EXERCISE- 8 (A)

Question 1:

Find the each case, the remainder when:

(i) $x^4 - 3x^2 + 2x + 1$ is divided by x – 1

(ii) $x^3 + 3x^2 - 12x + 4$ is divided by x – 2

(iii) $x^4 + 1$ is divided by x + 1

Solution 1:

By remainder theorem we know that when a polynomial f (x) is divided by x − a, then the remainder is f(a).

$(i) f(x) = x^4 - 3x^2 + 2x + 1$

$\text{Remainder} = f(1) = (1)^4 - 3(1)^2 + 2(1) + 1 = 1 - 3 + 2 + 1 = 1$

$(ii) f(x) = x^3 + 3x^2 - 12x + 4$

$\text{Remainder} = f(2) = (2)^3 + 3(2)^2 - 12(2) + 4$

$= 8 + 12 - 24 + 4$

$= 0$

$(iii) f(x) = x^4 + 1$

$\text{Remainder} = f(-1) = (-1)^4 + 1 = 1 + 1 = 2$

$(iv) f(x) = 4x^3 - 3x^2 + 2x - 4$

$$\text{Remainder} = f\left(\frac{-1}{2}\right)$$

$$= 4\left(\frac{-1}{2}\right)^3 - 3\left(\frac{-1}{2}\right)^2 + 2\left(\frac{-1}{2}\right) - 4$$

$$= \frac{-1}{2} - \frac{3}{4} - 1 - 4$$

$$= \frac{-2 - 3 - 20}{4}$$

$$= \frac{-25}{4} = -6\frac{1}{4}$$

$(v) f(x) = 4x^3 + 4x^2 - 27x + 16$

$$\text{Remainder} = f\left(\frac{3}{2}\right)$$

$$= 4\left(\frac{3}{2}\right)^3 + 4\left(\frac{3}{2}\right)^2 - 27\left(\frac{3}{2}\right) + 16$$

$= \frac{27}{2} + 9 - \frac{81}{2} + 16$

$= -27 + 25$

$= -2$

$(vi) f(x) = 2x^3 + 9x^2 - x - 15$

$\text{Remainder} = f\left(\frac{-3}{2}\right)$

$= 2\left(\frac{-3}{2}\right)^3 + 9\left(\frac{-3}{2}\right)^2 - \left(\frac{-3}{2}\right) - 15$

$= \frac{-27}{4} + \frac{81}{4} + \frac{3}{2} - 15$

$= \frac{27}{2} + \frac{3}{2} - 15$

$= \frac{30}{2} - 15 = 15 - 15 = 0$

Question 2:

Show that:

(i) $x - 2$ is a factor of $5x^2 + 15x - 50$.

(ii) $3x + 2$ is a factor of $3x^2 - x - 2$

Solution 2:

(x − a) is a factor of a polynomial f(x) if the remainder, when f(x) is divided by (x − a), is 0, i.e., if f (a) = 0.

$(i) f(x) = 5x^2 + 15x - 50$

$f(2) = 5(2)^2 + 15(2) - 50 = 20 + 30 - 50 = 0$

Hence, $x - 2$ is a factor of $5x^2 + 15x - 50$

$(ii) f(x) = 3x^2 - x - 2$

$f\left(\frac{-2}{3}\right) = 3\left(\frac{-2}{3}\right)^2 - \left(\frac{-2}{3}\right) - 2 = \frac{4}{3} + \frac{2}{3} - 2 = 2 - 2 = 0$

Hence, $3x + 2$ is a factor of $3x^2 - x - 2$

$(iii) f(x) = x^3 + 3x^2 + 3x + 1$

$f(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1 = -1 + 3 - 3 + 1 = 0$

Hence, $x + 1$ is a factor of $x^3 + 3x^2 + 3x + 1$

Question 3:

Use the remainder Theorem to find which of the following is a factor of $2x^3 + 3x^2 - 5x - 6$.

(i) $x + 1$ (ii) $2x - 1$ (iii) $x + 2$

Solution 3:

By remainder theorem we know that when a polynomial f (x) is divided by x - a, then the remainder is f(a).

Let $f(x) = 2x^3 + 3x^2 - 5x - 6$

(i) $f(-1) = 2(-1)^3 + 3(-1)^2 - 5(-1) - 6 = -2 + 3 + 5 - 6 = 0$

Thus, $(x + 1)$ is a factor of the polynomial f(x).

(ii)

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) - 6$$

$$= \frac{1}{4} + \frac{3}{4} - \frac{5}{2} - 6$$

$$= -\frac{5}{2} - 5 = \frac{-15}{2} \neq 0$$

Thus, $(2x - 1)$ is not a factor of the polynomial f(x).

(iii) $f(-2) = 2(-2)^3 + 3(-2)^2 - 5(-2) - 6 = -16 + 12 + 10 - 6 = 0$

Thus, $(x + 2)$ is a factor of the polynomial f(x).

(iv)

$$f\left(\frac{2}{3}\right) = 2\left(\frac{3}{2}\right)^3 + 3\left(\frac{3}{2}\right)^2 - 5\left(\frac{2}{3}\right) - 6$$

$$= \frac{16}{27} + \frac{4}{3} - \frac{10}{3} - 6$$

$$= \frac{16}{27} - 2 - 6$$

$$= \frac{16}{27} - 8 \neq 0$$

Thus, $(3x - 2)$ is not a factor of the polynomial f (x).

(v)

$$f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 + 3\left(\frac{3}{2}\right)^2 - 5\left(\frac{3}{2}\right) - 6$$

$$= \frac{27}{4} + \frac{27}{4} - \frac{15}{2} - 6$$

$$= \frac{27}{2} - \frac{15}{2} - 6$$

$$= 6 - 6 = 0$$

Thus, $(2x - 3)$ is a factor of the polynomial f(x).

Question 4:
(i) If 2x + 1 is a factor of $2x^2 + ax - 3$, find the value of a.
(ii) Find the value of k, if 3x – 4 is a factor of expression $3x^2 + 2x - k$.

