IMPORTANT CONCEPTS (TAKE A LOOK)

1. TRIGONOMETRY---A branch of mathematics in which we study the relationships between the sides and angles of a triangle, is called trigonometry.

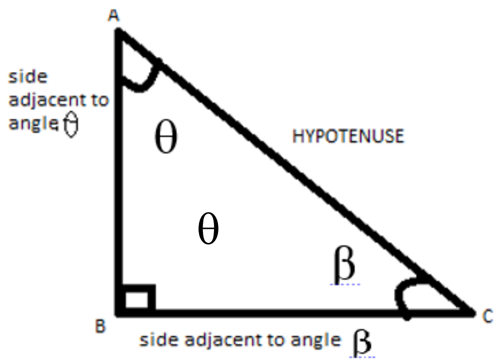
2. TRIGONOMETRIC RATIOS -----Trigonometric ratios of an acute angle in a right triangle express the relationship between the angle and length of its sides.

Trigonometric ratios of an acute angle in a right angled triangle ---

$$\sin\theta = \frac{\text{Side opposite to}\angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\cos\theta = \frac{\text{Side adjacent to}\angle\theta}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\tan\theta = \frac{\text{Side opposite to}\angle\theta}{\text{Side adjacent to}\angle\theta} = \frac{BC}{AB}$$



$$Cot\theta = \frac{1}{\tan\theta} = \frac{\text{side adjacent to}\angle\theta}{\text{Side opposite to}\angle\theta} = \frac{AB}{BC}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{Hypotenuse}}{\text{Side adjacent to}\angle\theta} = \frac{AC}{AB}$$

$$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{side Opposite to}\angle\theta} = \frac{AC}{BC}$$

For ∠β, sinβ= AB/ AC, cosβ = BC/AC, tanβ= AB/BC
Cosecβ = AC/AB, secβ= AC/BC, cotβ = BC/AB

3. Relationship between different trigonometric ratios

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}$$

$$\sin\theta = \frac{1}{\text{cosec}\,\theta}$$

4. Trigonometric Identity---- An equation involving trigonometric ratios of an angle is called a trigonometric identity if it is true for all values of the angle.

Important trigonometric identities:

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $1 + \tan^2\theta = \sec^2\theta$

(iii) $1 + \cot^2\theta = \mathrm{cosec}^2\theta$

5. Trigonometric Ratios of some specific angles.

θ	**0º**	**30º**	**45º**	**60º**	**90º**
sin θ	0	½	1/√2	√3/2	1
cosθ	1	√3/2	1/√2	1/2	0
tan θ	0	1/√3	1	√3	Not defined
cot θ	Not defined	√3	1	1/√3	0
sec θ	1	2/√3	√2	2	Not defined
cosec θ	Not defined	2	√2	2/√3	1

6. Trigonometric ratios of complementary angles.

(i) $\sin(90^\circ - \theta) = \cos\theta$

(ii) $\cos(90^\circ - \theta) = \sin\theta$

(iii) $\tan(90^\circ - \theta) = \cot\theta$

(iv) $\cot(90^\circ - \theta) = \tan\theta$

(v) $\sec(90^\circ - \theta) = \mathrm{cosec}\,\theta$

(vi) $\mathrm{cosec}(90^\circ - \theta) = \sec\theta$

Level – I

1. If θ and 3θ-30° are acute angles such that sinθ=cos (3θ-30°), then find the value of tanθ.

2. Find the value of $\frac{(\cos 30^\circ+\sin 60^\circ)}{(1+\cos 60^\circ+\sin 30^\circ)}$

3. Find the value of $(\sin\theta+\cos\theta)^2+(\cos\theta-\sin\theta)^2$

4. If $\tan\theta=\frac{3}{4}$ then find the value of $\cos^2\theta-\sin^2\theta$

5. If secθ+tanθ=p, then find the value of secθ-tanθ

6. Change $\sec^4\theta-\sec^2\theta$ in terms of tanθ.

7. Prove that $\frac{\sin^3\alpha+\cos^3\alpha}{\sin\alpha+\cos\alpha}+\sin\alpha\cos\alpha=1$ *(CBSE 2009)*

8. In a triangle ABC, it is given that < C= 90˚ and tanA=1/√3, find the value of (sinA cosB +cosA sinB) *(CBSE 2008)*

9. Find the value of $\mathit{cosec}^2 67^\circ-\tan^2 23^\circ$.

10. Ifcos x=cos60° cos30°+sin60° sin30°, then find the value of x

11. If $0^\circ\le x\le 90^\circ$ and $2\sin^2 x=1/2$, then find the value of x

12. Find the value of $\mathrm{cosec}^2 30^\circ-\sin^2 45^\circ-\sec^2 60^\circ$

13. Simplify (secθ+tanθ) (1-sinθ)

14. **Prove that cosA/ (1-sinA) +cosA/ (1+sinA) =2secA**

Level – II

1. If secα=5/4 then evaluate $\tan\alpha/(1+\tan^2\alpha)$.

2. If A+B =90°, then prove that $\sqrt{\frac{\tan A.\tan B+\tan A.\cot B}{\sin A.\sec B}-\frac{\sin^2 B}{\cos^2 A}}$ = tanA

