6. If $a_1, a_2, a_3 \dots$ is an arithmetic progression with common difference 1 and $a_1 + a_2 + a_3 + \cdots + a_{98} = 137$ then the value of $a_2 + a_4 + a_6 + \cdots + a_{98}$ is
(a) 100 (b) 85 (c) 93 (c) 107

7. If the sides of a right-angled triangle are in AP then the sines of the acute angles are
(a) $\frac{3}{5}, \frac{4}{5}$ (b) $\frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}}$
(c) $\frac{1}{2}, \frac{\sqrt{3}}{2}$ (d) none of these

8. If $x, 2y, 3z$ are in AP, where the distinct numbers x, y, z are in GP, then the common ratio of the GP is
(a) 3 (b) 1/3 (c) 2 (d) 1/2

9. If $n!$, $3 \times n!$, and $(n + 1)!$ are in GP, then $n!$, $5 \times n!$, and $(n + 1)!$ are in
(a) AP (b) GP
(c) HP (d) none of these

10. In a GP of even number of terms, the sum of all terms is 5 times the sum of the odd terms. The common ratio of the GP is
(a) –4/5 (b) 1/5
(c) 4 (d) none of these

11. Let $a = 1\,1\,1 \cdots 1$ (55 digits), $b = 1 + 10 + 10^2 + \cdots + 10^4$, $c = 1 + 10^5 + 10^{10} + 10^{15} + \cdots + 10^{50}$, then
(a) $a = b + c$ (b) $a = bc$
(c) $b = ac$ (d) $c = ab$

12. The arithmetic mean of two positive numbers is 6 and their geometric mean G and harmonic mean H satisfy the relation $G^2 + 3H = 48$, then product of the two numbers is
(a) 36 (b) 32 (c) 20 (d) 64

13. If $x = 1 + a + a^2 + \cdots +, \infty$, $y = 1 + b + b^2 + \cdots + \infty$, and $z = 1 + \frac{a}{b} + \left(\frac{a}{b}\right)^2 + \left(\frac{a}{b}\right)^3 \cdots \infty$, then
(a) $xy + yz + zx = x$ (b) $xy + yz - zx = y$
(c) $xy - yz + zx = x$ (d) none of these

14. If a, b, c are in AP, a, b, d are in GP then $a, a - b, d - c$ are in
(a) AP (b) GP
(c) HP (d) none of these

15. The sum: $(x + 2)^{n-1} + (x + 2)^{n-2}(x + 1) + (x + 2)^{n-3}(x + 1)^2 + \cdots + (x + 1)^{n-1}$
(a) $(x + 2)^{n-2} - (x + 1)^n$
(b) $(x + 2)^{n-1} - (x + 1)^{n-1}$
(c) $(x + 2)^n - (x + 1)^n$
(d) none of these

16. The harmonic mean of the roots of the equation $(5+\sqrt{2})x^2 - (4+\sqrt{5})x + 8 + 2\sqrt{5} = 0$ is
(a) 2 (b) 4 (c) 6 (d) 8

17. Consider an infinite geometric series with the first term a and common ratio r. If its sum is 4 and the second term is 3/4, then
(a) $a = \frac{4}{7}, r = \frac{3}{7}$ (b) $a = 2, r = \frac{3}{8}$
(c) $a = \frac{3}{2}, r = \frac{1}{2}$ (d) $a = 3$, $r = \frac{1}{4}$

18. If the sum of the first $2n$ terms of the AP 2, 5, 8, …, is equal to the sum of the first n terms of AP 57, 59, 61, …, then n equals
(a) 10 (b) 12 (c) 11 (d) 13

19. An infinite GP has the first term as a and sum 5, then
(a) $a < -10$
(b) $-10 < a < 10$
(c) $0 < a < 10$ and $a \neq 5$
(d) $a > 10$

20. Fifth term of a GP is 2, then the product of its first 9 terms is
(a) 256 (b) 512
(c) 1024 (d) none of these

21. Sum of infinite GP is 20 and sum of their squares is 100. The common ratio of GP is
(a) 5 (b) $\frac{3}{5}$ (c) $\frac{8}{5}$ (d) $\frac{1}{5}$

22. $1^3 - 2^3 + 3^3 - 4^3 + \cdots + 9^3 =$
(a) 425 (b) –425 (c) 475 (d) –475

23. If $f(x)$ is a polynomial function of the second degree. If $f(1) = f(-1)$ and a, b, c are in AP, then $f'(a), f'(b) f'(c)$ are in
(a) GP
(b) HP
(c) arithmetic–geometric progression
(d) AP

24. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the equation
(a) $x^2 + 18x + 16 = 0$ (b) $x^2 - 18x + 16 = 0$
(c) $x^2 + 18x - 16 = 0$ (d) $x^2 - 18x - 16 = 0$

25. If $x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} c^n$, where a, b, c are in AP and $|a| < 1, |b| < 1, |c| < 1$, then x, y, z are in
(a) AP (b) GP (c) HP
(d) arithmetic–geometric progression

26. Let $a_1, a_2, a_3, \ldots,$ be terms of an AP If $\frac{a_1 + a_2 + \cdots + a_p}{a_1 + a_2 + \cdots + a_q} = \frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals
(a) 41/11 (b) 7/2 (c) 2/7 (d) 11/41

27. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals
(a) $\frac{1}{2}\sqrt{5}$ (b) $\sqrt{2}$
(c) $\frac{1}{2}(\sqrt{5}-1)$ (d) $\frac{1}{2}-\sqrt{5}$

