Polynomials

MULTIPLE CHOICE QUESTIONS

Q1. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

a) 10 b) -10 c) 5 d) -5

Q2. A quadratic polynomial, the sum of whose zeros is 2 and one zero is 3 is

a) $x2-9$ b) $x2+9$ c) $x2+3$ d) $x2-3$

Q3. A quadratic polynomial, the sum of whose zeros is -5 and their product is 6 is

a) $x^2 + 5x + 6$ b) $x^2 + 5x + 6$ c) $x^2 - 5x + 6$ d) $-x^2 + 5x + 6$

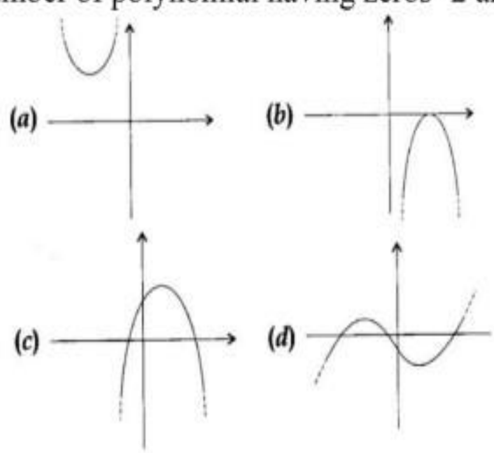
Q4. If one zero of the polynomial $f(x) = (k^2 + 4)x^2 + 13x + 4k$ is the reciprocal of the other, then k =

a) 2 b) -2 c) 1 d) -1

Q5. If $\propto$, β are the zeros of the polynomial $f(x) = x^2 + x + 1$, then $\frac{1}{\alpha} + \frac{1}{\beta} =$

a) 1 b) -1 c) 0 d) None of these

Q6. The number of polynomial having zeros -2 and 5 is



a) 1 b) 2 c) 3 d) More than 3

OBJECTIVE TYPE QUESTIONS (I MARK QUESTIONS)

Q1. Write the zeros of the polynomial $x^2 - x - 6$

Q2. Write a polynomial whose zeros are $(2+\sqrt{3})$ and $(2 - \sqrt{3})$

Q3. If α, β are the zeros of the polynomial, such that $\alpha+\beta=6$ and $\alpha\beta=4$, then write the polynomial.

Q4. If α and 1/α are the zeros of the polynomial $4x^2 - 2x + (k - 4)$, find the value of k
.
Q5. Check whether -2 is a zero of the polynomial $9x^3 - 18x^2 - x - 2$

SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)

Q1. Find the zeroes of the polynomial $2x^2 - 9$ and verify the relationship between zeros and coeffients.

Q2. Find a quadratic polynomial the sum and product of whose zeros are 3 and -2/5 respectively.

Q3. If α and β are zeros of $3x^2 + 5x + 13$,then find the value of $1/\alpha + 1/\beta$

Q4. Check whether $x = -3$ is a zero of $x^3 + 11x^2 + 23x - 35$.

Q5. Find p and q if p and q are the zeros of the quadratic polynomial $x^2 + px + q$.

SHORT ANSWER TYPE QUESTIONS(3 MARKS)

Q1. Find the zeroes of the following polynomial by factorisation method and verify the relations between the zeroes and their coefficients

i) $7y^2 - \frac{11}{3}y - \frac{2}{3}$

ii) $\sqrt{3}x^2 + 10x + 7\sqrt{3}$

iii) $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

Q2. If the sum of the zeroes of the polynomial $p(x) = (a+1)x^2 + (2a+3)x + (3a+4)$ is -1, then find the product of the zeroes.

Q3. If (x + a) is a factor of two polynomials $x^2 + px + q$ and $x^2 + mx + n$, then prove that $a = \frac{n-p}{m-P}$

Q4. Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integer k > 1?

Q5. If one zero of a polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

LONG ANSWER TYPE QUESTIONS(4 MARKS)

Q1. If α and β are the zeroes of the quadratic polynomial $p(s) = 3s^2 - 6s + 4$, find the value of

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$$

Q2. If the squared difference of the zeroes of the quadratic polynomial $f(x) = x^2 + px + 45$ is equal to 144, find the value of p.

