VECTORS & 3-DIMENSIONAL GEOMETRY

VECTORS

1. VECTORS & THEIR TYPES

Vector quantities are specified by definite magnitudes and definite directions. A vector is generally represented by a directed line segment, say $\overrightarrow{AB}$. A is called the **initial point** and B is called the **terminal point**. The magnitude of vector $\overrightarrow{AB}$ is expressed by $|\overrightarrow{AB}|$. $\overrightarrow{AB}$ may also be represented by $\vec{a}$ and its magnitude by $|\vec{a}|$.

1.1 Zero Vector

A vector of zero magnitude is a zero vector i.e. which has the same initial & terminal point, is called a **Zero Vector**. It is denoted by $\vec{O}$. The direction of zero vector is indeterminate.

1.2 Unit Vector

A vector of unit magnitude in direction of a vector $\vec{a}$ is called unit vector along $\vec{a}$ and is denoted by $\hat{a}$. Symbolically $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$.

1.3 Equal Vector

Two vectors are said to be equal if they have the same magnitude, direction & represent the same physical quantity.

1.4 Collinear Vector

Two vectors are said to be collinear if their directed line segments are parallel disregards to their direction. Collinear vectors are also called **Parallel Vectors** . If they have the same direction they are named as like vectors otherwise unlike vectors.

Symbolically, two non – zero vectors $\vec{a}$ and $\vec{b}$ are collinear if and only, if $\vec{a} = K\vec{b}$, where $K \in R - \{0\}$.

1.5 Coplanar Vector

A given number of vectors are called coplanar if their line segments are all parallel to the same plane. Note that **"Two Vectors Are Always Coplanar".**

1.6 Position Vector of A Point

Let O be a fixed origin, then the position vector (pv) of a point P is the vector $\overrightarrow{OP}$. If $\vec{a}$ and $\vec{b}$ are position vectors of two points A and B, then, $\overrightarrow{AB} = \vec{b} - \vec{a}$ = pv of B – pv of A.

1.7 Section Formula

If $\vec{a}$ and $\vec{b}$ are the position vectors to two points A and B then the p.v. of a point which divides AB in the ratio m : n is given by :

$\vec{r} = \dfrac{n\vec{a} + m\vec{b}}{m+n}$. **Note p.v. of mid point of AB =** $\dfrac{\vec{a}+\vec{b}}{2}$

2. ALGEBRA OF VECTORS

2.1 Addition of vectors

If two vectors $\vec{a}$ & $\vec{b}$ are represented by $\overrightarrow{OA}$ & $\overrightarrow{OB}$, then their sum $\vec{a}+\vec{b}$ is a vector represented by $\overrightarrow{OC}$, where OC is the diagonal of the parallelogram OACB.

- $\vec{a}+\vec{b} = \vec{b}+\vec{a}$ (commutative)
- $(\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})$ (associative)
- $\vec{a}+\vec{0} = \vec{a} = \vec{0}+\vec{a}$
- $\vec{a}+(-\vec{a}) = \vec{0} = (-\vec{a})+\vec{a}$

2.2 Multiplication of a Vector by a scalar

If $\vec{a}$ is a vector & m is a scalar, then m $\vec{a}$ is vector parallel to $\vec{a}$ whose modulus is |m| times that of $\vec{a}$. If m > 0 then $m\vec{a}$ and $\vec{a}$ have same direction and if m < 0, then they have opposite directions. This is multiplication is called **Scalar Multiplication**.

If $\vec{a}$ & $\vec{b}$ are vectors & m, n are scalars, then :

$$m\,(\vec{a}) = (\vec{a})\,m = m\,\vec{a}$$

$$m(n\,\vec{a}) = n(m\,\vec{a}) = (mn)\,\vec{a}$$

$$(m+n)\,\vec{a} = m\vec{a} + n\vec{a}$$

$$m(\vec{a}+\vec{b}) = m\,\vec{a} + m\vec{b}$$

2.3 Subtraction of Vectors

$\vec{a} - \vec{b}$ is defined as addition of vectors $\vec{a}$ and $(-\vec{b})$.

3. TEST OF COLLINEARITY

Three points A, B, C with position vectors $\vec{a}, \vec{b}, \vec{c}$ respectively are collinear, if & only if there exist scalar x, y, z not all zero simultaneously such that; $x\vec{a} + y\vec{b} + z\vec{c} = 0$, where x +y +z =0

4. TEST OF COPLANARITY

Four points A, B, C, D with position vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ respectively are coplanar if and only if there exist scalars x, y, z, w not all zero simultaneously such that $x\vec{a} + y\vec{b} + z\vec{c} + w\vec{d} = 0$ where, $x + y + z + w = 0$

5. SCALAR PRODUCT OF TWO VECTORS

- $\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta \; (0 \le \theta \le \pi)$

  note that if θ is acute then $\vec{a} \cdot \vec{b} > 0$ & if θ is obtuse then $\vec{a} \cdot \vec{b} < 0$

- $\vec{a} \cdot \vec{a} = |\vec{a}|^2 = \vec{a}^2$
- $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ (commutative)
- $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$ (distributive)
- $\vec{a} \cdot \vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b} \qquad (\vec{a} \ne 0, \vec{b} \ne 0)$
- $(m\vec{a}) \cdot \vec{b} = \vec{a} \cdot (m\vec{b}) = m\,(\vec{a} \cdot \vec{b})$, where m is scalar.
- $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$;

  $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

- projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$
- the angle ϕ between $\vec{a}$ & $\vec{b}$ is given by $\cos\phi = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}$

  $0 \le \phi \le \pi$

- if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then

  $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

- $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$
- $|\vec{b}| = \sqrt{b_1^2 + b_2^2 + b_3^2}$

NOTES :

(i) Maximum value of $\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}|$

(ii) Minimum values of $\vec{a} \cdot \vec{b} = -|\vec{a}|\,|\vec{b}|$

(iii) Any vector $\vec{a}$ can be written as,

$$\vec{a} = (\vec{a} \cdot \hat{i})\hat{i} + (\vec{a} \cdot \hat{j})\hat{j} + (\vec{a} \cdot \hat{k})\hat{k}.$$

(iv) A vector in the direction of the bisector of the angle between two vectors $\vec{a}$ & $\vec{b}$ is $\dfrac{\vec{a}}{|\vec{a}|} + \dfrac{\vec{b}}{|\vec{b}|}$.

Hence bisector of the angle between the two vectors $\vec{a}$ & $\vec{b}$ is $\lambda(\hat{a} + \hat{b})$, where $\lambda \in R^+$.

Bisector of the exterior angle between $\vec{a}$ & $\vec{b}$ is $\lambda(\hat{a} - \hat{b})$ $\lambda \in R - \{0\}$.

6. VECTOR PRODUCT OF TWO VECTORS

- If $\vec{a}$ & $\vec{b}$ are two vectors & θ is the angle between them then $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}| \sin\theta\ \hat{n}$, where $\hat{n}$ is the unit vector perpendicular to both $\vec{a}$ & $\vec{b}$ such that $\vec{a}$, $\vec{b}$ & $\hat{n}$ forms a right handed screw system.
- Geometrically $|\vec{a}\times\vec{b}|$ equals area of the parallelogram whose two adjacent sides are represented by $\vec{a}$ & $\vec{b}$
- $\vec{a}\times\vec{b}=\vec{0} \Leftrightarrow \vec{a}$ and $\vec{b}$ are parallel (collinear) (provided $\vec{a}\neq 0, \vec{b}\neq 0$) i.e. $\vec{a} = K\vec{b}$, where K is scalar.
- $\vec{a}\times\vec{b} \neq \vec{b}\times\vec{a}$ (not commutative)
- $(m\vec{a})\times\vec{b} = \vec{a}\times(m\vec{b}) = m(\vec{a}\times\vec{b})$ where m is scalar
- $\vec{a}\times(\vec{b}+\vec{c}) = (\vec{a}\times\vec{b})+(\vec{a}\times\vec{c})$ (distributive)
- $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$
- $\hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$
- If $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ then

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

- Unit vector perpendicular to the plane of $\vec{a}$ & $\vec{b}$ is

$$\hat{n} = \pm\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$$

- A vector of magnitude 'r' & perpendicular to the plane of $\vec{a}$ and $\vec{b}$ is $\pm\dfrac{r(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}$
- If θ is the angle between $\vec{a}$ & $\vec{b}$ then $\sin\theta = \dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$
- If $\vec{a}, \vec{b}$ & $\vec{c}$ are the pv's of 3 points A, B and C then the vector area of triangle ABC = $\frac{1}{2}[\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}]$.

  The point A, B & C are collinear if $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a} = 0$
- **Area of any quadrilateral** whose diagonal vectors are $\vec{d}_1$ & $\vec{d}_2$ is given by $\frac{1}{2}|\vec{d}_1\times\vec{d}_2|$
- **Lagranges Identity** : for any two vector $\vec{a}$ & $\vec{b}$;

$$(\vec{a}\times\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2 = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} \\ \vec{a}\cdot\vec{b} & \vec{b}\cdot\vec{b} \end{vmatrix}$$

7. SCALAR TRIPLE PRODUCT

- The scalar triple product of three vectors $\vec{a}, \vec{b}$ & $\vec{c}$ is defined as :

  $\vec{a}\cdot(\vec{b}\times\vec{c}) = |\vec{a}||\vec{b}||\vec{c}| \sin\theta\cos\phi$ where θ is the angle between $\vec{c}$ & $\vec{b}$ & ϕ is angle between $\vec{a}$ *and* $\vec{b}\times\vec{c}$

  It is also written as $[\vec{a}\ \vec{b}\ \vec{c}]$, spelled as **box product.**
- Scalar triple product geometrically represents the volume of the parallelopiped whose three coterminous edges are represented by $\vec{a}, \vec{b}$ & $\vec{c}$ i.e. $V = [\vec{a}\ \vec{b}\ \vec{c}]$
- In a scalar triple product the position of dot & cross can be interchanged i.e.

  $\vec{a}\cdot(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\cdot\vec{c}$ Also $[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$

  $\vec{a}\cdot(\vec{b}\times\vec{c}) = -\vec{a}\cdot(\vec{c}\times\vec{b})$ i.e. $[\vec{a}\ \vec{b}\ \vec{c}] = -[\vec{a}\ \vec{c}\ \vec{b}]$
- If $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$; $\vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and

  $\vec{c} = c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ then $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$.

In general, if $\vec{a} = a_1\vec{\ell} + a_2\vec{m} + a_3\vec{n}; \vec{b} = b_1\vec{\ell} + b_2\vec{m} + b_3\vec{n}$ and

$\vec{c} = c_1\vec{\ell} + c_2\vec{m} + c_3\vec{n}$ then $\left[\vec{a}\,\vec{b}\,\vec{c}\right] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \left[\vec{\ell}\,\vec{m}\,\vec{n}\right]$;

where $\vec{\ell}, \vec{m}$ & $\vec{n}$ are non - coplanar vectors.

- $\vec{a}, \vec{b}, \vec{c}$ are coplanar $\Leftrightarrow \left[\vec{a}\,\vec{b}\,\vec{c}\right] = 0$.
- Scalar product of three vectors, two of which are equal or parallel is 0.

NOTES :

- If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar then $[\vec{a}\,\vec{b}\,\vec{c}] > 0$ for right handed system & $[\vec{a}\,\vec{b}\,\vec{c}] < 0$ for left handed system.
- $\left[\hat{i}\ \ \hat{j}\ \ \hat{k}\right] = 1$
- $[K\,\vec{a}\,\vec{b}\,\vec{c}] = K\,[\vec{a}\,\vec{b}\,\vec{c}]$
- $[(\vec{a} + \vec{b})\,\vec{c}\,\vec{d}] = [\vec{a}\,\vec{c}\,\vec{d}] + [\vec{b}\,\vec{c}\,\vec{d}]$
- **The volume of the tetrahedron OABC** with O as origin & the pv's of A, B and C being $\vec{a}$, $\vec{b}$ & $\vec{c}$ respectively is given by $V = \frac{1}{6}[\vec{a}\,\vec{b}\,\vec{c}]$
- The position vector of the centroid of a tetrahedron if the pv's of its angular vertices are $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are given by $\frac{1}{4}[\vec{a} + \vec{b} + \vec{c} + \vec{d}]$.

  Note that this is also the point of concurrrency of the lines joining the vertices to the centroids of the opposite faces and is also called the centre of the tetrahedron. In case the tetrahedron is regular it is equidistant from the vertices and the four faces of the tetrahedron.
- **Remember that :** $\left[\left(\vec{a} - \vec{b}\right)\left(\vec{b} - \vec{c}\right)\left(\vec{c} - \vec{a}\right)\right] = 0$ &

  $\left[\left(\vec{a} + \vec{b}\right)\left(\vec{b} + \vec{c}\right)\left(\vec{c} + \vec{a}\right)\right] = 2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

8. VECTOR TRIPLE PRODUCT

Let $\vec{a}, \vec{b}, \vec{c}$ be any three vectors, then the expression $\vec{a} \times (\vec{b} \times \vec{c})$ is vector & is called vector triple product.

Geometrical Interpretation of $\vec{a} \times (\vec{b} \times \vec{c})$

Consider the expression $\vec{a} \times (\vec{b} \times \vec{c})$ which itself is a vector. Since it is a cross product of two vectors $\vec{a}$ and $(\vec{b} \times \vec{c})$. Now $\vec{a} \times (\vec{b} \times \vec{c})$ is a vector perpendicular to the plane containing $\vec{a}$ and $(\vec{b} \times \vec{c})$ but $\vec{b} \times \vec{c}$ is a vector perpendicular to the plane containing $\vec{b}$ & $\vec{c}$, therefore $\vec{a} \times (\vec{b} \times \vec{c})$ is a vector lying in the plane of $\vec{b}$ & $\vec{c}$ and perpendicular to $\vec{a}$. Hence we can express $\vec{a} \times (\vec{b} \times \vec{c})$ in terms of $\vec{b}$ & $\vec{c}$ i.e. $\vec{a} \times (\vec{b} \times \vec{c}) = x\vec{b} + y\vec{c}$ where x and y are scalars.

- $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\,\vec{b} - (\vec{a} \cdot \vec{b})\,\vec{c}$

  $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\,\vec{b} - (\vec{b} \cdot \vec{c})\,\vec{a}$
- $(\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a} \times (\vec{b} \times \vec{c})$

9. LINEAR COMBINATIONS

Given a finite set of vectors $\vec{a}, \vec{b}, \vec{c}, \ldots..$ then the vector $\vec{r} = x\vec{a} + y\,\vec{b} + z\vec{c} + \ldots.$ is called a linear combination of $\vec{a}, \vec{b}, \vec{c}, \ldots..$ for any x, y, z..... $\in$R. We have the following results :

(a) If $\vec{a}, \vec{b}$ are non zero, non-collinear vectors then $x\vec{a} + y\vec{b} = x'\,\vec{a} + y'\vec{b} \Rightarrow x = x'\,; y = y'$

(b) **Fundamental Theorem in plane :** Let $\vec{a}, \vec{b}$ be non-zero, non- collinear vectors. Then any vector $\vec{r}$ coplanar with $\vec{a}, \vec{b}$ can be expressed uniquely as a linear combination of $\vec{a}, \vec{b}$ i.e. There exist some unique x, y $\in$ R such that $x\vec{a} + y\vec{b} = \vec{r}$

(c) If $\vec{a}, \vec{b}, \vec{c}$ are non-zero, non-coplanar vectors then :

$x\vec{a} + y\vec{b} + z\vec{c} = x'\vec{a} + y'\vec{b} + z'\vec{c} \Rightarrow x = x', y = y', z = z'$

(d) **Fundamental Theorem in Space :** Let $\vec{a}, \vec{b}, \vec{c}$ be non-zero, non- coplanar vectors in space. Then any vector $\vec{r}$ can be uniquely expressed as a linear combination of $\vec{a}, \vec{b}, \vec{c}$ i.e. There exist some unique x, y, z $\in$ R such that $x\vec{a}+y\vec{b}+z\vec{c}=\vec{r}$.

(e) If $\vec{x}_1, \vec{x}_2$ .... $\vec{x}_n$ are n non zero vectors & k_1, k_2,............k_n are n scalars & if the linear combination $k_1\vec{x}_1+k_2\vec{x}_2+......k_n\vec{x}_n=0 \Rightarrow k_1=0, k_2=0,.......k_n=0$ then we say that vectors $\vec{x}_1, \vec{x}_2$ .... $\vec{x}_n$ are **Linearly Independent Vectors.**

(f) If $\vec{x}_1, \vec{x}_2$ .... $\vec{x}_n$ are not Linearly Independent then they are said to be Linearly Dependent vectors i.e. if $k_1\vec{x}_1+k_2\vec{x}_2+......k_n\vec{x}_n=0$ & if there exists at least one $k_r \neq 0$ then $\vec{x}_1, \vec{x}_2$ .... $\vec{x}_n$ are said to be **Linearly Dependent.**

NOTES :

- If $\vec{a} = 3\hat{i}+2\hat{j}+5\hat{k}$ then $\vec{a}$ is expressed as a Linear Combination of vectors $\hat{i}, \hat{j}, \hat{k}$. Also $\vec{a}$, $\hat{i}, \hat{j}, \hat{k}$ form a linearly dependent set of vectors. In general, **every set of four vectors is a linearly dependent system.**
- $\hat{i}, \hat{j}, \hat{k}$ are Linearly Independent set of vectors. For

  $K_1\hat{i}+K_2\hat{j}+K_3\hat{k}=0 \Rightarrow K_1=0=K_2=K_3$
- Two vectors $\vec{a}$ & $\vec{b}$ are linearly dependent $\Rightarrow \vec{a}$ is parallel to $\vec{b}$ i.e. $\vec{a}\times\vec{b}=0 \Rightarrow$ linear dependence of $\vec{a}$ & $\vec{b}$. Conversely if $\vec{a}\times\vec{b}\neq 0$ then $\vec{a}$ & $\vec{b}$ are linearly independent.
- **If three vectors $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent, then they are coplanar i.e.** $[\vec{a}, \vec{b}, \vec{c}]=0$ conversely, if $[\vec{a}, \vec{b}, \vec{c}]\neq 0$, then the vectors are linearly independent.

10. RECIPROCAL SYSTEM OF VECTORS

If $\vec{a}, \vec{b}, \vec{c}$ & $\vec{a}', \vec{b}', \vec{c}'$ are two sets of non-coplanar vectors such that $\vec{a}\cdot\vec{a}'=\vec{b}\cdot\vec{b}'=\vec{c}\cdot\vec{c}'=1$ then the two systems are called **Reciprocal System of vectors.**

$$\vec{a}'=\frac{\vec{b}\times\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]};\ \vec{b}'=\frac{\vec{c}\times\vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]};\ \vec{c}'=\frac{\vec{a}\times\vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$$

3 – DIMENSIONAL GEOMETRY

1. INTRODUCTION

There are infinite number of points in space. We want to identify each and every point of space with the help of three mutually perpendicular coordinate axes OX, OY and OZ.

1.1 Axes

Three mutually perpendicular lines OX, OY, OZ are considered as three axes.

1.2 Coordinate planes

Plane formed with the help of x and y axes is known as x–y plane similarly plane formed with y and z axes is known as y – z plane and plane formed with z and x axis z – x plane.

1.3 Coordinate of a Point

Consider any point P on the space and drop a perpendicular from that point to x – y plane; then the algebraic length of this perpendicular is considered as z–coordinate; and from the foot of the perpendicular drop perpendiculars to x and y axes the algebric length of these perpendiculars are considered as y and x coordinates respectively.

1.4 Vector Representation of a Point in Space

If the coordinates of a point P in space are (x, y, z) then the position vector of the point P with respect to the same origin is $x\hat{i}+y\hat{j}+z\hat{k}$.

2. DISTANCE FORMULA

Distance between any two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given as $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$

Vector method

We know that if position vector of two points A and B are given as $\overrightarrow{OA}$ and $\overrightarrow{OB}$ then

$$|AB| = |\overrightarrow{OB} - \overrightarrow{OA}|$$

$$\Rightarrow \quad |AB| = (x_2 i + y_2 j + z_2 k) - (x_1 i + y_1 j + z_1 k)|$$

$$\Rightarrow \quad |AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

2.1 Distance of a Point P From Coordinate Axes

Let PA, PB and PC be the distances of the point P(x, y, z) from the coordinate axes OX, OY and OZ respectively then

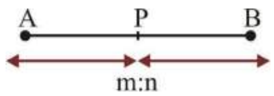
$$PA = \sqrt{y^2 + z^2},\ PB = \sqrt{z^2 + x^2},\ PC = \sqrt{x^2 + y^2}$$

3. SECTION FORMULA

(i) Internal Division

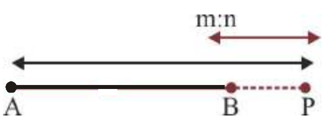
If point P divides the distance between the points A (x_1, y_1, z_1) and B (x_2, y_2, z_2) in the ratio of m : n (internally) then the coordinates of P are given by

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$



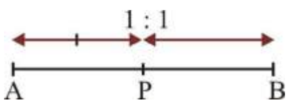
(ii) External division

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$$



(iii) Mid point

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

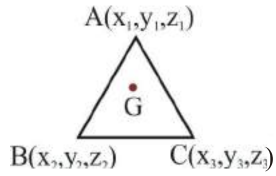


NOTES :

All these formulae are very much similar to two dimensional coordinate geometry.

3.1 Centroid of a Triangle

$$G \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$



3.2. In - Centre of Triangle ABC

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}, \frac{az_1 + bz_2 + cz_3}{a+b+c}\right)$$

Where $|AB| = a, |BC| = b, |CA| = c$

3.3 Centroid of a Tetrahedron

If A (x_1, y_1, z_1), B (x_2, y_2, z_2), C (x_3, y_3, z_3), and D (x_4, y_4, z_4) are the vertices of a tetrahedron then the coordinates of its centroid (G) are given by

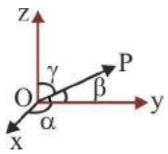
$$\left(\frac{\sum x_i}{4}, \frac{\sum y_i}{4}, \frac{\sum z_i}{4}\right)$$

4. RELATION BETWEEN TWO LINES

Two lines in the space may be coplanar and may be non- coplanar. Non- coplanar lines are called skew lines if they never intersect each other. Two parallel lines are also non intersecting lines but they are coplanar. Whether two lines are intersecting or non intersecting, the angle between them can be obtained.

5. DIRECTION COSINES AND DIRECTION RATIOS

(i) **Direction cosines :** Let α, β, γ be the angles which a directed line makes with the positive directions of the axes of x, y and z respectively, then cos α, cosβ, cosγ are called the direction cosines of the line. The direction cosines are usually denoted by (l, m, n).



Thus $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$.

(ii) If l, m, n, be the direction cosines of a lines, then $l^2 + m^2 + n^2 = 1$

(iii) **Direction ratios :** Let a, b, c be proportional to the direction cosines, l, m, n, then a, b, c are called the direction ratios.

If a, b, c are the direction ratio of any line L then $a\hat{i} + b\hat{j} + c\hat{k}$ will be a vector parallel to the line L.

If l, m, n are direction cosines of line L then $\ell\hat{i} + m\hat{j} + n\hat{k}$ is a unit vector parallel to the line L.

(iv) If l, m, n be the direction cosines and a, b, c be the direction ratios of a vector, then

$$\left(\ell = \frac{a}{\sqrt{a^2+b^2+c^2}}, m = \frac{b}{\sqrt{a^2+b^2+c^2}}, n\frac{c}{\sqrt{a^2+b^2+c^2}}\right)$$

$$\text{or } \left(\ell = \frac{-a}{\sqrt{a^2+b^2+c^2}}, m = \frac{-b}{\sqrt{a^2+b^2+c^2}}, n = \frac{-c}{\sqrt{a^2+b^2+c^2}}\right)$$

(v) If OP = r, when O is the origin and the direction cosines of OP are l, m, n then the coordinates of P are (lr, mr, nr).

If direction cosines of the line AB are l, m, n, $|AB| = r$, and the coordinates of A are (x_1, y_1, z_1) then the coordinates of B are given as $(x_1 + rl, y_1 + rm, z_1 + rn)$

(vi) If the coordinates of P and Q are (x_1, y_1, z_1) and (x_2, y_2, z_2) then the direction ratios of line PQ are $a = x_2 - x_1$, $b = y_2 - y_1$ and $c = z_2 - z_1$ and the direction cosines of line PQ are $l = \frac{x_2 - x_1}{|PQ|}$, $m = \frac{y_2 - y_1}{|PQ|}$ and $n = \frac{z_2 - z_1}{|PQ|}$

(vii) Direction cosines of axes : Since the positive x–axis makes angles 0°, 90°, 90° with axes of x, y and z respectively, therefore

Direction cosines of x–axis are (1, 0, 0)

Direction cosines of y–axis are (0, 1, 0)

Direction cosines of z–axis are (0, 0, 1)

6. ANGLE BETWEEN TWO LINES

If two lines having direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 respectively then we can consider two vectors parallel to the lines as $a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and $a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ and the angle between them can be given as

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}.$$

(i) The lines will be perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

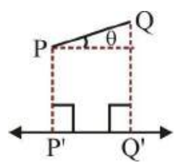
(ii) The lines will be parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(iii) Two parallel lines have same direction cosines i.e. $l_1 = l_2$, $m_1 = m_2$, $n_1 = n_2$

7. PROJECTION OF A LINE SEGMENT ON A LINE

(i) If the coordinates of P and Q are (x_1, y_1, z_1) and (x_2, y_2, z_2) then the projection of the line segments PQ on a line having direction cosines l, m, n is

$|l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)|$



(ii) Vector form : projection of a vector $\vec{a}$ on another vector $\vec{b}$ is $\vec{a}\cdot\hat{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$ In the above case we can consider $\overrightarrow{PQ}$ as $(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$ in place of $\vec{a}$ and $l\hat{i} + m\hat{j} + n\hat{k}$ in place of $\vec{b}$.

(iii) $l|\vec{r}|$, $m|\vec{r}|$, and $n|\vec{r}|$ are the projection of $\vec{r}$ in x, y and z axes.

(iv) $\vec{r} = |\vec{r}|(l\hat{i} + m\hat{j} + n\hat{k})$

8. EQUATION OF A LINE

(i) The equation of a line passing through the point (x_1, y_1, z_1) and having direction ratios a, b, c is

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=r$$ This form is called symmetric form. A general point on the line is given by (x_1+ar, y_1+br, z_1+cr).

(ii) Vector equation : Vector equation of a straight line passing through a fixed point with position vector $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\vec{r}=\vec{a}+\lambda\vec{b}$ where λ is a scalar.

(iii) The equation of the line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

(iv) Vector equation of a straight line passing through two points with position vectors $\vec{a}$ and $\vec{b}$ is $\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$.

(v) Reduction of cartesion form of equation of a line to vector form and vice versa

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

$$\Leftrightarrow \vec{r}=(x_1\hat{i}+y\hat{j}+z_1\hat{k})+\lambda(a\hat{i}+b\hat{j}+c\hat{k}).$$

NOTES :

Straight lines parallel to co–ordinate axes :

	Straight lines	Equation
(i)	Through origin	$y=mx, z=nx$
(ii)	x–axis	$y=0, z=0$
(iii)	y–axis	$x=0, z=0$
(iv)	z–axis	$x=0, y=0$
(v)	Parallel to x–axis	$y=p, z=q$
(vi)	Parallel to y–axis	$x=h, z=q$
(vii)	Parallel to z–axis	$x=h, y=p$

9. SKEW LINES

(i) The straight lines which are not parallel and non–coplanar i.e. non–intersecting are called skew lines.

If $\Delta=\begin{vmatrix}\alpha'-\alpha & \beta'-\beta & \gamma'-\gamma\\ \ell & m & n\\ \ell' & m' & n'\end{vmatrix}\neq 0$, then the lines are skew.

(ii) Vector Form : For lines $\vec{a}_1+\lambda\vec{b}_1$ and $\vec{a}_2+\lambda\vec{b}_2$ to be skew

$(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)\neq 0$ or $[\vec{b}_1\ \vec{b}_2\ (\vec{a}_2-\vec{a}_1)]\neq 0.$

(iii) Shortest distance between the two parallel lines

$\vec{r}=\vec{a}_1+\lambda\vec{b}$ and $\vec{r}=\vec{a}_2+\mu\vec{b}$ is $d=\left|\frac{(\vec{a}_2-\vec{a}_1)\times\vec{b}}{|\vec{b}|}\right|$.

10. PLANE

If a line joining any two points on a surface lies completely on it then the surface is a plane.

OR

If a line joining any two points on a surface is perpendicular to some fixed straight line, then the surface is called a plane. This fixed line is called the normal to the plane.

10.1 Equation of a Plane

(i) Normal form of the equation of a plane is $lx+my+nz=p$ where l, m, n are the direction cosines of the normal to the plane and p is the distance of the plane from the origin.

(ii) General form : $ax+by+cz+d=0$ is the equation of a plane where a, b, c are the direction ratios of the normal to the plane.

(iii) The equation of a plane passing through the point (x_1,y_1,z_1) is given by $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$ where a, b, c are the direction ratios of the normal to the plane.

(iv) Plane through three points : The equation of the plane through three non–collinear points (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) is $\begin{vmatrix}x & y & z & 1\\ x_1 & y_1 & z_1 & 1\\ x_2 & y_2 & z_2 & 1\\ x_3 & y_3 & z_3 & 1\end{vmatrix}=0$

(v) Intercept Form : The equation of a plane cutting intercepts a, b, c on the axes is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

(vi) Vector form : The equation of a plane passing through a point having position vector $\vec{a}$ and normal to vector $\vec{n}$ is

$(\vec{r}-\vec{a})\cdot\vec{n}=0$ or $\vec{r}\cdot\vec{n}=\vec{a}\cdot\vec{n}$

NOTES :

(a) Vector equation of a plane normal to unit vector $\hat{n}$ and at a distance d from the origin is $\vec{r}\cdot\hat{n}=d$

(b) **Planes parallel to the coordinate planes**

(i) Equation of yz–plane is $x=0$

(ii) Equation of xz–plane is $y=0$

(iii) Equation of xy–plane is $z=0$

(c) **Planes parallel to the axes :**

If $a=0$, the plane is parallel to x–axis i.e. equation of the plane parallel to the x–axis is $by+cz+d=0$.

Similarly, equation of planes parallel to y–axis and parallel to z–axis are $ax+cz+d=0$ and $ax+by+d=0$ respectively.

(d) **Plane through origin :** Equation of plane passing through origin is $ax+by+cz=0$.

(e) **Transformation of the equation of a plane to the normal form :** To reduce any equation $ax+by+cz-d=0$ to the normal form, first write the constant term on the right hand side and make it positive, then divided each term by $\sqrt{a^2+b^2+c^2}$, where a, b, c are coefficients of x, y and z respectively e.g.

$$\frac{ax}{\pm\sqrt{a^2+b^2+c^2}}+\frac{by}{\pm\sqrt{a^2+b^2+c^2}}+\frac{cz}{\pm\sqrt{a^2+b^2+c^2}}$$

$$=\frac{d}{\pm\sqrt{a^2+b^2+c^2}}$$

Where (+) sign is to be taken if $d>0$ and (–) sign is to be taken if $d<0$.

(f) Any plane parallel to the given plane $ax+by+cz+d=0$ is $ax+by+cz+\lambda=0$. Distance between two parallel planes $ax+by+cz+d_1=0$ and $ax+dy+xz+d_2=0$ is given as $\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$

(g) **Equation of a plane passing through a given point and parallel to the given vectors :** The equation of a plane passing through a point having position vector $\vec{a}$ and parallel to $\vec{b}$ and $\vec{c}$ is $\vec{r}=\vec{a}+\lambda\vec{b}+\mu\vec{c}$ (parametric form where λ and μ are scalars).

or $\vec{r}\cdot(\vec{b}\times\vec{c})=\vec{a}\cdot(\vec{b}\times\vec{c})$ (non- parametric form)

11 ANGLE BETWEEN TWO PLANES

(i) Consider two planes $ax+by+cz+d=0$ and $a'x+b'y+c'z+d'=0$. Angle between these planes is the angle between their normals. Since direction ratios of their normals are (a, b, c) and (a′, b′, c′) respectively, hence the angle θ between them is given by

$$\cos\theta=\frac{aa'+bb'+cc'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}$$

Planes are perpendicular if $aa'+bb'+cc'=0$ and planes are parallel if $\frac{a}{a'}=\frac{b}{b'}=\frac{c}{c'}$

(ii) The angle θ between the planes $\vec{r}\cdot\vec{n}_1=d_1$, $\vec{r}\cdot\vec{n}_2=d_2$ is given by $\cos\theta=\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$ Planes are perpendicular if $\vec{n}_1\cdot\vec{n}_2=0$ and Planes are parallel if $\vec{n}_1=\lambda\vec{n}_2$.

12. A PLANE AND A POINT

(i) Distance of the point (x',y',z') from the plane $ax+by+az+d=0$ is given by $\frac{|ax'+by'+cz'+d|}{\sqrt{a^2+b^2+c^2}}$

(ii) The length of the perpendicular from a point having position vector $\vec{a}$ to a plane $\vec{r}\cdot\vec{n}=d$ is given by

$$p=\frac{|\vec{a}\cdot\vec{n}-d|}{|\vec{n}|}.$$

13. ANGLE BISECTORS

(i) The equations of the planes bisecting the angle between two given planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ are

$$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}}=\pm\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}$$

(ii) **Equation of bisector of the angle containing origin :** First make both the constant terms positive. Then the positive sign in $\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}}=\pm\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}$ gives the bisector of the angle which contains the origin.

(iii) **Bisector of acute/obtuse angle :** First make both the constant terms positive. Then

$a_1a_2+b_1b_2+c_1c_2>0$

$\Rightarrow$ origin lies in obtuse angle

$a_1a_2+b_1b_2+c_1c_2<0$

$\Rightarrow$ origin lies in acute angle

14. FAMILY OF PLANES

(i) The equation of any plane passing through the line of intersection of non–parallel planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is given by $a_1x+b_1y+c_1z+d_1+\lambda(a_2x+b_2y+c_2z+d_2)=0$

(ii) The equation of plane passing through the intersection of the planes $\vec{r}\cdot\vec{n}_1=d_1$ and $\vec{r}\cdot\vec{n}_2=d_2$ is $\vec{r}\cdot(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ where λ is an arbitrary scalar

(iii) **Plane through a given line :** Equation of any plane through the line in symmetrical form.

$\frac{x-x_1}{\ell}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ is $A(x-x_1)+B(y-y_1)+C(z-z_1)=0$ where $Al+Bm+Cn=0$

NOTES :

A straight line in space is characterised by the intersection of two planes which are not parallel and therefore, the equation of a straight line is a solution of the system constituted by the equations of the two planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$. This form is also known as non–symmetrical form.

15. ANGLE BETWEEN A PLANE AND A LINE

(i) If θ is the angle between the line $\frac{x-x_1}{\ell}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ and the plane $ax+by+cz+d=0$, then

$$\sin\theta=\left[\frac{a\ell+bm+cn}{\sqrt{(a^2+b^2+c^2)}\sqrt{\ell^2+m^2+n^2}}\right]$$

(ii) Vector form : If θ is the angle between a line $\vec{r}=(\vec{a}+\lambda\vec{b})$ and $\vec{r}\cdot\vec{n}=d$ then $\sin\theta=\left[\frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}\right]$.

(iii) Condition for perpendicularity $\frac{\ell}{a}=\frac{m}{b}=\frac{n}{c}$, $\vec{b}\times\vec{n}=0$

(iv) Condition for parallelism $al+bm+cn=0$, $\vec{b}\cdot\vec{n}=0$

16. CONDITION FOR A LINE TO LIE IN A PLANE

(i) Cartesian form : Line $\frac{x-x_1}{\ell}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ would lie in a plane $ax+by+cz+d=0$ if $ax_1+by_1+cz_1+d=0$ and $al+bm+cn=0$.

(ii) Vector form : Line $\vec{r}=\vec{a}+\lambda\vec{b}$ would lie in the plane $\vec{r}\cdot\vec{n}=d$ if $\vec{b}\cdot\vec{n}=0$ and $\vec{a}\cdot\vec{n}=d$

17. COPLANER LINES

(i) If the given lines are $\frac{x-\alpha}{\ell}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ and $\frac{x-\alpha'}{\ell'}=\frac{y-\beta'}{m'}=\frac{z-\gamma'}{n'}$, then the condition for intersection/coplanarity is $\begin{vmatrix}\alpha-\alpha' & \beta-\beta' & \gamma-\gamma'\\ \ell & m & n\\ \ell' & m' & n'\end{vmatrix}=0$ and the plane containing the above two lines is

$$\begin{vmatrix}x-\alpha & y-\beta & z-\gamma\\ \ell & m & n\\ \ell' & m' & n'\end{vmatrix}=0$$

(ii) Condition of coplanarity if both the lines are in general assymetric form :-

$ax+by+cz+d=0=a'x+b'y+c'z+d'$ and

$\alpha x+\beta y+\gamma z+\delta=0=\alpha' x+\beta' y+\gamma' z+\delta'$

They are coplanar if $\begin{vmatrix} a & b & c & d \\ a' & b' & c' & d' \\ \alpha & \beta & \gamma & \delta \\ \alpha' & \beta' & \gamma' & \delta' \end{vmatrix}=0$

18. COPLANARITY OF FOUR POINTS

If the points $A(x_1\, y_1\, z_1)$, $B(x_2\, y_2\, z_2)$, $C(x_3\, y_3\, z_3)$ and $D(x_4\, y_4\, z_4)$ are coplaner then

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \\ x_4-x_1 & y_4-y_1 & z_4-z_1 \end{vmatrix}=0$$

Similarly, in vector method the points A $(\vec{r}_1)$, $B(\vec{r}_2)$, $C(\vec{r}_3)$ and $D(\vec{r}_4)$ are coplanar if there exists 4 scalars a, b, c and d such that

$a+b+c+d=0$ and $a\vec{r}_1+b\vec{r}_2+c\vec{r}_3+d\vec{r}_4=0$

19. SIDES OF A PLANE

(i) A plane divides the three dimensional space in two parts. Two points $A(x_1\, y_1\, z_1)$ and $B(x_2\, y_2\, z_2)$ are on the same side of the plane $ax+by+cz+d=0$ if $ax_1+by_1+cz_1+d$ and $ax_2+by_2+cz_2+d$ are both positive or both negative and are on opposite sides of plane if these values are opposite in signs.

(ii) A plane $ax+by+cz+d=0$ divides the line segment joining (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio $\left(-\dfrac{ax_1+by_1+cz_1+d}{ax_2+by_2+cz_2+d}\right)$

(iii) The xy–plane divides the line segment joining the point (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio $-\dfrac{z_1}{z_2}$. Similarly yz – plane in the ratio of $-\dfrac{x_1}{x_2}$ and zx– plane in the ratio of $-\dfrac{y_1}{y_2}$

20. FOOT OF PERPENDICULAR AND IMAGE OF A POINT W.R.T. A PLANE

Let $P(x_1, y_1, z_1)$ be a given point and $ax+by+cz+d=0$ be a given plane. Let (x', y', z') be the image point. Then,

$$\frac{x'-x_1}{a}=\frac{y'-y_1}{b}=\frac{z'-z_1}{c}=\frac{-2(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$$

And if (x_2, y_2, z_2) is foot of perpendicular from point P on given plane, then

$$\frac{x_2-x_1}{a}=\frac{y_2-y_1}{b}=\frac{z_2-z_1}{c}=\frac{-(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$$

SOLVED EXAMPLES

Example – 1

Show that the point $P(\vec{a}+2\vec{b}+\vec{c})$, $Q(\vec{a}-\vec{b}-\vec{c})$, $R(3\vec{a}+\vec{b}+2\vec{c})$ and $S(5\vec{a}+3\vec{b}+5\vec{c})$ are coplanar given that $\vec{a},\vec{b},\vec{c}$ are non–coplanar.