28. The difference between the sum of the first k terms of the series $1^3 + 2^3 + 3^3 + \cdots + n^3$ and the sum of the first k terms of $1 + 2 + 3 + \cdots + n$ is 1980. The value of k is
(a) 8 (b) 9 (c) 10 (d) 11

29. For $a, b > 0$, let $5a - b$. $2a + b$, $a + 2b$ be in AP and $(b + 1)^2$, $ab + 1$, $(a - 1)^2$ are in GP, then the value of $(a^{-1} + b^{-1})$ is
(a) 4 (b) 5 (c) 8 (d) 6

30. The value of $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{n}{5^n}\right)$ equals
(a) $\frac{5}{12}$ (b) $\frac{5}{24}$ (c) $\frac{5}{36}$ (d) $\frac{5}{16}$

31. If the first term of a GP $a_1, a_2, a_3, \ldots$ is unity such that the quantity $4a_2 + 5a_3$ has the least value then the sum to infinite number of terms of the sequence
(a) cannot be found out as diverges
(b) $\frac{5}{3}$ (c) $\frac{7}{5}$ (d) $\frac{5}{7}$

32. Let $a + ar_1 + ar_1^2 + \cdots + \infty$ and $a + ar_2 + ar_2^2 + \cdots + \infty$ be two infinite series of positive numbers with the same first term. The sum of the first series is r_1 and the sum of the second series is r_2. The value of $(r_1 + r_2)$ is
(a) 1/2 (b) 1
(c) $\frac{\sqrt{5}+1}{2}$ (d) 2

33. The value of x that satisfies the relation $x = 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots \infty$
(a) 2 cos 36° (b) 2 cos 144°
(c) 2 sin 18° (d) none of these

34. The value of n where n is a positive integer satisfying the equation $2 + (6.\ 2^2 - 4.\ 2) + (6.\ 3^2 - 4.3) + \cdots + (6.\ n^2 - 4 \cdot n) = 140$
(a) 3 (b) 4 (c) 5 (d) 7

35. Given the sequence $10^{\frac{1}{11}}, 10^{\frac{2}{11}}, 10^{\frac{3}{11}}, \ldots, 10^{\frac{n}{11}}$. The smallest value of $n \in N$ such that the product of the first n terms of the sequence exceeds one lac is
(a) 9 (b) 10 (c) 11 (d) 12

36. Consider the sequence $S = 7 + 13 + 21 + 31 + \cdots + T_n$. the value of T_{70} is
(a) 5013 (b) 5050 (c) 5113 (d) 5213

37. In an AP the series containing 99 terms, the sum of all the odd-numbered terms is 2550. the sum of all the 99 terms of the AP is
(a) 5010 (b) 5050 (c) 5100 (d) 5049

38. For which positive integers n is the ratio, $\sum_{k=1}^{n} k^2 \Big/ \sum_{k=1}^{n} k$ an integer?
(a) odd n only
(b) even n only
(c) $n = 1 + 6k$ only, where $k \geq 0$ and $k \in I$
(d) $n = 1 + 3k$, integer $k \geq 0$

39. The maximum value of the sum of the AP 50, 48, 46, 44, …, is
(a) 648 (b) 450 (c) 558 (d) 650

40. The coefficient of x^{49} in the product $(x - 1)(x - 3) \ldots (x - 99)$ is
(a) -99^2 (b) 1
(c) -2500 (d) none of these

41. If $\Sigma\, n = 55$, then $\Sigma\, n^2$ is equal to
(a) 385 (b) 506 (c) 1115 (d) 3025

42. The next term of the GP $x, x^2 + 2, x^3 + 10$ is
(a) 0 (b) 6 (c) $\frac{729}{16}$ (d) 54

43. Number of increasing geometrical progression(s) with first term unity, such that any three consecutive terms, on doubling the middle become in AP is
(a) 0 (b) 1 (c) 2 (d) infinity

44. Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - 2_{k-2}$ for $k = 3, 4, \ldots, 11$. If $\frac{a_1^2 + a_2^2 + \cdots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \cdots + a_{11}}{11}$ is equal to
(a) 1 (b) 2
(c) 0 (d) none of these

45. The sum of the first 24 terms of the AP a_1, a_2, a_3, ..., if it is known that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$ is
(a) 1800 (b) 1350
(c) 900 (c) none of these

46. The fourth, seventh, and the last term of a GP are 10, 80, and 2560 respectively. Then the first term and number of terms in the GP are
(a) $\frac{5}{4}$, 10 (b) $\frac{5}{4}$, 12
(c) $\frac{4}{5}$, 10 (d) $\frac{4}{5}$, 12

47. Two numbers whose arithmetic mean is 34 and the geometric mean is 16, then the ratio of number is
(a) 5 or 1/5 (b) 4 or 1/4
(c) 2 or 1/2 (d) 16 or 1/16

48. If the first two terms of an HP are 2/5 and 12/13 respectively. Then the largest term is
(a) 2nd term (b) 6th term
(c) 4th term (d) 6th term

49. The sum of the series $\frac{1}{3\times7}+\frac{1}{7\times11}+\frac{1}{11\times15}+$ ... to ∞ is
(a) 1/3 (b) 1/6 (c) 1/9 (d) 1/12

50. In a GP, $T_2 + T_5 = 216$ and $T_4 : T_6 = 1 : 4$ and all terms are integers, then its first term is
(a) 16 (b) 14
(c) 12 (d) none of these

