CONTINUITY
DPP - 1

1. If the function $f(x) = \begin{cases} (\cos x)^{1/x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at x = 0 ,then the value of k is

(A) 1 (B) -1 (C) 0 (D) e

2. If the function $f(x) = \begin{cases} 1 + \sin\frac{\pi x}{2} & \text{for } -\infty < x \leq 1 \\ ax + b & \text{for } 1 < x < 3 \\ 6\tan\frac{x\pi}{12} & \text{for } 3 \leq x < 6 \end{cases}$ is continuous in the interval $(-\infty, 6)$ then the values of a and b are respectively

(A) 0,2 (B) 1, 1 (C) 2, 0 (D) 2, 1

3. If $f(x) = \begin{cases} \dfrac{x^2 - 4x + 3}{x^2 - 1}, & \text{for } x \neq 1 \\ 2, & \text{for} x = 1 \end{cases}$, then

(A) $\lim_{x \to 1^-} f(x) = 2$ (B) $\lim_{x \to 1^-} f(x) = 3$

(C) f(x) is discontinuous at x = 1 (D) None of these

4. The function defined by $f(x) = \begin{cases} \left(x^2 + e^{\frac{1}{2-x}}\right)^{-1}, & x \neq 2 \\ k, & x = 2 \end{cases}$, is continuous from right at the point x = 2, then to

(A) 0 (B) 1/4 (C) -1/4 (D) None of these

5. The function $f(x) = \dfrac{1 - \sin x + \cos x}{1 + \sin x + \cos x}$ is not defined at $x = \pi$. The value of $f(\pi)$, so that f(x) is continuous is

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) -1 (D) 1

6. **Statement 1**
$f(x) = [x]^2 - 5[x] + 3$. $1 > x > 4$. where [ ] represents the greatest integer function is not continuous at x = 2 and 3 but is bounded.
and
Statement 2
A continuous function in a closed interval I is bounded.
(A) S-1 is correct, S-2 is correct and correct explanation of S-1
(B) S-1 is correct, S-2 is correct but not correct explanation of S-1
(C) S-1 is correct, S-2 is false
(D) S-1 is false, S-2 is correct

7. For the function $f(x) = \dfrac{1}{x + 2^{\frac{1}{(x-2)}}}$, $x \neq 2$ which of the following holds ?

(A) f(2) = 1/2 and f is continuous at x = 2
(B) f(2) ≠ 0, 1/2 and f is continuous at x = 2
(C) f can not be continuous at x = 2
(D) f(2) = 0 and f is continuous at x = 2

8. The function $f(x) = \dfrac{4 - x^2}{4x - x^3}$, is

(A) discontinuous at only one point in its domain
(B) discontinuous at two points in its domain
(C) discontinuous at three points in its domain
(D) continuous everywhere in its domain

Multiple Type

9. Let $f(x) = \dfrac{1 - \sin x}{(\pi - 2x)^2} \cdot \dfrac{\ell n(\sin x)}{\ell n(1 + \pi^2 - 4\pi x + 4x^2)}$ $x \neq \dfrac{\pi}{2}$. Value of $f\left(\dfrac{\pi}{2}\right)$ so that the function is continuous at $x = \pi/2$ is less than

(A) 1/16 (B) 1/32 (C) –1/64 (D) 1/128

10. Let $f(x) = [x^2 - x + 1]$ where [ ] denotes the greatest integer function. Then, in (0, 2),f(x) is discontinuous at the point

(A) $\dfrac{1 + \sqrt{5}}{2}$ (B) $\dfrac{1 - \sqrt{5}}{2}$ (C) 1 (D) Both (A) and (B)

CONTINUITY
DPP - 2

1. The poiints of discontinuity of $y = \dfrac{1}{u^2 + u - 2}$ where u = $\dfrac{1}{x-1}$ is

(A) $\dfrac{1}{2}, 1, 2$ (B) $\dfrac{-1}{2}, 1, -2$ (C) $\dfrac{1}{2}, -1, 2$ (D) None of these

2. The function f(x) = $\dfrac{\log(1+ax) - \log(1-bx)}{x}$ is not defined at x = 0. The value which should be assigned to f at x = 0 so that it is continuous at x = 0, is

(A) a = b (B) a + b (C) log a + log b (D) log a - log b

3. Let f(x) be defined for all x > 0 and be continuous. Let f(x) satisfy $f\left(\dfrac{x}{y}\right)$ = f(x) - f(y) for all x, y and f(e) = 1, then