Solution 4:
(i) 2x + 1 is a factor of $f(x) = 2x^2 + ax - 3$.

$$\therefore f\left(\frac{-1}{2}\right) = 0$$

$$\Rightarrow 2\left(\frac{-1}{2}\right)^2 + a\left(\frac{-1}{2}\right) - 3 = 0$$

$$\Rightarrow \frac{1}{2} - \frac{a}{2} = 3$$

$$\Rightarrow 1 - a = 6$$

$$\Rightarrow a = -5$$

(ii) 3x − 4 is a factor of $g(x) = 3x^2 + 2x - k$.

$$\therefore f\left(\frac{4}{3}\right) = 0$$

$$\Rightarrow 3\left(\frac{4}{3}\right)^2 + 2\left(\frac{4}{3}\right) - k = 0$$

$$\Rightarrow \frac{16}{3} + \frac{8}{3} - k = 0$$

$$\Rightarrow \frac{24}{3} = k$$

$$\Rightarrow k = 8$$

Question 5:
Find the values of constants a and b when x – 2 and x + 3 both are the factors of expression $x^3 + ax^2 + bx - 12$

Solution 5:
Let $f(x) = x^3 + ax^2 + bx - 12$

$x - 2 = 0 \Rightarrow x = 2$

x − 2 is a factor of f(x). So, remainder = 0

$$\therefore (2)^3 + a(2)^2 + b(2) - 12 = 0$$

$$\Rightarrow 8 + 4a + 2b - 12 = 0$$

$$\Rightarrow 4a + 2b - 4 = 0$$

$$\Rightarrow 2a + b - 2 = 0 \qquad \text{..........(1)}$$

$x + 3 = 0 \Rightarrow x = -3$

$x + 3$ is a factor of f(x). So, remainder = 0

$\therefore (-3)^3 + a(-3)^2 + b(-3) - 12 = 0$

$\Rightarrow -27 + 9a - 3b - 12 = 0$

$\Rightarrow 9a - 3b - 39 = 0$

$\Rightarrow 3a - b - 13 = 0 \qquad ....(2)$

Adding (1) and (2), we get,

$5a - 15 = 0$

$\Rightarrow a = 3$

Putting the value of a in (1), we get,

$6 + b - 2 = 0$

$\Rightarrow b = -4$

Question 6:

Find the value of k, if $2x + 1$ is a factor of $(3k + 2)x^3 + (k - 1)$

Solution 6:

Let $f(x) = (3k + 2)x^3 + (k - 1)$

$2x + 1 = 0 \Rightarrow x = \frac{-1}{2}$

Since, $2x + 1$ is a factor of f(x), remainder is 0.

$\therefore (3k+2)\left(\frac{-1}{2}\right)^3 + (k-1) = 0$

$\Rightarrow \frac{-(3k+2)}{8} + (k-1) = 0$

$\Rightarrow \frac{-3k - 2 + 8k - 8}{8} = 0$

$\Rightarrow 5k - 10 = 0$

$\Rightarrow k = 2$

Question 7:

Find the vaue of a, if $x - 2$ is a factor of $2x^5 - 6x^4 - 2ax^3 + 6ax^2 + 4ax + 8$

Solution 7:

$f(x) = 2x^5 - 6x^4 - 2ax^3 + 6ax^2 + 4ax + 8$

$x - 2 = 0 \Rightarrow x = 2$

Since, $x - 2$ is a factor of $f(x)$, remainder = 0.
$2(2)^5 - 6(2)^4 - 2a(2)^3 + 6a(2)^2 + 4a(2) + 8 = 0$
$64 - 96 - 16a + 24a + 8a + 8 = 0$
$-24 + 16a = 0$
$16a = 24$
$a = 1.5$

Question 8:

Find the value of m and n so that $x - 1$ and $x + 2$ both are factors of
$x^3 + (3m + 1)x^2 + nx - 18$

Solution 8:

Let $f(x) = x^3 + (3m + 1)x^2 + nx - 18$
$x - 1 = 0 \Rightarrow x = 1$
$x - 1$ is a factor of $f(x)$. So, remainder = 0

$$\therefore (1)^3 + (3m+1)(1)^2 + n(1) - 18 = 0$$
$$\Rightarrow 1 + 3m + 1 + n - 18 = 0$$
$$\Rightarrow 3m + n - 16 = 0 \qquad \text{...........}(1)$$

$x + 2 = 0 \Rightarrow x = -2$
$x + 2$ is a factor of $f(x)$. So, remainder = 0

$$\therefore (-2)^3 + (3m+1)(-2)^2 + n(-2) - 18 = 0$$
$$\Rightarrow -8 + 12m + 4 - 2n - 18 = 0$$
$$\Rightarrow 12m - 2n - 22 = 0$$
$$\Rightarrow 6m - n - 11 = 0 \qquad \text{...........}(2)$$

Adding (1) and (2), we get,
$9m - 27 = 0$
$m = 3$
Putting the value of m in (1), we get,
$3(3) + n - 16 = 0$
$9 + n - 16 = 0$
$n = 7$

Question 9:
When $x^3 + 2x^2 - kx + 4$ is divided by $x - 2$, the remainder is k. Find the value of constant k.

Solution 9:
Let $f(x) = x^3 + 2x^2 - kx + 4$

$x - 2 = 0 \Rightarrow x = 2$

On dividing f(x) by $x - 2$, it leaves a remainder k.

$\therefore f(2) = k$

$(2)^3 + 2(2)^2 - k(2) + 4 = k$

$8 + 8 - 2k + 4 = k$

$20 = 3k$

$k = \dfrac{20}{3} = 6\dfrac{2}{3}$

Question 10:
Find the value of a, if the division of $ax^3 + 9x^2 + 4x - 10$ by $x + 3$ leaves a remainder 5.

Solution 10:
Let $f(x) = ax^3 + 9x^2 + 4x - 10$

$x + 3 = 0 \Rightarrow x = -3$

On dividing f(x) by $x + 3$, it leaves a remainder 5.

$\therefore f(-3) = 5$

$a(-3)^3 + 9(-3)^2 + 4(-3) - 10 = 5$

$-27a + 81 - 12 - 10 = 5$

$54 = 27a$

$a = 2$

Question 11:
If $x^3 + ax^2 + bx + 6$ has $x - 2$ as a factor and leaves a remainder 3 when divided by $x - 3$, find the values of a and b.

Solution 11:
Let $f(x) = x^3 + ax^2 + bx + 6$

$x - 2 = 0 \Longrightarrow x = 2$

Since, x - 2 is a factor, remainder = 0

$\therefore f(2) = 0$

$(2)^3 + a(2)^2 + b(2) + 6 = 0$

$8 + 4a + 2b + 6 = 0$

$2a + b + 7 = 0$ ....(i)

$x - 3 = 0 \Longrightarrow x = 3$

On dividing f(x) by $x - 3$, it leaves a remainder 3.

