2. Inverse Trigonometric Function

Objective Type Questions

Q.1. Choose the correct option –

(1) If $\sin^{-1}\frac{1}{x} = y$ then –

(a) $0 \le y \le \pi$ (b) $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$

(c) $0 < y < \pi$ (d) $\frac{-\pi}{2} < y < \frac{\pi}{2}$

(2) $\sin^{-1}(2x\sqrt{1-x^2})$

(a) $2\sin^{-1}x$ (b) $2\cos^{-1}x$

(c) $\sin^{-1}2x$ (d) $\tan^{-1}2x$

(3) $\tan^{-1}\sqrt{3} - \sec^{-1}(-2) =$

(a) π (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

(4) $\text{con}^{-1}\left(\cos\frac{7\pi}{\pi}\right) =$

(a) $\frac{7\pi}{\pi}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

(5) $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right) =$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 1

(6) $\tan^{-1}\sqrt{3} - \cot^{-1}(\sqrt{3}) =$

(a) π (b) $-\frac{\pi}{2}$ (c) 0 (d) $2\sqrt{3}$

(7) $\sin(\tan^{-1}x)$, $|x| < 1 =$

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$

(c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

(8) $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$, then $x =$

(a) $0, \frac{1}{2}$ (b) $1, \frac{1}{2}$ (c) 0 (d) $\frac{1}{2}$

(9) $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\frac{x-y}{x+y} =$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{-3\pi}{4}$

Ans. (1) (b) (2) (a) (3) (b) (4) (b) (5) d (6) (a) (7) d (8) c (9) c.

Q.2. Fill in the blanks –

(1) $\text{Cos}^{-1}x$ is the domain of ....

(2) Principal value branch of $\tan^{-1}x$ ....

Ans. (1) (–1,1) (2) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

Q.3. Write True or False-

(1) The domain of $\text{Cos}^{-1}x$ is R-(-1,1)

(2) $\text{Sec}^{-1}x$. The principal value branch of $[0, \pi]$- $\left(\frac{\pi}{2}\right)$

Ans. (1) True (2) True.

Q.4. Write answer in one word/sentence.

(1) Write the domain of $\cot^{-1}x$

(2) Write the principal value branch of $\text{cosec}^{-1}x$

(3) Write the value of $\cos(\sec^{-1}x + \text{cosec}^{-1}x)$, $|x| \ge 1$

(4) Write the value of $\cot(\tan^{-1}a + \cot^{-1}a)$

Ans. (1) R (2) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right) - [0]$ (3) 0 (4) 0.

Short Answer Type Questions

Q.5. Find the principal value of $\sin^{-1}\left(\frac{1}{2}\right)$.

Solution. Let $\sin^{-1}\frac{1}{2} = x$, therefore $\sin x = \frac{1}{2}$. We know that, the range of the main branch of $\sin^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$. Therefore, $\frac{\pi}{6}$ is the main principal of $\sin^{-1}\frac{1}{2}$ **Ans.**

Q.6. Find the main principal of $\tan^{-1}(1)$

Solution : We know that the range of the principal branch of $\tan^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, and $\tan\frac{\pi}{4} = 1$.

Hence, the main principal of $\tan^{-1}(1)$ is $\frac{\pi}{4}$

Q.7. Find main principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$

Solution : Let, $\cos^{-1}\left(-\frac{1}{2}\right) = x$

$\cos x = \frac{-1}{2} \Rightarrow -\cos\frac{\pi}{3} = \cos\frac{2\pi}{3}$

Q.8. Find the main principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$

Solution : let, $\sin^{-1}\left(-\frac{1}{2}\right) = x$

$$\sin x = -\frac{1}{2} = \frac{-\pi}{6} \qquad \textbf{Ans.}$$

Q.9. Find the main principal value of-

$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

Solution : $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)\sin^{-1}\left(-\frac{1}{2}\right)$

Let,

$\tan^{-1}(1) = x \Rightarrow \tan x = \tan\frac{\pi}{4}$

$$x = \frac{\pi}{4}$$

Let,

$\cos^{-1}\left(-\frac{1}{2}\right) = y$

$$\cos y = -\frac{1}{2} = -\cos\frac{\pi}{3} = \cos\frac{2\pi}{3}$$

Like wise $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

Totally,

$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)\sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$

$$= \frac{3\pi}{4}$$

Q.10. Find the principal value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\cos^{-1}\left(\frac{1}{2}\right)$

