VECTOR ALGEBRA

CHAPTER – 10

VECTOR ALGEBRA

SCALARS

A quantity which have magnitude but no direction , is called scalars.

Example

Speed, Distance etc.

VECTORS

A quantity which have magnitude as well as direction , is called vector.

Example

Displacement, velocity etc.

MAGNITUDE OF A VECTOR

The length of the vector AB or a is called the magnitude of AB or a and it is represented by |AB| or |a|.

If $\vec{a} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then |a| = $\sqrt{x^2 + y^2 + z^2}$

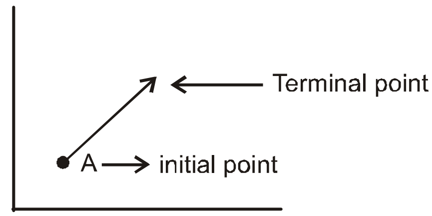
Note

Since, the length is never negative , so the notation |a| <0 has no meaning.

Example

Let $\vec{a}$=$\hat{\imath}$+2$\hat{\jmath}$ then |$\vec{a}$|=$\sqrt{5}$.

VECTORS AND THEIR REPRESENTATION



Vector quantities are specified by definite magnitude and definite direction. A vector is generally represented by a directed line segment, say $\overrightarrow{AB}$. A is called the initial point and B is called the terminal point.

TYPES OF VECTORS:

(i) Zero vector: A vector of zero magnitude i.e. which has the same initial and terminal point, is called a zero vector. It is denoted by O. The direction of zero vector is indeterminate.

(ii) Unit vector: A vector of unit magnitude in the direction of a vector $\vec{a}$ is called unit vector along $\vec{a}$ and is denoted by $\hat{a}$, symbolically $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$.

Example

Find the unit vector of $\vec{a} = 2\hat{\imath}+2\hat{\jmath}-5\hat{k}$.

Solution: $\hat{a} = \frac{2\hat{\imath}+2\hat{\jmath}-5\hat{k}}{\sqrt{33}}$

(iii) Equal vectors: Two vectors are said to be equal if they have the same magnitude, direction and represent the same physical quantity.

Note

If $\vec{a} = \vec{b}$, then $|\vec{a}| = |\vec{b}|$ but converse may not be true.

Example

Let $\vec{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + \hat{\jmath} + \hat{k}$ be two vectors . So , magnitude of $\vec{a}$ and $\vec{b}$ are $|\vec{a}| = \sqrt{3}$ and $|\vec{b}| = \sqrt{3}$. But $\vec{a} \neq \vec{b}$

(iv) Collinear vectors: Two vectors are said to be collinear if their directed line segments are parallel irrespective of their directions. Collinear vectors are also called parallel vectors. If they have the same direction they are named as like vectors otherwise unlike vectors.

Symbolically, two non-zero vectors $\vec{a}$ an $\vec{b}$ are collinear if and only if, $\vec{a} = \lambda\vec{b}$, where $\lambda \in R$

$$\vec{a} = \lambda\vec{b} \Leftrightarrow \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right) = \lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) \Leftrightarrow$$

$a_1 = \lambda b_1$, $a_2 = \lambda b_2$, $a_3 = \lambda b_3$

$$\Rightarrow \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = \lambda$$

Vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are collinear if $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

Example

Find if the given vectors are collinear vectors.
$\overrightarrow{PP}$ = i + j + k, $\overrightarrow{QQ}$ = - i - j - k

Solution: Two vectors are considered to be collinear vectors if one vector is a scalar multiple of the other vector.

Vector Q = - i - j - k = - (i + j + k) = - (Vector P)

⇒ Vector Q is a scalar multiple of vector P.

(v) Coplanar vectors: A given number of vectors are called coplanar if their line segments are all parallel to the same plane.

Note

"two vectors are always coplanar".

If the initial point of a vector is not specified , then it is called a free vector.

MULTIPLICATION OF A VECTOR BY A SCALAR

If $\vec{a}$ is a vector and m is a scalar, then m is a vector parallel to $\vec{a}$ whose magnitude is $|m|$ times that of $\vec{a}$. This multiplication is called scalar multiplication.

If $\vec{a}$ and $\vec{b}$ are vectors and m, n are scalars, then :

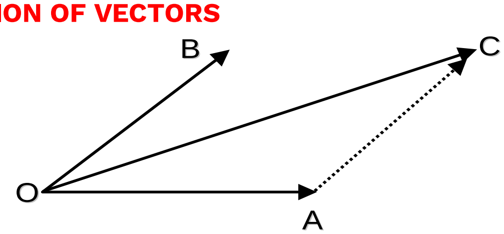
(i) $m\ (\vec{a}) = (\vec{a})\ m = m\vec{a}$

(ii) $m\ (n\vec{a}) = n(m\vec{a}) = (mn)\vec{a}$

(iii) $(m+n)\ \vec{a} = m\vec{a} + n\vec{a}$

(iv) $m\ (\vec{a}+\vec{b}) = m\vec{a} + m\vec{b}$

ADDITION OF VECTORS



(i) If two vectors $\vec{a}$ and $\vec{b}$ are represented by $\overrightarrow{OA}$ and $\overrightarrow{OB}$, then their sum $\vec{a}+\vec{b}$ is a vector represented by $\overrightarrow{OC}$, where OC is the diagonal of the parallelogram OACB.

(ii) $\vec{a}+\vec{b} = \vec{b}+\vec{a}$ (commutative)

(iii) $(\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})$ (associative)

(iv) $\vec{a}+\vec{0} = \vec{a} = \vec{0}+\vec{a}$

(v) $\vec{a}+(-\vec{a}) = \vec{0} = (-\vec{a})+\vec{a}$

(vi) $|\vec{a}+\vec{b}| \le |\vec{a}|+|\vec{b}|$

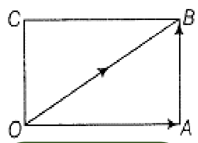
(vii) $|\vec{a}-\vec{b}| \ge ||\vec{a}|-|\vec{b}||$

Note

The vector sum of three sides of a triangle taken in order is $\vec{0}$.

PARALLELOGRAM LAW OF VECTOR ADDITION

If two vectors are represented along the two adjacent sides of a parallelogram, then their resultant is represented by the diagonal of the sides. If the sides OA and OC of parallelogram OABC represent $\overrightarrow{OA}$ and $\overrightarrow{OC}$ respectively, then we get $\overrightarrow{OA} + \overrightarrow{OC} = \overrightarrow{OB}$



Note

Both laws of vector addition are equivalent to each other.

POSITION VECTOR OF A POINT

Let O be a fixed origin, then the position vector of a point P is the vector $\overrightarrow{OP}$. If $\vec{a}$ and $\vec{b}$ are position vectors of two points A and B, then

$\overrightarrow{AB} = \vec{b} - \vec{a}$ = position vector (p.v.) of B – position vector (p.v.) of A.

To Find a Vector when its Position Vectors of End Points are Given:

Let a and b be the position vectors of end points A and B respectively of a line segment AB.

Then, $\overrightarrow{AB}$ = Position vector of $\vec{B}$ – Positron vector of $\vec{A}$ = $\overrightarrow{OB} - \overrightarrow{OA} = \vec{b} - \vec{a}$

Example

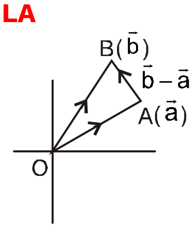
Find the vector joining the points P(2,3,0) and Q (-1,-2,-4) directed from P and Q.

Solution: Since the vector to be directed from P to Q, clearly P is the initial point and Q is the terminal points. So, the required vector joining P and Q is the vector $\overrightarrow{PQ}$, given by

$\overrightarrow{PQ} = (-1-2)\hat{\imath} + (-2-3)\hat{\jmath} + (-4-0)\hat{k}$

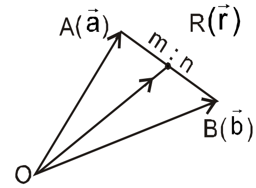
$\overrightarrow{PQ} = -3\hat{\imath} - 5\hat{\jmath} - 4\hat{k}$

DISTANCE FORMULA



Distance between the two points A($\vec{a}$) and B($\vec{b}$) is AB $= \left|\vec{a} - \vec{b}\right|$

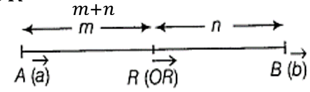
SECTION FORMULA



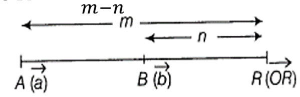
If $\vec{a}$ and $\vec{b}$ are the position vectors of two points A and B, then the p. v. of a point which divides AB in the ratio m: n is given by $\vec{r} = \dfrac{n\vec{a} + m\vec{b}}{m+n}$.

Cases:

for internally $\overrightarrow{OR} = \frac{m\vec{b}+n\vec{a}}{m+n}$



For externally $\overrightarrow{OR} = \frac{m\vec{b}-n\vec{a}}{m-n}$



Position vector of mid-point of the line segment joining end points A($\vec{a}$) and B($\vec{b}$) is given by $\overrightarrow{OR} = \frac{\vec{a}+\vec{b}}{2}$

Note

Position vector of mid point of AB $= \dfrac{\vec{a}+\vec{b}}{2}$ Hence EGHF is a parallelogram.

Example

Consider two points P and Q with position vectors $\overrightarrow{OP} = 3\vec{a} - 2\vec{b}$ and $\overrightarrow{OQ} = \vec{a} + \vec{b}$. Find the position vector formula of a point R which divides the line joining P and Q in the ratio 2:1, (i) internally, and (ii) externally.

Solution: Since point R divides PQ in the ratio 2:1. we have, m = 2 and n = 1

(i) R divides PQ internally

From equation (2), we have $\vec{r} = \frac{m\vec{b}+n\vec{a}}{m+n} \therefore \vec{r} = \frac{2(\vec{a}+\vec{b})+(3\vec{a}-2\vec{b})}{3} = \frac{5\vec{a}}{3}$

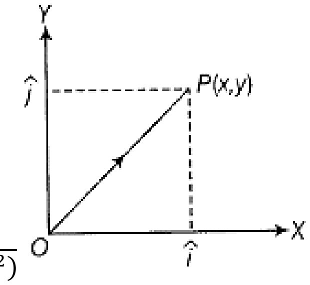
(ii) R divides PQ externally

From equation (3), we have $\vec{r} = \frac{m\vec{b}-n\vec{a}}{m-n} \therefore \vec{r} = \frac{2(\vec{a}+\vec{b})-(3\vec{a}-2\vec{b})}{} = 4\vec{b} - \vec{a}$

COMPONENTS OF VECTORS:

Let the position vector of P with reference to O is $\overrightarrow{OP}$ $= \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, this form of any vector is-called its component form. Here, x, y and z are called the scalar components of $\vec{r}$ and, $x\hat{i}$ and $z\hat{k}$ are called the vector components of $\vec{r}$ along the respective axes.

Two dimensions: If a point P in a plane has coordinates (x, y), then $\overrightarrow{OP} = x\hat{i} + y\hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors along OX and OY-axes, respectively.



Then, $|\overrightarrow{OP}| = \sqrt{(x^2 + y^2)}$

IMPORTANT RESULTS IN COMPONENT FORM

If $\vec{a}$ and $\vec{b}$ are any two vectors given in the component form as $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$.

Then , (a_1, a_2, a_3) and (b_1, b_2, b_3) are called direction ratios of $\vec{a}$ and $\vec{b}$ respectively .

(i) The sum of the vectors $\vec{a}$ and $\vec{b}$ is given by $\vec{a} + \vec{b} = (a_1 + b_1)\hat{\imath} + (a_2 + b_2)\hat{\jmath} + (a_3 + b_3)\hat{k}$

(ii) The difference of the vectors $\vec{a}$ and $\vec{b}$ is given by $\vec{a} - \vec{b} = (a_1 - b_1)\hat{\imath} + (a_2 - b_2)\hat{\jmath} + (a_3 - b_3)\hat{k}$

(iii) The vectors $\vec{a}$ and $\vec{b}$ are equal iff $a_1 = b_1, a_2 = b_2$ and $a_3 = b_3$

(iv) The multiplication of vector $\vec{a}$ by any scalar λ is given by $\lambda\vec{a} = (\lambda a_1)\hat{\imath} + (\lambda a_2)\hat{\jmath} + (\lambda a_3)\hat{k}$

(v) If $\frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = k$, Then vector $\vec{a}$ and $\vec{b}$ will be collinear.

