Chapter 1

Network Elements and Basic Laws

CHAPTER HIGHLIGHTS

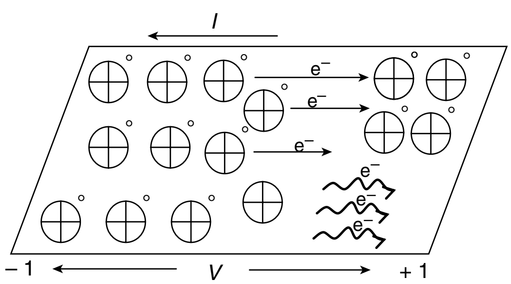
- Basic Concepts and Classification of Network Elements
- Circuit Elements
- Independent and Dependent Sources
- Voltage Controlled Voltage Source
- Voltage Controlled Current Source
- Current Controlled Voltage Source
- Current Controlled Current Source
- Network Element and Types of Elements
- Bilateral and Unilateral Networks
- Lumped and Distributed Networks
- D.C. And A.C. Networks
- Kirchhoff's Laws
- Circuit Elements in Series and Parallel
- Current Division and Nodal Analysis
- Source Transformation
- Mesh Analysis
- Star - Delta Transformation

BASIC CONCEPTS

The most basic quantity used in the analysis of electrical circuits is the electric charge.

BASIC QUANTITIES

1. **Electron:** Electron is a mobile charge carrier. The e^- is measured in Coulombs (C).
   $1\,e^- = 1.6 \times 10^{-19}$ C.
   (a) Multiples of electrons constitute charge (q).
   (b) The movement of charge (q) over time causes current.
2. **Current:** There are free electrons available in all semi-conductive and conductive materials, and these free e^- s move at random in all directions within the structure in the absence of external pressure or voltage. If a certain amount of voltage is applied across the material, all the free e^- s move in one direction depending on the polarity of the applied voltage.



3. **Voltage:** According to the structure of an atom, we know that there are two types of charges: positive and negative. A force of attraction exists between these charges. A certain amount of energy is required to overcome the force and move the charges through a specific distance. All opposite charges possess a certain amount of potential energy because of the separation between them. The difference in potential energy of the charges is called the potential difference. Potential difference in electrical terminology is known as voltage and is denoted by V, and it is expressed in terms of energy (W) per unit charges (Q).

$$\therefore\ V = \frac{W}{Q} \text{ or } \upsilon = \frac{dW}{dQ}$$

The voltage is defined as the work or energy required to move a unit charge through an element.
The time rate of change of charge produces an electrical current:

$$i(t) = \frac{dq(t)}{dt}$$

The electric current is measured in Ampere (A)

$$1 \text{ A} = 1\text{C}/1\text{s}$$

4. **Power and Energy:** Energy is the capacity for doing work, that is, energy is nothing but stored work.

Energy can be expressed as

$$W(t) = \int_{t_1}^{t_2} p(t).dt = \int_{t_1}^{t_2} v(t).i(t).dt$$

Power is the rate of change of energy and is denoted by P. If W amount of energy is used over a t amount of time, then

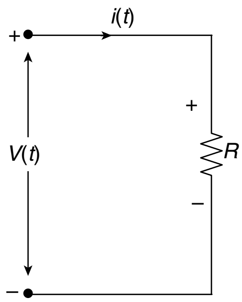
$$\text{Power } (P) = \frac{\text{Energy}}{\text{Time}} = \frac{W}{t} = \frac{dW}{dt}$$

$$P = \frac{dW}{dt} = \frac{dW}{dq} \times \frac{dq}{dt}$$

$$\therefore P = VI\ W.$$

One Watt is the amount of power generated when one Joule of energy is consumed in one second.

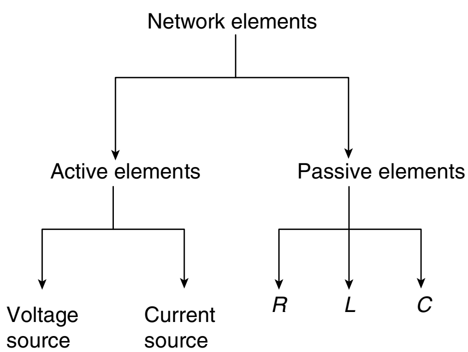
5. **Positive sign convention:** Current flows from the positive to the negative terminals.



Power is absorbed by element if the sign of power is positive. Therefore, current enters from the positive terminal and leaving from negative terminal of the element. Power is supplied (delivered) by element or source if the sign of power is negative, that is, current enters from the positive terminal of the element.

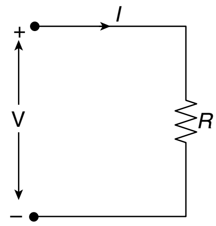
Classification of Network Elements

The network elements can be classified as follows:



Circuit elements: The basic elements of circuits are resistance, inductance, and capacitance.

1. ***Resistance:*** Electrical resistance is the property of material. It opposes the flow of electrons through the material. Thus, resistance restricts the flow of current through the material.



The unit of resistance (R) is 'ohm' (Ω).
According to Ohm's law,

$$J = \sigma E$$

$$J = \frac{I}{A} \text{ and electric field } E = \frac{V}{\ell}$$

$$\frac{I}{A} = \sigma \times \frac{V}{\ell} \Rightarrow V = \frac{\ell}{\sigma} \times \frac{I}{A}$$

$$V = \frac{\rho \ell}{A} \cdot I$$

$\therefore\ V = RI \Rightarrow$ Ohm's law in circuit theory

$$\therefore R = \frac{\rho \ell}{A} \Omega$$

where ρ is the resistivity (Ω-m); ℓ is the length of conductor (m); and A is the cross-sectional area (m^2).

When current flows through any conductor, heat is generated due to collision of free e^- s with atoms.

The power absorbed by the resistor is given by

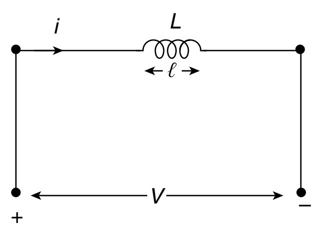
$$P = VI = I^2R = \frac{V^2}{R}\ W$$

$$P = \frac{dW}{dt}$$

$$dW = Pdt$$

$$W = \int_0^t P.dt = \int_0^t V.I\ dt = \frac{V^2 t}{R} \text{ Joules.}$$

2. **Inductance (L):** A wire of finite length when twisted into a coil, it becomes an inductor.



When a time varying current is flowing through the coil, magnetic flux will be produced.

$$\text{Total flux } \psi(t) = N\phi$$

$$\psi(t) = N\phi(t) \text{ wb}$$

$$V = \frac{d\psi(t)}{dt}$$

$$\psi(t) = Li$$

$$V = \frac{d}{dt}\{L.i\}$$

$$V_L = L.\frac{di_L(t)}{dt} \text{ Volts}$$

$$i_L = \frac{1}{L}\int_{-\infty}^{t} V.dt \text{ Amp.}$$

power $P = Vi$.

$$W = \int P.dt$$

$$W = \int L.i.di$$

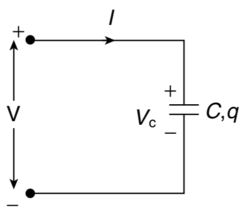
$$\boxed{W = \frac{1}{2}Li^2 \text{ Joules.}}$$

∴ The energy stored in the inductor at any instant will depend on the current flowing through the inductor at that instant.

$$L = \frac{\mu_o N^2 A}{\ell}$$

where l is the length of the inductor; N is the number of turns; and A is the cross-sectional area of coil.

3. **Capacitance:** It is the capability of an element to store electric charge within it.



$$i = \frac{dq}{dt} \text{ Amp.}$$

$$q \,\alpha\, V \Rightarrow q = CV$$

$$C = {}^{q}\!/_{V} \text{ Farads}$$

$$i_c = \frac{d}{dt}CV$$

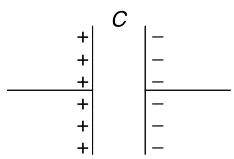
$$\boxed{\Rightarrow i_c = C\frac{dv}{dt} \text{Amp}}$$

$$V_c = \frac{1}{C}\int i_c \; dt \text{ Volts}$$

$$\Rightarrow P = Vi_c$$

$$\boxed{Wc = \int Pdt = {}^{1}\!/_{2}CV^2 \text{ Joules}}$$

For Parallel-plate Capacitance



$$\boxed{C = \frac{\varepsilon A}{d} \text{ Farad}}$$

where ε is the permittivity of material; A is the cross-sectional area of plates; and d is the distance between the plates.

Table 1.1: *Summary of relationships for the parameters*

Parameter	Basic relationship	Voltage–current relationships	Energy
R $G = {}^{1}\!/_{R}$	$V = iR$	$V_R = i_R R,\; i_R = G\cdot V_R$	$W_R = \int_{-\infty}^{t} p\cdot dt;\; W_R = \int_{-\infty}^{T} V_R.i_R dt$
L	$\psi = L.i$	$V_L = L\frac{di}{dt};\; i_L = \frac{1}{L}\int_{-\infty}^{t} V_L dt$	$W_L = 1/2\, Li^2 = \frac{1}{2}\psi.i$
C	$q = CV$	$V_C = \frac{1}{C}\int_{-\infty}^{t} i_C\cdot dt;\; i_C = C.\frac{dV_C}{dt}$	$W_C = 1/2\, CV^2 = 1/2\, q\cdot V$

NOTE

R, L, and C elements are a linear, passive, bilateral, and time-invariant at a constant temperature.

Solved Examples

Example 1

In the interval $0 > t > 4\pi$ ms, a 10 µF capacitance has a voltage $V = 25 \sin 200t$ V. The charge, power, and energy are

Solution

Charge $q = CV$

$$= 10 \times 25 \sin 200t \;(\mu C)$$

$$= 250 \sin 200t \;(\mu C)$$

Power $P = Vi$

However, $i_c = C.\dfrac{dV_c}{dt}$

$$= 10 \times 25 \times 200 \cos 200\, t \;\mu A$$

$$i_c = 50 \cos 200t \text{ mA}$$

$$\therefore P = Vi = 25 \sin 200t \,.50 \cos 200\, t \text{ mW}$$
$$= 25 \times 25 \sin 400\, t \text{ mW}$$
$$= 0.625 \sin 400\, t \text{ W}$$

$$\text{Energy } W_c = \int_{t1}^{t2} P \cdot dt$$

$$W_c = \int_{0}^{4\pi x 10^{-3}} 0.625 \;\sin 400\, t\, dt$$

$$= -\frac{0.625}{400}\left[\cos 400t\right]_0^{4\pi\times10^{-3}}$$

$$= 1.5 \text{ nJ}$$

Example 2

A capacitor of 100 μF stores 10 mJ of energy. Obtain the amount of charge stored in it. How much time does it take to build up this charge, if the charging current is 0.2 A?