Solution : $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

$$= \cos^{-1}\left[\cos\frac{\pi}{3}\right] + 2\sin^{-1}\left[\sin\frac{\pi}{6}\right]$$

$$= \frac{\pi}{3} + 2\times\frac{\pi}{6} = \frac{2\pi}{3}$$

Q.11. Show that $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\sin^{-1} x, \frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$

Solution : Let $\sin^{-1}x = \theta \Rightarrow \sin\theta = x$

Now, $\sin 2\theta = 2\sin\theta\cos\theta$

$$\Rightarrow \quad \sin 2\theta = 2\sin\theta\sqrt{1-\sin 2\theta}$$

$$\sin\theta = x$$

$$\sin 2\theta = 2x\sqrt{1-x^2}$$

$$2\theta = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$$

put the value of θ

$$2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$$

Q.12. Show that $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$= 2\cos_x^{-1} \cdot \frac{1}{\sqrt{2}} \leq x \leq 1$

Solution : Let $x = \cos\theta$, $\theta = \cos^{-1}x$

then $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$$= \sin^{-1}\left(2\cos\theta\sqrt{1-\cos^2\theta}\right)$$

$$= \sin^{-1}(2\cos\theta\sin\theta)$$

$$= \sin^{-1}(\sin 2\theta)$$

$$= 2\theta = 2\cos^{-1}x$$

Q.13. Prove that $\tan^{-1}x + \tan^{-1}\frac{2x}{1-x^2} = \tan{-1}\, x\left(\frac{3x-x^3}{1-3x^2}\right), |x| < \frac{1}{\sqrt{3}}$.

Solution : $\tan^{-1}x + \tan^{-1}\frac{2x}{1-x^2}$

$$\Rightarrow \tan^{-1}\frac{x+\frac{2x}{1-x^2}}{1-x\times\frac{2x}{1-x^2}}$$

$$\Rightarrow \tan^{-1}\frac{x(1-x^2)+2x}{\frac{(1-x^2)-2x^2}{1-x^2}}$$

$\Rightarrow \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ **Proved**

L.H.S. = R.H.S.

Q.14. Prove that

$$3\sin^{-1}x = \sin^{-1}(3x - 4x^3),\ x\left[\frac{1}{2},\frac{1}{2}\right]$$

Solution : $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$

Let $\sin^{-1}x = \theta \Rightarrow x = \sin\theta$, then

L.H.S. $= 3\sin^{-1}x = 3\sin^{-1}(\sin\theta) = 3\theta$

R.H.S. $= \sin^{-1}(3x - 4x^3)$

$= \sin^{-1}(3\sin\theta - 4\sin 3\theta)$

$= \sin^{-1}[\sin 3\theta] = 30$

$[\because \sin 3\theta - 3\sin\theta - 4\sin 3\theta]$

L.H.S. = R.H.S. **Proved**

Q.15. Prove that- $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x),\ x\left[\frac{1}{2},1\right]$

Solution : $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$

$x = \cos\theta$

Let : L.H.S. $= 3\cos^{-1}x$

$= 3\cos^{-1}(\cos\theta) = 3\theta$

R.H.S. $= \cos^{-1}(4x^3 - 3x)$

$= \cos^{-1}(4\cos 3\theta - 3\cos\theta)$

$= \cos^{-1}(\cos 3\theta) = 3$

$[\cos 3\theta = 4\cos 3\theta - 3\cos\theta)$

L.H.S. = R.H.S. **Proved**

Q.16. Prove that- $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{1}\left[\frac{1-x}{1+x}\right]$, $x \in [0,1]$

Solution : Let $\tan^{-1}\sqrt{x} = \theta$,

$\Rightarrow \sqrt{x} = \tan\theta \Rightarrow x = \tan^2\theta$

R.H.S. $= \frac{1}{2}\cos^{-1}\frac{1-x}{1+x}$

$= \frac{1}{2}\cos^{-1}\frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \frac{1}{2}\cos^{-1}(\cos^2\theta)$

$= \frac{1}{2}.2\theta = \tan^{-1}\sqrt{x}$ = L.H.S.