51. The sum of all the products of the first 10 natural numbers taken two at a time is
(a) 1320 (b) 1500
(c) 3300 (d) none of these

52. Sum of the series $S = \frac{4}{7}-\frac{5}{7^2}+\frac{4}{7^3}-\frac{5}{7^4}+\cdots$ up to ∞ is
(a) 23/24 (b) 23/48 (c) 4/43 (d) 3/11

53. If t_n denotes the nth term of the series 2 + 3 + 6 + 11 + 18 + ⋯ then t_{50} is
(a) $49^2 - 1$ (b) 49^2 (c) $50^2 + 1$(d) $49^2 + 2$

54. If a, x, b are in AP, a, y, b are in GP, and a, z, b are in HP such that $x = 9z$ and $a > 0$, $b > 0$ then
(a) $|y| = 3z$ (b) $x = 3|y|$
(c) $2y = x + z$ (d) none of these

55. Let sum of the first three terms of GP with real terms is 13/12 and their product is –1. Then the sum of infinite terms of GP is
(a) $\frac{4}{7}$ (b) $\frac{2}{3}$
(c) $\frac{7}{8}$ (d) none of these

56. If $\log_2 4$, $\log_{\sqrt{2}} 8$, and $\log_3 9^{k-1}$ are consecutive terms of a geometric sequence, then the number of integers that satisfy the system of inequalities $x^2 - x > 6$ and $|x| < k^2$ is
(a) 193 (b) 194 (c) 195 (d) 196

57. If the first term of a GP a_1, a_2, a_3, ... is unity such that the quantity $4a_2 + 5a_3$ has the least value then the sum to infinite number of terms of the sequence
(a) cannot be found out as diverges
(b) $\frac{5}{3}$ (c) $\frac{7}{5}$ (d) $\frac{5}{7}$

SOLUTIONS

1.(d) Given sum of 99 terms is 198

$\Rightarrow \quad \frac{99}{2}[T_1+T_{99}] = 198$

$\Rightarrow \quad \frac{T_1+T_{99}}{2} = 2$

Now T_1, T_{50}, T_{99} are in AP

$\Rightarrow \quad T_{50} = 2$

2.(a) If there are 9 odd terms then T_2, T_4, T_6, T_8 will be $\frac{9-1}{2} = 4$ in number.

Hence S_1 is an AP of n terms, but S_2 is an AP of $\frac{n-1}{2}$ terms with common difference $2d$

$$S_1 = \frac{n}{2}[T_1+T_n] \quad \text{(i)}$$

$$S_2 = \frac{1}{2}\left(\frac{n-1}{2}\right)[T_2+T_{n-1}]$$

$$= \frac{1}{2}\left(\frac{n-1}{2}\right)[T_1+T_n]$$

$$\therefore \quad \frac{S_1}{S_2} = \frac{2n}{n-1}$$

3.(b) According to the given condition,

$$\frac{\frac{n}{2}\left[2a+(n-1)d\right]}{\frac{n}{2}\left[2A+(n-1)D\right]} = \frac{5n+3}{3n+4}$$

$$\Rightarrow \quad \frac{a+\frac{n-1}{2}d}{A+\frac{n-1}{2}D} = \frac{5n+3}{3n+4}$$

Now, put $\frac{n-1}{2} = 16$ or $n = 33$.

4.(a) Since $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in AP

$\Rightarrow \quad \frac{a(b+c)}{bc}, \frac{b(c+a)}{ca}, \frac{c(a+b)}{ab}$ are in AP

$\Rightarrow \quad a^2(b+c), b^2(c+a), c^2(a+b)$ are in AP (multiplying each term by abc)

$\Rightarrow \quad b^2(c+a) - a^2(b+c) = c^2(a+b) - b^2(c+a)$

$\Rightarrow \quad b^2c - a^2c + b^2a - a^2b = c^2a - b^2a + c^2b - b^2c$

$\Rightarrow \quad c(b^2 - a^2) + ab(b-a) = a(c^2 - b^2) + cb(c-b)$

$\Rightarrow \quad (b-a)(c(b+a)+ab) = (c-b)[a(c+b)+cb]$

$\Rightarrow \quad (b-a)(bc+ca+ab) = (c-b)(ac+ab+bc)$

$\Rightarrow \quad b - a = c - b$

$\Rightarrow \quad a, b, c$ are in AP

5.(c) Series 17, 21, 25, …, 417 has common difference 4.

Series 16, 21, 26, …, 466 has common difference 5.

Hence the series with common terms has common difference as LCM of 4 and 5 which is 20.

The first common term is 21.

Hence the series is 21, 41, 61, …, 411 which has 20 terms.

6.(c) $d = 1$, let $a_1 = a$

$\therefore \quad a_1 + a_2 + \cdots + a_{98} = 137$

$\Rightarrow \quad \frac{98}{2}\left[2a+97\right] = 137 \qquad$ (i)

$\Rightarrow \quad 2a + 97 = \frac{137}{49} \qquad$ (ii)

To find $a_2 + a_4 + \cdots + a_{98}$ (49 terms)

$$= \frac{49}{2}\left[a_2 + a_{98}\right] = \frac{49}{2}\left[(a+1) + a + 97\right]$$

$$= \frac{49}{2}\left[2a+97+1\right] = \frac{49}{2}\left[\frac{137}{49}+1\right]$$

$$= \frac{137}{2} + \frac{49}{2} = \frac{186}{2} = 93$$

7.(a) Let $\angle C = 90°$ being greatest and $B = 90° - A$. The sides are $a - d$, a and $a + d$. We have $(a+d)^2 = (a-d)^2 + a^2$ (using Pythagoras theorem)

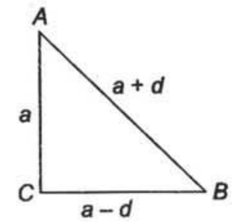

Fig. 1

$\therefore \quad 4ad - a^2 = 0 \Rightarrow a = 4d$

Hence the sides are $3d, 4d, 5d$.