$\therefore f(3) = 3$

$(3)^3 + a(3)^2 + b(3) + 6 = 3$

$27 + 9a + 3b + 6 = 3$

$3a + b + 10 = 0$ ....(ii)

Subtracting (i) from (ii), we get,

$a + 3 = 0$

$a = -3$

Substituting the value of a in (i), we get,

$-6 + b + 7 = 0$

$b = -1$

Question 12:

The expression $2x^3 + ax^2 + bx - 2$ leaves remainder 7 and 0 when divided by $2x - 3$ and $x + 2$ respectively. Calculate the values of a and b.

Solution 12:

Let $f(x) = 2x^3 + ax^2 + bx - 2$

$2x - 3 = 0 \quad x = \frac{3}{2}$

On dividing f(x) by $2x - 3$, it leaves a remainder 7.

$$\therefore 2\left(\frac{3}{2}\right)^3 + a\left(\frac{3}{2}\right)^2 + b\left(\frac{3}{2}\right) - 2 = 7$$

$$\frac{27}{4} + \frac{9a}{4} + \frac{3b}{2} = 9$$

$$\frac{27 + 9a + 6b}{4} = 9$$

$27 + 9a + 6b = 36$

$9a + 6b - 9 = 0$

$3a + 2b - 3 = 0$ ....(i)

$x + 2 = 0 \Rightarrow x = -2$

On dividing f(x) by x + 2, it leaves a remainder 0.

$\therefore 2(-2)^3 + a(-2)^2 + b(-2) - 2 = 0$

$-16 + 4a - 2b - 2 = 0$

$4a - 2b - 18 = 0 \qquad ....(ii)$

Adding (i) and (ii), we get,

$7a - 21 = 0$

$a = 3$

Substituting the value of a in (i), we get,

$3(3) + 2b - 3 = 0$

$9 + 2b - 3 = 0$

$2b = -6$

$b = -3$

Question 13:

What number should be added to $3x^3 - 5x^2 + 6x$ so that when resulting polynomial is divided by x – 3, the remainder is 8?

Solution 13:

Let the number k be added and the resulting polynomial be f(x).

So, $f(x) = 3x^3 - 5x^2 + 6x + k$

It is given that when f(x) is divided by (x − 3), the remainder is 8.

$\therefore f(3) = 8$

$3(3)^3 - 5(3)^2 + 6(3) + k = 8$

$81 - 45 + 18 + k = 8$

$54 + k = 8$

$k = -46$

Thus, the required number is −46

Question 14:

What number should be subtracted from $x^3 + 3x^2 - 8x + 14$ so that on dividing it by x – 2, the remainder is 10.

Solution 14:

Let the number to be subtracted be k and the resulting polynomial be f(x).

So, $f(x) = x^3 + 3x^2 - 8x + 14 - k$

It is given that when f(x) is divided by (x − 2), the remainder is 10.

$\therefore f(2) = 10$

$(2)^3 + 3(2)^2 - 8(2) + 14 - k = 10$

$8 + 12 - 16 + 14 - k = 10$

$18 - k = 10$

$k = 8$

Thus, the required number is 8.

Question 15:

The polynimials $2x^3 - 7x^2 + ax - 6$ and $x^3 - 8x^2 + (2a + 1)x - 16$ leave the same remainder when divided by x – 2. Find the value of ‘a’

Solution 15:

Let $f(x) = 2x^3 - 7x^2 + ax - 6$

$x - 2 = 0 \Rightarrow x = 2$

When f(x) is divided by (x − 2), remainder = f (2)

$\therefore f(2) = 2(2)^3 - 7(2)^2 + a(2) - 6$

$= 16 - 28 + 2a - 6$

$= 2a - 18$

Let $g(x) = x^3 - 8x^2 + (2a + 1)x - 16$

When g(x) is divided by (x − 2), remainder = g(2)

$\therefore g(2) = (2)^3 - 8(2)^2 + (2a + 1)(2) - 16$

$= 8 - 32 + 4a + 2 - 16$

$= 4a - 38$

By the given condition, we have:

f (2) = g (2)

$2a - 18 = 4a - 38$

$4a - 2a = 38 - 18$

$2a = 20$

$a = 10$

Thus, the value of a is 10.

EXERCISE. 8(B)

Question 1:

Using the factor Theorem, show that:

(i) (x -2) is a factor of $x^3 - 2x^2 - 9x + 18$. hence, factorise the expression. $x^3 - 2x^2 - 9x + 18$ Completely.

(ii) (x + 5) is a factor of $2x^3 + 5x^2 - 28x - 15$. Hence, factorise the expression $2x^3 + 5x^2 - 28x - 15$ completely.

(iii) (3x + 2) is a factor is $3x^3 + 2x^2 - 3x - 2$. Hence, factorise the expression $3x^3 + 2x^2 - 3x - 2$ completely.

(iv) 2x + 7 is a factor $2x^3 + 5x^2 - 11x - 14$. Hence, factorise the given expression completely.

Solution 1:

(i) Let $f(x) = x^3 - 2x^2 - 9x + 18$

$x - 2 = 0 \Rightarrow x = 2$

$\therefore$ Remainder = f (2)

$= (2)^3 - 2(2)^2 - 9(2) + 18$

$= 8 - 8 - 18 + 18$

$= 0$

Hence, (x − 2) is a factor of f(x).

Now, we have:

$$\begin{array}{r|l} & x^2 - 9 \\ \hline x-2 & x^3 - 2x^2 - 9x + 18 \\ & x^3 - 2x^2 \\ \hline & \quad -9x + 18 \\ & \quad -9x + 18 \\ \hline & \qquad 0 \end{array}$$

$\therefore$ $x^3 - 2x^2 - 9x + 18 = (x - 2)(x^2 - 9) = (x - 2)(x + 3)(x - 3)$

(ii) Let f (x) = $2x^3 + 5x^2 - 28x - 15$

$x + 5 = 0 \Rightarrow x = -5$

$\therefore$ Remainder = f (−5)

$= 2(-5)^3 + 5(-5)^2 - 28(-5) - 15$

$= -250 + 125 + 140 - 15$

$= -265 + 265$

$= 0$

Hence, (x + 5) is a factor of f(x).