Q.17. Prove that: $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x,\ -\frac{1}{\sqrt{2}} \le x \le 1$

Solution : L.H.S. $= \tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$

Let $\cos^{-1}x = \theta \Rightarrow x = \cos\theta$

and $0 \le \theta \le \frac{3\pi}{4}$

$\therefore \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}\right)$

$= \tan^{-1}\left[\frac{\sqrt{2}\cos\frac{\theta}{2}-\sqrt{2}\sin\frac{\theta}{2}}{\sqrt{2}\cos\frac{\theta}{2}+\sqrt{2}\sin\frac{\theta}{2}}\right]$

$[\because 1+\cos\theta = 2\cos^2(\theta/2)$

$1-\cos\theta = 2\sin^2(\theta/2)$

$= \tan^{-1}\left[\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right]$

Multiply and divide by $\cos\frac{\theta}{2}$

$= \tan^{-1}\left[\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}\right]$

$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right]$

$= \frac{\pi}{4}-\frac{\theta}{2}\left(0 \le \theta \le \frac{3\pi}{4} \Rightarrow \frac{\pi}{4} \ge \frac{\pi}{4}-\frac{\theta}{2} \ge -\frac{\pi}{4}\right)$

$= \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x =$ R.H.S.

Q.18. $\cot^{-1}\left[\frac{1}{\sqrt{x^2-1}}\right]$, $|x|>1$ Simplify and write.

Solution : $\cot^{-1}\left[\frac{1}{\sqrt{x^2-1}}\right]$

let $x=\sec\theta \Rightarrow \theta=\sec^{-1}x$

$$\Rightarrow \cot^{-1}\left(\frac{1}{\sqrt{x2-1}}\right)=\cot^{-1}\frac{1}{\sqrt{\sec^2\theta-1}}$$

$$\Rightarrow \cot^{-1}\frac{1}{\sqrt{\tan^2\theta}} \quad (\because \sec^2\theta-\tan^2\theta=1)$$

$$=\cot^{-1}(\cot\theta)=\theta=\sec^{-1}x \qquad \text{Ans.}$$

Q.19. Simplify and write - $\tan^{-1}\left[\frac{1}{\sqrt{x^2-1}}\right]$, $|x|>1$.

Solution : $\tan^{-1}\left[\frac{1}{\sqrt{x^2-1}}\right]$

Let $x=\sec\theta$, then $\theta=\sec^{-1}x$

$$\therefore \tan^{-1}\frac{1}{\sqrt{x^2-1}}=\tan^{-1}\frac{1}{\sqrt{\sec^2\theta-1}}$$

$$=\tan^{-1}\frac{1}{\sqrt{\tan^2\theta}}$$

$$=\tan^{-1}\left(\frac{1}{\tan\theta}\right)=\tan-1(\cot\theta)$$

$$=\tan^{-1}\left[\tan\left(\frac{\pi}{2}-\theta\right)\right]$$

$$\left[\because \tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta\right]$$

$$=\frac{\pi}{2}-\theta=\frac{\pi}{2}-\sec^{-1}x \qquad \text{Ans.}$$

Q.20. Write in the simplest form – $\tan^{-1}\left[\frac{\sqrt{1+x^2-1}}{x}\right]$, $x\neq 0$.

Solution : Let $x=\tan\theta \Rightarrow \tan^{-1}x=\theta$

$$=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$$

$$=\tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$=\tan^{-1}\left(\frac{\sqrt{\sec^2\theta}-1}{\tan\theta}\right)$$

$$=\tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$$

$$=\tan^{-1}\left(\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right)$$

$$=\tan^{-1}\left(\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\right)=\tan\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$=\tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$=\tan^{-1}\left(\tan-\frac{\theta}{2}\right)=\frac{\theta}{2}=\frac{1}{2}\tan^{-1}x \qquad \text{Ans.}$$

Q.21. Write in the simplest form $\tan^{-1}\left[\frac{x}{\sqrt{a^2-x^2}}\right]$, $|x|<1$.

Solution : $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$

Put $x=a\sin\theta$

$$=\tan^{-1}\frac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}$$

$$=\tan^{-1}\frac{a\sin\theta}{a\sqrt{1-\sin^2\theta}}$$

$$=\tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$=\tan^{-1}(\tan\theta)=\theta$$

but $x=a\sin\theta \Rightarrow \sin\theta=\frac{x}{a}$

$$\Rightarrow \theta=\sin^{-1}\frac{x}{a}$$

Hence $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}=\sin^{-1}\frac{x}{a}$

Q.22. Write in the simplest form $\tan^{-1}\left[\frac{3a^2x-x^3}{a^3-3ax^2}\right]$, $a>0$, $\frac{-a}{\sqrt{3}}\leq x\leq\frac{a}{\sqrt{3}}$.

