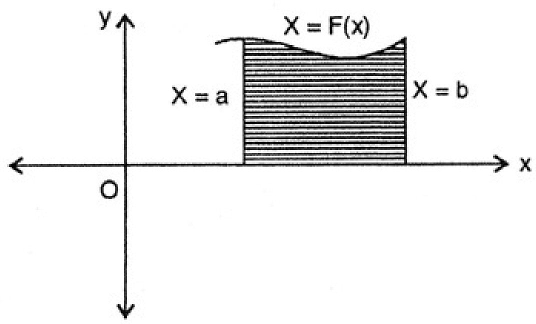
Application of Integrals

Teaching Points

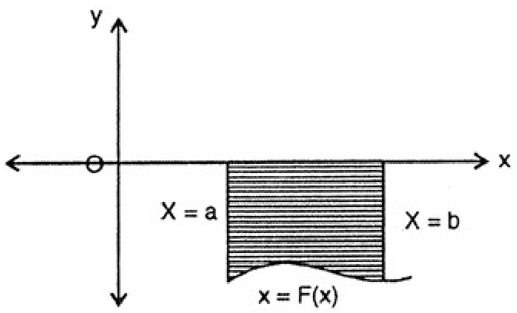
● The area bouned by the curve y = F(x) above the x-axis and between the lines x = a, x = b is given by

$$\int_a^b y\,dx = \int_a^b F(x)\,dx$$

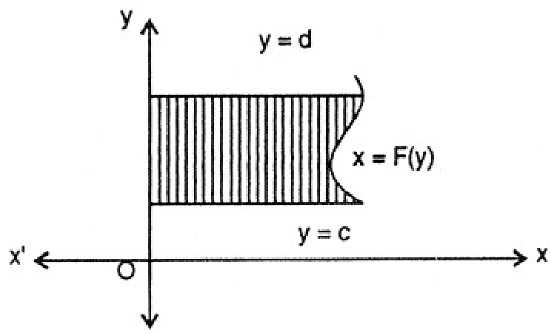


● If the curve between the lines x = a, x = b lies below the x-axis, then the required area is given

$$\int_a^b (-y)\,dx = \int_a^b y\,dx = \int_a^b F(x)\,dx$$



● The area bounded by curve y = F(x), x-axis and between lines
x = a, x = b to given by

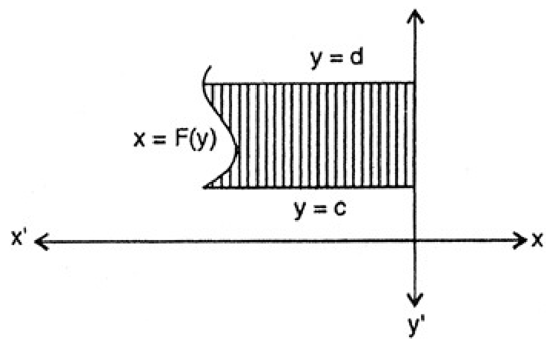


$$\int_a^b F(x)dx$$

● The area bounded by the curve x = F(y) above the y-axis and the lines y = c, y = d is given by

$$\int_c^d xdy = \int_c^d F(y)dy$$

● If the curve between the lines y = c, y = d lies below the y-axis (to the left of y-axis) then the area is given by

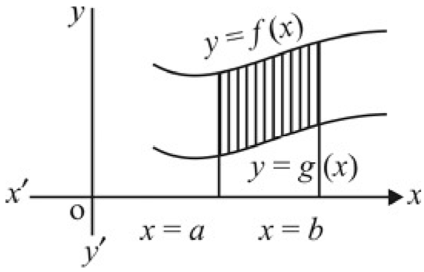


$$\int_c^d (-x)dy = -\int_c^d xdy = \int_c^d F(y)dy$$

● The area bounded by curve x = F(y), y-axis and between lines
y = c and y = d is gives by

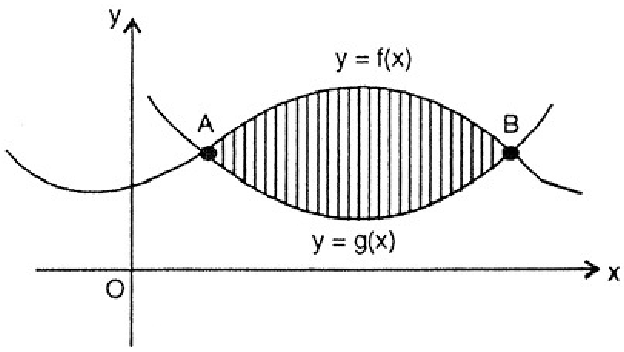
$$\int_c^d |F(y)|\,dy$$

● If 0 £ g(x) £ f(x), the area of region bounded between curves and ordinates x = a and x = b is given by



$$= \int_a^b [F(x) - g(x)]dx$$

● When we find the area bounded by the curves y = f(x) and y = g(x) and after drawing the graphs the shaded region is of such type.