(vi) If it is given that , l , m and n are direction cosines of a vector , then $l\hat{\imath} + m\hat{\jmath} + n\hat{k} = (\cos\alpha)\hat{\imath} + (\cos\beta)\hat{\jmath} + (\cos\Upsilon)\hat{k}$ is the unit vector in the direction of that vector , where α ,β and ϒ are the angle which the vector makes with x , y and z – axis , respectively.

ANGLE BETWEEN TWO VECTORS

It is the smaller angle formed when the initial points or the terminal points of the two vectors are brought together.

Example

Find the angle between the two vectors 2i + 3i + k, and 5i -2j + 3k.
Solution:
The two given vectors are:
$\vec{a}$ = 2i + 3i + k, and $\vec{b}$ = 5i -2j + 3k
$|a| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$
$|b| = \sqrt{5^2 + (-2)^2 + 3^2} = \sqrt{38}$
Using the dot product we have $\vec{a}.\vec{b}$ = 2.(5) + 3.(-2) + 1.(3) = 10 - 6 + 3 = 7
$\text{Cos } \theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|} = \frac{7}{\sqrt{14}.\sqrt{38}}$
$\theta = \text{Cos}^{-1}\left(\frac{7}{\sqrt{14}.\sqrt{38}}\right)$
$\theta = \text{Cos}^{-1}\, 0.304 = 72.3$

SCALAR PRODUCT OF TWO VECTORS

$\vec{a} \,.\, \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta, \ (0 \le \theta \le \pi)$

Note

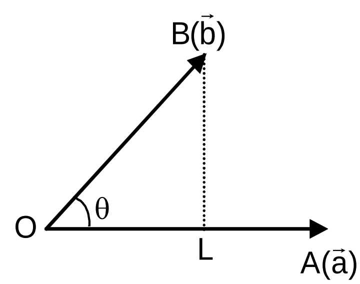
(i) If θ is acute, then $\vec{a}.\vec{b} > 0$ and if θ is obtuse, then $\vec{a}.\vec{b} < 0$.

(ii) $\vec{a} \,.\, \vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b} \ (\vec{a} \ne 0, \vec{b} \ne 0)$

(iii) Maximum value of $\vec{a}.\vec{b}$ is $|\vec{a}|\,|\vec{b}|$

(iv) Minimum value of $\vec{a}.\vec{b}$ is $-|\vec{a}|\,|\vec{b}|$

GEOMETRICAL INTERPRETATION OF SCALAR PRODUCT



Let $\vec{a}$ and $\vec{b}$ be vectors represented by $\overrightarrow{OA}$ and $\overrightarrow{OB}$ respectively. Let θ be the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Draw BL ⊥ OA and AM ⊥ OB

From ΔOBL and ΔOAM, we have OL = OB cos θ and OM = OA cosθ
Here OL are known as projections of $\vec{b}$ on $\vec{a}$ and $\vec{a}$ on $\vec{b}$ respectively.

Now, $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta = |\vec{a}|(|\vec{b}|\cos\theta) = |\vec{a}|$ (OB $\cos\theta$) = $|\vec{a}|$ (OL)

= (Magnitude of $\vec{b}$) (Projection of $\vec{a}$ on $\vec{b}$)

Thus geometrically interpreted, the scalar product of two vectors is the product of modulus of either vector and the projection of the other in its direction.

(i) Projection of $\vec{a}$ on $\vec{b}$ $= \dfrac{\vec{a}\,.\,\vec{b}}{|\vec{b}|}$

(ii) $\vec{a}\,.\,\vec{b} = \vec{b}\,.\,\vec{a}$ (commutative)

(iii) $\vec{a}\,.\,(\vec{b}+\vec{c}) = \vec{a}\,.\,\vec{b} + \vec{a}\,.\,\vec{c}$ (distributive)

(iv) $(m\vec{a}).\,\vec{b} = \vec{a}.(m\vec{b}) = m(\vec{a}\,.\,\vec{b})$, where m is a scalar.

(v) $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1;\ \hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

(vi) $\vec{a}\,.\,\vec{a} = |\vec{a}|^2 = \vec{a}^2$

(vii) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then $\vec{a}\,.\,\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$, $|\vec{b}| = \sqrt{b_1^2 + b_2^2 + b_3^2}$

(viii) $|\vec{a}\pm\vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 \pm 2|\vec{a}||\vec{b}|\cos\theta}$, where θ is the angle between the vectors

(ix) Any vector $\vec{a}$ can be written as $\vec{a} = (\vec{a}.\hat{i})\,\hat{i} + (\vec{a}.\hat{j})\,\hat{j} + (\vec{a}.\hat{k})\,\hat{k}$.

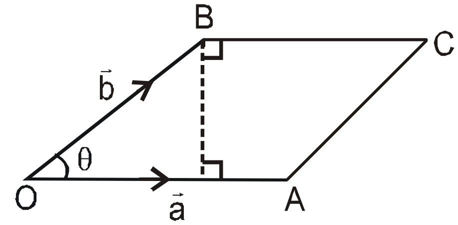
Example

Find the dot product of two vectors having magnitudes of 6 units and 7 units, and the angle between the vectors is 60°.
Solution:
The magnitudes of the two vectors are |a| = 6, |b| = 7, and the angle between the vectors is θ = 60°
The dot product of the two vectors is:
$\vec{a}.\vec{b}$ = |a|.|b|. Cos θ
= (6).(7).Cos60°
= (6).(7).(1/2)
= (3).(7)
= 21

VECTOR PRODUCT OF TWO VECTORS

(i) If $\vec{a}$, $\vec{b}$ are two vectors and θ is the angle between them, then $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$ where $\hat{n}$ is the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ such that $\vec{a}$, $\vec{b}$ and $\hat{n}$ forms a right handed screw system.

(ii) Geometrically $|\vec{a} \times \vec{b}|$ = area of the parallelogram whose two adjacent sides are represented by $\vec{a}$ and $\vec{b}$.



(iii) $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$ (not commutative)

(iv) $(m\vec{a})\times\vec{b} = \vec{a} \times (m\vec{b})$ $m(\vec{a}\times\vec{b})$, where m is a scalar.

(v) $\vec{a}\times(\vec{b}+\vec{c}) = (\vec{a}\times\vec{b}) + (\vec{a}\times\vec{c})$ (distributive)

(vi) $\vec{a}\times\vec{b} = \vec{0} \Leftrightarrow \vec{a}$ and $\vec{b}$ are parallel (collinear) $(\vec{a}\neq 0\ ,\ \vec{b}\neq 0)$ i.e. $\vec{a} = K\vec{b}$, where K is a scalar.

(vii) $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{0}$; $\hat{i}\times\hat{j} = \hat{k},\ \hat{j}\times\hat{k} = \hat{i},\ \hat{k}\times\hat{i} = \hat{j}$

(viii) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

(ix) A vector of magnitude 'r' and perpendicular to the plane of $\vec{a}$ and $\vec{b}$ is $\pm\dfrac{r(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}$

(x) If θ is the angle between $\vec{a}$ and $\vec{b}$, then $\sin\theta = \dfrac{|\vec{a} \times \vec{b}|}{|\vec{a}|\,|\vec{b}|}$

(xi) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are the position vectors of 3 points A, B and C respectively, then the vector area of ΔABC $= \frac{1}{2}(\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a})$. The points A, B and C are collinear if $\vec{a}\times\vec{b}\times\vec{b}\times\vec{c} + \vec{c}\times\vec{a} = \vec{0}$

(xii) Area of any quadrilateral whose diagonal vectors are $\vec{d}_1$ and $\vec{d}_2$ is given by $\frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$

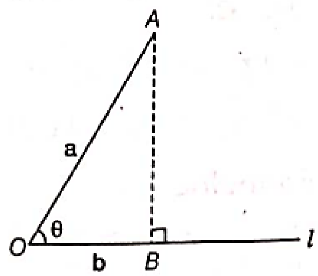
Example

Find $\vec{a}\times\vec{b}$ if $\vec{a} = 2\hat{\imath} + \hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath} + \hat{k}$

Solution: $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}$ = i(0-1)-j(2-1)+k(2-0) = $-\hat{\imath} - \hat{\jmath} + 2\hat{k}$

PROJECTION OF A VECTOR

Let $\vec{a}$ and $\vec{b}$ be two vectors represented by OA and OB respectively and let θ b the angle made by a with directed line l in the anti – clockwise direction.



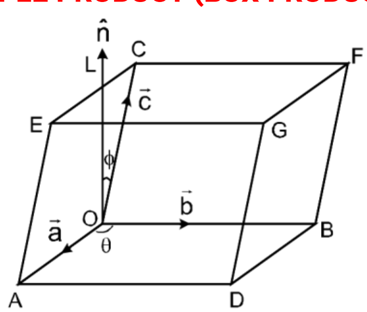
The ,the projection of OA on the line l is OB , which is given by $|OA|\cos\theta$ and the direction of $\vec{b}$, called projection vector, being the same (or opposite) to that of the line l , depending upon whether $\cos\theta$ is positive or negative.

Note

$\vec{b} = |OA|\cos\theta.\hat{b} = |\vec{a}|\cos\theta.\hat{b}$

$\therefore$ Projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$ and projection of $\vec{b}$ on $\vec{a}$ = $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

SCALAR TRIPLE PRODUCT (BOX PRODUCT)



(i) The scalar triple product of three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is defined as: $\vec{a} \times \vec{b} \,.\, \vec{c} = |\vec{a}|\ |\vec{b}|\ |\vec{c}|$, $\sin\theta \,.\, \cos\phi$ where θ is the angle between $\vec{a}$, $\vec{b}$ (i.e. $\vec{a} \wedge \vec{b} = \theta$) and ϕ is the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ $(\vec{a}\ \vec{b})\vec{c}\ \wedge = \phi)$. It is (i.e. $\vec{a} \times \vec{b} \,.\, \vec{c}$) also written as $[\vec{a}\ \vec{b}\ \vec{c}]$ and spelled as box product.

(ii) Scalar triple product geometrically represents the volume of
the parallelopiped whose three coterminous edges are
represented by $\vec{a}, \vec{b}$ and $\vec{c}$ i.e. $V = [\vec{a}\ \ \vec{b}\ \ \vec{c}]$

(iii) In a scalar triple product the position of dot and cross can be interchanged i.e.

$\vec{a}.(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b}).\vec{c} \quad\Rightarrow\quad [\,\vec{a}\vec{b}\,\vec{c}\,] = [\vec{b}\,\vec{c}\,\vec{a}] = [\vec{c}\,\vec{a}\vec{b}\ ]$

(iv) $\vec{a}.\,(\vec{b}\times\vec{c}) = -\vec{a}.(\vec{c}\times\vec{b})$ i.e. $[\vec{a}\vec{b}\vec{c}] = -[\ \vec{a}\,\vec{c}\,\vec{b}\ ]$

(v) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$; $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c}$ = $c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$, then $[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

In general, if $\vec{a} = a_1\vec{\ell} + a_2\vec{m} + a_3\vec{n}$; $\vec{b} = b_1\vec{\ell} + b_2\vec{m} + b_3\vec{n}$ and $\vec{c} = c_1\vec{\ell} + c_2\vec{m} + c_3\vec{n}$

then $\left[\vec{a}\vec{b}\vec{c}\right] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \left[\vec{\ell}\,\vec{m}\,\vec{n}\right]$, where

$\vec{\ell}$, $\vec{m}$ and $\vec{n}$ are non-coplanar vectors.

(vi) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar $\Leftrightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$

(vii) Scalar product of three vectors, two of which are equal or parallel is 0 $\Rightarrow [\vec{a}\,\vec{b}\vec{c}] = 0$

(viii) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar, then $[\vec{a}\vec{b}\vec{c}] > 0$ for right handed system and $[\vec{a}\,\vec{b}\,\vec{c}] < 0$ for left handed system.