Now, we have:

$$\begin{array}{r|l} & 2x^2 - 5x - 3 \\ \hline x+5 & 2x^3 + 5x^2 - 28x - 15 \\ & \underline{2x^3 + 10x^2} \\ & \quad -5x^2 - 28x \\ & \quad \underline{-5x^2 - 25x} \\ & \qquad -3x - 15 \\ & \qquad \underline{-3x - 15} \\ & \qquad\quad 0 \end{array}$$

$\therefore 2x^3 + 5x^2 - 28x - 15 = (x + 5)(2x^2 - 5x - 3)$

$= (x + 5)[2x^2 - 6x + x - 3]$

$= (x + 5)[2x(x - 3) + 1(x - 3)]$

$= (x + 5)(2x + 1)(x - 3)$

(iii) Let $f(x) = 3x^3 + 2x^2 - 3x - 2$

$3x + 2 = 0 \Rightarrow x = \frac{-2}{3}$

$\therefore$ Remainder = $f\left(\frac{-2}{3}\right)$

$= 3\left(\frac{-2}{3}\right)^3 + 2\left(\frac{-2}{3}\right)^2 - 3\left(\frac{-2}{3}\right) - 2$

$= \frac{-8}{9} + \frac{8}{9} + 2 - 2$

$= 0$

Hence, $(3x + 2)$ is a factor of $f(x)$.

Now, we have:

$$\begin{array}{r|l} & x^2 - 1 \\ \hline 3x+2 & 3x^3 + 2x^2 - 3x - 2 \\ & \underline{3x^3 + 2x^2} \\ & \qquad -3x - 2 \\ & \qquad \underline{-3x - 2} \\ & \qquad\quad 0 \end{array}$$

$\therefore 3x^3 + 2x^2 - 3x - 2 = (3x + 2)(x^2 - 1) = (3x + 2)(x + 1)(x - 1)$

(iv) $f(x) = 2x^3 + 5x^2 - 11x - 14$

$2x + 7 = 0 \Rightarrow x = \frac{-7}{2}$

$\therefore$ Remainder = $f\left(\frac{-7}{2}\right)$

$$= 2\left(\frac{-7}{2}\right)^3 + 5\left(\frac{-7}{2}\right)^2 - 11\left(\frac{-7}{2}\right) - 14$$
$$= \frac{-343}{4} + \frac{245}{4} + \frac{77}{2} - 14$$
$$= \frac{-49}{2} + \frac{77}{2} - 14$$
$$= \frac{28}{2} - 14$$
$$= 14 - 14 = 0$$

Hence, (2x + 7) is a factor of f(x).

Now, we have:

$$\begin{array}{r|l} & x^2 - x - 2 \\ \hline 2x+7 & 2x^3 + 5x^2 - 11x - 14 \\ & \underline{2x^3 + 7x^2} \\ & \quad -2x^2 - 11x \\ & \quad \underline{-2x^2 - 7x} \\ & \qquad -4x - 14 \\ & \qquad \underline{-4x - 14} \\ & \qquad\quad 0 \end{array}$$

$$\therefore 2x^3 + 5x^2 - 11x - 14 = (2x+7)\left(x^2 - x - 2\right)$$
$$= (2x+7)\left(x^2 - 2x + x - 2\right)$$
$$= (2x+7)\left[x(x-2) + (x-2)\right]$$
$$= (2x+7)(x-2)(x+1)$$

Question 2:

Using the Reminder Theorem, factorise each of the following completely.

(i) $3x^3 + 2x^2 - 19x + 6$

(ii) $2x^3 + x^2 - 13x + 6$

(iii) $3x^3 + 2x^2 - 23x - 30$

(iv) $4x^3 + 7x^2 - 36x - 63$

(v) $x^3 + x^2 - 4x - 4$

Solution 2:

(i)

For $x = 2$, the value of the given

expression $3x^3 + 2x^2 - 19x + 6$

$= 3(2)^3 + 2(2)^2 - 19(2) + 6$

$= 24 + 8 - 38 + 6$

$= 0$

$\Rightarrow x - 2$ is a factor of $3x^3 + 2x^2 - 19x + 6$

Now let us do long division.

$$\begin{array}{r|l} & 3x^2 + 8x - 3 \\ \hline x - 2 & 3x^3 + 2x^2 - 19x + 6 \\ & 3x^3 - 6x^2 \\ \hline & 8x^2 - 19x \\ & 8x^2 - 16x \\ \hline & -3x + 6 \\ & -3x + 6 \\ \hline & 0 \end{array}$$

Thus we have,

$3x^3 + 2x^2 - 19x + 6 = (x - 2)(3x^2 + 8x - 3)$

$= (x - 2)(3x^2 + 9x - x - 3)$

$= (x - 2)(3x(x + 3) - (x - 3))$

$= (x - 2)(3x - 1)(x + 3)$

(ii) Let f (x) = $2x^3 + x^2 - 13x + 6$

For x = 2,

$f(x) = f(2) = 2(2)^3 + (2)^2 - 13(2) + 6 = 16 + 4 - 26 + 6 = 0$

Hence, (x − 2) is a factor of f(x).

$$\begin{array}{r|l} & 2x^2 + 5x - 3 \\ \hline x - 2 & 2x^3 + x^2 - 13x + 6 \\ & 2x^3 - 4x^2 \\ \hline & 5x^2 - 13x \\ & 5x^2 - 10x \\ \hline & -3x + 6 \\ & -3x + 6 \\ \hline & 0 \end{array}$$

$$\therefore\ 2x^3+x^2-13x+6=(x-2)(2x^2+5x-3)$$
$$=(x-2)(2x^2+6x-x-3)$$
$$=(x-2)[2x(x+3)-(x-3)]$$
$$=(x-2)[2x(x+3)-(x+3)]$$

(iii) $f(x) = 3x^3 + 2x^2 - 23x - 30$

For $x = -2$,

$f(x) = f(-2) = 3(-2)^3 + 2(-2)^2 - 23(-2) - 30$

$= -24 + 8 + 46 - 30 = -54 + 54 = 0$

Hence, $(x + 2)$ is a factor of $f(x)$.

$$\begin{array}{r} 3x^2-4x-15 \\ x+2\overline{)3x^3+2x^2-23x-30} \\ \underline{3x^3+6x^2} \\ -4x^2-23x \\ \underline{-4x^2-8x} \\ -15x-30 \\ \underline{-15x-30} \\ 0 \end{array}$$

$$\therefore 3x^3+2x^2-23x-30=(x+2)(3x^2-4x-15)$$
$$=(x+2)(3x^2+5x-9x-15)$$
$$=(x+2)[x(3x+5)-3(3x+5)]$$
$$=(x+2)(3x+5)(x-3)$$