Sol. To show that P, Q, R, S are coplanar, we will show that $\overrightarrow{PQ}, \overrightarrow{PR}, \overrightarrow{PS}$ are coplanar.

$$\overrightarrow{PQ} = -3\,\vec{b} - 2\vec{c}$$

$$\overrightarrow{PR} = 2\,\vec{a} - \vec{b} + \vec{c}$$

$$\overrightarrow{PS} = 4\,\vec{a} + \vec{b} + 4\vec{c}$$

Let $\overrightarrow{PQ} = \lambda\,\overrightarrow{PR} + \mu\,\overrightarrow{PS}$

$\Rightarrow$ $-3\vec{b} - 2\vec{c} = \lambda\,(2\vec{a} - \vec{b} + \vec{c}) + \mu\,(4\vec{a} + \vec{b} + 4\vec{c})$

$\Rightarrow$ $-3\vec{b} - 2\vec{c} = (2\lambda + 4\mu)\,\vec{a} + (-\lambda + u)\,\vec{b} + (\lambda + 4\mu)\,\vec{c}$

As the vectors, $\vec{a}, \vec{b}, \vec{c}$ are non–coplanar, we can equate their coefficients.

$\Rightarrow$ $0 = 2\lambda + 4\mu$

$\Rightarrow$ $-3 = -\lambda + \mu$

$\Rightarrow$ $-2 = \lambda + 4\mu$

$\lambda = 2$, $\mu = -1$ is the unique solution for the above system of equations.

$\Rightarrow$ $\overrightarrow{PQ} = 2\,\overrightarrow{PR} - \overrightarrow{PS}$

$\overrightarrow{PQ}, \overrightarrow{PR}, \overrightarrow{PS}$ are coplanar because $\overrightarrow{PQ}$ is a linear combination of $\overrightarrow{PR}$ and $\overrightarrow{PS}$.

$\Rightarrow$ the points P, Q, R, S are also coplanar.

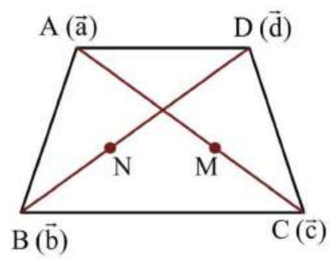
Example – 2

Prove that the segment joining mid–points of the diagonals of a trapezium is parallel to the parallel sides of the trapezium and is equal to half the difference of their lengths.

Sol. Let ABCD be the given trapezium and M, N be the mid points of the diagonals AC and BD.

Let $\vec{a},\vec{b},\vec{c},\vec{d}$ be the position vectors of A, B, C, D respectively.

using section formula, mid points of AC and BD are :



$$\vec{m} = \frac{\vec{a}+\vec{c}}{2}, \quad \vec{n} = \frac{\vec{b}+\vec{d}}{2}$$

$\Rightarrow$ $\overrightarrow{NM} = \vec{m} - \vec{n} = \dfrac{(\vec{a}+\vec{c}) - (\vec{b}+\vec{d})}{2}$

$\Rightarrow$ $\overrightarrow{NM} = \left(\dfrac{\vec{c}-\vec{b}}{2}\right) - \left(\dfrac{\vec{d}-\vec{a}}{2}\right)$

$\Rightarrow$ $\overrightarrow{NM} = 1/2\left(\overrightarrow{BC} - \overrightarrow{AD}\right)$

Let $\overrightarrow{BC} = k\,(\overrightarrow{AD})$

$\Rightarrow$ $\overrightarrow{NM} = 1/2\,(k-1)\,\overrightarrow{AD}$

NM || AD and NM = 1/2 (k – 1) AD

$\Rightarrow$ $NM = \dfrac{k(AD) - AD}{2} = \dfrac{BC - AD}{2}$

$\Rightarrow$ NM is parallel to AD (and BC) and is half the difference of BC and AD.

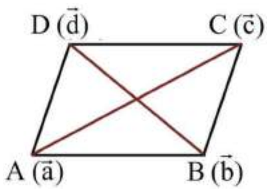
Example – 3

Show that the diagonals of a parallelogram bisect each other.

Sol. Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ be the position vectors of a vertices of a parallelogram ABCD.

AB = DC and AB || DC

(because ABCD is a parallelogram)



$\Rightarrow$ $\overrightarrow{AB} = \overrightarrow{DC}$

$\Rightarrow$ $\vec{b} - \vec{a} = \vec{c} - \vec{d}$

$\Rightarrow$ $(\vec{b} + \vec{d})/2 = (\vec{a} + \vec{c})/2$

$\Rightarrow$ P.V of mid point of BD = P.V of mid point of AC

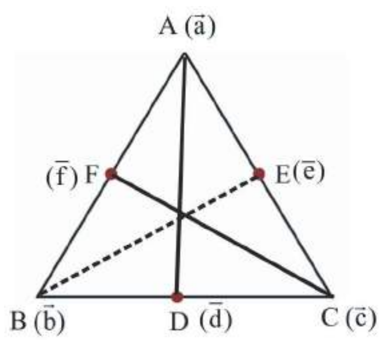
$\Rightarrow$ mid points of BD and AC coincide. Hence AC and BD bisect each other.

Example – 4

Show that the medians of the triangle are concurrent and the point of concurrence divides each median in the ratio 2 : 1.

Sol. Let $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of the vertices of a triangle ABC.

Let D, E, F be the mid–points of sides with P.V $\vec{d}, \vec{e}, \vec{f}$ as shown.



$\Rightarrow$ $\vec{d} = (\vec{b} + \vec{c})/2$ $\Rightarrow$ $2\vec{d} = \vec{b} + \vec{c}$

$\Rightarrow$ $\vec{e} = (\vec{c} + \vec{a})/2$ $\Rightarrow$ $2\vec{e} = \vec{c} + \vec{a}$

$\Rightarrow$ $\vec{f} = (\vec{a} + \vec{b})/2$ $\Rightarrow$ $2\vec{f} = \vec{a} + \vec{b}$

Now try to make the RHS of each equation equal.

$\Rightarrow$ $2\vec{d} + \vec{a} = \vec{a} + \vec{b} + \vec{c}$

$\Rightarrow$ $2\vec{e} + \vec{b} = \vec{b} + \vec{c} + \vec{a}$

$\Rightarrow$ $2\vec{f} + \vec{c} = \vec{c} + \vec{a} + \vec{b}$

$\Rightarrow$ $2\vec{d} + \vec{a} = 2\vec{e} + \vec{b} = 2\vec{f} + \vec{c} = \vec{a} + \vec{b} + \vec{c}$

Note that the sum of scalar coefficients of vectors is equal to 3 in each expression. We divide each term by 3.

$$\Rightarrow \frac{2\vec{d} + \vec{a}}{3} = \frac{2\vec{e} + \vec{b}}{3} = \frac{2\vec{f} + \vec{c}}{3} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$$

$$\Rightarrow \frac{2\vec{d} + \vec{a}}{2+1} = \frac{2\vec{e} + \vec{b}}{2+1} = \frac{2\vec{f} + \vec{c}}{2+1} = \frac{\vec{a} + \vec{b} + \vec{c}}{2+1}$$

$\Rightarrow$ the point G [$(\vec{a} + \vec{b} + \vec{c})/3$] divides AD, BE and CF each internally in ratio 2 : 1. Hence G is the common point of intersection of all medians.

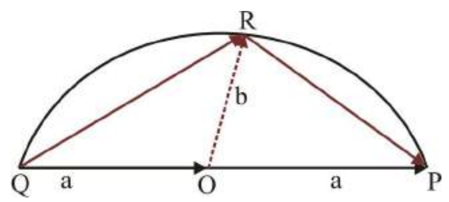
$\Rightarrow$ medians are concurrent and centroid G divides each median in 2 : 1.

$$\text{Centroid } G \equiv \left(\frac{\vec{a} + \vec{b} + \vec{c}}{3}\right)$$

Example – 5

Show that the angle in semi–circle is a right angle.

Sol. Let O be the centre and r be the radius of the semi–circle.



Let $\overrightarrow{OP} = \overrightarrow{QO} = \vec{a}$ and $\overrightarrow{OR} = \vec{b}$

$\Rightarrow$ $\overrightarrow{QR} = \vec{a} + \vec{b}$ and $\overrightarrow{RP} = \vec{a} - \vec{b}$

Now $\overrightarrow{QR} \cdot \overrightarrow{RP} = \left(\vec{a} + \vec{b}\right) \cdot \left(\vec{a} - \vec{b}\right)$

$= a^2 - b^2$

$= a^2 - b^2 = 0$

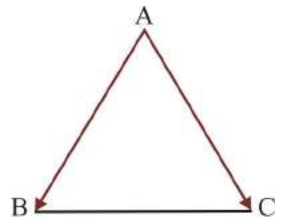
because a = b = radius of the semi–circle.

$\because \overrightarrow{QR} \cdot \overrightarrow{RP} = 0 \Rightarrow \theta = 90°$

Example – 6

The vertices of a triangle are A (2, 3, 0), B (–3, 2, 1) and C (4, –1, 0). Find the area of the triangle ABC and unit vector normal to the plane of this triangle.

Sol. Area of $\Delta ABC = 1/2\ |\overrightarrow{AB} \times \overrightarrow{AC}|$



$\overrightarrow{AB} = (-3-2)\,\hat{i} + (2-3)\,\hat{j} + (1-0)\,\hat{k}$

$\Rightarrow \quad \overrightarrow{AB} = -5\,\hat{i} - \hat{j} + \hat{k}$

and $\quad \overrightarrow{AC} = 2\hat{i} - 4\hat{j} + 0\hat{k}$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & -1 & 1 \\ 2 & -4 & 0 \end{vmatrix} = 4\hat{i} + 2\hat{j} + 22\hat{k}$$

$\Rightarrow \quad$ area of $\Delta ABC = \dfrac{1}{2}\sqrt{16+4+484} = \sqrt{126}$ sq. units

and unit vector normal to the plane of this triangle

$$= \frac{\overrightarrow{AB} \times \overrightarrow{AC}}{|\overrightarrow{AB} \times \overrightarrow{AC}|} = \frac{4\hat{i}+2\hat{j}+22\hat{k}}{2\sqrt{126}}$$

$$= \frac{2\hat{i}+\hat{j}+11\hat{k}}{\sqrt{126}}$$

Example – 7

A line makes angles α, β, γ and δ with the diagonals of a cube. Prove that :

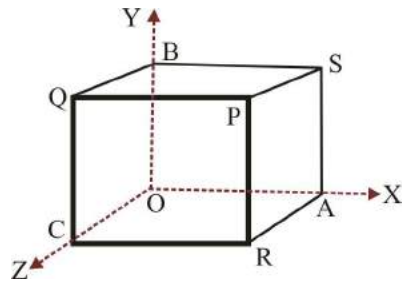
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$$

Sol. Let the origin O be one of the vertices of the cube and OA, OB, OC be the edges through O along the axes so that :

$\overrightarrow{OA} = a\,\hat{i}$, $\quad \overrightarrow{OB} = a\,\hat{j}$, $\quad \overrightarrow{OC} = a\,\hat{k}$

where a is the length of the edge of the cube

Let P, Q, R, S be the other vertices of the cube opposite to O, A, B, C respectively.



Hence the diagonals of the cube are OP, AQ, BR, & CS.

$\overrightarrow{OP} = a\,\hat{i} + a\,\hat{j} + a\,\hat{k}$

$\overrightarrow{AQ} = a\,\hat{i} + a\,\hat{j} + a\,\hat{k}$

$\overrightarrow{BR} = a\,\hat{i} - a\,\hat{j} + a\,\hat{k}$

$\overrightarrow{CS} = a\,\hat{i} + a\,\hat{j} - a\,\hat{k}$

If $\hat{n} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ is the unit vector along the line which makes the angles α, β, γ & δ with diagonals,

$$\cos\alpha = \frac{\hat{n}\,.\,\overrightarrow{OP}}{|\overrightarrow{OP}|\,|\hat{n}|} = \frac{ax+ay+az}{a\sqrt{3}} = \frac{x+y+z}{\sqrt{3}}$$

$$\cos\beta = \frac{-x+y+z}{\sqrt{3}}\ ; \quad \cos\gamma = \frac{x-y+z}{\sqrt{3}}$$

$$\cos\delta = \frac{x+y-z}{\sqrt{3}}$$

$\Rightarrow \quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$

$$\frac{1}{3}\,[(x+y+z)^2 + (-x+y+z)^2 + (x-y+z)^2 + (x+y-z)^2]$$

$$\frac{1}{3}\,4\,(x^2+y^2+z^2) = 4/3 \qquad [\because x^2+y^2+z^2 = 1]$$

Example – 8

Show that : $\hat{i}\times(\vec{a}\times\hat{i}) + \hat{j}\times(\vec{a}\times\hat{j}) + \hat{k}\times(\vec{a}\times\hat{k}) = 2\vec{a}$.

Sol. L.H.S. $= [(\hat{i}\cdot\hat{i})\vec{a} - (\hat{i}\cdot\vec{a})\hat{i}] + [(\hat{j}\cdot\hat{j})\vec{a} - (\hat{j}\cdot\vec{a})\hat{j}] + [(\hat{k}\cdot\hat{k})\,\vec{a}$

$-(\hat{k}\cdot\vec{a})\hat{k}]$

$= \vec{a} + \vec{a} + \vec{a} - [(\hat{i}\cdot\vec{a})\hat{i} + (\hat{j}\cdot\vec{a})\hat{j} + (\hat{k}\cdot\vec{a})\hat{k}]$

$= 3\,\vec{a} - [a_x\hat{i} + a_y\hat{j} + a_z\hat{k}] = 3\,\vec{a} - \vec{a} = 2\vec{a}$ = R.H.S.

Note : It is useful to remember that x–component of $\vec{a} = a_x\hat{i}$ etc ......

Example – 9

Find a vector of magnitude 5 units coplanar with vectors $3\hat{i}-\hat{j}-\hat{k}$ and $\hat{i}+\hat{j}-2\hat{k}$ and perpendicular to the vector $2\hat{i}+2\hat{j}+\hat{k}$.

Sol. Let $\vec{a}=3\hat{i}-\hat{j}-\hat{k}$

$\vec{b}=\hat{i}+\hat{j}-2\hat{k}$

and $\vec{c}=2\hat{i}+2\hat{j}+\hat{k}$

A vector coplanar with $\vec{a}$ and $\vec{b}$ and perpendicualr to $\vec{c}$ can be taken as

$\vec{r}=\ell\vec{c}\times(\vec{a}\times\vec{b})$ where l is a scalar

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & -1\\ 1 & 1 & -2\end{vmatrix}=3\hat{i}+5\hat{j}+4\hat{k}$$

$$\vec{c}\times(\vec{a}\times\vec{b})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 2 & 1\\ 3 & 5 & 4\end{vmatrix}=3\hat{i}-5\hat{j}+4\hat{k}$$

$\Rightarrow$ $\vec{r}=\ell\left(3\hat{i}-5\hat{j}+4\hat{k}\right)$

$|\vec{r}|=|\ell|\sqrt{9+25+15}=5$

$l=\pm\dfrac{5}{5\sqrt{2}}=\pm\dfrac{1}{\sqrt{2}}$

the required vector is $\vec{r}=\pm\dfrac{1}{\sqrt{2}}(3\hat{i}-5\hat{j}+4\hat{k})$

Example – 10

Show that the lines $\vec{r}=3\hat{i}-\hat{j}+\hat{k}+\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$ and $\vec{r}=2\hat{i}+2\hat{j}-2\hat{k}+\mu\left(\hat{i}-\hat{j}+2\hat{k}\right)$ are intersecting and hence find their point of intersection.

Sol. Let $\vec{p}$ be the position vector of their point of intersection.

$\Rightarrow$ $\vec{p}=3\hat{i}-\hat{j}+\hat{k}+\lambda(\hat{i}+\hat{j}+\hat{k})=2\hat{i}+2\hat{j}-2\hat{k}+\mu(\hat{i}-\hat{j}+2\hat{k})$

$\Rightarrow$ $(3+\lambda)\hat{i}+(\lambda-1)\hat{j}+(\lambda+1)\hat{k}=(\mu+2)\hat{i}+(2-\mu)\hat{j}+(2\mu-2)\hat{k}$

$\Rightarrow$ $3+\lambda=\mu+2$ ...(i)

$\Rightarrow$ $\lambda-1=2-\mu$ ...(ii)

$\Rightarrow$ $\lambda+1=2\mu-2$ ...(iii)

The lines are intersecting if these equations are consistent.

from (i) and (ii), we get

$\lambda=1,\ \mu=2$

Substituting these values in (iii), we get

$1+1=2(2)-2$

$\Rightarrow$ $2=2$

$\Rightarrow$ $\lambda=1,\mu=2$

satisfied (iii) also

Hence lines are intersecting and the point of intersection is :

$\vec{p}=3\hat{i}-\hat{j}+\hat{k}+\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$

$=3\hat{i}-\hat{j}+\hat{k}+\left(\hat{i}+\hat{j}+\hat{k}\right)$

$=4\hat{i}+2\hat{k}$

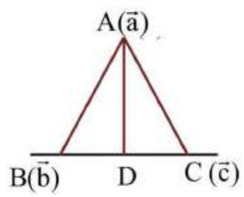
$\Rightarrow$ the coordinates of this point are (4, 0, 2).

Example – 11

The vertices of a triangle ABC are A (1, 0, 2), B (–2, 1, 3) and C (2, –1, 1). Find the equation of the line BC, the foot of the perpendicular from A to BC and the length of the perpendicular.

Sol. A vector parallel to BC is

$BC=\vec{c}-\vec{b}=4\hat{i}-2\hat{j}-2\hat{k}$



$\Rightarrow$ the equation of BC is : $\vec{r}=\vec{b}+t(\vec{c}-\vec{b})$

$\Rightarrow$ $\vec{r}=-2\hat{i}+\hat{j}+3\hat{k}+t\left(4\hat{i}-2\hat{j}-2\hat{k}\right)$

Let position vector of D be

$\vec{d} = -2\hat{i} + \hat{j} + 3\hat{k} + t\left(4\hat{i} - 2\hat{j} - 2\hat{k}\right)$

because D lies on line BC.

Now $\overrightarrow{AD} \perp \overrightarrow{BC}$

$\Rightarrow \quad \overrightarrow{AD} \,.\, \overrightarrow{BC} = 0$

$\Rightarrow \quad (\vec{d} - \vec{a}) \cdot (\vec{c} - \vec{b}) = 0$

$\Rightarrow \quad \left[-3\hat{i} + \hat{j} + \hat{k} + t\left(4\hat{i} - 2\hat{j} - 2\hat{k}\right)\right] . \left(4\hat{i} - 2\hat{j} - 2\hat{k}\right) = 0$

$\Rightarrow \quad (4t - 3)\,4 + (1 - 2t)\,(-2) + (1 - 2t)\,(-2) = 0$

$\Rightarrow \quad 24t - 16 = 0$

$\Rightarrow \quad t = 2/3$

$\Rightarrow \quad \vec{d} = -2\hat{i} + \hat{j} + 3\hat{k} + (2/3)\,(4\hat{i} - 2\hat{j} - 2\hat{k})$

$\vec{d} = \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{5}{3}\hat{k}$

$\Rightarrow \quad D \equiv \left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{3}\right)$

$\overrightarrow{AD} = \vec{d} - \vec{a} = \left(\frac{2}{3} - 1\right)\hat{i} - \frac{1}{3}\hat{j} + \left(\frac{5}{3} - 2\right)\hat{k}$

$= -\frac{1}{3}\hat{i} - \frac{1}{3}\hat{j} - \frac{1}{3}\hat{k} \qquad AD = |\overrightarrow{AD}| = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$ units

Example – 12

Find the equation of the plane passing through the points A (2, 1, 3), B (–1, 2, 4) and C (0, 2, 1). Hence find the coordinate of the point of intersection of the plane ABC and the line $\vec{r} = \hat{i} - \hat{j} + \hat{k} + \lambda\,(2\hat{i} + \hat{k})$.

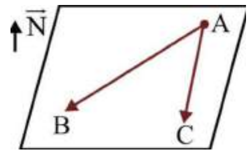
Sol. Let $\vec{N}$ be a vector perpendicular to the plane of ΔABC.

$\Rightarrow \quad \vec{N} = \overrightarrow{AB} \times \overrightarrow{AC}$

$= \left(-3\hat{i} + \hat{j} + \hat{k}\right) \times \left(-2\hat{i} + \hat{j} - 2\hat{k}\right)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 1 \\ -2 & 1 & -2 \end{vmatrix}$$

$= -3\hat{i} - 8\hat{j} - \hat{k}$



$\Rightarrow$ the equation of the plane is

$(\vec{r} - \vec{a}).\vec{N} = 0$ where $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$

$\Rightarrow \quad \vec{r} \,.\, \vec{N} = \vec{a} \,.\, \vec{N}$

$\Rightarrow \quad \vec{r} \,.\left(-3\hat{i} - 8\hat{j} - \hat{k}\right) = -6 - 8 - 3$

$\Rightarrow \quad \vec{r} \,.\left(-3\hat{i} - 8\hat{j} - \hat{k}\right) = -17$

The given line is $\vec{r} = \hat{i} - \hat{j} + \hat{k} + \lambda\,(2\hat{i} + \hat{k})$

To find the point of intersection, we solve these equations simultaneously.

$\Rightarrow \quad \left[\hat{i} - \hat{j} + \hat{k} + \lambda\left(2\hat{i} + \hat{k}\right)\right] . \left(-3\hat{i} - 8\hat{j} - \hat{k}\right) = -17$

$\Rightarrow \quad (2\lambda + 1)\,(-3) + 8 + (\lambda + 1)\,(-1) = -17$

$\Rightarrow \quad \lambda = 3$

$\Rightarrow$ the point of intersection is

$\vec{r} = \hat{i} - \hat{j} + \hat{k} + 3\,(2\hat{i} + \hat{k})$

$\Rightarrow \quad \vec{r} = 7\hat{i} - \hat{j} + 4\hat{k}$

$\Rightarrow$ coordinates are (7, –1, 4)

Example – 13

From the point A (1, 2, 0), perpendicular is drawn to the plane $\vec{r} \,.\left(3\hat{i} - \hat{j} + \hat{k}\right) = 2$ meeting it at the point P. Find the coordinates of point P and the distance AP.

Sol. Let us first find the equation of line AP. As AP is normal to the plane, the vector $\vec{N} = 3\hat{i} - \hat{j} + \hat{k}$ is parallel to AP.

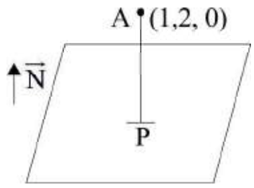
$\Rightarrow$ equation of AP is $\vec{r} = \hat{i} + 2\hat{j} + t\,(3\hat{i} - \hat{j} + \hat{k})$

Now we solve equations of AP and plane to get point P.

$\Rightarrow \quad \left[\hat{i} + 2\hat{j} + t\left(3\hat{i} - \hat{j} + \hat{k}\right)\right] . \left(3\hat{i} - \hat{j} + \hat{k}\right) = 2$

$\Rightarrow \quad (3t + 1)\,3 + (2 - t)\,(-1) + t = 2$

$\Rightarrow \quad t = 1/11$



$\Rightarrow$ point P is $\vec{r} = \hat{i} + 2\hat{j} + 1/11(3\hat{i} - \hat{j} + \hat{k})$

$$\vec{r} = \frac{14}{11}\hat{i} + \frac{21}{11}\hat{j} + \frac{1}{11}\hat{k}$$

$$\Rightarrow \quad P \equiv \left(\frac{14}{11}, \frac{21}{11}, \frac{1}{11}\right)$$

$$AP = \sqrt{\left(\frac{14}{11} - 1\right)^2 + \left(\frac{21}{11} - 2\right)^2 + \left(\frac{1}{11} - 0\right)^2}$$

$$= \frac{1}{\sqrt{11}}$$

Example – 14

The position vectors of the points P and Q are $5\hat{i} + 7\hat{j} - 2\hat{k}$ and $-3\hat{i} + 3\hat{j} + 6\hat{k}$ respectively. The vector $\vec{A} = 3\hat{i} - \hat{j} + \hat{k}$ passes through the point P and the vector $\vec{B} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ passes through the point Q. A third vector $2\hat{i} + 7\hat{j} - 5\hat{k}$ intersects vectors $\vec{A}$ and $\vec{B}$. Find the position vectors of the points of intersection.

Sol. Equation of line AP $\equiv$ $\vec{r} = 5\hat{i} + 7\hat{j} - 2\hat{k} + \lambda_1\left(3\hat{i} - \hat{j} + \hat{k}\right)$

Equation of line BQ $\equiv$

$$\vec{r} = -3\hat{i} + 3\hat{j} + 6\hat{k} + \lambda_3\left(-3\hat{i} + 2\hat{j} + 4\hat{k}\right)$$

Since Point D lies on AP, its position vector can be taken as : $\vec{d} = 5\hat{i} + 7\hat{j} - 2\hat{k} + \lambda_1\left(3\hat{i} - \hat{j} + \hat{k}\right)$

A vector parallel to line CD is $2\hat{i} + 7\hat{j} - 5\hat{k}$

Equation of line CD $\equiv$

$$\vec{r} = 5\hat{i} + 7\hat{j} - 2\hat{k} + \lambda_1\left(3\hat{i} - \hat{j} + \hat{k}\right) + \lambda_2\left(2\hat{i} + 7\hat{j} - 5\hat{k}\right)$$

solve equation of line BQ with equation of line CD to get point of intersection C.

Solve BQ and CD to get :

$$5\hat{i} + 7\hat{j} - 2\hat{k} + \lambda_1\left(3\hat{i} - \hat{j} + \hat{k}\right) + \lambda_2\left(2\hat{i} + 7\hat{j} - 5\hat{k}\right)$$

$$= -3\hat{i} + 3\hat{j} + 6\hat{k} + \lambda_3\left(-3\hat{i} + 2\hat{j} + 4\hat{k}\right)$$

Equating the coefficients of i, j and k, we get

$$5 + 3\lambda_1 + 2\lambda_2 = -3\,(1 + \lambda_3) \quad \text{....(1)}$$

$$7 - \lambda_1 + 7\lambda_2 = 3 + 2\lambda_3 \quad \text{....(2)}$$

$$-2 + \lambda_1 - 5\lambda_2 = 6 + 4\lambda_3 \quad \text{....(3)}$$

Solve equations (1), (2) and (3) to get :

$\lambda_2 = -1, \lambda_3 = -1$ and $\lambda_1 = -1$

$\Rightarrow$ $D \equiv (2, 8, -3)$ and $C \equiv (0, 1, 2)$

Example – 15

The volume of the parallelopiped whose sides are given by $\overrightarrow{OA} = 2\hat{i} - 3\hat{j}, \overrightarrow{OB} = \hat{i} + \hat{j} - \hat{k}, \overrightarrow{OC} = 3\hat{i} - \hat{k}$, is :

(a) $\frac{4}{13}$ (b) 4

(c) $\frac{2}{7}$ (d) none of these

Ans. (b)

Sol. The volume of parallelopiped $= \left[\vec{a}\ \vec{b}\ \vec{c}\right] = \begin{vmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix}$

$$= 2(-1) + 3(-1 + 3) = -2 + 6 = 4$$

Example – 16

The points with position vectors

$60\hat{i} + 3\hat{j}$, $40\hat{i} - 8\hat{j}$, $a\hat{i} - 52\hat{j}$ are collinear, if :

(a) $a = -40$ (b) $a = 40$

(c) $a = 20$ (d) none of these

Ans. (a)

Sol. if $\overrightarrow{AB} = -20\hat{i} - 11\hat{j}$

and $\overrightarrow{AC} = (a - 60)\hat{i} - 55\hat{j}$, then the three points A. B and C will be collinear if

$$\frac{a - 60}{-20} = \frac{-55}{-11} \Rightarrow a = -40$$

Example – 17

A vector $\vec{a}$ has components 2p and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to the new system, $\vec{a}$ has components p + 1 and 1, then :

(a) $p = 0$ (b) $p = 1$ or $p = -\frac{1}{3}$

(c) $p = 1$ or $p = \frac{1}{3}$ (d) $p = 1$ or $p = -1$

Ans. (b)

Sol. Here, $\vec{a} = (2p)\hat{i} + \hat{j}$, when a system is rotated,the new component of $\vec{a}$ *are* $(p+1)$ and 1.

$$i.e.\ \vec{b} = (p+1)\hat{i} + \hat{j} \Rightarrow |\vec{a}|^2 = |\vec{b}|^2$$

$$or\ 4p^2 + 1 = (p+1)^2 + 1 \Rightarrow 4p^2 = p^2 + 2p + 1$$

$$\Rightarrow 3p^2 - 2p - 1 = 0 \Rightarrow (3p+1)(p-1) = 0$$

$$\Rightarrow p = 1, -\frac{1}{3}$$

Example – 18

Let a, b, c be distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then c is:

(a) the Arithmetic Mean of a and b

(b) the Geometric Mean of a and b

(c) the Harmonic Mean of a and b

(d) equal to zero

Ans. (b)

Sol. Since, three vectors are coplanar.

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

Applying $C_1 \to C_1 - C_2$ $\begin{vmatrix} 0 & a & c \\ 1 & 0 & 1 \\ 0 & c & b \end{vmatrix} = 0$

$$\Rightarrow -1\left(ab - c^2\right) = 0 \Rightarrow ab = c^2$$

Example – 19

If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\left(\vec{b} + \vec{c}\right)}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

Ans. (a)

Sol. Since, $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\left(\vec{b} + \vec{c}\right)}{\sqrt{2}}$

$$\Rightarrow \left(\vec{a}\cdot\vec{c}\right)\vec{b} - \left(\vec{a}\cdot\vec{b}\right)\vec{c} = \frac{1}{\sqrt{2}}\vec{b} + \frac{1}{\sqrt{2}}\vec{c}$$

On equating the coefficient of $\vec{c}$,we get

$$\vec{a}\cdot\vec{b} = -\frac{1}{\sqrt{2}} \Rightarrow |\vec{a}||\vec{b}|\cos\theta = -\frac{1}{\sqrt{2}}$$

$$\therefore \cos\theta = -\frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{3\pi}{4}$$

Example – 20

Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors such that $\vec{u} + \vec{v} + \vec{w} = 0$. If $|\vec{u}| = 3$, $|\vec{v}| = 4$ and $|\vec{w}| = 5$, then $\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{w} + \vec{w}\cdot\vec{u}$ is :

(a) 47 (b) – 25

(c) 0 (d) 25

Ans. (b)

Sol. Since, $\vec{u} + \vec{v} + \vec{w} = \vec{0}$

$$\Rightarrow \left|\vec{u} + \vec{v} + \vec{w}\right|^2 = 0$$

$$\Rightarrow |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 + 2\left(\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{w} + \vec{w}\cdot\vec{u}\right) = 0$$

$$\Rightarrow 9 + 16 + 25 + 2\left(\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{w} + \vec{w}\cdot\vec{u}\right) = 0$$

$$\Rightarrow \vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{w} + \vec{w}\cdot\vec{u} = -25$$

Example – 21

If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplanar vectors, then $(\vec{a}+\vec{b}+\vec{c}).[(\vec{a}+\vec{b}) \times (\vec{a}+\vec{c})]$ equals :

(a) 0 (b) $[\vec{a}\ \vec{b}\ \vec{c}]$

(c) $2 \cdot [\vec{a}\ \vec{b}\ \vec{c}]$ (d) $-[\vec{a}\ \vec{b}\ \vec{c}]$

Ans. (d)

Sol. $\left(\vec{a}+\vec{b}+\vec{c}\right)\cdot\left[\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}+\vec{c}\right)\right]$

$$=\left(\vec{a}+\vec{b}+\vec{c}\right)\cdot\left[\vec{a}\times\vec{a}+\vec{a}\times\vec{c}+\vec{b}\times\vec{a}+\vec{b}\times\vec{c}\right]$$

$$=\left\{\vec{a}\cdot\left(\vec{a}\times\vec{c}\right)+\vec{a}\cdot\left(\vec{b}\times\vec{a}\right)+\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)\right\}$$

$$+\left\{\vec{b}\cdot\left(\vec{a}\times\vec{c}\right)+\vec{a}\cdot\left(\vec{b}\times\vec{a}\right)+\vec{b}\cdot\left(\vec{b}\times\vec{c}\right)\right\}$$

$$\left\{\vec{c}\cdot\left(\vec{a}\times\vec{c}\right)+\vec{c}\cdot\left(\vec{b}\times\vec{a}\right)+\vec{c}\cdot\left(\vec{b}\times\vec{c}\right)\right\}$$

$$\Rightarrow\left(\vec{a}\ \vec{b}\ \vec{c}\right)+\left(\vec{b}\ \vec{a}\ \vec{c}\right)+\left(\vec{c}\ \vec{b}\ \vec{a}\right)\Rightarrow\therefore\left(\vec{a}\ \vec{b}\ \vec{c}\right)$$

Example – 22

If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=4\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{c}=\hat{i}+\alpha\hat{j}+\beta\hat{k}$ are linearly dependent vectors and $|\vec{c}|=\sqrt{3}$, then :

(a) $\alpha=1, \beta=-1$ (b) $\alpha=1, \beta=\pm 1$

(c) $\alpha=-1, \beta=\pm 1$ (d) $\alpha=\pm 1, \beta=1$

Ans. (d)

Sol. Since, $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent vectors.

$$\Rightarrow\left[\vec{a}\ \vec{b}\ \vec{c}\right]=0$$

$$\Rightarrow\begin{vmatrix}1 & 1 & 1\\ 4 & 3 & 4\\ 1 & \alpha & \beta\end{vmatrix}=0$$

Applying, $C_1 \to C_2-C_1, C_3 \to C_3-C_1,$

$$\begin{vmatrix}1 & 0 & 0\\ 4 & -1 & 0\\ 1 & \alpha-1 & \beta-1\end{vmatrix}=0\Rightarrow-(\beta-1)=0\Rightarrow\beta=1$$

Also, $|\vec{c}|=\sqrt{3}$ [given]

$$\Rightarrow 1+\alpha^2+\beta^2=3\ \left[given, c=\hat{i}+\alpha\hat{j}+\beta\hat{k}\right]$$

$$\Rightarrow 1+\alpha^2+1=3\Rightarrow\alpha^2=1\Rightarrow\alpha=\pm 1$$

Example – 23

Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c}=|\vec{c}|, |\vec{c}-\vec{a}|=2\sqrt{2}$ and the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$ is 30º, then $|(\vec{a}\times\vec{b})\times\vec{c}|$ is equal to :

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

(c) 2 (d) 3

Ans. (b)

Sol. In this equation, vector $\vec{c}$ is not given therefore, we cannot apply the formulae of $\vec{a}\times\vec{b}\times\vec{c}$ (vector triple product).

$$\left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right|=\left|\vec{a}\times\vec{b}\right|\left|\vec{c}\right|\sin 30° \quad ...(i)$$

$$\left|\vec{a}\times\vec{b}\right|=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -2\\ 1 & 1 & 0\end{vmatrix}=2\hat{i}-2\hat{j}+\hat{k}$$

$$\Rightarrow\left|\vec{a}\times\vec{b}\right|=\sqrt{2^2+(-2)^2+1}=\sqrt{4+4+1}=\sqrt{9}=3$$

Since, $\left|\vec{c}-\vec{a}\right|=2\sqrt{2}$ [given]

$$\Rightarrow\left|\vec{c}-\vec{a}\right|^2=8$$

$$\Rightarrow\left(\vec{c}-\vec{a}\right)\cdot\left(\vec{c}-\vec{a}\right)=8$$

$$\Rightarrow\vec{c}\cdot\vec{c}-\vec{c}\cdot\vec{a}-\vec{a}\cdot\vec{c}+\vec{a}\cdot\vec{a}=8$$

$$\Rightarrow\left|\vec{c}\right|^2+\left|\vec{a}\right|^2-2\vec{a}\cdot\vec{c}=8$$

$$\Rightarrow\left|\vec{c}\right|^2+9-2\left|\vec{c}\right|=8$$

$$\Rightarrow |\vec{c}|^2 - 2|\vec{c}| + 1 = 0$$

$$\Rightarrow \left(|\vec{c}| - 1\right)^2 = 0 \Rightarrow |\vec{c}| = 1$$

From Eq.(1), $\left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right| = (3)(1).\left(\frac{1}{2}\right) = \frac{3}{2}$

Example – 24

Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is perpendicular to $\vec{a}$, then $\vec{c}$ is equal to :

(a) $\frac{1}{\sqrt{2}}\left(-\hat{j}+\hat{k}\right)$ (b) $\frac{1}{\sqrt{3}}\left(-\hat{i}-\hat{j}-\hat{k}\right)$

(c) $\frac{1}{\sqrt{5}}\left(\hat{i}-2\hat{j}\right)$ (d) $\frac{1}{\sqrt{5}}\left(\hat{i}-\hat{j}-\hat{k}\right)$

Ans. (a)

Sol. It is given that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, we take

$\vec{c} = p\vec{a} + q\vec{b}$ ...(*i*)

Where, p and q are scalars.

Since, $\vec{c} \perp \vec{a} \Rightarrow \vec{c}\cdot\vec{a} = 0$

Taking dot product of $\vec{a}$ in Eq.(i), we get

$$\vec{c}\cdot\vec{a} = p\vec{a}\cdot\vec{a} + q\vec{b}\cdot\vec{a} \Rightarrow 0 = p|\vec{a}|^2 + q|\vec{b}\cdot\vec{a}|$$

$$\left[\begin{array}{l} \because \vec{a} = 2\hat{i}+\hat{j}+\hat{k} \\ |\vec{a}| = \sqrt{2^2+1+1} = \sqrt{6} \\ \vec{a}\cdot\vec{b} = \left(2\hat{i}+\hat{j}+\hat{k}\right)\cdot\left(\hat{i}+2\hat{j}-\hat{k}\right) \\ = 2+2-1 = 3 \end{array}\right]$$

$$\Rightarrow 0 = p\cdot 6 + q\cdot 3 \Rightarrow q = -2p$$

On putting in Eq.(i), we get

$$\vec{c} = p\vec{a} + b(-2p)$$

$$\Rightarrow \vec{c} = p\vec{a} - 2p\vec{b} \Rightarrow \vec{c} = p\left(\vec{a} - 2\vec{b}\right)$$

$$\Rightarrow \vec{c} = p\left[\left(2\hat{i}+\hat{j}+\hat{k}\right) - 2\left(\hat{i}+2\hat{j}-\hat{k}\right)\right]$$

$$\Rightarrow \vec{c} = p\left(-3\hat{j}+3\hat{k}\right) \Rightarrow |\vec{c}| = p\sqrt{(-3)^2+3^2}$$

$$\Rightarrow |\vec{c}|^2 = p^2\left(\sqrt{18}\right)^2 \Rightarrow |\vec{c}| = p^2.18$$

$$\Rightarrow 1 = p^2.18 \left[\because |\vec{c}| = 1\right]$$

$$\Rightarrow p^2 = \frac{1}{18} \Rightarrow p = \pm\frac{1}{3\sqrt{2}}$$

$$\therefore \vec{c} = \pm\frac{1\left(-\hat{j}+\hat{k}\right)}{\sqrt{2}}$$

Example – 25

Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$, whose projection on $\vec{a}$ is of magnitude $\sqrt{2/3}$, is

(a) $2\hat{i}+3\hat{j}-3\hat{k}$ (b) $2\hat{i}+3\hat{j}+3\hat{k}$

(c) $-2\hat{i}-\hat{j}+5\hat{k}$ (d) $2\hat{i}+\hat{j}+5\hat{k}$

Ans. (a,c)

Sol. Given vectors are $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and

$\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$

Any vector $\vec{r}$ in the plane of $\vec{b}$ and $\vec{c}$ is

$$\vec{r} = \vec{b} + t\left(\vec{c}\right) = \hat{i} + 2\hat{j} - \hat{k} + t\left(\hat{i} + \hat{j} - 2\hat{k}\right)$$

$$= (1+t)\hat{i} + (2+t)\hat{j} - (1+2t)\hat{k} \quad ....(i)$$

Since, projection $\vec{r}$ *on* $\vec{a}$ *is* $\sqrt{\frac{2}{3}}$.

$$\therefore \frac{\vec{r}\cdot\vec{a}}{|\vec{a}|} = \sqrt{\frac{2}{3}}$$

$$\Rightarrow \left|\frac{2(1+t)-(2+t)-(1+2t)}{\sqrt{6}}\right| = \sqrt{\frac{2}{3}}$$

$$\Rightarrow |-(1+t)| = 2 \Rightarrow t = 1 \text{ or } -3$$

On putting t=1,-3 in Eq.(i) respectively,we get

$$\hat{r} = 2\hat{i} + 3\hat{j} - 3\hat{k}$$

$$or\ \hat{r} = -2\hat{i} - \hat{j} + 5\hat{k}$$

Example – 26

Which of the following expressions are meaningful questions?