(ix) (i) $[\ \hat{i}\ \ \hat{j}\ \ \hat{k}\ ] = 1$ (ii) $[K\vec{a}\ \vec{b}\vec{c}] = K[\vec{a}\,\vec{b}\,\vec{c}]$

(iii) $[(\vec{a}+\vec{b})\ \vec{c}\ \ \vec{d}] = [\vec{a}\vec{c}\,\vec{d}] + [\vec{b}\vec{c}\ \vec{d}]$

(x) $\left[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}\right] = 0$ and $\left[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}\right] = 2\left[\vec{a}\vec{b}\,\vec{c}\right]$

(xi) $\left[\ \vec{a}\ \vec{b}\ \vec{c}\ \right]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$

QUESTIONS

MCQ

Q1. Let $\vec{A} = \hat{\imath} + 3\hat{\jmath} + \alpha\hat{k}, \vec{B} = \hat{\imath} + 3\hat{\jmath} + \beta\hat{k}, \vec{C} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $|\vec{B} \times \vec{Q}| = 5\sqrt{3}$. If $\vec{A}$ is perpendicular to $\vec{B}$ then the maximum value of $|\vec{A}|$ is

(a) $\sqrt{35}$ (b) $\sqrt{50}$
(c) $\sqrt{40}$ (d) $\sqrt{80}$

Q2. The scalar product of $5i + j - 3k$ and $3i - 4j + 7k$ is:

(a) 15 (b) -15
(c) 10 (d) -10

Q3. Find a unit vector in the direction of the vector $(3\hat{\imath} - 2\hat{\jmath} + 6\hat{k})$

(a) $\frac{3}{7}\hat{\imath} - \frac{2}{7}\hat{\jmath} + \frac{6}{7}\hat{k}$ (b) $\frac{3}{7}\hat{\imath} + \frac{2}{7}\hat{\jmath} + \frac{6}{7}\hat{k}$
(c) $-\frac{3}{7}\hat{\imath} - \frac{2}{7}\hat{\jmath} - \frac{6}{7}\hat{k}$ (d) None

Q4. If $\vec{a} = (2\hat{\imath} - 4\hat{\jmath} + 5\hat{k})$ then find the value of λ so that $\lambda\vec{a}$ may be a unit vector.

(a) $\lambda = \pm\frac{1}{\sqrt{5}}$ (b) $\lambda = \pm\frac{1}{5\sqrt{5}}$
(c) $\lambda = \pm\frac{1}{3\sqrt{5}}$ (d) None

Q5. Find a vector of magnitude 9 units in the direction of t" he vector $(-2\hat{\imath} + \hat{\jmath} + 2\hat{k})$.

(a) $-6\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$ (b) $-6\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$
(c) $-6\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$ (d) None

Q6. Find the position vector of the point which divides the join of the points $(2\vec{a} - 3\vec{b})$ and $(3\vec{a} - 2\vec{b})$ externally in the ratio $2:3$.

(a) $-4\vec{b}$ (b) $-3\vec{b}$
(c) $-2\vec{b}$ (d) $-5\vec{b}$

Q7. If $\vec{a} = (\hat{\imath} - \hat{\jmath} + 7\hat{k})$ and $\vec{b} = (5\hat{\imath} - \hat{\jmath} + \lambda\hat{k})$ then find the value of λ so that $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are orthogonal vectors.

(a) $\lambda = \pm 3$ (b) $\lambda = \pm 2$
(c) $\lambda = \pm 5$ (d) $\lambda = \pm 7$

Q8. Write the projection of the vector $(\hat{\imath} + \hat{\jmath})$ on the vector $(\hat{\imath} - \hat{\jmath})$.

(a) 0 (b) $\hat{\imath} + \hat{\jmath}$
(c) $\hat{\imath} - \hat{\jmath}$ (d) 1

Q9. If $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$, find $|\vec{a} - \vec{b}|$.

(a) 5 (b) $\sqrt{5}$
(c) 3 (d) $\sqrt{3}$

Q10. Find the unit vectors perpendicular to the plane of the vecto rs $\vec{a} = 2\hat{\imath} - 6\hat{\jmath} - 3\hat{k}$ and $\vec{b} = 4\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$.

(a) $\pm\frac{1}{7}(3\hat{\imath} - 2\hat{\jmath} - 6\hat{k})$ (b) $\pm\frac{1}{7}(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$
(c) $\pm\frac{1}{7}(-3\hat{\imath} - 2\hat{\jmath} - 6\hat{k})$ (d) None

Q11. If $|\vec{a}| = 2, |\vec{b}| = 7$ and $(\vec{a} \times \vec{b}) = (3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$, find the angle between $\vec{a}$ and $\vec{b}$

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

Q12. Find the values of x for which $x(\hat{\imath} + \hat{\jmath} + \hat{k})$ is a unit vector.

(a) $x = \pm\frac{1}{\sqrt{5}}$ (b) $x = \pm\frac{1}{\sqrt{3}}$
(c) $x = \pm\frac{1}{\sqrt{7}}$ (d) $x = \pm\frac{1}{\sqrt{8}}$

Q13. Find the vector equation of the plane through the point $3\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and parallel to the lines $\vec{r} = -\hat{\jmath} + 3\hat{k} + \lambda(2\hat{\imath} - 5\hat{\jmath} - \hat{k})$ and $\vec{r} = \hat{\imath} - 3\hat{\jmath} + \hat{k} + \mu(-5\hat{\imath} + 4\hat{\jmath})$

(a) $\vec{r} \cdot (4\hat{\imath} + 5\hat{\jmath} - 17\hat{k}) + 27 = 0$
(b) $\vec{r} \cdot (4\hat{\imath} + 5\hat{\jmath} - 17\hat{k}) = 0$
(c) $\vec{r} \cdot (4\hat{\imath} + 5\hat{\jmath} - 17\hat{k}) + 17 = 0$
(d) None

Q14. Compute the magnitude of $\vec{c} = \frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} - \frac{1}{\sqrt{3}}\hat{k}$

(a) 0 (b) 3
(c) 2 (d) 1

Q15. Find the direction cosine of $\hat{\imath} + \hat{\jmath} + \hat{k}$

(a) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ (b) $\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$
(c) $\left(\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$ (d) $\left(\frac{-2}{\sqrt{3}}, \frac{-2}{\sqrt{3}}, \frac{-2}{\sqrt{3}}\right)$

Q16. For given vector $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{b} = -\hat{\imath} + \hat{\jmath} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.

(a) $\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{k}$ (b) $\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{k}$
(c) $\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{\jmath}$ (d) None of these

Q17. $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$ =?

(a) $2(\vec{a} \times \vec{b})$ (b) $(\vec{a} \times \vec{b})$
(c) $-2(\vec{a} \times \vec{b})$ (d) $-(\vec{a} \times \vec{b})$

Q18. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$

(a) $\frac{3}{\sqrt{22}}\hat{\imath} + \frac{3}{\sqrt{22}}\hat{\jmath} + \frac{2}{\sqrt{22}}\hat{k}$
(b) $\frac{3}{\sqrt{22}}\hat{\imath} + \frac{3}{\sqrt{22}}\hat{\jmath} - \frac{2}{\sqrt{22}}\hat{k}$
(c) $\frac{3}{\sqrt{22}}\hat{\imath} - \frac{3}{\sqrt{22}}\hat{\jmath} + \frac{2}{\sqrt{22}}\hat{k}$
(d) None

Q19. Find the value of $\boldsymbol{\lambda}$ when the projection of $\vec{a} = (\lambda\hat{\imath} + \hat{\jmath} + 4\hat{k})$ on $\vec{b} = (2\hat{\imath} + 6\hat{\jmath} + 3\hat{k})$ is 4 units

(a) 3 (b) 2
(c) 5 (d) 7

Q20. Write the projection of the vector $(\hat{\imath}+\hat{\jmath}+\hat{k})$ along the vector $\hat{\jmath}$.

(a) 1 (b) $\hat{\imath}$

(c) $\hat{\jmath}$ (d) $\hat{k}$

Q21. Find $\vec{a}\cdot(\vec{b}\times\vec{c})$ *where* $\vec{a}=(2\hat{\imath}+\hat{\jmath}+3\hat{k}), \vec{b}=(-\hat{\imath}+2\hat{\jmath}+\hat{k})$ *and* $\vec{c}=(3\hat{\imath}+\hat{\jmath}+2\hat{k})$

(a) -5 (b) -7

(c) -9 (d) -10

Q22. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively, when $|\vec{a}\times\vec{b}|=\sqrt{3}$.

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{2\pi}{3}$

Q23. Write the value of $\hat{\imath}\cdot(\hat{\jmath}\times\hat{k})+\hat{\jmath}\cdot(\hat{\imath}\times\hat{k})+\hat{k}\cdot(\hat{\imath}\times\hat{\jmath})$

(a) -1 (b) 0

(c) 1 (d) None

Q24. If $\boldsymbol{\theta}$ is the angle between $\vec{a}$ and $\vec{b}$, and $|\vec{a}\cdot\vec{b}|=|\vec{a}\times\vec{b}|$ then what is the value of $\boldsymbol{\theta}$?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

Q25. Find the direction cosines of the vector $\vec{a}=(\hat{\imath}+2\hat{\jmath}+3\hat{k})$

(a) $\frac{1}{\sqrt{14}},\frac{-2}{\sqrt{14}},\frac{3}{\sqrt{14}}$ (b) $\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{-3}{\sqrt{14}}$

(c) $\frac{-1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}$ (d) $\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}$

Q26. If $\vec{a}=(2\hat{\imath}+4\hat{\jmath}-\hat{k})$ and $\vec{b}=(3\hat{\imath}-2\hat{\jmath}+\lambda\hat{k})$ be such that $\vec{a}\perp\vec{b}$ then $\boldsymbol{\lambda}$ =?

(a) 2 (b) -2

(c) 3 (d) -3

Q27. For any two vectors $\vec{a}$ and $\vec{b}$, $(\vec{a}\times\vec{b})^2+(\vec{a}\cdot\vec{b})^2$ =?

(a) $|\vec{b}|^2$ (b) $|\vec{a}|^2$

(c) $|\vec{a}|^2|\vec{b}|^2$ (d) None

Q28. The distance of point $(2,1,-3)$ parallel to the vector $(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}-6\hat{\mathbf{k}})$ from the plane $2x+y+z+8=0$ is :

(a) 50 (b) 70

(c) 90 (d) 100

Q29. Given, vectors $\vec{a}=2\hat{\imath}+3\hat{\jmath}-\hat{k}$ and $\vec{b}=\hat{\jmath}+\hat{k}$. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a}$ and $\vec{b}$.If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}.\vec{b}=12$, then $\vec{u}$ is equal to :

(a) $2\hat{\imath}+8\hat{\jmath}+16\hat{k}$ (b) $4\hat{\imath}+4\hat{\jmath}+16\hat{k}$

(c) $-4\hat{\imath}+8\hat{\jmath}+8\hat{k}$ (d) $-4\hat{\imath}+8\hat{\jmath}+16\hat{k}$

Q30. the points A, B and C having position vectors $(3\hat{\imath}-4\hat{\jmath}-4\hat{k})$, $(2\hat{\imath}-\hat{\jmath}+\hat{k})$ and $(\hat{\imath}-3\hat{\jmath}-5\hat{k})$ are:

(a) vertices of a right-angled triangle

(b) are collinear

(c) No relation

(d) None

SUBJECTIVE QUESTIONS

Q1. Find unit vector of $\hat{i}-2\hat{j}+3\hat{k}$

Q2. Find values of x & y for which the vectors $\vec{a}=(x+2)\,\hat{i}-(x-y)\,\hat{j}+\hat{k}$, $\vec{b}=(x-1)\,\hat{i}+(2x+y)\,\hat{j}+2\hat{k}$ are parallel

Q3. If $\vec{a}=2\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}+\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram.

Q4. The midpoint of two opposite sides of quadrilateral and the midpoint of the diagonals are vertices of a parallelogram. Prove using vectors.

Q5. Show by using distance formula that the points (4, 5, –5), (0, –11, 3) and (2, –3,–1) are collinear.

SUBJECTIVE QUESTIONS

Q1. The vertices of a triangle are A(5, 4, 6), B(1, –1, 3) and C(4, 3, 2). The internal bisector of ∠BAC meets BC in D. Find AD.________

Q2. Find the value of p for which the vectors $\vec{a}=3\hat{i}+2\hat{j}+9\hat{k}$ and $\vec{b}=\hat{i}+p\hat{j}+3\hat{k}$ are parallel _____.

Q3. Find the projection of the line joining (1, 2, 3) and (–1, 4, 2) on the line having direction ratios 2, 3, –6.________.

Q4. Find the volume of a parallelopiped whose sides are given by $-6\hat{\imath}+7\hat{\jmath}-3\hat{k}$, $-5\hat{i}+7\hat{j}-3\hat{k}$ and $7\hat{i}-5\hat{j}-3\hat{k}$ ________.

Q5. If the line through the points (4, 1, 2) and (5, λ, 0) is parallel to the line through the points (2, 1, 1) and (3, 3, –1), find λ.__________.

TRUE AND FALSE

Q1. Distance between any two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given as $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$

Q2. If $\vec{a}=\vec{b}$, ⇔ $|\vec{a}|=|\vec{b}|$

Q3. If $\frac{b_1}{a_1}=\frac{b_2}{a_2}=\frac{b_3}{a_3}=k$, Then vector $\vec{a}$ and $\vec{b}$ will be non - collinear

Q4. If θ is acute, then $\vec{a}.\vec{b}>0$ and if θ is obtuse, then $\vec{a}.\vec{b}<0$.