(A) $Q = 1.414$ mC, $t = 1.4$ ms
(B) $Q = 1.414$ mC, $t = 7.07$ ms
(C) $Q = 2$ μC, $t = 0.1$ ms
(D) $Q = 2$ mC, $t = 10$ ms

Solution

The energy stored in a capacitor is given by

$$W_c = \frac{CV^2}{2} = \frac{1}{2}QV = \frac{Q^2}{2C} \quad W_c = \frac{CV^2}{2} = \frac{1}{2}QV = \frac{Q^2}{2C}$$

$$\therefore Q = \sqrt{2CW} = \sqrt{2\times10\times10^{-3}\times100\times10^{-6}}$$

$$= 1.414 \text{ mC; we know } i = \frac{dq}{dt}$$

$$\therefore Q = It$$

$$t = \frac{Q}{I} = \frac{1.414\times10^{-3}}{0.2} = 7.07 \text{ ms}$$

Example 3

The strength of current in 2 H inductor changes at a rate of 3 A/s. The voltage across it and the magnitude of energy stored in Inductor after 4 secs are

(A) $V_L = 6$ V, $W_L = 144$ J (B) $V_L = 1.5$ V, $W_L = 12$ J
(C) $V_L = 6$ V, $W_L = 77$ J (D) $V_L = 1.5$ V, $W_L = 144$ J

Solution

From the given data,

$$L = 2H, \frac{di_L}{dt} = 3 \text{ A/s}$$

$$V_L = L\frac{di_L}{dt} = 2\times3 = 6\text{ V}$$

$$W = \frac{1}{2}L\cdot i^2$$

$$\frac{di}{dt} = 3 \text{ A/s}$$

$$di = 3\, dt$$

$$I = 3t \text{ A/s; however, } t = 4 \text{ s}$$

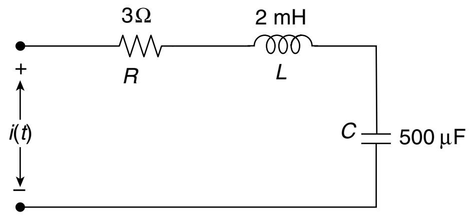
$$I = 3\times4 = 12 \text{ A}$$

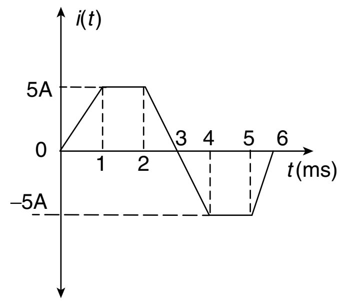
$$W = \frac{1}{2}\times2\times(12)^2$$

$$= 144 \text{ Joules}$$

Example 4

A current source $i(t)$ is applied to a series RLC circuit shown in the following figures.





The maximum voltage across resistor is

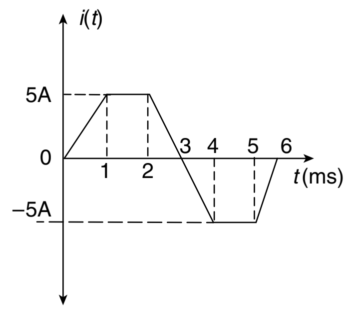
(A) 10 V (B) 15 V (C) 5 V (D) 20 V

Solution

The drop across resistance $V_R(t) = i(t)\,R$

$$V_{R(max)} = i_{max}\, R \text{ V}$$

$$V(t) = 5\times3 = 15 \text{ V}$$



Example 5

The total voltage across indicator is

(A) 0 V (B) −10 V
(C) 10 V (D) None of these

Solution

$$V_L = L.\frac{di(t)}{dt}$$

1. **For $0 \le t \le 1$ ms:**

$$i(t) = \frac{5}{1\times10^{-3}}t \text{ Amp} = 5\times10^3 t \text{ amp}$$

$$V_L = 2\times10^{-3}\times5\times10^3 = 10 \text{ V}$$

2. **For 1 ms $\le t \le$ 2 ms:**

$$i(t) = 5 \text{ A constant}$$

$$\therefore V_L = 0 \text{ V}$$

3. **For $2 \le t \le 4$ ms:**

$$A(2\times10^{-3},5)\ B\ (4\times10^{-3},-5)$$

$$I(t) = \frac{-5-5}{2\times10^{-3}}(t-2\times10^{-3}) + 5$$

$$i(t) = 5 - 5\times10^3\,(t - 2\times10^{-3}) \text{ A}$$

$$= 15 - 5\times10^3\, t \text{ A}$$

$$V_L = L\frac{di(t)}{dt} = 2\times10^{-3}[0-5\times10^3] = -10 \text{ Volts}$$

4. **For 4 ms $\le t \le$ 5 ms:**
$i(t) = 5$ and $\Rightarrow$ constants
so $V_L = 0$

5. **For 5 ms $\le t \le$ 6 ms:**

$$A\ (5\times10^{-3},-5)\ B\ (6\times10^{-3},0)$$

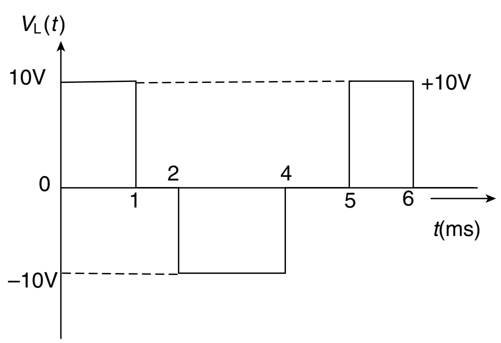
$$i_L\ (t) = 5\times10^3\, t \text{ A}$$

$$V_L(t) = L\frac{di_L(t)}{dt}$$

$$= 2\times10^{-3}\times5\times10^3$$

$$= 10 \text{ V}$$

$\therefore V_L\ (t)$ is shown in the following figure.



$$\Rightarrow \sum V_L = 10 + 0 + (-10) + 0 + 10$$

$$V_L = 10 \text{ V}$$

Example 6

A voltage $V(t) = 2\sin\omega t$ V is applied across a capacitor having time varying capacitance given by $c(t) = (2 + 0.5 \sin t)$ F. Find $i(t)$.

Solution

$$Q = CV \Rightarrow q(t) = C(t)\ V(t)$$

$$i(t) = \frac{dq(t)}{dt}$$

$$i(t) = \frac{d}{dt}\{(2+0.5 \sin t)\times 2 \sin \omega t\}$$

$$= \frac{d}{dt}\{4 \sin \omega t + \sin t \sin \omega t\}$$

$$= 4\cos \omega t\omega + \sin t \cos \omega t.\ \omega + \sin \omega t \cos t$$

$$I(t) = \sin \omega t \cos t + \omega\,[4 + \sin t] \cos \omega t$$

Example 7

If ω = 2 rad/s, the value of $i(t)$ at $t = \frac{1}{2}$ s in the above mentioned problem is

(A) 2 A (B) 4 A
(C) 6 A (D) 8 A

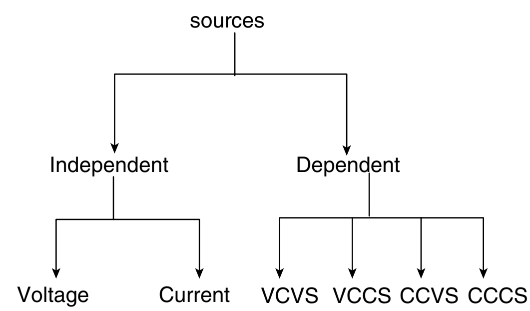
Solution

$$i(t) = [4 + \sin t]\ \omega \cos \omega t + \sin \omega t \cos t$$

$$= [4 + \sin 0.5\}\ 2\times\cos 1 + \sin 1\times\cos 0.5$$

$$= 8.016 + 0.174 = 8.033 \approx 8 \text{ A}$$

Energy Sources



Independent Sources

Ideal Voltage Source

Terminal voltage of an ideal voltage source is independent of the current supplied by it. Internal resistance of an ideal voltage source is zero.

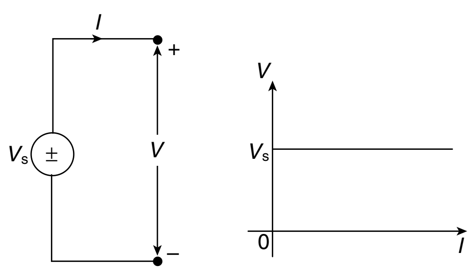


Figure 1 Ideal voltage source and *V*–*I* characteristics.

Practical Voltage Source

Practical voltage source has some finite internal resistance. Due to the presence of an internal resistance, the terminal voltage of a practical voltage source reduces with increase in current supplied.

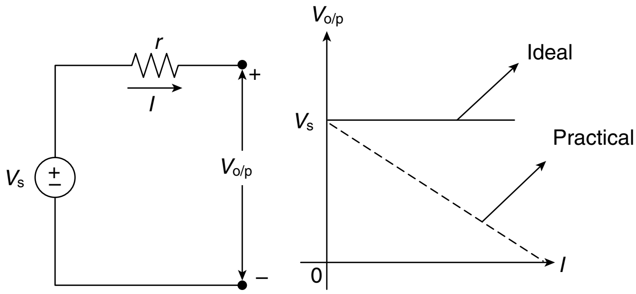


Figure 2 Practical voltage source and *V*–*I* characteristics.

By KVL

$$V_s - Ir = V_{o/p}$$
$$V_{o/p} = V_s - Ir$$

When current through any element is zero, then the potential difference is zero.

Ideal Current Source

Current delivered by any ideal current source is independent of voltage across its terminals. Internal resistance of an ideal current source is infinite.

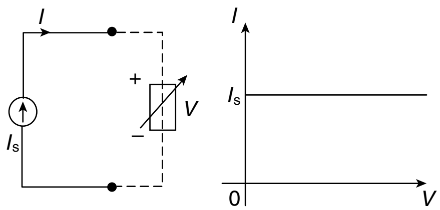
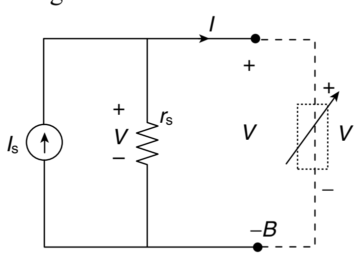


Figure 3 Ideal current source and *I*–*V* characteristics, i.e., $I = I_s$, for all *V*.

Practical Current Source

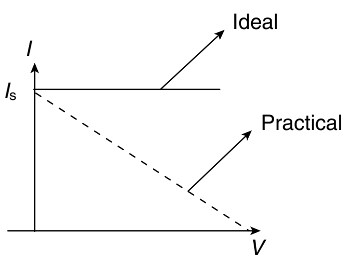
Practical current source has some finite internal resistance. Due to the presence of an internal resistance, current delivered by the practical current source reduces with increase in its terminal voltage.