(a) $\vec{u} \cdot (\vec{v} \times \vec{w})$ (b) $(\vec{u} \cdot \vec{v}) \cdot \vec{w}$

(c) $(\vec{u} \cdot \vec{v})\, \vec{w}$ (d) $\vec{u} \times (\vec{v} \cdot \vec{w})$

Ans. (a,c)

Sol. (a) $\vec{u} \cdot \left(\vec{v} \times \vec{w}\right)$ is a meaningful operation.

Therefore, (a) is the answer.

(b) $\vec{u} \cdot \left(\vec{v} \cdot \vec{w}\right)$ is not meaningful, since $\vec{v} \cdot \vec{w}$ is a scalar quantity and for dot product both quantities should be vector.

Therefore, (b) is not the answer.

(c) $\left(\vec{u} \cdot \vec{v}\right)\vec{w}$ is meaningful, since it is a simple multiplication of vector and scalar quantity.

Therefore, (c) is the answer.

(d) $\vec{u} \times \left(\vec{v} \cdot \vec{w}\right)$ is not meaningful, since $\vec{v} \cdot \vec{w}$ is a scalar quantity and for cross product, both quantities should be vector.

Therefore (d) is not the answer.

Example – 27

Let $\vec{a}$ and $\vec{b}$ be two non–collinear unit vectors. If $\vec{u} = \vec{a} - (\vec{a} \cdot \vec{b})\, \vec{b}$ and $\vec{v} = \vec{a} \times \vec{b}$, then $|\vec{v}|$ is

(a) $|\vec{u}|$ (b) $|\vec{u}| + |\vec{u} \cdot \vec{a}|$

(c) $|\vec{u}| + |\vec{u} \cdot \vec{b}|$ (d) $|\vec{u}| + \vec{u} \cdot (\vec{a} + \vec{b})$

Ans. (a,c)

Sol. Let θ be the angle between $\vec{a}$ and $\vec{b}$ .Since, $\vec{a}$ and $\vec{b}$ are non-collinear vectors, then $\theta \neq 0\ and\ \theta \neq \pi.$

We have, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$$= \cos\theta \quad \left[\because |\vec{a}| = 1, |\vec{b}| = 1, given\right]$$

$$Now,\ \vec{u} = \vec{a} - \left(\vec{a} \cdot \vec{b}\right)\vec{b} \Rightarrow |\vec{u}| = \left|\vec{a} - \left(\vec{a} \cdot \vec{b}\right)\vec{b}\right|$$

$$\Rightarrow |\vec{u}|^2 = \left|\vec{a} - \left(\vec{a} \cdot \vec{b}\right)\vec{b}\right|^2$$

$$\Rightarrow |\vec{u}|^2 = \left|\vec{a} - \cos\theta\, \vec{b}\right|^2$$

$$\Rightarrow |\vec{u}|^2 = |\vec{a}|^2 + \cos^2\theta |\vec{b}|^2 - 2\cos\theta\left(\vec{a} \cdot \vec{b}\right)$$

$$\Rightarrow |\vec{u}|^2 = 1 + \cos^2\theta - 2\cos^2\theta$$

$$\Rightarrow |\vec{u}|^2 = 1 - \cos^2\theta \Rightarrow |\vec{u}|^2 = \sin^2\theta$$

$$Also,\ \vec{v} = \vec{a} \times \vec{b} \quad [given]$$

$$\Rightarrow |\vec{v}|^2 = \left|\vec{a} \times \vec{b}\right|^2 \Rightarrow |\vec{v}|^2 = |\vec{a}|^2 |\vec{b}|^2 \cdot \sin^2\theta$$

$$\Rightarrow |\vec{v}|^2 = \sin^2\theta$$

$$\therefore |\vec{u}|^2 = |\vec{v}|^2$$

$$Now,\ \vec{u} \cdot \vec{a} = \left[\vec{a} - \left(\vec{a} \cdot \vec{b}\right)\vec{b}\right] \cdot \vec{a} = \vec{a} \cdot \vec{a} - \left(\vec{a} \cdot \vec{b}\right)\left(\vec{b} \cdot \vec{a}\right)$$

$$= |\vec{a}|^2 - \cos^2\theta = 1 - \cos^2\theta = \sin^2\theta$$

$$\therefore |\vec{u}| + |\vec{u} \cdot \vec{a}| = \sin\theta + \sin^2\theta \neq |\vec{v}|$$

$$\vec{u} \cdot \vec{b} = \left[\vec{a} - \left(\vec{a} \cdot \vec{b}\right)\vec{b}\right] \cdot \vec{b}$$

$$= \vec{a} \cdot \vec{b} - \left(\vec{a} \cdot \vec{b}\right)\left(\vec{b} \cdot \vec{b}\right) = \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{b}|\vec{b}|^2$$

$$= \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{b} = 0 \quad ...(i)$$

$$\therefore |\vec{u}| + |\vec{u} \cdot \vec{b}| = |\vec{u}| + 0 = |\vec{u}| = |\vec{v}|$$

$$Also,\ \vec{u} \cdot \left(\vec{a} + \vec{b}\right) = \vec{u} \cdot \vec{a} + \vec{u} \cdot \vec{b} = \vec{u} \cdot \vec{a}$$

$$\Rightarrow |\vec{u}| + \vec{u} \cdot \left(\vec{a} + \vec{b}\right) = |\vec{u}| + \vec{u} \cdot \vec{a} \neq |\vec{v}|$$

Example – 28

Let $\vec{A},\vec{B},\vec{C}$ be three vectors of lengths 3, 4, 5 respectively. Let $\vec{A}$ be perpendicular to $\vec{B}+\vec{C}, \vec{B}$ to $\vec{C}+\vec{A}$ and $\vec{C}$ to $\vec{A}+\vec{B}$. Then the length of vector $\vec{A}+\vec{B}+\vec{C}$ is...

Ans. $(5\sqrt{2})$

Sol. Given, $|\vec{A}|=3, |\vec{B}|=4, |\vec{C}|=5$

Since, $\vec{A}\cdot(\vec{B}+\vec{C})=\vec{B}\cdot(\vec{C}+\vec{A})=\vec{C}\cdot(\vec{A}+\vec{B})=0$ ....(i)

$$\therefore |\vec{A}+\vec{B}+\vec{C}|^2=|\vec{A}|^2+|\vec{B}|^2+|\vec{C}|^2+2(\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{C}+\vec{C}\cdot\vec{A})$$

$$=9+16+25+0$$

$$\left[\textit{from Eq.}(i)\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{C}+\vec{C}\cdot\vec{A}=0\right]$$

$$\therefore |\vec{A}+\vec{B}+\vec{C}|^2=50$$

$$\therefore |\vec{A}+\vec{B}+\vec{C}|=5\sqrt{2}$$

Example – 29

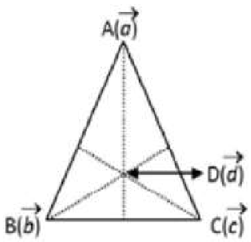
A, B, C and D are four points in a plane with position vectors $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ respectively such that $(\vec{a}-\vec{d}).(\vec{b}-\vec{c})=(\vec{b}-\vec{d}).(\vec{c}-\vec{a})=0$. The point D, then, is the.... of the triangle ABC.

Ans. (Orthocentre)

Sol. $As, (\vec{a}-\vec{d})\cdot(\vec{b}-\vec{c})=(\vec{b}-\vec{d})\cdot(\vec{c}-\vec{a})=0$

$$\Rightarrow AD\perp BC \text{ and } BD\perp CA$$

which clearly represents from figure that D is orthocentre of ΔABC



Example – 30

If $\vec{A}, \vec{B}, \vec{C}$ are three non-coplanar vectors, then

$$\frac{\vec{A}.(\vec{B}\times\vec{C})}{(\vec{C}\times\vec{A}).\vec{B}}+\frac{\vec{B}.(\vec{A}\times\vec{C})}{\vec{C}.(\vec{A}\times\vec{B})}= ....$$

Ans. (0)

Sol. $$\frac{\vec{A}\cdot(\vec{B}\times\vec{C})}{(\vec{C}\times\vec{A})\cdot\vec{B}}+\frac{\vec{B}\cdot(\vec{A}\times\vec{C})}{\vec{C}\cdot(\vec{A}\times\vec{B})}$$

$$=\frac{(\vec{A}\,\vec{B}\,\vec{C})}{(\vec{C}\,\vec{A}\,\vec{B})}+\frac{(\vec{B}\,\vec{A}\,\vec{C})}{(\vec{C}\,\vec{A}\,\vec{B})}=\frac{(\vec{A}\,\vec{B}\,\vec{C})-(\vec{A}\,\vec{B}\,\vec{C})}{(\vec{C}\,\vec{A}\,\vec{B})}=0$$

Example – 31

If $\vec{A}=(1,1,1), \vec{C}=(0,1,-1)$ are given vectors, then a vector $\vec{B}$ satisfying the equations $\vec{A}\times\vec{B}=\vec{C}$ and $\vec{A}.\vec{B}=3$ is........

Ans. $\Rightarrow \frac{5}{2}\hat{i}+\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}$

Sol. Let $\vec{B}=x\hat{i}+y\hat{j}+z\hat{k}$

Given $\vec{A}=\hat{i}+\hat{j}+\hat{k}, \vec{C}=\hat{i}-\hat{j}$

Also, given $\vec{A}\times\vec{B}=\vec{C}$

$$\Rightarrow (z-y)\hat{i}-(x-z)\hat{j}+(y-x)\hat{k}=\hat{j}-\hat{k}$$

$$\Rightarrow z-y=0,\ x-z=1, y-x=-1$$

Also, $\vec{A}\cdot\vec{B}=3\Rightarrow x+y+z=3$

On solving above equations, we get

$$x=\frac{5}{3}, y=z=\frac{2}{3}$$

$$\Rightarrow \frac{5}{2}\hat{i}+\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}$$

Example – 32

If the vectors $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ $(a\neq b\neq c\neq 1)$ are coplanar, then the value of

$$\frac{1}{(1-a)}+\frac{1}{(1-b)}+\frac{1}{(1-c)}=.....$$

Ans. (1)

Sol. Since, vectors are coplanar

$$\therefore \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}=0$$

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\begin{vmatrix} a & 1 & 1 \\ 1-a & b-1 & 0 \\ 1-a & 0 & c-1 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} \frac{a}{(1-a)} & \frac{1}{(1-b)} & \frac{1}{(1-c)} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix}=0$$

$$\therefore \frac{a}{1-a}(1)-\frac{1}{1-b}(-1)+\frac{1}{1-c}(1)=0$$

$$\therefore \frac{a}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=0$$

$$\therefore -1+\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=0$$

$$\therefore \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$$

Example – 33

Let $\vec{b}=4\hat{i}+3\hat{j}$ and $\vec{c}$ be two vectors perpendicular to each other in the xy-plane. All vectors in the same plane having projections 1 and 2 along $\vec{b}$ and $\vec{c}$, respectively, are given by ....

Ans. $\left(2\hat{i}-\hat{j}\right)$

Sol. Let $\vec{c}=a\hat{i}+b\hat{j}$

Since, $\vec{b}$ and $\vec{c}$ are perpendiculars to each other.

Then, $\vec{b}\cdot\vec{c}=0 \Rightarrow \left(4\hat{i}+3\hat{j}\right)\cdot\left(a\hat{i}+b\hat{j}\right)=0$

$\Rightarrow 4a+3b=0 \Rightarrow a:b=3:-4$

$\therefore \vec{c}=\lambda\left(3\hat{i}+4\hat{j}\right)$, where λ is constant to ratio.

Let the required vectors be $\vec{a}=p\hat{i}+q\hat{j}$

Projection $\vec{a}$ on $\vec{b}$ is $\dfrac{\vec{a}\cdot\vec{b}}{\left|\vec{b}\right|}$

$$\therefore 1=\frac{4p+3q}{5} \Rightarrow 4p+3q=5 \quad ....(i)$$

Also, projection of $\vec{a}$ on $\vec{c}$ is $\dfrac{\vec{a}\cdot\vec{c}}{\left|\vec{c}\right|}$

$$\Rightarrow 2=\frac{3\lambda p-4\lambda q}{5\lambda} \Rightarrow 3p-4q=10$$

On solving above equations , we get p=2,q=-1

$\therefore \vec{c}=2\hat{i}-\hat{j}$

Example – 34

A unit vector coplanar with $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$ and perpendicular to $\hat{i}+\hat{j}+\hat{k}$ is.......

Ans. $\frac{1}{\sqrt{2}}\cdot\left(-\hat{j}+\hat{k}\right)$

Sol. Any vector coplanar with $\hat{i}+\hat{j}+2\hat{k}\ and\ \hat{i}+2\hat{j}+\hat{k}$ is given by

$$\vec{a}=x\left(\hat{i}+\hat{j}+2\hat{k}\right)+y\left(\hat{i}+2\hat{j}+\hat{k}\right)$$

$$=(x+y)\hat{i}+(x+2y)\hat{j}+(2x+y)\hat{k}$$

This vector is perpendicular to $\hat{i}+\hat{j}+\hat{k}$,if

$$(x+y)1+(x+2y)1+(2x+y)1=0$$

$$\Rightarrow 4x+4y=0\Rightarrow -x=y$$

$$\therefore \vec{a}=-x\hat{j}+x\hat{k}=x\left(-\hat{j}+\hat{k}\right)\Rightarrow\left|\vec{a}\right|=\sqrt{2}|x|$$

Hence, the required unit vector is

$$\frac{1}{\sqrt{2}}\cdot\left(-\hat{j}+\hat{k}\right)$$

Example – 35

A non-zero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i},\hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j},\ \hat{i}+\hat{k}$. The angle between $\vec{a}$ and the vector $\hat{i}-2\hat{j}+2\hat{k}$ is........

Ans. $\frac{\pi}{4} or \frac{3\pi}{4}$

Sol. Equation of plane containing $\hat{i}$ and $\hat{i}+\hat{j}$ is

$$\left[\left(\vec{r}-\hat{i}\right)\quad \hat{i}\quad \left(\hat{i}+\hat{j}\right)\right]=0$$

$$\Rightarrow\left(\vec{r}-\hat{i}\right)\cdot\left[\hat{i}\times\left(\hat{i}+\hat{j}\right)\right]=0$$

$$\Rightarrow\left\{\left(x\hat{i}+y\hat{j}+z\hat{k}\right)-\hat{i}\right\}\cdot\left[\hat{i}\times\hat{i}+\hat{i}\times\hat{j}\right]=0$$

$$\Rightarrow\left\{(x-1)\hat{i}+y\hat{j}+z\hat{k}\right\}\cdot\left[\hat{k}\right]=0$$

$$\Rightarrow (x-1)\hat{i}\cdot\hat{k}+y\hat{j}\cdot\hat{k}+z\hat{k}\cdot\hat{k}=0$$

$$\Rightarrow z=0\quad ...(i)$$

Equation of plane containing $\hat{i}-\hat{j}$ and $\hat{i}+\hat{k}$ is

$$\left[\left(\vec{r}-\left(\hat{i}-\hat{j}\right)\right)\left(\hat{i}-\hat{j}\right)\left(\hat{i}+\hat{k}\right)\right]=0$$

$$\Rightarrow\left(\vec{r}-\hat{i}-\hat{j}\right)\cdot\left[\left(\hat{i}-\hat{j}\right)\times\left(\hat{i}+\hat{k}\right)\right]=0$$

$$\Rightarrow\left\{\left(x\hat{i}+y\hat{j}+z\hat{k}\right)-\left(\hat{i}-\hat{j}\right)\right\}\cdot\left[\hat{i}\times\hat{i}+\hat{i}\times\hat{k}-\hat{j}\times\hat{i}-\hat{j}\times\hat{k}\right]=0$$

$$\Rightarrow\left\{(x-1)\hat{i}+(y+1)\hat{j}+z\hat{k}\right\}\cdot\left[-\hat{j}+\hat{k}-\hat{i}\right]=0$$

$$\Rightarrow -(x-1)-(y+1)+z=0\quad ...(ii)$$

Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

Since, $\vec{a}$ is parallel to Eqs.(i) and (ii), we get

$$a_3=0$$

and $a_1+a_2-a_3=0\Rightarrow a_1=-a_2, a_3=0$

Thus, a vector in the direction $\vec{a}\,is\,\hat{i}-\hat{j}$

If θ is the angle between $\vec{a}\ and\ \hat{i}-2\hat{j}+2\hat{k}$, , then

$$\cos\theta=\pm\frac{(1)(1)+(-1)(-2)}{\sqrt{1+1}\sqrt{1+4+4}}=\pm\frac{3}{\sqrt{2}\cdot 3}$$

$$\Rightarrow\cos\theta=\pm\frac{1}{\sqrt{2}}\Rightarrow\theta=\frac{\pi}{4}or\frac{3\pi}{4}$$

Example – 36

Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = 10\vec{a} + 2\vec{b}$, and $\overrightarrow{OC} = \vec{b}$, where O, A and C are non-collinear points. Let p denote the area of the quadrilateral OABC, and let q denote the area of the parallelogram with OA and OC as adjacent sides. If $p = kq$, then $k =$ ....

Ans. (6)

Sol. Since, q=area of parallelogram with $\overrightarrow{OA}$ and $\overrightarrow{OC}$ as adjacent sides

$$= \left|\overrightarrow{OA} \times \overrightarrow{OC}\right| = \left|\vec{a} \times \vec{b}\right|$$

and p=area of quadrilateral OABC

$$= \frac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right| + \frac{1}{2}\left|\overrightarrow{OB} \times \overrightarrow{OC}\right|$$

$$= \frac{1}{2}\left|\vec{a} \times \left(10\vec{a} + 2\vec{b}\right)\right| + \frac{1}{2}\left|\left(10\vec{a} + 2\vec{b}\right) \times \vec{b}\right|$$

$$= \left|\vec{a} \times \vec{b}\right| + 5\left|\vec{a} \times \vec{b}\right| = 6\left|\vec{a} \times \vec{b}\right|$$

$$\therefore p = 6q$$

$$\Rightarrow k = 6$$

Example – 37

Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1, 1 and 2 respectively. If $\vec{a} \times (\vec{a} \times \vec{c}) + \vec{b} = \vec{0}$, then the acute angle between $\vec{a}$ and $\vec{c}$ is ....

Ans. $\frac{\pi}{6}$

Sol. Given, $\vec{a} \times \left(\vec{a} \times \vec{c}\right) + \vec{b} = \vec{0}$

$$\Rightarrow \left(\vec{a} \cdot \vec{c}\right)\vec{a} - \left(\vec{a} \cdot \vec{a}\right)\vec{c} + \vec{b} = \vec{0}$$

$$\Rightarrow (2\cos\theta)\vec{a} - \vec{c} + \vec{b} = \vec{0}$$

$$\Rightarrow \left(2\cos\theta\vec{a} - \vec{c}\right)^2 = \left(-\vec{b}\right)^2$$

$$\Rightarrow 4\cos^2\theta \cdot \left|\vec{a}\right|^2 + \left|\vec{c}\right|^2 - 2.2\cos\theta\left|\vec{a}\right| \cdot \left|\vec{c}\right| = \left|\vec{b}\right|^2$$

$$\Rightarrow 4\cos^2\theta + 4 - 8\cos^2\theta = 1$$

$$\Rightarrow 4\cos^2\theta = 3$$

$$\Rightarrow \cos\theta = \pm\frac{\sqrt{3}}{2}$$

For θ to be acute, $\cos\theta = \frac{\sqrt{3}}{2}$

$$\Rightarrow \theta = \frac{\pi}{6}$$

Example – 38

Find all the values of λ such that $x, y, z \neq (0, 0, 0)$ and $\left(\hat{i} + \hat{j} + 3\hat{k}\right)x + \left(3\hat{i} - 3\hat{j} + \hat{k}\right)y + \left(-4\hat{i} + 5\hat{j}\right)z = \lambda\left(\hat{i}x + \hat{j}y + \hat{k}z\right)$

where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the coordinate axes.

Ans. $(0, -1)$

Sol. Since, $\left(\vec{i} + \vec{j} + 3\vec{k}\right)x + \left(3\vec{i} - 3\vec{j} + \vec{k}\right)y + \left(-4\vec{i} + 5\vec{j}\right)z$

$$= \lambda\left(\vec{i}x + \vec{j}y + \vec{k}z\right)$$

$$\Rightarrow x + 3y - 4z = \lambda x, x - 3y + 5z = \lambda y, 3x + y + 0z = \lambda z$$

$$\Rightarrow (1-\lambda)x + 3y - 4z = 0, x - (3+\lambda)y + 5z = 0 \text{ and}$$

$$3x + y - \lambda z = 0$$

Since, $(x, y, z) \neq (0, 0, 0)$

$\therefore$ Non-trivial solution.

$$\Rightarrow \Delta = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 3 & -4 \\ 1 & -(3+\lambda) & 5 \\ 3 & 1 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\left(3\lambda + \lambda^2 - 5\right) - 3(-\lambda - 15)$$

$$-4(1 + 9 + 3\lambda) = 0$$

$$\Rightarrow \lambda(\lambda+1)^2 = 0$$

$$\therefore \lambda = 0, -1$$

Example – 39

The position vectors of the points A, B, C and D are $3\hat{i}-2\hat{j}-\hat{k},\ 2\hat{i}+3\hat{j}-4\hat{k},\ -\hat{i}+\hat{j}+2\hat{k}$ and $4\hat{i}+5\hat{j}+\lambda\hat{k}$, respectively. If the points A, B, C and D lie on a plane, find the value of λ.

Ans. $-\dfrac{146}{17}$

Sol. Here, $\overrightarrow{AB}=-\hat{i}+5\hat{j}-3\hat{k}$

$\overrightarrow{AC}=-4\hat{i}+3\hat{j}+3\hat{k}$ *and* $\overrightarrow{AD}=\hat{i}+7\hat{j}+(\lambda+1)\hat{k}$

We know that A, B, C, D lie in a plane if $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$ are coplanar i.e.

$$\left[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}\right]=0\Rightarrow\begin{vmatrix}-1 & 5 & -3\\ -4 & 3 & 3\\ 1 & 7 & \lambda+1\end{vmatrix}=0$$

$$\Rightarrow -(3\lambda+3-21)-5(-4\lambda-4-3)-3(-28-3)=0$$

$$\Rightarrow 17\lambda+146=0$$

$$\therefore \lambda=\frac{-146}{17}$$

Example – 40

If A, B, C, D are any four points in space, prove that

$$\left|\overrightarrow{AB}\times\overrightarrow{CD}+\overrightarrow{BC}\times\overrightarrow{AD}+\overrightarrow{CA}\times\overrightarrow{BD}\right|=4\ (\text{area of}\ \Delta ABC).$$

Sol. Let the position vectors of points A,B,C,D be $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ respectively.

Then, $\overrightarrow{AB}=\vec{b}-\vec{a},\quad \overrightarrow{BC}=\vec{c}-\vec{b},\quad \overrightarrow{AD}=\vec{d}-\vec{a},$

$\overrightarrow{BD}=\vec{d}-\vec{b},\ \overrightarrow{CA}=\vec{a}-\vec{c},\ \overrightarrow{CD}=\vec{d}-\vec{c}$

$$Now,\left|\overrightarrow{AB}\times\overrightarrow{CD}+\overrightarrow{BC}\times\overrightarrow{AD}+\overrightarrow{CA}\times\overrightarrow{BD}\right|$$

$$=\left|\left(\vec{b}-\vec{a}\right)\times\left(\vec{d}-\vec{c}\right)+\left(\vec{c}-\vec{b}\right)\times\left(\vec{d}-\vec{a}\right)+\left(\vec{a}-\vec{c}\right)\times\left(\vec{d}-\vec{b}\right)\right|$$

$$=\left|\vec{b}\times\vec{d}-\vec{a}\times\vec{d}-\vec{b}\times\vec{c}+\vec{a}\times\vec{c}+\vec{c}\times\vec{d}-\vec{c}\times\vec{a}-\vec{b}\times\vec{d}\right.$$

$$\left.+\vec{b}\times\vec{a}+\vec{a}\times\vec{d}-\vec{a}\times\vec{b}-\vec{c}\times\vec{d}+\vec{c}\times\vec{b}\right|$$

$$=2\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|\quad ....(i)$$

Also, are of ΔABC

$$=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|=\frac{1}{2}\left(\vec{b}-\vec{a}\right)\times\left(\vec{c}-\vec{a}\right)$$

$$=\frac{1}{2}\left|\vec{b}\times\vec{c}-\vec{b}\times\vec{a}-\vec{a}\times\vec{c}+\vec{a}\times\vec{a}\right|$$

$$=\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|\quad ....(ii)$$

From Eqs. (i) and (ii)

$$\left|\overrightarrow{AB}\times\overrightarrow{CD}+\overrightarrow{BC}\times\overrightarrow{AD}+\overrightarrow{CA}\times\overrightarrow{BD}\right|=2(2\,area\ of\ \Delta ABC)$$

$$=4\left(area\ of\ \Delta ABC\right)$$

Example – 41

If vectors $\vec{a},\vec{b},\vec{c}$ are coplanar, show that

$$\begin{vmatrix}\vec{a} & \vec{b} & \vec{c}\\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c}\\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c}\end{vmatrix}=\vec{0}$$

Sol. Given that, $\vec{a}\ \vec{b}\ \vec{c}$ are coplanar vectors.

$\therefore$ There exists scalars x,y,z not all zero, such that

$$x\vec{a}+y\vec{b}+z\vec{c}=0\quad ...(i)$$

Taking dot with $\vec{a}$ and $\vec{b}$ respectively, we get

$$x\left(\vec{a}\cdot\vec{a}\right)+y\left(\vec{a}\cdot\vec{b}\right)+z\left(\vec{a}\cdot\vec{c}\right)=0\quad ...(ii)$$

$$and\ x\left(\vec{a}\cdot\vec{b}\right)+y\left(\vec{b}\cdot\vec{b}\right)+z\left(\vec{c}\cdot\vec{b}\right)=0\quad ...(iii)$$

Since, Eqs.(i), (ii) and (iii) represent homogeneous equations with $(x,y,z)\neq(0,0,0)$

$\Rightarrow$ Non-trivial solutions

$\therefore \Delta=0$

$$\Rightarrow\begin{vmatrix}\vec{a} & \vec{b} & \vec{c}\\ \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c}\\ \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c}\end{vmatrix}=\vec{0}$$

Example – 42

Let $\vec{A}=2\hat{i}+\hat{k},\ \vec{B}=\hat{i}+\hat{j}+\hat{k},$ and $\vec{C}=4\hat{i}-3\hat{j}+7\hat{k}.$

Determine a vector $\vec{R}$ satisfying

$\vec{R}\times\vec{B}=\vec{C}\times\vec{B}$ and $\vec{R}.\vec{A}=0$.

Ans. $\left(-\hat{i}-8\hat{j}+2\hat{k}\right)$

Sol. Let $\vec{R}=x\hat{i}+y\hat{j}+z\hat{k}$

$\therefore \vec{R}\times\vec{B}=\vec{C}\times\vec{B}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & 7 \\ 1 & 1 & 1 \end{vmatrix}$$

$\Rightarrow (y-z)\hat{i}-(x-z)\hat{j}+(x-y)\hat{k}=-10\hat{i}+3\hat{j}+7\hat{k}$

$\Rightarrow y-z=-10, z-x=3, x-y=7$

and $\vec{R}\cdot\vec{A}=0$

$\Rightarrow 2x+z=0$

On solving above equations,

$x=-1, y=-8\ and\ z=2$

$\therefore \vec{R}=-\hat{i}-8\hat{j}+2\hat{k}$

Example – 43

Determine the value of 'c' so that for all real x, the vector $cx\hat{i}-6\hat{j}-3\hat{k}$ and $x\hat{i}+2\hat{j}+2cx\hat{k}$ make an obtuse angle with each other.

Ans. $\left(\frac{-4}{3},0\right)$

Sol. Let $\vec{a}=cx\hat{i}-6\hat{j}+3\hat{k}\ and\ \vec{b}=x\hat{i}+2\hat{j}+2cx\hat{k}.$

Since, $\vec{a}$ and $\vec{b}$ make an obtuse angle.

$\Rightarrow \vec{a}\cdot\vec{b}<0 \Rightarrow cx^2-12+6cx<0$

$\Rightarrow c<0$ and discriminant **<0**

$\Rightarrow c<0\ and\ 36c^2-4.(-12)c<0$

$\Rightarrow c<0\ and\ 12c(3c+4)<0$

$\Rightarrow c<0\ and\ c>-\frac{4}{3}$

$\therefore c\in\left(\frac{-4}{3},0\right)$

Example – 44

For any two vectors $\vec{u}$ and $\vec{v}$ prove that

(a) $|\vec{u}.\vec{v}|^2+|\vec{u}\times\vec{v}|^2=|\vec{u}|^2|\vec{v}|^2$ and

(b) $\left(1+|\vec{u}|^2\right)\left(1+|\vec{v}|^2\right)$

$= |1-\vec{u}.\vec{v}|^2+|\vec{u}+\vec{v}+(\vec{u}\times\vec{v})|^2$

Sol. (a) Since, $\vec{u}\cdot\vec{v}=|\vec{u}||\vec{v}|\cos\theta$

and $\vec{u}\times\vec{v}=|\vec{u}||\vec{v}|\sin\theta\hat{n}$

where, θ is the angle between $\vec{u}$ and $\vec{v}$ and $\hat{n}$ is unit vector perpedicular to the plane of $\vec{u}\ and\ \vec{v}$.

Again, $|\vec{u}\cdot\vec{v}|^2=|\vec{u}|^2|\vec{v}|^2\cos^2\theta$

$|\vec{u}\times\vec{v}|^2=|\vec{u}|^2|\vec{v}|^2\sin^2\theta$

$\therefore |\vec{u}\cdot\vec{v}|^2+|\vec{u}\times\vec{v}|^2=|\vec{u}|^2|\vec{v}|^2\left(\cos^2\theta+\sin^2\theta\right)$

$=|\vec{u}|^2|\vec{v}|^2 \quad ....(i)$

(b) $\left|\vec{u}+\vec{v}+\left(\vec{u}\times\vec{v}\right)\right|^2$

$=|\vec{u}+\vec{v}|^2+|\vec{u}\times\vec{v}|^2+2|\vec{u}+\vec{v}|\cdot|\vec{u}\times\vec{v}|$

$=|\vec{u}|^2+|\vec{v}|^2+2\vec{u}\cdot\vec{v}+|\vec{u}\times\vec{v}|^2+0$

[$\therefore$ $\vec{u}\times\vec{v}$ is perpendicular to the plane of $\vec{u}\ and\ \vec{v}$]

$\therefore \left|\vec{u}+\vec{v}+\left(\vec{u}\times\vec{v}\right)\right|^2+|1-\vec{u}\cdot\vec{v}|^2$

$$=|\vec{u}|^2+|\vec{v}|^2+2\vec{u}\cdot\vec{v}+|\vec{u}\times\vec{v}|^2+1-2\vec{u}\cdot\vec{v}+|\vec{u}\cdot\vec{v}|^2$$

$$=|\vec{u}|^2+|\vec{v}|^2+1+|\vec{u}|^2|\vec{v}|^2\quad[from\ Eq.(i)]$$

$$=|\vec{u}|^2\left(1+|\vec{v}|^2\right)+\left(1+|\vec{v}|^2\right)=\left(1+|\vec{v}|^2\right)\left(1+|\vec{u}|^2\right)$$

Example – 45

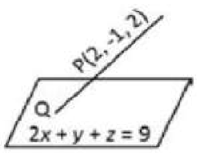
A line with positive direction cosines passes through the point P(2, –1, 2) and makes equal angles with the coordinate axes. The line meets the plane 2x + y + z = 9 at point Q. The length of the line segment PQ equals

(a) 1 (b) $\sqrt{2}$

(c) $\sqrt{3}$ (d) 2

Ans. (c)

Sol. Since, $l=m=n=\dfrac{1}{\sqrt{3}}$



$\therefore$ Equation of line is $\dfrac{x-2}{\frac{1}{\sqrt{3}}}=\dfrac{y+1}{\frac{1}{\sqrt{3}}}=\dfrac{z-2}{\frac{1}{\sqrt{3}}}$

$\Rightarrow x-2=y+1=z-2=r\ [say]$

$\therefore$ Any point on the line is

$Q\equiv(r+2,r-1,r+2)$

$\because Q$ lies on the plane $2x+y+z=9$

$\therefore 2(r+2)+(r-1)+(r+2)=9$

$\Rightarrow 4r+5=9$

$\Rightarrow r=1$

$\Rightarrow Q(3,0,3)$

$\therefore PQ=\sqrt{(3-2)^2+(0+1)^2+(3-2)^2}=\sqrt{3}$

Example – 46

The area of the triangle whose vertices are A (1, –1, 2), B(2, 1, –1), C(3, –1, 2) is ......

Ans. $\sqrt{13}\ sq.unit$

Sol. Area of $\Delta ABC=\dfrac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$

$\overrightarrow{AB}=\hat{i}+2\hat{j}-3\hat{k}$

$and\ \overrightarrow{AC}=2\hat{i}$

$$\therefore\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&-3\\2&0&0\end{vmatrix}=2\left(-3\hat{j}-2\hat{k}\right)$$

$\Rightarrow Area\ of\ triangle=\dfrac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$

$=\dfrac{1}{2}\cdot 2\cdot\sqrt{9+4}$

$=\sqrt{13}\ sq\ units$

Example – 47

The unit vector perpendicular to the plane determined by P(1, –1, 2), Q(2, 0, –1) & R(0, 2, 1) is ......