3. If 7 $\sin^2 A+3\cos^2 A=4$, show that tanA =1/√3. (CBSE 2008)

4. Prove that $\sqrt{\frac{secA-1}{secA+1}}$. + $\sqrt{\frac{secA+1}{secA-1}}$ = 2cosecA

5. Prove that $(\sin\theta+\operatorname{cosec}\theta)^2 + (\cos\theta+\sec\theta)^2 = 7+\tan^2\theta+\cot^2\theta$. (CBSE 2008, 2009C)

6. Evalute $\frac{11\sin 70^\circ}{7\cos 20^\circ}$-- $\frac{4\cos 53^\circ \operatorname{cosec} 37^\circ}{7\tan 15^\circ \tan 35^\circ \tan 55^\circ \tan 75^\circ}$

7. Find the value of sin30˚ geometrically.

8. If tan (A-B) =√3, and sinA =1, then find A and B.

9. If θ is an acute angle and sinθ=cosθ, find the value of $3\tan^2\theta + 2\sin^2\theta - 1$.

10. If $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ and $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$, prove that $x^2/a^2 + y^2/b^2 = 2$.

11. Provethat $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$ = tanθ.

Level - III

1. Evaluate the following: - $\sin^2 25^\circ + \sin^2 65^\circ + \sqrt{3}(\tan 5^\circ \tan 15^\circ \tan 30^\circ \tan 75^\circ \tan 85^\circ)$

2. If $\frac{\cos\alpha}{\cos\beta} = m$, and $\frac{\cos\alpha}{\sin\beta} = n$, show that $(m^2+n^2)\cos^2\beta = n^2$. (CBSE 2012)

3. Prove that $\tan^2\theta + \cot^2\theta + 2 = \operatorname{cosec}^2\theta \sec^2\theta$

4. If cosθ + sinθ = √2 cos θ, then show that (cosθ-sinθ) = √2 sinθ.

(CBSE 1997, 2002, 2007)

5. Prove that $(\sin\theta+\sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2 = (1+\sec\theta \operatorname{cosec}\theta)^2$.

6. Prove that sinθ/ (1-cosθ) + tanθ/ (1+cosθ) = secθcosecθ + cotθ.

7. If x = asinθ and y = btanθ. Prove that $a^2/x^2 - b^2/y^2 = 1$.

8. Prove that $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$.

9. Prove that (secθ+tanθ – 1)/ (tanθ – secθ+1) = cosθ/ (1 – sinθ).

10. Prove that (1 +cotθ - cosec θ) (1+tanθ+secθ) = 2 (CBSE 2005, 07, 08)

11.Evaluate $\frac{\sin^2\theta+\sin^2(90°-\theta)}{3(\sec^2 61°-\cot^2 29°)}$ - $\frac{3\cot^2 30°\sin^2 54°\sec^2 36°}{2(\operatorname{cosec}^2 65°-\tan^2 25°)}$

12. If sinθ +cosθ =m and secθ+ cosecθ =n, then prove that n $(m^2 – 1)$ =2m.

Self-Evaluation

1. If a cosθ + b sinθ = c, then prove that asinθ – bcosθ = $\mp\sqrt{a^2+b^2-c^2}$.

2. If A,B,C are interior angles of **triangle ABC, show that** $\operatorname{cosec}^2\left(\frac{B+C}{2}\right) - \tan^2\frac{A}{2}$ = 1.

3. Ifsinθ + $\sin^2\theta$ + $\sin^3\theta$ = 1, prove that $\cos^6\theta$ – $4\cos^4\theta$ + $8\cos^2\theta$ = 4.

4. IftanA = ntanB, sinA = msinB, prove that $\cos^2 A = (m^2 - 1)/(n^2-1)$.

5. Evaluate: $\frac{\sec\theta\operatorname{cosec}(90°-\theta) - \tan\theta\cot(90°-\theta) + \sin^2 55° + \sin^2 35°}{\tan 10°\tan 20°\tan 60°\tan 70°\tan 80°}$

6. If secθ + tanθ=p, prove that sinθ = $(p^2-1)/(p^2+1)$.

7. Prove that $\frac{1}{\sec\theta-\tan\theta} - \frac{1}{\cos\theta} = \frac{1}{\cos\theta} - \frac{1}{\sec\theta+\tan\theta}$.

8. Prove that: $\frac{\cos\theta}{1-\tan\theta} + \frac{\sin^2\theta}{\sin\theta-\cos\theta}$ = sinθ + cosθ

9. Prove that $\frac{1+\cos A+\sin A}{1+\cos A-\sin A} = \frac{1+\sin A}{\cos A}$.

10. Prove that(1 + cosθ + sinθ) / (1+ cosθ – sinθ) = (1 + sinθ)/ cosθ