NEWTON'S LAWS OF MOTION & FRICTION

KEY CONCEPTS

FORCE

A push or pull that one object exerts on another.

FORCES IN NATURE

There are four fundamental forces in nature :

1. Gravitational force 2. Electromagnetic force
3. Strong nuclear force 4. Weak force

TYPES OF FORCES ON MACROSCOPIC OBJECTS

(a) Field Forces or Range Forces :

These are the forces in which contact between two objects is not necessary.

Ex. (i) Gravitational force between two bodies. (ii) Electrostatic force between two charges.

(b) Contact Forces :

Contact forces exist only as long as the objects are touching each another.

Ex. (i) Normal forces. (ii) Frictional force

(c) Attachment to Another Body :

Tension (T) in a string and spring force (F = kx) comes in this group.

NEWTON'S FIRST LAW OF MOTION (OR GALILEO'S LAW OF INERTIA)

Every body continues in its state of rest or uniform motion in a straight line unless compelled by an external force to change that state.

Definition of force from Newton's first law of motion *"Force is that push or pull which changes or tends to change the state of rest or of uniform motion in a straight line"*.

Inertia : Inertia is the property of the body due to which body oppose the change of its state. Inertia of a body is measured by mass of the body. $\boxed{\text{inertia} \propto \text{mass}}$

TYPES OF INERTIA

Inertia of rest : It is the inability of a body to change, its state of rest by itself.

Examples :

- When we shake a branch of a mango tree, the mangoes fall down.
- When a bus or train starts suddenly the passengers sitting inside tends to fall backwards.
- The dust particles in a blanket fall off when it is beaten with a stick.
- When a stone hits a window pane, the glass is broken into a number of pieces whereas if the high speed bullet strikes the pane, it leaves a clean hole.

Inertia of motion : It is the inability of a body to change its state of uniform motion by itself.

Examples :

- When a bus or train stop suddenly, a passenger sitting inside tends to fall forward.
- A person jumping out of a speeding train may fall forward.
- A ball thrown upwards in a running train continues to move along with the train.

Inertia of direction : It is the inability of a body to change its direction of motion by itself.

Examples :

- When a car rounds a curve suddenly, the person sitting inside is thrown outwards.
- Rotating wheels of vehicle throw out mud, mudguard over the wheels stop this mud.
- A body released from a balloon rising up, continues to move in the direction of balloon.

NEWTON'S SECOND LAW OF MOTION

Rate of change of momentum of a body is directly proportional to the external force applied on it and the change in momentum takes place in the direction of force

$$\vec{F} \propto \frac{d\vec{p}}{dt} \quad \text{or} \quad \vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

if m = constant then $\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}$

- Newton's Second Law Provides the Definition of the Concept of Force.
- **Definition of the 1 Newton (N) :–** If an object of mass one kilogram has an acceleration of $1\,ms^{-2}$ relative to an inertial reference frame, then the net force exerted on the object is one newton.

CONSEQUENCES OF NEWTON'S II LAW OF MOTION

- **Concept of inertial mass :** From Newton's II law of motion $a = \frac{F}{M}$

  i.e., the magnitude of acceleration produced by a given body is inversely proportional to mass i.e. greater the mass, smaller is the acceleration produced in the body. Thus, mass is the measure of inertia of the body. The mass given by above equation is therefore called the inertial mass.

- **An accelerated motion is the result of application of the force :**

  There may be two types of accelerated motion :

(i) When only the magnitude of velocity of the body changes : In this types of motion the force is applied along the direction of motion or opposite to the direction of motion.

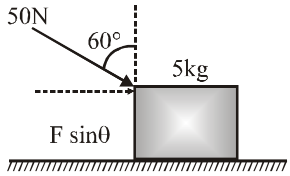
(ii) When only the direction of motion of the body changes : In this case the force is applied at right angles to the direction of motion of the body, e.g. uniform circular motion.

- **Acceleration produced in the body depends only on its mass and not on the final or initial velocity.**

Ex. A force $\vec{F} = (6\hat{i} - 8\hat{j} + 10\hat{k})\,N$ produces acceleration of 1 ms^{-2} in a body. Calculate the mass of the body.

Sol. $\because$ Acceleration $a = \frac{|\vec{F}|}{m}$ $\quad\therefore$ mass $m = \frac{|\vec{F}|}{a} = \frac{\sqrt{6^2 + 8^2 + 10^2}}{1} = 10\sqrt{2}\text{ kg}$

Ex. A force of 50 N acts in the direction as shown in figure. The block of mass 5kg, resting on a smooth horizontal surface. Find out the acceleration of the block.



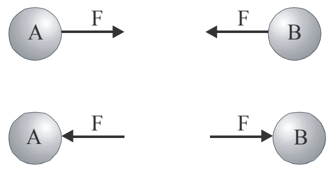
Sol. Horizontal component of the force = Fsinθ = 50sin60° = $\frac{50\sqrt{3}}{2}$ N

acceleration of the block $a = \frac{F\sin\theta}{m} = \frac{50\sqrt{3}}{2} \times \frac{1}{5} = 5\sqrt{3}$ m/s^2

NEWTON'S THIRD LAW OF MOTION

The first and second laws are statements about a single object, whereas the third law is a statement about two objects.

- According to this law, every action has equal and opposite reaction. Action and reaction act on different bodies and they are simultaneous. There can be no reaction without action.
- If an object A exerts a force F on an object B, then B exerts an equal and opposite force (–F) on A.



The forces between two objects A and B are equal and opposite, whether they are attractive or repulsive.

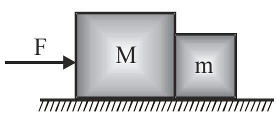
- Action and reaction never cancel each other, since they act on different bodies.

- **First law** : If no net force acts on a particle, then it is possible to select a set of reference frames, called inertial reference frames, observed from which the particle moves without any change in velocity.
- **Second law** : Observed from an inertial reference frame, the net force on a particle is proportional to the time rate of change of its linear momentum: $\frac{d(m\vec{v})}{dt}$
- **Third law** : Whenever a particle A exerts a force on another particle B, B simultaneously exerts a force on A with the same magnitude in the opposite direction.

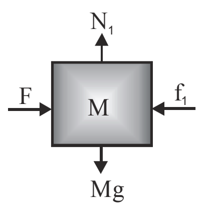
FREE BODY DIAGRAM

A diagram showing all external forces acting on an object is called "Free Body Diagram" (F.B.D.) In a specific problem, first we are required to choose a body and then we find the number of forces acting on it, and all the forces are drawn on the body, considering it as a point mass. The resulting diagram is known as free body diagram (FBD).

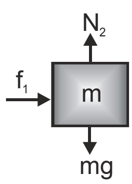
For example, if two bodies of masses m and M are in contact and a force F on M is applied from the left as shown in figure (a), the free body diagrams of M and m will be as shown in figure (b) and (c).