Ans. $\pm\dfrac{\left(2\hat{i}+\hat{j}+\hat{k}\right)}{\sqrt{6}}$

Sol. A unit vector perpendicular to the plane determined by P,Q,R

$$=\pm\frac{\left(\overrightarrow{PQ}\right)\times\left(\overrightarrow{PR}\right)}{\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right|}$$

$$\therefore Unit\ vector=\pm\frac{\left(\overrightarrow{PQ}\right)\times\left(\overrightarrow{PR}\right)}{\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right|}$$

$\overrightarrow{PQ}=\hat{i}+\hat{j}-3\hat{k}\ and\ \overrightarrow{PR}=-i+3j-k$

$$\therefore\overrightarrow{PQ}\times\overrightarrow{PR}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&-3\\-1&3&-1\end{vmatrix}$$

$= \hat{i}(-1+9)+\hat{j}(-1-3)+\hat{k}(3+1)$

$= 8\hat{i}-4\hat{j}+4\hat{k}$

$\Rightarrow \left|\overline{PQ}\times\overline{PR}\right| = 4\sqrt{4+1+1} = 4\sqrt{6}$

Hence the unit vector is

$= \pm\dfrac{\left(2\hat{i}+\hat{j}+\hat{k}\right)}{\sqrt{6}}$

Example – 48

If the angle between the line $x=\dfrac{y-1}{2}=\dfrac{z-3}{\lambda}$ and the plane $x+2y+3z=4$ is $\cos^{-1}\left(\sqrt{\dfrac{5}{14}}\right)$, then λ equals to

(a) $\dfrac{3}{2}$ (b) $\dfrac{2}{5}$

(c) $\dfrac{5}{3}$ (d) $\dfrac{2}{3}$

Ans. (d)

Sol. $\dfrac{x-0}{1}=\dfrac{y-1}{2}=\dfrac{z-3}{\lambda}$

$x+2y+3z=4$

Angle between line and plane (by definition)

$= \sin^{-1}\left(\dfrac{1\cdot1+2\cdot2+\lambda\cdot3}{\sqrt{1+4+9}\sqrt{1+4+\lambda^2}}\right) = \sin^{-1}\left(\dfrac{5+3\lambda}{\sqrt{14}\sqrt{5+\lambda^2}}\right)$

So, $\dfrac{(5+3\lambda)^2}{14\left(5+\lambda^2\right)}+\dfrac{5}{14}=1 \quad \left(\because \sin^2\theta+\cos^2\theta=1\right)$

$\Rightarrow \dfrac{(5+3\lambda)^2}{5+\lambda^2}+5=14$

$\Rightarrow (5+3\lambda)^2+5\left(5+\lambda^2\right)=14\left(5+\lambda^2\right)$

$\Rightarrow 25+30\lambda+9\lambda^2+25+5\lambda^2=70+14\lambda^2$

$\Rightarrow 30\lambda+50=70$

$\Rightarrow 30\lambda=20 \quad \therefore \lambda=\dfrac{2}{3}$

Example – 49

The length of the perpendicular drawn from the point $(3,-1,11)$ to the line $\dfrac{x}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}$ is

(a) $\sqrt{66}$ (b) $\sqrt{29}$

(c) $\sqrt{33}$ (d) $\sqrt{53}$

Ans. (d)

Sol. Let feet of perpendicular is

$(2\alpha, 3\alpha+2, 4\alpha+3)$

$\Rightarrow$ direction ratio of lines are $2,3,4$

$\Rightarrow 2(2\alpha-3)+3(3\alpha+3)+4(4\alpha-8)=0$

$\Rightarrow \alpha=1$

$\Rightarrow$ Foot of perpendicular is $(2,5,7)$

$\Rightarrow$ length of perpendicular $=\sqrt{1+6^2+4^2}=\sqrt{53}$

Example – 50

An equation of a plane parallel to the plane $x-2y+2z-5=0$ and at a unit distance from the origin is

(a) $x-2y+2z-3=0$ (b) $x-2y+2z+1=0$

(c) $x-2y+2z-1=0$ (d) $x-2y+2z+5=0$

Ans. (a)

Sol. Equation of a plane parallel to $x-2y+2z-5=0$ and at a unit distance from origin is

$x-2y+2z+k=0$

$\Rightarrow \dfrac{|k|}{3}=1 \Rightarrow |k|=3$

$\therefore x-2y+2z-3=0$ or $x-2y+2z+3=0$

Example – 51

If the line $\dfrac{x-1}{2}=\dfrac{y+1}{3}=\dfrac{z-1}{4}$ and $\dfrac{x-3}{1}=\dfrac{y-k}{2}=\dfrac{z}{1}$ intersect, then k is equal to

(a) -1 (b) $\dfrac{2}{9}$

(c) $\dfrac{9}{2}$ (d) 0

Ans. (c)

Sol. $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}=r_1$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}=r_2$

or $2r_1+1=r_2+3, 3r_1-1=2r_2+k, 4r_1+1=r_2$

$\Rightarrow 2r_1-r_2=2$ and $4r_1-r_2=-1$

$-2r_1=3\Rightarrow r_1=\frac{-3}{2}$ and $r_2=-5$

$\therefore -\frac{9}{2}-1=-10+k\Rightarrow k=10-\frac{11}{2}=\frac{9}{2}$

Example – 52

The image of the line $\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane $2x-y+z+3=0$ is the line :

(a) $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$

(b) $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

(c) $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$

(d) $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$

Ans. (b)

Sol. DR's of line = 3,1,-5

DR's of the normal of plane = 2,-1,1

$\Rightarrow 3(2)+1(-1)-5=0$

So, the line is parallel to the plane

Image is point (1,3,4) in plane

$$\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-4}{1}=-\frac{2(2-3+4+3)}{6}$$

$$\Rightarrow \frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-4}{1}=-2$$

$$\Rightarrow x=-3, y=5, z=2$$

$\Rightarrow$ Equation of image of the line is

$$\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$$

Example – 53

The angle between the lines whose direction cosines satisfy the equations $l+m+n=0$ and $l^2=m^2+n^2$ is :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

Ans. (b)

Sol. $l+m+n=0$ ....(1)

$l^2=m^2+n^2$ .....(2)

From (1) and (2)

$(m+n)^2=m^2+n^2$

$\Rightarrow 2mn=0\Rightarrow m=0$, or $n=0$

If $m=0$ then from (1)

$$l+n=0\Rightarrow l=-n\Rightarrow \frac{l}{1}=\frac{n}{-1}$$

If $n=0$ then from (1)

$$l+m=0\Rightarrow l=-m\Rightarrow \frac{l}{1}=\frac{m}{-1}$$

$\Rightarrow$ angle between the lines

$$\cos\theta=\frac{(1)(1)+(-1)(0)+(-1)(0)}{\sqrt{1+0+1}\sqrt{1+0+1}}$$

$$=\frac{1}{2}$$

$$\Rightarrow \theta=\frac{\pi}{3}$$

Example – 54

A line in the 3-dimensional space makes and angle $\theta\left(0<\theta\le\frac{\pi}{2}\right)$ with both the x and y axis. Then the set of all values of θ is the interval:

(a) $\left(0,\frac{\pi}{4}\right]$ (b) $\left[\frac{\pi}{6},\frac{\pi}{3}\right]$

(c) $\left[\frac{\pi}{4},\frac{\pi}{2}\right]$ (d) $\left(\frac{\pi}{3},\frac{\pi}{2}\right]$

Ans. (c)

Sol. We know that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\Rightarrow \cos^2\theta+\cos^2\theta+\cos^2\gamma=1$

$\Rightarrow \cos^2\gamma=1-2\cos^2\theta$

$\because 0\le\cos^2\gamma\le1$

$\Rightarrow 0\le 1-2\cos^2\theta\le1$

$\Rightarrow -1\le-2\cos^2\theta\le0$

$\Rightarrow 0\le2\cos^2\theta\le1$

$\Rightarrow 0\le\cos^2\theta\le\frac{1}{2}\Rightarrow 0\le\cos\theta\le\frac{1}{\sqrt{2}}$

$\Rightarrow \theta\in\left[\frac{\pi}{4},\frac{\pi}{2}\right]$

Example – 55

The plane containg the line $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and parallel to the line $\frac{x}{1}=\frac{y}{1}=\frac{z}{4}$ passes through the point:

(a) (1, -2, 5) (b) (1, 0, 5)

(c) (0, 3, -5) (d) (-1, -3, 0)

Ans. (b)

Sol. DR's of the given lines are (1,2,3) and (1,1,4)

$\Rightarrow$ DR's of the normal to the plane

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 1 & 1 & 4\end{vmatrix}$$

$=\hat{i}(5)-\hat{j}(1)+\hat{k}(-1)$

$=5\hat{i}-\hat{j}-\hat{k}$

It contains the point (1,2,3)

$\Rightarrow$ Equation of the plane is

$5(x-1)-1(y-2)-1(z-3)=0$

$\Rightarrow 5x-y-z=0$

It also passes through (1,0,5)

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

VECTORS

Basics of Vectors

1. In a regular hexagon ABCDEF, $\overrightarrow{AB} = a$, $\overrightarrow{BC} = \vec{b}$ and $\overrightarrow{CD} = \vec{c}$. Then, $\overrightarrow{AE} =$

(a) $\vec{a}+\vec{b}+\vec{c}$ (b) $2\vec{a}+\vec{b}+\vec{c}$

(c) $\vec{b}+\vec{c}$ (d) $\vec{a}+2\vec{b}+2\vec{c}$

2. If ABCDE is a pentagon, then $\overrightarrow{AB}+\overrightarrow{AE}+\overrightarrow{BC}+\overrightarrow{DC}+\overrightarrow{ED}+\overrightarrow{AC}$ equals

(a) $3\overrightarrow{AD}$ (b) $3\overrightarrow{AC}$

(c) $3\overrightarrow{BE}$ (d) $3\overrightarrow{CE}$

3. Which of the following is unit vectors

(a) $\hat{i}+\hat{j}$ (b) $\dfrac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{2}}$

(c) $\hat{i}+\hat{j}+\hat{k}$ (d) $\dfrac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}$

4. If $\vec{a}=2\hat{i}+5\hat{j}$ and $\vec{b}=2\hat{i}-\hat{j}$, then unit vector in the direction of $\vec{a}+\vec{b}$ is

(a) $\hat{i}+\hat{j}$ (b) $\sqrt{2}(\hat{i}+\hat{j})$

(c) $(\hat{i}+\hat{j})/\sqrt{2}$ (d) $(\hat{i}-\hat{j})/\sqrt{2}$

5. For any two vector $\vec{a}$ and $\vec{b}$, correct statement is

(a) $|\vec{a}-\vec{b}|=|\vec{a}|-|\vec{b}|$ (b) $|\vec{a}+\vec{b}|\geq|\vec{a}|-|\vec{b}|$

(c) $|\vec{a}+\vec{b}|=|\vec{a}|+|\vec{b}|$ (d) $|\vec{a}-\vec{b}|\leq|\vec{a}|-|\vec{b}|$

6. If $\hat{i},\hat{j},\hat{k}$ are position vectors of A, B, C and $\overrightarrow{AB}=\overrightarrow{CX}$, then position vector of X is

(a) $-\hat{i}+\hat{j}+\hat{k}$ (b) $\hat{i}-\hat{j}+\hat{k}$

(c) $\hat{i}+\hat{j}-\hat{k}$ (d) $\hat{i}+\hat{j}+\hat{k}$

7. If the position vector of points A and B with respect to point P are respectively $\vec{a}$ and $\vec{b}$ then the position vector of middle point of AB is

(a) $\dfrac{\vec{b}-\vec{a}}{2}$ (b) $\dfrac{\vec{a}+\vec{b}}{2}$

(c) $\dfrac{\vec{a}-\vec{b}}{2}$ (d) none of these

8. The points with position vectors $3\hat{i}-4\hat{j}-4\hat{k}$, $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}-3\hat{j}-5\hat{k}$ form

(a) an equilateral triangle

(b) an isosceles triangle

(c) a right angle triangle

(d) none of these

9. The position vector of two points P and Q are respectively $\vec{p}$ and $\vec{q}$ then the position vector of the point dividing PQ in 2 : 5 is

(a) $\dfrac{\vec{p}+\vec{q}}{2+5}$ (b) $\dfrac{5\vec{p}+2\vec{q}}{2+5}$

(c) $\dfrac{2\vec{p}+5\vec{q}}{2+5}$ (d) $\dfrac{\vec{p}-\vec{q}}{2+5}$

10. The position vector of the vertices of triangle ABC are $\hat{i},\hat{j}$ and $\hat{k}$ then the position vector of its orthocentre is

(a) $\hat{i}+\hat{j}+\hat{k}$ (b) $2(\hat{i}+\hat{j}+\hat{k})$

(c) $\frac{1}{3}(\hat{i}+\hat{j}+\hat{k})$ (d) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$

11. If, D, E, F are mid points of sides BC, CA and AB respectively of a triangle ABC, and $\hat{i}+\hat{j}$, $\hat{j}+\hat{k}$, $\hat{k}+\hat{i}$ are p.v. of points A, B and C respectively, then p.v. of centroid of ΔDEF is

(a) $\dfrac{\hat{i}+\hat{j}+\hat{k}}{3}$ (b) $\hat{i}+\hat{j}+\hat{k}$

(c) $2(\hat{i}+\hat{j}+\hat{k})$ (d) $\dfrac{2(\hat{i}+\hat{j}+\hat{k})}{3}$

12. If $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$, then the unit vector parallel to $\vec{a} + \vec{b}$, is

(a) $\frac{1}{3}\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$ (b) $\frac{1}{5}\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$

(c) $\frac{1}{\sqrt{3}}\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$ (d) none of these

13. A parallelopiped is formed by planes drawn through the points (2, 3, 5) and (5, 9, 7), parallel to the co-ordinate planes. The length of a diagonal of the parallelopiped is:

(a) 7 (b) $\sqrt{38}$

(c) $\sqrt{155}$ (d) None of these

14. The vector $\vec{c}$, directed along the internal bisector of the angle between the vectors, $\vec{a} = 7\hat{i} - 4\hat{j} - 4\hat{k}$ and $\vec{b} = -2\hat{i} - \hat{j} + 2\hat{k}$ with $|\vec{c}| = 5\sqrt{6}$ is :

(a) $\frac{5}{3}(\hat{i} - 7\hat{j} + 2\hat{k})$ (b) $\frac{5}{3}(5\hat{i} + 5\hat{j} + 2\hat{k})$

(c) $\frac{5}{3}(\hat{i} + 7\hat{j} + 2\hat{k})$ (d) $\frac{5}{3}(-5\hat{i} + 5\hat{j} + 2\hat{k})$

15. The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β ?

(a) $\alpha = 1, \beta = 1$ (b) $\alpha = 2, \beta = 2$

(c) $\alpha = 1, \beta = 2$ (d) $\alpha = 2, \beta = 1$

Collinearity and Coplanarity Conditions

16. If position vectors of A, B, C, D are respectively $2\hat{i} + 3\hat{j} + 5\hat{k}, \hat{i} + 2\hat{j} + 3\hat{k}, -5\hat{i} + 4\hat{j} - 2\hat{k}$ and $\hat{i} + 10\hat{j} + 10\hat{k}$, then

(a) AB || CD (b) DC || AD

(c) A, B, C are collinear (d) B, C, D are collinear

17. If the vector $\vec{b}$ is collinear with the vector $\vec{a} = \left(2\sqrt{2}, -1, 4\right)$ & $|\vec{b}| = 10$, then

(a) $\vec{a} \pm \vec{b} = 0$ (b) $\vec{a} \pm 2\vec{b} = 0$

(c) $2\vec{a} \pm \vec{b} = 0$ (d) none

18. If $\vec{a} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = -8\hat{i} + 4\hat{j} - 6\hat{k}$ are two vectors then $\vec{a}$, $\vec{b}$ are

(a) like parallel (b) unlike parallel

(c) non-collinear (d) perpendicular

19. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $\left(1, a, a^2\right), \left(1, b, b^2\right)$ and $\left(1, c, c^2\right)$ are non–coplanar, then the product abc equals

(a) 2 (b) –1

(c) 1 (d) 0

Scalar and vector products

20. If the moduli of vectors $\vec{a}$ and $\vec{b}$ are 1 and 2 respectively and $\vec{a}.\vec{b} = 1$, then the angle θ between them is :

(a) $\theta = \pi/6$ (b) $\theta = \pi/3$

(c) $\theta = \pi/2$ (d) $\theta = 2\pi/3$

21. If the angle between $\vec{a}$ and $\vec{b}$ is θ then for $\vec{a}.\vec{b} \geq 0$

(a) $0 \leq \theta \leq \pi$ (b) $0 < \theta$ or $\theta > \pi/2$

(c) $\pi/2 \leq \theta \leq \pi$ (d) $0 \leq \theta \leq \pi/2$

22. $(\vec{A} + \vec{B})^2 + (\vec{A} - \vec{B})^2$ equals

(a) $2\,(\vec{A}^2 + \vec{B}^2)$ (b) $4\,\vec{A}\,.\,\vec{B}$

(c) $\vec{A}^2 + \vec{B}^2$ (d) none of these

23. If $|\vec{a}| = |\vec{b}|$, then $(\vec{a} + \vec{b})\,.\,(\vec{a} - \vec{b})$ is

(a) positive (b) negative

(c) zero (d) none of these

24. If $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = 4\hat{i} - 2\hat{j} + 4\hat{k}$, then $(2\vec{a} + \vec{b})\,.\,(\vec{a} - 2\vec{b})$ equals

(a) 14 (b) –14

(c) 0 (d) none of these

25. Angle between the vectors $2\hat{i}+6\hat{j}+3\hat{k}$ and $12\hat{i}-4\hat{j}+3\hat{k}$ is

(a) $\cos^{-1}\left(\frac{1}{10}\right)$ (b) $\cos^{-1}\left(\frac{9}{11}\right)$

(c) $\cos^{-1}\left(\frac{9}{91}\right)$ (d) $\cos^{-1}\left(\frac{1}{9}\right)$

26. If $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+5\hat{j}$, $3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ be p.v. of four points A,B,C and D respectively, then the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$ is

(a) $\pi/4$ (b) $\pi/2$

(c) π (d) none of these

27. Projection vector of $\vec{a}$ on $\vec{b}$ is

(a) $\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\vec{b}$ (b) $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

(c) $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$ (d) $\frac{\vec{a}.\vec{b}}{|\vec{a}|^2}\hat{b}$

28. The angle between the vectors $\vec{a}+\vec{b}$ & $\vec{a}-\vec{b}$, given $|\vec{a}|=2, |\vec{b}|=1$ and angle between $\vec{a}$ & $\vec{b}$ is $\pi/3$, is

(a) $\tan^{-1}\frac{2}{\sqrt{3}}$ (b) $\tan^{-1}\sqrt{\frac{2}{3}}$

(c) $\tan^{-1}\sqrt{\frac{3}{7}}$ (d) none

29. Given the vectors $\vec{a}$ & $\vec{b}$ the angle between which equals 120°. If $|\vec{a}|=3$ & $|\vec{b}|=4$ then the length of the vector $2\vec{a}-\frac{3}{2}\vec{b}$ is

(a) $6\sqrt{3}$ (b) $7\sqrt{2}$

(c) $4\sqrt{5}$ (d) none

30. $\left[\frac{\vec{a}}{|\vec{a}|^2}-\frac{\vec{b}}{|\vec{b}|^2}\right]^2 =$

(a) $|\vec{a}|^2-|\vec{b}|^2$ (b) $\left[\frac{\vec{a}-\vec{b}}{|\vec{a}||\vec{b}|}\right]^2$

(c) $\left[\frac{\vec{a}|\vec{a}|-\vec{b}|\vec{b}|}{|\vec{a}||\vec{b}|}\right]^2$ (d) none

31. If the coordinates of the points A, B, C be (–1, 3, 2), (2, 3, 5) and (3, 5, –2) respectively, then $\angle A =$

(a) 0° (b) 45°

(c) 60° (d) 90°

32. The coordinates of the points A, B, C, D are (4, α, 2), (5, –3, 2), (β, 1, 1) & (3, 3, –1). Line AB would be perpendicular to line CD when

(a) $\alpha = -1, \beta = -1$ (b) $\alpha = 1, \beta = 2$

(c) $\alpha = 2, \beta = 1$ (d) $\alpha = 2, \beta = 2$

33. If $\vec{a}$ and $\vec{b}$ are vectors of equal magnitude 2 and α be the angle between them, then magnitude of $\vec{a}+\vec{b}$ will be 2 if

(a) $\alpha = \pi/3$ (b) $\alpha = \pi/4$

(c) $\alpha = \pi/2$ (d) $\alpha = 2\pi/3$

34. If θ be the angle between vectors $\hat{i}+2\hat{j}+3\hat{k}$ and $3\hat{i}+2\hat{j}+\hat{k}$, then the value of sin θ is

(a) $\sqrt{6/7}$ (b) $\frac{2\sqrt{6}}{7}$

(c) 1/7 (d) none of these

35. Two non zero vectors $\vec{a}$ and $\vec{b}$ will be parallel, if

(a) $\vec{a}.\vec{b}=0$ (b) $\vec{a}\times\vec{b}\neq\vec{0}$

(c) $\vec{a}=\vec{b}$ (d) none of these

36. If $\vec{a}$ and $\vec{b}$ are two vectors, then -

(a) $|\vec{a}\times\vec{b}| \geq |\vec{a}|\,|\vec{b}|$ (b) $|\vec{a}\times\vec{b}| \leq |\vec{a}|\,|\vec{b}|$

(c) $|\vec{a}\times\vec{b}| > |\vec{a}|\,|\vec{b}|$ (d) $|\vec{a}\times\vec{b}| < |\vec{a}|\,|\vec{b}|$

37. If $|\vec{a}\times\vec{b}| = \vec{a}.\vec{b}$ then angle between a and b is

(a) 0° (b) 90°

(c) 60° (d) 45°

38. The unit vector perpendicular to vectors $\hat{i}+\hat{j}$ and $\hat{j}+\hat{k}$ is

(a) $\frac{1}{\sqrt{3}}\left(\hat{i}-\hat{j}+\hat{k}\right)$ (b) $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}-\hat{k}\right)$

(c) $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$ (d) none of these

39. If $|\vec{a}.\vec{b}| = 3$ and $|\vec{a}\times\vec{b}| = 4$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\cos^{-1} 3/4$ or $\pi-\cos^{-1}\frac{3}{4}$

(b) $\cos^{-1} 3/5$ or $\pi-\cos^{-1}\frac{3}{5}$

(c) $\sin^{-1}\frac{3}{5}$ or $\pi-\sin^{-1}\frac{3}{5}$

(d) $\pi/4$ or $\frac{3\pi}{4}$

40. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are any vectors then which one of the following is a **wrong statement.**

(a) $\vec{a}+\vec{b}=\vec{b}+\vec{a}$ (b) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$

(c) $\vec{a}\times\vec{b}=\vec{b}\times\vec{a}$ (d) $|\vec{a}\times\vec{b}|=|\vec{b}\times\vec{a}|$

41. If for vectors $\vec{a}$ and $\vec{b}$, $\vec{a}\times\vec{b}=0$ and $\vec{a}.\vec{b}=0$, then

(a) $\vec{a}\parallel\vec{b}$ (b) $\vec{a}\perp\vec{b}$

(c) $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$ (d) none of these

42. If $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ then correct statement is

(a) $\vec{a}=0$

(b) $\vec{b}=0=\vec{c}$

(c) $\vec{b}=\vec{c}$

(d) above three are not necessary

43. For any vectors $\vec{a}$, $\vec{b}$; $|\vec{a}\times\vec{b}|^2+(\vec{a}.\vec{b})^2$ is equal to

(a) $|\vec{a}|^2|\vec{b}|^2$ (b) $\vec{a}^2+\vec{b}^2$

(c) $\vec{a}^2-\vec{b}^2$ (d) 0

44. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$ and $\vec{a}\neq\vec{d}, \vec{b}\neq\vec{c}$, Then

(a) $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$.

(b) $\vec{a}-\vec{d}$ is perpendicular to $\vec{b}-\vec{c}$.

(c) $\vec{a}-\vec{d}$ is equal to $\vec{b}-\vec{c}$.

(d) none of these

45. Vectors $\vec{a}\,\&\,\vec{b}$ make an angle $\theta=\frac{2\pi}{3}$. If $|\vec{a}|=1, |\vec{b}|=2$ then $\{(\vec{a}+3\vec{b})\times(3\vec{a}-\vec{b})\}^2=$

(a) 225 (b) 250

(c) 275 (d) 300

46. Unit vector perpendicular to the plane of the triangle ABC with pv's $\vec{a}, \vec{b}, \vec{c}$ of the vertices A, B, C is

(a) $\frac{(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})}{\Delta}$ (b) $\frac{(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})}{2\Delta}$

(c) $\frac{(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})}{4\Delta}$ (d) none

47. The area of the quadrilateral ABCD, where A(0, 4, 1), B(2, 3, –1), C(4, 5, 0) and D(2, 6, 2), is equal to

(a) 9 sq. units (b) 18 sq. units

(c) 27 sq. units (d) 81 sq. units

48. If the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ from the sides BC, CA and AB respectively of a triangle ABC, then

(a) $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{b}=0$ (b) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

(c) $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$ (d) $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=0$

Scalar and vector triple product.

49. For non-zero vectors $\vec{a}, \vec{b}, \vec{c}$, $|\vec{a}\times\vec{b}.\vec{c}|=|\vec{a}||\vec{b}||\vec{c}|$ holds if and only if

(a) $\vec{a}.\vec{b}=0. \vec{b}.\vec{c}=0$ (b) $\vec{c}.\vec{a}=0, \vec{a}.\vec{b}=0$

(c) $\vec{a}.\vec{c}=0. \vec{b}.\vec{c}=0$ (d) $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$

50. The volume of the parallelopiped whose sides are given by $\overrightarrow{OA} = 2\hat{i} - 3\hat{j}$. $\overrightarrow{OB} = \hat{i} + \hat{j} - \hat{k}$ and $\overrightarrow{OC} = 3\hat{i} - \hat{k}$ is :

(a) 4/13 (b) 4

(c) 2/7 (d) none

51. If $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$, where $\vec{a}, \vec{b}, \vec{c}$ are any three vectors such that $\vec{a} \cdot \vec{b} \neq 0$, $\vec{b} \cdot \vec{c} \neq 0$, then $\vec{a}$ and $\vec{c}$ are

(a) inclined at an angle of $\frac{\pi}{6}$ between them

(b) perpendicular

(c) parallel

(d) inclined at an angle of $\frac{\pi}{3}$ between them

52. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = 0$ and $\vec{a} . \vec{b} = 3$, is

(a) $-\hat{i} + \hat{j} - 2\hat{k}$ (b) $2\hat{i} - \hat{j} + 2\hat{k}$

(c) $\hat{i} - \hat{j} - 2\hat{k}$ (d) $\hat{i} + \hat{j} - 2\hat{k}$

53. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}||\vec{a}|$. If θ is the angle between vector $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is:

(a) $\frac{2}{3}$ (b) $\frac{-2\sqrt{3}}{3}$

(c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{-\sqrt{2}}{3}$

54. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(a) $\frac{\pi}{2}$ (b) $\frac{2\pi}{3}$

(c) $\frac{5\pi}{6}$ (d) $\frac{3\pi}{4}$

Numerical Value Type Questions

55. If G is the intersection of diagonals of a parallelogram ABCD and O is any point and $\overline{OA} + \overline{OB} + \overline{OC} + \overline{OD} = k\ \overline{OG}$. Then the value of k is

56. If vector $2\hat{i} + 3\hat{j} - 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$ represents the adjacent sides of any parallelogram then the sum of squares of lengths of diagonals of parallelogram is

57. The 2 vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represents the two sides AB and AC, respectively of a ΔABC. Then twice of square of length of the median through A is

58. P is a point on the line segment joining the points (3, 2, –1) and (6, 2, –2). If x co-ordinate of P is 5, then its y co-ordinate is

59. The plane XOZ divides the join of (1, –1, 5) & (2, 3, 4) in the ratio λ : 1, then 6λ is :

60. If vectors (x – 2) $\hat{i} + \hat{j}$ and (x + 1) $\hat{i} + 2\hat{j}$ are collinear, then the value of x is

61. If points $\hat{i} + 2\hat{k}$, $\hat{j} + \hat{k}$ and $\lambda\hat{i} + \mu\hat{j}$ are collinear, then $\lambda + \mu$ equals

62. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors, then the points with p.v. $\vec{a} - 2\vec{b} + 3\vec{c}$, $2\vec{a} + \lambda\vec{b} - 4\vec{c}$, $-7\vec{b} + 10\vec{c}$ will be collinear if the value of λ is

63. The number of distinct real values of λ for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar is

64. If $\vec{a}$ and $\vec{b}$ are unit vectors and 60° is the angle between them, then $(2\vec{a} - 3\vec{b}) . (4\vec{a} + \vec{b})$ equals

65. If vectors $3\hat{i} + 2\hat{j} + 8\hat{k}$ and $2\hat{i} + x\hat{j} + \hat{k}$ are perpendicular then -x is equal to

66. If vector $\vec{a} + \vec{b}$ is perpendicular to $\vec{b}$ and $2\vec{b} + \vec{a}$ is perpendicular to $\vec{a}$, then $\left(\frac{|\vec{a}|}{|\vec{b}|}\right)^2$ equals

67. If $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} - 4\hat{j} + 2\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + 2\hat{k}$ then 3 times the projection of $\vec{a} + \vec{b}$ on $\vec{c}$ is

68. If the angle between two vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{j} + a\hat{k}$ is $\pi/3$, then the non-negative value of a is

69. The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is

70. Let $\vec{A}, \vec{B}, \vec{C}$ be vectors of length 3, 4 and 5 respectively. Let $\vec{A}$ be perpendicular to $\vec{B} + \vec{C}$, $\vec{B}$ to $\vec{C} + \vec{A}$ and $\vec{C}$ to $\vec{A} + \vec{B}$. Then square of length of the vector, $\vec{A} + \vec{B} + \vec{C}$ is:

71. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + 3\hat{k}$, then $|\vec{a} \times \vec{b}|^2$ is

72. If $(\vec{a} \times \vec{b})^2 + (\vec{a}.\vec{b})^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}|$ is equal to

73. $(\hat{i} + \hat{j}).\left[(\hat{j} + \hat{k}) \times (\hat{k} + \hat{i})\right]$ equals

74. If the diagonals of a parallelogram are respectively $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$, then the square of the area of parallelogram is

75. Twice of the area of the parallelogram constructed on the vectors $\vec{a} = \vec{p} + 2\vec{q}$ & $\vec{b} = 2\vec{p} + \vec{q}$ where $\vec{p}$ & $\vec{q}$ are unit vectors forming an acute angle of 30° is

76. If the value of $[(\vec{a} + 2\vec{b} - \vec{c})\ \ (\vec{a} - \vec{b})\ \ (\vec{a} - \vec{b} - \vec{c})]$ is equal to $k\ [\vec{a}\ \vec{b}\ \vec{c}]$. Then the value of k is

77. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors & $\vec{p}, \vec{q}, \vec{r}$ are vectors defined by the relations

$$\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]},\ \vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]},\ \vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}.$$

Then the value of the expression; $(\vec{a} + \vec{b}).\vec{p} + (\vec{b} + \vec{c}).\vec{q} + (\vec{c} + \vec{a}).\vec{r}$ is

78. Volume of the tetrahedron whose vertices are represented by the position vectors, A (0, 1, 2) ; B (3, 0, 1); C (4, 3, 6) and D (2, 3, 2) is :

3-DIMENSIONAL GEOMETRY

Introduction and Concept of DC & DR

79. If a line makes angles 90°, 60° and 30° with positive direction of x, y and z-axis respectively, then its direction-cosines are :

(a) $< 0, 0, 0 >$ (b) $< 1, 1, 1 >$

(c) $< 0, \frac{1}{2}, \frac{\sqrt{3}}{2} >$ (d) $< \frac{\sqrt{3}}{2}, \frac{1}{2}, 0 >$

80. A line makes equal angles with co-ordinate axes. Direction cosines of this line are

(a) $\pm 1, \pm 1, \pm 1$ (b) $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

(c) $\pm\frac{1}{3}, \pm\frac{1}{3}, \pm\frac{1}{3}$ (d) $\pm\sqrt{3}, \pm\sqrt{3}, \pm\sqrt{3}$

81. If a line passes through the points (–2, 4, –5) and (1, 2, 3) then its direction-cosines will be :

(a) $< -\frac{2}{\sqrt{77}}, \frac{8}{\sqrt{77}}, \frac{3}{\sqrt{77}} >$ (b) $< -3, 2, +8 >$

(c) $< +\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}} >$ (d) $< 3, -2, 8 >$

82. A line makes acute angles of α, β and γ with the co-ordinate axes such that $\cos\alpha \cos\beta = \cos\beta \cos\gamma = \frac{2}{9}$ and $\cos\gamma \cos\alpha = \frac{4}{9}$, then $\cos\alpha + \cos\beta + \cos\gamma$ is equal to :

(a) 25/9 (b) 5/9

(c) 5/3 (d) 2/3

83. If a line has direction ratios $< 2, -1, -2 >$, then its direction-cosines will be :

(a) $< \frac{2}{3}, \frac{1}{3}, \frac{2}{3} >$ (b) $< \frac{1}{3}, \frac{1}{3}, \frac{1}{3} >$

(c) $< \frac{-2}{3}, -\frac{1}{3}, \frac{-2}{3} >$ (d) $< \frac{2}{3}, -\frac{1}{3}, -\frac{2}{3} >$

84. Two lines, whose direction ratios are :

$< a_1, b_1, c_1 >$ and $< a_2, b_2, c_2 >$ respectively are perpendicular if

(a) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{c_1}{c_2}$ (b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(c) $a_1a_2 + b_1b_2 + c_1c_2 = 0$ (d) $a_1a_2 + b_1b_2 + c_1c_2 = 1$

85. A line makes angles α,β,γ with the coordinate axes. If $\alpha + \beta = 90^o$, then $\gamma =$

(a) 0 (b) 90^o

(c) 180^o (d) None of these

86. The direction cosines of the line, $x = y = z$ are :

(a) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ (b) $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$

(c) $\sqrt{5}, \sqrt{13}, \sqrt{10}$ (d) $\frac{1}{\sqrt{13}}, \frac{1}{\sqrt{10}}, \frac{1}{\sqrt{5}}$

Equation of lines in 3-D

87. A line passes through a point A with p.v. $3\hat{i} + \hat{j} - \hat{k}$ and is parallel to the vector $2\hat{i} - \hat{j} + 2\hat{k}$. If P is a point on this line such that AP = 15 units, then the p.v. of the point P may be:

(a) $13\hat{i} + 4\hat{j} - 9\hat{k}$ (b) $13\hat{i} - 4\hat{j} + 9\hat{k}$

(c) $7\hat{i} - 6\hat{j} + 11\hat{k}$ (d) $+7\hat{i} + 6\hat{j} + 11\hat{k}$

88. Image of the point P with position vector $7\hat{i} - \hat{j} + 2\hat{k}$ in the line whose vector equation is, $\vec{r} = 9\hat{i} + 5\hat{j} + 5\hat{k} + \lambda \ (\hat{i} + 3\hat{j} + 5\hat{k})$ has the position vector :

(a) (–9, 5, 2) (b) (9, 5, –2)

(c) (9, –5, –2) (d) none

89. Find the angle between the two straight lines, $\vec{r} = 3\hat{i} - 2\hat{j} + 4\hat{k} + \lambda \ (-2\hat{i} + \hat{j} + 2\hat{k})$ and

$\vec{r} = \hat{i} + 3\hat{j} - 2\hat{k} + \mu \ (3\hat{i} - 2\hat{j} + 6\hat{k})$:

(a) $\cos^{-1}(4/21)$ (b) $\sin^{-1}(4/21)$

(c) $\sin^{-1}(17/21)$ (d) $\cos^{-1}(17/21)$

90. The lines,

$\vec{r}_1 = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda \ (2\hat{i} + 3\hat{j} + 4\hat{k})$

and $\vec{r}_2 = 2\hat{i} + 3\hat{j} + 4\hat{k} + \mu \ (3\hat{i} + 4\hat{j} + 5\hat{k})$ are :

(a) coplanar (b) skew

(c) such that shortest distance between them is 1

(d) none

91. The equations of x-axis in space are

(a) $x = 0, y = 0$ (b) $x = 0, z = 0$

(c) $x = 0$ (d) $y = 0, z = 0$

92. Two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ would be coplanar if:

(a) $[\vec{a}_1 \ \vec{b}_1 \ \vec{b}_2] = [\vec{a}_2 \ \vec{b}_1 \ \vec{b}_2]$

(b) $(\vec{a}_1 \cdot \vec{b}_1)\vec{b}_2 = (\vec{a}_2 \cdot \vec{b}_1)\vec{b}_2$

(c) $\vec{a}_1 \ (\vec{b}_1 \cdot \vec{b}_2) = \vec{a}_2 \ (\vec{b}_1 \cdot \vec{b}_2)$

(d) $\vec{a}_1 \cdot \vec{b}_1 - \vec{a}_1 \cdot \vec{b}_2 = \vec{a}_2 \cdot \vec{b}_1 - \vec{a}_2 \cdot \vec{b}_2$

93. The point of intersection of lines,

$\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ & $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is :

(a) (–1, –1, –1) (b) (–1, –1, 1)

(c) (1, –1, –1) (d) (–1, 1, –1)

94. The straight lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and

$\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{-2}$ are

(a) parallel lines (b) Intersecting at 60°

(c) Skew lines (d) Intersecting at right angle

95. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The co-ordinates of each of the points of intersection are given by

(a) (3a, 3a, 3a), (a, a, a) (b) (3a, 2a, 3a), (a, a, a)

(c) (3a, 2a, 3a), (a, a, 2a) (d) (2a, 3a, 3a), (2a, a, a)

96. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is

(a) 30° (b) 45°

(c) 90° (d) 0°

97. If the straight lines $x = 1 + s$, $y = -3 - \lambda s$, $z = 1 + \lambda s$ and $x = \frac{t}{2}$, $y = 1 + t$, $z = 2 - t$, with parameters s and t respectively, are coplanar, then λ equals

(a) -2 (b) -1

(c) $-\frac{1}{2}$ (d) 0

98. If the lines

$$\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k} \text{ and } \frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$$

are coplanar, then k can have

(a) any value (b) exactly one value

(c) exactly two values (d) exactly three value

99. The shortest distance between the skew lines $\ell_1 : \vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\ell_2 : \vec{r} = \vec{a}_2 + \mu \vec{b}_2$ is:

(a) $\frac{\left|(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$ (b) $\frac{\left|(\vec{a}_2-\vec{a}_1).(\vec{a}_2\times\vec{b}_2)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$

(c) $\frac{\left|(\vec{a}_2-\vec{b}_2).(\vec{a}_1\times\vec{b}_1)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$ (d) $\frac{\left|(\vec{a}_1-\vec{b}_2).(\vec{b}_1\times\vec{a}_2)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$

Various form of Equation of Planes

100. The line, $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is parallel to the plane :

(a) $3x + 4y + 5z = 7$ (b) $2x + y - 2z = 0$

(c) $x + y - z = 2$ (d) $2x + 3y + 4z = 0$

101. The equation of the plane passing through the points (3, 2, 2) and (1, 0 –1) and parallel to the line $\frac{x-1}{2}=\frac{y-1}{-2}=\frac{z-2}{3}$, is

(a) $4x - y - 2z + 6 = 0$ (b) $4x - y + 2z + 6 = 0$

(c) $4x - y - 2z - 6 = 0$ (d) $3x - 2z - 5 = 0$

102. The equation of a plane which passes through (2, –3, 1) and is normal to the line joining the points (3, 4, –1) and (2, –1, 5) is given by :

(a) $x + 5y - 6z + 19 = 0$ (b) $x - 5y + 6z - 19 = 0$

(c) $x + 5y + 6z + 19 = 0$ (d) $x - 5y - 6z - 19 = 0$

103. A plane meets the coordinate axes in A, B, C and (α,β,γ) is the centroid of the triangle ABC. Then the equation of the plane is

(a) $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$ (b) $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$

(c) $\frac{3x}{\alpha}+\frac{3y}{\beta}+\frac{3z}{\gamma}=1$ (d) $\alpha x + \beta y + \gamma z = 1$

104. The plane $ax + by + cz = 1$ meets the co-ordinate axes in A, B and C. The centroid of the triangle is :

(a) (3a, 3b, 3c) (b) $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$

(c) $\left(\frac{3}{a},\frac{3}{b},\frac{3}{c}\right)$ (d) $\left(\frac{1}{3a},\frac{1}{3b},\frac{1}{3c}\right)$

105. The equation of the plane which is right bisector of the line joining (2, 3, 4) and (6, 7, 8), is :

(a) $x + y + z - 15 = 0$ (b) $x - y + z - 15 = 0$

(c) $x - y - z - 15 = 0$ (d) $x + y + z + 15 = 0$

106. The equation of the plane containing the line $\frac{x-x_1}{\ell}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$, where :

(a) $ax_1 + by_1 + cz_1 = 0$ (b) $a\ell + bm + cn = 0$

(c) $\frac{a}{\ell}=\frac{b}{m}=\frac{c}{n}$ (d) $\ell x_1 + my_1 + nz_1 = 0$

107. The vector equation of the plane passing through the origin and the line of intersection of the plane $\vec{r}.\vec{a}=\lambda$ and $\vec{r}.\vec{b}=\mu$ is :

(a) $\vec{r}.\left(\lambda\vec{a}-\mu\vec{b}\right)=0$ (b) $\vec{r}.\left(\lambda\vec{b}-\mu\vec{a}\right)=0$

(c) $\vec{r}.\left(\lambda\vec{a}+\mu\vec{b}\right)=0$ (d) $\vec{r}.\left(\lambda\vec{b}+\mu\vec{a}\right)=0$

108. Equation of the plane passing through the line of intersection of the planes $P : ax + by + cz + d = 0$, $P' : a'x + b'y + c'z + d' = 0$, and parallel to x-axis is :

(a) $Pa - P'a' = 0$ (b) $P/a = P'/a' = 0$

(c) $Pa + P'a' = 0$ (d) $P/a = P'/a'$

109. The Plane $2x - (1 + \lambda) y + 3\lambda z = 0$ passes through the intersection of the planes

(a) $2x - y = 0$ and $y - 3z = 0$

(b) $2x + 3z = 0$ and $y = 0$

(c) $2x - y + 3z = 0$ and $y - 3z = 0$

(d) none of these

110. The equation of the plane through the intersection of the planes $x + 2y + 3z = 4$ and $2x + y - z = -5$ and perpendicular to the plane $5x + 3y + 6z + 8 = 0$ is

(a) $7x - 2y + 3z + 81 = 0$ (b) $23x + 14y - 9z + 48 = 0$

(c) $51x - 15y - 50z + 173 = 0$ (d) none of these

111. The equation of the plane containing the two lines

$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-0}{3}$ and $\frac{x}{-2} = \frac{y-2}{11} = \frac{z+1}{-1}$ is

(a) $8x + y - 5z - 7 = 0$ (b) $8x + y + 5z - 7 = 0$

(c) $8x - y - 5z - 7 = 0$ (d) none of these

112. The equation of the plane through the intersection of the planes $ax + by + cz + d = 0$ and $lx + my + nz + p = 0$ and parallel to the line $y = 0, z = 0$

(a) $(bl - am) y + (cl - an) z + dl - ap = 0$

(b) $(am - bl) x + (mc - bn) z + md - bp = 0$

(c) $(na - cl) x + (bn - cm) y + nd - cp = 0$

(d) none of these

113. Equation of the plane which passes through the point of intersection of lines $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2}$ and $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and has the largest distance from the origin is:

(a) $7x + 2y + 4z = 54$ (b) $3x + 4y + 5z = 49$

(c) $4x + 3y + 5z = 50$ (d) $5x + 4y + 3z = 57$

Some Formulae of Plane

114. The distance of the point, $(-1, -5, -10)$ from the point of intersection of the line, $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane, $x - y + z = 5$, is :

(a) 10 (b) 11

(c) 12 (d) 13

115. The point at which the line joining the points $(2, -3, 1)$ & $(3, -4, -5)$ intersects the plane $2x + y + z = 7$ is :

(a) $(1, 2, 7)$ (b) $(1, -2, 7)$

(c) $(-1, 2, 7)$ (d) $(1, -2, -7)$

116. The point of intersection of the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+2}{3}$ and the plane, $2x + 3y + z = 0$, is :

(a) $(0, 1, -2)$ (b) $(1, 2, 3)$

(c) $(-1, 9, -25)$ (d) $\left(\frac{-1}{11}, \frac{9}{11}, \frac{-25}{11}\right)$

117. The direction ratio of normal to the plane through $(1, 0, 0), (0, 1, 0)$, which makes an angle $\frac{\pi}{4}$ with plane $x + y = 3$, are :

(a) $1, \sqrt{2}, 1$ (b) $1, 1, \sqrt{2}$

(c) $1, 1, 2$ (d) $\sqrt{2}, 1, 1$

118. If line $\vec{r} = (\hat{i} - 2\hat{j} - \hat{k}) + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$ is parallel to the plane $\vec{r} \cdot (3\hat{i} - 2\hat{j} - m\hat{k}) = 14$ then the value of m is