Q5. Projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

ASSERTION AND REASONING

Directions : (Q1 -5) In the following questions , A statement of Assertion (A) is followed by a statement of Reason (R).
(a) Both A and R are true but R is the correct explanation of A
(b) Both A and R are true but R is Not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Q1. **Assertion (A) :** The position of a particle in a rectangular coordinate system is (3 , 2, 5) .Then its position vector be $2\hat{\imath} + 5\hat{\jmath} + 3\hat{k}$
Reason(R) : The displacement vector of the particle that moves from point P(2 , 3, 5) to point Q(3 ,4, 5) is $\hat{\imath} + \hat{\jmath}$

Q2. **Assertion (A) :** The direction of cosine of vector $\vec{a} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ are $\frac{2}{\sqrt{45}}, \frac{4}{\sqrt{45}}, \frac{-5}{\sqrt{45}}$
Reason (R) :A vector having zero magnitude and arbitrary direction is called ' zero vector' or 'null vector'.

Q3. **Assertion (A) :** Let $\vec{a} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{b} = -\hat{\imath} + \hat{\jmath} - \hat{k}$ be two vectors then $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$
Reason(R) : Let $\vec{a}$ and $\vec{b}$ be two vectors then $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$

Q4. **Assertion (A) : The unit vector in the direction of the sum of the vectors** $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ **and** $\vec{b} = -\hat{\imath} + \hat{\jmath} + 3\hat{k}$ **is** $\frac{1}{\sqrt{26}}\hat{\imath} + \frac{3}{\sqrt{26}}\hat{k}$
Reason(R) : A vector of unit magnitude in the direction of a vector $\vec{a}$ is called unit vector along $\vec{a}$ and is denoted by $\hat{a}$, symbolically $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$.

Q5. **Assertion (A) :** A vector in the direction of a vector $5\hat{\imath} - \hat{\jmath} + 2\hat{k}$ which has a magnitude of 8 units is $\hat{a} = \frac{5\hat{\imath} - \hat{\jmath} + 2\hat{k}}{\sqrt{30}}$
Reason(R) : If $\vec{a}$ and $\vec{b}$ are position vectors of two points A and B, then $\overrightarrow{AB} = \vec{b} - \vec{a}$ = position vector **(p. v.)** of B – position vector **(p. v.)** of A.

HOMEWORK

MCQ

Q1. The points A(1, –1, – 5), B(3, 1, 3) and C(9, 1, –3) are the vertices of-
(a) an equilateral triangle
(b) an isosceles triangle
(c) a right angled triangle
(d) none of these

Q2. The distance of a point P(x, y, z) from its image in xy plane is-
(a) 2|y|
(b) 2|z|
(c) 2|x|
(d) $2\sqrt{x^2 + y^2 + z^2}$

Q3. Find the ratio in which the segment joining the points (2, 4, 5), (3, 5, –4) is divided by the yz-plane.
(a) 3 : 1
(b) 2 : 3
(c) 1 : 3
(d) 1 : 2

Q4. If points A (3, 2, –4); B(5,4, –6) and C(9, 8,–10) are collinear then B divides AC in the ratio-
(a) 2 : 1 (b) 1 : 2
(c) 2 : 3 (d) 3 : 2

Q5. If the ZX- plane divides the line segment joining (1, -1, 5) and (2, 3, 4) in the ratio p:1 , the p+1 is equal to
(a) $\frac{1}{3}$ (b) 3
(c) $\frac{4}{3}$ (d) $\frac{3}{4}$

Q6. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitude $\sqrt{3}$ and 2 respectively and $\vec{a}.\vec{b} = \sqrt{6}$
(a) 45° (b) 90°
(c) 50° (d) 30°

Q7. Let $\vec{a} = \hat{\imath} + 3\hat{\jmath} + 7\hat{k}$ and $\vec{b} = 7\hat{\imath} - \hat{\jmath} + 8\hat{k}$, find the projection of $\vec{a}$ on $\vec{b}$
(a) $\frac{60}{\sqrt{114}}$ (b) $\frac{10}{\sqrt{114}}$
(c) $\frac{50}{\sqrt{114}}$ (d) $\frac{30}{\sqrt{114}}$

Q8. Find λ when the scalar projection of $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ is 4 units.
(a) 6 (b) 5
(c) 7 (d) 8

Q9. If $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$, then $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$
(a) Perpendicular to each other
(b)Parallel
(c)Non- collinear
(d)Collinear

Q10. Distance of the point (x, y, z) from y-axis is-

(a) y

(b) $\sqrt{x^2+y^2}$

(c) $\sqrt{y^2+z^2}$

(d) $\sqrt{z^2+x^2}$

SUBJECTIVE QUESTIONS

Q1. ABCDE is a pentagon. Prove that the resultant of the forces $\overrightarrow{AB}, \overrightarrow{AE}, \overrightarrow{BC}, \overrightarrow{DC}, \overrightarrow{ED}$ and $\overrightarrow{AC}$ is 3. $\overrightarrow{AC}$

Q2. Show that the points A(2, 3, 4), B(−1, 2, −3) and C(−4, 1, −10) are collinear. Also find the ratio in which C divides AB.

Q3. If the points P, Q, R, S are (4, 7, 8), (− 1, − 2, 1), (2, 3, 4) and (1,2,5) respectively, show that PQ and RS intersect. Also find the point of intersection.

Q4. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, $|\vec{a}|=3$, $|\vec{b}|=5$ and $|\vec{c}|=6$, find the angle between $\vec{a}$ and $\vec{b}$.

Q5. Find the values of x for which the angle between the vectors $\vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k}$ and $\vec{b} = 7\hat{i} - 2\hat{j} + x\hat{k}$ is obtuse.

NUMERICAL TYPE QUESTIONS

Q1. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $= 2\vec{a} - \hat{j} + 3\hat{k}$, then Component of $\vec{b}$ along $\vec{a}$ is $\lambda(\hat{i} + \hat{j} + \hat{k})$, λ = _____

Q2. A vector of magnitude 9, which is perpendicular to both the vectors $\hat{i}-7\hat{j}+7\hat{k}$ and $3\hat{i}-2\hat{j}+2\hat{k}$ is = ?$\frac{K}{\sqrt{2}}$ $(\hat{j}+\hat{k})$ then K = ________.

Q3. For any three vectors $\vec{a}, \vec{b}, \vec{c}$, then $\vec{a}\times(\vec{b}+\vec{c}) + \vec{b}\times(\vec{c}+\vec{a}) + \vec{c}\times(\vec{a}+\vec{b})$ = _____.

Q4. For any vector $\vec{a}$, then $|\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2 = P|\vec{a}|^2$ then P = ____.

Q5. The lines will be perpendicular if $a_1a_2 + b_1b_2 + c_1c_2$ = ______.

TRUE AND FALSE

Q1. If $\vec{a}$, $\vec{b}$ are two vectors and θ is the angle between them, then $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\cos\theta.\hat{n}$ where $\hat{n}$ is the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ such that $\vec{a}$, $\vec{b}$ and $\hat{n}$ forms a right handed screw system.

Q2. A given number of vectors are called coplanar if their line segments are all parallel to the same plane

Q3. $[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}] = 2\vec{a}.\vec{b}.\vec{c}$

Q4. The vectors $\vec{a}=-4\hat{i}+8\hat{j}-4\hat{k}$, $\vec{b}=4\hat{i}-2\hat{j}-2\hat{k}$ and $\vec{c}=-2\hat{i}-2\hat{j}+4\hat{k}$ are coplanar.

Q5. $\vec{a}\times\{\vec{b}\times(\vec{c}\times\vec{d})\} = (\vec{b}.\vec{d})(\vec{a}\times\vec{c}) - (\vec{b}.\vec{c})(\vec{a}\times\vec{d})$

ASSERTION AND REASONING

Directions : (Q1 -5) In the following questions , A statement of Assertion (a) is followed by a statement of Reason (R).

(a) Both A and R are true but R is the correct explanation of A

(b) Both A and R are true but R is Not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

Q1. **Assertion (a) :** If a , b and c are unit vectors , then $|a-b|^2+|b-c|^2+|c-a|^2$ does not exceed 9.
Reason(R) : $|a-b|^2 + |b-c|^2 + |c-a|^2 = 2(a^2+b^2+c^2) - 2(a\times b + b\times c + c\times a)$
$= 2\times3 - 2(a\times b + b\times c + c\times a)$
$= 6 - \{(a+b+c)^2 - a^2 - b^2 - c^2\}$
$= 9 - |a+b+c|^2 \leq 10$

Q2. **Assertion (a) :** Let a , b , c be distinct non – negative numbers .If the vectors $a\hat{\imath}+a\hat{\jmath}+c\hat{k}$ and $c\hat{\imath}+c\hat{\jmath}+b\hat{k}$ lie in a plane , the c is arithmetic mean of a and b
Reason (R) : [(b + c) × (a + b + c)] is equal to 0

Q3. **Assertion (a) :** Let a, b and c be vectors with magnitudes 3, 4 and 5 respectively and a + b + c = 0, then the values of a . b + b . c + c . a is -25
Reason (R): The magnitudes of mutually perpendicular forces a, b and c are 2, 10 and 11, respectively. Then the magnitude of its resultant is 15

Q4. **Assertion (a) :** The sum of $\vec{a}=2\hat{\imath}+4\hat{\jmath}+7\hat{k}$ and $\vec{b}=-2\hat{\imath}+3\hat{\jmath}+2\hat{k}$ is $7\hat{\jmath}+9\hat{k}$
Reason (R) : The sum of the vectors $\vec{a}$ and $\vec{b}$ is given by $\vec{a}+\vec{b}=(a_1+b_1)\hat{\imath}+(a_2+b_2)\hat{\jmath}+(a_3+b_3)\hat{k}$

Q5. **Assertion (a) :** The vectors $2\vec{a}-\vec{b}+3\vec{c}$, $\vec{a}+\vec{b}-2\vec{c}$ and $\vec{a}+\vec{b}-3\vec{c}$ are non-coplanar vectors.
Reason (R) : Four points $2\vec{a}+3\vec{b}-\vec{c}$, $\vec{a}-2\vec{b}+3\vec{c}$, $3\vec{a}+4\vec{b}-2\vec{c}$ and $\vec{a}-6\vec{b}+6\vec{c}$ are coplanar.

SOLUTIONS

MCQ

S1. **(a)**

$\vec{A}\cdot\vec{B}=0$
$1+9+\alpha\beta=0$
$\alpha\beta=-10$

$\vec{B}\times\vec{C}=\begin{vmatrix} i & j & k \\ 1 & 3 & \beta \\ 1 & -2 & 3 \end{vmatrix}$

$=i(9+2\beta)-j(3-\beta)+k(-2-3)$
$|\vec{B}\times\vec{C}|^2=75$
$\Rightarrow (9+2\beta)^2+(3-\beta)^2+25=75$
$\Rightarrow \beta^2+6\beta+8=0$
$\Rightarrow \beta=-2,-4$
for $\beta=-2, \alpha=5$ and for $\beta=-4, \alpha=\frac{5}{2}$
For maximum value of $|\vec{A}|^2, at\ \alpha=5$
Therefore, maximum value of $|\vec{A}|$ is $=\sqrt{1+9+25}=\sqrt{35}$

S2. **(d)**

Let $A=5i+j-3k$
$B=3i-4j+7k$
$A\cdot B=(5i+j-3k)\cdot(3i-4j+7k)$
$=5\cdot 3+1\cdot(-4)+(-3)\cdot 7$
$=15-4-21$
$=-10$

S3. **(a)** $\vec{a}=3\hat{\imath}-2\hat{\jmath}+6\hat{k}$

$\therefore \hat{a}=\frac{3\hat{\imath}-2\hat{\jmath}+6\hat{k}}{\sqrt{3^2+2^2+6^2}}$
$=\frac{3}{7}\hat{\imath}-\frac{2}{7}\hat{\jmath}+\frac{6}{7}\hat{k}$

S4. **(c)**

$\vec{a}=2\hat{\imath}-4\hat{\jmath}+5\hat{k}$
$\therefore \lambda\vec{a}=2\lambda\hat{\imath}-4\lambda\hat{\jmath}+5\lambda\hat{k}$
For a unit vector, its magnitude equals to 1.
$\therefore |\lambda\vec{a}|=\sqrt{(2\lambda)^2+(4\lambda)^2+(5\lambda)^2}=1$
$\Rightarrow 45\lambda^2=1$
$\Rightarrow \lambda^2=\frac{1}{45}=\frac{1}{(3\sqrt{5})^2}$
$\Rightarrow \lambda=\pm\frac{1}{3\sqrt{5}}$