(A) f(x) = In x (B) f(x) is bounded (C) $f\left(\dfrac{1}{x}\right) \to 0$ as $x \to 0$ (d) $xf(x) \to 1$ as $x \to 0$

4. Lef f(x) = $[x]\sin\left(\dfrac{\pi}{[x+1]}\right)$, where [.] denotes the greatest integer funciton. The domain of f is........ and the points of discontinuity of f in the domain are

(A) $[x \in R]\ x \in [-1, 0)$, I - {0] (B) $\{x \in R]\ x \notin [1, 0)\}$, I - {0)

(C) $\{x \in R]\ x \notin [-1, 0)\}$, I - {0) (D) None of these

5. The set of all points of discontinuity of the inverse of f(x) = $\dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ is

(A) ϕ (B) $(-\infty, -1]$ (C) $[1, \infty)$ (D) R - (-1, 1)

6. Let f(x) = $\begin{cases} x\dfrac{e^{[x]+|x|} - 4}{[x]+|x|}, & x \neq 0 \\ 3 & x = 0 \end{cases}$

Where [ ] denotes the greatest integer function. Then,

(A) f(x) is discontinuous at x = 0 (B) f(x) is continuous at x = 0

(C) f(x) is left continuous at x = 0 (D) f(x) is right continuous at x = 0

7. If f(x) = $\begin{bmatrix} -4\sin x + \cos x & \text{for} & x \leq -\dfrac{\pi}{2} \\ a\sin x + b & \text{for} & -\dfrac{\pi}{2} < x < \dfrac{\pi}{2} \\ \cos x + 2 & \text{for} & x \geq \dfrac{\pi}{2} \end{bmatrix}$ is continuous then

(A) a = −1, b = 3 (B) a = 1, b = −3 (C) a = 1, b = 3 (D) a = −1, b = −3

8. $f(x) = \begin{cases} \sin\left(\dfrac{a-x}{2}\right)\tan\left[\dfrac{\pi x}{2a}\right] & \text{for } x > a \\ \dfrac{\left[\cos\left(\dfrac{\pi x}{2a}\right)\right]}{a-x} & \text{for } x < a \end{cases}$

where [x] is the greatest integer function of x, and a > 0, then
(A) $f(a^-) < 0$
(B) f has a removable discontinuity at x = a
(C) f has an irremovable discontinuity at x = a
(D) $f(a^+) > 0$

Multiple Type

9. Let [x] be the greatest integer less than or equals to x. Then, at which of the following points (s) the function $f(x) = x\cos(\pi(x + [x]))$ is discontinuous ?
(A) x = − 1 (B) x = 1 (C) x = 0 (D) x = 2

10.

Column – I	Column – II
(A) If $f(x) = \begin{cases} \dfrac{a+3\cos x}{x^2}, & x < 0 \\ b\tan\left(\dfrac{\pi}{[x+3]}\right) & x \ge 0 \end{cases}$	(P) $\lvert a+b\rvert = 0$
is continuous at x = 0, then (where [*] denotes the greatest integer function)	(Q) $\lvert a-b\rvert = 2$
(B) If $f(x) = \begin{cases} -2\sin x, & -\pi \le x \le -\pi/2 \\ a\sin x + b, & -\pi/2 < x < \pi/2 \\ \cos x, & \pi/2 \le x \le \pi \end{cases}$	(R) $\lvert a+2b\rvert = 1$
is continuous in [-π, π], then	(S) $\lvert a+2b\rvert = 4$
(C) If $f(x) = \begin{cases} (3/2)^{(\cot 3x)/\cot 2x}, & 0 < x < \pi/2 \\ b+3, & x = \pi/2 \\ (1+\lvert\cos x\rvert)^{\left(\frac{a\lvert\tan x\rvert}{b}\right)}, & \dfrac{\pi}{2} < x < \pi \end{cases}$	
is continuous at $x = \dfrac{\pi}{2}$, then (where [*] denotes the greatest integer function)	(T) [a - 2b] = -2

CONTINUITY
DPP - 3

1. If $f(x)=\dfrac{x^2-bx+25}{x^2-7x+10}$ for $x \neq 5$ and f is continuous at $x = 5$, then f(5) has the value equal to

(A) 0 (B) 5 (C) 10 (D) 25

2. Consider the function $f(x) = \begin{cases} 1-x, & 0 \le x \le 1 \\ x+2, & 1 < x < 2 \\ 4-x, & 2 \le x \le 4 \end{cases}$ Find $a = \lim\limits_{x\to 1} f(f(x))$ and $b = \lim\limits_{x\to 2} f(f(x))$