Solution : Let $x=a\tan\theta$ then $\theta=\tan^{-1}\frac{x}{a}$

Hence $\tan^{-1}\left[\frac{3a^2x-x3}{a(a^2-3x^2)}\right]$

$$= \tan^{-1}\left[\frac{3a^3\tan\theta - a^3\tan^3\theta}{a(a^2 - 3a^2\tan^2\theta)}\right]$$

$$= \tan^{-1}\left[\frac{a^3(3\tan\theta - \tan^3\theta)}{a^3(1-3\tan^2\theta)}\right]$$

$$= \tan^{-1}(\tan 3\theta) = 3\theta$$

$$= 3\tan^{-1}\frac{x}{a}$$

Q.23. Write in the simplest form $\tan^{-1}\left[\frac{\cos x}{1-\sin x}\right]$, $\frac{-\pi}{2} < x\frac{3\pi}{2}$.

Solution : $\tan^{-1}\frac{\cos x}{1-\sin x} = \tan^{-1}\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}$

$$= \tan^{-1}\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}$$

$$= \tan^{-1}\left[\left\{\cot\left(\frac{\pi}{4}-\frac{x}{z}\right)\right\}\right]$$

$$= \tan^{-1}\left[\tan\left\{\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right\}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right] = \frac{\pi}{4}+\frac{x}{2}$$ **Ans.**

Q.24. Write in the simplest form $\tan^{-1}\left[\sqrt{\frac{1-\cos x}{1+\cos x}}\right]$, $x < \pi$

Solution : $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$

here $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}} = \tan^{-1}\sqrt{\frac{2\sin^2\frac{1}{2}x}{2\cos^2\frac{1}{2}x}}$

$$= \tan^{-1}\left(\tan\frac{x}{2}\right) = \frac{x}{2}$$ **Ans.**

Q.25. Write in the simplest form

$$\tan^{-1}\left[\frac{\cos x - \sin x}{\cos x + \sin x}\right],\ 0 < x < \pi$$

Solution : $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$

$$= \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2}-x\right) - \sin x}{\sin\left[\frac{\pi}{2}-x\right] + \sin x}\right]$$

$$= \tan^{-1}\left[\frac{2\cos\frac{\pi}{4}.\sin\left(\frac{\pi}{4}-x\right)}{2\sin\frac{\pi}{4}.\cos\left(\frac{\pi}{4}-x\right)}\right]$$

$$= \tan^{-1}\left[\frac{2.\frac{1}{\sqrt{2}}.\sin\left(\frac{\pi}{4}-x\right)}{2.\frac{1}{\sqrt{2}}.\cos\left(\frac{\pi}{4}-x\right)}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}-x\right)\right] = \frac{\pi}{4}-x$$ **Ans.**

Q.26. Simplify– $\tan^{-1}\left[\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right]$, if $\frac{a}{b}\tan x > -1$.

Solution : $\tan^{-1}\left[\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right]$

$$\tan^{-1}\frac{\frac{a}{b}-\tan x}{1+\frac{a}{b}\tan x}$$

$$\tan^{-1}\frac{a}{b} - \tan^{-1}(\tan x) = \tan^{-1}\frac{a}{b} - x$$ **Ans.**

Q.27. Prove that–

$$\tan^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right] = \frac{x}{2},\ x \in \left[0,\frac{\pi}{4}\right]$$

Solution : L.H.S. $\tan^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$

$$= \tan^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}} \times \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$$

[On rationalising the denominator]

$$= \tan^{-1}\left[\frac{\left(\sqrt{1+\sin x}\right)^2-\left(\sqrt{1-\sin x}\right)^2}{\left(\sqrt{1+\sin x}-\sqrt{1-\sin x}\right)^2}\right]$$

$$= \tan^{-1}\left[\frac{1+\sin x-1+\sin x}{\left(\sqrt{1+\sin x}\right)^2-2\sqrt{1-\sin x}.\sqrt{1+\sin x}+\left(\sqrt{1+\sin x}\right)^2}\right]$$

$$= \tan^{-1}\left[\frac{2\sin x}{1+\sin x+1-\sin x-2\sqrt{1-\sin^2 x}}\right]$$

$$= \tan^{-1}\left[\frac{2\sin x}{2-2\sqrt{1-\sin^2 x}}\right]$$

$$= \tan^{-1}\left[\frac{2\sin x/2\cos x/2}{2\cos^2 x/2}\right]$$

$$= \tan^{-1}\left(\frac{\sin x/2}{\cos x/2}\right) = \tan^{-1}(\tan x/2)$$

$= \frac{x}{2} =$ R.H.S.