Clearly, $\sin A = \frac{BC}{AB} = \frac{a-d}{a+d} = \frac{3d}{5d} = \frac{3}{5}$

$$\sin B = \frac{AC}{AB} = \frac{a}{a+d} = \frac{4d}{5d} = \frac{4}{5}$$

8.(b) Given $x, 2y, 3z$ are in AP $\Rightarrow 4y = x + 3z$

Also x, y, z are in GP $\Rightarrow y = xr, z = xr^2$, where r is common ratio.

$\Rightarrow \quad 4xr = x + 3xr^2 \Rightarrow 3r^2 - 4r + 1 = 0$

$\Rightarrow \quad (3r-1)(r-1) = 0 \Rightarrow r = 1/3$ or $r = 1$

9.(a) $n!$, $3 \times n!$, and $(n+1)!$ are in GP

$\Rightarrow \quad (3 \times n!)^2 = n! \times (n+1)!$

$\Rightarrow \quad 9 = n + 1 \Rightarrow n = 8$

$\Rightarrow \quad n!, 5 \times n!$, and $(n+1)!$ are $8!, 5 \times 8!$, and $9!$

Clearly $8! + 9! = 8!(1+9) = 2(5 \times 8!)$

Hence $n!, 5 \times n!$, and $(n+1)!$ are in AP.

10.(c) Consider the GP of $2n$ terms: $a, ar, ar^2, \ldots, ar^{2n-1}$

Now sum of all terms $= \frac{a(r^{2n}-1)}{r-1}$, sum of odd terms $= \frac{a((r^2)^n - 1)}{r^2 - 1}$

According to the question

$$\frac{a(r^{2n}-1)}{r-1} = 5\frac{a((r^2)^n-1)}{r^2-1}$$

$\Rightarrow \quad 1 = \frac{5}{r+1} \Rightarrow r = 4$

11.(b) $a = 1 + 10 + 10^2 + \cdots + 10^{54}$

$$= \frac{10^{55}-1}{10-1} = \frac{10^{55}-1}{10^5-1} \cdot \frac{10^5-1}{10-1} = bc$$

12.(b) $a + b = 12,\ ab + \dfrac{6ab}{a+b} = 48,\ ab + \dfrac{ab}{2} = 48$

$\therefore \quad ab = 32$

13.(c) $x = 1 + a + a^2 + \cdots \infty \Rightarrow x = \dfrac{1}{1-a}$

$\Rightarrow \quad a = \dfrac{x-1}{x}$, similarly $y = \dfrac{1}{1-b}$

$\Rightarrow \quad b = \dfrac{y-1}{y}$

Then $\quad z = 1 + \dfrac{a}{b} + \left(\dfrac{a}{b}\right)^2 + \left(\dfrac{a}{b}\right)^3 \cdots \infty$

$$= \frac{1}{1-\frac{a}{b}} = \frac{b}{b-a} = \frac{\frac{y-1}{y}}{\frac{y-1}{y} - \frac{x-1}{x}}$$

$$= \frac{xy-x}{y-x} \Rightarrow xy + xz - yz = x$$

14.(b) According to the given conditions $2b = a + c$ and $b^2 = ad$ then numbers $a,\ a - b,\ d - c$ becomes a, $a - b,\ \dfrac{b^2}{a} - 2b + a$ or $a,\ a - ar,\ ar^2 - 2ar + a$

Clearly $(a - ar)^2 = a\ (ar^2 - 2ar + a)$. Hence numbers are in GP.

15.(c) Since $\dfrac{(x+2)^n - (x+1)^n}{(x+2) - (x+1)}$

$= (x + 2)^{n-1} + (x + 2)^{n-2}(x + 1) + (x + 2)^{n-3}(x + 1)^2 + \cdots + (x + 1)^{n-1}$

$\therefore$ Required sum $= (x + 2)^n - (x + 1)^n$

$[\because (x + 2) - (x + 1) = 1]$

16.(b) Harmonic mean H or roots α and β is

$$H = \frac{2\alpha\beta}{\alpha+\beta} = \frac{2\frac{8+2\sqrt{5}}{5+\sqrt{2}}}{\frac{4+\sqrt{5}}{5+\sqrt{2}}} = 4$$

17.(d) Sum = 4 and the second term = 3/4, it is given that the first term is a and common ratio r.