By KCL

$$I_S = \frac{V}{r_s} + I$$

$$\boxed{I = I_S - \frac{V}{r_s}}$$



→ Current always choose minimum resistance path.

Dependent Sources

Voltage-controlled Voltage Source (VCVS)

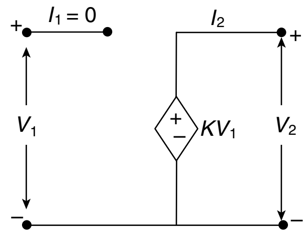
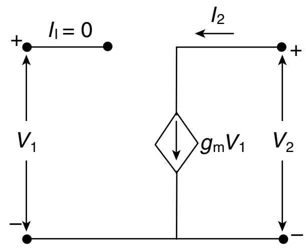


Figure 4 VCVS.

Voltage-controlled Current Source (VCCS)

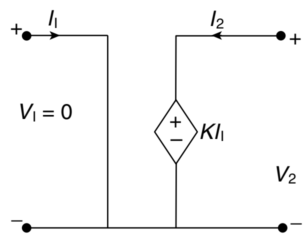


VCCS

i.e., $I_2 = g_m V_1$

∴ Output current depends on the input voltage.

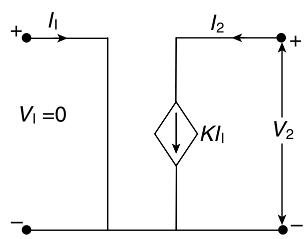
Current-controlled Voltage Source



CCVS

i.e. $V_2 = KI_1$

Current-controlled Current Source



CCCS

∴ $I_2 = KI_1$

∴ output current depends on the input current, so it is called current-controlled current source, where *K* is the constant.

Basic Definitions

Network Terminology

In this section, some of the basic terms that are commonly associated with a network are defined.

Network Element

Network elements can be either active elements or passive elements. Active elements supply power or energy to the network (outside world). For example, voltage source and current source. Passive elements either store the energy or dissipate energy in the form of heat. For example, *R*, *L*, and *C*.

Branch

A part of the network that connects the various points of the network with one another is called a branch.

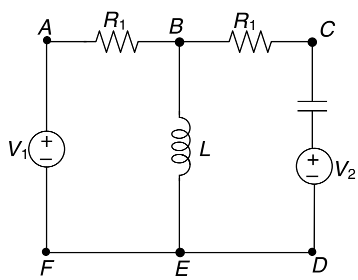
Node

A point at which two or more elements are connected together is called node. The junction points are also the nodes of network.

Mesh or Loop

Mesh is a set of branches forming a closed path in a network

Example:



Branches ⇒ $A - B$, $B - E$, etc.

Nodes ⇒ *A*, *B*, *C*, etc.

Mesh ⇒ *ABEFA*, *ABCDEFA*, *BCDEB*

Types of Elements

1. Linear and non-linear
2. Active and passive
3. Bilateral and unilateral
4. Distributed and lumped
5. Time invariant

Linear and Non-linear

A two-terminal element is said to be linear for all time '*t*', if its characteristics is a straight line through the origin, otherwise it is non-linear.

For example, a linear element must satisfy superposition and homogeneity principles.

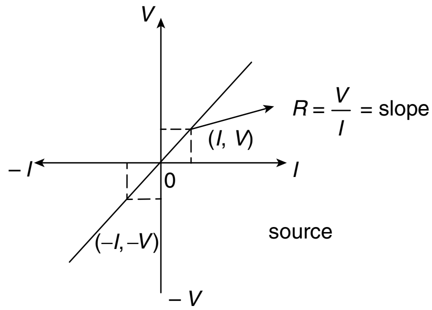


Figure 5 A bilateral, linear characteristics.

Active Network

A circuit that contains at least one source of energy is called active. An energy source may be a voltage or current. An element is said to be active if it delivers a net amount of energy to the outside world is non-zero.

For example, transistors, op-amps, batteries, etc.

Passive Network

A circuit that contains no energy source is called passive circuit. These networks consist of passive elements only. For example, *R*, *L*, *C* and thermistors, etc.

Bilateral and Unilateral Networks

A circuit whose characteristics and behaviours are same irrespective of the direction of current through various elements is called bilateral network. Otherwise, it is said to be unilateral. Example for unilateral and Diode bilateral are diode and resistors, respectively.

Lumped and Distributed Networks

A network in which all the network elements are physically separable is known as lumped network. Otherwise, it is called distributed network. Examples for lumped and distributed networks are simple *RLC* circuits and transmission lines, respectively.

DC Network and AC Network

A network consists of DC sources that are fixed polarity sources varying with time is called a DC network. A network consist of AC sources that are alternating sources periodically varying with time is called an AC network.

Kirchhoff's Laws

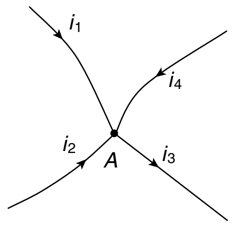
In 1847, a German physicist, Kirchhoff, formulated two fundamental laws:

1. Kirchhoff's current law (KCL)
2. Kirchhoff's Voltage law (KVL)

Kirchhoff's Current Law (KCL)

In any network, the algebraic sum of currents meeting at a point or node is always zero. Therefore, the total current leaving a junction is equal to the total current entering that junction.

Example:



For example, let us assume currents entering a junction is positive and currents leaving away from the junction is negative (vice versa).

By applying KCL at node A

By KCL ⇒ Σ Leaving currents = 0

$$i_1 + i_2 + i_4 - i_3 = 0 \Rightarrow i_3 = i_1 + i_2 + i_4$$

We know $i = \dfrac{dq}{dt}$

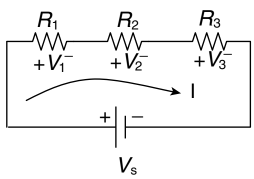
$$\frac{dq_3}{dt} = \frac{dq_1}{dt} + \frac{dq_2}{dt} + \frac{dq_4}{dt}$$

$$\boxed{q_3 = q_1 + q_2 + q_4}$$

Therefore, sum of entering charges is equal to sum of leaving charges. KCL is based on the principle of law of conservation of charge.

Kirchhoff's Voltage Law (KVL)

The algebraic sum of all branch voltages around any closed path is always zero at all instants of time.



By applying KVL to the abovementioned circuit

$$-V_1 - V_2 - V_3 + V_S = 0$$

$$\text{or} \quad V_S = V_1 + V_2 + V_3$$

$$V_S = IR_1 + IR_2 + IR_3$$

$$V_S = I(R_1 + R_2 + R_3)$$

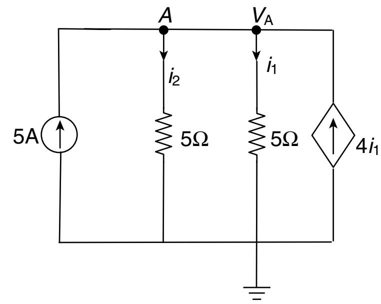
Properties

1. KCL and KVL are applied to any lumped electric circuits.
2. KCL expresses conservation of charge at each and every node. Further, KVL expresses conservation of flux or energy in every loop of electric circuit.

NOTE

Ohm's law is not applicable for active elements. It is applicable only for linear and passive elements.

Example 8



The values of i_1 and i_2 are, respectively,

(A) $i_1 = -10$ A, $i_2 = 5$ A
(B) $i_1 = i_2 = -2.5$ A
(C) $i_1 = 2.5$ A and $i_2 = -2.5$ A
(D) None of these

Solution

Applying KCL at node 'A'

$$5 + 4i_1 = i_1 + i_2$$

$$3i_1 - i_2 + 5 = 0$$

$$3\left[\frac{V_A}{5}\right] - \frac{V_A}{5} + 5 = 0$$

$$\frac{2V_A}{5} + 5 = 0$$

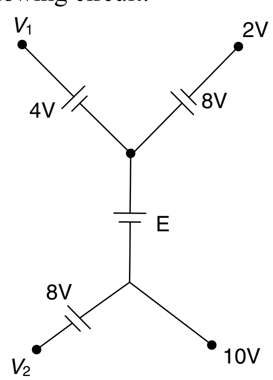
$$V_A = -12.5 \text{ V.}$$

$$i_1 = -\frac{12.5}{5} = -2.5 \text{ Amp}$$

$$i_2 = i_1 = -2.5 \text{ A}$$

Example 9

Consider the following circuit:



The node voltages V_1, V_2, and E are, respectively,

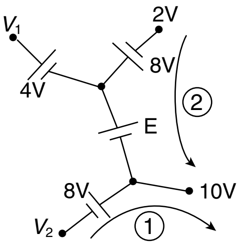
(A) $V_1 = -14$ V, $V_2 = 18$ V, $E = -2$ V
(B) $V_1 = +14$ V, $V_2 = -2$ V, $E = -2$ V
(C) $V_1 = +14$ V, $V_2 = 2$ V, $E = 0$ V
(D) $V_1 = -14$ V, $V_2 = -2$ V, $E = 0$ V

Solution

By applying KVL in loop 1

$$V_2 + 8 - 10 = 0$$

$$V_2 = 2 \text{ V}$$



By applying KVL in loop 2

$$2 + 8 - E - 10 = 0$$

$$E = 0 \text{ V}$$

By applying KVL in loop 3

$$V_1 - 4 - 0 - 10 = 0$$

$$V_1 = 14 \text{ v}$$

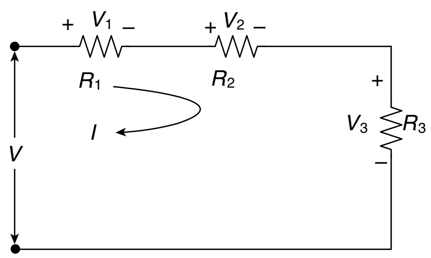
$$\therefore V_1 = 14 \text{ V and } V_2 = 2 \text{ V}$$

$$E = 0 \text{ V}$$

Circuit Elements in Series

When sending end of an element is connected to receiving end of another element and no other element is connected at that node, then those two elements are said to be connected in series. Current flowing through series-connected elements is equal.

Series-connected Resistors



In the abovementioned network R_1, R_2, and R_3 are connected in series. By applying KVL for the loop,

$$V = V_1 + V_2 + V_3$$

$$= IR_1 + IR_2 + IR_3$$

$$= [R_1 + R_2 + R_3].I$$

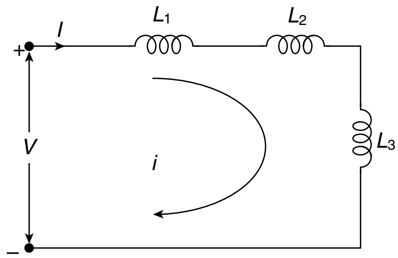
But $V = I.\ R_{eq}$.

$$\therefore I.\ R_{eq} = I\,[R_1 + R_2 + R_3]$$

$$\boxed{R_{eq} = R_1 + R_2 + R_3 + \ldots + R_n}$$

Series-connected Inductors

L_1, L_2, and L_3 are the three inductances connected in series, as shown in the figure.