(iv) $f(x) = 4x^3 + 7x^2 - 36x - 63$

For $x = 3$,

$f(x) = f(3) = 4(3)^3 + 7(3)^2 - 36(3) - 63$

$= 108 + 63 - 108 - 63 = 0$

Hence, $(x + 3)$ is a factor of $f(x)$.

$$\begin{array}{r} 4x^2-5x-21 \\ x+3\overline{)4x^3+7x^2-36x-63} \\ \underline{4x^3+12x^2} \\ -5x^2-36x \\ \underline{-5x^2-15x} \\ -21x-63 \\ \underline{-21x-63} \\ 0 \end{array}$$

$\therefore 4x^3 + 7x^2 - 36x - 63 = (x+3)(4x^2 - 5x - 21)$

$= (x+3)(4x^2 - 12x + 7x - 21)$

$= (x+3)[4x(x-3) + 7(x-3)]$

$= (x+3)(4x+7)(x-3)$

(v) f (x) = $x^3 + x^2 - 4x - 4$

For x = −1,

$f(x) = f(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4$

$= -1 + 1 + 4 - 4 = 0$

Hence, (x + 1) is a factor of f(x).

$$
\begin{array}{r|l}
 & x^2 - 4 \\
\hline
x+1 & x^3 + x^2 - 4x - 4 \\
 & \underline{x^3 + x^2} \\
 & \quad -4x - 4 \\
 & \quad \underline{-4x - 4} \\
 & \qquad 0
\end{array}
$$

$\therefore x^3 + x^2 - 4x - 4 = (x+1)(x^2 - 4)$

$= (x+1)(x+2)(x-2)$

Question 3:

Using the remainder Theorem, factorise the expression $3x^3 + 10x^2 + x - 6$. Hence, solve the equation $3x^3 + 10x^2 + x - 6 = 0$

Solution 3:

Let $f(x) = 3x^3 + 10x^2 + x - 6$

For x = − 1,

$f(x) = f(-1) = 3(-1)^3 + 10(-1)^2 + (-1) - 6 = -3 + 10 - 1 - 6 = 0$

Hence, (x + 1) is a factor of f(x).

$$
\begin{array}{r|l}
 & 3x^2 + 7x - 6 \\
\hline
x+1 & 3x^3 + 10x^2 + x - 6 \\
 & \underline{3x^3 + 3x^2} \\
 & \quad 7x^2 + x \\
 & \quad \underline{7x^2 + 7x} \\
 & \qquad -6x - 6 \\
 & \qquad \underline{-6x - 6} \\
 & \qquad\quad 0
\end{array}
$$

$\therefore\ 3x^3 + 10x^2 + x - 6 = (x+1)(3x^2 + 7x - 6)$

$= (x+1)(3x^2 + 9x - 2x - 6)$

$= (x+1)[3x(x+3) - 2(x+3)]$

$= (x+1)(x+3)(3x-2)$

Now, $3x^3 + 10x^2 + x - 6 = 0$

$\Rightarrow (x+1)(x+3)(3x-2) = 0$

$\Rightarrow x = -1, -3, \frac{2}{3}$

Question 4:

Factorise the expression

f (x) = $2x^3 - 7x^2 - 3x + 18$

Hence, find all possible values of x for which f(x) = 0

Solution 4:

f (x) = $2x^3 - 7x^2 - 3x + 18$

For x = 2,

f(x) = f(2) = $2(2)^3 - 7(2)^2 - 3(2) + 18$

= 16 − 28 − 6 + 18 = 0

Hence, (x − 2) is a factor of f(x).

$$\begin{array}{r|l} & 2x^2 - 3x - 9 \\ \hline x-2 & 2x^3 - 7x^2 - 3x + 18 \\ & \underline{2x^3 - 4x^2} \\ & \quad -3x^2 - 3x \\ & \quad \underline{-3x^2 + 6x} \\ & \qquad -9x + 18 \\ & \qquad \underline{-9x + 18} \\ & \qquad\quad 0 \end{array}$$

$\therefore\ 2x^3 - 7x^2 - 3x + 18 = (x-2)(2x^2 - 3x - 9)$

$= (x-2)(2x^2 - 6x + 3x - 9)$

$= (x-2)[2x(x-3) + 3(x-3)]$

$= (x-2)(x-3)(2x+3)$

Now, $f(x) = 0$

$$\Rightarrow 2x^3 - 7x^2 - 3x + 18 = 0$$
$$\Rightarrow (x-2)(x-3)(2x+3) = 0$$
$$\Rightarrow x = 2, 3, \frac{-3}{2}$$

Question 5:

Given that x – 2 and x + 1 are factors of $f(x) = x^3 + 3x^2 + ax + b$; calculate the values of a and b. Hence, find all the factors of f(x)

Solution 5:

$f(x) = x^3 + 3x^2 + ax + b$

Since, $(x - 2)$ is a factor of f (x), f (2) = 0

$\Longrightarrow (2)^3 + 3(2)^2 + a(2) + b = 0$

$\Longrightarrow 8 + 12 + 2a + b = 0$

$\Longrightarrow 2a + b + 20 = 0$ ...(i)

Since, $(x + 1)$ is a factor of f (x), f (−1) = 0

$\Longrightarrow (-1)^3 + 3(-1)^2 + a(-1) + b = 0$

$\Longrightarrow -1 + 3 - a + b = 0$

$\Longrightarrow -a + b + 2 = 0$ .... (ii)

Subtracting (ii) from (i), we get,

$3a + 18 = 0$

$\Longrightarrow a = -6$

Substituting the value of a in (ii), we get,

$b = a - 2 = -6 - 2 = -8$

$\therefore f(x) = x^3 + 3x^2 - 6x - 8$

Now, for $x = -1$,

$f(x) = f(-1) = (-1)^3 + 3(-1)^2 - 6(-1) - 8 = -1 + 3 + 6 - 8 = 0$

Hence, $(x + 1)$ is a factor of f (x).

$$\begin{array}{r|l}
 & x^2 + 2x - 8 \\
\hline
x+1 & x^3 + 3x^2 - 6x - 8 \\
 & x^3 + x^2 \\
\hline
 & \quad 2x^2 - 6x \\
 & \quad 2x^2 + 2x \\
\hline
 & \qquad -8x - 8 \\
 & \qquad -8x - 8 \\
\hline
 & \qquad\quad 0
\end{array}$$

$$\therefore x^3+3x^2-6x-8=(x+1)(x^2+2x-8)$$
$$=(x+1)(x^2+4x-2x-8)$$
$$=(x+1)[x(x+4)-2(x+4)]$$
$$=(x+1)(x+4)(x-2)$$

Question 6:

The expression $4x^3 - bx^2 + x - c$ leaves remainders 0 and 30 when divided by x + 1 and 2x – 3 respectively. Calculate the values of b and c. Hence, factorise the expression completely.