(a) $+2$

(b) -2

(c) 0

(d) can not be predicted with this much informations

119. The distance of the point $(1, -5, 9)$ from the plane $x - y + z = 5$ measured along a straight line $x = y = z$ is

(a) $3\sqrt{5}$ (b) $10\sqrt{3}$

(c) $5\sqrt{3}$ (d) $3\sqrt{10}$

120. The reflection of the point (α, β, γ) in the xy – plane is

(a) $(\alpha, \beta, 0)$ (b) $(0, 0, \gamma)$

(c) $(-\alpha, -\beta, \gamma)$ (d) $(\alpha, \beta, -\gamma)$

121. The plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}(\alpha)$ with x-axis. The value of α is equal to

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{2}}{3}$

(c) $\frac{2}{7}$ (d) $\frac{3}{7}$

122. If a plane passes through the point (1, 1, 1) and is perpendicular to the line $\frac{x-1}{3}=\frac{y-1}{0}=\frac{z-1}{4}$ then its perpendicular distance from the origin is

(a) 3/4 (b) 4/3

(c) 7/5 (d) 1

123. The angle between the planes, $2x - y + z = 6$ and $x + y + 2z = 7$ is

(a) 30º (b) 45º

(c) 0º (d) 60º

124. The length of the perpendicular from origin on plane $\vec{r}\cdot(3\hat{i}-4\hat{j}+12\hat{k})=5$ is

(a) $\frac{5}{69}$ (b) $\frac{25}{69}$

(c) $\frac{5}{13}$ (d) $\sqrt{\frac{5}{13}}$

125. The angle between the line, $\frac{x+1}{3}=\frac{y-1}{2}=\frac{z-2}{4}$ and the plane, $2x + y - 3z + 4 = 0$, is :

(a) $\sin^{-1}\left(\frac{4}{\sqrt{406}}\right)$ (b) $\sin^{-1}\left(\frac{14}{\sqrt{406}}\right)$

(c) $\sin^{-1}\left(\frac{4}{14\sqrt{29}}\right)$ (d) None of these

126. If the given planes, $ax + by + cz + d = 0$ and $a'x+b'y+c'z+d'=0$ be mutually perpendicular, then:

(a) $\frac{a}{a'}=\frac{b}{b'}=\frac{c}{c'}$ (b) $\frac{a}{a'}+\frac{b}{b'}+\frac{c}{c'}=0$

(c) $aa' + bb' + cc' + dd' = 0$

(d) $aa' + bb' + cc' = 0$

127. The angle between the line $\frac{x-2}{a}=\frac{y-2}{b}=\frac{z-2}{c}$ and the plane $ax + by + cz + 6 = 0$ is

(a) $\sin^{-1}\frac{1}{\sqrt{a^2+b^2+c^2}}$ (b) 45º

(c) 60º (d) 90º

128. The image of the point P (1, 3, 4) in the plane $2x - y + z + 3 = 0$ is

(a) (3, 5, –2) (b) (–3, 5, 2)

(c) (3, –5, 2) (d) (–1, 4, 2)

129. The image of the points (–1, 3, 4) in the plane $x - 2y = 0$ is

(a) (15, 11, 4) (b) $\left(-\frac{17}{3},-\frac{19}{3},1\right)$

(c) (8, 4, 4) (d) $\left(\frac{9}{5},-\frac{13}{5},4\right)$

130. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$

(c) $\frac{7}{2}$ (d) $\frac{9}{2}$

131. If the distance between planes, $4x - 2y - 4z + 1 = 0$ and $4x - 2y - 4z + d = 0$ is 7, then d is:

(a) 41 or - 42 (b) 42 or - 43

(c) -41 or 43 (d) -42 or 44

132. If the angle between the line $2(x+1) = y = z + 4$ and the plane $2x-y+\sqrt{\lambda}z+4=0$ is $\frac{\pi}{6}$, then the value of λ is

(a) $\frac{135}{7}$ (b) $\frac{45}{11}$

(c) $\frac{45}{7}$ (d) $\frac{135}{11}$

133. A symmetrical form of the line of intersection of the planes $x = ay + b$ and $z = cy + d$ is:

(a) $\frac{x-b}{a}=\frac{y-1}{1}=\frac{z-d}{c}$

(b) $\frac{x-b-a}{a}=\frac{y-1}{1}=\frac{z-d-c}{c}$

(c) $\frac{x-a}{b}=\frac{y-0}{1}=\frac{z-c}{d}$

(d) $\frac{x-b-a}{b}=\frac{y-1}{0}=\frac{z-d-c}{d}$

Numerical Value Type Questions

134. If a line makes angles α, β, γ with the co-ordinate axes, then $-(\cos 2\alpha + \cos 2\beta + \cos 2\gamma)$ is :

135. If the straight lines $x = 1 + s$, $y = -3 - \lambda s$, $z = 1 + \lambda s$ and $x = t/2$, $y = 1 + t$, $z = 2 - t$, with parameter s and t respectively, are coplanar, then $-\lambda$ equals to

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

VECTORS

1. The length of the projection of the line segment joining the points (5, -1, 4) and (4, -1, 3) on the plane, $x+y+z=7$ is : **(2018)**

(a) $\sqrt{\frac{2}{3}}$ (b) $\frac{2}{\sqrt{3}}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

2. If the position vectors of the vertices A, B and C of a ΔABC are respectively $4\hat{i}+7\hat{j}+8\hat{k}$, $2\hat{i}+3\hat{j}+4\hat{k}$ and $2\hat{i}+5\hat{j}+7\hat{k}$, then the position vector of the point, where the bisector of $\angle A$ meets BC is : **(2018/Online Set–2)**

(a) $\frac{1}{2}\left(4\hat{i}+8\hat{j}+11\hat{k}\right)$ (b) $\frac{1}{3}\left(6\hat{i}+11\hat{j}+15\hat{k}\right)$

(c) $\frac{1}{3}\left(6\hat{i}+13\hat{j}+18\hat{k}\right)$ (d) $\frac{1}{4}\left(8\hat{i}+14\hat{j}+19\hat{k}\right)$

3. The magnitude of the projection of the vector $2\hat{i}+3\hat{j}+\hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$, is

(2019-04-08/Shift-1)

(a) $\frac{\sqrt{3}}{2}$ (b) $\sqrt{6}$

(c) $3\sqrt{6}$ (d) $\sqrt{\frac{3}{2}}$

4. Let $\vec{a}=3\hat{i}+2\hat{j}+x\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$, for some real x. Then $\left|\vec{a}\times\vec{b}\right|=r$ is possible if: **(2019-04-08/Shift-2)**

(a) $\sqrt{\frac{3}{2}}<r\le 3\sqrt{\frac{3}{2}}$ (b) $r\ge 5\sqrt{\frac{3}{2}}$

(c) $0<r\le\sqrt{\frac{3}{2}}$ (d) $3\sqrt{\frac{3}{2}}<r<5\sqrt{\frac{3}{2}}$

5. Let $\vec{\alpha}=3\hat{i}+\hat{j}$ and $\vec{\beta}=2\hat{i}-\hat{j}+3\hat{k}$. If $\vec{\beta}=\vec{\beta}_1-\vec{\beta}_2$ where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$ then $\vec{\beta}_1\times\vec{\beta}_2$ is equal to: **(2019-04-09/Shift-1))**

(a) $-3\hat{i}+9\hat{j}+5\hat{k}$ (b) $3\hat{i}-9\hat{j}-5\hat{k}$

(c) $\frac{1}{2}\left(-3\hat{i}+9\hat{j}+5\hat{k}\right)$ (d) $\frac{1}{2}\left(3\hat{i}-9\hat{j}+5\hat{k}\right)$

6. Let $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ has the magnitude 12 then one such vector is

(2019-04-12/Shift-1)

(a) $4\left(2\hat{i}+2\hat{j}+2\hat{k}\right)$ (b) $4\left(2\hat{i}-2\hat{j}-\hat{k}\right)$

(c) $4\left(2\hat{i}+2\hat{j}-\hat{k}\right)$ (d) $4\left(-2\hat{i}-2\hat{j}+\hat{k}\right)$

7. If the volume of parallelepiped formed by the vectors $\hat{i}+\lambda\hat{j}+\hat{k}$, $\hat{j}+\lambda\hat{k}$ and $\lambda\hat{i}+\hat{k}$ is minimum, then λ is equal to **(2019-04-12/Shift-1)**

(a) $-\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $-\sqrt{3}$

8. Let $\alpha\in R$ and the three vectors

$\vec{a}=\alpha\hat{i}+\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}-\alpha\hat{k}$ and

$\vec{c}=\alpha\hat{i}-2\hat{j}+3\hat{k}$.

Then the set $S=\left(\alpha:\vec{a},\vec{b}\text{ and }\vec{c}\text{ are coplanar}\right)$

(2019-04-12/Shift-2)

(a) is singleton

(b) is empty

(c) contains exactly two positive numbers

(d) contains exactly two numbers only one of which is positive

9. Let $\vec{a}=\hat{i}-\hat{j}, \vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a}\times\vec{c}+\vec{b}=\vec{0}$ and $\vec{a}.\vec{c}=4$, then $|\vec{c}|^2$ is equal to: **(2019-01-09/Shift-1)**

(a) $\frac{19}{2}$ (b) 9

(c) 8 (d) $\frac{17}{2}$

10. Let $\vec{a}=\hat{i}+\hat{j}+\sqrt{2}\hat{k}, \vec{b}=b_1\hat{i}+b_2\hat{j}+\sqrt{2}\hat{k}$ and $\vec{c}=5\hat{i}+\hat{j}+\sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $|\vec{a}|$. If $\vec{a}+\vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to: **(2019-01-09/Shift-2)**

(a) $\sqrt{32}$ (b) 6

(c) $\sqrt{22}$ (d) 4

11. Let $\vec{a}=2\hat{i}+\lambda_1\hat{j}+3\hat{k}, \vec{b}=4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}$ and $\vec{c}=3\hat{i}+6\hat{j}+(\lambda_3-1)\hat{k}$ be three vectors such that $\vec{b}=2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1,\lambda_2,\lambda_3)$ is: **(2019-01-10/Shift-1)**

(a) (1, 3, 1) (b) $\left(-\frac{1}{2},4,0\right)$

(c) $\left(\frac{1}{2},4,-2\right)$ (d) (1, 5, 1)

12. Let $\vec{\alpha}=(\lambda-2)\vec{a}+\vec{b}$ and $\vec{\beta}=(4\lambda-2)\vec{a}+3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is: **(2019-01-10/Shift-2)**

(a) -4 (b) -3

(c) 4 (d) 3

13. Let $\vec{a}=\hat{i}+2\hat{j}+4\hat{k}, \vec{b}=\hat{i}+\lambda\hat{j}+4\hat{k}$ and $\vec{c}=2\hat{i}+4\hat{j}+(\lambda^2-1)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a}\times\vec{c}$ is: **(2019-01-11/Shift-1)**

(a) $-10\hat{i}-5\hat{j}$ (b) $-14\hat{i}-5\hat{j}$

(c) $-14\hat{i}+5\hat{j}$ (d) $-10\hat{i}+5\hat{j}$

14. Let $\sqrt{3}\hat{i}+\hat{j}, \hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is **(2019-01-11/Shift-2)**

(a) 4 (b) 3

(c) 2 (d) 1

15. The sum of the distinct real values of μ, for which the vectors, $\mu\hat{i}+\hat{j}+\hat{k}, \hat{i}+\mu\hat{j}+\hat{k}, \hat{i}+\hat{j}+\mu\hat{k}$ are co-planar, is _. **(2019-01-12/Shift-2)**

(a) -1 (b) 0

(c) 1 (d) 2

16. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}\vec{b}$, then $|\alpha-\beta|$ is equal to **(2019-01-12/Shift-2)**

(a) 30° (b) 90°

(c) 60° (d) 45°

17. Let $\vec{a}, \vec{b}$ *and* $\vec{c}$ be three unit vectors such that $|\vec{a}-\vec{b}|^2+|\vec{a}-\vec{c}|^2=8$. Then $|\vec{a}+2\vec{b}|^2+|\vec{a}+2\vec{c}|^2$ is equal to : **(2020-09-02/Shift-1)**

18. Let the position vectors of points 'A' and 'B' be $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio $\lambda:1 (\lambda>0)$. If O is the origin and $\overrightarrow{OB}.\overrightarrow{OP}-3\left|\overrightarrow{OA}\times\overrightarrow{OP}\right|^2=6$, then λ is equal to .... **(2020-09-02/Shift-2)**

19. Let $a,b,c \in R$ be such that $a^2+b^2+c^2=1$, If $a\cos\theta = b\cos\left(\theta+\frac{2\pi}{3}\right) = c\cos\left(\theta+\frac{4\pi}{3}\right)$, where $\theta=\frac{\pi}{9}$, then the angle between the vectors $a\hat{i}+b\hat{j}+c\hat{k}$ and $b\hat{i}+c\hat{j}+a\hat{k}$ is **(2020-09-03/Shift-2)**

(a) $\frac{\pi}{2}$ (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{9}$ (d) 0

20. Let x_0 be the point of local maxima of $f(x) = \vec{a}.(\vec{b}\times\vec{c})$, where $\vec{a}=x\hat{i}-2\hat{j}+3\hat{k}, \vec{b}=-2\hat{i}+x\hat{j}-\hat{k}$ and $\vec{c}=7\hat{i}-2\hat{j}+x\hat{k}$. Then the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ at $x=x_0$ is : **(2020-09-04/Shift-1)**

(a) – 22 (b) – 4

(c) – 30 (d) 14

21. If $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$, then the value of $\left|\hat{i}\times(\vec{a}\times\hat{i})\right|^2+\left|\hat{j}\times(\vec{a}\times\hat{j})\right|^2+\left|\hat{k}\times(\vec{a}\times\hat{k})\right|^2$ is equal to ... . **(2020-09-04/Shift-2)**

22. If the volume of a parallelopiped, whose coterminous edges are given by the vectors $\vec{a}=\hat{i}+\hat{j}+n\hat{k}, \vec{b}=2\hat{i}+4\hat{j}-n\hat{k}$ and $\vec{c}=\hat{i}+n\hat{j}+3\hat{k}(n\ge 0)$, is 158 cu. units, then: **(2020-09-05/Shift-1)**

(a) $\vec{a}.\vec{c}=17$ (b) $\vec{b}.\vec{c}=10$

(c) n = 9 (d) n = 7

23. Let the vectors $\vec{a},\vec{b},\vec{c}$ be such that $|\vec{a}|=2, |\vec{b}|=4$ and $|\vec{c}|=4$. If the projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$, then the value of $|\vec{a}+\vec{b}-\vec{c}|$ is______ **(2020-09-05/Shift-2)**

24. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$ is_____. **(2020-09-06/Shift-1)**

25. If $\vec{x}$ and $\vec{y}$ be two non-zero vectors such that $|\vec{x}+\vec{y}|=|\vec{x}|$ and $2\vec{x}+\lambda\vec{y}$ is perpendicular to $\vec{y}$, then the value of λ is _______ **(2020-09-06/Shift-2)**

26. A vector $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}(\alpha,\beta\in R)$ lies in the plane of the vectors, $\vec{b}=\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}-\hat{j}+4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then **(2020-01-07/Shift-1)**

(a) $\vec{a}.\hat{i}+3=0$ (b) $\vec{a}.\hat{k}+4=0$

(c) $\vec{a}.\hat{i}+1=0$ (d) $\vec{a}.\hat{k}+2=0$

27. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$. If $\lambda=\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ and $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$, then the ordered pair $(\lambda,\vec{d})$ is equal to : **(2020-01-07/Shift-2)**

(a) $\left(\frac{3}{2}, 3\vec{a}\times\vec{c}\right)$ (b) $\left(-\frac{3}{2}, 3\vec{c}\times\vec{b}\right)$

(c) $\left(-\frac{3}{2}, 3\vec{a}\times\vec{b}\right)$ (d) $\left(\frac{3}{2}, 3\vec{b}\times\vec{c}\right)$

28. If the foot of perpendicular drawn from the point (1,0,3) on a line passing through $(\alpha,7,1)$ is $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$, then α equal to_____ **(2020-01-07/Shift-2)**

29. If volume of parallelopiped whose coterminous edges are given by $\vec{u}=\hat{i}+\hat{j}+\lambda\hat{k}$, $\vec{v}=\hat{i}+\hat{j}+3\hat{k}$ and $\vec{w}=2\hat{i}+\hat{j}+\hat{k}$ be 1 cu. Unit. If θ be the angle between the edges $\vec{u}$ and $\vec{w}$, then, $\cos\theta$ can be: **(2020-01-08/Shift-1)**

(a) $\frac{7}{6\sqrt{6}}$ (b) $\frac{5}{7}$

(c) $\frac{7}{6\sqrt{3}}$ (d) $\frac{5}{3\sqrt{3}}$

30. Let $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b}\times\vec{c}=\vec{b}\times\vec{a}$ and $\vec{c}.\vec{a}=0$ then $\vec{c}.\vec{b}$ is equal to: **(2020-01-08/Shift-2)**

(a) $\frac{1}{2}$ (b) $-\frac{3}{2}$

(c) $-\frac{1}{2}$ (d) -1

31. If the vectors $\vec{p}=(a+1)\hat{i}+a\hat{j}+a\hat{k}$, $\vec{q}=a\hat{i}+(a+1)\hat{j}+a\hat{k}$ and $\vec{r}=a\hat{i}+a\hat{j}+(a+1)\hat{k}$ $(a\in R)$ are coplanar and $3(\vec{p}.\vec{q})^2-\lambda|\vec{r}\times\vec{q}|^2=0$, then value of λ is____. **(2020-01-09/Shift-1)**

32. The projection of the line segment joining the points (1,-1,3) and (2,-4,11) on the line joining the points (-1,2,3) and (3,-2,10) is ________ . **(2020-01-09/Shift-1)**

33. $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=\sqrt{3}, |\vec{b}|=5, \vec{b}.\vec{c}=10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $\vec{a}$ is `perpendicular to vector $\vec{b}\times\vec{c}$, then $|\vec{a}\times(\vec{b}\times\vec{c})|$ is equal to **(2020-01-09/Shift-2)**

34. Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}.\vec{c}=|\vec{c}|$, $|\vec{c}-\vec{a}|=2\sqrt{2}$ and the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $|(\vec{a}\times\vec{b})\times\vec{c}|$ is: **(2021-07-20/Shift-1)**

(a) $\frac{2}{3}$ (b) 4

(c) 3 (d) $\frac{3}{2}$

35. Let $\vec{a},\vec{b},\vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ with the vector $\vec{a}+\vec{b}+\vec{c}$. Then $36\cos^2 2\theta$ is equal to ____ **(2021-07-20/Shift-1)**

36. For $p>0$ a vector $\vec{v}_2=2\hat{i}+(p+1)\hat{j}$ is obtained by rotating the vector $\vec{v}_1=\sqrt{3}p\hat{i}+\hat{j}$ by an angle θ about the origin in a counter clockwise direction. If $\tan\theta=\frac{(\alpha\sqrt{3}-2)}{4\sqrt{3}+3}$, then the value of α is equal to ? **(2021-07-20/Shift-2)**

37. In a triangle ABC, if $|\overline{BC}|=3, |\overline{CA}|=5$ and $|\overline{BA}|=7$, then the projection of the vector $\overrightarrow{BA}$ on $\overrightarrow{BC}$ is equal to ? **(2021-07-20/Shift-2)**

(a) $\frac{11}{2}$ (b) $\frac{13}{2}$

(c) $\frac{19}{2}$ (d) $\frac{15}{2}$

38. Let the vectors $(2+a+b)\hat{i}+(a+2b+c)\hat{j}-(b+c)k$, $(1+b)\hat{i}+2b\hat{j}-bk$ and $(2+b)\hat{i}+2b\hat{j}+(1-b)k$ $a,b,c\in R$ be co-planar. Then which of the following is true ? **(2021-07-25/Shift-1)**

(a) 2a = b + c (b) 2b = a + c

(c) 3c = a + b (d) a = b + 2c

39. Let $\vec{p}=2\hat{i}+3\hat{j}+k$ and $\vec{q}=\hat{i}+2\hat{j}+k$ be two vectors. If a vector $\vec{r}=(\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k})$ is perpendicular to each of the vectors $(\vec{p}+\vec{q})$ and $(\vec{p}-\vec{q})$ and $|\vec{r}|=\sqrt{3}$, then $|\alpha|+|\beta|+|\gamma|$ is equal to ________ ? **(2021-07-25/Shift-1)**

40. Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + 3\hat{k}$. Then the vector product $(\vec{a}+\vec{b})\times(\vec{a}\times((\vec{a}-\vec{b})\times\vec{b}))\times\vec{b}$ is equal to: **(2021-07-27/Shift-1)**

(a) $5(30\hat{i}-5\hat{j}+7\hat{k})$ (b) $5(34\hat{i}-5\hat{j}+3\hat{k})$

(c) $7(30\hat{i}-5\hat{j}+7\hat{k})$ (d) $7(34\hat{i}-5\hat{j}+3\hat{k})$

41. Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}, \vec{b}$ and $\vec{c} = \hat{j}-\hat{k}$ be three vectors such that $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{a}.\vec{b}=1$. If the length of projection vector of the vector $\vec{b}$ on the vector $\vec{a}\times\vec{c}$ is l, then the value of $3l^2$ is equal to ______. **(2021-07-27/Shift-1)**

42. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} = \vec{b}\times(\vec{b}\times\vec{c})$. If magnitudes of the vectors $\vec{a},\vec{b}$ and $\vec{c}$ are $\sqrt{2},1$ and 2 respectively and the angle between $\vec{b}$ and $\vec{c}$ is $\theta\left(0<\theta<\frac{\pi}{2}\right)$, then the value of $\theta\left(0<\theta<\frac{\pi}{2}\right)$ is equal to: **(2021-07-27/Shift-2)**

(a) $\frac{\sqrt{3}+1}{\sqrt{3}}$ (b) 2

(c) $\sqrt{3}+1$ (d) 1

43. Let $\vec{a} = \hat{i}-\alpha\hat{j}+\beta\hat{k}, \vec{b} = 3\hat{i}+\beta\hat{j}-\alpha\hat{k}$ and $\vec{c} = -\alpha\hat{i}-2\hat{j}+\hat{k}$, where α and β are integers. If $\vec{a}.\vec{b}=-1$ and $\vec{b}.\vec{c}=10$, then $(\vec{a}\times\vec{b}).\vec{c}$ is equal to ______. **(2021-07-27/Shift-2)**

44. Let three vectors $\vec{a},\vec{b}$ and $\vec{c}$ be such that $\vec{a}\times\vec{b}=\vec{c}, \vec{b}\times\vec{c}=\vec{a}$ and $|\vec{a}|=2$. Then which one of the following is not true? **(2021-07-22/Shift-2)**

(a) Projection of $\vec{a}$ on $(\vec{b}\times\vec{c})$ is 2

(b) $|3\vec{a}+\vec{b}-2\vec{c}|^2 = 51$

(c) $[\vec{a}\ \vec{b}\ \vec{c}]+[\vec{c}\ \vec{a}\ \vec{b}]=8$

(d) $\vec{a}\times((\vec{b}+\vec{c})\times(\vec{b}-\vec{c}))=\vec{0}$

45. Let a vector $\vec{a}$ be coplanar with vectors $\vec{b} = 2\hat{i}+\hat{j}+\hat{k}$ and $\vec{c} = \hat{i}-\hat{j}+\hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d} = 3\hat{i}+2\hat{j}+6\hat{k}$, and $|\vec{a}|=\sqrt{10}$. Then a possible value of $[\vec{a}\ \vec{b}\ \vec{c}]+[\vec{a}\ \vec{d}\ \vec{b}]+[\vec{a}\ \vec{c}\ \vec{d}]$ is equal to : **(2021-07-22/Shift-2)**

(a) –40 (b) –42

(c) –29 (d) –38

46. Let a, b and c be distinct positive numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}, \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are co-planar, then c is equal to: **(2021-07-25/Shift-2)**

(a) $\sqrt{ab}$ (b) $\frac{a+b}{2}$

(c) $\frac{1}{a}+\frac{1}{b}$ (d) $\frac{2}{\frac{1}{a}+\frac{1}{b}}$

47. If $|\vec{a}|=2, |\vec{b}|=5$ and $|\vec{a}\times\vec{b}|=8$, then $|\vec{a}.\vec{b}|$ is equal to: **(2021-07-25/Shift-2)**

(a) 5 (b) 4

(c) 6 (d) 3

48. If $(\vec{a}+3\vec{b})$ is perpendicular to $(7\vec{a}-5\vec{b})$ and $(\vec{a}-4\vec{b})$ is perpendicular to $(7\vec{a}-2\vec{b})$, then the angle between $\vec{a}$ and $\vec{b}$ (in degrees) is___ **(2021-07-25/Shift-2)**

49. Let $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$. Let a vector $\vec{v}$ in the plane containing $\vec{a}$ and $\vec{b}$. If $\vec{v}$ is perpendicular to the vector $3\vec{i}+2\hat{j}-\hat{k}$ and its projection on $\vec{a}$ is 19 units, then $|2\vec{v}|^2$ is equal to _____ **(2021-09-01/Shift-2)**

50. If the projection of the vector $\hat{i}+2\hat{j}+\hat{k}$ on the sum of the two vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $-\lambda\hat{i}+2\hat{j}+3\hat{k}$ is 1, then λ is equal to ________. **(2021-08-26/Shift-2)**

51. Let $\vec{a}=\hat{i}+5\hat{j}+\alpha\hat{k}$, $\vec{b}=\hat{i}+3\hat{j}+\beta\hat{k}$ and $\vec{c}=-\hat{i}+2\hat{j}-3\hat{k}$ be three vectors such that, $|\vec{b}\times\vec{c}|=5\sqrt{3}$ and $\vec{a}$ is perpendicular to $\vec{b}$. Then the greatest amongst the values of $|\vec{a}|^2$ is ______. **(2021-08-27/Shift-1)**

52. Let $\bar{a}=\hat{i}+\hat{j}+\hat{k}$ and $\bar{b}=\hat{j}-\hat{k}$. If $\bar{c}$ is a vector such that $\bar{a}\times\bar{c}=\bar{b}$ and $\bar{a}.\bar{c}=3$, then $\bar{a}.(\bar{b}\times\bar{c})$ is equal to : **(2021-08-26/Shift-1)**

(a) -2 (b) 6

(c) 2 (d) -6

53. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|2\vec{a}+3\vec{b}|=|3\vec{a}+\vec{b}|$ and the angle between $\vec{a}$ and $\vec{b}$ is 60°. If $\frac{1}{8}\vec{a}$ is a unit vector, then $|\vec{b}|$ is equal to ? **(2021-08-31/Shift-1)**

(a) 5 (b) 8

(c) 4 (d) 6

54. Let $\vec{a},\vec{b},\vec{c}$ be three vectors mutually perpendicular to each other and have same magnitude. If a vector $\vec{r}$ satisfies

$$\vec{a}\times\{(\vec{r}-\vec{b})\times\vec{a}\}+\vec{b}\times\{(\vec{r}-\vec{c})\times\vec{b}\}+\vec{c}\times\{(\vec{r}-\vec{a})\times\vec{c}\}=\vec{0},$$

then $\vec{r}$ is equal to: **(2021-08-31/Shift-2)**

(a) $\frac{1}{2}(\vec{a}+\vec{b}+2\vec{c})$ (b) $\frac{1}{2}(2\vec{a}+\vec{b}-\vec{c})$

(c) $\frac{1}{3}(\vec{a}+\vec{b}+\vec{c})$ (d) $\frac{1}{2}(\vec{a}+\vec{b}+\vec{c})$

55. Let $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+5\hat{k}$.

If $\vec{r}\times\vec{a}=\vec{b}\times\vec{r}$, $\vec{r}\cdot(\alpha\hat{i}+2\hat{j}+\hat{k})=3$ and

$\vec{r}\cdot(2\hat{i}+5\hat{j}-\alpha\hat{k})=-1, \alpha\in R$, then the value of $\alpha+|\vec{r}|^2$ is equal to **(2021-03-16/Shift-2)**

(a) 13 (b) 15

(c) 9 (d) 11

56. Let $\vec{c}$ be a vector perpendicular to the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$. If $\vec{c}\cdot(\hat{i}+\hat{j}+3\hat{k})=8$ then the value of $\vec{c}\cdot(\vec{a}\times\vec{b})$ is equal to ___ **(2021-03-16/Shift-2)**

57. Let a vector $\alpha\hat{i}+\beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i}+\hat{j}$ by an angle 45° about the origin in counter clockwise direction in the first quadrant. Then the area of triangle having vertices (α, β), $(0, \beta)$ and (0, 0) is equal to **(2021-03-16/Shift-1)**

(a) 1 (b) $\frac{1}{\sqrt{2}}$

(c) $2\sqrt{2}$ (d) $\frac{1}{2}$

58. Let the position vectors of two points P and Q be $3\hat{i}-\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}-4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are (4, –1, 2) and (–2, 1, –2), respectively. Let lines PR and QS intersect at T. If the vector $\overline{TA}$ is perpendicular to both $\overline{PR}$ and $\overline{QS}$ and the length of vector $\overline{TA}$ is $\sqrt{5}$ units, then the modulus of a position vector of A is **(2021-03-16/Shift-1)**

(a) $\sqrt{5}$ (b) $\sqrt{171}$

(c) $\sqrt{227}$ (d) $\sqrt{482}$

59. Let O be the origin. Let $\overline{OP}=x\hat{i}+y\hat{j}-\hat{k}$ and $\overline{OQ}=-\hat{i}+2\hat{j}+3x\hat{k}$, $x, y\in R$, $x>0$, be such that $|\overline{PQ}|=\sqrt{20}$ and the vector $\overline{OP}$ is perpendicular to $\overline{OQ}$. If $\overline{OR}=3\hat{i}+z\hat{j}-7\hat{k}$, $z\in R$, is coplanar with $\overline{OP}$ and $\overline{OQ}$, then the value of $x^2+y^2+z^2$ is equal to : **(2021-03-17/Shift-2)**

(a) 1 (b) 9

(c) 2 (d) 7

60. Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$. If the vector $\vec{x}$ is perpendicular to $(3\hat{i}+2\hat{j}-\hat{k})$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $|\vec{x}|^2$ is equal to .... **(2021-03-17/Shift-2)**

61. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 7\hat{i} + \hat{j} - 6\hat{k}$.

If $\vec{r} \times \vec{a} = \vec{r} \times \vec{b}$, $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -3$, then $\vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k})$ is equal to : **(2021-03-17/Shift-1)**

(a) 12 (b) 13
(c) 10 (d) 8

62. Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors perpendicular to each other and $|\vec{a}| = |\vec{b}|$. If $|\vec{a} \times \vec{b}| = |\vec{a}|$, then the angle between the vectors $(\vec{a} + \vec{b} + (\vec{a} \times \vec{b}))$ and $\vec{a}$ is equal to : **(2021-03-18/Shift-2)**

(a) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$ (b) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(c) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

63. A vector $\vec{a}$ has components 3p and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, $\vec{a}$ has components p + 1 and $\sqrt{10}$, then a value of p is equal to : **(2021-03-18/Shift-1)**

(a) 1 (b) $\frac{4}{5}$

(c) $-\frac{5}{4}$ (d) –1

64. Let three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{c} = 7$ and $\vec{b}$ is perpendicular to $\vec{c}$ where $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{k}$, then the value of $2|\vec{a} + \vec{b} + \vec{c}|^2$ is __________. **(2021-02-24/Shift-1)**

65. Let $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a} \cdot \vec{b}$ is equal to: **(2021-02-25/Shift-2)**

66. Let $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $\vec{r} \cdot \vec{a}$ is equal to ____. **(2021-02-25/Shift-1)**

67. If vectors $\vec{a}_1 = x\hat{i} - \hat{j} + \hat{k}$ and $\vec{a}_2 = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i} + y\hat{j} + z\hat{k}$ is **(2021-02-26/Shift-2)**

(a) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$ (b) $\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$

(c) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$ (d) $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$

68. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a} \times (\vec{a} \times (\vec{a} \times (\vec{a} \times \vec{b})))$ is equal to **(2021-02-26/Shift-1)**

(a) $\vec{0}$ (b) $\frac{1}{2}|\vec{a}|^4 \vec{b}$

(c) $\vec{a} \times \vec{b}$ (d) $|\vec{a}|^4 \vec{b}$

3-DIMENSIONAL GEOMETRY

69. The shortest distance between the z-axis and the line x +y + 2z - 3 = 0 = 2x + 3y + 4z - 4, is **(2015/Online Set–2)**

(a) 1 (b) 2
(c) 4 (d) 3

70. A plane containing the point (3, 2, 0) and the line $\frac{x-1}{1} = \frac{y-2}{5} = \frac{z-3}{4}$ also contains the point : **(2015/Online Set–2)**

(a) (0, 3, 1) (b) (0, 7, -10)
(c) (0, -3, 1) (d) (0, 7, 10)

71. If the line, $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in the plane, $lx + my - z = 9$, then $l^2 + m^2$ is equal to : **(2016)**

(a) 18 (b) 5
(c) 2 (d) 26

72. The distance of the point (1, –5, 9) from the plane x – y + z = 5 measured along the line x = y = z is **(2016)**

(a) $10\sqrt{3}$ (b) $\frac{10}{\sqrt{3}}$

(c) $\frac{20}{3}$ (d) $3\sqrt{10}$

73. The shortest distance between the lines lies $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$ in the interval : **(2016/Online Set–1)**

(a) [0, 1)
(b) [1, 2)
(c) (2, 3]
(d) (3, 4]

74. The distance of the point (1, -2, 4) from the plane passing through the point (1, 2, 2) and perpendicular to the planes x-y+2z=3 and 2x-2y+z+12=0, is : **(2016/Online Set–1)**

(a) $2\sqrt{2}$
(b) 2
(c) $\sqrt{2}$
(d) $\frac{1}{\sqrt{2}}$

75. The number of distinct real values of λ for which the lines

$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{\lambda^2}$ and $\frac{x-3}{1}=\frac{y-2}{\lambda^2}=\frac{z-1}{2}$

are coplanar is : **(2016/Online Set–2)**

(a) 4
(b) 1
(c) 2
(d) 3

76. If the image of the point P(1, –2, 3) in the plane, 2x + 3y – 4z + 22 = 0 measured parallel to the line, $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$ is Q then PQ is equal to : **(2017)**

(a) $3\sqrt{5}$
(b) $2\sqrt{42}$
(c) $\sqrt{42}$
(d) $6\sqrt{5}$

77. The coordinates of the foot of the perpendicular from the point (1, –2, 1) on the plane containing the lines,

$\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ and $\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$ is :

(2017/Online Set–1)

(a) (2, –4, 2)
(b) (-1, 2, –1)
(c) (0, 0, 0)
(d) (1, 1, 1)

78. The line of intersection of the planes $=\vec{r}.\left(3\hat{i}-\hat{j}+\hat{k}\right)$ and $\vec{r}.\left(\hat{i}+4\hat{j}-2\hat{k}\right)=2,$ is: **(2017/Online Set–1)**

(a) $\frac{x-\frac{4}{7}}{-2}=\frac{y}{7}=\frac{z-\frac{5}{7}}{13}$

(b) $\frac{x-\frac{4}{7}}{2}=\frac{y}{-7}=\frac{z+\frac{5}{7}}{13}$

(c) $\frac{x-\frac{6}{13}}{2}=\frac{y+\frac{5}{13}}{-7}=\frac{z}{-13}$

(d) $\frac{x-\frac{6}{13}}{2}=\frac{y-\frac{5}{13}}{-7}=\frac{z}{-13}$

79. If the line, $\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z+\lambda}{-2}$ lies in the plane, 2x – 4y + 3z = 2, then the shortest distance between this line and the line, $\frac{x-1}{12}=\frac{y}{9}=\frac{z}{4}$ is : **(2017/Online Set–2)**

(a) 2
(b) 1
(c) 0
(d) 3

80. If a variable plane, at a distance of 3 units from the origin, intersects the coordinate axes at A, B and C, then the locus of the centroid of ΔABC is : **(2017/Online Set–2)**

(a) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=1$
(b) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=3$
(c) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{9}$
(d) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=9$

81. If L_1 is the line of intersection of the planes $2x-2y+3z-2=0,\ x-y+z+1=0$ and L_2 is the line of intersection of the planes

$x+2y-z-3=0,\ 3x-y+2z-1=0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is : **(2018)**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{4\sqrt{2}}$

(c) $\frac{1}{3\sqrt{2}}$ (d) $\frac{1}{2\sqrt{2}}$

82. A variable plane passes through a fixed point (3, 2, 1) and meets x, y and z axes at A, B and C respectively. A plane is drawn parallel to yz-plane through A, a second plane is drawn parallel zx-plane through B and a third plane is drawn parallel to xy-plane through C. Then the locus of the point of intersection of these three planes, is: **(2018/Online Set–1)**

(a) $\frac{x}{3}+\frac{y}{2}+\frac{z}{1}=1$ (b) x + y + z = 6

(c) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{11}{6}$ (d) $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$

83. An angle between the plane, x + y + z = 5 and the line of intersection of the planes, 3x + 4y + z -1 = 0 and 5x + 8y + 2z + 14 = 0, is: **(2018/Online Set–1)**

(a) $\sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$ (b) $\cos^{-1}\left(\sqrt{\frac{3}{17}}\right)$

(c) $\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)$ (d) $\sin^{-1}\left(\frac{3}{\sqrt{17}}\right)$

84. An angle between the lines whose direction cosines are given by the equations,

l + 3m + 5n = 0 and 5 lm – 2mn + 6nl = 0, is : **(2018/Online Set–2)**

(a) $\cos^{-1}\left(\frac{1}{3}\right)$ (b) $\cos^{-1}\left(\frac{1}{4}\right)$

(c) $\cos^{-1}\left(-\frac{1}{6}\right)$ (d) $\cos^{-1}\left(\frac{1}{8}\right)$

85. A plane bisects the line segment joining the points (1, 2, 3) and (-3, 4, 5) at right angles. Then this plane also passes through the point : **(2018/Online Set–2)**

(a) (-3, 2, 1) (b) (3, 2, 1)

(c) (-1, 2, 3) (d) (1, 2, -3)

86. The sum of the intercepts on the coordinate axes of the plane passing through the point (-2, -2, 2) and containing the line joining the points (1, -1, 2) and (1, 1, 1), is : **(2018/Online Set–3)**

(a) 4 (b) -4

(c) -8 (d) 12

87. If the angle between the lines, $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{5-x}{-2}=\frac{7y-14}{p}=\frac{z-3}{4}$ is $\cos^{-1}\left(\frac{2}{3}\right)$, then p is equal to : **(2018/Online Set–3)**

(a) $\frac{7}{2}$ (b) $\frac{2}{7}$

(c) $-\frac{7}{4}$ (d) $-\frac{4}{7}$

88. The equation of a plane containing the line of intersection of the planes 2x – y – 4 = 0 and y + 2z – 4 = 0 and passing through the point (1, 1, 0) is : **(2019-04-08/Shift-1)**

(a) x - 3y - 2z = -2 (b) 2x - z = 2

(c) x - y - z = 0 (d) x + 3y + z = 4

89. The length of the perpendicular from the point (2, –1, 4) on the straight line, $\frac{x+3}{10}=\frac{y-2}{-7}=\frac{z}{1}$ is: **(2019-04-08Shift-1)**

(a) greater than 3 but less than 4

(b) less than 2

(c) greater than 2 but less than 3

(d) greater than 4

90. The vector equation of the plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5which is perpendicular to the plane x – y + z = 0 is : **(2019-04-08/Shift-2)**

(a) $\vec{r}\times(\hat{i}-\hat{k})+2=0$ (b) $\vec{r}.(\hat{i}-\hat{k})-2=0$

(c) $\vec{r}\times(\hat{i}+\hat{k})+2=0$ (d) $\vec{r}.(\hat{i}-\hat{k})+2=0$

91. If a point R(4, y, z) lies on the line segment joining the points P(2, –3, 4) and Q(8, 0, 10), then distance of R from the origin is: **(2019-04-08/Shift-2)**

(a) $2\sqrt{14}$ (b) $2\sqrt{21}$

(c) 6 (d) $\sqrt{53}$

92. If the line, $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{4}$ meets the plane, $x+2y+3z=15$ at a point P, then the distance of P from the origin is: **(2019-04-09/Shift-1)**

(a) $\frac{\sqrt{5}}{2}$ (b) $2\sqrt{5}$

(c) $\frac{9}{2}$ (d) $\frac{7}{2}$

93. A plane passing through the points (0, -1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane $y-z+5=0$, also passes through the point: **(2019-04-09/Shift-1))**

(a) $(-\sqrt{2},1,-4)$ (b) $(\sqrt{2},-1,4)$

(c) $(-\sqrt{2},-1,-4)$ (d) $(\sqrt{2},1,4)$

94. The vertices B and C of a ΔABC lie on the line, $\frac{x+2}{3}=\frac{y-1}{0}=\frac{z}{4}$ such that BC = 5 units.Then the area (in sq. units) of this triangle, given that the point $A(1,-1,2)$, is: **(2019-04-09/Shift-2))**

(a) $5\sqrt{17}$ (b) $2\sqrt{34}$

(c) 6 (d) $\sqrt{34}$

95. Let P be the plane, which contains the line of intersection of the plane, $x+y+z-6=0$ and $2x+3y+z+5=0$ and it is perpendicular to the xy-plane. Then the distance of the point (0, 0, 256) from P is equal to: **(2019-04-09/Shift-2))**

(a) $\frac{17}{\sqrt{5}}$ (b) $63\sqrt{5}$

(c) $205\sqrt{5}$ (d) $\frac{11}{\sqrt{5}}$

96. If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and $\theta\in(0,\pi)$ with $\hat{k}$ then a value of θ is: **(2019-04-09/Shift-2)**

(a) $\frac{5\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{5\pi}{12}$ (d) $\frac{2\pi}{3}$

97. If Q (0, –1, –3) is the image of the point P in the plane 3x – y + 4z = 2 and R is the point (3,–1 , –2), then the area (in sq. units) of ΔPQR is : **(2019-04-10/Shift-1))**

(a) $2\sqrt{13}$ (b) $\frac{\sqrt{91}}{4}$

(c) $\frac{\sqrt{91}}{2}$ (d) $\frac{\sqrt{65}}{2}$

98. If the length of the perpendicular from the point $(\beta,0,\beta)$ $(\beta\neq 0)$ to the line, $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to: **(2019-04-10/Shift-1)**

(a) 1 (b) 2

(c) –1 (d) –2

99. The distance of the point having position vector $-\hat{i}+2\hat{j}+6\hat{k}$ from the straight line passing through the point (2, 3, -4) and parallel to the vector, $6\hat{i}+3\hat{j}-4\hat{k}$ is:

(2019-04-10/Shift-2)

(a) 7 (b) $4\sqrt{3}$

(c) 6 (d) $2\sqrt{13}$

100. A perpendicular is drawn from a point on the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{1}$ to the plane $x+y+z=3$ such that the foot the perpendicular Q also lies on the plane $x-y+z=3$. Then the co-ordinates of Q are:

(2019-04-10/Shift-2)

(a) (1, 0, 2) (b) (2, 0, 1)

(c) (-1, 0, 4) (d) (4, 0, -1)

101. If the plane $2x-y+2z+3=0$ has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes $4x-2y+4z+\lambda=0$ and $2x-y+2z+\mu=0$ respectively, then the maximum value of $\lambda+\mu$ is equal to : **(2019-04-10)/Shift-2**

(a) 9 (b) 15

(c) 5 (d) 13

102. If the line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the plane 2x + 3y – z + 13 = 0 at a point P and the plane 3x + y + 4z = 16 at a point Q, then PQ is equal to

(2019-04-12/Shift-1)

(a) 14 (b) $\sqrt{14}$

(c) $2\sqrt{7}$ (d) $2\sqrt{14}$

103. A plane which bisects the angle between the two given planes $2x-y+2z-4=0$ and $x+2y+2z-2=0$ passes through the point: **(2019-04-12/Shift-2)**

(a) (1, -4, 1) (b) (1, 4, -1)

(c) (2, 4, 1) (d) (2, -4, 1)

104. The length of the perpendicular drawn from the point (2, 1, 4) to the plane containing the lines $\vec{r}=(\hat{i}+\hat{j})+\lambda(\hat{i}+2\hat{j}-\hat{k})$ and

$\vec{r}=(\hat{i}+\hat{j})+\mu(-\hat{i}+\hat{j}-2\hat{k})$ is ______.