By applying KVL to the circuit,

$$V = L_1\frac{di}{dt} + L_2\cdot\frac{di}{dt} + L_3\frac{di}{dt}$$

$$= (L_1 + L_2 + L_3)\frac{di}{dt}$$

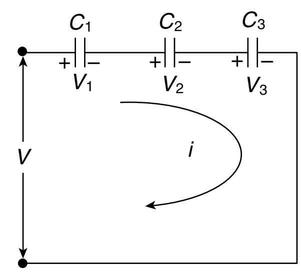
However, we know

$$V_L = L_{eq}.\ \frac{di}{dt}$$

$$\boxed{\therefore L_{eq} = L_1 + L_2 + L_3}$$

Series-connected Capacitors

If three circuit elements are capacitances connected in series, assume zero initial charges. By applying KVL for the circuit, we get



$$V = V_1 + V_2 + V_3$$

$$= \frac{1}{C_1}\int idt + \frac{1}{C_2}\cdot\int i.dt + \frac{1}{C_3}\int idt$$

$$= \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]\int idt$$

$$V = \frac{1}{C_{eq}}\cdot\int idt$$

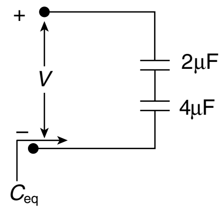
$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots + \frac{1}{C_n}}$$

Example 10

Two capacitors $C_1 = 2$ μF and $C_2 = 4$ μF are connected in series. The equivalent capacitance is

(A) 6 μF (B) 8 μF (C) 2 μF (D) 4/3 μF

Solution



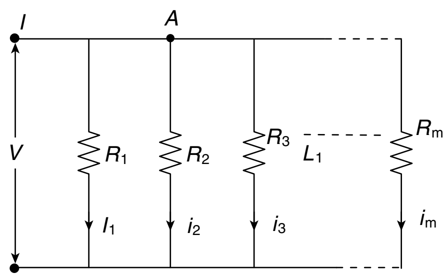
$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{4} \Rightarrow C_{eq} = \frac{2 \times 4}{2+4} = 8/6\ \mu\text{F}$$

$$C_{eq} = 4/3\ \mu\text{F}$$

Circuit Elements are Connected in Parallel

Resistors in Parallel

Resistors R_1, R_2, R_3, … R_n are connected in parallel, as shown in the following figure.



By applying KCL at node A,

$$I = i_1 + i_2 + i_3 + \ldots i_m$$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \ldots + \frac{V}{R_m}$$

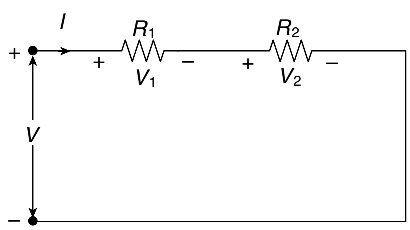
$$\boxed{\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \ldots + \frac{V}{R_m}}$$

Let $m = 2$

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

Voltage Division

A set of series-connected resistors, as shown in figure, is referred as a voltage divider.



Applied voltage V is divided into V_1 and V_2 across R_1 and R_2, respectively.

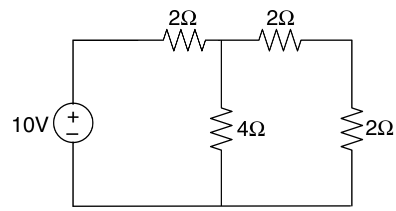
$$I = \frac{V}{R_1 + R_2}$$

$$V_1 = IR_1;\ V_2 = IR_2$$

$$\boxed{V_1 = \frac{V}{R_1 + R_2} R_1 \text{Volts}}$$

$$\text{and } V_2 = \frac{V}{R_1 + R_2} R_2 \text{ Volts}$$

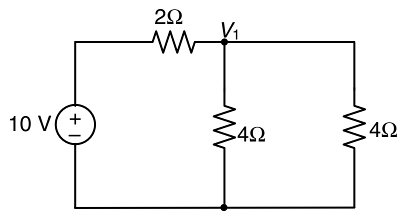
Example 11



The voltage across 4 Ω resistance is

(A) 5 V (B) 2.5 V
(C) 7.5 V (D) None of these

Solution

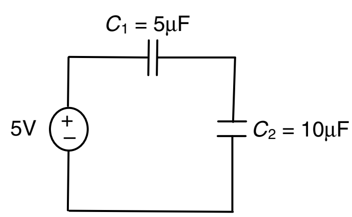


$$\frac{V_1 - 10}{2} + \frac{V_1}{4} + \frac{V_1}{4} = 0$$

$$2(V_1 - 10) + 2V_1 = 4V_1 = 20$$

$$V_1 = 5 \text{ Volts}$$

Example 12



The value of V_{C2} and V_{C1} are as follows:

(A) $V_1 = \frac{5}{3}$ volts, $V_2 = \frac{10}{3}$ volts

(B) $V_1 = \frac{10}{3}$ volts, $V_2 = \frac{5}{3}$ volts

(C) $V_1 = 2.5$ volts, $V_2 = 2.5$ volts

(D) $V_1 = \frac{3}{5}$ volts, $V_2 = \frac{10}{3}$ volts

Solution

$$V_{c2} = \frac{V}{C_1 + C_2} \times C_1$$

$$= \frac{5}{15} \times 5$$

$$= \frac{5}{3} \text{ Volts}$$

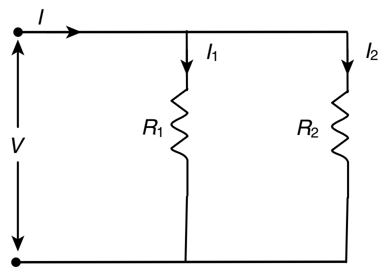
$$V_{c1} = \frac{V}{C_1 + C_2} \times C_2$$

$$= \frac{5}{15} \times 10$$

$$= \frac{10}{3} \text{ Volts}$$

Current Division

A parallel arrangement of resistors shown in the figure results in a current divider.



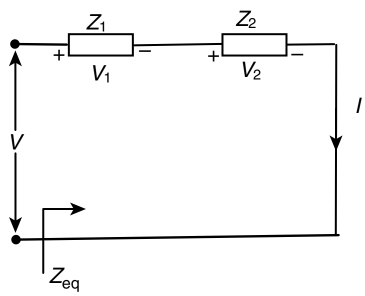
Total current I is divided into I_1 and I_2 through R_1 and R_2, respectively. I_1 and I_2 are expressed as in the following figure.

$$\boxed{I_1 = \frac{R_2}{R_1 + R_2} \cdot I}$$

$$\boxed{I_2 = \frac{R_1}{R_1 + R_2} . I}$$

NOTE

When two impedances are connected in series, voltage division across those impedances depends on elements in it.



1. If $Z = R$ or $Z = L$

$$Z_{eq} = Z_1 + Z_2$$

$$V_1 = \frac{V}{Z_{eq}} \times Z_1 \text{ volts}$$

$$V_2 = \frac{V}{Z_{eq}} \times Z_2 \text{ volts}$$

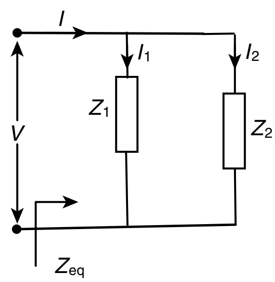
2. If $Z_1 = C_1$ and $Z_2 = C_2$, then

$$Z_{eq} = \frac{C_1 \times C_2}{C_1 + C_2}$$

$$\boxed{\begin{aligned} V_2 &= \frac{V}{C_1 + C_2} \times C_1 \\ V_1 &= \frac{V}{C_1 + C_2} \times C_2 \end{aligned}}$$

NOTE

When two impedances are connected in parallel, current division through those impedances depends on the elements present in it.



$$I = I_1 + I_2.$$

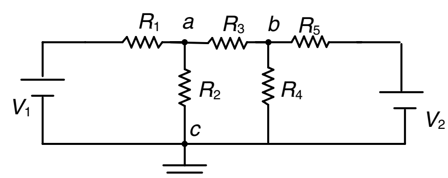
1. If $Z_1 = R_1\ \Omega$ and $Z_2 = R_2\ \Omega$ or

$$Z_{eq} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} \quad Z_1 = L_1 \text{ and } Z_2 = L_2$$

2. If $Z_1 = C_1$ and $Z_2 = C_2$, then

$$Z_{eq} = C_1 + C_2$$

Nodal Analysis



By applying KCL at node a,

$$\frac{V_a - V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a - V_b}{R_3} = 0$$

$$V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-\frac{V_b}{R_3}=\frac{V_1}{R_1} \quad (1)$$

By applying KCL at node 'b'

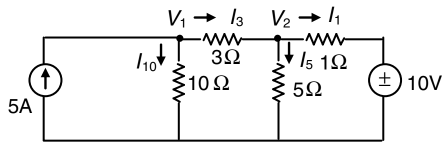
$$\frac{V_b-V_a}{R_3}+\frac{V_b}{R_4}+\frac{V_b-V_2}{R_5}=0$$

$$\left(\frac{-1}{R_3}\right)V_a+\left[\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right]V_b=\frac{V_2}{R_5} \quad (2)$$

By solving Eqs (1) and (2), the currents can be estimated.

Example 13

Write the node voltage equations and determine the currents in each branch of the network shown in figure.



Solution

By applying KCL at node 1

$$5=\frac{V_1}{10}+\frac{V_1-V_2}{3}$$

$$5=V_1\left[\frac{1}{10}+\frac{1}{3}\right]-V_2\left[\frac{1}{3}\right] \quad (1)$$

By applying KCL at node 2

$$\frac{V_2-V_1}{3}+\frac{V_2}{5}+\frac{V_2-10}{1}=0$$

$$-V_1\left[\frac{1}{3}\right]+V_2\left[\frac{1}{3}+\frac{1}{5}+1\right]=10 \quad (2)$$

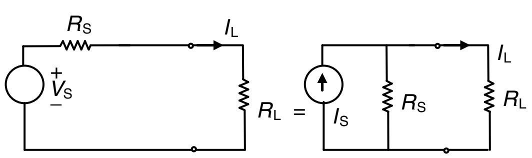
Solving Eqs (1) and (2)

$$V_1=19.85\text{ V and } V_2=10.9\text{ V}$$

$$I_{10}=\frac{V_1}{10}=1.985\text{ A}, I_3=\frac{V_1-V_2}{3}=2.98\,A$$

$$I_5=\frac{V_2}{5}=\frac{10.9}{5}=2.18\text{ A}, I_1=\frac{V_2-10}{1}=0.9\text{ A}$$

Source Transformation



V_S is the voltage and R_S is the series resistance.