Q.28. Show that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11} = \tan^{-1}\frac{3}{4}$

Solution : L.H.S. $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{2}{11}$

$$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{2}{11}}{1-\frac{1}{2}\times\frac{2}{11}}\right)$$

$$\left[\because \tan^{-1}x+\tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= \tan^{-1}\left(\frac{\frac{11+4}{22}}{\frac{22-2}{22}}\right)$$

$$= \tan^{-1}\left(\frac{15}{22}\times\frac{22}{20}\right)$$

$$= \tan^{-1}\frac{3}{4} = \text{R.H.S.}$$

Q.29. Prove that: $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

Solution : L.H.S. $= \tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}$

$$= \tan^{-1}\left(\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\times\frac{7}{24}}\right)$$

$$\left[\because \tan^{-1}x+\tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= \tan^{-1}\left(\frac{48+77}{264-14}\right) = \tan^{-1}\left(\frac{125}{250}\right)$$

$$= \tan^{-1}\frac{1}{2} = \text{R.H.S.}$$

Q.30. Solve this : $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

Solution : L.H.S.

$$= \tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}$$

$$= \tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{8}$$

$$= \tan^{-1}\frac{\frac{1}{3}+\frac{1}{5}}{1-\frac{1}{3}.\frac{1}{5}}+\tan^{-1}\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}.\frac{1}{8}}$$

$$= \tan^{-1}\frac{\frac{8}{15}}{\frac{14}{15}}+\tan^{-1}\frac{\frac{15}{56}}{\frac{55}{56}}$$

$$= \tan^{-1}\frac{8}{14}+\tan^{-1}\frac{3}{11} = \tan^{-1}\frac{4}{7}+\tan^{-1}\frac{3}{11}$$

$$= \tan^{-1}\frac{\frac{4}{7}+\frac{3}{11}}{1-\frac{4}{7}\times\frac{3}{11}} = \tan^{-1}\frac{\frac{44+21}{77}}{\frac{77-12}{77}}$$

$$= \tan^{-1}\frac{65}{65} = \tan^{-1}1 = \frac{\pi}{4} = \quad \text{R.H.S.}$$

Q.31. Prove that: $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

Solution : $\because \quad 2\tan^{-1}x + \tan^{-1}\frac{2x}{1-x^2}$

$$\therefore \quad 2\tan^{-1}\frac{1}{2} = \tan^{-1}\frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2} = \tan^{-1}\frac{4}{3}$$

$$\text{L.H.S.} = 2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}$$

$$= \tan^{-1}\frac{28+3}{21-4}$$

$$= \tan^{-1}\frac{31}{17} = \text{R.H.S.}$$

Q.32. Find the value of tan [2 cos (2sin^{-1} $\frac{1}{2}$)]

Solution : $\because \quad \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

$$\therefore \quad \tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\}$$

$$= \tan^{-1}\left\{2\cos\left(2\times\frac{\pi}{6}\right)\right\}$$

$$= \tan^{-1}\left(2\cos\frac{\pi}{3}\right)$$

$$= \tan^{-1}\left(2\times\frac{1}{2}\right)$$

$$= \tan^{-1}(1)$$

$$= \tan^{-1}\left(\tan\frac{\pi}{4}\right) = \frac{\pi}{4}$$ **Ans.**

Q.33. Find the value of tan$\frac{1}{2}$ [sin^{-1} $\frac{2x}{1+x^2}$ + cos^{-1} $\frac{1-y^2}{1+y^2}$], $|x|<1$, $y>0$ and $xy<1$

Solution : $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]$

$$= \tan\frac{1}{2}\left[2\tan^{-1}x+2\tan^{-1}y\right]$$

$$= \tan\left(\tan^{-1}x+\tan^{-1}y\right)$$

$$= \tan\left[\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= \frac{x+y}{1-xy}$$ **Ans.**

Q.34. If sin (sin^{-1} $\frac{1}{5}$ + cos^{-1} x) = 1 then, find the value of x.