(2019-04-12/Shift-2)

(a) 3 (b) $\frac{1}{3}$

(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

105. The equation of the line passing through (–4, 3, 1) parallel to the plane x + 2y – z – 5 = 0 and intersecting the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z-2}{-1}$ is: **(2019-01-09/Shift-1)**

(a) $\frac{x-4}{2}=\frac{y+3}{1}=\frac{z+1}{4}$ (b) $\frac{x+4}{1}=\frac{y-3}{1}=\frac{z-1}{3}$

(c) $\frac{x+4}{3}=\frac{y-3}{-1}=\frac{z-1}{1}$ (d) $\frac{x+4}{-1}=\frac{y-3}{1}=\frac{z-1}{1}$

106. The equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is:

(2019-01-09/Shift-2)

(a) x - 2y + z = 0 (b) 3x + 2y - 3z = 0

(c) x + 2y - 2z = 0 (d) 5x + 2y - 4z = 0

107. If the lines x = ay + b, z = cy + d and x = a′z + b′, y = c′z + d′ are perpendicular, then:

(2019-01-09/Shift-2)

(a) ab′ + bc′ + 1 = 0 (b) cc′ + a + a′ = 0

(c) bb′ + cc′ + 1 = 0 (d) aa′ + c + c′ = 0

108. The plane passing through the point (4, -1, 2) and parallel to the lines $\frac{x+2}{3}=\frac{y-2}{-1}=\frac{z+1}{2}$ and $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-4}{3}$ also passes through the point:

(2019-01-10/Shift-1)

(a) (1, 1, -1) (b) (1, 1, 1)

(c) (-1, -1, -1) (d) (-1, -1, 1)

109. Let A be a point on the line

$\vec{r}=(1-3\mu)\hat{i}+(\mu-1)\hat{j}+(2+5\mu)\hat{k}$ and B(3, 2, 6) be a point in the space. Then the value of μ for which the vector $\overrightarrow{AB}$ is parallel to the plane $x-4y+3z=1$ is

(2019-01-10/Shift-1)

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$

(c) $\frac{1}{2}$ (d) $-\frac{1}{4}$

110. The plane which bisects the line segment joining the points (–3, –3, 4) and (3, 7, 6) at right angles, passes through which one of the following points?

(2019-01-10/Shift-2)

(a) (–2, 3, 5) (b) (4, –1, 7)

(c) (2, 1, 3) (d) (4, 1, –2)

111. On which of the following lines lies the point of intersection of the line $\frac{x-4}{2}=\frac{y-5}{2}=\frac{z-3}{1}$ and the plane, $x+y+z=2$?

(2019-01-10/Shift-2)

(a) $\frac{x+3}{3}=\frac{4-y}{3}=\frac{z+1}{-2}$ (b) $\frac{x-4}{1}=\frac{y-5}{1}=\frac{z-5}{-1}$

(c) $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z+4}{-5}$ (d) $\frac{x-2}{2}=\frac{y-3}{2}=\frac{z+3}{3}$

112. The plane containing the line $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z-1}{3}$ and also containing its projection on the plane $2x+3y-z=5$, contains which one of the following points?

(2019-01-11/Shift-1)

(a) (2,2,0) (b) (–2,2,2)

(c) (0,–2,2) (d) (2,0,–2)

113. Two lines $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$ and $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates : **(2019-01-11/Shift-2)**

(a) $(2,-4,-7)$ (b) $(2,-4,7)$

(c) $(2,4,7)$ (d) $(-2,4,7)$

114. If the point (2, α, β) lies on the plane which passes through the points (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane 2x – 5y = 15, then 2α – 3β is equal to :

(2019-01-11/Shift-2)

(a) 12 (b) 7

(c) 5 (d) 17

115. A tetrahedron has vertices P(l, 2, 1), Q(2, 1, 3), R(-1, 1, 2) and O(0, 0, 0). The angle between the faces OPQ and PQR is:

(2019-01-12/Shift-1)

(a) $\cos^{-1}\left(\frac{-17}{31}\right)$ (b) $\cos^{-1}\left(\frac{-19}{35}\right)$

(c) $\cos^{-1}\left(\frac{-9}{35}\right)$ (d) $\cos^{-1}\left(\frac{-7}{31}\right)$

116. The perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$, is **(2019-01-12/Shift-1)**

(a) $11\sqrt{6}$ (b) $\frac{11}{\sqrt{6}}$

(c) 11 (d) $6\sqrt{11}$

117. If an angle between the line, $\frac{x+1}{2}=\frac{y-2}{1}=\frac{z-3}{-2}$ and the plane, $x-2y-Kz=3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ then a value of K is **(2019-01-12/Shift-2)**

(a) $\sqrt{\frac{5}{3}}$ (b) $\sqrt{\frac{3}{5}}$

(c) $-\frac{3}{5}$ (d) $-\frac{5}{3}$

118. Let S be the set of all real values of λ such that a plane passing through the points $(-\lambda^2, 1, 1), (1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ also passes through the point $(-1, -1, 1)$ Then S is equal to **(2019-01-12/Shift-2)**

(a) $\{\sqrt{3}\}$ (b) $\{\sqrt{3}, -\sqrt{3}\}$

(c) $\{1, -1\}$ (d) $\{3, -3\}$

119. The plane passing through the points (1, 2,1), (2, 1, 2) and parallel to the line, $2x = 3y$, $z = 1$ also passes through the point : **(2020-09-02/Shift-1)**

(a) (0, –6, 2) (b) (0, 6, – 2)

(c) (– 2, 0, 1) (d) (2, 0, –1)

120. A plane passing through the point (3, 1, 1) contains two lines whose direction ratios are 1, –2, 2 and 2, 3, –1 respectively. If this plane also passes through the point $(\alpha, -3, 5)$, then α is equal to : **(2020-09-02/Shift-2)**

(a) – 5 (b) 10

(c) 5 (d) – 10

121. The foot of the perpendicular drawn from the point (4, 2, 3) to the line joining the points (1, – 2, 3) and (1, 1, 0) lies on the plane : **(2020-09-03/Shift-1)**

(a) $x - y - 2z = 1$ (b) $x - 2y + z = 1$

(c) $2x + y - z = 1$ (d) $x + 2y - z = 1$

122. The lines $\vec{r} = (\hat{i} - \hat{j}) + l(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$ **(2020-09-03/Shift-1)**

(a) do not intersect for any values of l and m

(b) intersect when $l = 1$ and $m = 2$

(c) intersect when $l = 2$ and $m = \frac{1}{2}$

(d) intersect for all values of l and m

123. The plane which bisects the line joining the points (4, –2, 3) and (2, 4, –1) at right angles also passes through the point : **(2020-09-03/Shift-2)**

(a) (0, –1, 1) (b) (4, 0, 1)

(c) (4, 0, –1) (d) (0, 1, –1)

124. Let a plane P contain two lines $\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j}), \lambda \in R$ and $\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k}), \mu \in R$. If $Q(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn from the point M (1,0, 1) to P, then $3(\alpha + \beta + \gamma)$ equals .... **(2020-09-03/Shift-2)**

125. If the equation of a plane P, passing through the intersection of the planes, $x+4y - z+7=0$ and $3x+y+5z=8$ is $ax+by+6z=15$ for some a, b $\in$ R, then the distance of the point (3,2, –1) from the plane P is....... units. **(2020-09-04/Shift-1)**

126. The distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is: **(2020-09-04/Shift-2)**

(a) $\frac{1}{7}$ (b) 7

(c) $\frac{7}{5}$ (d) 1

127. If (a,b,c) is the image of the point (1,2, –3) in the line, $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then a+b+c is **(2020-09-05/Shift-1)**

(a) 2 (b) 3

(c) –1 (d) 1

128. If for some $\alpha \in$ R, the lines $L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point: **(2020-09-05/Shift-2)**

(a) (2, –10, –2) (b) (10, –2, –2)

(c) (10,2,2) (d) (–2,10,2)

129. The shortest distance between the lines $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$ and $x + y + z + 1 = 0$, $2x - y + z + 3 = 0$ is: **(2020-09-06/Shift-1)**

(a) 1 (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$

130. A plane P meets the coordinate axes at A, B and C respectively. The centroid of ΔABC is given to be $(1, 1, 2)$. Then the equation of the line through this centroid and perpendicular to the plane P is: **(2020-09-06/Shift-2)**

(a) $\frac{x-1}{2}=\frac{y-1}{1}=\frac{z-2}{1}$ (b) $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$

(c) $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ (d) $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-2}{2}$

131. Let P be a plane passing through the points (2,1,0), (4,1,1) and (5,0,1) and R be any point (2,1,6). Then the image of R in the plane P is: **(2020-01-07/Shift-1)**

(a) (6,5,2) (b) (6,5, -2)

(c) (4,3,2) (d) (3, 4, -2)

132. The shortest distance between the lines

$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is:

(2020-01-08/Shift-1)

(a) $2\sqrt{30}$ (b) $\frac{7}{2}\sqrt{300}$

(c) 3 (d) $3\sqrt{30}$

133. The mirror image of the point (1, 2, 3) in a plane is $\left(-\frac{7}{3},-\frac{4}{3},-\frac{1}{3}\right)$. Which of the following points lies on this plane? **(2020-01-08/Shift-2)**

(a) (1, -1, 1) (b) (-1, -1, 1)

(c) (1, 1, 1) (d) (-1, -1, -1)

134. If the distance between the plane, $23x - 10y - 2z + 48 = 0$ and the plane containing the lines $\frac{x+1}{2}=\frac{y-3}{4}=\frac{z+1}{3}$ and $\frac{x+3}{2}=\frac{y+2}{6}=\frac{z-1}{\lambda}, (\lambda \in R)$, is equal to $\frac{k}{\sqrt{633}}$, then k is equal to________ **(2020-01-09/Shift-2)**

135. Let P be a plane passing through the points (1, 0, 1), (1, –2, 1) and (0, 1, –2). Let a vector $\vec{a}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ be such that $\vec{a}$ is parallel to the plane P, perpendicular to $\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and $\vec{a}.\left(\hat{i}+\hat{j}+2\hat{k}\right)=2$, then $(\alpha-\beta+\gamma)^2$ equals ____. **(2021-07-20/Shift-1)**

136. If the shortest distance between the lines

$\vec{r}_1=\alpha\hat{i}+2\hat{j}+2\hat{k}+\lambda\left(\hat{i}-2\hat{j}+2\hat{k}\right), \lambda \in R, \alpha > 0$

and $\vec{r}_2=-4\hat{i}-\hat{k}+\mu\left(3\hat{i}-2\hat{j}-2\hat{k}\right), \mu \in R$ is 9, then α is equal to _______. **(2021-07-20/Shift-1)**

137. The lines $x = ay - 1 = z - 2$ and $x = 3y - 2 = bz - 2$, $(ab \neq 0)$ are coplanar, if ? **(2021-07-20/Shift-2)**

(a) $b=1, a \in R-\{0\}$ (b) $a=2, b=3$

(c) $a=2, b=2$ (d) $a=1, b \in R-\{0\}$

138. Consider the line L given by the equation $\frac{x-3}{2}=\frac{y-1}{1}=\frac{z-2}{1}$. Let Q be the mirror image of the point $(2, 3, -1)$ with respect to L. Let a plane P be such that it passes through Q, and the line L is perpendicular to P. Then which of the following points is on the plane P ? **(2021-07-20/Shift-2)**

(a) (1, 2, 2) (b) (–1, 1, 2)

(c) (1, 1, 1) (d) (1, 1, 2)

139. Let the foot of perpendicular from a point $P(1, 2, -1)$ to the straight line $L=\frac{x}{1}=\frac{y}{0}=\frac{z}{-1}$ be N. Let a line be drawn from P parallel to the plane $x+y+2z=0$ which meets L at point Q. If α is the acute angle between the lines PN and PQ, then $\cos\alpha$ is equal to _______________ ? **(2021-07-25/Shift-1)**

(a) $\frac{1}{2\sqrt{3}}$ (b) $\frac{1}{\sqrt{5}}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

140. Let the plane passing through the point (–1,0, –2) and perpendicular to each of the planes $2x + y - z = 2$ and $x - y - z = 3$ be $ax+by+cz+8=0$, then the value of $a+b+c$ is equal to: **(2021-07-27/Shift-1)**

(a) 8 (b) 4

(c) 3 (d) 5

141. Let a plane P pass through the point (3,7, –7) and contain the line, $\frac{x-2}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$. If distance of the plane P from the origin is d, then d^2 is equal to ______. **(2021-07-27/Shift-1)**

142. For real numbers α and $\beta \neq 0$, if the point of intersection of the straight lines $\frac{x-\alpha}{1}=\frac{y-1}{2}=\frac{z-1}{3}$ and $\frac{x-4}{\beta}=\frac{y-6}{3}=\frac{z-7}{3}$, lies on the plane $x+2y-z=8$, then $\alpha-\beta$ is equal to: **(2021-07-27/Shift-2)**

(a) 5 (b) 3
(c) 7 (d) 9

143. The distance of the point P(3, 4, 4) from the point of intersection of the line joining the points Q(3, –4, –5) and R(2, –3, 1) and the plane $2x+y+z=7$, is equal to ______. **(2021-07-27/Shift-2)**

144. Let L be the line of intersection of planes $\vec{r}.\left(\hat{i}-\hat{j}+2\hat{k}\right)=2$ and $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)=2$. If $P(\alpha,\beta,\gamma)$ is the foot of perpendicular on L from the point (1, 2, 0), then the value of $35(\alpha+\beta+\gamma)$ is equal to : **(2021-07-22/Shift-2)**

(a) 134 (b) 119
(c) 143 (d) 101

145. If the shortest distance between the straight lines $3(x-1)=6(y-2)=2(z-1)$ and $4(x-2)=2(y-\lambda)=(z-3), \lambda \in R$ is $\frac{1}{\sqrt{38}}$, then the integral value of λ is equal to : **(2021-07-22/Shift-2)**

(a) –1 (b) 2
(c) 3 (d) 5

146. If the lines $\frac{x-k}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x+1}{3}=\frac{y+2}{2}=\frac{z+3}{1}$ are co-planar, then the value of k is ______________. **(2021-07-25/Shift-2)**

147. The distance of line $3y-2z-1=0=3x-z+4$ from the point $(2,-1,6)$ is **(2021-09-01/Shift-2)**

(a) $2\sqrt{6}$ (b) $4\sqrt{2}$
(c) $2\sqrt{5}$ (d) $\sqrt{26}$

148. Let the acute angle bisector of the two planes $x-2y-2z+1=0$ and $2x-3y-6z+1=0$ be the plane P. Then which of the following points lies on P **(2021-09-01/Shift-2)**

(a) $(4,0,-2)$ (b) $\left(-2,0,-\frac{1}{2}\right)$
(c) $\left(3,1,-\frac{1}{2}\right)$ (d) $(0,2,-4)$

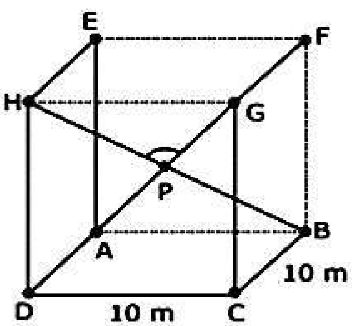
149. Let Q be the foot of the perpendicular from the point P(7, –2, 13) on the plane containing the lines, $\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ and $\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$.

Then $(PQ)^2$, is equal to ________. **(2021-08-26/Shift-2)**

150. Let P be the plane passing through the point (1, 2, 3) and the line of intersection of the planes $\vec{r}.\left(\hat{i}+\hat{j}+4\hat{k}\right)=16$ and $\vec{r}.\left(-\hat{i}+\hat{j}+\hat{k}\right)=6$. Then which of the following points lie on P? **(2021-08-26/Shift-2)**

(a) (3, 3, 2) (b) (-8, 8, 6)
(c) (4, 2, 2) (d) (6, 6, 2)

151. A hall has a square floor of dimension $10m \times 10m$ (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}\frac{1}{5}$, then the height of the hall (in meters) is: **(2021-08-26/Shift-2)**



(a) $5\sqrt{3}$ (b) 5
(c) $2\sqrt{10}$ (d) $5\sqrt{2}$

152. Equation of a plane at a distance $\sqrt{\frac{2}{21}}$ from the origin, which contains the line of intersection of the planes $x-y-z-1=0$ and $2x+y-3z+4=0$, is : **(2021-08-27/Shift-1)**

(a) $-x+2y+2z-3=0$ (b) $3x-4z+3=0$

(c) $3x-y-5z+2=0$ (d) $4x-y-5z+2=0$

153. The distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured parallel to a line, whose direction ratios are 2, 3, –6 is: **(2021-08-27/Shift-1)**

(a) 2 (b) 5

(c) 3 (d) 1

154. Let the line L be the projection of the line :

$$\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{2}$$

In the plane $x-2y-z=3$. If d is the distance of the point (0, 0, 6) from L, then d^2 is equal to _____. **(2021-08-26/Shift-1)**

155. A plane P contains the line $x + 2y + 3z + 1 = 0 = x - y - z - 6$, and is perpendicular to the plane $-2x+y+z+8=0$. Then which of the following points lies on P? **(2021-08-26/Shift-1)**

(a) (1, 0, 1) (b) (2, –1, 1)

(c) (–1, 1, 2) (d) (0, 1,1)

156. The angle between the straight lines, whose direction cosines are given by the equation $2l+2m-n=0$ and $mn+nl+lm=0$, is : **(2021-08-27/Shift-2)**

(a) $\frac{\pi}{3}$ (b) $\pi-\cos^{-1}\left(\frac{4}{9}\right)$

(c) $\cos^{-1}\left(\frac{8}{9}\right)$ (d) $\frac{\pi}{2}$

157. Let S be the mirror image of the point Q (1,3,4) with respect to the plane $2x-y+z+3=0$ and let $R(3,5,\gamma)$ be a point on this plane. Then the square of the length of the line segment SR is _______. **(2021-08-27/Shift-2)**

158. The equation of the plane passing through the line of intersection of the planes $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ and $\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$ and parallel to the x-axis is : **(2021-08-27/Shift-2)**

(a) $\vec{r}.\left(\hat{i}+3\hat{k}\right)+6=0$ (b) $\vec{r}.\left(\hat{j}-3\hat{k}\right)-6=0$

(c) $\vec{r}.\left(\hat{i}-3\hat{k}\right)+6=0$ (d) $\vec{r}.\left(\hat{j}-3\hat{k}\right)+6=0$

159. The square of the distance of the point of intersection of the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+1}{6}$ and the plane $2x-y+z=6$ from the point (–1, –1, 2) is __________ ? **(2021-08-31/Shift-1)**

160. Let the equation of the plane, that passes through the point (1, 4, –3) and contains the line of intersection of the planes $3x-2y+4z-7=0$ and $x+5y-2z+9=0$, be $\alpha x+\beta y+\gamma Z+3=0$ then $\alpha+\beta+\gamma$ is equal to ? **(2021-08-31/Shift-1)**

(a) –15 (b) 15

(c) –23 (d) 23

161. The distance of the point (–1, 2, –2) from the line of intersection of the planes $2x + 3y + 2z = 0$ and $x – 2y + z = 0$ is: **(2021-08-31/Shift-2)**

(a) $\frac{\sqrt{42}}{2}$ (b) $\frac{5}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{34}}{2}$

162. Suppose the line $\frac{x-2}{\alpha}=\frac{y-2}{-5}=\frac{z+2}{2}$ lies on the plane $x+3y-2z+\beta=0$, then $(\alpha+\beta)$ is equal to ____ **(2021-08-31/Shift-2)**

163. If the foot of the perpendicular from point (4, 3, 8) on the line $L_1 : \frac{x-a}{1} = \frac{y-2}{3} = \frac{z-b}{4}$, $l \neq 0$ is (3, 5, 7), then the shortest distance between the line L1 and line $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is equal to

(2021-03-16/Shift-2)

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{6}}$ (d) $\sqrt{\frac{2}{3}}$

164. If (x, y, z) be an arbitrary point lying on a plane P which passes through the points (42, 0, 0), (0, 42, 0) and (0, 0, 42), then the value of the expression

$$3+\frac{x-11}{(y-19)^2(z-12)^2}+\frac{y-19}{(x-11)^2(z-12)^2}$$

$$+\frac{z-12}{(x-11)^2(y-19)^2}-\frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

is equal to **(2021-03-16/Shift-2)**

(a) 3 (b) 0

(c) –45 (d) 39

165. If the distance of the point (1, –2, 3) from the plane x + 2y – 3z + 10 = 0 measured parallel to the line $\frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1}$ is $\sqrt{\frac{7}{2}}$, then the value of $|m|$ is equal to ____. **(2021-03-16/Shift-2)**

166. Let P be plane $lx+my+nz=0$ containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If plane P divides the line segment AB joining points $A(-3, -6, 1)$ and $B(2, 4, -3)$ in ratio k : 1 then the value of k is equal to

(2021-03-16/Shift-1)

(a) 4 (b) 3

(c) 1.5 (d) 2

167. If for a > 0, the feet of perpendiculars from the points $A(a, -2a, 3)$ and $B(0, 4, 5)$ on the plane $lx+my+nz=0$ are points $C(0, -a, -1)$ and D respectively, then the length of line segment CD is equal to **(2021-03-16/Shift-1)**

(a) $\sqrt{66}$ (b) $\sqrt{41}$

(c) $\sqrt{55}$ (d) $\sqrt{31}$

168. If the equation of plane passing through the mirror image of a point (2, 3, 1) with respect to line $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1}$ and containing the line $\frac{x-2}{3} = \frac{1-y}{2} = \frac{z+1}{1}$ is $\alpha x+\beta y+\gamma z=24$, then $\alpha+\beta+\gamma$ is equal to : **(2021-03-17/Shift-2)**

(a) 19 (b) 18

(c) 21 (d) 20

169. Let P be an arbitrary point having sum of the squares of the distances from the planes $x+y+z=0$, $lx-nz=0$ and $x-2y+z=0$, equal to 9. If the locus of the point P is $x^2+y^2+z^2=9$, then the value of l – n is equal to .... **(2021-03-17/Shift-2)**

170. The equation of the plane which contains the y-axis and passes through the point (1, 2, 3) is :

(2021-03-17/Shift-1)

(a) 3x + z = 6 (b) x + 3z = 0

(c) x + 3z = 10 (d) 3x – z = 0

171. If the equation of the plane passing through the line of intersection of the planes $2x-7y+4z-3=0, 3x-5y+4z+11=0$ and the point (–2, 1, 3) is $ax+by+cz-7=0$, then the value of 2a + b + c – 7 is .... **(2021-03-17/Shift-1)**

172. If $\vec{a}=\alpha\hat{i}+\beta\hat{j}+3\hat{k}$, $\vec{b}=-\beta\hat{i}-\alpha\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}-\hat{k}$ Such that $\vec{a}\cdot\vec{b}=1$ and $\vec{b}\cdot\vec{c}=-3$, then $\frac{1}{3}\left(\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}\right)$ is equal to .... **(2021-03-17/Shift-1)**

173. Let the mirror image of the point (1,3,a) with respect to the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})-b=0$ be $(-3, 5, 2)$. Then, the value of $|a+b|$ is equal to _______.

(2021-03-18/Shift-2)

174. Let P be a plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4}=\frac{y-2}{-3}=\frac{z+5}{7}$. If the point $(1,-1,\alpha)$ lies on the plane, P, then the value of $|5\alpha|$ is equal to ________. **(2021-03-18/Shift-2)**

175. The equation of the planes parallel to the plane $x-2y+2z-3=0$ which are at unit distance from the point (1, 2, 3) is $ax+by+cz+d=0$.

If $(b-d)=K(c-a)$, then the positive value of K is .... **(2021-03-18/Shift-1)**

176. Let the plane $ax+by+cz+d=0$ bisect the line joining the points (4, –3, 1) and (2, 3, –5) at the right angles. If a,b,c,d are integers, then the minimum value of $(a^2+b^2+c^2+d^2)$ is ....

(2021-03-18/Shift-1)

177. Let $a.b \in R$. If the mirror image of the point P(a, 6, 9) with respect to the line $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9}$ is $(20, b, -a-9)$, then $|a+b|$ is equal to

(2021-02-24/Shift-2)

(a) 88 (b) 84

(c) 90 (d) 86

178. The vector equation of the plane passing through the intersection of the planes $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}\cdot(\hat{i}-2\hat{j})=-2$, and the point (1, 0, 2) is:

(2021-02-24/Shift-2)

(a) $\vec{r}\cdot(\hat{i}+7\hat{j}+3\hat{k})=7$ (b) $\vec{r}\cdot(3\hat{i}+7\hat{j}+3\hat{k})=7$

(c) $\vec{r}\cdot(\hat{i}-7\hat{j}+3\hat{k})=\frac{7}{3}$ (d) $\vec{r}\cdot(\hat{i}+7\hat{j}+3\hat{k})=\frac{7}{3}$

179. Let λ be an integer. If the shortest distance between the lines $x-\lambda=2y-1=-2z$ and $x=y+2\lambda=z-\lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is ____.

(2021-02-24/Shift-2)

180. The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}$ and the plane $x+y+z=17$ is **(2021-02-24/Shift-1)**

(a) $2\sqrt{19}$ (b) 38

(c) $19\sqrt{2}$ (d) $\sqrt{38}$

181. The equation of the plane passing through the point (1, 2, –3) and perpendicular to the planes $3x+y-2z=5$ and $2x-5y-z=7$, is **(2021-02-24/Shift-1)**

(a) $6x-5y+2z+10=0$ (b) $6x-5y+2z-2=0$

(c) $3x-10y+2z+11=0$ (d) $11x+y+17z+38=0$

182. A plane passes through the points A(1,2,3), B(2,3,1) and C(2,4,2). If O is the origin and P is (2, –1, 1), then the projection of $\overrightarrow{OP}$ on this plane is of length:

(2021-02-25/Shift-2)

(a) $\sqrt{\frac{2}{3}}$ (b) $\sqrt{\frac{2}{5}}$

(c) $\sqrt{\frac{2}{7}}$ (d) $\sqrt{\frac{2}{11}}$

183. A line 'l' passing through origin is perpendicular to the lines

$l_1 : \vec{r}=(3+t)\hat{i}+(-1+2t)\hat{j}+(4+2t)\hat{k}$

$l_2 : \vec{r}=(3+2s)\hat{i}+(3+2s)\hat{j}+(2+s)\hat{k}$

If the co-ordinates of the point in the first octant on 'l_2' at a distance of $\sqrt{17}$ from the point of intersection of 'l' and 'l_1' are (a,b,c), then 18 (a + b + c) is equal to:

(2021-02-25/Shift-2)

184. Let α be the angle between the lines whose direction cosines satisfy the equations $l+m-n=0$ and $l^2+m^2-n^2=0$. Then the value of $\sin^4\alpha+\cos^4\alpha$ is:

(2021-02-25/Shift-1)

(a) $\frac{5}{8}$ (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) $\frac{3}{8}$

185. The equation of the line through the point (0, 1, 2) and perpendicular to the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{-2}$ is:

(2021-02-25/Shift-1)

(a) $\frac{x}{-3}=\frac{y-1}{4}=\frac{z-2}{3}$
(b) $\frac{x}{3}=\frac{y-1}{-4}=\frac{z-2}{3}$
(c) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{3}$
(d) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{-3}$

186. If the mirror image of the point (1, 3, 5) with respect to the plane $4x-5y+2z=8$ is (α, β, γ), then $5(\alpha+\beta+\gamma)$ equals : **(2021-02-26/Shift-2)**

(a) 39
(b) 41
(c) 47
(d) 43

187. Let L be a line obtained from the intersection of two planes $x+2y+z=6$ and $y+2z=4$. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from (3, 2, 1) on L, then the value of $21(\alpha+\beta+\gamma)$ equals: **(2021-02-26/Shift-2)**

(a) 68
(b) 102
(c) 142
(d) 136

188. If $(1,5,35),(7,5,5),(1,\lambda,7)$ and $(2\lambda,1,2)$ are coplanar, then the sum of all possible values of λ is :

(2021-02-26/Shift-1)

(a) $\frac{44}{5}$
(b) $\frac{39}{5}$
(c) $-\frac{44}{5}$
(d) $-\frac{39}{5}$

189. Consider the three planes

$P_1: 3x+15y+21z=9$

$P_2: x-3y-z=5$, and

$P_3: 2x+10y+14z=5$

Then, which one of the following is true?

(2021-02-26/Shift-1)

(a) P_1 and P_2 are parallel
(b) P_1, P_2 and P_3 are parallel
(c) P_1 and P_3 are parallel
(d) P_2 and P_3 are parallel

190. Let (λ, 2, 1) be a point on the plane which passes through the point (4, –2, 2). If the plane is perpendicular to the line joining the points (–2, –21, 29) and (–1, –16, 23), then $\left(\frac{\lambda}{11}\right)^2-\frac{4\lambda}{11}-4$ is equal to ____ . **(2021-02-26/Shift-1)**

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

VECTORS

Objective Questions I [Only one correct option]

1. If $\vec{a}$ & $\vec{b}$ lie on a plane normal to the plane containing $\vec{c}$ & $\vec{d}$ then, $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})$ is equal to

(a) –1 (b) 1
(c) 0 (d) none

2. 'P' is a point inside the triangle ABC such that $BC\,(\overrightarrow{PA})+CA\,(\overrightarrow{PB})+AB\,(\overrightarrow{PC})=0$ then for the triangle ABC the point P is its

(a) incentre (b) circumcentre
(c) centroid (d) orthocentre

3. Let $\vec{p}$ is the p.v. of the orthocentre & $\vec{g}$ is the p.v. of the centroid of the triangle ABC where circumcentre is the origin. If $\vec{p}=K\vec{g}$ then K =

(a) 3 (b) 2
(c) 1/3 (d) 2/3

4. If the unit vectors $\vec{e}_1$ and $\vec{e}_2$ are inclined at an angle 2θ and $|\vec{e}_1-\vec{e}_2|<1$, then for $\theta\in[0,\pi]$, θ may lie in the interval :

(a) $\left[0,\frac{\pi}{6}\right)$ (b) $\left[\frac{\pi}{6},\frac{\pi}{2}\right]$
(c) $\left(\frac{5\pi}{6},\pi\right]$ (d) $\left[\frac{\pi}{2},\frac{5\pi}{6}\right]$

5. Two given points P and Q in the rectangular cartesian co–ordinates lie on $y = 2^{x+2}$ such that $\overrightarrow{OP}.\hat{i}=-1$ and $\overrightarrow{OQ}.\hat{i}=+2$ where $\hat{i}$ is a unit vector along the x–axis. The magnitude of $\overrightarrow{OQ}-4\,\overrightarrow{OP}$ will be :

(a) 10 (b) 20
(c) 30 (d) none

6. A, B, C and D are four points in a plane with pv's $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ respectively such that

$(\vec{a}-\vec{d}).(\vec{b}-\vec{c})=(\vec{b}-\vec{d}).(\vec{c}-\vec{a})=0$.