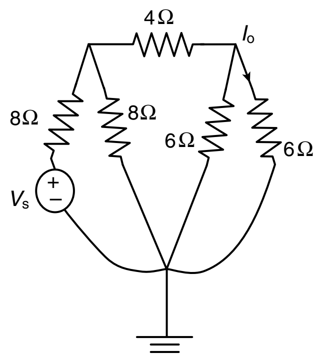
$$I_L=\frac{V_S}{R_S+R_L}$$

When transformed to a current source

$$I_S=\frac{V_S}{R_S}\text{ and } I_L=\frac{V_S}{R_S+R_L}$$

Example 14

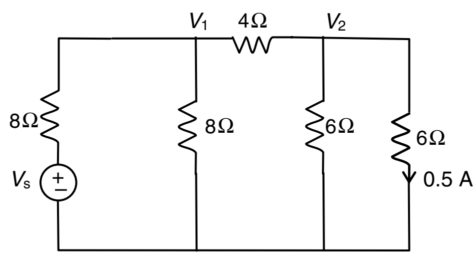
For the network shown in the figure, find V_s when $I_o = 0.5$ A.



(A) $V_s = 22$ V
(B) $V_s = 20$ V
(C) $V_s = -22$ V
(D) $V_s = +70$ V

Solution

Redraw the above network.



Given $I_o = 0.5$ A

Therefore, $I_o=\frac{V_2}{6}\Rightarrow V_2=6\times 0.5=3$ V

By applying KCL at node V_1

$$\frac{V_1-V_S}{8}+\frac{V_1}{8}+\frac{V_1-3}{4}=0$$

$$V_1-V_S+V_1+2(V_1-3)=0$$

$$4V_1-V_S=6 \quad (1)$$

By applying KCL at node (2)

$$\frac{V_2-V_1}{4}+\frac{V_2}{3}=0$$

$$3(V_2-V_1)+4V_2=0$$

$$7V_2=3\,V_1$$

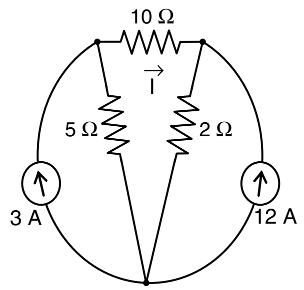
$$V_1 = \frac{7 \times 3}{3} = 7 \text{ volts}$$

$$V_S = 4V_1 - 6$$

$$= 4 \times 7 - 6 = 22 \text{ V}$$

Example 15

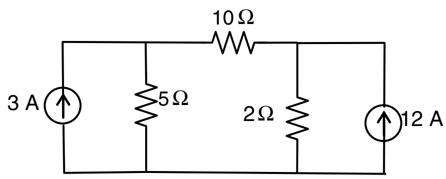
In the network shown in figure, find the current in the 10 Ω resistor.



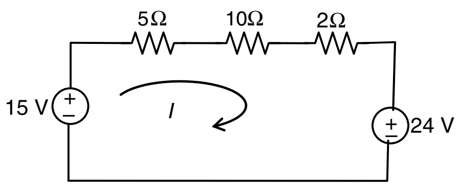
(A) $I = 0.53$ A (B) $I = 9$ A
(C) $I = -1$ A (D) $I = -0.53$ A

Solution

Redraw the above circuit



By applying source transformation



$$I = \frac{15 - 24}{17} = -\frac{9}{17}$$

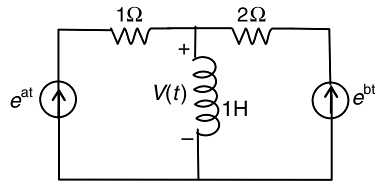
$$= -0.53 \text{ A}$$

NOTES

1. **Simple node:** It is an inter-connection of only two branches.
2. **Principle node:** It is an inter-connection of at least three branches.
3. **Super node:** If a branch between two essential non-reference nodes contain a voltage source, this is called 'super-node'.

Example 16

In the circuit, the voltage $V(t)$ is



(A) $e^{at} - e^{bt}$ (B) $e^{at} + e^{bt}$
(C) $a.e^{at} - b.e^{bt}$ (D) $a.e^{at} + b\,e^{bt}$

Solution

$$V_L = L.\frac{di_L}{dt}$$

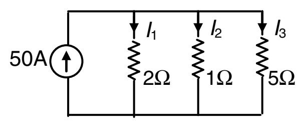
$$I_L = e^{at} + e^{bt}$$

$$V_L(t) = 1\frac{d}{dt}\left[e^{at} + e^{bt}\right]$$

$$= ae^{at} + be^{b}$$

Example 17

Determine the current in all resistors in the circuit shown in figure.



Solution

By applying KCL,

$$I = I_1 + I_2 + I_3$$

$$I = \frac{V}{2} + \frac{V}{1} + \frac{V}{5}$$

$$50 = V\left[\frac{1}{2} + \frac{1}{1} + \frac{1}{5}\right]$$

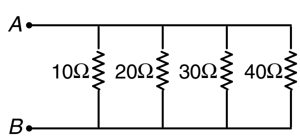
$$V = \frac{50}{1.7} = 29.41 \text{ V}$$

$$I_1 = \frac{29.41}{2} = 14.705 \text{ A}$$

$$I_2 = \frac{29.41}{1} = 29.41 \text{ A}$$

$$I_3 = \frac{29.41}{5} = 5.882 \text{ A}$$

Example 18



Determine the parallel resistance between points A and B of the circuit shown in the figure.
(A) 8 Ω (B) 6 Ω (C) 4.8 Ω (D) 3 Ω

Solution

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$= \frac{1}{10} + \frac{1}{20} + \frac{1}{30} + \frac{1}{40}$$

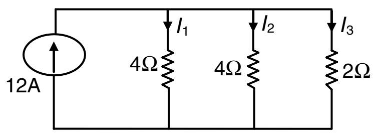
$$= 0.1 + 0.05 + 0.033 + 0.025$$

$$\frac{1}{R_T} = 0.208$$

$$R_T = 4.8\,\Omega$$

Example 19

Determine the current through each resistor in the following circuit.



Solution

$$I_1 = 12 \times \frac{4 \parallel 2}{4 \parallel 2 + 4} = 12 \times \frac{\frac{4\times 2}{4+2}}{\frac{4\times 2}{4+2} + 4} = 12 \times \frac{1.3333}{1.3333+4} = 3\text{ A}$$

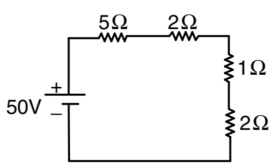
$$I_2 = 3\text{A}$$

$$I_3 = 12 - 3 - 3$$

$$I_3 = 6\text{ A}$$

Example 20

Determine the total amount of power dissipated in the following circuit.



(A) 100 W (B) 250 W (C) 150 W (D) 200 W

Solution

Total resistance $R = 5 + 2 + 1 + 2$

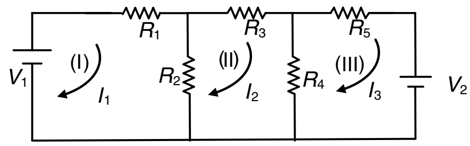
$$= 10\ \Omega$$

$$\text{Power} = \frac{V^2}{R}$$

$$= \frac{(50)^2}{10}$$

$$= 250\text{ W}$$

Mesh Analysis



Consider the network shown in the figure. V_1 and V_2 are the voltage sources. The loop currents are I_1, I_2, and I_3 in their direction as shown in the figure.

By applying KVL in loop 1

$$-I_1R_1 - I_2R_2 + V_1 = 0$$

$$V_1 = I_1(R_1 + R_2) - I_2R_2 \tag{1}$$

By applying KVL in loop 2,

$$-I_2R_3 - I_2R_4 + I_3R_4 - I_2R_2 + I_1R_2 = 0$$

$$-I_1R_2 + I_2(R_2 + R_3 + R_4) - I_3R_4 = 0 \tag{2}$$

By applying KVL in loop 3,

$$-I_3R_5 - V_2 - I_3R_4 + I_2R_4 = 0$$

$$-I_2R_4 + I_3(R_4 + R_5) = -V_2 \tag{3}$$

From Eqs (1), (2), and (3),

$$\begin{bmatrix} R_1+R_2 & -R_2 & 0 \\ -R_2 & R_2+R_3+R_4 & -R_4 \\ 0 & -R_4 & R_4+R_5 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ V_2 \end{bmatrix}$$

$$I_1 = \frac{\begin{vmatrix} V_1 & -R_2 & 0 \\ 0 & R_2+R_3+R_4 & -R_4 \\ -V_2 & -R_4 & R_4+R_5 \end{vmatrix}}{|R|}$$

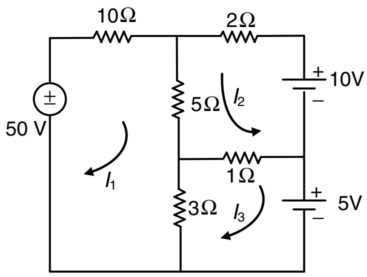
$$\text{where } [R] = \begin{bmatrix} R_1+R_2 & -R_2 & 0 \\ -R_2 & R_2+R_3+R_4 & -R_4 \\ 0 & -R_4 & R_4+R_5 \end{bmatrix}$$

$$I_2 = \frac{\begin{vmatrix} R_1+R_2 & V_1 & 0 \\ -R_2 & 0 & -R_4 \\ 0 & -V_2 & R_4+R_5 \end{vmatrix}}{|R|}$$

$$I_3 = \frac{\begin{vmatrix} R_1+R_2 & -R_2 & V_1 \\ -R_2 & R_2+R_3+R_4 & 0 \\ 0 & -R_4 & -V_2 \end{vmatrix}}{|R|}$$

Example 21

Determine the mesh currents I_1, I_2, and I_3 in the circuit shown in the figure.



Solution

By applying KVL in loop 1

$$50 = (10 + 5 + 3)\, I_1 + 5I_2 - 3I_3 \qquad (1)$$

By applying KVL in loop 2

$$10 = 2I_2 + 5(I_1 + I_2) + 1(I_2 + I_3)$$

$$10 = 5\, I_1 + 8\, I_2 + I_3 \qquad (2)$$

By applying KVL in loop 3

$$1(i_2 + i_3) + 5 + 3\,(i_3 - i_1) = 0$$

$$-\,3i_1 + i_2 + 4i_3 = -\,5 \qquad (3)$$

By Cramer's rule

$$I_1 = \frac{\begin{vmatrix} 50 & 5 & -3 \\ 10 & 8 & 1 \\ -5 & 1 & 4 \end{vmatrix}}{\begin{vmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{vmatrix}}$$

$I_1 = 3.3$ A

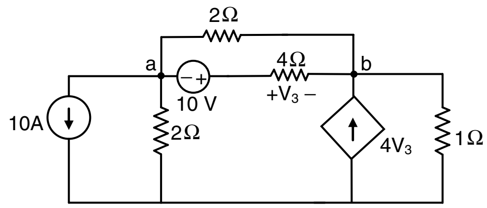
$$I_2 = \frac{\begin{vmatrix} 18 & 50 & -3 \\ 5 & 10 & 1 \\ -3 & -5 & 4 \end{vmatrix}}{\begin{vmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{vmatrix}}$$

$I_2 = -\,0.997$ A

$$I_3 = \frac{\begin{vmatrix} 18 & 5 & 50 \\ 5 & 8 & 10 \\ -3 & 1 & -5 \end{vmatrix}}{\begin{vmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{vmatrix}}$$

$I_3 = 1.47$ A

Example 22



Write nodal equations for the circuit shown in figure and find the power supplied by the 10 V source.