(a)



(b)



(c)

Important Point :

Two forces in Newton's third law never occur in the same free–body diagram. This is because a free–body diagram shows forces acting on a single object, and the action–reaction pair in Newton's third law always act on different objects.

MOTION OF BODIES IN CONTACT

Case I :

When two bodies of masses m_1 and m_2 are kept on the frictionless surface and a force F is applied on one body, then the force with which one body presses the other at the point of contact is called Force of Contact. These two bodies will move with same acceleration a.

(i) When the force F acts on the body with mass m_1 as shown in fig. (1) $F = (m_1 + m_2)\, a$.

If the force exerted by m_2 on m_1 is f_1 (force of contact) then for body m_1 : $(F - f_1) = m_1\, a$

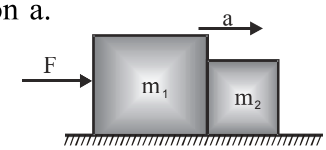


Fig.(1) : When the force F acts on mass m_1.

for body m_2 : $f_1 = m_2 a$

$\Rightarrow$ action of m_1 on m_2 : $f_1 = \dfrac{m_2 F}{m_1 + m_2}$

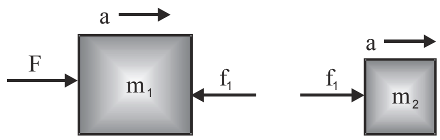


Fig. 1(a) : F.B.D. representation of action and reaction forces.

(ii) When the force F acts on the body with mass m_2 as shown in figure 2

$F = (m_1 + m_2)\, a$

for body with mass m_2

$F - f_2 = m_2\, a$

for body m_1, $f_2 = m_1 a$

$\Rightarrow$ action on m_1 , $f_2 = \dfrac{m_1 F}{m_1 + m_2}$

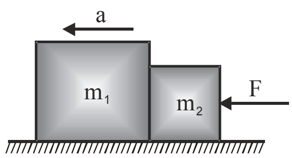


Fig. (2) : When the force F acts on mass m_2.

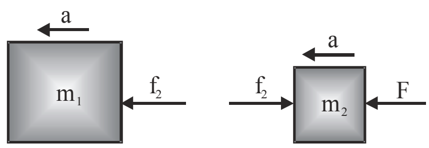
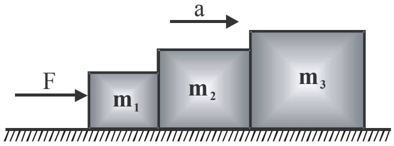


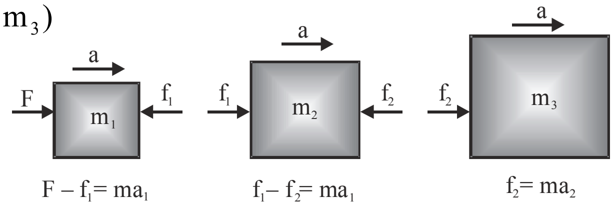
Fig. 2 (a) : F.B.D. representation of action and reaction forces.

Case II :

Three bodies of masses m_1, m_2 and m_3 placed one after another and in contact with each other. Suppose a force F is applied horizontally on mass m_1



then $F = (m_1 + m_2 + m_3)\, a \;\Rightarrow\; a = \dfrac{F}{(m_1 + m_2 + m_3)}$

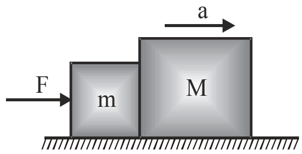


$f_1 = \dfrac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$ (action on both m_2 and m_3)

and $f_2 = \dfrac{m_3 F}{(m_1 + m_2 + m_3)}$ (action on m_3 alone) when the force F is applied on m_3, then

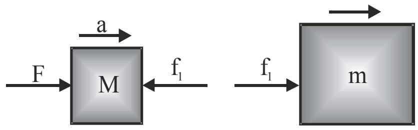
$$f_1 = \frac{m_1 F}{(m_1 + m_2 + m_3)} \text{ (action on } m_1 \text{ alone) and } f_2 = \frac{(m_1 + m_2)F}{(m_1 + m_2 + m_3)} \text{ (action on } m_1 \text{ and } m_2)$$

Ex. Two blocks of mass m = 2 kg and M = 5 kg are in contact on a frictionless table. A horizontal force F (= 35 N) is applied to m. Find the force of contact between the block, will the force of contact remain same if F is applied to M ?



Sol. As the blocks are rigid under the action of a force F, both will move with same acceleration

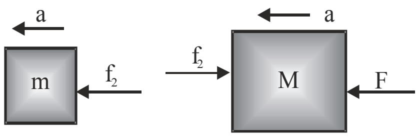
$$a = \frac{F}{m+M} = \frac{35}{2+5} = 5 \text{ m/s}^2$$



force of contact f_1 = Ma = 5 × 5 = 25 N

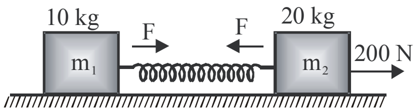
If the force is applied to M then its action on m will be

f_2 = ma = 2 × 5 = 10 N.



From this problem it is clear that acceleration does not depends on the fact that whether the force is applied to m or M, but force of contact does.

Ex. Two masses 10 kg and 20 kg respectively are connected by a massless spring as shown in figure force of 200 N acts on the 20 kg mass. At the instant shown in figure the 10 kg mass has acceleration of 12 m/s², what is the acceleration of 20 kg mass?



Sol. Equation of motion for m_1 is $F = m_1 a_1 = 10 \times 12 = 120$ N.

Force on 10 kg–mass is 120 N to the right. As action and reaction are equal and opposite, the reaction force F on 20 kg mass F = 120 N to the left.

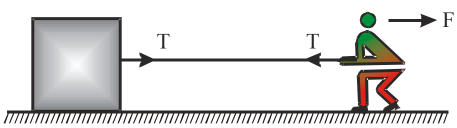
∴ Equation of motion for m_2 is $200 - F = 20\,a_2$

$$\Rightarrow 200 - 120 = 20\,a_2 \qquad \Rightarrow 20\,a_2 = 80 \qquad \Rightarrow a_2 = \frac{80}{20} = 4 \text{ ms}^{-2}$$

SYSTEM OF MASSES TIED BY STRINGS

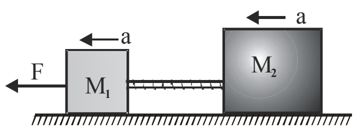
Tension in a String :

It is an intermolecular force between the atoms of a string, which acts or reacts when the string is streched.

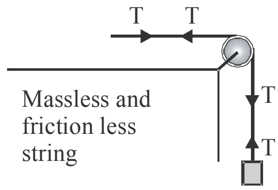


Important points about the tension in a string :

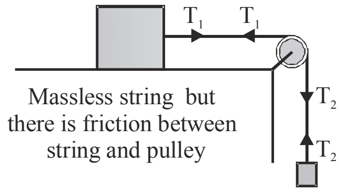
- Force of tension act on a body in the direction away from the point of contact or tied ends of the string.
- String is assumed to be inextensible so that the magnitude of accelerations of any number of masses connected through strings is always same.