$\Rightarrow \quad \dfrac{a}{1-r} = 4$ and $ar = 3/4 \Rightarrow r = \dfrac{3}{4a}$

Therefore, $\dfrac{a}{1-\frac{3}{4a}} = 4 \Rightarrow \dfrac{4a^2}{4a-3} = 4$

$\Rightarrow \quad a^2 - 4a + 3 = 0$

$\Rightarrow \quad (a - 1)(a - 3) = 0$

$\Rightarrow \quad a = 1$ or 3

When $a = 1,\ r = 3/4$ and when $a = 3,\ r = 1/4$

18.(c) Given $2 + 5 + 8 + \cdots 2n$ terms

$= 57 + 59 + 61 + \cdots n$ terms

$\Rightarrow \quad \dfrac{2n}{2}[4 + (2n - 1)3] = \dfrac{n}{2}[114 + (n - 1)2]$

$\Rightarrow \quad 6n + 1 = n + 56$

$\Rightarrow \quad 5n = 55$

$\Rightarrow \quad n = 11$

19.(c) $S_\infty = \dfrac{a}{1-r} = 5$ (given)

$\Rightarrow \quad r = \dfrac{5-a}{5}$

But $\quad 0 < |r| < 1$

$\Rightarrow \quad 0 < \left|\dfrac{5-a}{5}\right| < 1$

$\Rightarrow \quad -1 < \dfrac{5-a}{5} < 1$ and $a \neq 5$

$\Rightarrow \quad -5 < 5 - a < 5$ and $a \neq 5$

$\Rightarrow \quad -10 < -a < 0$ and $a \neq 5$

$\Rightarrow \quad 10 > a > 0$ and $a \neq 5$

$\Rightarrow \quad 0 < a < 10$ and $a \neq 5$

20.(b) $t_5 = ar^4 = 2$

Product of its first 9 terms

$= a(ar)(ar^2) \cdots (ar^8)$

$= a^9 r^{1+2+\cdots+8}$

$= a^9 r^{\frac{8}{2}(1+8)} = a^9 r^{36}$

$= (ar^4)^9 = 2^9 = 512$

21.(b) $a + ar + ar^2 + \cdots$ to $\infty = 20 \Rightarrow \dfrac{a}{1-r} = 20 \qquad$ (i)

$a^2 + a^2r^2 + a^2r^4 + \cdots$ to $\infty = 100$

$\Rightarrow \quad \dfrac{a^2}{1-r^2} = 100 \qquad$ (ii)

Squaring (i), $\dfrac{a^2}{(1-r)^2} = 400 \qquad$ (iii)

Dividing (iii) by (ii), $\dfrac{\frac{a^2}{(1-r)^2}}{\frac{a^2}{1-r^2}} = \dfrac{400}{100}$

$\Rightarrow \quad \dfrac{1-r^2}{(1-r)^2} = 4 \Rightarrow \dfrac{1+r}{1-r} = 4$

$\Rightarrow \quad 1 + r = 4 - 4r$

$\Rightarrow \quad 5r = 3$

$\Rightarrow \quad r = \dfrac{3}{5}$

22.(a) Required sum

$= (1^3 + 2^3 + \cdots + 9^3) - 2(2^3 + 4^3 + 6^3 + 8^3)$

$= \left(\dfrac{9(9+1)}{2}\right)^2 - 2 \times 2^3\,(1 + 2^3 + 3^3 + 4^3)$

$= (45)^2 - 16\left(\dfrac{4(4+1)}{2}\right)^2$

$= 2025 - 16 \times 4 \times 25 = 2025 - 1600 = 425$

23.(d) Let $f(x) = ax^2 + bx + c\ (a \neq 0)$.

Now $f(1) = f(-1) \Rightarrow a + b + c = a - b + c$

$\Rightarrow \quad b = 0$

$\therefore \quad f(x) = ax^2 + c \ \therefore f'(x) = 2ax$

$\therefore \quad f'(a) = 2a^2, f'(b) = 2ab, f'(c) = 2ac$

Since a, b, c are in AP [given]

$\therefore \quad f'(a), f'(b)$ are $f'(c)$ are also in AP

24.(b) Let "a" and "b" be two numbers

By the question, $A = 9$ and $G = 4$

$\Rightarrow \quad \dfrac{a+b}{2} = 9$ and $\sqrt{ab} = 4$

$\Rightarrow \quad a + b = 18$ and $ab = 16$

$\therefore$ The required equation is $x^2 - (a+b)x + ab = 0$

$\Rightarrow \quad x^2 - 18x + 16 = 0$

25.(c) Here $x = \dfrac{1}{1-a}, y = \dfrac{1}{1-b}, z = \dfrac{1}{1-c}$

Since a, b, c are in AP,

$\therefore \quad 1 - a, 1 - b, 1 - c$ are in AP

$\Rightarrow \quad \dfrac{1}{1-a}, \dfrac{1}{1-b}, \dfrac{1}{1-c}$ are in HP

$\Rightarrow \quad x, y, z$ are in HP

26.(d) Given, $a_1, a_2, a_3, \cdots$ be terms of AP

$\therefore \quad \dfrac{a_1 + a_2 + \cdots + a_p}{a_1 + a_2 + \cdots + a_q} = \dfrac{p^2}{q^2}$

$\Rightarrow \quad \dfrac{\frac{p}{2}[2a_1 + (p-1)d]}{\frac{q}{2}[2a_1 + (q-1)d]} = \dfrac{p^2}{q^2}$

$\Rightarrow \quad \dfrac{2a_1 + (p-1)d}{2a_1 + (q-1)d} = \dfrac{p}{q}$

$\Rightarrow \quad [2a_1 + (p-1)d]\, q = [2a_1 + (q-1)d]p$

$\Rightarrow \quad 2a_1\,(q - p) = d[(q-1)p - (p-1)q]$

$\Rightarrow \quad 2a_1\,(q - p) = d(q - p)$

$\Rightarrow \quad 2a_1 = d$

$\therefore \quad \dfrac{a_6}{a_{21}} = \dfrac{a_1 + 5d}{a_1 + 20d} = \dfrac{a_1 + 10a_1}{a_1 + 40a_1} = \dfrac{11}{41}$