Q3. If α and β are the roots of the equation $ax^2+bx+c=0$ and if $px^2+qx+r=0$ has roots $\frac{1-\alpha}{\alpha}$ and $\frac{1-\beta}{\beta}$, then r is

Q4. If a and b are the zeroes of the quadratic polynomial $f(x)=x^2-px+q$, prove that

$$\frac{a^2}{b^2} + \frac{b^2}{a^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2.$$

Q5. If l and m are zeroes of the polynomial $p(x)=2x^2-5x+7$, find a polynomial whose zeroes are $2l+3$ and $2m+3$.

ANSWERS

MULTIPLE CHOICE QUESTIONS

1 (b) Since 2 is zero P(2)=0 P(2) = $2^2 + 3\times2 + k = 0$ which gives k= −10

2

(a) Given $\alpha + \beta = 0$ $\alpha = 3$ so $\beta = -3$

$$p(x) = k(x^2 - (\alpha+\beta)x + \alpha\beta)$$

$$p(x) = k(x^2 - 9)$$

3

(a)

$$P(x) = k(x^2 - (-5x) + 6)$$
$$P(x) = k(x^2 + 5x + 6)$$
$$when\ k = 1\ p(x) = x^2 + 5x + 6$$

4

(a) Let the zeros be α , $\frac{1}{\alpha}$

So $\alpha \times \frac{1}{\alpha} = 1 = \frac{4k}{k^2+4}$

cross multiplying we get $k^2 - 4k + 4 = 0 \Rightarrow (k-2)^2 = 0$ which gives k = 2

5

(b) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-1}{1} = -1 \quad \{\alpha + \beta = -1\ and\ \alpha\beta = 1\}$

6

(d)

$$P(x) = k(x^2 - (-2+5)x + -2 X 5) = k(x^2 = 3x - 10)$$

Since k can take infinite number of values, there can be more than three polynomials.

OBJECTIVE TYPE QUESTIONS (I MARK QUESTIONS)

Q1. $X^2 - x - 6 = (x - 3)(x + 2)$ so the zeros are 3 and −2

Q2. $= K(x^2 - (\alpha + \beta)x + \alpha\beta)$

$= K(x^2 - (2 + \sqrt{3} + 2 - \sqrt{3})x + (2 + \sqrt{3})(2 - \sqrt{3}))$

$=K (x^2 - (4)x + 2^2 - (\sqrt{3})^2)$

$= K(x^2 - 4x + (4 - 3))$

$= K(x^2 - 4x + 1)$

Q3. $P(x) = K (x^2 - (\alpha + \beta)x + \alpha\beta) = K(x^2 - 6x + 4)$

Q4.

Given $\alpha, \frac{1}{\alpha}$ are the zeros of the polynomial. Product of the zeros $= \frac{c}{a} = \frac{k-4}{4}$

$$\alpha X \frac{1}{\alpha} = \frac{k-4}{4}$$

$$1 = \frac{k-4}{4}$$

Cross multiplying we get k = 8

Q5.

$f(x) = 9x^3 - 18x^2 - x - 2$ If -2 is a zero then $f(-2) = 0$

$$f(-2) = 9X((-2)^3 + 18X(-2)^2 - (-2) - 2$$
$$= 9X(-8) + 18X(4) + 2 - 2$$
$$= -72 + 72 + 2 - 2 = 0$$

Since $f(-2) = 0$ -2 is a zero of the given polynomial.

SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)

Q1. ± 3/√2

Q2. $x^2 - 15x - 2$

Q3. − 5/13

Q4. x= -3 is not a zero

Q5. 5 p = 1; q = −2

SHORT ANSWER TYPE QUESTIONS(3 MARKS)

1 i) y = 14/21, − 1/7

ii) x = −√3 , −7/√3

iii) x = − 2/√3, 3/ 4√3⁄2

2 Product = -2

3 Correct proof

4 The quadratic polynomial cannot have equal zeros for any odd integer k > 1

5 k = − 1/9

LONG ANSWER TYPE QUESTIONS(4 MARKS)

1

Sum of zeroes = $\alpha+\beta = \frac{-b}{a} = -(\frac{-6}{3}) = 2$ ....(i)