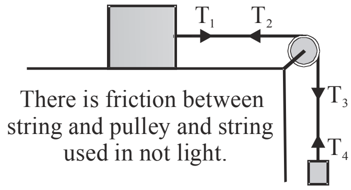
- If the string is extensible the acceleration of different masses connected through it will be different until the string can stretch.
- String is massless and frictionless so that tension throughout the string remains same.



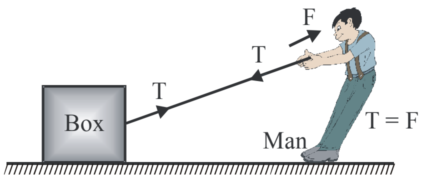
- If the string is massless but not frictionless, at every contact tension changes.



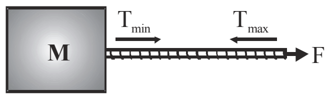
- If the string is not light, tension at each point will be different depending on the acceleration of the string.



- If a force is directly applied on a string as say man is pulling a tied string from the other end with some force the tension will be equal to the applied force irrespective of the motion of the pulling agent, irrespective of whether the box will move or not, man will move or not.

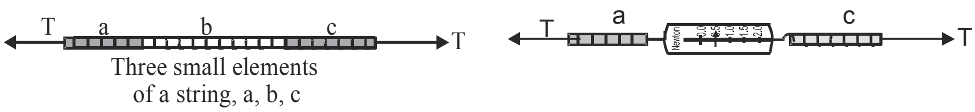


- String is assumed to be massless unless stated, hence tension in it everywhere remains the same and equal to applied force. However, if a string has a mass, tension at different points will be different being maximum (= applied force) at the end through which force is applied and minimum at the other end connected to a body.



- In order to produce tension in a string two equal and opposite stretching forces must be applied. The tension thus produced is equal in magnitude to either applied force (i.e., T = F) and is directed inwards opposite to F. Here it must be noted that a string can never be compressed like a spring.

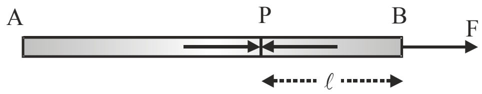
- If string is cut so that element b is replaced by a spring scale (the rest of the string being undisturbed), the scale reads the tension T.



- Every string can bear a maximum tension, i.e. if the tension in a string is continuously increased it will break if the tension is increased beyond a certain limit. The maximum tension which a string can bear without breaking is called "breaking strength". It is finite for a string and depends on its material and dimensions.

Ex. A uniform rope of length L is pulled by a constant force F. What is the tension in the rope at a distance ℓ from the end where it is applied?

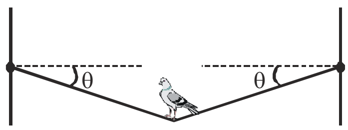
Sol. Let mass of rope is M and T be tension in the rope at point P, then. Acceleration of rope, $a = \frac{F}{M}$



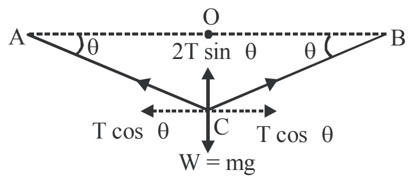
Equation of motion of part PB is F – T = (mℓ) a

$$\Rightarrow T = F - (m\ell)\,a = F - \left(\frac{M}{L}\right)(\ell)\left(\frac{F}{M}\right) = \left[1 - \frac{\ell}{L}\right] F$$

Ex. A bird with mass m perches at the middle of a stretched string Show that the tension in the string is give by $T = \frac{mg}{2\sin\theta}$. Assume that each half of the string is straight.



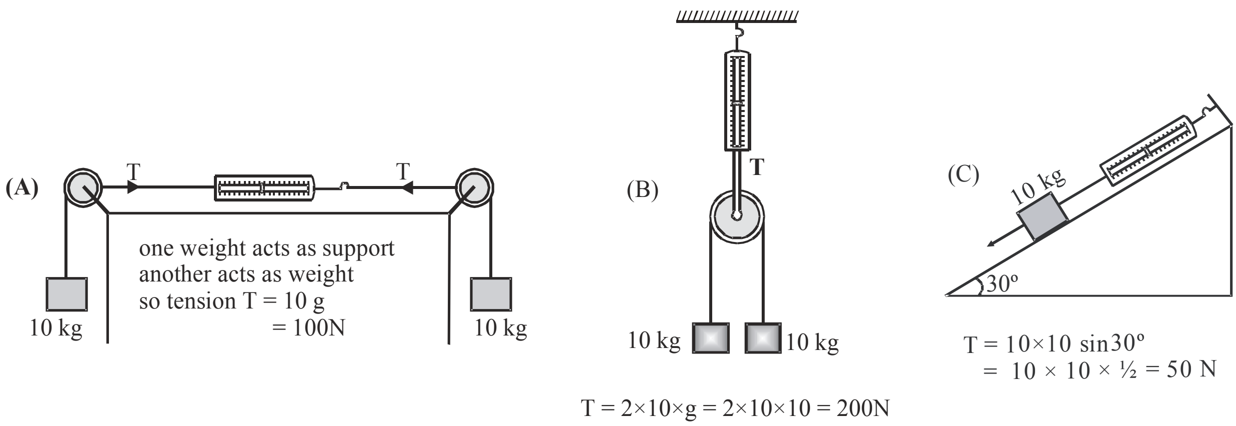
Sol. Initial position of wire = AOB. Final position of wire = ACB Let θ be the angle made by wire with horizontal, which is very small. Resolving tension T of string in horizontal and vertical directions, we note that the horizontal components cancel while vertical components add and balance the weight.



For equilibrium 2T sinθ = W = mg $\Rightarrow T = \frac{W}{2\sin\theta}$ = mg/2sinθ

Ex. The system shown in figure are in equilibrium. If the spring balance is calibrated in newtons, what does it record in each case? (g = 10 m/s²)

Sol.