S5. **(a)**

Let λ be an arbitrary constant and the required vector is $-2\lambda\hat{\imath}+\lambda\hat{\jmath}+2\lambda\hat{k}$
For any vector $\vec{a}=a_x\hat{\imath}+a_y\hat{\jmath}+a_z\hat{k}$ the magnitude $|\vec{a}|=\sqrt{a_x^2+a_y^2+a_z^2}$
$\therefore \sqrt{(2\lambda)^2+(\lambda)^2+(2\lambda)^2}=9$
$\Rightarrow 3\lambda=9$
$\Rightarrow \lambda=3$
Therefore, Required vector $=-6\hat{\imath}+3\hat{\jmath}+6\hat{k}$

S6. **(d)**

The point dividing a line joining points a and b in a ratio m:n internally or externally is given by $\frac{mb\pm na}{m+b}$ respectively.
The position vector of the point dividing the line externally
$=\frac{2\times(3\vec{a}-2\vec{b})-3\times(2\vec{a}-3\vec{b})}{2-3}$
$=-5\vec{b}$

S7. **(c)**

$\vec{a}=\hat{\imath}-\hat{\jmath}+7\hat{k}$
$\vec{b}=5\hat{\imath}-\hat{\jmath}+\lambda\hat{k}$
$(\vec{a}+\vec{b})=\hat{\imath}-\hat{\jmath}+7\hat{k}+5\hat{\imath}-\hat{\jmath}+\lambda\hat{k}$
$\Rightarrow \vec{a}+\vec{b}=6\hat{\imath}-2\hat{\jmath}+(7+\lambda)\hat{k}$
$\vec{a}-\vec{b}=\hat{\imath}-\hat{\jmath}+7\hat{k}-(5\hat{\imath}-\hat{\jmath}+\lambda\hat{k})$
$\Rightarrow \vec{a}-\vec{b}=-4\hat{\imath}+0\hat{\jmath}+(7-\lambda)\hat{k}$
Now $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=(6\hat{\imath}-2\hat{\jmath}+(7+\lambda)\hat{k})\cdot(-4\hat{\imath}+0\hat{\jmath}+(7-\lambda)\hat{k})$
Since these two vectors are orthogonal, their dot product is zero
$\Rightarrow (6\times -4)+(-2\times 0)+((7+\lambda)\times(7-\lambda)=0$
$\Rightarrow \lambda^2=25$
$\Rightarrow \lambda=\pm 5$

S8. **(a)** Let,

$\vec{a}=(\hat{\imath}+\hat{\jmath})$
$\vec{b}=(\hat{\imath}-\hat{\jmath})$
$|\vec{b}|=\sqrt{1^2+(-1)^2}=\sqrt{1+1}=\sqrt{2}$
$\hat{b}=\frac{\vec{b}}{|\vec{b}|}=\frac{\hat{\imath}-\hat{\jmath}}{\sqrt{2}}$
$\therefore$ The projection of $\hat{\imath}+\hat{\jmath}$ on $(\hat{\imath}-\hat{\jmath})$ is: $(\hat{\imath}+\hat{\jmath})\cdot\frac{\hat{\imath}-\hat{\jmath}}{\sqrt{2}}=\frac{1-1}{\sqrt{2}}=0$

S9. **(b)** We know that,

$\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$
& $\Rightarrow 4=2\times 3\cos\theta$
$\Rightarrow 4=6\cos\theta$
$\Rightarrow \cos\theta=4/6$
$\Rightarrow \cos\theta=2/3$
$\Rightarrow \left|\vec{a}-\vec{b}\right|^2=|\vec{a}|^2+|\vec{b}|^2-2|\vec{a}||\vec{b}|\cos\theta$
$\Rightarrow \left|\vec{a}-\vec{b}\right|^2=2^2+3^2-(2\times 2\times 3)\times\frac{2}{3}$
$\Rightarrow \left|\vec{a}-\vec{b}\right|^2=4+9-8=5$
$\Rightarrow |\vec{a}-\vec{b}|=\sqrt{5}$

S10. **(d)** Let $\vec{a} = (2\hat{\imath} - 6\hat{\jmath} - 3\hat{k}), \vec{b} = (4\hat{\imath} + 3\hat{\jmath} - \hat{k})$

$$(\vec{a} \times \vec{b}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix} = \hat{\imath}\begin{vmatrix} -6 & -3 \\ 3 & -1 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 2 & -3 \\ 4 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & -6 \\ 4 & 3 \end{vmatrix}$$

$$= \hat{\imath}(6+9) - \hat{\jmath}(-2+12) + \hat{k}(6+24) = (15\hat{\imath} - 10\hat{\jmath} + 30\hat{k})$$

$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(15)^2 + (-10)^2 + (30)^2} = \sqrt{225 + 100 + 900} = \sqrt{1225} = 35$

Hence, the required unit vector $= \frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|} = \frac{5(3\hat{\imath} - 2\hat{\jmath} + 6\hat{k})}{35} = \pm\frac{1}{7}(3\hat{\imath} - 2\hat{\jmath} + 6\hat{k})$

S11. **(a)** Given:- $|\vec{a}| = 2, |\vec{b}| = 7 \& \vec{a} \times \vec{b} = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$

Let, $\theta =$ Angle between $\vec{a}$ and $\vec{b}$

Now, $|\vec{a} \times \vec{b}| = \sqrt{3^2 + 2^2 + 6^2} = 7$

Now, $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \cdot \sin\theta \Rightarrow 7 = 2 \times 7 \times \sin\theta \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

S12. **(b)** The given vector is $\vec{a} = x\hat{\imath} + x\hat{\jmath} + x\hat{k}$.

It will be a unit vector if $|\vec{a}| = 1$

$\Rightarrow \sqrt{x^2 + x^2 + x^2} = 1 \Rightarrow 3x^2 = 1$

$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$

S13. **(a)** The required plane is:

$(\vec{r} - \vec{r_1}) \cdot (\vec{b} \times \vec{d}) = 0$

Now $\vec{b} \times \vec{d} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -5 & -1 \\ -5 & 4 & 0 \end{vmatrix} = (0+4)\hat{\imath} -$

$(0-5)\hat{\jmath} + (8-25)\hat{k} = 4\hat{\imath} + 5\hat{\jmath} - 17\hat{k}$

$(\vec{r} - (3\hat{\imath} - \hat{\jmath} + 2\hat{k})) \cdot (4\hat{\imath} + 5\hat{\jmath} - 17\hat{k}) = 0$

$\vec{r} \cdot (4\hat{\imath} + 5\hat{\jmath} - 17\hat{k}) - (3 \cdot 4 + (-1) \cdot 5 + 2 \cdot (-17)) = 0$

$\vec{r} \cdot (4\hat{\imath} + 5\hat{\jmath} - 17\hat{k}) + 27 = 0$

S14. **(d)** $|\vec{c}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2} \qquad = \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 1$

S15. **(a)** Now, let α, β, and γ be the angles formed by $\vec{a}$ with the positive directions of x, y, and z

$\cos\alpha = \frac{1}{\sqrt{3}}, \cos\beta = \frac{1}{\sqrt{3}}, \cos\gamma = \frac{1}{\sqrt{3}}$.

S16. **(b)** For given vector $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{b} = -\hat{\imath} + \hat{\jmath} - \hat{k}$

Therefore , $\vec{a} + \vec{b} = (2 - (-1))\hat{\imath} + (-1 + 1)\hat{\jmath} + (2 - (-1))\hat{k}$

$|\vec{a} + \vec{b}| = \sqrt{2}$

Thus unit vector in the direction of $\vec{a} + \vec{b} = \frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{k}$

S17. **(a)**

$(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$

$= (\vec{a} - \vec{b}) \times \vec{a} + (\vec{a} - \vec{b}) \times \vec{b}$

$= \vec{a} \times \vec{a} - \vec{b} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{b}$

$= \vec{0} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b} - \vec{0}$ $\{since\ \vec{b} \times \vec{a} = -\vec{a} \times \vec{b}\}$

$= 2(\vec{a} \times \vec{b})$

S18. **(c)**

$\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

$2\vec{a} - \vec{b} + 3\vec{c} \quad = 2(\hat{\imath} + \hat{\jmath} + \hat{k}) - (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + 3(\hat{\imath} - 2\hat{\jmath} + \hat{k})$

$= 2\hat{\imath} + 2\hat{\jmath} + 2\hat{k} - 2\hat{\imath} + \hat{\jmath} - 3\hat{k} + 3\hat{\imath} - 6\hat{\jmath} + 3\hat{k}$

$= 3\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$

$|2\vec{a} - \vec{b} + 3\vec{c}| \quad = \sqrt{3^2 + (-3)^2 + 2^2} = \sqrt{9 + 9 + 4} = \sqrt{22}$

Hence, the unit vector along $2\vec{a}-\vec{b}+3\vec{c}$ is

$\frac{2\vec{a}-\vec{b}+3\vec{c}}{|2\vec{a}-\vec{b}+3\vec{c}|}=\frac{3\hat{\imath}-3\hat{\jmath}+2\hat{k}}{\sqrt{22}}=\frac{3}{\sqrt{22}}\hat{\imath}-\frac{3}{\sqrt{22}}\hat{\jmath}+\frac{2}{\sqrt{22}}\hat{k}$

S19. (c) $\vec{a}=\lambda\hat{\imath}+\hat{\jmath}+4\hat{k}$

$\vec{b}=2\hat{\imath}+6\hat{\jmath}+3\hat{k}$ projection of a on b is given by: $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

$|\vec{b}|=(2^2+6^2+3^2)^{1/2}$

$|\vec{b}|=(4+36+9)^{1/2}=(49)^{1/2}=7$

a unit vector in the direction of the sum of the vectors is given by:

$\hat{b}=\frac{b}{|\vec{b}|}=\frac{2\hat{\imath}+6\hat{\jmath}+3\hat{k}}{7}$

Now it is given that: $\vec{a}.\hat{b}=4$

$\Rightarrow (\lambda\hat{\imath}+\hat{\jmath}+4\hat{k})\cdot\left(\frac{2\hat{\imath}+6\hat{\jmath}+3\hat{k}}{7}\right)=4$

$\Rightarrow 2\lambda+6+(3\times 4)=28$

$\Rightarrow \lambda=(28-12-6)/2$

$\Rightarrow \lambda=10/2=5$

S20. (a) projection of a on b is given by: $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$ the projection of the vector $(\hat{\imath}+\hat{\jmath}+\hat{k})$ along with vector j is:

$(\hat{\imath}+\hat{\jmath}+\hat{k})\cdot\hat{\jmath}=0+1+0=1$ the projection of the vector $(i+j+k)$ along the vector $\hat{\jmath}$ is 1.