Solution : Given that $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right) = 1$

$$\Rightarrow \sin^{-1}\frac{1}{5}+\cos^{-1}x = \sin^{-1}(1)$$

$$\left[\because \sin\theta = x \Rightarrow \theta = \sin^{-1}x\right]$$

$$\Rightarrow \sin^{-1}\frac{1}{5}+\cos^{-1}x = \sin^{-1}\left(\sin\frac{\pi}{2}\right)\theta$$

$$\Rightarrow \sin^{-1}\frac{1}{5}+\cos^{-1}x = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\frac{1}{5} = \frac{\pi}{2}-\cos^{-1}x$$

$$\Rightarrow \sin^{-1}\frac{1}{5} = \sin^{-1}x \qquad \left[\because \sin^{-1}x+\cos^{-1}x = \frac{\pi}{2}\right]$$

$$\Rightarrow x = \frac{1}{5}$$ **Ans.**

Q.35. If tan^{-1} $\frac{x-1}{x-2}$ + tan^{-1} $\frac{x+1}{x+2}$ = $\frac{\pi}{4}$ then, find the value of x.

Solution : $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$

$$\tan^{-1}\left[\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right] = \frac{\pi}{4}$$

$$\tan^{-1}\left[\frac{(x-1)(x+2)+(x+1)(x-2)}{(x^2-4)-(x^2-1)}\right] = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{2x^2-4}{-3}\right) = \tan^{-1}(1)$$

$$\frac{2x^2-4}{-3}=1$$

$$2x^2-4=3 \Rightarrow 2x^2=1$$

$$\Rightarrow x=\pm\sqrt{\frac{1}{2}}=\pm\frac{1}{\sqrt{2}}$$ **Ans.**

Q.36. Solve $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$

Solution : Given that $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left(\frac{2x+3x}{1-6x^2}\right)=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{5x}{1-6x^2}\right)=\frac{\pi}{4}$$

$$\Rightarrow \frac{5x}{1-6x^2}=\tan\frac{\pi}{4}$$

$$\Rightarrow \frac{5x}{1-6x^2}=1$$

$$\Rightarrow 5x=1-6x^2$$

$$\Rightarrow 6x^2+5x-1=0$$

$$\Rightarrow (x+1)(6x-1)=0$$

$\Rightarrow x+1=0$ या $6x-1=0$

$$\therefore x=-1,\frac{1}{6}$$

Keeping $x=-1$ makes the left side of the equation negative, which is impossible.

Hence $x=\frac{1}{6}$ is required solution. **Ans.**

Q.37. Find the value of $\sin^{-1}[\sin\frac{2\pi}{3}]$

Solution : $\sin^{-1}\left[\sin\frac{2\pi}{3}\right]$

$$\sin^{-1}\left[\sin\left(\pi-\frac{\pi}{3}\right)\right]$$

$$\sin^{-1}\left[\sin\frac{\pi}{3}\right]=\frac{\pi}{3}$$ **Ans.**

Q.38. Find the value of $\sin^{-1}[\sin\frac{3\pi}{5}]$

Solution : $\sin^{-1}\left[\sin\frac{3\pi}{5}\right]$

$$=\sin^{-1}\left\{\sin\left(\pi-\frac{2\pi}{5}\right)\right\}$$

$$=\sin^{-1}\left(\sin\frac{2\pi}{5}\right)=\frac{2\pi}{5}$$ **Ans.**

Q.39. Find the value of $\tan^{-1}[\tan\frac{3\pi}{4}]$

Solution : $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$$=\tan^{-1}\left\{\tan\left(\pi-\frac{\pi}{4}\right)\right\}$$

$$=\tan^{-1}\left\{-\tan\frac{\pi}{4}\right\}$$

$$=\tan^{-1}\left\{\tan\left(-\frac{\pi}{4}\right)\right\}$$

$$[\because \tan(-\theta)=-\tan\theta]$$

$$=-\frac{\pi}{4}$$ **Ans.**

Q.40. Find the value of $\tan^{-1}[\tan\frac{7\pi}{6}]$

Solution : $\tan^{-1}\left[\tan\frac{7\pi}{6}\right]\neq\frac{7\pi}{6}$

since, $\frac{7\pi}{6}\notin\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Now, $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)=\tan^{-1}\left\{\tan\left(\pi+\frac{\pi}{6}\right)\right\}$

$$=\tan^{-1}\left\{\tan\left(\frac{\pi}{6}\right)\right\}=\frac{\pi}{6}$$ **Ans.**