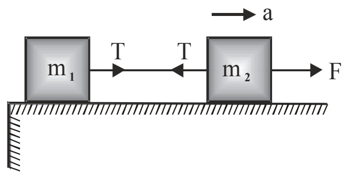


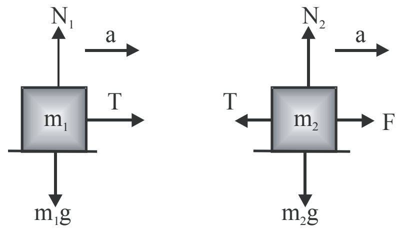
MOTION OF BODIES CONNECTED BY STRINGS

Case : I

Two bodies :

Let us consider the case of two bodies of masses m_1 and m_2 connected by a thread and placed on a smooth horizontal surface as shown in figure. A force F is applied on the body of mass m_2 in forward direction as shown. Our aim is to consider the acceleration of the system and the tension T in the thread. The forces acting separately on two bodies are also shown in the figure:





From figure $T = m_1 a$

and $F - T = m_2 a$

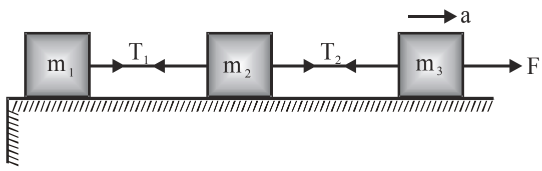
$\Rightarrow$ $F = (m_1 + m_2)a$

$\Rightarrow$ $a = \dfrac{F}{m_1 + m_2}$ & $T = \dfrac{m_1 F}{m_1 + m_2}$

Case II:

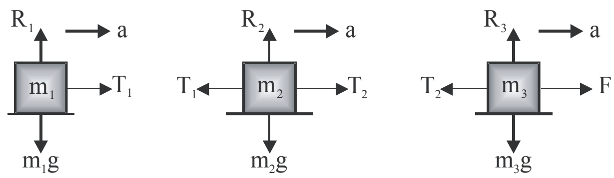
Three bodies

In case of three bodies, the situation is shown in figure



Acceleration $a = \dfrac{F}{m_1 + m_2 + m_3}$,

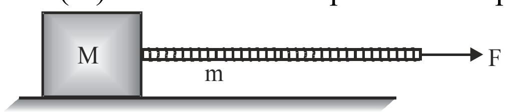
$$T_1 = m_1 a = \frac{m_1 F}{m_1 + m_2 + m_3}$$



$\because$ for block of mass m_3 $F - T_2 = m_3 a$

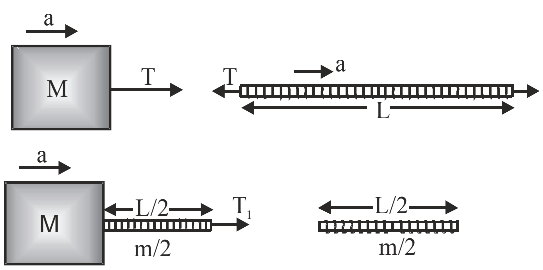
$$\therefore\ T_2 = F - \frac{m_3 F}{m_1 + m_2 + m_3} = \frac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$$

Ex. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m as shown in fig. A horizontal force F is applied to one end of the rope. Find (i) The acceleration of the rope and block (ii) The force that the rope exerts on the block. (iii) Tension in the rope at its mid point.



Sol. (i) Accelaration $a = \dfrac{F}{(m + M)}$

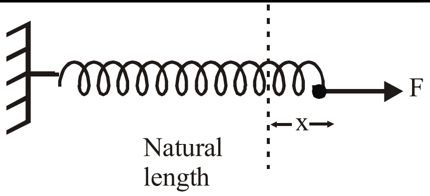
(ii)Force exerted by rope $T = Ma = \dfrac{M.F}{(m + M)}$



(iii) $T_1 = \left(\dfrac{m}{2} + M\right)\ a = \left(\dfrac{m + 2M}{2}\right)\left(\dfrac{F}{m + M}\right)$

Tension in rope at midpiont $T_1 = \dfrac{(m + 2M)F}{2(m + M)}$

Spring Force (According to Hooke's law) :

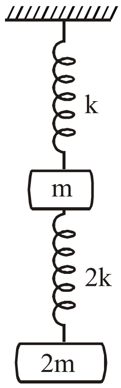


In equilibrium $F = kx$

k is spring constant

Note : Spring force is non impulsive in nature.

Ex. If the lower spring is cut, find acceleration of the blocks, immediately after cutting the spring.

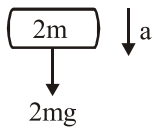


Sol. Intial stretches

$$x_{upper} = \frac{3mg}{k} \quad \& \quad x_{lower} = \frac{mg}{k}$$

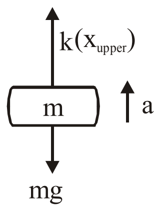
On cutting the lower spring, by virture of non-impulsive nature of spring the stretch in upper spring remains same. Thus,

Lower block :



$$2mg = 2ma \Rightarrow a = g$$

Upper block :



$$k\left(\frac{3mg}{k}\right) - mg = ma \Rightarrow a = 2g$$

FRAME OF REFERENCE

It is a conveniently chosen co–ordinate system which describes the position and motion of a body in space.

INERTIAL AND NON–INERTIAL FRAMES OF REFERENCE

Inertial frames of reference :

A reference frame which is either at rest or in uniform motion along the straight line. A non–accelerating frame of reference is called an inertial frame of reference.

- All the fundamental laws of physics have been formulated in respect of inertial frame of reference.
- All the fundamental laws of physics can be expressed as to have the same mathematical form in all the inertial frames of reference.
- The mechanical and optical experiments performed in an inertial frame in any direction will always yield the same results. It is called isotropic property of the inertial frame of reference.

Examples of inertial frames of reference :

- A frame of reference remaining fixed w.r.t. distant stars is an inertial frame of reference.
- A space–ship moving in outer space without spinning and with its engine cut–off is also inertial frame of reference.
- For practical purposes, a frame of reference fixed to the earth can be considered as an inertial frame. Strictly speaking, such a frame of reference is not an inertial frame of reference, because the motion of earth around the sun is accelerated motion due to its orbital and rotational motion. However, due to negligibly small effects of rotation and orbital motion, the motion of earth may be assumed to be uniform and hence a frame of reference fixed to it may be regarded as inertial frame of reference.

Non–inertial frame of reference :

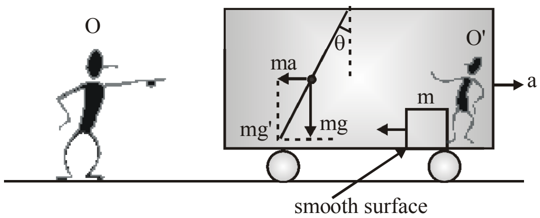
An accelerating frame of reference is called a non–inertial frame of reference.

Newton's laws of motion are not directly applicable in such frames, before application we must add pseudo force.

Note : A rotating frame of references is a non–inertial frame of reference, because it is also an accelerating one due to its centripetal acceleration.

PSEUDO FORCE

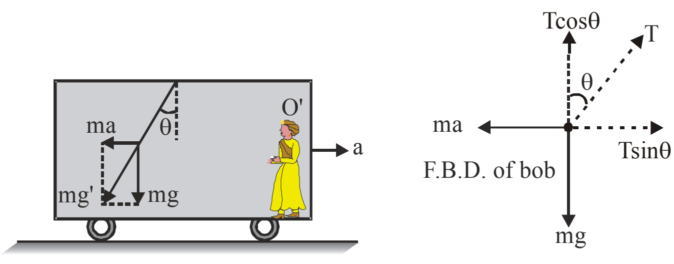
The force on a body due to acceleration of non–inertial frame is called fictitious or apparent or pseudo force and is given by $\vec{F} = -m\vec{a}_0$, where $\vec{a}_0$ is acceleration of non–inertial frame with respect to an inertial frame and m is mass of the particle or body. The direction of pseudo force must be opposite to the direction of acceleration of the non–inertial frame.