(A) a = does not exist & b = 0
(B) a = 1 & b = does not exist
(C) a = does not exist & b = 0
(D) a = 1 & b = 4

3. On the interval I = [–2, 2], function $f(x) = \begin{cases} (x+1)e^{-\left[\frac{1}{|x|}+\frac{1}{x}\right]} & (x \neq 0) \\ 0 & (x = 0) \end{cases}$, then which of the following is in correct

(A) is continuous for all values of $x \in I$
(B) is continuous for $x \in I - (0)$
(C) assumes all intermediate values from f(–2) & f(2)
(D) has a maximum value equal to 3/e.

4. Consider $f(x) = \begin{cases} x[x]^2 \log_{(1+x)} 2 & \text{for } -1 < x < 0 \\ \dfrac{\ell n\,(e^{x^2} + 2\sqrt{\{x\}})}{\tan\sqrt{x}} & \text{for } 0 < x < 1 \end{cases}$

where [*] & {*} are the greatest integer function & fractional part function respectively, then

(A) $f(0) = \ell n\, 2 \Rightarrow$ f is continuous at x = 0
(B) $f(0) = 2 \Rightarrow$ f is continuous at x = 0
(C) $f(0) = e^2 \Rightarrow$ f is continuous at x = 0
(D) f has an irremovable discontinuity at x = 0

5. Let $f : R \to R$ be a continuous onto function satisfying $f(x) + f(-x) = 0$, $\forall\ x \in R$. If (–3) = 2 and f(5) = 4 in [–5, 5], then the equation f(x) = 0 has

(A) exactly three real roots
(B) exactly two real roots
(C) atleast five real roots
(D) atleast three real roots

6. Consider the function $f(x) = \begin{cases} \dfrac{x}{[x]} & \text{if } 1 \le x < 2 \\ 1 & \text{if } x = 2 \\ \sqrt{6-x} & \text{if } 2 < x \le 3 \end{cases}$

Where [x] denotes step up function then at x = 2 function
(A) has missing point removable discontinuity
(B) has isolated point removable discontinuity
(C) has non removable discontinuity finite type
(D) is continuous

7. If f(x) = 1/(2 – x), then the points of discontinuity of the composite function y = f(f(f(x))) are
(A) 2, 3/4 (B) 1, 2 (C) 2, 3 (D) 2, 3/2

MULTIPLE CORRECT

8. $\lim\limits_{x\to\infty} \dfrac{1-(2\sin x)^{2n}}{1+(2\sin x)^{2n}}$ is continuous at x =

(A) $\pi/6$ (B) $\pi/4$ (C) $\pi/2$ (D) None of these

9. Function whose jump (non-negative difference of LHL & RHL) of discontinuity is greater than or equal to one is/are

(A) $f(x) = \begin{cases} \dfrac{(e^{1/x}+1)}{(e^{1/x}-1)}; & x < 0 \\ \dfrac{(1-\cos x)}{x}; & x > 0 \end{cases}$

(B) $g(x) = \begin{cases} \dfrac{(x^{1/3}-1)}{(x^{1/2}-1)}; & x < 1 \\ \dfrac{\ell n x}{x-1}; & \dfrac{1}{2} < x < 1 \end{cases}$

(C) $u(x) = \begin{cases} \dfrac{\sin^{-1} 2x}{\tan^{-1} 3x}; & x \in \left(0, \dfrac{1}{2}\right] \\ \dfrac{|\sin x|}{x}; & x < 0 \end{cases}$

(D) $v(x) = \begin{cases} \log_3(x+2) \ ; & x > 2 \\ \log_{1/2}(x^2+5); & x < 2 \end{cases}$

Integer Type

10. Let f(x) = cosec 2x + cosec2^2x + cosec 2^3x + ...+ cosec 2^nx, x Î $\left(0, \dfrac{\pi}{2}\right)$ and g(x) = f(x) + cot2^nx.

If $H(x) = \begin{cases} (\cos x)^{g(x)} + (\sec x)^{\operatorname{cosec} x} & \text{if } x > 0 \\ p & \text{if } x = 0 \\ \dfrac{e^x + e^{-x} - 2\cos x}{x \sin x} & \text{if } x < 0 \end{cases}$. Find the value of p, if possible to make the function H(x) continuous at x = 0.