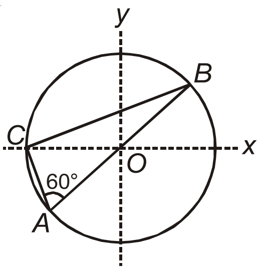
(3) Zero

(4) Depends on the path connecting the initial and final positions

Sol. Answer (3)

Points A and B lies on equipotential surface.

30. Consider a system of three charges $\frac{q}{3}$, $\frac{q}{3}$ and $-\frac{2q}{3}$ placed at points A, B and C, respectively, as shown in the figure. Take O to be the centre of the circle of radius R and angle $CAB = 60°$.



(1) The electric field at point O is $\dfrac{q}{8\pi\varepsilon_0 R^2}$ directed along the negative x-axis

(2) The potential energy of the system is zero

(3) The magnitude of the force between the charges at C and B is $\dfrac{q^2}{54\pi\varepsilon_0 R^2}$

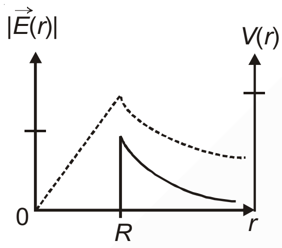
(4) The potential at point O is $\dfrac{q}{12\pi\varepsilon_0 R}$

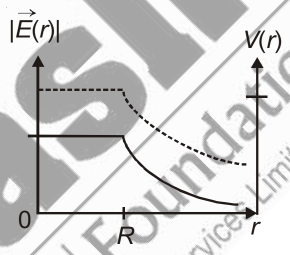
Sol. Answer (3)

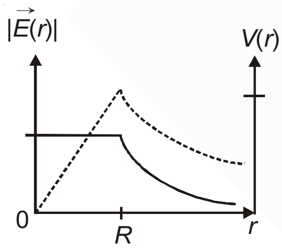
Force between C and B is

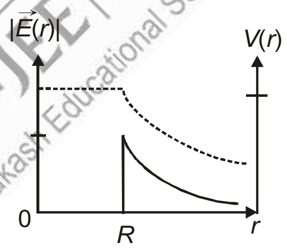
$$\frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{3}\right)\left(\frac{q}{3}\right)\times\frac{1}{(R\sqrt{3})^2}=\frac{q^2}{54\pi\varepsilon_0 R^2}$$

31. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential $V(r)$ with the distance r from the centre, is best represented by whcih graph?

(1)


(2)


(3)


(4)


Sol. Answer (4)

Inside, $E(r)$ = 0, outside $E(r)\propto\dfrac{1}{r^2}$

Inside $V(r)$ = constant, outside $V(r)\propto\dfrac{1}{r}$

32. Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then,

(1) Negative and distributed uniformly over the surface of the sphere

(2) Negative and appears only at the point on the sphere closest to the point charge

(3) Negative and distributed non-uniformly over the entire surface of the sphere

(4) Zero

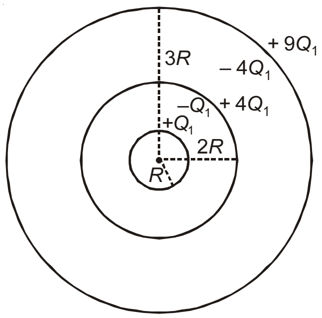
Sol. Answer (4)

The sphere is initially neutral. So its net charge on induction will be zero.

33. Three concentric metallic spherical shells of radii R, $2R$, $3R$, are given charges Q_1, Q_2, Q_3, respectively.; It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$, is

(1) 1 : 2 : 3 (2) 1 : 3 : 5 (3) 1 : 4 : 9 (4) 1 : 8 : 18

Sol. Answer (2)



Charge distribution will be as shown.

$Q_2 = 4Q_1 - Q = 3Q_1$

$Q_3 = 9Q_1 - 4Q_1 = 5Q_1$

$Q_1 : Q_2 : Q_3 = 1 : 3 : 5$

34. A long, hollow conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral.

(1) A potential difference appears between the two cylinders when a charge density is given to the inner cylinder

(2) A potential difference appears between the two cylinders when a charge density is given to the outer cylinder

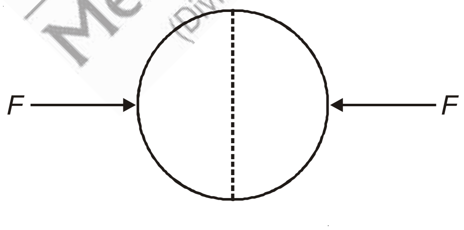
(3) No potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders

(4) No potential difference appears between the two cylinders when same charge density is given to both the cylinders

Sol. Answer (1)

The potential difference between long, hollow concentric cylinders is independent of charge on outer cylinder.

35. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to



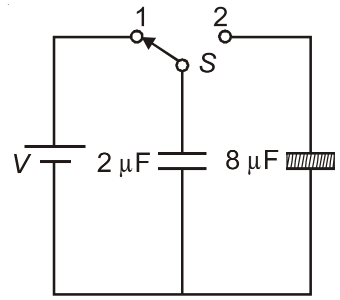
(1) $\frac{1}{\varepsilon_0}\sigma^2 R^2$ (2) $\frac{1}{\varepsilon_0}\sigma^2 R$ (3) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (4) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

Sol. Answer (1)

By dimensional analysis

$$[F] = \left[\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}\right] = \left[\frac{1}{\varepsilon_0}\frac{\sigma^2 A^2}{R^2}\right] = \left[\frac{1}{\varepsilon_0}\sigma^2 R^2\right]$$

36. A 2 μF capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is



(1) 0% (2) 20% (3) 75% (4) 80%

Sol. Answer (4)

$$U_1 = \frac{1}{2}C_1V^2$$

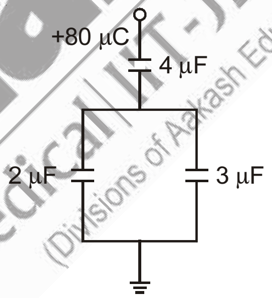
$q = C_1V$

$$V' = \frac{q}{C_1 + C_2} = \frac{C_1V}{C_1 + C_2} = \frac{V}{5}$$

$$U_f = \frac{1}{2}(C_1 + C_2)V'^2 = \frac{1}{10}C_1V^2$$

$$\frac{\Delta U}{U_i} \times 100 = 80\%$$

37. In the given circuit, a charge of +80 μC is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is



(1) +32 μC (2) +40 μC (3) +48 μC (4) +80 μC

Sol. Answer (3)

For the 2 μF and 3 μF capacitor, equivalent capacitance is 5 μF.

$$\Rightarrow \quad V = \frac{Q}{C} = 16 \text{ V}$$

Now, $q = CV = 48$ μC

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