For observer O on ground train is moving with acceleration on "a" for observer O' in side the train there is pseudo force in opposite direction shown in figure.

- When we draw the free body diagram of a mass, with respect to an **inertial frame of reference** we apply only the real forces (forces which are actually acting on the mass).
- But when the free body diagram is drawn from a non–inertial frame of reference a pseudo force (in addition to all real forces) has to be applied to make the equation $\vec{F} = m\vec{a}$ to be valid in this frame also.

Ex. A pendulum of mass m is suspended from the ceiling of a train moving with an acceleration 'a' as shown in figure. Find the angle θ in equilibrium position.



Sol. Non–inertial frame of reference (Train)

F.B.D. of bob w.r.t. train. (real forces + pseudo force) : with respect to train, bob is in equilibrium

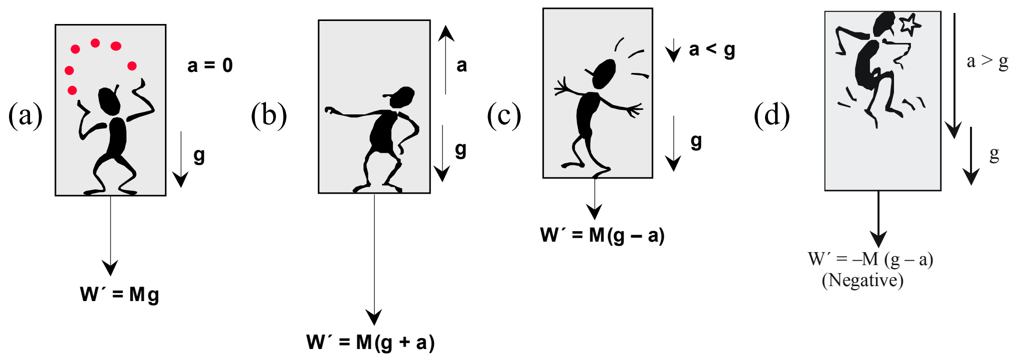
$\therefore\ \Sigma F_y = 0 \Rightarrow T\cos\theta = mg$ and

$\Sigma F_x = 0 \Rightarrow T\sin\theta = ma$

$$\Rightarrow\ \tan\theta = \frac{a}{g}\ \Rightarrow\ \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

MOTION IN A LIFT

The weight of a body is simply the force exerted by earth on the body. If body is on an accelerated platform, the body experiences fictitious force, so the weight of the body appears changed and this new weight is called apparent weight. Let a man of weight W = Mg be standing in a lift.
We consider the following cases :



Case (a) :
If the lift moving with constant velocity v upwards or downwards.
In this case there is no accelerated motion hence no pseudo force experienced by observer inside the lift.
So apparent weight W′ = Mg (Actual weight).

Case (b) :
If the lift is accelerated upward with constant acceleration a.
Then net forces acting on the man are (i) weight W = Mg downward (ii) fictitious force F_0 = Ma downward.
So apparent weight $W' = W + F_0 = Mg + Ma = M(g + a)$

Case (c) :
If the lift is accelerated downward with acceleration a < g
Then fictitious force F_0 = Ma acts upward while weight of man W = Mg always acts downward.
So apparent weight $W' = W + F_0 = Mg - Ma = M(g - a)$

Special Case : If a = g then W′ = 0 (condition of weightlessness).
Thus, in a freely falling lift the man will experience weightlessness.

Case (d) :
If lift accelerates downward with acceleration a > g :
Then as in Case c . Apparent weight W′ = M(g – a) is negative, i.e., the man will be accelerated upward and will stay at the ceiling of the lift.

Ex. A spring weighing machine inside a stationary lift reads 50 kg when a man stands on it. What would happen to the scale reading if the lift is moving upward with (i) constant velocity, and (ii) constant acceleration?

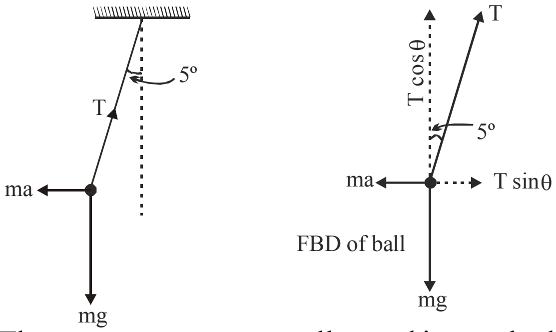
Sol. (i) In the case of constant velocity of lift, there is no fictitious force; therefore the apparent weight = actual weight. Hence the reading of machine is 50 kgwt.
(ii) In this case the acceleration is upward, the fictitious force ma acts downward,therefore apparent weight is more than actual weight i.e. W′= m (g + a).

Hence scale shows a reading = $m(g + a) = \dfrac{mg\left(1 + \dfrac{a}{g}\right)}{g} = \left(50 + \dfrac{50a}{g}\right)$ kgwt.

Ex. Two objects of equal mass rest on the opposite pans of an arm balance. Does the scale remain balanced when it is accelerated up or down in a lift?

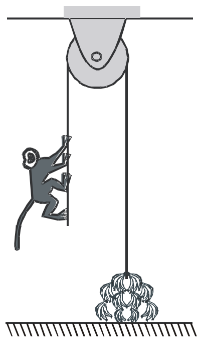
Sol. Yes, since both masses experience equal fictitious forces in magnitude as well as direction.

Ex. A passenger on a large ship sailing in a quiet sea hangs a ball from the ceiling of her cabin by means of a long thread. Whenever the ship accelerates, she notes that the pendulum ball lags behind the point of suspension and so the pendulum no longer hangs vertically. How large is the ship's acceleration when the pendulum stands at an angle of 5° to the vertical?



Sol. The ball is accelerated by the force T sin5°.
Therefore T sin5° = ma.
Vertical component ΣF = 0, so T cos5° = mg.
By solving a = g tan 5° = 0.0875 g
= 0.86 ms^{-2}.

Ex. A 12 kg monkey climbs a light rope as shown in figure. The rope passes over a pulley and is attached to a 16 kg bunch of bananas. Mass and friction in the pulley are negligible so that the pulley's only effect is to reverse the direction of the rope. What is the maximum acceleration the monkey can have without lifting the bananas?