Q.41. Find the value of $\cos^{-1}[\cos\frac{13\pi}{6}]$

Solution : $\cos^{-1}\left[\cos\frac{13\pi}{6}\right]=\cos^{-1}\left[\cos\left(2\pi+\frac{\pi}{6}\right)\right]$

$$=\cos^{-1}\left(\cos\frac{\pi}{6}\right)\ [\because \cos(2\pi+\theta)=\cos\theta]=\frac{\pi}{6}$$ **Ans.**

Q.42. Find the value of $\tan[\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}]$

Solution : $\tan\left[\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right]$

$$= \tan\left[\tan^{-1}\frac{\frac{3}{5}}{\sqrt{1-\left(\frac{3}{5}\right)^2}}+\tan^{-1}\frac{2}{3}\right]$$

$$\left[\because \sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}} \text{ and } \cot^{-1}\frac{x}{y} = \tan^{-1}\frac{y}{x}\right]$$

$$= \tan\left[\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right] = \tan\left[\tan^{-1}\frac{\left(\frac{3}{4}+\frac{2}{3}\right)}{1-\frac{3}{4}\times\frac{2}{3}}\right]$$

$$= \tan\left[\tan^{-1}\frac{\frac{17}{12}}{\frac{1}{2}}\right] = \tan\left(\tan^{-1}\frac{17}{6}\right) = \frac{17}{6}$$ **Ans.**

Q.43. Show that $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$

Solution : $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$

$$\text{L.H.S.} = \sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17}$$

$$= \cos^{-1}\sqrt{1-\left(\frac{3}{5}\right)^2} - \cos^{-1}\sqrt{1-\left(\frac{8}{17}\right)^2}$$

$$= \cos^{-1}\frac{\sqrt{25-9}}{5} - \cos\frac{\sqrt{289-64}}{17}$$

$$= \cos^{-1}\frac{4}{5} - \cos^{-1}\frac{15}{17}$$

$$= \cos^{-1}\left\{\frac{4}{5}\times\frac{15}{17}+\sqrt{1-\left(\frac{4}{5}\right)^2}\sqrt{1-\left(\frac{15}{17}\right)^2}\right\}$$

$$= \cos^{-1}\frac{60}{85}+\frac{\sqrt{25-16}}{5}\times\frac{\sqrt{289-225}}{17}$$

$$= \cos^{-1}\left[\frac{60}{85}+\frac{3}{5}\times\frac{8}{17}\right] = \cos^{-1}\left(\frac{60+24}{85}\right)$$

$$= \cos^{-1}\frac{84}{85} = \text{R.H.S.}$$

Q.44. Show that $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

Solution : $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

$$\text{L.H.S.} = \sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \sin^{-1}$$

$$\left[\frac{8}{17}\sqrt{1-\left(\frac{3}{5}\right)^2}+\frac{3}{5}\sqrt{1-\left(\frac{8}{17}\right)^2}\right]$$

$$\left[\because \sin^{-1}x + \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)\right]$$

$$= \sin^{-1}\left(\frac{8}{17}\times\frac{4}{5}+\frac{3}{5}\times\frac{15}{17}\right) = \sin^{-1}\left(\frac{77}{85}\right)$$

$$= \tan^{-1}\left[\frac{\frac{77}{85}}{\sqrt{1-\left(\frac{77}{85}\right)^2}}\right]$$

$$= \tan^{-1}\left[\frac{77}{85}\times\frac{85}{36}\right] = \tan^{-1}\frac{77}{36} = \text{R.H.S.}$$

Q.45. Show that $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{35}$

Solution : :

$$\cos\left(\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}\right) = \cos\left(\cos^{-1}\frac{4}{5}\right)\cos\left(\cos^{-1}\frac{12}{13}\right)$$

$$-\sin\left(\cos^{-1}\frac{4}{5}\right)\sin\left(\cos^{-1}\frac{12}{13}\right)$$

$$= \frac{4}{5}\times\frac{12}{13} - \sqrt{1-\frac{16}{25}}\sqrt{1-\frac{144}{169}}$$

$$= \frac{48}{65} - \frac{3}{5}\times\frac{5}{13} = \frac{33}{65}$$

Hence, $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$ □