S21. (d)

$\vec{a}=2\hat{\imath}+\hat{\jmath}+3\hat{k}$

$\vec{b}=-\hat{\imath}+2\hat{\jmath}+\hat{k}$

$\vec{c}=3\hat{\imath}+\hat{\jmath}+2\hat{k}$

$\vec{b}\times\vec{c}=(-\hat{\imath}+2\hat{\jmath}+\hat{k})\times(3\hat{\imath}+\hat{\jmath}+2\hat{k})=\begin{bmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ -1 & 2 & 1\\ 3 & 1 & 2\end{bmatrix}$

$\begin{bmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ -1 & 2 & 1\\ 3 & 1 & 2\end{bmatrix}=\hat{\imath}(4-1)-\hat{\jmath}(-2-3)+\hat{k}(-1-6)=3\hat{\imath}+5\hat{\jmath}-7\hat{k}$

$\therefore \vec{b}\times\vec{c}=3\hat{\imath}+5\hat{\jmath}-7\hat{k}$

$\therefore \vec{a}.(\vec{b}\times\vec{c})=(2\hat{\imath}+\hat{\jmath}+3\hat{k})\cdot(3\hat{\imath}+5\hat{\jmath}-7\hat{k})=(2\times 3)+(1\times 5)+(3\times -7)$

$=6+5-21=-10$

S22. (a)

$|\vec{a}|=1$

$|\vec{b}|=2$

Since, $|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta$

Substituting the given values, we get:

$\Rightarrow \sqrt{3}=1\times 2\times\sin\theta$

$\Rightarrow \sin\theta=\frac{\sqrt{3}}{2}$

$\Rightarrow^{\theta}=\sin^{-1}\frac{\sqrt{3}}{2}$

$\Rightarrow \theta=60^\circ=\frac{\pi}{3}$

S23. (c) We know that:

$\hat{\imath}\times\hat{\jmath}=\hat{k},\hat{\jmath}\times\hat{k}=\hat{\imath},\hat{k}\times\hat{\imath}=\hat{\jmath}$

$\hat{\jmath}\times\hat{\imath}=-\hat{k},\hat{k}\times\hat{\jmath}=-\hat{\imath},\hat{\imath}\times\hat{k}=-\hat{\jmath}$

$\hat{\imath}.\hat{\imath}=\hat{\jmath}\cdot\hat{\jmath}=\hat{k}\cdot\hat{k}=1$

$\therefore \hat{\imath}\cdot(\hat{\jmath}\times\hat{k})+\hat{\jmath}\cdot(\hat{\imath}\times\hat{k})+\hat{k}\cdot(\hat{\imath}\times\hat{\jmath})=$

$\hat{\imath}\cdot\hat{\imath}+\hat{\jmath}\cdot(-\hat{\jmath})+\hat{k}\cdot\hat{k}=1-1+1=1$

S24. (a) It is given that:

$|\vec{a} \times \vec{b}| = |\vec{a} \cdot \vec{b}|$

$\Rightarrow |\vec{a}||\vec{b}|\sin\,\theta = |\vec{a}||\vec{b}|\cos\,\theta$

$\Rightarrow \sin\,\theta = \cos\,\theta$

$\Rightarrow \tan\,\theta = 1$

$\Rightarrow^{\theta} = \tan^{-1}\,1 = \frac{\pi}{4}$

S25. **(d)** For a vector $\vec{a} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

For a vector $\vec{a} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

$l = \frac{a}{\sqrt{a^2+b^2+c^2}}, m = \frac{b}{\sqrt{a^2+b^2+c^2}}, l = \frac{c}{\sqrt{a^2+b^2+c^2}}$

$\therefore l = \frac{1}{\sqrt{1^2+2^2+3^2}} = \frac{1}{\sqrt{1+4+9}} = \frac{1}{\sqrt{14}}.$

$\therefore m = \frac{2}{\sqrt{1^2+2^2+3^2}} = \frac{2}{\sqrt{1+4+9}} = \frac{2}{\sqrt{14}}$

$\therefore n = \frac{3}{\sqrt{1^2+2^2+3^2}} = \frac{3}{\sqrt{1+4+9}} = \frac{3}{\sqrt{14}}$

Ans: $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

S26. **(b)** Given : $\vec{a} = 2\hat{\imath} + 4\hat{\jmath} - \hat{k}, \vec{b} = 3\hat{\imath} - 2\hat{\jmath} + \lambda\hat{k}$ and $\vec{a} \perp \vec{b}$

Formula to be used $\vec{p} \cdot \vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip - For perpendicular vectors, $\theta = \frac{\pi}{2}$ i.e.

$\cos\,\theta = 0$ i.e. the dot product $= 0$

Hence, $\vec{a}.\vec{b} = 0$

$\therefore (2\hat{\imath} + 4\hat{\jmath} - \hat{k}) \cdot (3\hat{\imath} - 2\hat{\jmath} + \lambda\hat{k}) = 0$

$\Rightarrow 6 - 8 - \lambda = 0$

$\Rightarrow \lambda = -2$

S27. **(c)** $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = (|\vec{a}||\vec{b}|\sin\,\theta)^2 + (|\vec{a}||\vec{b}|\cos\,\theta)^2$

$= |\vec{a}|^2|\vec{b}|^2\sin^2\,\theta + |\vec{a}|^2|\vec{b}|^2\cos^2\,\theta$

$= |\vec{a}|^2|\vec{b}|^2(\sin^2\,\theta + \cos^2\,\theta)$ $= |\vec{a}|^2|\vec{b}|^2$

S28. **(a)**

Equation of line AB

$\frac{x-2}{2} = \frac{y-1}{3} = \frac{z+3}{-6} = \lambda$

Let B $(2\lambda + 2, 3\lambda + 1, -6\lambda - 3)$

B satisfy plane $2x + y + z + 8 = 0$

$2(2\lambda + 2) + (3\lambda + 1) - 6\lambda - 3 + 8 = 0$

$\lambda = -10$

B $(-18, -29, 57)$

Distance AB $= \sqrt{400 + 900 + 3600} = 70$

S29. **(d)** Let $\vec{u} = u_1\hat{\imath} + u_2\hat{\jmath} + u_3\hat{k}$

As $\vec{u}$ is coplanar with the vectors $\vec{a}$ and $\vec{b}$,

$\begin{vmatrix} u_1 & u_2 & u_3 \\ 2 & 3 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 0$

$4u_1 - 2u_2 + 2u_3 = 0$

$2u_1 - u_2 + u_3 = 0 \ldots (1)$

$\vec{u} \cdot \vec{a} = 0$

$\Rightarrow 2u_1 + 3u_2 - u_3 = 0 \ldots (2)$

$u \cdot b = 12$

$\Rightarrow u_2 + u_3 = 12 \ldots (3)$

Solving (1),(2) and (3),

$u_1 = -4, u_2 = 8, u_3 = 16$

Thus, $\vec{u} = -4\hat{\imath} + 8\hat{\jmath} + 16\hat{k}$

S30. **(a)**

A$= 3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$

$B = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$C = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$

$\therefore \overrightarrow{AB}$

$= (2\hat{\imath} - \hat{\jmath} + \hat{k}) - (3\hat{\imath} - 4\hat{\jmath} - 4\hat{k})$

$= -\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$

$\therefore \overrightarrow{BC}$

$= (\hat{\imath} - 3\hat{\jmath} - 5\hat{k}) - (2\hat{\imath} - \hat{\jmath} + \hat{k})$

$= -\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$

$\therefore \overrightarrow{CA}$

$= (3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}) - (\hat{\imath} - 3\hat{\jmath} - 5\hat{k})$

$= 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\therefore \overrightarrow{AB} \cdot \overrightarrow{CA}$

$= (-\hat{\imath} + 3\hat{\jmath} + 5\hat{k}) \cdot (2\hat{\imath} - \hat{\jmath} + \hat{k})$

$= -2 - 3 + 5$

$= 0$

The triangle is right-angled

SUBJECTIVE QUESTIONS

S1. $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$

if $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

then $|\vec{a}| = \sqrt{a_x^{\,2} + a_y^{\,2} + a_z^{\,2}}$

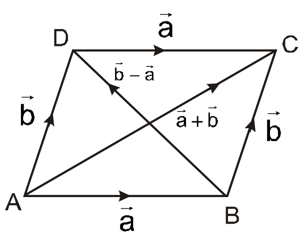
$\therefore \quad |\vec{a}| = \sqrt{14} \Rightarrow \hat{a} = \dfrac{\vec{a}}{|\vec{a}|} = \dfrac{1}{\sqrt{14}}$

$\hat{i} - \dfrac{2}{\sqrt{14}}\hat{j} + \dfrac{3}{\sqrt{14}}\hat{k}$

S2. $\vec{a}$ and $\vec{b}$ are parallel if $\dfrac{x+2}{x-1} = \dfrac{y-x}{2x+y} = \dfrac{1}{2}$

$x = -5, y = -20$

S3. Let ABCD be a parallelogram such that $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{BC} = \vec{b}$.



Then, $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\Rightarrow \overrightarrow{AC} = \vec{a} + \vec{b} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$|\overrightarrow{AC}| = \sqrt{9+16+25} = \sqrt{50}$

$\overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}$

$\Rightarrow \overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB} = \vec{b} - \vec{a} = -(\hat{i} + 2\hat{j} + 3\hat{k})$

$|\overrightarrow{BD}| = \sqrt{1+4+9} = \sqrt{14}$

$\therefore$ Unit vector along $\overrightarrow{AC} = \dfrac{\overrightarrow{AC}}{|\overrightarrow{AC}|} = \dfrac{1}{\sqrt{50}}$

$\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$

and Unit vector along $\overrightarrow{BD} = \dfrac{\overrightarrow{BD}}{|\overrightarrow{BD}|} = -\dfrac{1}{\sqrt{14}}$

$\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

S4. Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ be the position vectors of vertices A, B, C, D respectively.

Let E, F, G, H be midpoint of AB, CD, AC and BD respectively

P.V of E = $\dfrac{\vec{a}+\vec{b}}{2}$

P.V of F = $\dfrac{\vec{c}+\vec{d}}{2}$

P.V. of G = $\dfrac{\vec{a}+\vec{c}}{2}$

P.V. of H = $\dfrac{\vec{b}+\vec{d}}{2}$

$\overrightarrow{EG} = \left(\dfrac{\vec{a}+\vec{c}}{2}\right) - \left(\dfrac{\vec{a}+\vec{b}}{2}\right) = \dfrac{\vec{c}-\vec{b}}{2}$

$\overrightarrow{HF} = \dfrac{\vec{c}+\vec{d}}{2} - \left(\dfrac{\vec{b}+\vec{d}}{2}\right) = \dfrac{\vec{c}-\vec{b}}{2}$

$\overrightarrow{EG} = \overrightarrow{HF}$

$\Rightarrow \quad \overrightarrow{EG} \,||\, \overrightarrow{HF}$ and EG = HF

S5. Let A ≡ (4, 5, –5), B ≡ (0, –11, 3), C ≡ (2, –3, –1).

AB =

$\sqrt{(4-0)^2 + (5+11)^2 + (-5-3)^2} = \sqrt{336} = \sqrt{4\times 84} = 2\sqrt{84}$

BC = $\sqrt{(0-2)^2 + (-11+3)^2 + (3+1)^2} = \sqrt{84}$

AC = $\sqrt{(4-2)^2 + (5+3)^2 + (-5+1)^2} = \sqrt{84}$

BC + AC = AB

Hence points A, B, C are collinear and C lies between A and B.

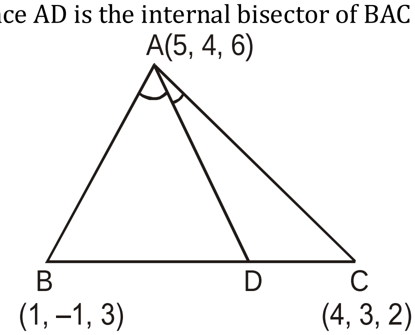
NUMERICAL TYPE QUESTIONS

S1. ($\dfrac{\sqrt{1530}}{8}$)

AB = $\sqrt{4^2 + 5^2 + 3^2} = 5\sqrt{2}$

AC = $\sqrt{1^2 + 1^2 + 4^2} = 3\sqrt{2}$

Since AD is the internal bisector of BAC



$\therefore \dfrac{BD}{DC} = \dfrac{AB}{AC} = \dfrac{5}{3}$

$\therefore$ D divides BC internally in the ratio 5 : 3

$\therefore \quad D \equiv \left(\dfrac{5\times4+3\times1}{5+3}, \dfrac{5\times3+3(-1)}{5+3}, \dfrac{5\times2+3\times3}{5+3}\right)$

or, D = $\left(\dfrac{23}{8}, \dfrac{12}{8}, \dfrac{19}{8}\right)$

$\therefore$ AD = $\sqrt{\left(5-\frac{23}{8}\right)^2+\left(4-\frac{12}{8}\right)^2+\left(6-\frac{19}{8}\right)^2} = \frac{\sqrt{1530}}{8}$ unit

S2. ($\frac{2}{3}$) vectors $\vec{a} = 3\hat{i}+2\hat{j}+9\hat{k}$ and $\vec{b} = \hat{i}+p\hat{j}+3\hat{k}$ are parallel iff

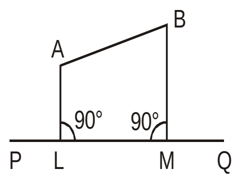
$\frac{3}{1} = \frac{2}{p} = \frac{9}{3} \Rightarrow 3 = \frac{2}{p} \Rightarrow p = \frac{2}{3}$

S3. ($\frac{8}{7}$) Let A ≡ (1, 2, 3), B ≡ (–1, 4, 2)

Direction ratios of the given line PQ are 2, 3, – 6

$\sqrt{2^2+3^2+(-6)^2} = 7$

$\therefore$ direction cosines of PQ are $\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}$



Projection of AB on PQ

$= |\lambda\,(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)|$

$= \left|\frac{2}{7}(-1-1)+\frac{3}{7}(4-2)-\frac{6}{7}\ (2-3)\right| = \left|\frac{-4+6+6}{7}\right|$

$= \frac{8}{7}$

S4. (528)

Let $\vec{a} = -6\hat{i}+14\hat{j}+10\hat{k}$, $\vec{b} = -5\hat{i}+7\hat{j}-3\hat{k}$ and $\vec{c} = 7\hat{i}-5\hat{j}-3\hat{k}$.