Sol. Effective weight of monkey $W_m = M_m(g + a)$

As per given condition $W_m = W_b$

$\Rightarrow$ $M_m (g + a) = M_b\, g$

$$\Rightarrow \quad a = \frac{(M_b - M_m)\ g}{M_m} = \left(\frac{16-12}{12}\right)\times 9.8$$

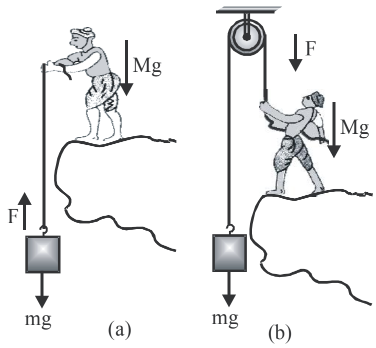
$$= \frac{9.8}{3} = 3.26\ \text{m/s}^2$$

PULLEY SYSTEM

A single fixed pulley changes the direction of force only and in general, assumed to be massless and frictionless.

It is clear from example given below.

Ex. A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in figure. What is the action on the floor by the man in the two cases ? If the floor yields to a normal force of 700 N, which mode should be the man adopt to lift the block without the floor yielding ?



Sol. Mass of the block, m = 25 kg ; mass of the man, M = 50 kg
Force applied to lift the block F = mg = 25 × 9.8 = 245 N
Weight of the man, M g = 50 × 9.8 = 490 N

(a) When the block is raised by the man by applying force F in upward direction, reaction equal and opposite to F will act on the floor in addition to the weight of the man.

$\therefore$ action on the floor Mg + F = 490 + 245 = 735 N

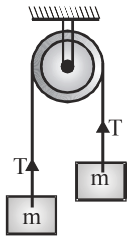
(b) When the block is raised by the mass applying force F over the rope (passed over the pulley) in downward direction, reaction equal and opposite to F will act on the floor,

$\therefore$ action on the floor $\quad Mg - F = 490 - 425 = 245$ N

floor yields to a normal force of 700 N, the mode (b) should be adopted by the man to lift block.

Some cases of pulley

I Case



$m_1 = m_2 = m$

Tension in the string $T = mg$

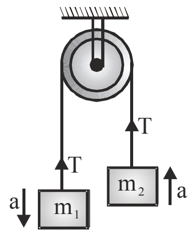
Acceleration 'a' = zero

Reaction at the suspension of the pulley

$R = 2T = 2\ mg$.

$$\text{Acceleration} = \frac{\text{net pulling force}}{\text{total mass to be pulled}}$$

II Case



$m_1 > m_2$

now for mass m_1, $m_1g - T = m_1a$

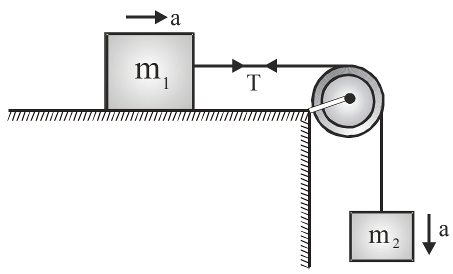
for mass m_2, $T - m_2g = m_2a$

$$a = \frac{(m_1 - m_2)}{(m_1 + m_2)}g \text{ and } T = \frac{2m_1m_2}{(m_1 + m_2)}g$$

$$\text{Tension} = \frac{2 \times \text{Product of masses}}{\text{Sum of two masses}}g$$

Reaction at the suspension of pulley $R = 2T = \dfrac{4m_1m_2\,g}{(m_1 + m_2)}$

III Case :

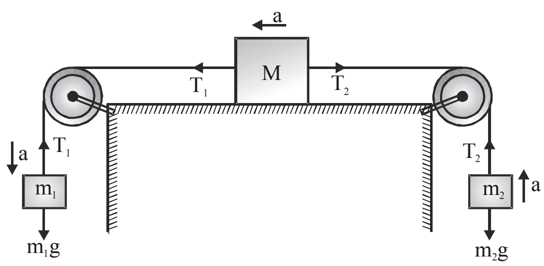


For mass m_1 : $T = m_1\,a$

For mass m_2 : $m_2g - T = m_2\,a$

$$\text{acceleration } a = \frac{m_2g}{(m_1 + m_2)} \text{ and } T = \frac{m_1m_2}{(m_1 + m_2)}g$$

IV Case :



$(m_1 > m_2)$

For m_1, $m_1g - T_1 = m_1a$

For m_2, $T_2 - m_2g = m_2a$

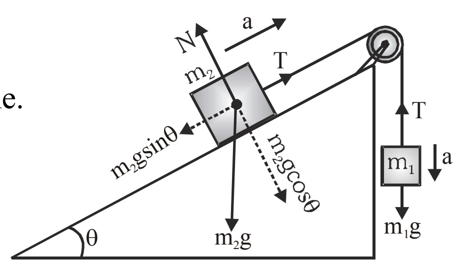
For M $T_1 - T_2 = Ma$

$$\Rightarrow a = \frac{(m_1 - m_2)}{(m_1 + m_2 + M)}g,$$

$$T_1 = \frac{(2m_2 + M)m_1g}{m_1 + m_2 + M},\ T_2 = \frac{(2m_1 + M)m_2g}{m_1 + m_2 + M}$$

V Case :

Mass suspended over a pulley from another on an inclined plane.



For mass m_1 : $m_1g - T = m_1\,a$

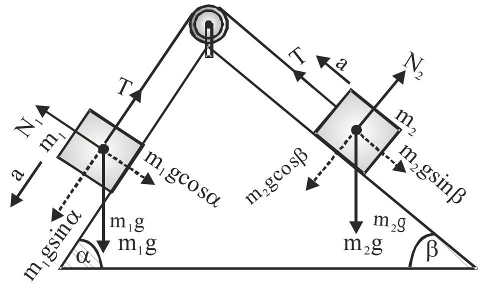
For mass m_2 : $T - m_2g\sin\theta = m_2\,a$

$$\text{acceleration } a = \frac{(m_1 - m_2\sin\theta)}{(m_1 + m_2)}g$$

$$T = \frac{m_1m_2(1 + \sin\theta)}{(m_1 + m_2)}g$$

VI Case :

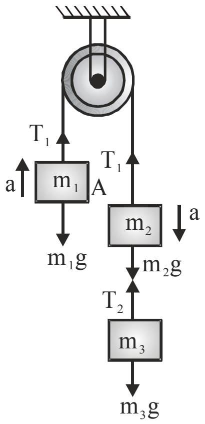
Masses m_1 and m_2 are connected by a string passing over a pulley ($m_1 > m_2$)



Acceleration $a = \dfrac{(m_1 \sin\alpha - m_2 \sin\beta)}{(m_1 + m_2)} g$

Tension $T = \dfrac{m_1 m_2 (\sin\alpha + \sin\beta)}{(m_1 + m_2)} g$

VII Case :



For mass m_1 : $T_1 - m_1g = m_1a$

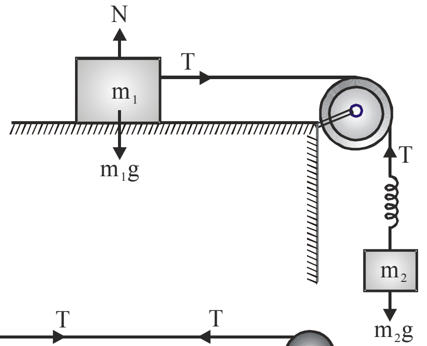
For mass m_2 : $m_2g + T_2 - T_1 = m_2a$