Solution

By applying KCL at node 'a'

$$10 + \frac{V_a}{2} + \frac{V_a - V_b}{2} + \frac{V_a + 10 - V_b}{4} = 0$$

$$1.25\ V_a - 0.75\ V_b = -12.5 \qquad (1)$$

By applying KCL at node 'b'

$$\frac{V_b - 10 - V_a}{4} + \frac{V_b - V_a}{2} - 4V_3 + \frac{V_b}{1} = 0$$

$$-\,4.75\ V_a + 5.75\ V_b = 42.b \qquad (2)$$

$$V_3 = V_a + 10 - V_b \qquad (3)$$

Solving Eqs (1), (2), and (3)

$$V_a = -11.03 \text{ Volts.}$$

and $V_b = -1.724$ V.

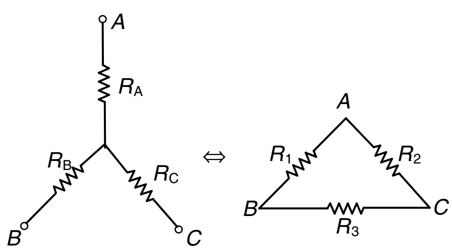
The current delivered by 10 V source is

$$I_{10} = \frac{V_a - V_b + 10}{4}$$

The power supplied by the 10 V source is

$$P_{10} = (10)I_{10} = 10\left(\frac{V_a - V_b + 10}{4}\right) = 1.735 \text{ Watts}$$

Star–Delta Transformation



If Delta is given, corresponding star elements are

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_B = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

$$R_C = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

If star elements are given, corresponding delta network elements are

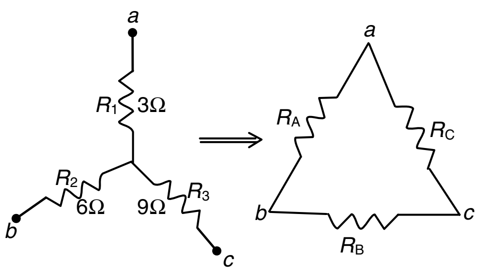
$$R_1 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

$$R_2 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

$$R_3 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

Example 23

A star-connected network with is delta equivalent is shown in the figure. R_A, R_B, and R_C (in ohms) are, respectively,



(A) 99 Ω, 33 Ω, 16.5 Ω (B) 11 Ω, 16.5 Ω, 33 Ω
(C) 11 Ω, 33 Ω, 16.5 Ω (D) 1 Ω, 3 Ω, 1.5 Ω

Solution

$$\Sigma R_A = R_1 R_2 + R_2 R_3 + R_3 R_1$$
$$= 6 \times 3 + 3 \times 9 + 9 \times 6$$
$$= 99$$

$$R_A = \frac{\sum R}{R_3} = \frac{99}{9} = 11\ \Omega$$

$$R_B = \frac{\sum R}{R_1} = \frac{99}{3} = 33\ \Omega$$

$$R_C = \frac{\sum R}{R_2} = \frac{99}{6} = 16.5\ \Omega$$

NOTES

Twelve $Z\ \Omega$ impedances are used as edges to form a cube. The equivalent impedance seen between the two diagonally opposite corners of the cube is

1. If $Z = R\ \Omega$, then $z_{eq} = \frac{5}{6} R\Omega$
2. If $Z = L$, then $z_{eq} = \frac{5}{6}\ LH$
3. If $Z = C$, then $z_{eq} = \frac{6}{5}\ CF$

Example 24

Twelve 3 H inductors are used as edges to form a cube. Determine the equivalent inductance seen between the two diagonally opposite corners of the cube.

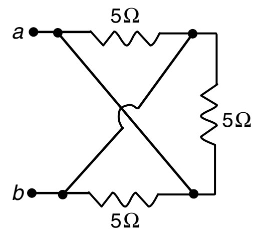
Solution

We know

$$Z_{eq} = \frac{5}{6} L = \frac{5}{6} \times 3 = 2.5\ \text{H}$$

Example 25

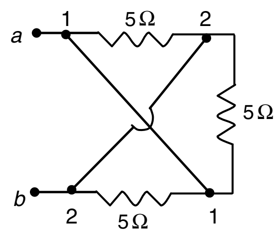
Consider the circuit shown in figure. Determine R_{ab}.

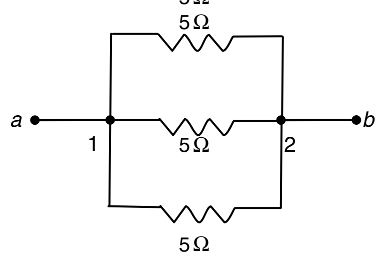


(A) 2.5 Ω (B) $\frac{5}{3}$ Ω
(C) 7.5 Ω (D) None of these

Solution

Redraw the abovementioned circuit

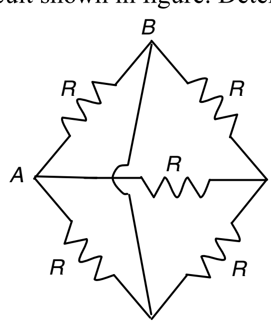




$$R_{ab} = (5 \parallel 5 \parallel 5)\ \Omega$$
$$= \frac{2.5 \times 5}{7.5} = \frac{5}{3} \Omega$$

Example 26

Consider the circuit shown in figure. Determine R_{AB}.

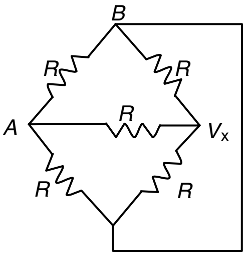


(A) $R_{AB} = 0\ \Omega$ (B) $R_{AB} = R\ \Omega$

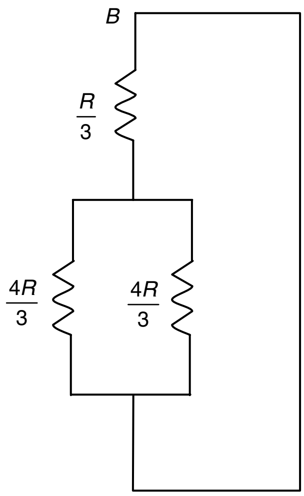
(C) $R_{AB} = 2\ R\Omega$ (D) $R_{AB} = \frac{R}{2}\ \Omega$

Solution

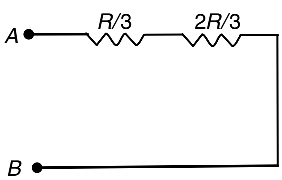
Redraw the given circuit



Convert $\Delta - y$



⇒



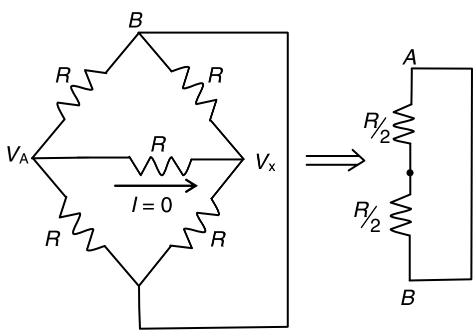
$R_{AB} = R\ \Omega$, for all values of ‘R’

The second method is as follows:

From the given circuit,

Since $R_1 \times R_4 = R_2\ R_{3,}$ bridge is in balanced condition.

$$R \times R = R \times R$$



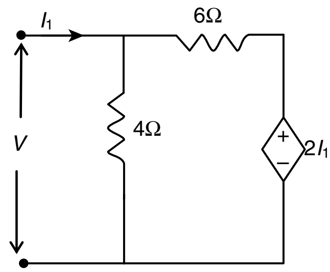
$$\Rightarrow R_{AB} = \frac{R}{2} + \frac{R}{2} = R\Omega$$

Here, $V_x = V_A$.

Therefore, $I = 0$.

Example 27

The circuit shown in figure will act as a load resistor of



(A) $\frac{1}{5}\Omega$ (B) $\frac{16}{5}\Omega$

(C) $\frac{5}{16}\Omega$ (D) None of these

Solution

From the given circuit,

$$I_1 = \frac{V}{4} + \frac{V - 2I_1}{6}$$

$$12I_1 = 3V + 2V - 2V - 4I_1$$

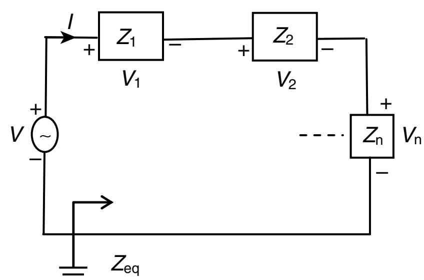
$$16_1 = 5\text{V}$$

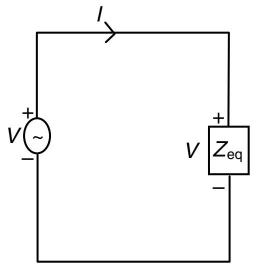
$$\frac{V}{I_1} = R = \frac{16}{5}\Omega$$

Equivalent Circuits w.r.t. Passive R, L, C, S

⇒ Two elements are said to be in series only when currents through the elements are same and two elements are said to be in parallel only when voltages across the elements are same.