Solution 6:

Let $f(x) = 4x^3 - bx^2 + x - c$

It is given that when f(x) is divided by (x + 1), the remainder is 0.

$f(-1) = 0$

$4(-1)^3 - b(-1)^2 + (-1) - c = 0$

$-4 - b - 1 - c = 0$

$b + c + 5 = 0$ ...(i)

It is given that when f(x) is divided by (2x − 3), the remainder is 30.

$$\therefore f\left(\frac{3}{2}\right)=30$$

$$4\left(\frac{3}{2}\right)^3-b\left(\frac{3}{2}\right)^2+\left(\frac{3}{2}\right)-c=30$$

$$\frac{27}{2}-\frac{9b}{4}+\frac{3}{2}-c=30$$

$$54-9b+6-4c-120=0$$

$$9b+4c+60=0 \qquad \text{...........(ii)}$$

Multiplying (i) by 4 and subtracting it from (ii), we get,

$5b + 40 = 0$

$b = -8$

Substituting the value of b in (i), we get,

$c = -5 + 8 = 3$

Therefore, $f(x) = 4x^3 + 8x^2 + x - 3$

Now, for $x = -1$, we get,

$f(x) = f(-1) = 4(-1)^3 + 8(-1)^2 + (-1) - 3 = -4 + 8 - 1 - 3 = 0$

Hence, (x + 1) is a factor of f(x).

$$\begin{array}{r|l} & 4x^2+4x-3 \\ \hline x+1 & 4x^3+8x^2+x-3 \\ & 4x^3+4x^2 \\ \hline & 4x^2+x \\ & 4x^2+4x \\ \hline & -3x-3 \\ & -3x-3 \\ \hline & 0 \end{array}$$

$\therefore 4x^3+8x^2+x-3=(x+1)(4x^2+4x-3)$

$=(x+1)(4x^2+6x-2x-3)$

$=(x+1)[2x(2x+3)-(2x+3)]$

$=(x+1)(2x+3)(2x-1)$

Question 7:

If x + a is a common factor of expressions f(x) = x^2 + px + q and g(x) = x^2 + mx + n;

Show that : $a=\dfrac{n-q}{m-p}$

Solution 7:

f (x) = x^2 + px + q

It is given that (x + a) is a factor of f (x).

$\therefore f(-a)=0$

$\Rightarrow (-a)^2+p(-a)+q=0$

$\Rightarrow a^2-pa+q=0$

$\Rightarrow a^2=pa-q \qquad ....(i)$

g(x) = x^2 + mx + n

It is given that (x + a) is a factor of g(x).

$\therefore g(-a)=0$

$\Rightarrow (-a)^2+m(-a)+n=0$

$\Rightarrow a^2-ma+n=0$

$\Rightarrow a^2=ma-n \qquad ....(ii)$

From (i) and (ii), we get,

pa − q = ma − n

$n - q = a(m - p)$

$$a = \frac{n-q}{m-p}$$

Hence, proved.

Question 8:

The polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$, when divided by $x - 4$, leave the same remainder in each case. Find the value of a.

Solution 8:

Let $f(x) = ax^3 + 3x^2 - 3$

When f(x) is divided by $(x - 4)$, remainder = f (4)

$f(4) = a(4)^3 + 3(4)^2 - 3 = 64a + 45$

Let $g(x) = 2x^3 - 5x + a$

When g(x) is divided by $(x - 4)$, remainder = g (4)

$g(4) = 2(4)^3 - 5(4) + a = a + 108$

It is given that f (4) = g (4)

$64a + 45 = a + 108$

$63a = 63$

$a = 1$

Question 9:

Find the value of 'a', if $(x - a)$ is factor of $x^3 - ax^2 + x + 2$.

Solution 9:

Let $f(x) = x^3 - ax^2 + x + 2$

It is given that $(x - a)$ is a factor of f(x).

∴ Remainder = f (a) = 0

$a^3 - a^3 + a + 2 = 0$

$a + 2 = 0$

$a = -2$

Question 10:

Find the number that must be subtracted from the polynomial $3y^3 + y^2 - 22y + 15$, so that the resulting polynomial is completely divisible by $y + 3$.

Solution 10:

Let the number to be subtracted from the given polynomial be k.

Let $f(y) = 3y^3 + y^2 - 22y + 15 - k$

It is given that f(y) is divisible by (y + 3).

Remainder = $f(-3) = 0$

$3(-3)^3 + (-3)^2 - 22(-3) + 15 - k = 0$

$-81 + 9 + 66 + 15 - k = 0$

$9 - k = 0$

$k = 9$

EXERCISE. 8 (C)

Question 1:

Show that $(x - 1)$ is a factor of

$x^3 - 7x^2 + 14x - 8$

Hence, completely factorise that given expression

Solution 1:

Let $f(x) = x^3 - 7x^2 + 14x - 8$

$f(1) = (1)^3 - 7(1)^2 + 14(1) - 8 = 1 - 7 + 14 - 8 = 0$

Hence, $(x - 1)$ is a factor of f (x).

$$\begin{array}{r|l} & x^2 - 6x + 8 \\ \hline x - 1 & x^3 - 7x^2 + 14x - 8 \\ & x^3 - x^2 \\ \hline & -6x^2 + 14x \\ & -6x^2 + 6x \\ \hline & 8x - 8 \\ & 8x - 8 \\ \hline & 0 \end{array}$$

$\therefore x^3 - 7x^2 + 14x - 8 = (x-1)(x^2 - 6x + 8)$

$= (x-1)(x^2 - 2x - 4x + 8)$

$= (x-1)[x(x-2) - 4(x-2)]$

$= (x-1)(x-2)(x-4)$

Question 2:

Using remainder Theorem, factorise:

$2x^3 + 7x^2 - 8x - 28$ Completely.