For mass m_3 : $m_3g - T_2 = m_3a$

Acceleration $a = \dfrac{(m_2 + m_3 - m_1)}{(m_1 + m_2 + m_3)} g$

we can calculate tensions T_1 and T_2 from above equations

VIII Case :



From case (iii)

tension $T = \dfrac{m_1 m_2}{(m_1 + m_2)} g$

If x is the extension in the spring,

then $T = kx$

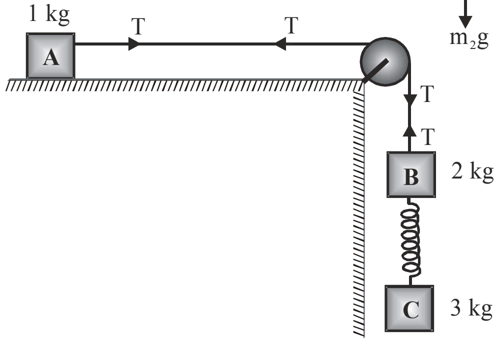
$x = \dfrac{T}{k} = \dfrac{m_1 m_2 g}{k(m_1 + m_2)}$

Ex. In the system shown in figure all surface are smooth, string is massless and inextensible. Find:

(a) acceleration of the system

(b) tension in the string and

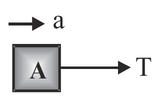
(c) extension in the spring if force constant of spring is $k = 50$ N/m (Take $g = 10$ m/s^2)



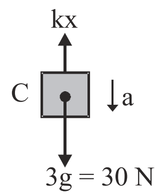
Sol. (a) In this case net pulling force = $m_cg + m_Bg = 50$ N and total mass to be pulled is (1 + 2 + 3) kg = 6 kg.

$\therefore$ Acceleration of the system is $a = \dfrac{50}{6}$ ms^{-2}

(b) Free body diagram of 1 kg block gives $T = ma = (1)\left(\dfrac{50}{6}\right)N = \dfrac{50}{6}N$



(c) Free body diagram of 3 kg block gives



$30 - kx = ma$ but $ma = 3 \times \dfrac{50}{6} = 25N$

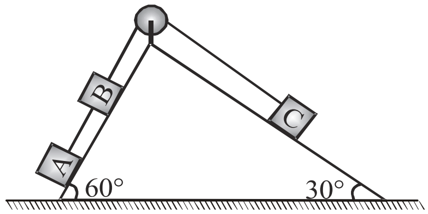
$x = \dfrac{30 - 25}{k} = \dfrac{5}{50} = 0.1$ m $= 10$ cm

Ex. In the adjacent figure, masses of A, B and C are 1 kg, 3 kg and 2 kg respectively.

Find : (a) the acceleration of the system and

(b) tensions in the string

Neglect friction. ($g = 10\ ms^{-2}$)



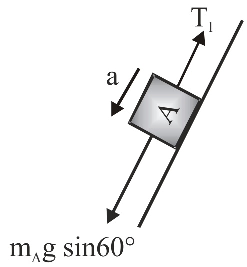
Sol. (a) In this case net pulling force

$= m_A g \sin 60° + m_B g \sin 60° - m_C g \sin 30°$

$= (m_A + m_B)\, g \sin 60° - m_C\, g \sin 30°$

$= (1 + 3) \times 10 \times \frac{\sqrt{3}}{2} - 2 \times 10 \times \frac{1}{2}$

$= 20\sqrt{3} - 10 = 20 \times 1.732 - 10 = 24.64$ N



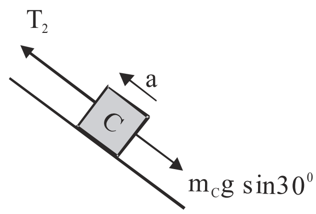
Total mass being pulled $= 1 + 3 + 2 = 6$ kg

$\therefore$ Acceleration of the system $a = \frac{24.64}{6} = 4.1\ m/s^2$

(b) For the tension T_1 in the string between A and B, $m_A g \sin 60° - T_1 = (m_A)(a)$

$\therefore T_1 = m_A\, g \sin 60° - m_A\, a = m_A\,(g \sin 60° - a)$

$\Rightarrow T_1 = (1)\left(10 \times \frac{\sqrt{3}}{2} - 4.1\right) = 4.56$ N

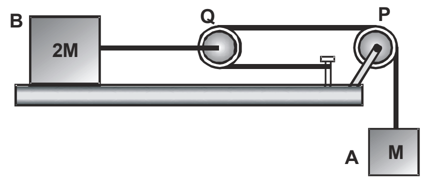


For the tension T_2 in the string between B and C.

$T_2 - m_C\, g \sin 30° = m_C a$

$\Rightarrow T_2 = m_C\,(a + g \sin 30°) = 2\left[4.1 + 10\left(\frac{1}{2}\right)\right] = 18.2$ N

Ex.. Consider the situation shown in figure. Both the pulleys and the strings are light and all the surfaces are frictionless. Calculate



(a) the acceleration of mass M,

(b) tension in the string PQ and

(c) force exerted by the clamp on the pulley P.

Sol. As pulley Q is not fixed so if it moves a distance d the length of string between P and Q will changes by 2d (d from above and d from below), i.e., M will move 2d. This in turn implies that if a is the acceleration of M the acceleration of Q and of 2M will be $\frac{a}{2}$.

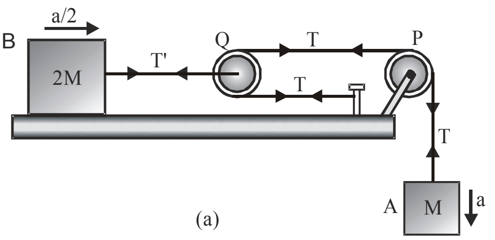
Now if we consider the motion of mass M, it is accelerated downward, so

$T = M(g - a)$ ....(i)

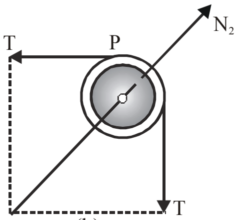
And for the motion of Q

$2T - T' = 0 \times \frac{a}{2} = 0 \quad \Rightarrow T' = 2T$ ....(ii)

And for the motion of mass 2M $\quad T' = 2M\left(\frac{a}{2}\right), \quad \Rightarrow \quad T' = Ma$ ....(iii)



(a)



(b)

(a) From equation (ii) and (iii) as $T = \frac{1}{2}Ma$, so equation (i) reduces to

$$T = \frac{1}{2}\ Ma = M(g - a) \quad \Rightarrow a = \frac{2}{3}\ g$$

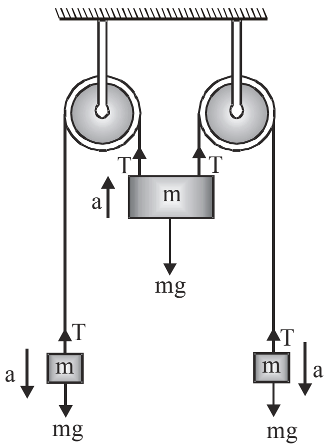
(b) So the acceleration of mass M is $\frac{2}{3}$ g while tension in the string PQ from equation (1) will be

$$T = M(g - \frac{2}{3}g) = \frac{1}{3}\ Mg$$

(c) Now from figure (b), it is clear that force on pulley by the clamp will be equal and opposite to the resultant of T and T at 90° to each other, i.e.,

$$(N_2) = \sqrt{T^2 + T^2} = \sqrt{2}\,T = \frac{\sqrt{2}}{3}\ Mg$$

Ex. Consider the double Atwood's machine as shown in the figure



(a) What is acceleration of the masses?