We find the x-cordinate of their point of inter-sections. Let for point A and B the values of x are a and b.

Then Required Area = $\int_a^b [f(x) - g(x)]dx$

Note : If the power of ‘x’ is even in the given curve then the graph of the curve is symmetric about y-axis. If equation of curve contains only even power of ‘y’ then the graph is symmertic about x-axis. If curve contains even power in both ‘x’ and ‘y’ then graph is symmetric about both axis.

Question for Practice

Q1. Find the common area bounded by the circles $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$.

Q2. Using integration find area of region bounded by the triangle whsoe vertices are

(a) (–1, 0), (1, 3) and (3, 2)

(b) (–2, 2) (0, 5) and (3, 2)

Q3. Using integration find the area bounded by the lines.

(i) x + 2y = 2, y – x = 1 and 2x + y – 7 = 0

(ii) y = 4x + 5, y = 5 – x and 4y – x = 5.

Q4. Find the area of the region $\left\{(x, y) : |x-1| \le y \le \sqrt{5-x^2}\right\}$.

Q5. Find the area of the region bounded by

y2 = x and line x + y = 2

Q6. Find the area enclosed by the curve y = sin x between x = 0 and x = 3π/2 and x-axis.

Q7. Find the area bounded by semi circle y = $\sqrt{25-x^2}$ and x-axis.

Q8. Find area of region given by $[(x, y) : x^2 \le y \le |x|]$.

Q9. Find area of smaller region bounded by ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and straight line 2x + 3y = 6.

Q10. Find the area of region bounded by the curve x2 = 4y and line x = 4y – 2.

Q11. Using integration find the area of region in first quadrant enclosed by x-axis the line $x = \sqrt{3}\, y$ and the circle x2 + y2 = 4.

Q12. Draw a rough sketch of the region $[(x, y) : x^2 + y^2 \le 4 \le x + y]$ and find its area.

Q13. Find the smaller of two areas bounded by the curve y = |x| and x2 + y2 = 8.

Q14. Find the area lying above x-axis and included between the circle x2 + y2 = 8x and the parabola y2 = 4x.

Q15. Using integration find the area enclosed by the curve y = cos x, y = sin x and x-axis in the interval

$\left(0, \frac{\pi}{2}\right)$.

Q16. Sketch the graph y = |x – 5|. Evaluate $\int_0^6 |x-5|\, dx$.

Answers

1. $\left(8\frac{\pi}{3}-2\sqrt{3}\right)$ sq units

2. (a) 4 sq units, (b) 2 sq units

3. (a) 6 sq units (b) $\frac{15}{2}$ sq units

4. $\left[\frac{5}{2}\left(\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)+\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)-\frac{1}{2}\right]$ sq units

5. $\frac{9}{2}$ sq units

6. 3 sq units

7. $\frac{25}{2}$ sq units

8. $\frac{1}{3}$ sq units

9. $\frac{3}{2}(\pi-2)$ sq units

10. $\frac{9}{8}$ sq units

11. $\frac{\pi}{3}$ sq units

12. $(\pi - 2)$ sq units

13. 2π sq units

14. $\frac{8}{3}(8+3\pi)$ sq units

15. $(2-\sqrt{2})$ sq units

16. 5 sq units

Hints

1. Required area = $2\left\{\int_0^1 \sqrt{4-(x-2)^2}\,dx+\int_1^2 \sqrt{4-x^2}\,dx\right\}$

4. Required area = $\int_{-1}^{2} \sqrt{5-x^2}\,dx - \int_{-1}^{1} (-x+1)\,dx - \int_{1}^{2} (x-1)\,dx$

5. Required area = $\int_{-2}^{1} (2-y)\,dy - \int_{-2}^{1} y^2\,dy$

14. Required area = $\int_{0}^{4} 2\sqrt{x}\,dx + \int_{4}^{8} \sqrt{16-(x-4)^2}\,dx$