We know that the volume of a parallelopiped whose three adjacent edges are $\vec{a}, \vec{b}, \vec{c}$ is $\left|[\vec{a}\ \vec{b}\ \vec{c}]\right|$.

Now $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} -6 & 14 & 10 \\ -5 & 7 & -3 \\ 7 & -5 & -3 \end{vmatrix} = -6(-21 - 15) - 14(15 + 21) + 10(25 - 49) = -528$

So required volume of the parallelopiped = $\left|[\vec{a}\ \vec{b}\ \vec{c}]\right| = |-528| = 528$ cubic units.

S5. (3) If the line through the points (4, 1, 2) and (5, λ, 0) is parallel to the line through the points (2, 1, 1) and (3, 3, –1), find λ.__________.

For parallel lines, Direction cosines must be equal.

$\therefore$ Direction cosine for 1st $\Rightarrow 5-4, \lambda-1, 0-2 \Rightarrow 1, k-1, -2$

$\therefore$ Direction cosine for 2nd $\Rightarrow 3-2, 3-1, -1-1 \Rightarrow 1, 2, -2$

$\therefore\ \lambda - 1 = 2 \Rightarrow \lambda = 3$

TRUE AND FALSE

S1. (True)

By definition

Distance between any two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given as $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$

S2. (False)

If $\vec{a} = \vec{b}$, then $|\vec{a}| = |\vec{b}|$ but converse may not be true.

Example: Let $\vec{a} = \hat{\imath}-\hat{\jmath}+\hat{k}$, $\vec{b} = \hat{\imath}+\hat{\jmath}+\hat{k}$ be two vectors. So, magnitude of $\vec{a}$ and $\vec{b}$ are $|\vec{a}| = \sqrt{3}$ and $|\vec{b}| = \sqrt{3}$. But $\vec{a} \neq \vec{b}$

S3. (False)

If $\frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = k$, Then vector $\vec{a}$ and $\vec{b}$ will be collinear

S4.(True)

$\vec{a}\ .\ \vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta,\ (0 \le \theta \le \pi)$

(i) If θ is acute, then $\vec{a}.\vec{b} > 0$ and if θ is obtuse, then $\vec{a}.\vec{b} < 0$.

S5. (True)

By definition

Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\ .\ \vec{b}}{|\vec{b}|}$

ASSERTION AND REASON

S1. (d) **Assertion (A)** is wrong

The position of a particle in a rectangular coordinate system is (3 , 2, 50.Then its position vector be $3\hat{\imath}+2\hat{\jmath}+5\hat{k}$

Reason (R) is correct.

The displacement vector of the particle that moves from point P(2, 3, 5) to point Q (3 , 4, 5)

$= (3-2)\hat{\imath}+(4-3)\hat{\jmath}+(5-5)\hat{k}$

$= \hat{\imath}+\hat{\jmath}$

S2. (b) Assertion (A) is correct

Direction cosine of $\vec{a} = 2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ are : $\frac{2}{\sqrt{4+16+25}}, \frac{4}{\sqrt{4+16+25}}, \frac{-5}{\sqrt{4+16+25}}$ or $\frac{2}{\sqrt{45}}, \frac{4}{\sqrt{45}}, \frac{-5}{\sqrt{45}}$

Reason (R) : A vector having zero magnitude and arbitrary direction is called ' zero vector' or 'null vector'. Thus R is correct.

A and R both are correct but R is not correct explanation of A.

S3. **(d)** **Assertion (A) :** Let $\vec{a} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{b} = -\hat{\imath} + \hat{\jmath} - \hat{k}$ be two vectors then $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & -1 \\ -1 & 1 & -1 \end{vmatrix} = \hat{\imath}(1+2) - \hat{\jmath}(-1-1) + \hat{k}(1+2) = 3\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\vec{b} \times \vec{a} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 1 & -1 \\ 1 & 2 & -1 \end{vmatrix} = \hat{\imath}(-2-1) - \hat{\jmath}(1+1) + \hat{k}(-2-1) = -3\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$

$\therefore \vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$

Thus A is not true

Reason(R) : Let $\vec{a}$ and $\vec{b}$ be two vectors then $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$

R is the property of vectors , therefore R is correct.

S4. **(d)** **Assertion (A) :** Let $\vec{a} = 5\hat{\imath} - \hat{\jmath} + 2\hat{k}$, $|\vec{a}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{30}$

$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{5\hat{\imath} - \hat{\jmath} + 2\hat{k}}{\sqrt{30}}$

Hence the vector in direction of vector $5\hat{\imath} - \hat{\jmath} + 2\hat{k}$ which has magnitude of 8 units is given by

$8\hat{a} = 8 \times \frac{5\hat{\imath} - \hat{\jmath} + 2\hat{k}}{\sqrt{30}} = \frac{40}{\sqrt{30}}\hat{\imath} - \frac{8}{\sqrt{30}}\hat{\jmath} + \frac{16}{\sqrt{30}}\hat{k}$

$\therefore$ A is not true.

Reason (R): If $\vec{a}$ and $\vec{b}$ are position vectors of two points A and B, then $\overrightarrow{AB} = \vec{b} - \vec{a}$ = position vector **(p.v.)** of B – position vector **(p.v.)** of A.

R is true.

HOMEWORK

MCQ

S1. **(a)**

Given , $A(1, -1, -5), B(3, 1, 3)$ and $C(9, 1, -3)$

$\therefore \overrightarrow{AB} = \sqrt{(1-3)^2 + (-1-1)^2 + (-5-3)^2} = \sqrt{72}$

$\therefore \overrightarrow{BC} = \sqrt{(3-9)^2 + (1-1)^2 + (3+3)^2} = \sqrt{72}$

$\therefore \overrightarrow{AC} = \sqrt{(1-9)^2 + (-1-1)^2 + (-5+3)^2} = \sqrt{72}$

$\therefore \overrightarrow{AB} = \overrightarrow{BC} = \overrightarrow{AC}$

$\therefore$ The points A , B and C are the vertices of an equilateral triangle.

S2. **(b)** Distance of a point (x , y, z) from x y – plane is gibe by |z| .

Therefore , the distance of the point (x, y, z) from its image in x y – plane = 2 | z|

S3. **(b)** Let the yz plane divide the line segment joining the points (2 , 4 , 5) and (3 , -5, 4) is m : 1.

Now , we know that on yz – plane the co – ordinate of x is 0

$\therefore \frac{3m+2}{m+1} = 0$

$\Rightarrow 3m + 2 = 0$

$\Rightarrow m = \frac{-2}{3}$

Hence , y z – plane divide the line segment joining the points (2 , 4, 5) and (3 , -5 ,4) in 2: 3 externally.

S4. **(b)** Given that

Point A (3 ,2 ,-4) , B(5 , 4, -6) and C(9 ,8,-10) are collinear

B must divide line segment AC in some ratio externally and internally

We know that

Co -ordinate of point A (x ,y ,z) that divides line segment joining (x_1, y_1, z_1) and (x_2, y_2, z_2) in ratio m : n is (x ,y ,z) = $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

Let point B(5 ,4, -6) divide line segment A(3 , 2,-4) and C(9 , 8, -10) in the ratio k :1

B(5 ,4,-6) = $\left(\frac{9k+3}{k+1}, \frac{8k+2}{k+1}, \frac{-10k-4}{k+1}\right)$

Comparing x - coordinate of B

$\Rightarrow 5 = \frac{9k+3}{k+1}$

$\Rightarrow 5k + 5 = 9k + 3$

$\Rightarrow k = \frac{1}{2}$

Thus point B divides AC in the ratio 1 : 2

S5. **(c)** Let the point be given by (1 , -1, 5) and (2, 3, 4)

A point that divides the line joining these 2 points in the ration p : 1 given by $\left(\frac{2p+1}{p+1}, \frac{3p-1}{p+1}, \frac{4p+5}{p+1}\right)$.

Since this point has to lie on the ZX – plane ,so 3p – 1=0

$\Rightarrow p = \frac{1}{3}$

$\Rightarrow 1 + p = 1 + \frac{1}{3} = \frac{4}{3}$

S6. **(a)** Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$.

Given $\vec{a}.\vec{b} = \sqrt{6} \Rightarrow |\vec{a}||\vec{b}|\cos\theta = \sqrt{6}$

$\Rightarrow \sqrt{3}.2\cos\theta = \sqrt{6}$

$\Rightarrow \cos\theta = \frac{\sqrt{6}}{2\sqrt{3}} = \frac{1}{\sqrt{2}} \Rightarrow \cos\theta = \cos 45° \Rightarrow \theta = 45°$

S7. **(a)** Let $\vec{a} = \hat{\imath} + 3\hat{\jmath} + 7\hat{k}$ and $\vec{b} = 7\hat{\imath} - \hat{\jmath} + 8\hat{k}$

$\therefore\ \vec{a}.\vec{b} = 1.7 + 3.(-1) + 7.8 = 60$

$|\vec{b}| = \sqrt{7^2 + 1 + 64} = \sqrt{114}$

The projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\vec{b}}{|\vec{b}|} = \frac{60}{\sqrt{114}}$

S8. **(b)** The scalar projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|} = 4$

Here , $\vec{a}.\vec{b} = \lambda.2. +1.6 + 4.3 = 2\lambda + 18$

And $|\vec{b}| = \sqrt{2^2 + 6^2 + 3^2} = \sqrt{49} = 7$

From (i) , we get $\frac{2\lambda+18}{7} = 4 \Rightarrow \lambda = 5$

S9. **(a)** Given $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$,

$\therefore\ \vec{a} + \vec{b} = 4\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{a} - \vec{b} = -2\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$

$\therefore\ (\vec{a} + \vec{b})(\vec{a} - \vec{b}) = 0$

Thus , the dot product of two non -zero vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ is 0 , therefore , the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular to each other.

S10. **(d)** The given point (x ,y ,z)

Now distance of point (x, y, z) from the y -axis i.e., (0 ,y,0)

$\therefore$ By distance formula

Distance

$=\sqrt{(x-0)^2 + (y-y)^2 + (z-0)^2}$

$= \sqrt{x^2 + z^2}$

SUBJECTIVE QUESTIONS

S1. Let $\vec{R}$ be the resultant force

$\therefore\ \vec{R} = \overrightarrow{AB} + \overrightarrow{AE} + \overrightarrow{BC} + \overrightarrow{DC} + \overrightarrow{ED} + \overrightarrow{AC}$

$\Rightarrow \vec{R} = (\overrightarrow{AB} + \overrightarrow{BC}) + (\overrightarrow{AE} + \overrightarrow{ED} + \overrightarrow{DC}) + \overrightarrow{AC}$

$\Rightarrow \vec{R} = \overrightarrow{AC} + \overrightarrow{AC} + \overrightarrow{AC}$

$\Rightarrow \vec{R} = 3\overrightarrow{AC}$. Hence proved.

S2. Given A ≡ (2, 3, 4), B ≡ (–1, 2, –3), C ≡ (–4, 1, –10).

A (2, 3, 4) B (–1, 2, –3)

Let C divide AB internally in the ratio k : 1, then

$C \equiv \left(\frac{-k+2}{k+1}, \frac{2k+3}{k+1}, \frac{-3k+4}{k+1}\right)$ $\therefore\ \frac{-k+2}{k+1} = -4 \Rightarrow 3k = -6 \Rightarrow k = -2$

For this value of k, $\frac{2k+3}{k+1} = 1$, and $\frac{-3k+4}{k+1} = -10$

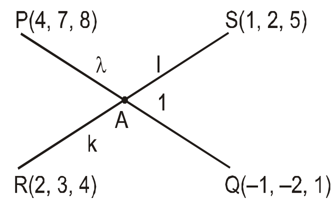
Since k < 0, therefore C divides AB externally in the ratio 2 : 1 and points A, B, C are collinear.

S3. Let the lines PQ and RS intersect at point A.