(b) What is the tension in each string?

Sol. (a) Here the system behaves as a rigid system, therefore every part of the system will move with same acceleration. Thus Applying newton's law

$mg - T = ma$ ...(1)

$2T - mg = ma$ ...(2)

Doubling the first equation and adding

$mg = 3\ ma \Rightarrow$ acceleration $a = \frac{1}{3}g$

(b) Tension in the string $T = m\ (g - a) = m\left(g - \frac{g}{3}\right) = \frac{2}{3}\,mg$

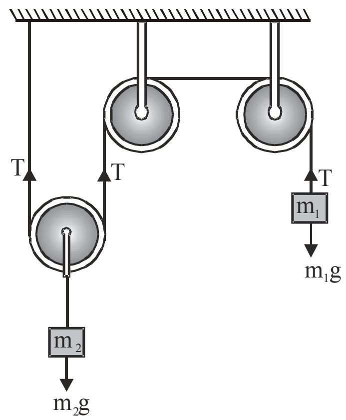
Ex. Consider the system of masses and pulleys shown in fig. with massless string and frictionless pulleys.

(a) Give the necessary relation between masses m_1 and m_2 such that system is in equilibrium and does not move.

(b) If m_1 = 6 kg and m_2 = 8 kg, calculate the magnitude and direction of the acceleration of m_1.

Sol. (a) Applying newton's law $m_2g - 2T = 0$ (because there is no acceleration) and $T - m_1g = 0$

$\Rightarrow (m_2 - 2m_1)\, g = 0 \Rightarrow m_2 = 2m_1$



(b) If the upwards acceleration of m_1 is a,

then acceleration of m_2 is $\frac{a}{2}$ downwards

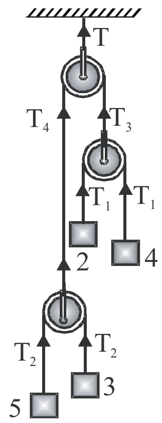
for mass m_2: $m_2g - 2T = m_2\left(\frac{a}{2}\right) \Rightarrow 2m_2g - 4T = m_2a$

for mass m_1 : $T - m_1g = m_1a$

$$\Rightarrow \quad a = \left(\frac{2m_2 - 4m_1}{m_2 + 4m_1}\right) g = \frac{2(8-12)}{8+24} g = -\frac{g}{4}$$

Negative sign shows that acceleration is opposite to considered direction i.e. it is downwards for m_1 and upwards for m_2.

Ex. In the given figure If $T_1 = 2T_2 = 50$ N then find the value of T.



Sol. As given in figure,

$T_3 = 2T_1 = 2(2T_2) = 4T_2$

and $T_4 = 2T_2$

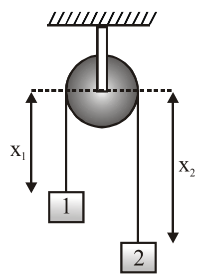
$\therefore \quad T = T_3 + T_4 = 4T_2 + 2T_2 = 6T_2$

$$= 6 \times \frac{50}{2} = 150\text{N}$$

CONSTRAINT RELATIONS

These equations establish the relation between accelerations (or velocities) of different masses attached by string(s). Normally number of constraint equations are equal to number of strings in the system under consideration.

Ex. Find the relation between acceleration of 1 and 2.



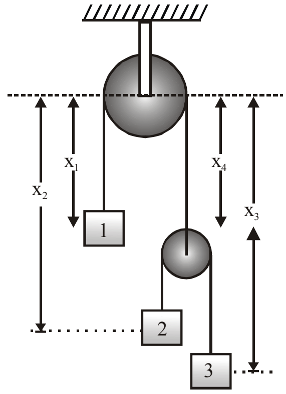
Sol. At any instant of time let x_1 and x_2 be the displacements of 1 and 2 from a fixed line. Then $x_1 + x_2$ = constant

Differentiating w.r.t. time, $v_1 + v_2 = 0$

Again differentiating w.r.t. time, $a_1 + a_2 = 0 \Rightarrow a_1 = -a_2$

So acceleration of 1 and 2 are equal but in opposite directions.

Ex. At certain moment of time, velocities of 1 and 2 both are 1 ms^{-1} upwards. Find the velocity of 3 at that moment.



Sol. $x_1 + x_4 = \ell_1$ (length of first string)

$x_2 - x_4 + x_3 - x_4 = \ell_2$ (length of second string)

$\Rightarrow v_1 + v_4 = 0$ & $v_2 + v_3 - 2v_4 = 0$

$\Rightarrow v_2 + v_3 + 2v_1 = 0$

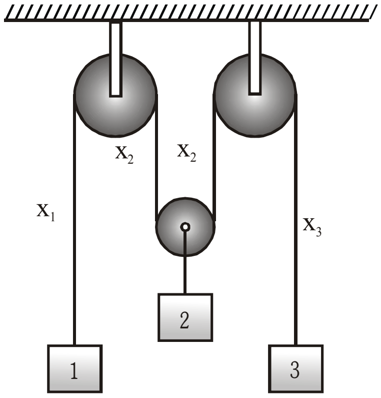
Taking upward direction as positive

$v_1 = v_2 = 1$

so $1 + v_3 + 2 \times 1 = 0 \Rightarrow v_3 = -3\ ms^{-1}$

i.e. velocity of block 3 is 3 ms^{-1} downwards.

Ex. Find the relation between acceleration of blocks a_1, a_2 and a_3.

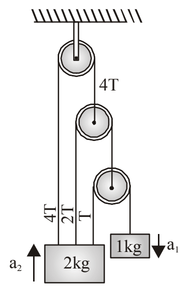


Sol. $x_1 + 2\ x_2 + x_3 = \ell$

$v_1 + 2v_2 + v_3 = 0$

$a_1 + 2a_2 + a_3 = 0$

Ex. Using contraint equation. Find the relation between a_1 and a_2.



Sol. For this system $a_1T = a_2(4T+2T+T) \Rightarrow a_1=7a_2$