Product of zeroes = $\alpha\beta = \frac{c}{a} = \frac{4}{3}$ -..............(ii)

Now, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2(\frac{1}{\alpha} + \frac{1}{\beta}) + 3\alpha\beta = \frac{\alpha^2+\beta^2}{\alpha\beta} + 2(\frac{\alpha+\beta}{\alpha\beta}) + 3\alpha\beta$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} + 2(\frac{\alpha+\beta}{\alpha\beta}$

$= \frac{(2)^2 - 2(\frac{4}{3})}{\frac{4}{3}} + 2(\frac{2}{\frac{4}{3}}) + 3(\frac{4}{3})$

$= 1 + 3 + 4 = 8$

2

$f(x) = x^2 + px + 45$

Sum of zeroes = $\alpha+\beta = \frac{-b}{a} = -p$ (i)

Product of zeroes = $\alpha\beta = \frac{c}{a} = 45$...(ii)

Given $(\alpha - \beta)^2 = 144$

$(\alpha + \beta)^2 - 4\alpha\beta = 144$

$(-p)^2 - 4(45) = 14$

$P^2 = 144 + 180 = 3$

$P = \sqrt{324} = 18$

3

Since α and β are the roots of the equation $ax^2 + bx + c = 0$, so,

$$\alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$$

The equation with roots $\frac{1-\alpha}{\alpha}$ and $\frac{1-\beta}{\beta}$ can be written as

$$x^2 - \{\frac{1-\alpha}{\alpha} + \frac{1-\beta}{\beta}\} x + \{\frac{1-\alpha}{\alpha} * \frac{1-\beta}{\beta}\} = 0 \quad .... \quad 1$$

Now, sum of zeroes , $\{\frac{1-\alpha}{\alpha} + \frac{1-\beta}{\beta}\} = \frac{\alpha + \beta - 2\alpha\beta}{\alpha\beta}$

$$= \frac{\alpha+\beta}{\alpha\beta} + \frac{-2\alpha\beta}{\alpha\beta} = \frac{\alpha+\beta}{\alpha\beta} - 2, \quad ....2$$

$$= \frac{-b}{c} - 2 = \frac{-b-2c}{c} \quad \text{, since } \alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$$

Product of zeroes

$$\frac{1-\alpha}{\alpha} * \frac{1-\beta}{\beta} = \frac{1-(\alpha+\beta)+\alpha\beta}{\alpha\beta} = \frac{1-\frac{-b}{a}+\frac{c}{a}}{\frac{c}{a}} = \frac{a+b+c}{c} \quad ....3$$

Putting 2 and 3 in 1

The required equation is $x^2 - \{\frac{-b-2c}{c}\}x + \frac{a+b+c}{c} = 0$

$cx^2 + (b + 2c) x + (a + b + c) = 0$ —- (i)

On comparing equation (i) with the equation given $px^2 + qx + r = 0$, $r = a + b + c$.

Sum of zeroes = a+ b = p

Product of zeroes = ab = q

$$\frac{a^2}{b^2} + \frac{b^2}{a^2} = \frac{a^4 + b^4}{a^2b^2} = \frac{(a^2 + b^2)^2 - 2a^2b^2}{a^2b^2}$$

$$= \frac{[(a+b)^2 - 2ab]^2 - 2a^2b^2}{a^2b^2} = \frac{[p^2 - 2q]^2 - 2q^2}{q^2}$$

$$= \frac{p^4 - 4p^2q + 4q^2 \;\; -2q^2}{q^2} = \frac{p^4 - 4p^2q + 2q^2}{q^2}$$

$$= \frac{p^4}{q^2} - \frac{-4p^2q}{q^2} + \frac{2q^2}{q^2}$$

$$= \frac{p^4}{q^2} - \frac{-4p^2q}{a^2} + 2$$

5

$l + m = \frac{5}{2}$

$lm = \frac{7}{2}$

a polynomial whose zeroes are $2l + 3$ *and* $2m + 3$ is

$x^2 - (2l + 3 + 2m + 3)x + (2l + 3)(2m + 3)$

$= x^2 - [2(l + m) + 6)]x + (4lm + 6(l + m) + 9)$

$= x^2 - 5x + 6x + 14 + 15 + 9$

$= x^2 + x + 38$