27.(c) Here $a = ar + ar^2$

$\Rightarrow \quad r^2 + r - 1 = 0$

$\Rightarrow \quad r = \dfrac{-1 \pm \sqrt{1+4}}{2}$

$= \dfrac{-1+\sqrt{5}}{2} = \dfrac{\sqrt{5}-1}{2}$ [as $r > 0$]

28.(b) $\left[\dfrac{k(k+1)}{2}\right]^2 - \dfrac{k(k+1)}{2} = 1980$

$\dfrac{k(k+1)}{2}\left[\dfrac{k(k+1)}{2} - 1\right] = 1980$

$k(k + 1)\,(k^2 + k - 2) = 1980 \times 4$

$(k - 1)\,k(k + 1)\,(k + 2) = 8.\ 9.\ 10.\ 11$

$\therefore \quad k - 1 = 8 \Rightarrow k = 9$

29.(d) For the given AP, we have

$2(2a + b) = (5a - b) + (a + 2b)$

$\Rightarrow \quad b = 2a$ (i)

Also for the given GP, we have

$(ab + 1)^2 = (a - 1)^2\,(b + 1)^2$ (ii)

$\therefore$ Putting $b = 2a$ from (i) in (ii), we get

$a = 0, -2$ or $\dfrac{1}{4}$

But $a > 0$, so $a = \dfrac{1}{4}$ and $b = 2a = \dfrac{1}{2}$

Hence $(a^{-1} + b^{-1}) = 2 + 4 = 6$

30.(c) We have $S = \dfrac{1}{5} - \dfrac{2}{5^2} + \dfrac{3}{5^3} - \dfrac{4}{5^4} + \dfrac{5}{5^5} \cdots \infty$

and $\dfrac{S}{5} = \dfrac{1}{5^2} - \dfrac{2}{5^3} + \dfrac{3}{5^4} - \dfrac{4}{5^5} + \cdots \infty$ (adding)

$\dfrac{6S}{5} = \dfrac{1}{5} - \dfrac{1}{5^2} + \dfrac{1}{5^3} - \dfrac{1}{5^4} + \dfrac{1}{5^5} \cdots \infty$

$\dfrac{6S}{5} = \dfrac{\frac{1}{5}}{1 + \frac{1}{5}} = \left(\dfrac{1}{5}\right)\left(\dfrac{5}{6}\right) \Rightarrow S = \dfrac{5}{36}$

31.(d) Let common ratio of GP be r.

$\therefore$ Series is $1, r, r^2, r^3, \ldots$

Let $y = 4a_2 + 5a_3 = 4r + 5r^2$ is least for y_{least}

$r = \dfrac{-2}{5}$

$\therefore \quad S_0 = \dfrac{1}{1+\frac{2}{5}} = \dfrac{5}{7}$

32.(b) $\dfrac{a}{1-r_1} = r_1$ and $\dfrac{a}{1-r_2} = r_2$

Hence r_1 and r_2 are the roots of $\dfrac{a}{1-r} = r$

$\Rightarrow \quad r^2 - r + a = 0$

$\Rightarrow \quad r_1 + r_2 = 1$

33.(c) $x = \dfrac{1}{1+x} \Rightarrow x^2 + x - 1 = 0$

$x = \dfrac{-1+\sqrt{5}}{2}$ or $\dfrac{-1-\sqrt{5}}{2}$ (rejected)

Hence $x = \left(\dfrac{\sqrt{5}-1}{4}\right)\cdot 2 = 2\sin 18°$

34.(b) $S = 1 + 6(2^2 + 3^2 + 4^2 + \cdots + n^2) + 1 - 4(2 + 3 + 4 + \cdots + n)$

$$6\sum_{r=1}^{n} r^2 - 4\sum_{r=1}^{n} r = 140$$

$\Rightarrow \quad n(n+1)(2n+1) - 2n(n+1) = 140$

$\Rightarrow \quad n(n+1)(2n-1) = 4.5.7$

$\Rightarrow \quad n = 4$

35.(c) $10^{\frac{n(n+1)}{22}} > 10^5 \Rightarrow \dfrac{n(n+1)}{22} > 5$

$n^2 + n > 110$

$(n + 11)(n - 10) > 0$

$\Rightarrow \quad n > 10 \Rightarrow n = 11$

36.(c) $S = 7 + 13 + 21 + 31 + \cdots + T_n$

$S = 7 + 13 + 21 + \cdots + T_{n-1} + T_n$

$T_n = 7 + 6 + 8 + 10 + \cdots + (T_n - T_{n-1})$

$= 7 + \dfrac{n-1}{2}[12 + (n-2)2]$

$= 7 + \dfrac{n-1}{2}[6 + n - 2]$

$= 7 + (n-1)(n+4)$

$= 7 + n^2 + 3n - 4$

$T_n = n^2 + 3n + 3$

$T_{70} = 4900 + 210 + 3 = 5113$

37.(d) AP is $a, (a+d), (a+2d) \cdots (a+98d)$

Sum of odd terms = 2550

$\underbrace{a + (a+2d) + (a+4d) + \cdots + (a+98d)}_{\text{50 terms}} = 2550$

$\dfrac{50}{2}[2a + 98d] = 2550$ or $50[a + 49d]$

$= 2550$ or $a + 49d = 51$

This is the 50th terms of AP. Hence

$S_{99} = 51 \times 99 = 5049$

38.(d) $\dfrac{n(n+1)(2n+1)2}{6\cdot n(n+1)}$ must be an integer

$\dfrac{2n+1}{3}$ must be an integer

$\Rightarrow \quad (2n + 1)$ is divisible by 3

$\Rightarrow \quad n \in 1, 4, 7, 10, \ldots, n$ is of the form of $(3k + 1)$, $k \geq 0$, $k \in I$