Let A divide PQ in the ratio λ : 1, (λ ≠ –1)

then $A \equiv \left(\frac{-\lambda+4}{\lambda+1}, \frac{-2\lambda+7}{\lambda+1}, \frac{\lambda+8}{\lambda+1}\right)$. ...(a)

Let A divide RS in the ratio k : 1, then $A \equiv \left(\frac{k+2}{k+1}, \frac{2k+3}{k+1}, \frac{5k+4}{k+1}\right)$ ...(b)



From (a) and (b), we have,

$\frac{-\lambda+4}{\lambda+1} = \frac{k+2}{k+1} \Rightarrow -\lambda k - \lambda + 4k + 4 = \lambda k + 2\lambda + k + 2 \Rightarrow 2\lambda k + 3\lambda - 3k - 2 = 0$ ...(c)

$\frac{-2\lambda+7}{\lambda+1} = \frac{2k+3}{k+1} \Rightarrow -2\lambda k - 2\lambda + 7k + 7 = 2\lambda k + 3\lambda + 2k + 3 \Rightarrow 4\lambda k + 5\lambda - 5k - 4 = 0$ .....(d)

$\frac{\lambda+8}{\lambda+1} = \frac{5k+4}{k+1}$ ..... (5)

Multiplying equation (c) by 2, and subtracting from equation (d), we get $-\lambda + k = 0$ or, $\lambda = k$

Putting λ = k in equation (c), we get $2\lambda^2 + 3\lambda - 3\lambda - 2 = 0 \Rightarrow \lambda = 1 = k$

Clearly λ = k = 1 satisfies eqn. (5), hence our assumption is correct.

$\therefore\ A \equiv \left(\frac{-1+4}{2}, \frac{-2+7}{2}, \frac{1+8}{2}\right)$ or,

$A \equiv \left(\frac{3}{2}, \frac{5}{2}, \frac{9}{2}\right)$.

S4. We have, $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow \vec{a} + \vec{b} = -\vec{c} \Rightarrow (\vec{a} + \vec{b}) . (\vec{a} + \vec{b}) = (-\vec{c}) . (-\vec{c})$

$\Rightarrow \left|\vec{a}+\vec{b}\right|^2 = |\vec{c}|^2 \Rightarrow \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2\vec{a}\,.\,\vec{b} = \left|\vec{c}\right|^2$

$\Rightarrow \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2\,|\vec{a}|\left|\vec{b}\right| \cos\theta = \left|\vec{c}\right|^2$

$\Rightarrow 9 + 25 + 2\,(c)\,(5)\cos\theta = 36 \Rightarrow \cos\theta = \dfrac{2}{30}$

$\Rightarrow \theta = \cos^{-1}\dfrac{1}{15}$

S5. The angle θ between vectors $\vec{a}$ and $\vec{b}$ is given by $\cos\theta = \dfrac{\vec{a}\,.\,\vec{b}}{|\vec{a}||\vec{b}|}$

Now, θ is obtuse $\Rightarrow \cos\theta < 0 \Rightarrow \dfrac{\vec{a}\,.\,\vec{b}}{|\vec{a}||\vec{b}|} < 0 \Rightarrow \vec{a}\,.\,\vec{b} < 0$ $[\because |\vec{a}|, |\vec{b}| > 0]$

$\Rightarrow 14x^2 - 8x + x < 0 \Rightarrow 7x\,(2x - 1) < 0 \Rightarrow x(2x-1) < 0 \Rightarrow 0 < x < \dfrac{1}{2}$

Hence, the angle between the given vectors is obtuse if $x \in (0, 1/2)$

NUMERICAL TYPE QUESTIONS

S1. $\left(\frac{4}{3}\right)$ Component of $\vec{b}$ along $\vec{a}$ is $\left(\dfrac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right)\vec{a}$;

Here $\vec{a}.\vec{b} = 2 - 1 + 3 = 4$ and $|\vec{a}|^2 = 3$

Hence $\left(\dfrac{\vec{a}\,.\,\vec{b}}{|\vec{a}|^2}\right)\vec{a} = \dfrac{4}{3}\vec{a} = \dfrac{4}{3}(\hat{i} + \hat{j} + \hat{k})$, then $\lambda = \frac{4}{3}$

S2. **(9)** Let $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$. Then

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix} = (-14 + 14) - (2 - 21) + (-2 + 21)\,\hat{k} = 19\hat{j} + 19\hat{k}$$

$\Rightarrow |\vec{a}\times\vec{b}| = 19\sqrt{2}$

$\therefore$ Required vector $= \pm 9\left(\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}\right) = \pm\dfrac{9}{\sqrt{2}}(\hat{j}+\hat{k})$

S3. **(0)** We have, $\vec{a} \times (\vec{b}+\vec{c}) + \vec{b} \times (\vec{c}+\vec{a}) + \vec{c} \times (\vec{a}+\vec{b})$

$= \vec{a}\times\vec{b} + \vec{a}\times\vec{c} + \vec{b}\times\vec{c} + \vec{b}\times\vec{a} + \vec{c}\times\vec{a} + \vec{c}\times\vec{b}$ [Using distributive law]

$= \vec{a}\times\vec{b} + \vec{a}\times\vec{c} + \vec{b}\times\vec{c} - \vec{a}\times\vec{b} - \vec{a}\times\vec{c} - \vec{b}\times\vec{c} = \vec{0}$ [Since $\vec{b}\times\vec{a} = -\vec{a}\times\vec{b}$]

S4. **(2)** Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$. Then

$\vec{a}\times\hat{i} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) \times \hat{i} = a_1(\hat{i}\times\hat{i}) + a_2(\hat{j}\times\hat{i}) + a_3(\hat{k}\times\hat{i}) = -a_2\hat{k} + a_3\hat{j}$

$\Rightarrow |\vec{a}\times\hat{i}|^2 = a_2^2 + a_3^2$

$\vec{a}\times\hat{j} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) \times \hat{j} = a_1\hat{k} - a_3\hat{i}$

$\Rightarrow |\vec{a}\times\hat{j}|^2 = a^2{}_1 + a_3^2$

$\vec{a}\times\hat{k} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) \times \vec{k} = -a_1\hat{j} + a_2\hat{i}$

$\Rightarrow |\vec{a}\times\hat{k}|^2 = a_1^2 + a_2$

$\therefore |\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2 = a_2^2 + a_3^3 + a_1^2 + a_3^2 + a_1^2 + a_2^2$

$= 2\,(a_1^2 + a_2^2 + a_3^2) = 2\,|\vec{a}|^2$

S5. **(0)** The lines will be perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

TRUE AND FALSE

S1. **(False)** If $\vec{a}$, $\vec{b}$ are two vectors and θ is the angle between them, then $\vec{a} \times \vec{b} = \left|\vec{a}\right|\left|\vec{b}\right| \sin\theta\,\hat{n}$ where $\hat{n}$ is the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ such that $\vec{a}$, $\vec{b}$ and $\hat{n}$ forms a right handed screw system.

S2. **(True)** A given number of vectors are called coplanar if their line segments are all parallel to the same plane

S3. **(False)** $\left[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}\right] = \left(\vec{a}+\vec{b}\right) . \left[\left(\vec{b}+\vec{c}\right)\times\left(\vec{c}+\vec{a}\right)\right] = \left(\vec{a}+\vec{b}\right) \left[\vec{b}\times\vec{c} + \vec{b}\times\vec{a} + \vec{c}\times\vec{a}\right].$

$= \left[\vec{a} \;\; \vec{b} \;\; \vec{c}\right] + \left[\vec{a} \;\; \vec{b} \;\; \vec{c}\right] = 2\left[\vec{a} \;\; \vec{b} \;\; \vec{c}\right]$

S4. **(True)** $[\vec{a} \;\; \vec{b} \;\; \vec{c}] = \begin{vmatrix} -4 & 8 & -4 \\ 4 & -2 & -2 \\ -2 & -2 & 4 \end{vmatrix} = -4(-8 - 4) - 8(16 - 4) - 4(-8 - 4) = 0$

So vectors , are coplanar

S5. **(True)** We have, $\vec{a}\times\{\vec{b}\times(\vec{c}\times\vec{d})\}$

$= \vec{a}\times\{(\vec{b}\ .\ \vec{d})\ \vec{c}-(\vec{b}\ .\ \vec{c})\ \vec{d}\}$

$=\vec{a}\times\{(\vec{b}\ .\ \vec{d})\ \vec{c}\}-\vec{a}\times\{(\vec{b}\ .\ \vec{c})\ \vec{d}\}$ [by dist. law]

$= (\vec{b}\ .\ \vec{d})\ (\vec{a}\times\vec{c})-(\vec{b}\ .\ \vec{c})\ (\vec{a}\times\vec{d})$.

ASSERTION AND REASONING

S1. **(c)** **Assertion (A) :** If a , b and c are unit vectors , then $|a-b|^2+|b-c|^2+|c-a|^2$ does not exceed 9.

$|a - b|^2 + |b - c|^2 + |c - a|^2 = 2 (a^2 + b^2 + c^2) - 2 (a \times b + b \times c + c \times a)$

$= 2 \times 3 - 2 (a \times b + b \times c + c \times a)$

$= 6 - \{(a + b + c)^2 - a^2 - b^2 - c^2\}$

$= 9 - |a + b + c|^2 \leq 9$

Thus A it true but R is false

S2. **(d)** **Assertion (A):**

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

$\Rightarrow\ -ac-ab+ab+\ c^2=0$

$\Rightarrow\ c^2=ab$

Hence , c is the geometric mean of a and b.

Thus A is false

Reason (R):

a. [(b + c) × (a + b + c)]

= a . (b × a + b × b + b × c) + a . (c × a + c × b + c × c)

= [aba] + [abb] + [a b c] + [aca] + [a c b] + [a c c]

= 0 + 0 + [a b c] + 0 − [a b c] + 0

= 0

Thus R is true.

S3. **(b)** **Assertion (A) :** Since a + b + c = 0

On squaring both sides, we get

$|a|^2 + |b|^2 + |c|^2 + 2 (a . b + b . c + c . a) = 0$

$\Rightarrow 2 (a . b + b . c + c . a) = - (9 + 16 + 25)$

$\Rightarrow a . b + b . c + c . a = -25$

Thus A is true.

Reason (R) $R = \sqrt{[2^2 + 10^2 + 11^2]}$.

$= \sqrt{[4 + 100 + 121]}$

= 15

Thus A and R are true but R is not correct explanation of A.

S4. **(a)** **Assertion (A) :** The sum of $\vec{a}=2\hat{\imath}+4\hat{\jmath}+7\hat{k}$ and $\vec{b}=-2\hat{\imath}+3\hat{\jmath}+2\hat{k}$ is $7\hat{\jmath}+9\hat{k}$

Reason (R) : The sum of the vectors $\vec{a}$ and $\vec{b}$ is given by $\vec{a}+\vec{b}=(a_1+b_1)\hat{\imath}+(a_2+b_2)\hat{\jmath}+(a_3+b_3)\hat{k}$

Both A and R are true and R is correct explanation of A.

S5. **(b)** **Assertion (A) :**

Let, the given vectors be coplanar.

Then one of the given vectors is expressible in terms of the other two.

Let $2\vec{a}-\vec{b}+3\vec{c} = x\left(\vec{a}+\vec{b}-2\vec{c}\right) + y\left(\vec{a}+\vec{b}-3\vec{c}\right)$, for some scalars x and y.

$\Rightarrow 2\vec{a}-\vec{b}+3\vec{c} = (x + y)\ \vec{a} + (x + y)\ \vec{b} + (-2x - 3y)\ \vec{c}$

$\Rightarrow 2 = x + y, -1 = x + y$ and $3 = - 2x - 3y$.

Solving first and third of these equations, we get x = 9 and y = −7.

Clearly these values do not satisfy the second equation. Hence the given vectors are not coplanar.

Thus A is true

Reason (R): Let the given four points be P, Q, R and S respectively. These points are coplanar if the vectors $\overrightarrow{PQ}$, $\overrightarrow{PR}$ and $\overrightarrow{PS}$ are coplanar. These vectors are coplanar iff one of them can be expressed as a linear combination of other two.

So let $\overrightarrow{PQ} = x\overrightarrow{PR} + y\ \overrightarrow{PS}$

$\Rightarrow\ -\vec{a}-5\vec{b}+4\vec{c} = x\left(\vec{a}+\vec{b}-\vec{c}\right) + y\left(-\vec{a}-9\vec{b}+7\vec{c}\right)$

$\Rightarrow\ -\vec{a}-5\vec{b}+4\vec{c} = (x - y)\ \vec{a} + (x - 9y)\ \vec{b} + (-x + 7y)\ \vec{c}$

$\Rightarrow\ x - y = -1,\ x - 9y = -5,\ -x + 7y = 4$ [Equating coff. of $\vec{a}, \vec{b}, \vec{c}$ on both sides]

Solving the first two equations of these three equations, we get $x = -\frac{1}{2}$, $y = \frac{1}{2}$

These values also satisfy the third equation. Hence the given four points are coplanar.

Thus R is true.