Series Circuits





Let $n = 2$

$$\Rightarrow Z = Z_R = R\Omega$$

$$\Rightarrow Z = Z_L = j\omega L\Omega$$

$$\Rightarrow Z = Z_C = \frac{1}{j\omega c}\ \Omega$$

$$Z_{eq} = Z_1 + Z_2$$

If Z is equal to

i) R:

$$R_{eq} = R_1 + R_2$$

ii) L:

$$L_{eq} = L_1 + L_2$$

iii) C:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Voltage Division

$$V = Z_{eq}\ I$$

$$I = \frac{V}{Z_{eq}}$$

$\Rightarrow V_1 = IZ_1$ (by Ohm's law)

$$\Rightarrow \boxed{V_1 = \frac{V}{Z_1 + Z_2} Z_1}$$

$$\Rightarrow V_2 = Z_2\, I$$

$$\Rightarrow \boxed{V_2 = \frac{V}{Z_1 + Z_2} Z_2}$$

1. $Z = R$:

$$V_1 = \frac{V \cdot R_1}{R_1 + R_2}:$$

$$V_2 = \frac{V.R_2}{R_1 + R_2}$$

2. $Z = j\omega L$:

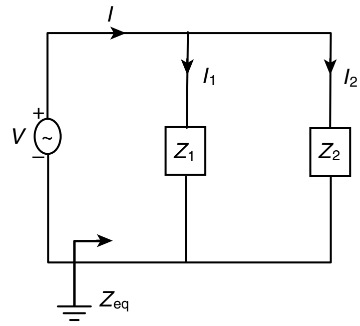
$$V_1 = \frac{V \cdot L_1}{L_1 + L_2}; V_2 = \frac{V \cdot L_2}{L_1 + L_2}$$

3. $Z = \frac{1}{j\omega C}$:

$$V_1 = \frac{V \cdot C_2}{C_1 + C_2}:$$

$$V_2 = \frac{V \cdot C_1}{C_1 + C_2}$$

Parallel Circuits



$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

If $Z = R$

$$R_{eq} = \frac{R_1\ R_2}{R_1 + R_2}$$

If $R_1 = R_2 = R\ \Omega$

$$R_{eq} = \frac{R}{2}\Omega$$

If $Z = j\omega L$: $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$

$$L_{eq} = \frac{L_1 \cdot L_2}{L_1 + L_2}$$

If $Z = Z_c$, $C_{eq} = C_1 + C_2$

Current Division

$$V = z_{eq}\, I$$

$$\Rightarrow \boxed{V = \frac{Z_1\, Z_2}{Z_1 + Z_2} \times I}$$

$$I_1 = \frac{V}{Z_1}$$

$$I_1 = Z_2 * I/(Z_1 + Z_2)$$

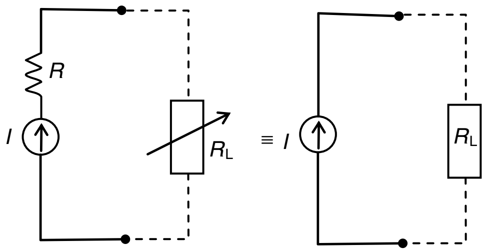
$$I_2 = Z_1 * I/(Z_1 + Z_2)$$

For example, $Z_1 = \frac{1}{j\omega c1}; Z_2 = \frac{1}{j\omega c2}$

$$I_1 = \frac{I \cdot C_1}{C_1 + C_2}; I_2 = \frac{I \cdot C_2}{C_1 + C_2}$$

NOTES

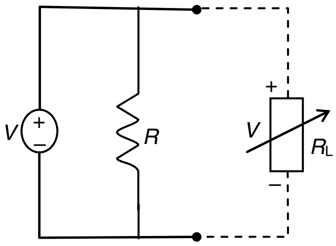
1. Resistance connected in series with an ideal current source (internal resistance of an ideal current source) does not have any effect on current supplied by it. Therefore, it can be neglected.



$R \neq \infty$

The load current is independent of R value, $i^2 R \neq 0$.

2. Resistance connected in parallel with an ideal voltage source does not have any effect on voltage offered by it. Therefore, it can be neglected.



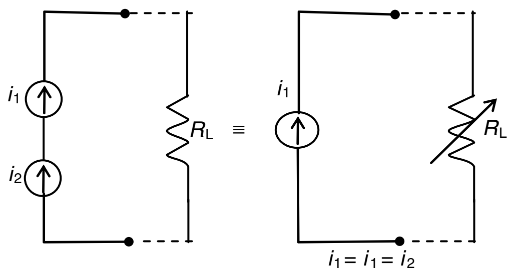
Here $R \neq 0$

Therefore, a resistor in parallel with an ideal voltage source can be neglected in the analysis.

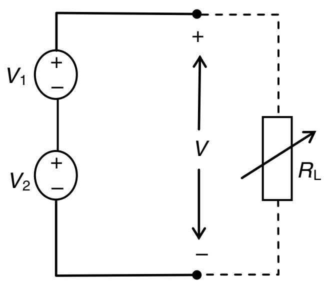
Therefore, the load voltage is independent of R value

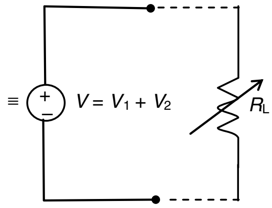
$\frac{V^2}{R} \neq 0$

3. Equivalent of series-connected current sources is a single current source and are of same magnitude.



4. Equivalent of two series-connected voltage sources is the sum of those two with their polarity.



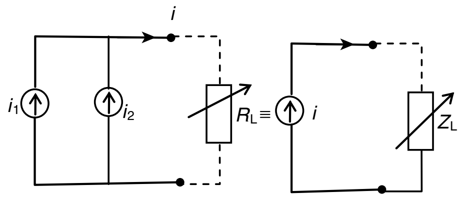


By applying KVL,

$$\Rightarrow V_2 + V_1 - V = 0$$

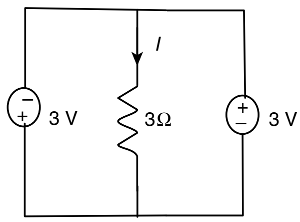
$$V = V_1 + V_2$$

5. Equivalent of two parallel-connected current sources is equal to sum of those two with their relevant polarity.



$$\therefore i = i_1 + i_2$$

Example 28



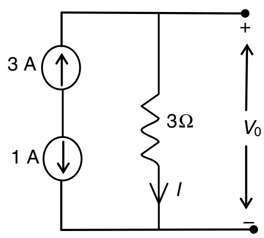
The current I is

(A) 1 A (B) −1 A
(C) 2 A (D) Indeterminate

Solution

Two voltage sources are in parallel and different this is violates KVL and KCL and therefore indeterminate.

Example 29



Solution

Determine voltage V_o

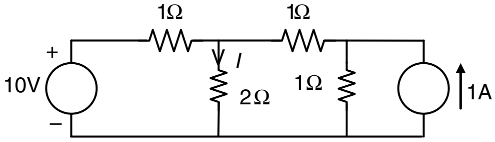
$$V_o = I \times 3$$
$$= (3 - 1)\,3$$
$$= 6 \text{ Volts}$$

Exercises

Practice Problems I

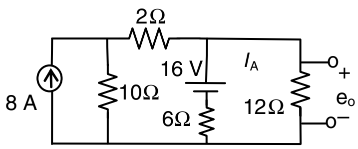
Direction for questions 1 to 21: Select the correct alternative from the given choices.

1. The current in the 2 Ω resistor '*I*' is



(A) $\frac{1}{8}$ A (B) $\frac{3}{8}$ A (C) $\frac{21}{8}$ A (D) $\frac{23}{8}$ A

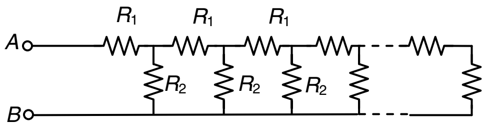
2. The voltage e_o in the figure is



(A) 48 V (B) 24 V (C) 36 V (D) 28 V

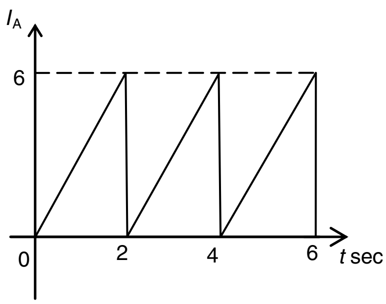
3. The driving point impedance of the infinite ladder network shown in the figure is ______.

Given $R_1 = 3\ \Omega$ and $R_2 = 2\ \Omega$



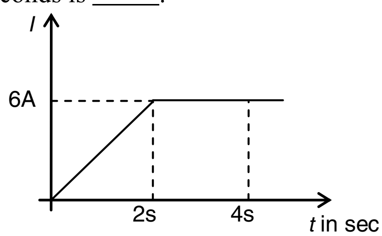
(A) $\frac{3}{2}\Omega$ (B) $\frac{2}{3}\Omega$ (C) $\left[3+\frac{2}{3}\right]\Omega$ (D) $\sqrt{21}$

4. The current wave formed in a pure resistor of 5 Ω is shown in the figure. The power dissipated in the resistor is



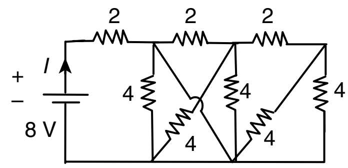
(A) 20 W (B) 45 W (C) 60 W (D) 90 W

5. Figure shows the waveform of the current passing through an inductor of resistance 1 Ω and inductance 2 H. The energy absorbed by the inductor in the first four seconds is ______.



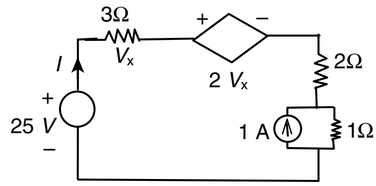
(A) 132 J (B) 98 J (C) 144 J (D) 168 J

6. In the circuit of the given figure, the source current 'I' is



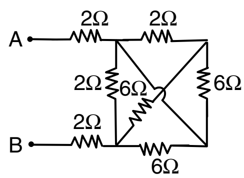
(A) 2 A (B) 4 A (C) 1 A (D) $\frac{8}{3}$ A

7. In the circuit shown in the figure, the current *I* is



(A) $\frac{25}{3}$ A (B) $\frac{25}{6}$ A (C) 2 A (D) 3 A

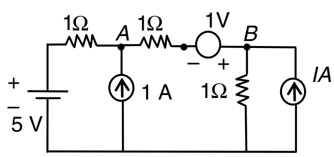
8. In the circuit shown in the figure,



R_{AB} =?