Solution 2:

Let $f(x) = 2x^3 + 7x^2 - 8x - 28$

For x = 2,

$f(x) = f(2) = 2(2)^3 + 7(2)^2 - 8(2) - 28 = 16 + 28 - 16 - 28 = 0$

Hence, $(x - 2)$ is a factor of $f(x)$.

$$
\begin{array}{r|l}
 & 2x^2 + 11x + 14 \\
\hline
x - 2 & 2x^3 + 7x^2 - 8x - 28 \\
 & 2x^3 - 4x^2 \\
\hline
 & \quad 11x^2 - 8x \\
 & \quad 11x^2 - 22x \\
\hline
 & \qquad 14x - 28 \\
 & \qquad 14x - 28 \\
\hline
 & \qquad\quad 0
\end{array}
$$

$\therefore 2x^3 + 7x^2 - 8x - 28 = (x - 2)(2x^2 + 11x + 14)$

$= (x - 2)(2x^2 + 4x + 7x + 14)$

$= (x - 2)[2x(x + 2) + 7(x + 2)]$

$= (x - 2)(x + 2)(2x + 7)$

Question 3:

When $x^3 + 3x^2 - mx + 4$ is divided by $x - 2$, the remainder is $m + 3$. Find the value of m.

Solution 3:

Let $f(x) = x^3 + 3x^2 - mx + 4$

According to the given information,

$f(2) = m + 3$

$(2)^3 + 3(2)^2 - m(2) + 4 = m + 3$

$8 + 12 - 2m + 4 = m + 3$

$24 - 3 = m + 2m$

$3m = 21$

$m = 7$

Question 4:

What should be subtracted from $3x^3 - 8x^2 + 4x - 3$, so that the resulting expression has $x + 2$ as a factor?

Solution 4:

Let the required number be k.

Let $f(x) = 3x^3 - 8x^2 + 4x - 3 - k$

According to the given information,

$f(-2) = 0$

$3(-2)^3 - 8(-2)^2 + 4(-2) - 3 - k = 0$

$-24 - 32 - 8 - 3 - k = 0$

$-67 - k = 0$

$k = -67$

Thus, the required number is -67.

Question 5:

If $(x + 1)$ and $(x - 2)$ are factors of $x^3 + (a + 1)x^2 - (b - 2)x - 6$, find the values of a and b. And then, factorise the given expression completely.

Solution 5:

Let $f(x) = x^3 + (a + 1)x^2 - (b - 2)x - 6$

Since, $(x + 1)$ is a factor of f(x).

$\therefore$ Remainder = $f(-1) = 0$

$(-1)^3 + (a + 1)(-1)^2 - (b - 2)(-1) - 6 = 0$

$-1 + (a + 1) + (b - 2) - 6 = 0$

$a + b - 8 = 0$ ...(i)

Since, $(x - 2)$ is a factor of f(x).

$\therefore$ Remainder = $f(2) = 0$

$(2)^3 + (a + 1)(2)^2 - (b - 2)(2) - 6 = 0$

$8 + 4a + 4 - 2b + 4 - 6 = 0$

$4a - 2b + 10 = 0$

$2a - b + 5 = 0$ ...(ii)

Adding (i) and (ii), we get,

$3a - 3 = 0$

$a = 1$

Substituting the value of a in (i), we get,

$1 + b - 8 = 0$

$b = 7$

$\therefore f(x) = x^3 + 2x^2 - 5x - 6$

Now, (x + 1) and (x - 2) are factors of f (x). Hence, (x + 1) (x - 2) = x^2 - x - 2 is a factor of f (x).

$$\begin{array}{r|l} & x+3 \\ \hline x^2-x-2 & x^3+2x^2-5x-6 \\ & x^3-x^2-2x \\ \hline & 3x^2-3x-6 \\ & 3x^2-3x-6 \\ \hline & 0 \end{array}$$

∴ f(x) = $x^3 + 2x^2 - 5x - 6 = (x + 1)(x - 2)(x + 3)$

Question 6:

If x – 2 is a factor of x^2 + ax + b and a + b = 1, find the values of a and b.

Solution 6:

Let f(x) = x^2 + ax + b

Since, (x − 2) is a factor of f(x).

∴ Remainder = f(2) = 0

$(2)^2 + a(2) + b = 0$

4 + 2a + b = 0

2a + b = −4 ...(i)

It is given that:

a + b = 1 ...(ii)

Subtracting (ii) from (i), we get,

a = −5

Substituting the value of a in (ii), we get,

b = 1 − (−5) = 6

Question 7:

Factorise $x^3 + 6x^2 + 11x + 6$ completely using factor theorem.

Solution 7:

Let f(x) = $x^3 + 6x^2 + 11x + 6$

For x = −1

f(−1) = $(−1)^3 + 6(-1)^2 + 11(−1) + 6$

= −1 + 6 − 11 + 6 = 12 − 12 = 0

Hence, (x + 1) is a factor of f(x).

$$\begin{array}{r|l} & x^2+5x+6 \\ \hline x+1 & x^3+6x^2+11x+6 \\ & x^3+x^2 \\ \hline & \quad 5x^2+11x \\ & \quad 5x^2+5x \\ \hline & \qquad\quad 6x+6 \\ & \qquad\quad 6x+6 \\ \hline & \qquad\qquad 0 \end{array}$$

$\therefore x^3+6x^2+11x+6=(x+1)(x^2+5x+6)$

$=(x+1)(x^2+2x+3x+6)$

$=(x+1)[x(x+2)+3(x+2)]$

$=(x+1)(x+2)(x+3)$

Question 8:

Find the value of 'm', if $mx^3 + 2x^2 - 3$ and $x^2 - mx + 4$ leave the same remainder when each is divided by $x - 2$.

Solution 8:

Let $f(x) = mx^3 + 2x^2 - 3$

$g(x) = x^2 - mx + 4$

It is given that f (x) and g (x) leave the same remainder when divided by $(x - 2)$. Therefore, we have:

$f(2) = g(2)$

$m(2)^3 + 2(2)^2 - 3 = (2)^2 - m(2) + 4$

$8m + 8 - 3 = 4 - 2m + 4$

$10m = 3$

$m = \frac{3}{10}$

Question 9:

The polynominal $px^3 + 4x^2 - 3x + q$ is completely divisible by $x^2 - 1$; find the values of p and q. Also, for these values of p and q factorize the given polynominal completely.