39.(d) For maximum value of the given sequence to n terms, when the nth term is either zero or the smaller positive number of the sequence

i.e., $50 + (n-1)(-2) = 0$

$\Rightarrow \quad n = 26$

$\therefore \quad S_{26} = \dfrac{26}{2}(50+0) = 26 \times 25 = 650$

40.(c) Here, number of factors = 50

$\therefore$ The coefficient of $x^{49} = -1 - 3 - 5 - \cdots - 99$

$= -\dfrac{50}{2}(1+99)$

$= -2500$

41.(a) $\Sigma n = 55 \Rightarrow \dfrac{n(n+1)}{2} = 55$

$\Rightarrow \quad n^2 + n - 110 = 0$

$\therefore \quad n = 10 \qquad (\because n \neq -11)$

$\therefore \quad \Sigma n^2 = \dfrac{n(n+1)(2n+1)}{6}$

$= \dfrac{10 \times 11 \times 21}{6} = 385$

42.(c) $x, x^2 + 2, x^3 + 10$ are in GP

$\Rightarrow \quad x(x^3 + 10) = (x^2 + 2)^2 = x^4 + 4x^2 + 4$

$\Rightarrow \quad 4x^2 - 10x + 4 = 0$

$\Rightarrow \quad 2x^2 - 5x + 2 = 0 \Rightarrow x = 2, \dfrac{1}{2}$

4th term of GP

$= (x^3 + 10)r = (x^3 + 10)\left(\frac{x^2+2}{x}\right)$

$= 54$ when $x = 2$

$= \frac{729}{16}$, when $x = \frac{1}{2}$

43.(b) Let the GP be $1, r, r^2, r^3, \ldots$, where $r > 1$

Let r^m, r^{m+1}, r^{m+2} be three consecutive terms of the GP

Given, $r^m, 2r^{m+1}, r^{m+2}$ are in AP

$\Rightarrow \quad r^m + r^{m+2} = 4r^{m+1} \Rightarrow r^2 - 4r + 1 = 0$

$\Rightarrow \quad r = \frac{4 \pm 2\sqrt{3}}{2} = 2 + \sqrt{3}$ $(\because r > 1)$

44.(c) $a_k = 2a_{k-1} - a_{k-2} \Rightarrow a_1, a_2, \ldots, a_{11}$ are in AP

$\therefore \quad \frac{a_1^2 + a_2^2 + \cdots + a_{11}^2}{11}$

$= \frac{11a^2 + 35 \times 11d^2 + 10ad}{11} = 90$

$\Rightarrow \quad 225 + 35d^2 + 150d = 90$

$35d^2 + 150d + 135 = 0 \Rightarrow d = -3, -9/7$

Given $a_2 < \frac{27}{2} \therefore d = -3$ and $d \neq -9$

$\Rightarrow \quad \frac{a_1 + a_2 + \cdots + a_{11}}{11} = \frac{11}{2}[30 - 10 \times 3] = 0$

45.(c) We know that in an AP the sum of the terms equidistant from the beginning and end is always same and is equal to the sum of first and last term, i.e., $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \cdots$ So, if an AP consists of 24 terms, then $a_1 + a_{24} = a_5 + a_{20} = a_{10} + a_{15}$.

Now, $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

$\Rightarrow \quad (a_1 + a_{24}) + (a_5 + a_{20}) + (a_{10} + a_{15}) = 225$

$\Rightarrow \quad 3(a_1 + a_{24}) = 225$

$\Rightarrow \quad a_1 + a_{24} = \frac{225}{3} = 75 \quad$ (i)

$\Rightarrow \quad S_{24} = \frac{24}{2}(a_1 + a_{24})$

$\left[\text{using } S_n = \frac{n}{2}(a_1 + a_n)\right]$

$= 12\ (75) = 900 \quad$ [using (i)]

46.(b) Let a be the first term and r be the common ratio of the given GP then

$a_4 = 10, a_7 = 80$

$\Rightarrow \quad ar^3 = 10$ and $ar^6 = 80$

$\Rightarrow \quad \frac{ar^6}{ar^3} = \frac{80}{10}$

$\Rightarrow \quad r^3 = 8$

$\Rightarrow \quad r = 2$

Putting $r = 2$ in $ar^3 = 10$, we get $a = \frac{10}{8}$.

Let there be n terms in the given GP, then, $a_n = 2650 \Rightarrow ar^{n-1} = 2560$.