Then for the triangle ABC, D is its :

(a) incentre (b) circumcentre
(c) othocentre (d) centroid

7. Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ be three non–zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$, then $\begin{vmatrix} a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}^2$ is equal to :

(a) 0
(b) 1
(c) $\frac{1}{4}(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)$
(d) none of these

8. The vectors $\vec{p}$ & $\vec{q}$ satisfy the system of equations $2\vec{p}+\vec{q}=\vec{a}, \vec{p}+2\vec{q}=\vec{b}$ and the angle between $\vec{p}$ & $\vec{q}$ is θ. If it is known that in the rectangular system of co-ordinates the vectors $\vec{a}$ & $\vec{b}$ have the forms $\vec{a}=(1,1)$ & $\vec{b}=(1,-1)$ then $\cos\theta=$

(a) $\frac{4}{5}$ (b) $-\frac{4}{5}$
(c) $-\frac{3}{5}$ (d) none

9. Let A $(\vec{a})$ and B $(\vec{b})$ be points on two skew lines $\vec{r} = \vec{a} + \lambda\vec{p}$ and $\vec{r} = \vec{b} + u\vec{q}$ and the shortest distance between the skew lines is 1, where $\vec{p}$ and $\vec{q}$ are unit vectors forming adjacent sides of a parallelogram enclosing an area of $\frac{1}{2}$ units. If an angle between AB and the line of shortest distance is 60°, then AB =

(a) $\frac{1}{2}$ (b) 2

(c) 1 (d) $\lambda \in R - \{0\}$

10. If $\vec{b}$ & $\vec{c}$ are any two perpendicular unit vectors and $\vec{a}$ is any vector, then, $(\vec{a}.\vec{b})\,\vec{b} + (\vec{a}.\vec{c})\,\vec{c} + \frac{\vec{a}.(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|^2}\,(\vec{b}\times\vec{c})$ is equal to :

(a) $\vec{a}$ (b) $\vec{b}$

(c) $\vec{c}$ (d) none of these

11. If $A_1, A_2, A_3, \ldots\ldots, A_n$ are the vertices of a regular plane polygon with n sides & O is its centre then

$$\sum_{i=1}^{n-1} (\overrightarrow{OA_i} \times \overrightarrow{OA_{i+1}}) =$$

(a) $(1-n)\,\overrightarrow{OA_2} \times \overrightarrow{OA_1}$ (b) $(n-1)\,\overrightarrow{OA_2} \times \overrightarrow{OA_1}$

(c) $n\,\overrightarrow{OA_2} \times \overrightarrow{OA_1}$ (d) none

12. If the vector product of a constant vector $\overrightarrow{OA}$ with a variable vector $\overrightarrow{OB}$ in a fixed plane OAB be a constant vector, then locus of B is :

(a) a straight line perpendicular to $\overrightarrow{OA}$

(b) a circle with centre O radius equal to $|\overrightarrow{OA}|$

(c) a straight line parallel to $\overrightarrow{OA}$

(d) none of these

13. Let $\vec{A}, \vec{B}, \vec{C}$ be vectors of length 3, 4 and 5 respectively. Let $\vec{A}$ be perpendicular to $\vec{B}+\vec{C}$, $\vec{B}$ to $\vec{C}+\vec{A}$ and $\vec{C}$ to $\vec{A}+\vec{B}$. Then length of the vector, $\vec{A}+\vec{B}+\vec{C}$ is :

(a) $-5\sqrt{2}$ (b) $\sqrt{2}$

(c) $5\sqrt{2}$ (d) none of these

14. If $\vec{a}$ & $\vec{b}$ are unit vectors such that $(\vec{a}+\vec{b})\cdot(2\vec{a}+3\vec{b})\times(3\vec{a}-2\vec{b}) = 0$, then angle between $\vec{a}$ and $\vec{b}$

(a) 0 (b) $\pi/2$

(c) π (d) indeterminate

15. If a plane cuts off intercepts OA = a, OB = b, OC = c from the coordinate axes, then the area of the triangle ABC =

(a) $\frac{1}{2}\sqrt{b^2c^2 + c^2a^2 + a^2b^2}$

(b) $\frac{1}{2}(bc + ca + ab)$

(c) $\frac{1}{2}abc$

(d) $\frac{1}{2}\sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}$

16. Let $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$. Then $[\vec{a}\ \vec{b}\ \vec{c}]$ depends on

(a) only x (b) only y

(c) neither x nor y (d) both x and y

17. The value of a so that the volume of the parallelopiped formed by the vectors $\hat{i} + a\hat{j} - \hat{k}$, $\hat{j} + a\hat{k}$, $\hat{i} + \hat{j} + \hat{k}$ becomes minimum is

(a) $\sqrt{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{7}{4}$

18. If the vectors $a\,\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ $(a \neq b \neq c \neq 1)$ are coplanar then the value of

$$\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=$$

(a) 1 (b) –1

(c) 0 (d) none

19. $[(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c})\ (\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})\ (\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b})]=$

(a) $[\vec{a}\,\vec{b}\,\vec{c}]^2$ (b) $[\vec{a}\,\vec{b}\,\vec{c}]^3$

(c) $[\vec{a}\,\vec{b}\,\vec{c}]^4$ (d) none

20. The triple product $(\vec{d}+\vec{a}).[\vec{a}\times(\vec{b}\times(\vec{c}\times\vec{d}))]$ simplifies to

(a) $(\vec{b}.\vec{d})[\vec{a}\,\vec{c}\,\vec{d}]$ (b) $(\vec{b}\cdot\vec{c})[\vec{a}\,\vec{b}\,\vec{d}]$

(c) $(\vec{b}.\vec{a})[\vec{a}\,\vec{b}\,\vec{d}]$ (d) none

21. Let $\vec{r},\vec{a},\vec{b}$ and $\vec{c}$ be four non–zero vectors such that $\vec{r}\cdot\vec{a}=0$, $|\vec{r}\times\vec{b}|=|\vec{r}||\vec{b}|, |\vec{r}\times\vec{c}|=|\vec{r}||\vec{c}|$, then $[a\ b\ c]=$

(a) |a| |b| |c| (b) –|a| |b| |c|

(c) 0 (d) none of these

22. If $\vec{a},\vec{b},\vec{c}$ are such that $[\vec{a}\ \vec{b}\ \vec{c}]=1, \vec{c}=\lambda\,\vec{a}\times\vec{b}$, $\vec{a}\wedge\vec{b}<\frac{2\pi}{3}$ and $|\vec{a}|=\sqrt{2},|\vec{b}|=\sqrt{3}$, $|\vec{c}|=\frac{1}{\sqrt{3}}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

23. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{o}$, then $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is equal to

(a) $6(\vec{b}\times\vec{c})$ (b) $6(\vec{c}\times\vec{a})$

(c) $6(\vec{a}\times\vec{b})$ (d) none of these

24. If $((\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}))\cdot(\vec{a}\times\vec{d})=0$, then which of the following is always true

(a) $\vec{a},\vec{b},\vec{c},\vec{d}$ are necessarily coplanar

(b) either $\vec{a}$ or $\vec{d}$ must lie in the plane of $\vec{b}$ and $\vec{c}$

(c) either $\vec{b}$ or $\vec{c}$ must lie in plane of $\vec{a}$ and $\vec{d}$

(d) either $\vec{a}$ or $\vec{b}$ must lie in plane of $\vec{c}$ and $\vec{d}$

25. If $P_1:\vec{r}\cdot\vec{n}_1-d_1=0, P_2:\vec{r}\cdot\vec{n}_2-d_2=0$ and $P_3:\vec{r}\cdot\vec{n}_3-d_3=0$ are three planes and $\vec{n}_1,\vec{n}_2$ and $\vec{n}_3$ are three non–coplanar vectors then, the three lines $P_1=0$, $P_2=0$ and $P_2=0$, $P_3=0$ and $P_3=0$, $P_1=0$ are

(a) parallel lines (b) coplanar lines

(c) coincident lines (d) concurrent lines

26. If $\vec{a},\vec{b}$ and $\vec{c}$ are three unit vectors equally inclined to each other at an angle α. Then the angle between $\vec{a}$ and plane of $\vec{b}$ and $\vec{c}$ is

(a) $\theta=\cos^{-1}\left(\frac{\cos\alpha}{\cos\frac{\alpha}{2}}\right)$ (b) $\theta=\sin^{-1}\left(\frac{\cos\alpha}{\cos\frac{\alpha}{2}}\right)$

(c) $\theta=\cos^{-1}\left(\frac{\sin\frac{\alpha}{2}}{\sin\alpha}\right)$ (d) $\theta=\sin^{-1}\left(\frac{\sin\frac{\alpha}{2}}{\sin\alpha}\right)$

27. For any vector $\vec{A}, \hat{i}\times(\hat{i}\times\vec{A})+\hat{j}\times(\hat{j}\times\vec{A})+\hat{k}\times(\hat{k}\times\vec{A})$ simplifies to

(a) $3\vec{A}$ (b) $\vec{A}$

(c) $-\vec{A}$ (d) $-2\vec{A}$

28. If $\vec{a},\vec{b},\vec{c}$ be the unit vectors such that $\vec{b}$ is not parallel to $\vec{c}$ and $\vec{a}\times(2\vec{b}\times\vec{c})=\vec{b}$ then the angle that $\vec{a}$ makes with $\vec{b}\,\&\,\vec{c}$ are respectively

(a) $\frac{\pi}{3}\,\&\,\frac{\pi}{4}$ (b) $\frac{\pi}{3}\,\&\,\frac{2\pi}{3}$

(c) $\frac{\pi}{2}\,\&\,\frac{2\pi}{3}$ (d) $\frac{\pi}{2}\,\&\,\frac{\pi}{3}$

29. If a vector $\vec{a}$ is expressed as the sum of two vectors $\vec{a}'$ and $\vec{a}''$ along and perpendicular to a given vector $\vec{b}$ then $\vec{a}''$ is

(a) $\dfrac{(\vec{a}\times\vec{b})\times\vec{b}}{\vec{b}}$ (b) $\dfrac{\vec{b}\times(\vec{a}\times\vec{b})}{|\vec{b}|^2}$

(c) $\dfrac{\vec{b}\times(\vec{a}\times\vec{b})}{\vec{b}}$ (d) $\dfrac{\vec{a}\times\vec{b}}{|\vec{b}|^2}\vec{b}$

30. If $\vec{a}, \vec{b}$ and $\vec{c}$ are any three vectors, then

$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\times\vec{c}$ is true if :

(a) $\vec{b}\,\&\,\vec{c}$ are collinear (b) $\vec{a}\,\&\,\vec{c}$ are collinear

(c) $\vec{a}\,\&\,\vec{b}$ are collinear (d) none of these

31. $(\vec{r}\,.\,\hat{i})\,(\hat{i}\times\vec{r}) + (\vec{r}\,.\,\hat{j})\,(\hat{j}\times\vec{r}) + (\vec{r}\,.\,\hat{k})\,(\hat{k}\times\vec{r}) =$

(a) 0 (b) $\vec{r}$

(c) $2\,\vec{r}$ (d) $3\,\vec{r}$

32. For a non zero vector $\vec{A}$ if the equations $\vec{A}\,.\,\vec{B} = \vec{A}\,.\,\vec{C}$ and $\vec{A}\times\vec{B} = \vec{A}\times\vec{C}$ hold simultaneously, then

(a) $\vec{A}$ is perpendicular to $\vec{B}-\vec{C}$

(b) $\vec{A} = \vec{B}$

(c) $\vec{B} = \vec{C}$

(d) $\vec{C} = \vec{A}$

33. Let $\vec{a} = \hat{i}+\hat{j}$ and $\vec{b} = 2\hat{i}-\hat{k}$. The point of intersection of the lines $\vec{r}\times\vec{a} = \vec{b}\times\vec{a}$ and $\vec{r}\times\vec{b} = \vec{a}\times\vec{b}$ is :

(a) $-\hat{i}+\hat{j}+\hat{k}$ (b) $3\hat{i}-\hat{j}+\hat{k}$

(c) $3\hat{i}+\hat{j}-\hat{k}$ (d) $\hat{i}-\hat{j}-\hat{k}$

Objective Questions II [One or more than one correct option]

34. The vectors $\vec{a}, \vec{b}, \vec{c}$ are of the same length & pairwise form equal angles. If $\vec{a} = \hat{i}+\hat{j}$ & $\vec{b} = \hat{j}+\hat{k}$ then $\vec{c}$ can be

(a) (1, 0, 1) (b) $\left(-\dfrac{4}{3}, \dfrac{1}{3}, -\dfrac{4}{3}\right)$

(c) $\left(\dfrac{1}{3}, -\dfrac{4}{3}, \dfrac{1}{3}\right)$ (d) $\left(-\dfrac{1}{3}, \dfrac{4}{3}, -\dfrac{1}{3}\right)$

35. The vector $\vec{a}\times(\vec{b}\times\vec{a})$ is

(a) perpendicular to $\vec{a}$

(b) perpendicular to $\vec{b}$

(c) coplanar with $\vec{a}$ & $\vec{b}$

(d) perpendicular to $\vec{a}\times\vec{b}$

36. $(\vec{u}\times\vec{v})\,.\,(\vec{u}\times\vec{v})$ is equal to

(a) $\begin{vmatrix} \vec{u}\,.\,\vec{u} & \vec{u}\,.\,\vec{v} \\ \vec{u}\,.\,\vec{v} & \vec{v}\,.\,\vec{v} \end{vmatrix}$ (b) $(\vec{u}\,.\,\vec{v})^2 - \vec{u}^2\,.\,\vec{v}^2$

(c) $|\vec{u}|^2\,|\vec{v}|^2 - (\vec{u}\,.\,\vec{v})^2$ (d) none

37. If $\vec{z}_1 = a\,\hat{i} + b\,\hat{j}$ & $\vec{z}_2 = c\,\hat{i} + d\,\hat{j}$ are two vectors in $\hat{i}$ & $\hat{j}$ system where $|\vec{z}_1| = |\vec{z}_2| = r$ & $\vec{z}_1\,.\,\vec{z}_2 = 0$ then $\vec{w}_1 = a\,\hat{i} + c\,\hat{j}$ and $\vec{w}_2 = b\,\hat{i} + d\,\hat{j}$ satisfy

(a) $|\vec{w}_1| = r$ (b) $|\vec{w}_2| = r$

(c) $\vec{w}_1\,.\,\vec{w}_2 = 0$ (d) none of these

38. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the pv's of the point A, B, C and D respectively in three dimensional space and satisfy the relation $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$, then :

(a) A, B, C and D are coplanar

(b) the line joining the points B and D divides the line joining the point A and C in the ratio 2 : 1.

(c) the line joining the points A and C divides the line joining the points B and D in the ratio 1 : 1.

(d) the four vectors $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ are linearly dependents.

39. $\vec{a}$ and $\vec{b}$ are two non collinear unit vectors. Then $\vec{a}, \vec{b}, x\vec{a} - y\vec{b}$ form a triangle, if:

(a) $x = -1; y = 1$ and $\left|\vec{a}+\vec{b}\right| = 2\cos\left(\frac{\vec{a} \wedge \vec{b}}{2}\right)$

(b) $x = -1; y = 1$ and $\cos(\vec{a} \wedge \vec{b}) +$

$|\vec{a}+\vec{b}| \cos[\vec{a} \wedge -(\vec{a}+\vec{b})] = -1$

(c) $\left|\vec{a}+\vec{b}\right| = -2\cot\left(\frac{\vec{a} \wedge \vec{b}}{2}\right)\cos\left(\frac{\vec{a} \wedge \vec{b}}{2}\right)$ and

$x = -1, y = 1$

(d) none of these

40. If $\overrightarrow{OA} = \vec{a}; \overrightarrow{OB} = \vec{b}; \overrightarrow{OC} = 2\vec{a} + 3\vec{b}; \overrightarrow{OD} = \vec{a} - 2\vec{b}$, the length of $\overrightarrow{OA}$ is three times the length of $\overrightarrow{OB}$ and $\overrightarrow{OA}$ is prependicular to $\overrightarrow{DB}$ then $(\overrightarrow{BD} \times \overrightarrow{AC}).(\overrightarrow{OD} \times \overrightarrow{OC})$ is :

(a) $7|\vec{a} \times \vec{b}|^2$ (b) $42|\vec{a} \times \vec{b}|^2$

(c) 0 (d) none of these

41. $\hat{a}$ & $\hat{b}$ are two given unit vectors at right angle. The unit vector equally inclined with $\hat{a}$ & $\hat{b}$ and $\hat{a} \times \hat{b}$ will be :

(a) $-\frac{1}{\sqrt{3}}(\hat{a}+\hat{b}+\hat{a}\times\hat{b})$ (b) $\frac{1}{\sqrt{3}}(\hat{a}+\hat{b}+\hat{a}\times\hat{b})$

(c) $\frac{1}{\sqrt{3}}(\hat{a}+\hat{b}-\hat{a}\times\hat{b})$ (d) $-\frac{1}{\sqrt{3}}(\hat{a}+\hat{b}-\hat{a}\times\hat{b})$

42. If $\vec{a}$ and $\vec{b}$ unequal unit vectors such that $(\vec{a}-\vec{b})\times[(\vec{b}+\vec{a})\times(2\vec{a}+\vec{b})] = \vec{a}+\vec{b}$, then angle θ between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{2}$ (b) 0

(c) π (d) $\frac{\pi}{4}$

43. A vector $(\vec{d})$ is equally inclined to three vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{j} - 2\hat{k}$. Let $\vec{x}, \vec{y}, \vec{z}$ be three vector in the plane of $\vec{a}, \vec{b}; \vec{b}, \vec{c}; \vec{c}, \vec{a}$ respectively then

(a) $\vec{x} \cdot \vec{d} = 14$

(b) $\vec{y} \cdot \vec{d} = 3$

(c) $\vec{z} \cdot \vec{d} = 0$

(d) $\vec{r} \cdot \vec{d} = 0$ where $\vec{r} = \lambda\vec{x} + \mu\vec{y} + \delta\vec{z}$

44. Identify the statement(s) which is/are INCORRECT ?

(a) $\vec{a} \times [\vec{a} \times (\vec{a} \times \vec{b})] = (\vec{a} \times \vec{b})(\vec{a}^2)$

(b) If $\vec{a}, \vec{b}, \vec{c}$ are non–zero, non coplanar vectors, and $\vec{v}.\vec{a} = \vec{v}.\vec{b} = \vec{v}.\vec{c} = 0$ then $\vec{v}$ must be a null vector

(c) If $\vec{a}$ and $\vec{b}$ lie in a plane normal to the plane containing the vectors $\vec{a} - \vec{b}, \vec{c} - \vec{d}$, where $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are non–zero vectors, then $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{o}$

(d) If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}'\vec{c}'$ are reciprocal system of vectors then $\vec{a} \cdot \vec{b}' + \vec{b} \cdot \vec{c}' + \vec{c} \cdot \vec{a}' = 3$

Numerical Value Type Questions

45. If $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$ and $\vec{a} \perp (\vec{b} + \vec{c}), \vec{b} \perp (\vec{c} + \vec{a})$, $\vec{c} \perp (\vec{a} + \vec{b})$ then $|\vec{a} + \vec{b} + \vec{c}|^2$ is .....

46. $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors and every two are inclined to each other at an angle $\cos^{-1}(3/5)$. If $\vec{a} \times \vec{b} = p\vec{a} + q\vec{b} + r\vec{c}$, where p, q, r are scalars, then $55q^2$ is equal to

Assertion & Reason

Use the following codes to answer the questions

(A) If both assertion and reason are correct and reason is the correct explanation of assertion.

(B) If both assertion and reason are true but and reason is not the correct explanation of assertion.

(C) If assertion is true but reason is false.

(D) If assertion is false but reason is true.

(E) If assertion and reason are both false.

47. **Assertion :** Let $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}, \vec{b} = 4\hat{i} + 6\hat{j} - 2\hat{k},$ then $\vec{a} \times \vec{b} = \vec{o}$

Reason : If $\vec{a} \neq \vec{o}$, $\vec{b} \neq \vec{o}$ and $\vec{a}$ and $\vec{b}$ are non–collinear vectors, then $\vec{a} \times \vec{b}$ = ab sin θ $\hat{n}$, where θ is the smaller angle between the vectors $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is unit vector such that $\vec{a}, \vec{b}, \hat{n}$ taken in this order form right handed orientation

(a) A (b) B
(c) C (d) D
(e) E

48. **Assertion :** Let $\vec{a} = 3\hat{i} - \hat{j}$, $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$. If $\vec{b} = \vec{b}_1 + \vec{b}_2$ such that $\vec{b}_1$ is collinear with $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$ is possible, then $\vec{b}_2 = \hat{i} + 3\hat{j} - 3\hat{k}$.

Reason : If $\vec{a}$ and $\vec{b}$ are non–zero, non–collinear vectors, then $\vec{b}$ can be expressed as $\vec{b} = \vec{b}_1 + \vec{b}_2$, where $\vec{b}_1$ is collinear with $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$.

(a) A (b) B
(c) C (d) D
(e) E

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.For each question, choose the option corresponding to the correct matching.

49.

Column–I	**Column–II**
(A) If $\vec{a} + \vec{b} = \hat{j}$ and $2\vec{a} - \vec{b} = 3\hat{i} + \frac{\hat{j}}{2}$, then cosine of the angle between $\vec{a}$ and $\vec{b}$ is	**(P)** 1
(B) If $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = \lvert\vec{c}\rvert$, angle between each pair of vectors is $\frac{\pi}{3}$ and $\lvert\vec{a} + \vec{b} + \vec{c}\rvert = \sqrt{6}$, then $\lvert\vec{a}\rvert =$	**(Q)** $5\sqrt{3}$
(C) Area of the parallelogram whose diagonals represent the vectors $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$ is	**(R)** 7
(D) If $\vec{a}$ is perpendicular to $\vec{b} + \vec{c}, \vec{b}$ is perpendicular to $\vec{c} + \vec{a}$, $\vec{c}$ is perpendicular to $\vec{a} + \vec{b}, \lvert\vec{a}\rvert = 2, \lvert\vec{b}\rvert = 3$ and $\lvert\vec{c}\rvert = 6$, then $\lvert\vec{a} + \vec{b} + \vec{c}\rvert =$	**(S)** $-\frac{3}{5}$

The correct matching is

(a) A-R; B-P; C-Q; D-S
(b) A-P; B-R; C-Q; D-S
(c) A-Q; B-R; C-R; D-P
(d) A-S; B-P; C-Q; D-R

50. Match the following

	Column-I		**Column-II**
(A)	If the vectors $\vec{a}, \vec{b}, \vec{c}$ form sides $\overrightarrow{BA}, \overrightarrow{CA}, \overrightarrow{AB}$ of ΔABC, then	**(P)**	$\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a}$
(B)	If $\vec{a}, \vec{b}, \vec{c}$ are forming three adjacent sides of regular tetrahedron, then	**(Q)**	$\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$
(C)	If $\vec{a} \times \vec{b} = \vec{c}$, $\vec{b} \times \vec{c} = \vec{a}$ then	**(R)**	$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$
(D)	$\vec{a}, \vec{b}, \vec{c}$ are unit vectors and $\vec{a} + \vec{b} + \vec{c} = 0$ then	**(S)**	$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -\frac{3}{2}$

The correct matching is

(a) A $\to$ R ; B $\to$ P; C $\to$P,Q; D $\to$ S

(b) A $\to$ R,S ; B $\to$ P; C $\to$S; D $\to$Q

(c) A $\to$P ; B $\to$R; C $\to$ Q; D $\to$P,S

(d) A $\to$ R ; B $\to$Q; C $\to$P; D $\to$ S

Paragraph Type Questions

Using the following passage, solve Q.51 to Q.54

PASSAGE

Three vectors $\hat{a}$, $\hat{b}$ and $\hat{c}$ are such that

$\hat{a} \times \hat{b} = \hat{c}, \hat{b} \times \hat{c} = \hat{a}, \hat{c} \times \hat{a} = \hat{b}.$

Answer the following questions :

51. If vector $3\hat{a} - 2\hat{b} + 2\hat{c}$ and $-\hat{a} - 2\hat{c}$ are adjacent sides of a parallelogram, then an angle between the diagonals is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

52. Vectors $2\hat{a} - 3\hat{b} + 4\hat{c}$, $\hat{a} + 2\hat{b} - \hat{c}$ and $x\hat{a} - \hat{b} + 2\hat{c}$ are coplanar, then x =

(a) $\frac{8}{5}$ (b) $\frac{5}{8}$

(c) 0 (d) 1

53. Let $\vec{x} = \hat{a} + \hat{b}, \vec{y} = 2\hat{a} - \hat{b}$, then the point of intersection of straight lines $\vec{r} \times \vec{x} = \vec{y} \times \vec{x}, \vec{r} \times \vec{y} = \vec{x} \times \vec{y}$ is

(a) $2\hat{b}$ (c) $3\hat{b}$

(c) $3\hat{a}$ (d) $2\hat{a}$

54. $\hat{a} \cdot (\hat{b} \times \hat{c}) + \hat{b} \cdot (\hat{c} \times \hat{a}) + \hat{c} \cdot (\hat{a} \times \hat{b})$ is equal to

(a) 1 (b) 3

(c) 0 (d) – 12

3-DIMENSIONAL GEOMETRY

Objective Questions I [Only one correct option]

55. Consider a tetrahedron with faces f_1, f_2, f_3, f_4. Let $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ be the vectors whose magnitudes are respectively equal to the areas of f_1, f_2, f_3, f_4 & whose directions are perpendicular to these faces in the outward direction. Then

(a) $\vec{a}_1 + \vec{a}_2 + \vec{a}_3 + \vec{a}_4 = \vec{0}$ (b) $\vec{a}_1 + \vec{a}_3 = \vec{a}_2 + \vec{a}_4$

(c) $\vec{a}_1 + \vec{a}_2 = \vec{a}_3 + \vec{a}_4$ (d) none

56. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non–coplanar unit vectors equally inclined to one another at an acute angle θ. Then $[\vec{a}\ \vec{b}\ \vec{c}]$ in terms of θ is equal to :

(a) $(1+\cos\theta)\sqrt{\cos 2\theta}$ (b) $(1+\cos\theta)\sqrt{1-2\cos 2\theta}$

(c) $(1-\cos\theta)\sqrt{1+2\cos\theta}$ (d) none of these

57. If line makes angle $\alpha,\beta,\gamma,\delta$ with the four diagonals of a cube, then the value of $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta =$

(a) 1 (b) 4/3

(c) 2/3 (d) Variable

58. If the sum of the squares of the distance of a point from the three coordinate axes be 36, then its distance from the origin is

(a) 6 (b) $3\sqrt{2}$

(c) $2\sqrt{3}$ (d) $6\sqrt{2}$

59. The direction ratio's of the line $x-y+z-5=0=x-3y-6$ are

(a) 3, 1, –2 (b) 2, –4, 1

(c) $\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}$ (d) $\frac{2}{\sqrt{21}}, \frac{-4}{\sqrt{21}}, \frac{1}{\sqrt{21}}$

60. A variable plane passes through a fixed point (1, 2, 3). The locus of the foot of the perpendicular drawn from origin to this plane is :

(a) $x^2 + y^2 + z^2 - x - 2y - 3z = 0$

(b) $x^2 + 2y^2 + 3z^2 - x - 2y - 3z = 0$

(c) $x^2 + 4y^2 + 9z^2 + x + 2y + 3 = 0$

(d) $x^2 + y^2 + z^2 + x + 2y + 3z = 0$

61. The straight lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and

$\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{-2}$ are

(a) parallel lines (b) Intersecting at 60º

(c) Skew lines (d) Intersecting at right angle

62. The equation of the plane which bisects the angle between the planes $3x - 6y + 2z + 5 = 0$ and $4x - 12y + 3z - 3 = 0$ which contains the origin is

(a) $33x - 13y + 32z + 45 = 0$

(b) $x - 3y + z - 5 = 0$

(c) $33x + 13y + 32z + 45 = 0$

(d) None of these

63. P is fixed point (a, a, a) on a line through the origin equally inclined to the axes, then any plane through P perpendicular to OP, makes intercepts on the axes, the sum of whose reciprocals is equal to

(a) a (b) a/2

(c) 3a/2 (d) 1/a

64. If $\vec{a}_1, \vec{a}_2$ and $\vec{a}_3$ are non–coplanar vectors and

$(x + y - 3)\ \vec{a}_1 + (2x - y + 2)\ \vec{a}_2 + (2x + y + \lambda)\ \vec{a}_3 = \vec{0}$

holds for some 'x' and 'y' then 'λ' is

(a) $\frac{7}{3}$ (b) 2

(c) $-\frac{10}{3}$ (d) $\frac{5}{3}$

65. If the foot of the perpendicular from the origin to a plane is P (a, b, c), the equation of the plane is

(a) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$

(b) $ax + by + cz = 3$

(c) $ax + by + cz = a^2 + b^2 + c^2$

(d) $ax + by + cz = a + b + c$

66. Equation of line in the plane $P : 2x - y + z - 4 = 0$ which is perpendicular to the line l whose equation is $\frac{x-2}{1} = \frac{y-2}{-1} = \frac{z-3}{-2}$ and which passes through the point of intersection of l and P is

(a) $\frac{x-2}{3} = \frac{y-1}{5} = \frac{z-1}{-1}$ (b) $\frac{x-1}{3} = \frac{y-3}{5} = \frac{z-5}{-1}$

(c) $\frac{x+2}{2} = \frac{y+1}{-1} = \frac{z+1}{1}$ (d) $\frac{x-2}{2} = \frac{y-1}{-1} = \frac{z-1}{1}$

67. Equation of plane which passes through the point of intersection of lines $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2}$ and $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and at greatest distance from the point (0, 0, 0) is :

(a) $4x + 3y + 5z = 25$ (b) $4x + 3y + 5z = 50$

(c) $3x + 4y + 5z = 49$ (d) $x + 7y - 5z = 2$

68. Let A(1, 1, 1), B(2, 3, 5), C(–1, 0, 2) be three points, then equation of a plane parallel to the plane ABC which is at distance 2 from plane ABC

(a) $2x - 3y + z + 2\sqrt{14} = 0$

(b) $2x - 3y + z - \sqrt{14} = 0$

(c) $2x - 3y + z + 2 = 0$

(d) $2x - 3y + z - 2 = 0$

Numerical Value Type Questions

69. If equation of the plane through the straight line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z}{5}$ and perpendicular to the plane $x - y + z + 2 = 0$ is $ax - by + cz + 4 = 0$, then find the value of $10^3a + 10^2b + 10c$.

70. In a regular tetrahedron let θ be the angle between any edge and a face not containing the edge. If $\cos^2\theta = \frac{a}{b}$ where $a, b \in I^+$ also a and b are coprime, then find the value of $(10a + b)$

71. If the planes $x - cy - bz = 0$, $cx - y + az = 0$ and $bx + ay - z = 0$ pass through a straight line, then find the value of $a^2 + b^2 + c^2 + 2abc$.

72. If the reflection of the point P(1, 0, 0) in the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ is (α, β, γ). Find $-(\alpha + \beta + \gamma)$

Assertion & Reason

Use the following codes to answer the questions

(A) If both assertion and reason are correct and reason is the correct explanation of assertion.

(B) If both assertion and reason are true but and reason is not the correct explanation of assertion.

(C) If assertion is true but reason is false.

(D) If assertion is false but reason is true.

(E) If assertion and reason are both false.

73. **Assertion :** If $\vec{a} = 3\hat{i} + \hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}, \vec{c} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{d} = 2\hat{i} - \hat{j}$, then there exist real numbers α, β, γ such that $\vec{a} = \alpha\vec{b} + \beta\vec{c} + \gamma\vec{d}$

Reason : $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are four vectors in a 3–dimensional space. If $\vec{b}, \vec{c}, \vec{d}$ are non–coplanar, then there exist real numbers α, β, γ such that $\vec{a} = \alpha\vec{b} + \beta\vec{c} + \gamma\vec{d}$

(a) A (b) B
(c) C (d) D
(e) E

74. **Assertion :** Let $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are position vectors of four points A, B, C and D and $3\vec{a} - 2\vec{b} + 5\vec{c} - 6\vec{d} = \vec{0}$, then points A, B, C and D are coplanar.

Reason : Three non zero, linearly dependent co–initial vectors $\left(\overrightarrow{PQ}, \overrightarrow{PR} \text{ and } \overrightarrow{PS}\right)$ are coplanar.

(a) A (b) B
(c) C (d) D
(e) E

75. **Assertion :** A point on the straight line $2x + 3y - 4z = 5$, $3x - 2y + 4z = 7$ can be determined by taking $x = k$ and then solving the two equations for y and z, where k is any real number except 12/5.

Reason : If $c' \neq kc$, then the straight line $ax + by + cz + d = 0$, $kax + kby + c'z + d' = 0$, does not intersect the plane $z = \alpha$, where α is any real number except $\frac{d' - kd}{kc - c'}$.

(a) A (b) B
(c) C (d) D
(e) E

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching. For each question, choose the option corresponding to the correct matching.

76.

Column–I	Column–II
(A) Foot of perp. drawn for point (1, 2, 3) to the line $\frac{x-2}{2}=\frac{y-1}{3}=\frac{z-2}{4}$ is	**(P)** $\left(\frac{107}{29},\frac{30}{29},\frac{69}{29}\right)$
(B) Image of line point (1, 2, 3) in the line $\frac{x-2}{2}=\frac{y-1}{3}=\frac{z-2}{4}$ is	**(Q)** $\left(\frac{88}{29},\frac{125}{29},\frac{69}{29}\right)$
(C) Foot of perpendicular from the point (2, 3, 5) to the plane $2x + 3y - 4z + 17 = 0$ is	**(R)** $\left(\frac{68}{29},\frac{44}{29},\frac{78}{29}\right)$
(D) Image of the point (2, 5, 1) in the plane $3x - 2y + 4z - 5 = 0$ is	**(S)** $\left(\frac{38}{29},\frac{57}{29},\frac{185}{29}\right)$

The correct matching is

(a) A-R; B-P; C-S; D-Q

(b) A-R; B-S; C-P; D-Q

(c) A-R; B-P; C-Q; D-S

(d) A-P; B-R; C-S; D-Q

77. Consider three planes

$P_1 : 2x + y + z = 1$

$P_2 : x - y + z = 2$

$P_3 : \alpha x + y + 3z = 5$

The three planes intersects each other at point P on XOY plane and at point Q on YOZ plane. O is the origin.

	Column-I	Column-II
(A)	The value of α is	**(P)** 1
(B)	The length of projection of PQ on x-axis is	**(Q)** 2
(C)	If the coordinates of point R on the line PQ situated at minimum distance from point ‘O’ are (a, b, c), then the value of 7a + 14b +14c is	**(R)** 4
(D)	If the area of ΔPOQ is $\sqrt{\frac{a}{b}}$ Then the value of a – b is where a & b are co prime numbers	**(S)** 3

The correct matching is

(a) $A \to S$; $B \to P$; $C \to Q$; $D \to R$

(b) $A \to P$; $B \to R$; $C \to Q$; $D \to S$

(c) $A \to R$; $B \to Q$; $C \to P$; $D \to S$

(d) $A \to R$; $B \to P$; $C \to Q$; $D \to S$

Paragraph Type Questions

Using the following passage, solve Q. 78 to Q. 80

Passage

Let $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ be two planes, where $d_1, d_2 > 0$. Then origin lies in acute angle if $a_1a_2 + b_1b_2 + c_1c_2 < 0$ and origin lies in obtuse angle if $a_1a_2 + b_1 b_2 + c_1c_2 > 0$.

Further point (x_1, y_1, z_1) and origin both lie either in acute angle or in obtuse angle, if $(a_1x_1 + b_1y_1 + c_1z_1 + d_1)(a_2x_1 + b_2y_1 + c_2z_1 + d_2) > 0$. One of (x_1, y_1, z_1) and origin lie in acute angle and the other in obtuse angle, if

$(a_1x_1 + b_1y_1 + c_1z_1 + d_1)(a_2x_1 + b_2y_1 + c_2z_1 + d_2) < 0$

78. Given the planes $2x + 3y - 4z + 7 = 0$ and $x - 2y + 3z - 5 = 0$, if a point P is $(1, -2, 3)$, then

(a) O and P both lie in acute angle between the planes

(b) O and P both lie in obtuse angle

(c) O lies in acute angle, P lies in obtuse angle

(d) O lies in obtuse angle, P lies an acute angle.

79. Given the planes $x + 2y - 3z + 5 = 0$ and $2x + y + 3z + 1 = 0$. If a point P is $(2, -1, 2)$, then

(a) O and P both lie in acute angle between the planes

(b) O and P both lie in obtuse angle

(c) O lies in acute angle, P lies in obtuse angle

(d) O lies in obtuse angle, P lies an acute angle.

80. Given the planes $x + 2y - 3z + 2 = 0$ and $x - 2y + 3z + 7 = 0$, if the point P is $(1, 2, 2)$, then

(a) O and P both lie in acute angle between the planes

(b) O and P both lie in obtuse angle

(c) O lies in acute angle, P lies in obtuse angle

(d) O lies in obtuse angle, P lies an acute angle.

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

VECTORS

Objective Question I [Only one correct option]

1. If the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ form the sides BC, CA and AB respectively of a triangle ABC, then : **(2000)**

(a) $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=0$ (b) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

(c) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}$ (d) $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=0$

2. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit coplanar vectors, then the scalar triple product $\left[\left(2\vec{a}-\vec{b}\right)\left(2\vec{b}-\vec{c}\right)\left(2\vec{c}-\vec{a}\right)\right]$ is equal to : **(2000)**

(a) 0 (b) 1

(c) $-\sqrt{3}$ (d) $\sqrt{3}$

3. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, then

$\left|\vec{a}-\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}-\vec{a}\right|^2$ does not exceed : **(2001)**

(a) 4 (b) 9

(c) 8 (d) 6

4. Let $\vec{a}=\hat{i}-\hat{k},\ \vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}$ and

$\vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$ Then $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ depends on : **(2001)**

(a) only x (b) only y

(c) neither x nor y (d) both x and y

5. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+2\vec{b}$ and $5\vec{a}-4\vec{b}$ are perpendicular to each other, then the angle between $\vec{a}$ and $\vec{b}$ is : **(2002)**

(a) 45^o (b) 60^o

(c) $\cos^{-1}\left(\frac{1}{3}\right)$ (d) $\cos^{-1}\left(\frac{2}{7}\right)$

6. Let $\vec{V}=2\vec{i}+\vec{j}-\vec{k}$ and $\vec{W}=\vec{i}+3\vec{k}$. If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $\left[\vec{U}\ \vec{V}\ \vec{W}\right]$ is : **(2002)**

(a) -1 (b) $\sqrt{10}+\sqrt{6}$

(c) $\sqrt{59}$ (d) $\sqrt{60}$

7. If $\vec{a}=\left(\hat{i}+\hat{j}+\hat{k}\right)$, $\vec{a}\cdot\vec{b}=1$ and $\vec{a}\times\vec{b}=\hat{j}-\hat{k}$, then $\vec{b}$ is : **(2003)**

(a) $\hat{i}-\hat{j}+\hat{k}$ (b) $2\hat{j}-\hat{k}$

(c) $\hat{i}$ (d) $2\hat{i}$

8. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero, non-coplanar vectors and

$\vec{b}_1=\vec{b}-\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}$, $\quad\vec{b}_2=\vec{b}+\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}$,

$\vec{c}_1=\vec{c}-\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}-\frac{\vec{c}\cdot\vec{b}}{|\vec{b}|^2}\vec{b}$, $\quad\vec{c}_2=\vec{c}-\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}-\frac{\vec{c}\cdot\vec{b}_1}{|\vec{b}|^2}\vec{b}_1$,

$\vec{c}_3=\vec{c}-\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}-\frac{\vec{c}\cdot\vec{b}_2}{|\vec{b}|^2}\vec{b}_2$, $\quad\vec{c}_4=\vec{a}-\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}$.