Solution 9:

Let $f(x) = px^3 + 4x^2 - 3x + q$

It is given that f(x) is completely divisible by $(x^2 - 1) = (x + 1)(x - 1)$.
Therefore, $f(1) = 0$ and $f(-1) = 0$
$f(1) = p(1)^3 + 4(1)^2 - 3(1) + q = 0$
$p + q + 1 = 0$ ...(i)
$f(-1) = p(-1)^3 + 4(-1)^2 - 3(-1) + q = 0$
$-p + q + 7 = 0$ ...(ii)
Adding (i) and (ii), we get,
$2q + 8 = 0$
$q = -4$
Substituting the value of q in (i), we get,
$p = -q - 1 = 4 - 1 = 3$
$\therefore f(x) = 3x^3 + 4x^2 - 3x - 4$
Given that f(x) is completely divisible by $(x^2 - 1)$

$$
\begin{array}{r|l}
 & 3x+4 \\ \hline
x^2-1 & 3x^3+4x^2-3x-4 \\
 & 3x^3 \qquad\quad -3x \\ \cline{2-2}
 & \qquad 4x^2 \qquad -4 \\
 & \qquad 4x^2 \qquad -4 \\ \cline{2-2}
 & \qquad\qquad 0
\end{array}
$$

$$
\begin{aligned}
\therefore\ 3x^3 + 4x^2 - 3x - 4 &= (x^2 - 1)(3x - 4) \\
&= (x - 1)(x + 1)(3x + 4)
\end{aligned}
$$

Question 10:

Find the number which should be added to $x^2 + x + 3$ so that the resulting polynomial is completely divisible by $(x + 3)$

Solution 10:

Let the required number be k.
Let $f(x) = x^2 + x + 3 + k$
It is given that f(x) is divisible by $(x + 3)$.
$\therefore$ Remainder = 0
$f(-3) = 0$
$(-3)^2 + (-3) + 3 + k = 0$
$9 - 3 + 3 + k = 0$
$9 + k = 0$
$k = -9$
Thus, the required number is -9.

Question 11:

When the polynomial $x^3 + 2x^2 – 5ax – 7$ is divided by $(x – 1)$, the remainder is A and when the polynomial $x^3 + ax^2 − 12x + 16$ is divided by $(x + 2)$, the remainder is B. Find the value of 'a' if $2A + B = 0$

Solution 11:

It is given that when the polynomial $x^3 + 2x^2 − 5ax − 7$ is divided by $(x − 1)$, the remainder is A.

$\therefore (1)^3 + 2(1)^2 − 5a(1) − 7 = A$

$1 + 2 − 5a − 7 = A$

$− 5a − 4 = A$ ...(i)

It is also given that when the polynomial $x^3 + ax^2 − 12x + 16$ is divided by $(x + 2)$, the remainder is B.

$\therefore x^3 + ax^2 − 12x + 16 = B$

$(−2)^3 + a(−2)^2 − 12(−2) + 16 = B$

$−8 + 4a + 24 + 16 = B$

$4a + 32 = B$ ...(ii)

It is also given that $2A + B = 0$

Using (i) and (ii), we get,

$2(−5a − 4) + 4a + 32 = 0$

$− 10a − 8 + 4a + 32 = 0$

$− 6a + 24 = 0$

$6a = 24$

$a = 4$

Question 12:

$(3x + 5)$ is a factor of the polynomial $(a – 1)x^3 + (a + 1)x^2 – (2a + 1)x – 15$. Find the value of 'a'. For this value of 'a', factorise the given polynominal completely.

Solution 12:

Let $f(x) = (a − 1)x^3 + (a + 1)x^2 − (2a + 1)x − 15$

It is given that $(3x + 5)$ is a factor of f(x).

$\therefore \text{Remainder} = 0$

$$f\left(\frac{-5}{3}\right) = 0$$

$$(a-1)\left(\frac{-5}{3}\right)^3 + (a+1)\left(\frac{-5}{3}\right)^2 - (2a+1)\left(\frac{-5}{3}\right) - 15 = 0$$

$$(a-1)\left(\frac{-125}{27}\right)+(a+1)\left(\frac{25}{9}\right)-(2a+1)\left(\frac{-5}{3}\right)-15=0$$

$$\frac{-125(a-1)+75(a+1)+45(2a+1)-405}{27}=0$$

$$-125a+125+75a+75+90a+45-405=0$$

$$40a-160=0$$

$$401=160$$

$$a=4$$

$\therefore$ f(x) = (a - 1)x^3 + (a + 1)x^2 - (2a + 1)x - 15

= $3x^3 + 5x^2 - 9x - 15$

$$\begin{array}{r|l} & x^2-3 \\ \hline 3x+5 & 3x^3+5x^2-9x-15 \\ & 3x^3+5x^2 \\ \hline & \quad -9x-15 \\ & \quad -9x-15 \\ \hline & \quad 0 \end{array}$$

$$\therefore 3x^3+5x^2-9x-15=(3x+5)(x^2-3)$$

$$=(3x+5)(x+\sqrt{3})(x-\sqrt{3})$$

Question 13:

When divided by x – 3 the polynomials $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p + 3)x - 6$ leave the same remainder. Find the value of ‘ p’.

Solution 13:

If (x − 3) divides f(x) = $x^3 - px^2 + x + 6$, then,

Remainder = f(3) = $3^3 - p(3)^2 + 3 + 6 = 36 - 9p$

If (x − 3) divides g(x) = $2x^3 - x^2 - (p + 3)x - 6$, then

Remainder = g(3) = $2(3)^3 - (3)^2 - (p + 3)(3) - 6 = 30 - 3p$

Now, f(3) = g(3)

$\Longrightarrow 36 - 9p = 30 - 3p$

$\Longrightarrow -6p = -6$

$\Longrightarrow p = 1$

Question 14:

Use the Remainder Theorem to factorise the following expression:

$2x^3 + x^2 - 13x + 6$

Solution 14:

$f(x) = 2x^3 + x^2 - 13x + 6$

Factors of constant term 6 are ±1, ± 2, ± 3, ± 6.

Putting x = 2, we have:

$f(2) = 2(2)^3 + 2^2 - 13 (2) + 6 = 16 + 4 - 26 + 6 = 0$

Hence $(x - 2)$ is a factor of $f(x)$.

$$\begin{array}{r|l} & 2x^2 + 5x - 3 \\ \hline x-2 & 2x^3 + x^2 - 13x + 6 \\ & 2x^3 - 4x^2 \\ \hline & \quad 5x^2 - 13x \\ & \quad 5x^2 - 10x \\ \hline & \qquad -3x + 6 \\ & \qquad -3x + 6 \\ \hline & \qquad 0 \end{array}$$

$$\begin{aligned} 2x^3 + x^2 - 13x + 6 &= (x-2)(2x^2 + 5x - 3) \\ &= (x-2)(2x^2 + 6x - x - 3) \\ &= (x-2)(2x(x+3) - 1(x+3)) \\ &= (x-2)(2x-1)(x+3) \end{aligned}$$