$\Rightarrow \quad \frac{10}{8}(2^{n-1}) = 2560 \Rightarrow 2^{n-4} = 256$

$\Rightarrow \quad 2^{n-4} = 2^8 \Rightarrow n - 4 = 8 \Rightarrow n = 12$

47.(d) Let the two numbers be a and b. Then, AM = 34

$\Rightarrow \quad \frac{a+b}{2} = 34 \Rightarrow a + b = 68$

and GM = 16 $\Rightarrow \sqrt{ab} = 16 \Rightarrow ab = 256 \quad$ (i)

$\therefore \quad (a - b)^2 = (a + b)^2 - 4ab$

$\Rightarrow \quad (a - b)^2 = (68)^2 - 4 \times 256 = 3600$

$\Rightarrow \quad a - b = 60 \quad$ (ii)

On solving (i) and (ii), we get $a = 64$ and $b = 4$. Hence ratio of number is 16 or 1/16.

48.(a) Let the HP be $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \frac{1}{a+3d}$.

Then, $\frac{1}{a} = \frac{2}{5}$ and $\frac{1}{a+d} = \frac{12}{13}$

$\Rightarrow \quad a = \frac{5}{2}$ and $d = -\frac{17}{12}$

Now, nth term of the HP is

$\frac{1}{a + (n-1)d} = \frac{12}{47 - 17n}$

So, the nth term is largest when $47 - 17n$ has the least value

Clearly, $\frac{12}{47 - 17n}$ is least for $n = 2$.

Hence, 2nd term is the largest term.

49.(d) $S = \frac{1}{4}\left[\left(\frac{1}{3} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{11}\right) + \left(\frac{1}{11} - \frac{1}{15}\right) + \cdots \infty\right]$

$S_\infty = \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$

50.(c) $ar(1 + r^3) = 216$ and $\frac{ar^3}{ar^5} = \frac{1}{4}$

$\Rightarrow \quad r^2 = 4 \Rightarrow r = 2, -2$

When $r = 2$ then $2a(9) = 216 \therefore a = 12$ and when $r = -2$, then $-2a(1-8) = 216$

$\therefore \quad a = \dfrac{216}{16} = \dfrac{27}{2}$, which is not an integer

51.(a) To find the required sum, consider

$(1+2+3+\cdots+10)^2 = 1^2+2^2+3^2+\cdots+10^2 + 2$ (required sum)

$\Rightarrow \quad \left[\dfrac{10(11)}{2}\right]^2 = \dfrac{10(11)(21)}{6} + 2S$

where S = required sum

$\Rightarrow \quad \left[\dfrac{10(11)}{2}\right]^2 - \dfrac{10(11)(21)}{6} = 2S$

$\Rightarrow \quad \dfrac{110}{12}[330-42] = 2S$

$\Rightarrow \quad S = 1320$

52.(b) $S = \dfrac{4}{7} - \dfrac{5}{7^2} + \dfrac{4}{7^3} - \dfrac{5}{7^4} + \cdots$

$= \left(\dfrac{4}{7} + \dfrac{4}{7^3} + \cdots\right) - \left(\dfrac{5}{7^2} + \dfrac{5}{7^4} + \cdots\right)$

$= \dfrac{\frac{4}{7}}{1-\frac{1}{7^2}} - \dfrac{\frac{5}{7^2}}{1-\frac{1}{7^2}} = \dfrac{28}{48} - \dfrac{5}{48} = \dfrac{23}{48}$

53.(d) $2 + 3 + 6 + 11 + 18 + \cdots$

$= (0^2+2) + (1^2+2) + (2^2+2) + (3^2+2) + \cdots$

Hence $t_{50} = 49^2 + 2$.

54.(b) x is AM of a and b, y is GM of a and b, z is HM of a and b

$\Rightarrow \quad y^2 = xz$

Also given $x = 9z$

$\Rightarrow \quad x = 9y^2/x \Rightarrow 9y^2 = x^2 \Rightarrow x = 3|y|$

$\Rightarrow \quad x \in (-8, 1] - \{0\}$

55.(a) Let $\dfrac{a}{r}$, a, ar be three terms in GP.

$\therefore$ Product of terms $= a^3 = -1$ (given)

$\Rightarrow \quad a = -1$

Now, sum of terms $= \dfrac{a}{r} + a + ar$

$= \dfrac{13}{12}$ (given)

$\Rightarrow \quad \dfrac{-1}{r} - 1 - r = \dfrac{13}{12}$

$\Rightarrow \quad 12r^2 + 25r + 12 = 0$

$\therefore \quad (3r+4)(4r+3) = 0$

$\Rightarrow \quad r = \dfrac{-4}{3}, \dfrac{-3}{4}$

But $r \neq \dfrac{-4}{3}$

$\therefore \quad S_\infty = \dfrac{a}{1-r} = \dfrac{-1}{1-\left(\frac{-3}{4}\right)} = \dfrac{4}{7}$

56.(a) 2, 6, $2(k-1)$ are in GP $\Rightarrow k = 10$

Now $x^2 - x - 6 > 0$

$\Rightarrow \quad (x-3)(x+2) > 0$ (i)

and $|x| < 100$

$\Rightarrow \quad -100 < x < 100$ (ii)

From (i) and (ii)

$\Rightarrow \quad x \in (-100, -2) \cup (3, 100)$

$\therefore$ Number of integers -99 to -3 and 4 to 99

i.e., $97 + 96 = 193$

57.(d) Let common ratio of GP be r.

$\therefore$ Series is $1, r, r^2, r^3, \ldots$

Let $y = 4a_2 + 5a_3 = 4r + 5r^2$ is least for y_{least}

$r = \dfrac{-2}{5}$

$\therefore \quad S_\infty = \dfrac{1}{1+\frac{2}{5}} = \dfrac{5}{7}$