Then which of the following is a set of mutually orthogonal vectors ? **(2005)**

(a) $\left\{\vec{a},\vec{b}_1,\vec{c}_1\right\}$ (b) $\left\{\vec{a},\vec{b}_1,\vec{c}_2\right\}$

(c) $\left\{\vec{a},\vec{b}_2,\vec{c}_3\right\}$ (d) $\left\{\vec{a},\vec{b}_2,\vec{c}_4\right\}$

9. Let $\vec{a}=\hat{i}+2\hat{j}+\hat{k},\ \vec{b}=\hat{i}-\hat{j}+\hat{k},\ \vec{c}=\hat{i}+\hat{j}-\hat{k}$.

A vector coplanar to $\vec{a}$ and $\vec{b}$ has a projection along $\vec{c}$ of magnitude $\frac{1}{\sqrt{3}}$, then the vector is : **(2006)**

(a) $4\hat{i}-\hat{j}+4\hat{k}$ (b) $4\hat{i}+\hat{j}-4\hat{k}$

(c) $2\hat{i}+\hat{j}+\hat{k}$ (d) none of these

10. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Which one of the following is correct ? **(2007)**

(a) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}=\vec{0}$

(b) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq\vec{0}$

(c) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{a}\times\vec{c}\neq 0$

(d) $\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}$ are mutually perpendicular

11. Let two non–collinear unit vectors $\hat{a}$ and $\hat{b}$ form an acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}$ cos t + $\hat{b}$ sin t. When P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overline{OP}$. Then, **(2008)**

(a) $\hat{u}=\dfrac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M=(1+\hat{a}\cdot\hat{b})^{1/2}$

(b) $\hat{u}=\dfrac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M=(1+\hat{a}\cdot\hat{b})^{1/2}$

(c) $\hat{u}=\dfrac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M=(1+2\hat{a}\cdot\hat{b})^{1/2}$

(d) $\hat{u}=\dfrac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M=(1+2\hat{a}\cdot\hat{b})^{1/2}$

12. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the unit vectors such that $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=1$ and $\vec{a}\cdot\vec{c}=\dfrac{1}{2}$, then **(2009)**

(a) $\vec{a}, \vec{b}, \vec{c}$ are non–coplanar

(b) $\vec{a}, \vec{b}, \vec{d}$ are non–coplanar

(c) $\vec{b}, \vec{d}$ are non–parallel

(d) $\vec{a}, \vec{d}$ are parallel and $\vec{b}, \vec{c}$ are parallel

13. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}, 4\hat{i}, 3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively. The quadrilateral PQRS must be a **(2010)**

(a) parallelogram, which is neither a rhombus nor a rectangle

(b) square

(c) rectangle, but not a square

(d) rhombus, but not a square

14. Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB}=2\hat{i}+10\hat{j}+11\hat{k}$ and $\overrightarrow{AD}=-\hat{i}+2\hat{j}+2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD′. If AD′′makes a right angle with the side AB, then the cosine of the angle α is given by **(2010)**

(a) $\dfrac{8}{9}$ (b) $\dfrac{\sqrt{17}}{9}$

(c) $\dfrac{1}{9}$ (d) $\dfrac{4\sqrt{5}}{9}$

15. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\dfrac{1}{\sqrt{3}}$, is given by **(2011)**

(a) $\hat{i}-3\hat{j}+3\hat{k}$ (b) $-3\hat{i}-3\hat{j}-\hat{k}$

(c) $3\hat{i}-\hat{j}+3\hat{k}$ (d) $\hat{i}+3\hat{j}-3\hat{k}$

16. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}+\vec{b}|=\sqrt{29}$ and $\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k})=(2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$, then a possible value of $(\vec{a}+\vec{b}).(-7\hat{i}+2\hat{j}+3\hat{k})$ is **(2012)**

(a) 0 (b) 3

(c) 4 (d) 8

17. The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$, is **(2017)**

(a) $-14x + 2y + 15z = 3$ (b) $14x - 2y + 15z = 27$

(c) $14x + 2y - 15z = 1$ (d) $14x + 2y + 15z = 31$

18. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

$$\overrightarrow{OP}.\overrightarrow{OQ}+\overrightarrow{OR}.\overrightarrow{OS}=\overrightarrow{OR}.\overrightarrow{OP}+\overrightarrow{OQ}.\overrightarrow{OS}=\overrightarrow{OQ}.\overrightarrow{OR}+\overrightarrow{OP}.\overrightarrow{OS}$$

Then the triangle PQR has S as its **(2017)**

(a) incentre (b) circumcentre

(c) orthocenter (d) centroid

19. Let O be the origin, and $\overrightarrow{OX},\overrightarrow{OY},\overrightarrow{OZ}$ be three unit vectors in the directions of the sides $\overrightarrow{QR},\overrightarrow{RP},\overrightarrow{PQ}$, respectively, of a triangle PQR. Then **(2017)**

$|\overrightarrow{OX}\times\overrightarrow{OY}|=$

(a) sin(P + R) (b) sin 2R

(c) sin (P + Q) (d) sin (Q + R)

20. If the triangle PQR varies, then the minimum value of cos (P + Q) + cos (Q + R) + cos (R + P) **(2017)**

(a) $-\frac{3}{2}$ (b) $\frac{3}{2}$

(c) $\frac{5}{3}$ (d) $-\frac{5}{3}$

Objective Questions II [One or more than one correct option]

21. Let $\vec{A}$ be a vector parallel to line of intersection of planes P_1 and P_2 through the origin. P_1 is parallel to the vectors $2\hat{j}+3\hat{k}$ and $4\hat{j}-3\hat{k}$ and P_2 is parallel to $\hat{j}-\hat{k}$ and $3\hat{i}+3\hat{j}$, then the angle between vector $\vec{A}$ and $2\hat{i}+\hat{j}-2\hat{k}$ is **(2006)**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{3\pi}{4}$

22. The vector(s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, and are perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is/are **(2011)**

(a) $\hat{j}-\hat{k}$ (b) $-\hat{i}+\hat{j}$

(c) $\hat{i}-\hat{j}$ (d) $-\hat{j}+\hat{k}$

23. A line l passing through the origin is perpendicular to the lines

$l_1:(3+t)\hat{i}+(-1+2t)\hat{j}+(4+2t)\hat{k},\ -\infty<t<\infty$

$l_2:(3+2s)\hat{i}+(3+2s)\hat{j}+(2+s)\hat{k},\ -\infty<s<\infty$

Then, the coordinate(s) of the point(s) on l_2 at a distance of $\sqrt{17}$ from the point of intersection of l and l_1 are **(2013)**

(a) $\left(\frac{7}{3},\frac{7}{3},\frac{5}{3}\right)$ (b) (–1, –1, 0)

(c) (1, 1, 1) (d) $\left(\frac{7}{9},\frac{7}{9},\frac{8}{9}\right)$

24. Let $\vec{x},\vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a non-zero vector perpendicular to $\vec{x}$ and $\vec{y}\times\vec{z}$ and $\vec{b}$ is a non-zero vector perpendicular to $\vec{y}$ and $\vec{z}\times\vec{x}$, then **(2014)**

(a) $\vec{b}=(\vec{b}.\vec{z})(\vec{z}-\vec{x})$

(b) $\vec{a}=(\vec{a}.\vec{y})(\vec{y}-\vec{z})$

(c) $\vec{a}.\vec{b}=-(\vec{a}.\vec{y})(\vec{b}.\vec{z})$

(d) $\vec{a}=(\vec{a}.\vec{y})(\vec{z}-\vec{y})$

25. Let ΔPQR be a triangle. Let $\vec{a}=\overrightarrow{QR}$, $\vec{b}=\overrightarrow{RP}$ and $\vec{c}=\overrightarrow{PQ}$. If $|\vec{a}|=12$, $|\vec{b}|=4\sqrt{3}$, $\vec{b}.\vec{c}=24$, then which of the following is (are) true ? **(2015)**

(a) $\frac{|\vec{c}|^2}{2}-|\vec{a}|=12$ (b) $\frac{|\vec{c}|^2}{2}+|\vec{a}|=30$

(c) $|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}|=48\sqrt{3}$ (d) $\vec{a}.\vec{b}=-72$

26. Let $\hat{u}=u_1\hat{i}+u_2\hat{j}+u_3\hat{k}$ be a unit vector in R^3 and $\hat{w}=\frac{1}{\sqrt{6}}\left(\hat{i}+\hat{j}+2\hat{k}\right)$. Given that there exists a vector $\vec{v}$ in R^3 such that $|\hat{u}\times\vec{v}|=1$ and $\hat{w}.|\hat{u}\times\vec{v}|=1$. Which of the following statement(s) is(are) correct ? **(2016)**

(a) There is exactly one choice for such $\vec{v}$

(b) There are infinitely many choices for such

(c) If $\hat{u}$ lies in the xy-plane then $|u_1|=|u_2|$

(d) If $\hat{u}$ lies in the xz-plane then $2|u_1|=|u_3|$

27. Let L_1 and L_2 be the following straight line.

$$L_1:\frac{x-1}{1}=\frac{y}{-1}=\frac{z-1}{3} \quad and \quad L_2:\frac{x-1}{-3}=\frac{y}{-1}=\frac{z-1}{1}$$

Suppose the straight line

$$L:\frac{x-\alpha}{l}=\frac{y-1}{m}=\frac{z-\gamma}{-2}$$

lies in the plane containing L_1 and L_2, and passes through the point of intersection of L_1 and L_2. If the line L bisects the acute angle between the lines L_1 and L_2, then which of the following statements is/are TRUE ? **(2020)**

(a) $\alpha-\gamma=3$ (b) $l+m=2$

(c) $\alpha-\gamma=1$ (d) $l+m=0$

28. Let a and b be positive real numbers. Suppose $\overrightarrow{PQ}=a\hat{i}+b\hat{j}$ and $\overrightarrow{PS}=a\hat{i}-b\hat{j}$ are adjacent sides of a parallelogram PQRS. Let $\vec{u}$ and $\vec{v}$ be the projection vectors of $\vec{w}=\hat{i}+\hat{j}$ along $\overrightarrow{PQ}$ and $\overrightarrow{PS}$, respectively. If $|\vec{u}|+|\vec{v}|=|\vec{w}|$ and if the area of the parallelogram PQRS is 8, then which of the following statements is/are TRUE? **(2020)**

(a) $a+b=4$

(b) $a-b=2$

(c) The length of the diagonal PQ of the parallelogram PQRS is 4

(d) $\vec{w}$ is an angle bisector of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PS}$

29. Let O be the origin and $\overline{OA}=2\hat{i}+2\hat{j}+\hat{k}$, $\overline{OB}=\hat{i}-2\hat{j}+2\hat{k}$ and $\overline{OC}=\frac{1}{2}\left(\overline{OB}-\lambda\overline{OA}\right)$ for some $\lambda>0$. If $\left|\overline{OB}\times\overline{OC}\right|=\frac{9}{2}$, then which of the following statements is (are) TRUE? **(2021)**

(a) Projection of $\overline{OC}$ on $\overline{OA}$ is $-\frac{3}{2}$

(b) Area of the triangle OAB is $\frac{9}{2}$

(c) Area of triangle ABC is $\frac{9}{2}$

(d) The acute angle between the diagonals of the parallelogram with adjacent sides $\overline{OA}$ and $\overline{OC}$ is $\frac{\pi}{3}$

Numerical Value Type Questions

30. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a}=\frac{\hat{i}-2\hat{j}}{\sqrt{5}}$ and $\vec{b}=\frac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$, then the value of

$\left(2\vec{a}+\vec{b}\right).\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}-2\vec{b}\right)\right]$ is.... **(2010)**

31. Let $\vec{a}=-\hat{i}-\hat{k}$, $\vec{b}=-\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}+2\hat{j}+3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}.\vec{a}=0$, then the value of $\vec{r}.\vec{b}$ is..... **(2011)**

32. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying

$|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=9$, then

$|2\vec{a}+5\vec{b}+5\vec{c}|$ is **(2012)**

33. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, where p,q and r are scalars, then the value of $\frac{p^2 + 2q^2 + r^2}{q^2}$ is **(2014)**

34. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a}.\vec{b} = 0$. For some $x, y \in R$, let $\vec{c} = x\vec{a} + y\vec{b} + (\vec{a} \times \vec{b})$. If $|\vec{c}| = 2$ and the vector $\vec{c}$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$ and then the value of $8\cos^2\alpha$ is ____. **(2018)**

35. Let P be a point in the first octant, whose image Q in the plane $x + y = 3$ lies on the z-axis (that is, the line segment PQ is perpendicular to the plane $x + y = 3$ and the mid-point of PQ lies in the plane $x + y = 3$). Let the distance of P from the x-axis be 5. If R is the image of P in the xy-plane, then the length of PR is _____. **(2018)**

36. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors, consider a vector $\vec{c} = \alpha\vec{a} + \beta\vec{b}, \alpha, \beta \in R$. If the projection of $\vec{c}$ on the vector $(\vec{a} + \vec{b})$ is $3\sqrt{2}$, then the minimum value of $(\vec{c} - (\vec{a} \times \vec{b})).\vec{c}$ equals **(2019)**

37. In a triangle PQR, let $\vec{a} = \overline{QR}, \vec{b} = \overline{RP}$ and $\vec{c} = \overline{PQ}$. If $|\vec{a}| = 3, |\vec{b}| = 4$ and $\frac{\vec{a}.(\vec{c} - \vec{b})}{\vec{c}.(\vec{a} - \vec{b})} = \frac{|\vec{a}|}{|\vec{a}| + |\vec{b}|}$, then the value of $|\vec{a} \times \vec{b}|^2$ is ……… . **(2020)**

38. Let α, β and γ be real numbers such that the system of linear equations

$$x + 2y + 3z = \alpha$$

$$4x + 5y + 6z = \beta$$

$$7x + 8y + 9z = \gamma - 1$$

is consistent. Let |M| represent the determinant of the matrix

$$M = \begin{bmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Let P be the plane containing all those (α, β, γ) for which the above system of linear equations is consistent, and D be the square of the distance of the point (0, 1, 0) from the plane P. The value of D is ____. **(2021)**

39. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors in three-dimensional space, where $\vec{u}$ and $\vec{v}$ are unit vectors which are not perpendicular to each other and $\vec{u} \cdot \vec{w} = 1, \vec{v} \cdot \vec{w} = 1, \vec{w} \cdot \vec{w} = 4$. If the volume of the parallelopiped, whose adjacent sides are represented by the vectors $\vec{u}, \vec{v}$ and $\vec{w}$, is $\sqrt{2}$, then the value of $|3\vec{u} + 5\vec{v}|$ is ____. **(2021)**

Assertion & Reason

For the following questions choose the correct answer from the codes (a), (b), (c) and (d) defined as follows.

(a) Statement I is true, Statement II is also true; Statement II is the correct explanation of Statement I.

(b) Statement I is true, Statement II is also true; Statement II is not the correct explanation of Statement I.

(c) Statement I is true, Statement II is false.

(d) Statement I is false, Statement II is true.

40. Let the vectors $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RS}, \overrightarrow{ST}, \overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.

Assertion : $\overrightarrow{PQ} \times (\overrightarrow{RS} + \overrightarrow{ST}) \neq \vec{0}$.

Reason : $\overrightarrow{PQ} \times \overrightarrow{RS} = \vec{0}$ and $\overrightarrow{PQ} \times \overrightarrow{ST} \neq \vec{0}$. **(2007)**

(a) A (b) B

(c) C (d) D

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.For each question, choose the option corresponding to the correct matching.

41. Match List–I with List–II and select the correct answer using the code given below the lists.

List–I	List–II
P. Volume of parallelopiped determined by vectors $\vec{a}, \vec{b}$ & $\vec{c}$ is 2. Then, the volume of the parallelopied determined by vectors $2\left(\vec{a}\times\vec{b}\right), 3\left(\vec{b}\times\vec{c}\right)$ and $\left(\vec{c}\times\vec{a}\right)$ is	1. 100
Q. Volume of parallelopiped determined by vectors $\vec{a}, \vec{b}$ & $\vec{c}$ is 5. Then, the volume of the parallelopiped determined by vectors $3\left(\vec{a}+\vec{b}\right), \left(\vec{b}+\vec{c}\right)$ and $2\left(\vec{c}+\vec{a}\right)$ is	2. 30
R. Area of a triangle with adjacent sides determined by vector $\vec{a}$ and $\vec{b}$ is 20. Then, the area of the triangle with adjacent sides determined by vectors $\left(2\vec{a}+3\vec{b}\right)$ and $\left(\vec{a}-\vec{b}\right)$ is	3. 24
S. Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then, the area of the parallelogram with adjacent sides determined by vectors $\left(\vec{a}+\vec{b}\right)$ and $\vec{a}$ is	4. 60

(2015)

	P	Q	R	S		P	Q	R	S
(a)	4	2	3	1	(b)	2	3	1	4
(c)	3	4	1	2	(d)	1	4	3	2

Text

42. If $\vec{u}, \vec{v}, \vec{w}$ are three non–coplanar unit vectors and α, β, γ are the angles between $\vec{u}$ and $\vec{v}, \vec{v}$ and $\vec{w}, \vec{w}$ and $\vec{u}$ respectively and $\vec{x}, \vec{y}, \vec{z}$ are unit vectors along the bisectors of the angles α, β, γ respectively. Prove that

$$[\vec{x}\times\vec{y} \;\; \vec{y}\times\vec{z} \;\; \vec{z}\times\vec{x}]$$

$$= \frac{1}{16}[\vec{u}\ \vec{v}\ \vec{w}]^2 \sec^2\frac{\alpha}{2}\sec^2\frac{\beta}{2}\sec^2\frac{\gamma}{2}.$$ **(2003)**

43. A plane is parallel to two lines whose direction ratios are (1, 0, –1) & (–1, 1, 0) and it contains the point (1, 1, 1,). If it cuts coordinate axes at A, B, C. Then find the volume of the tetrahedron OABC. **(2004)**

44. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are four distinct vectors satisfying the conditions $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$, then prove that $\vec{a}.\vec{b} + \vec{c}.\vec{d} \neq \vec{a}.\vec{c} + \vec{b}.\vec{d}$. **(2004)**

45. Consider the cube in the first octant with sides OP, OQ and OR of length 1, along the x-axis, y-axis and z-axis, respectively, where O(0, 0, 0) is the origin. Let $S\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ be the centre of the cube and T be the vertex of the cube opposite to the origin O such that S lies on the diagonal OT. If $\vec{p} = \overrightarrow{SP}, \vec{q} = \overrightarrow{SQ}, \vec{r} = \overrightarrow{SR}$ and $\vec{t} = \overrightarrow{ST}$, then the value of $\left|\left(\vec{p}\times\vec{q}\right)\times\left(\vec{r}\times\vec{t}\right)\right|$ is ____ **(2018)**

3-DIMENSIONAL GEOMETRY

Objective Question I [Only one correct option]

46. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ be such that $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right) = 0$. Let P_1 and P_2 be planes determined by the pairs of vectors $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ respectively, then the angle between P_1 and P_2 is: **(2000)**

(a) 0 (b) $\pi/4$

(c) $\pi/3$ (d) $\pi/2$

47. The positive value of 'a' so that the volume of the parallelopiped formed by $\hat{i}+a\hat{j}+\hat{k}, \hat{j}+a\hat{k}$ and $a\hat{i}+\hat{k}$ becomes minimum is : **(2003)**

(a) 4 (b) 3

(c) $1/\sqrt{3}$ (d) $\sqrt{3}$

48. The value of k such that $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies in the plane $2x - 4y + z = 7$, is **(2003)**

(a) 7 (b) –7

(c) No real value (d) 4

49. The unit vector which is orthogonal to the vector $3\hat{i}+2\hat{j}+6\hat{k}$ and is coplanar with the vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ is : **(2004)**

(a) $\frac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$ (b) $\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$

(c) $\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$ (d) $\frac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$

50. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then the value of k is **(2004)**

(a) $\frac{3}{2}$ (b) $\frac{9}{2}$

(c) $-\frac{2}{9}$ (d) $-\frac{3}{2}$

51. A variable plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ at a unit distance from origin cuts the coordinate axes at A, B and C. Centroid (x, y, z) satisfies the equation $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=K$. Then value of K is **(2005)**

(a) 9 (b) 3

(c) 1/9 (d) 1/3

52. A plane passes through (1, –2, 1) and is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$, then the distance of the plane from the point (1, 2, 2) is **(2006)**

(a) 0 (b) 1

(c) $\sqrt{2}$ (d) $2\sqrt{2}$

53. The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i}+\hat{j}+\hat{k}, \hat{i}-\lambda^2\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-\lambda^2\hat{k}$ are coplanar, is : **(2007)**

(a) zero (b) one

(c) two (d) three

54. The edges of a parallelopiped are of unit length and are parallel to non–coplanar unit vectors $\hat{a},\hat{b},\hat{c}$ such that $\hat{a}\cdot\hat{b}=\hat{b}\cdot\hat{c}=\hat{c}\cdot\hat{a}=\frac{1}{2}$. Then the volume of the parallelopiped is **(2008)**

(a) $\frac{1}{\sqrt{2}}$ cu unit (b) $\frac{1}{2\sqrt{2}}$ cu unit

(c) $\frac{\sqrt{3}}{2}$ cu unit (d) $\frac{1}{\sqrt{3}}$ cu unit

55. Let P (3, 2, 6) be a point in space and Q be a point on the line $\vec{r}=(\hat{i}-\hat{j}+2\hat{k})+\mu(-3\hat{i}+\hat{j}+5\hat{k})$. Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$ is **(2009)**

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

56. The point P is the intersection of the straight line joining the points Q(2, 3, 5) and R(1, –1, 4) with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point T (2, 1, 4) to QR, then the length of the line segment PS is **(2012)**

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$

(c) 2 (d) $2\sqrt{2}$

57. The equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ & $x - y + z = 3$ & at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, –1) is **(2012)**

(a) $5x - 11y + z = 17$ (b) $\sqrt{2}x+y=3\sqrt{2}-1$

(c) $x+y+z=\sqrt{3}$ (d) $x-\sqrt{2}y=1-\sqrt{2}$

58. Let $\overrightarrow{PR}=3\hat{i}+\hat{j}-2\hat{k}$ and $\overrightarrow{SQ}=\hat{i}-3\hat{j}-4\hat{k}$ determine diagonals of a parallologram PQRS and $\overrightarrow{PT}=\hat{i}+2\hat{j}+3\hat{k}$ be another vector. Then, the volume of the parallelopiped determined by the vectors $\overrightarrow{PT}, \overrightarrow{PQ}$ and $\overrightarrow{PS}$ is **(2013)**

(a) 5 (b) 20
(c) 10 (d) 30

59. Perpendiculars are drawn from points on the line $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z}{3}$ to the plane x + y + z = 3. The feet of perpendiculars lie on the line. **(2013)**

(a) $\frac{x}{5}=\frac{y-1}{8}=\frac{z-2}{-13}$ (b) $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{-5}$

(c) $\frac{x}{4}=\frac{y-1}{3}=\frac{z-2}{-7}$ (d) $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

60. From a point $P(\lambda, \lambda, \lambda)$, perpendiculars PQ and PR are drawn respectively on the lines y = x, z = 1 and y = – x, z = – 1. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is(are) **(2014)**

(a) $\sqrt{2}$ (b) 1

(c) –1 (d) $-\sqrt{2}$

61. Let P be the image of the point (3, 1, 7) with respect to the plane x – y + z = 3. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1}=\frac{y}{2}=\frac{z}{1}$ is **(2016)**

(a) x + y – 3z = 0 (b) 3x + z = 0
(c) x – 4y + 7z = 0 (d) 2x – y = 0

Objective Questions II [One or more than one correct option]

62. If the straight lines $\frac{x-1}{2}=\frac{y+1}{K}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{K}$ are coplanar, then the plane(s) containing these two lines is/are **(2012)**

(a) y + 2z = –1 (b) y + z = –1
(c) y – z = –1 (d) y – 2z = –1

63. Two lines $L_1 : x = 5, \frac{y}{3-\alpha}=\frac{z}{-2}$ and $L_2 : x = \alpha, \frac{y}{-1}=\frac{z}{2-\alpha}$ are coplanar. Then, α can take value(s). **(2013)**

(a) 1 (b) 2
(c) 3 (d) 4

64. In R^3, consider the planes $P_1 : y = 0$ and $P_2 : x + z = 1$. Let P_3 be the plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0, 1, 0) from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relations is (are) true ? **(2015)**

(a) $2\alpha + \beta + 2\gamma + 2 = 0$ (b) $2\alpha - \beta + 2\gamma + 4 = 0$
(c) $2\alpha + \beta - 2\gamma - 10 = 0$ (d) $2\alpha - \beta + 2\gamma - 8 = 0$

65. In R^3, let L be a straight line passing through the origin. Suppose that all the points on L at a constant distance from the two planes $P_1 : x + 2y - z + 1 = 0$ & $P_2 : 2x - y + z - 1 = 0$. Let M be the locus of the foot of the perpendiculars drawn from the points on L to plane P_1. Which of the following point(s) lie(s) on M. **(2015)**

(a) $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$ (b) $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$

(c) $\left(-\frac{5}{6}, 0, \frac{1}{6}\right)$ (d) $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

66. Consider a pyramid OPQRS located in the first octant ($x \geq 0, y \geq 0, z \geq 0$) with O as origin, and OP and OR along the x-axis and the y-axis, respectively. The base OPQR of the pyramid is a square with OP = 3. The point S is directly above the mid-point T of diagonal OQ such that TS =3. Then **(2016)**

(a) the acute angle between OQ and OS is $\frac{\pi}{3}$

(b) the equation of the plane containing the triangle OQS is x – y = 0

(c) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$

(d) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

67. Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes. Then, which of the following statements(s) is (are) TRUE? **(2018)**

(a) The line of intersection of P_1 and P_2 has direction ratios 1, 2, –1

(b) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2

(c) The acute angle between P_1 and P_2 is 60°

(d) If P_3 is the plane passing through the point (4, 2, –2) and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point (2, 1, 1) from the plane P_3 is $\frac{2}{\sqrt{3}}$.

68. Let L_1 and L_2 denote the lines

$\vec{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k}), \lambda \in R$ and

$\vec{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k}), \mu \in R$

respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe (s) L_3? **(2019)**

(a) $\vec{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

(b) $\vec{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

(c) $\vec{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

(d) $\vec{r} = t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$

69. Three lines

$L_1 : \vec{r} = \lambda\hat{i}, \lambda \in R$

$L_2 : \vec{r} = \hat{k} + \mu\hat{j},\ \mu \in R$ and

$L_3 : \vec{r} = \hat{i} + \hat{j} + \gamma\hat{k}, \gamma \in R.$

are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P, Q and R are collinear. **(2019)**

(a) $\hat{k} + \frac{1}{2}\hat{j}$ (b) $\hat{k} + \hat{j}$

(c) $\hat{k}$ (d) $\hat{k} - \frac{1}{2}\hat{j}$

70. Let $\alpha, \beta, \gamma, \delta$ be real numbers such that $\alpha^2 + \beta^2 + \gamma^2 \neq 0$ and $\alpha + \gamma = 1$. Suppose the point (3,2,-1) is the mirror image of the point (1,0,-1) with respect to the plane $\alpha x + \beta y + yz = \delta$. Then which of the following statements is/are TRUE? **(2020)**

(a) $\alpha + \beta = 2$ (b) $\delta - y = 3$

(c) $\delta + \beta = 4$ (d) $\alpha + \beta + \gamma = \delta$

Numerical Value Type Questions

71. Consider the set of eight vectors $V = \left[a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\right]$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then, p is **(2013)**

72. Three lines are given by

$\vec{r} = \lambda\hat{i}, \lambda \in R$

$\vec{r} = \mu(\hat{i} + \hat{j}), \mu \in R$ and $\vec{r} = \gamma(\hat{i} + \hat{j} + \hat{k}), \gamma \in R$

Let the lines cut the plane x + y + z = 1 at points A,B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals **(2019)**

Assertion & Reason

For the following questions choose the correct answer from the codes (a), (b), (c) and (d) defined as follows.

(a) Statement I is true, Statement II is also true; Statement II is the correct explanation of Statement I.

(b) Statement I is true, Statement II is also true; Statement II is not the correct explanation of Statement I.

(c) Statement I is true, Statement II is false.

(d) Statement I is false, Statement II is true.

73. Consider the planes

$3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.

Statement–I : The parametric equations of the line of intersection of the given planes are $x = 3 + 14t$, $y = 1 - 2t$, $z = 15t$.

Statement–II : The vector $14\hat{i}+2\hat{j}+15\hat{k}$ is parallel to the line of intersection of the given planes. **(2007)**

(a) A (b) B

(c) C (d) D

74. Consider three planes

$P_1 : x - y + z = 1$

$P_2 : x + y - z = 1$ and $P_3 : x - 3y + 3z = 2$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, P_1 and P_2, respectively.

Statement–I : At least two of the lines L_1, L_2 and L_3 are non-parallel.

Statement–II : The three planes do not have a common point.

(2008)

(a) A (b) B

(c) C (d) D

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching. For each question, choose the option corresponding to the correct matching.

75. Consider the following linear equations

$ax + by + cz = 0$,

$bx + cy + az = 0$,

$cx + ay + bz = 0$ **(2007)**

Column–I	**Column–II**
(A) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	**(P)** the equations represent planes meeting only at a single point
(B) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	**(Q)** the equations represent the lines $x = y = z$
(C) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	**(R)** the equations represent identical planes
(D) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	**(S)** the equations represent the whole of the three dimensional space.

The correct matching is

(a) A-R, B-Q; C-P; D-S

(b) A-Q, B-R; C-P; D-S

(c) A-S, B-Q; C-P; D-R

(d) A-R, B-Q; C-S; D-P

76. Consider the lines

$L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$, $L_2 : \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1 : 7x + y + 2z = 3$, $P_2 : 3x + 5y - 6z = 4$. Let $ax + by + cz = d$ the equation of the plane passing through the point of intersection of lines L_1 and L_2 and perpendicular to planes P_1 and P_2.

Match List–I with List–II and select the correct answer using the code give below the lists. **(2013)**

List–I	**List–II**
P. a =	**1.** 13
Q. b =	**2.** –3
R. c =	**3.** 1
S. d =	**4.** –2

	P	**Q**	**R**	**S**
(a)	3	2	4	1
(b)	1	3	4	2
(c)	3	2	1	4
(d)	2	4	1	3

77. Match the following. **(2015)**

Column–I	**Column–II**
(A) In a triangle Δ XYZ, let a, b, and c be the lengths of the sides opposite to the angles X, Y and Z, respectively. If $2(a^2-b^2)=c^2$ and $\lambda = \frac{\sin(X-Y)}{\sin Z}$, then possible value of n for which $\cos(n\pi\lambda) = 0$ is	**(P)** 1
(B) In a triangle ΔXYZ, let a, b and c be the lengths of the sides opposite to the angles X, Y and Z respectively. If $1 + \cos 2X - 2\cos 2Y = 2 \sin X \sin Y$, then possible value(s) of $\frac{a}{b}$ is are	**(Q)** 2
(C) In R^2, let $\sqrt{3}i+\hat{j}, \hat{i} + \sqrt{3}\hat{j}$ and $\beta\hat{i} + (1-\beta)\hat{j}$ be the position vectors X, Y and Z with respect to the origin O, respectively. If the distance of Z from the bisector of the acute angle of $\overline{OX}$ with $\overline{OY}$ is $\frac{3}{\sqrt{2}}$, the possible value(s) of $\lvert\beta\rvert$ is(are)	**(R)** 3
(D) Suppose that $F(\alpha)$ denotes the area of the region bounded by $x = 0$, $x = 2$, $y^2 = 4x$ and $y = \lvert ax - 1\rvert + \lvert \alpha x - 2\rvert + ax$, where $\alpha \in \{0, 1\}$. Then the value(s) of $F(\alpha) + \frac{8}{3}\sqrt{2}$, when $\alpha = 0$ and $\alpha = 1$, is (are)	**(S)** 5
	(T) 6

The correct matching is

(a) A-P,R,S; B-P; C-P,Q; D-S,T
(b) A-P,R,S; B-P; C-T,Q; D-S
(c) A-P,S; B-Q; C-P; D-S,T
(d) A-S,T; B-P; C-P,Q; D-P,R,S

Passage Based Problem

Using the following passage, solve Q.78 to Q.80

Consider the lines

$$L_1: \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}, \quad L_2: \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$$ **(2008)**

78. The unit vector perpendicular to both L_1 and L_2 is

(a) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (b) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(c) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (d) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

79. The shortest distance between L_1 and L_2 is

(a) 0 unit (b) $17/\sqrt{3}$ unit

(c) $41/5\sqrt{3}$ unit (d) $17/5\sqrt{3}$ unit

80. The distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2 is

(a) $2/\sqrt{75}$ unit (b) $7/\sqrt{75}$ unit

(c) $13/\sqrt{75}$ unit (d) $23/\sqrt{75}$ unit

Text

81. Find 3–dimensional vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ satisfying

$$\vec{v}_1 \cdot \vec{v}_1 = 4, \vec{v}_1 \cdot \vec{v}_2 = -2, \vec{v}_1 \cdot \vec{v}_3 = 6,$$

$$\vec{v}_2 \cdot \vec{v}_2 = 2, \vec{v}_2 \cdot \vec{v}_3 = -5, \vec{v}_3 \cdot \vec{v}_3 = 29$$ **(2001)**

82. (a) Find the equation of the plane passing through the points (2, 1, 0), (5, 0, 1) and (4, 1, 1).

(b) If P is the point (2, 1, 6), then the point Q such that PQ is perpendicular to the plane in (a) and the mid point of PQ lies on it. **(2003)**

83. T is a parallelopiped in which A, B, C and D are vertices of one face and the face just above it has corresponding vertices A', B', C', D', T is now compressed to S with face ABCD remaining same and A', B', C', D' shifted to A'', B'', C'', D'' in S. The volume of parallelopiped S is reduced to 90% of T. Prove that locus of A'' is a plane.

(2004)

84. Find the equation of the plane containing the lines $2x - y + z - 3 = 0$, $3x + y + z = 5$ and at a distance of $\frac{1}{\sqrt{6}}$ from the point $(2, 1, -1)$. **(2005)**

85. Incident ray is along the unit vector $\hat{v}$ and the reflected ray is along the unit vector $\hat{w}$. The normal is along unit vector $\hat{a}$ outwards. Express $\hat{w}$ in terms of $\hat{a}$ and $\hat{v}$.

(2005)

Answer Key

CHAPTER -8 | VECTORS & 3-DIMENSIONAL GEOMETRY

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (c)	**2.** (b)	**3.** (d)	**4.** (c)	**77.** (3)	**78.** (6)	**79.** (c)	**80.** (b)
5. (b)	**6.** (a)	**7.** (b)	**8.** (c)	**81.** (c)	**82.** (c)	**83.** (d)	**84.** (c)
9. (b)	**10.** (c)	**11.** (d)	**12.** (a)	**85.** (b)	**86.** (a)	**87.** (b)	**88.** (b)
13. (a)	**14.** (a)	**15.** (a)	**16.** (a)	**89.** (a)	**90.** (a)	**91.** (d)	**92.** (a)
17. (c)	**18.** (b)	**19.** (b)	**20.** (b)	**93.** (a)	**94.** (d)	**95.** (b)	**96.** (c)
21. (d)	**22.** (a)	**23.** (c)	**24.** (b)	**97.** (a)	**98.** (c)	**99.** (a)	**100.** (b)
25. (c)	**26.** (c)	**27.** (a)	**28.** (a)	**101.** (d)	**102.** (a)	**103.** (a)	**104.** (d)
29. (a)	**30.** (b)	**31.** (d)	**32.** (a)	**105.** (a)	**106.** (b)	**107.** (b)	**108.** (d)
33. (d)	**34.** (b)	**35.** (c)	**36.** (b)	**109.** (a)	**110.** (d)	**111.** (a)	**112.** (a)
37. (d)	**38.** (a)	**39.** (b)	**40.** (c)	**113.** (c)	**114.** (d)	**115.** (b)	**116.** (d)
41. (c)	**42.** (d)	**43.** (a)	**44.** (a)	**117.** (b)	**118.** (a)	**119.** (b)	**120.** (d)
45. (d)	**46.** (b)	**47.** (a)	**48.** (b)	**121.** (c)	**122.** (c)	**123.** (d)	**124.** (c)
49. (d)	**50.** (b)	**51.** (c)	**52.** (a)	**125.** (a)	**126.** (d)	**127.** (d)	**128.** (b)
53. (c)	**54.** (c)	**55.** (4)	**56.** (46)	**129.** (d)	**130.** (c)	**131.** (c)	**132.** (c)
57. (17)	**58.** (2)	**59.** (2)	**60.** (5)	**133.** (b)	**134.** (1)	**135.** (2)	
61. (1)	**62.** (3)	**63.** (2)	**64.** (0)				
65. (7)	**66.** (2)	**67.** (17)	**68.** (0)				
69. (2)	**70.** (50)	**71.** (70)	**72.** (3)				
73. (2)	**74.** (14)	**75.** (3)	**76.** (3)				

CHAPTER -8 | VECTORS & 3-DIMENSIONAL GEOMETRY

EXERCISE - 2 :
PREVIOUS YEAR JEE MAIN QUESTIONS

1. (a)	**2.** (c)	**3.** (d)	**4.** (b)
5. (c)	**6.** (b)	**7.** (b)	**8.** (b)
9. (a)	**10.** (b)	**11.** (b)	**12.** (a)
13. (d)	**14.** (d)	**15.** (a)	**16.** (a)
17. (2)	**18.** (0.8)	**19.** (a)	**20.** (a)
21. (18)	**22.** (b)	**23.** (6)	**24.** (4)
25. (1)	**26.** (BONUS)	**27.** (c)	**28.** (4)
29. (c)	**30.** (c)	**31.** (1)	**32.** (81)
33. (30)	**34.** (d)	**35.** (4)	**36.** (6)
37. (a)	**38.** (b)	**39.** (3)	**40.** (d)
41. (2)	**42.** (b)	**43.** (9)	**44.** (b)
45. (b)	**46.** (a)	**47.** (c)	**48.** (60)
49. (1494)	**50.** (5)	**51.** (90)	**52.** (a)
53. (a)	**54.** (d)	**55.** (b)	**56.** (28)
57. (d)	**58.** (b)	**59.** (b)	**60.** (486)
61. (a)	**62.** (b)	**63.** (d)	**64.** (75)
65. (2)	**66.** (12)	**67.** (c)	**68.** (d)
69. (b)	**70.** (d)	**71.** (c)	**72.** (a)
73. (c)	**74.** (a)	**75.** (d)	**76.** (b)
77. (c)	**78.** (d)	**79.** (c)	**80.** (a)
81. (c)	**82.** (d)	**83.** (a)	**84.** (c)
85. (a)	**86.** (b)	**87.** (a)	**88.** (c)
89. (a)	**90.** (d)	**91.** (a)	**92.** (c)
93. (d)	**94.** (d)	**95.** (d)	**96.** (d)

97. (c)	**98.** (c)	**99.** (a)	**100.** (b)
101. (d)	**102.** (d)	**103.** (d)	**104.** (c)
105. (c)	**106.** (a)	**107.** (d)	**108.** (b)
109. (a)	**110.** (d)	**111.** (c)	**112.** (d)
113. (a)	**114.** (b)	**115.** (b)	**116.** (b)
117. (a)	**118.** (b)	**119.** (c)	**120.** (c)
121. (c)	**122.** (a)	**123.** (c)	**124.** (5)
125. (3)	**126.** (d)	**127.** (a)	**128.** (a)
129. (c)	**130.** (b)	**131.** (b)	**132.** (d)
133. (a)	**134.** (3)	**135.** (81)	**136.** (6)
137. (a)	**138.** (a)	**139.** (d)	**140.** (b)
141. (3)	**142.** (c)	**143.** (7)	**144.** (b)
145. (c)	**146.** (1)	**147.** (a)	**148.** (b)
149. (96)	**150.** (d)	**151.** (d)	**152.** (d)
153. (d)	**154.** (26)	**155.** (d)	**156.** (d)
157. (72)	**158.** (d)	**159.** (61)	**160.** (c)
161. (d)	**162.** (7)	**163.** (c)	**164.** (a)
165. (2)	**166.** (d)	**167.** (a)	**168.** (a)
169. (0)	**170.** (d)	**171.** (4)	**172.** (2)
173. (1)	**174.** (38)	**175.** (4)	**176.** (28)
177. (a)	**178.** (a)	**179.** (1)	**180.** (d)
181. (d)	**182.** (d)	**183.** (44)	**184.** (a)
185. (a)	**186.** (c)	**187.** (b)	**188.** (a)
189. (c)	**190.** (8)		

CHAPTER -8 | VECTORS & 3-DIMENSIONAL GEOMETRY

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. (c)	**2.** (a)	**3.** (a)	**4.** (a)
5. (a)	**6.** (c)	**7.** (c)	**8.** (b)
9. (b)	**10.** (a)	**11.** (a)	**12.** (c)
13. (c)	**14.** (d)	**15.** (a)	**16.** (c)
17. (b)	**18.** (a)	**19.** (c)	**20.** (a)
21. (c)	**22.** (b)	**23.** (a)	**24.** (c)
25. (d)	**26.** (a)	**27.** (d)	**28.** (d)
29. (b)	**30.** (b)	**31.** (a)	**32.** (c)
33. (c)	**34.** (a,d)	**35.** (a,c,d)	**36.** (a,c)
37. (a,b,c)	**38.** (a,c,d)	**39.** (a, b)	**40.** (a,b,c)
41. (a,b)	**42.** (a,c)	**43.** (c,d)	**44.** (a,c,d)
45. (50)	**46.** (9)	**47.** (b)	**48.** (d)
49. (d)	**50.** (a)	**51.** (a)	**52.** (a)
53. (c)	**54.** (b)	**55.** (a)	**56.** (c)
57. (b)	**58.** (b)	**59.** (a)	**60.** (a)
61. (d)	**62.** (d)	**63.** (d)	**64.** (c)
65. (c)	**66.** (b)	**67.** (b)	**68.** (a)
69. (1710)	**70.** (13)	**71.** (1)	**72.** (7)
73. (b)	**74.** (b)	**75.** (d)	**76.** (a)
77. (d)	**78.** (b)	**79.** (c)	**80.** (b)

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

1. (b)	**2.** (a)	**3.** (b)	**4.** (c)
5. (b)	**6.** (c)	**7.** (c)	**8.** (b)
9. (a)	**10.** (b)	**11.** (a)	**12.** (c)
13. (a)	**14.** (b)	**15.** (c)	**16.** (c)
17. (d)	**18.** (c)	**19.** (c)	**20.** (a)
21. (b,d)	**22.** (a, d)	**23.** (b, d)	**24.** (a,b,c)
25. (a,c,d)	**26.** (b,c)	**27.** (a,b)	**28.** (a,c)
29. (a,b,c)	**30.** (5)	**31.** (9)	**32.** (3)
33. (4)	**34.** (3)	**35.** (8)	**36.** (18.00)
37. (108)	**38.** (1.50)	**39.** (7)	**40.** (c)
41. (c)	**43.** $\frac{9}{2} cu\ unit$	**45.** $\frac{1}{2}$	**46.** (a)
47. (c)	**48.** (a)	**49.** (c)	**50.** (b)
51. (a)	**52.** (d)	**53.** (c)	**54.** (a)
55. (a)	**56.** (a)	**57.** (a)	**58.** (c)
59. (d)	**60.** (c)	**61.** (c)	**62.** (b, c)
63. (a,d)	**64.** (b,d)	**65.** (a,b)	**66.** (b,c,d)
67. (c,d)	**68.** (a,b,c)	**69.** (a, d)	**70.** (a,b,c)
71. (5)	**72.** (0.75)	**73.** (d)	**74.** (d)
75. (a)	**76.** (a)	**77.** (a)	
78. (b)	**79.** (d)	**80.** (c)	

81. $\left(\vec{v}_1 = 2\hat{i}, \vec{v}_2 = -\hat{i}+\hat{j} \ and\ \vec{v}_3 = 3\hat{i}-2\hat{j}\pm 4\hat{k}\right)$

82. (a) $x+y-2z=3$, (b) Q(6, 5,-2)

84. $2x-y+z-3=0 \ and\ 62x+29y+19z-105=0$

85. $\hat{w} = \hat{v} - 2\left(\hat{a}.\hat{v}\right)\hat{a}$