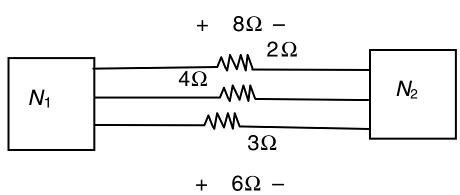
(A) $\frac{21}{4}$ Ω (B) $\frac{5}{6}$ Ω (C) 10 Ω (D) 8 Ω

9. What should be the value of current '*I*' to have zero current flowing through *AB*?



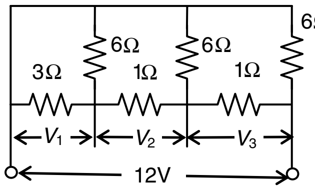
(A) 2 A (B) 4 A (C) −4 A (D) −2 A

10. The two electrical sub-networks N_1 and N_2 are connected through three resistors, as shown in the figure. The voltage across the 2 Ω resistor is 8 V and the 3 Ω resistor is 6 V. The voltage across the 4 Ω resistor is



(A) 24 V (B) −24 V (C) 8 V (D) −8 V

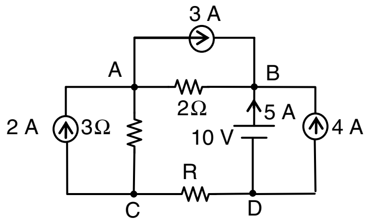
11. In the circuit shown in the figure, the voltages V_1, V_2, and V_3 are



(A) $\frac{8}{3}, \frac{16}{3}, 4$ (B) $\frac{16}{3}, \frac{8}{3}, \frac{8}{3}$

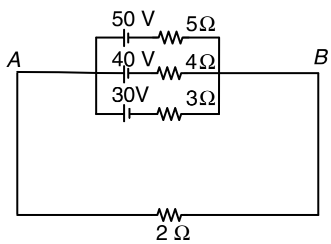
(C) $\frac{16}{3}, \frac{8}{3}, 4$ (D) $4, \frac{16}{3}, \frac{8}{3}$

12. In the network shown in the figure, the current in resistor R is



(A) 2 A (B) 3 A (C) 4 A (D) 9 A

13. In the following circuit, the voltage AB is



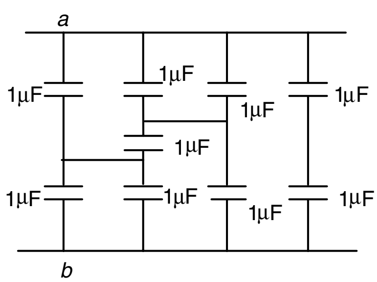
(A) 35 V (B) 28.2 V (C) 38.3 V (D) 42.6 V

14. A network contains linear resistors that are connected in series across an ideal voltage source. If all the resistances are halved and the voltage is doubled, then the voltage across each resistor becomes

(A) doubled (B) halved

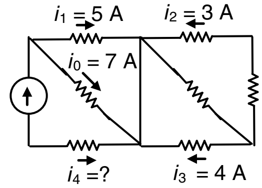
(C) not changed (D) None of these

15. Obtain the equivalent capacitance of the network given



(A) 1 μF (B) 0.8 μF (C) 1.9 μF (D) 2.6 μF

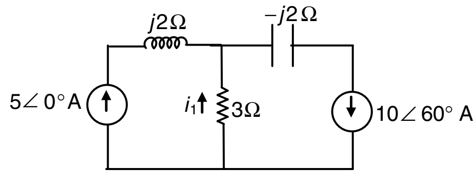
16. The current i_4 in the circuit of the figure equal to



(A) 12 A (B) −12 A

(C) 4 A (D) None of these

17.

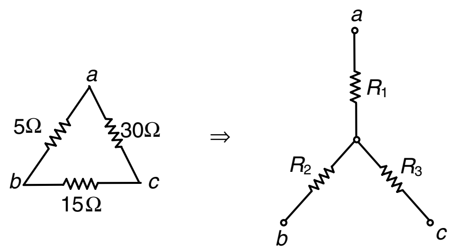


For the circuit shown in the figure, the instantaneous current $i_1(t)$ is

(A) $\frac{10\sqrt{3}}{2}\angle 90°$ A (B) $\frac{10\sqrt{3}}{2}\angle -90°$Amp

(C) $5\angle 60°$ A (D) $5\angle -60°$ A

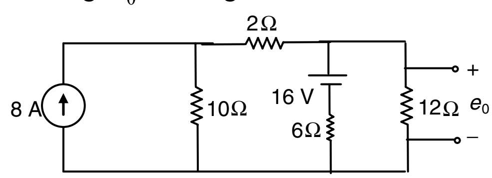
18. A delta-connected network with its Wye equivalent is shown in the given figure. The resistance R_1, R_2, and R_3 (in Ohm) are, respectively,



(A) 1.5, 3, and 9 (B) 3, 9, and 1.5

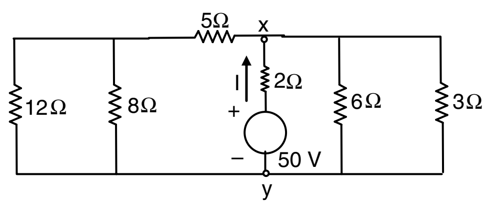
(C) 9, 3, and 1.5 (D) 3, 1.5, and 9

19. The voltage e_0 in the figure



(A) 48 V (B) 24 V (C) 36 V (D) 28 V

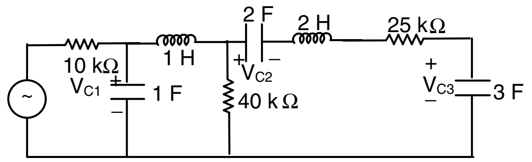
20.



The current I supplied by the source 50 V is

(A) 25 A (B) 13.7 A (C) 9.8 A (D) 3.66 A

21. The voltages V_{C1}, V_{C2}, and V_{C3} across the capacitors in the circuit in the given figure, under steady state are, respectively,



(A) 80 V, 32 V, 48 V (B) 80 V, 48 V, 32 V
(C) 20 V, 8 V, 12 V (D) 20 V, 12 V, 8 V

Practice Problems 2

Direction for questions 1 to 19: Select the correct alternative from the given choices.

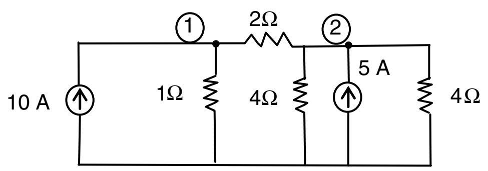
1. A network contains linear resistors that are connected in series across an ideal voltage source. If all the resistances are halved and the voltage is doubled, then the voltage across each resistor becomes
(A) doubled (B) halved
(C) not changed (D) None of these

2. Twelve similar conductors of 1 Ω resistance form a cubical framework. Then, the resistance between two adjacent corners, two opposite corners of one face, and two opposite corners of the cube are

(A) $\frac{3}{4}, \frac{5}{6}, \frac{7}{12}$ (B) $\frac{7}{4}, \frac{5}{6}, \frac{3}{4}$

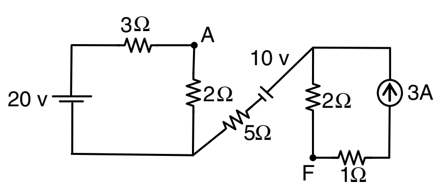
(C) $\frac{7}{12}, \frac{3}{4}, \frac{5}{6}$ (D) $\frac{12}{7}, \frac{4}{3}, \frac{6}{5}$

3. In the network shown in the figure, the voltage at node 2 is



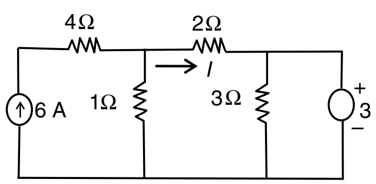
(A) 2 V (B) 10 V (C) 6 V (D) 4 V

4. In the network shown in the figure, the voltage AF is



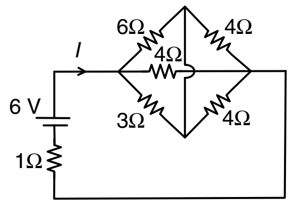
(A) 4 V (B) –4 V (C) 6 V (D) 2 V

5. For the circuit shown in the figure, the current '*I*' is given by



(A) 2 A (B) 3 A (C) 1 A (D) zero

6. The current '*I*' supplied by the source in the figure is



(A) 2 A (B) 3 A (C) $\frac{3}{2}$ A (D) $\frac{2}{3}$ A

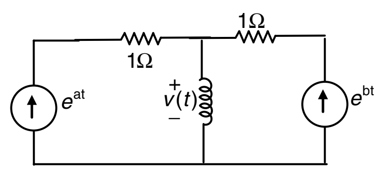
7. A resistance of 10 Ω is connected in series with two resistances of 20 Ω arranged in parallel. What resistance should be shunted across this parallel combination so that the total current taken shall be 2 A with 30 V applied?
(A) 5 Ω (B) 10 Ω (C) 20 Ω (D) 25 Ω

8. The resistance of a strip of conductor is *R*Ω. If the strip is elongated such that its length is doubled, the resistance of the strip is given by
(A) 4 *R* (B) 2 *R* (C) $\frac{R}{2}$ (D) *R*

9. The current $i(t)$ through a 10 Ω resistor in series with an inductance is given by $i(t) = 3 + 4\sin(100t + 45°) + 4\sin(300t + 60°)$ A. The RMS value of the current and the power dissipated in the circuit are
(A) $\sqrt{41}$ A, 410 W, respectively
(B) $\sqrt{35}$ A, 350 W, respectively
(C) 5 A, 250 W, respectively
(D) 11 A, 1210 W, respectively

10. The nodal method of circuit analysis is based on
(A) KVL and Ohm's law
(B) KCL and Ohm's law
(C) KCL and KVL
(D) KCL, KVL, and Ohm's law

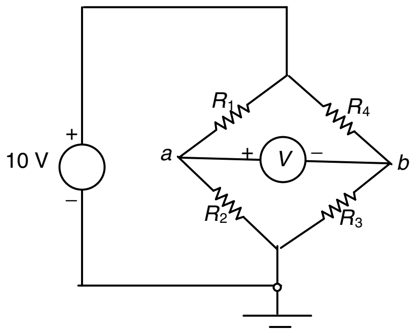
11. In the given circuit, the voltage $v(t)$ is



(A) $e^{at} - e^{bt}$ (B) $e^{at} + e^{bt}$
(C) $a\,e^{at} - b\,e^{bt}$ (D) $a\,e^{at} + b\,e^{bt}$

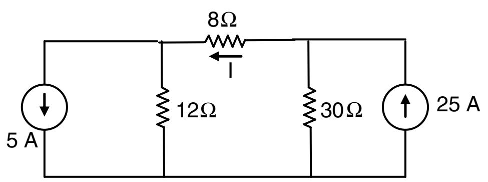
12. The rms value of the voltage defined by $v(t) = 5 + 5\sin(314t + \pi/6)$ is
(A) 5 V (B) 2.5 V (C) 6.12 V (D) 10 V

13. If $R_1 = R_2 = R_4 = R$ and $R_3 = 1.1R$ in the bridge circuit shown in the figure, then the reading in the ideal voltmeter connected between a and b is



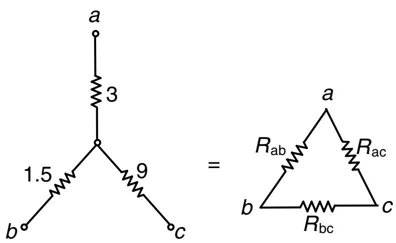
(A) 0.238 V (B) 0.138 V
(C) –0.238 V (D) 1 V

14.



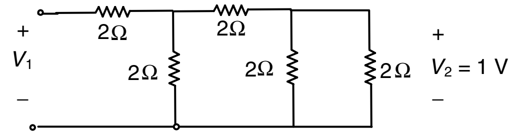
The current I in the circuit is
(A) 20 A (B) –20 A
(C) 16.2 A (D) –16.12 A

15.



The value of R_{ab}, R_{ac}, and R_{bc} are
(A) 30 Ω, 15 Ω, 5 Ω (B) 5 Ω, 30 Ω, 15 Ω
(C) 5 Ω, 30 Ω, 15 Ω (D) 15 Ω, 5 Ω, 30 Ω

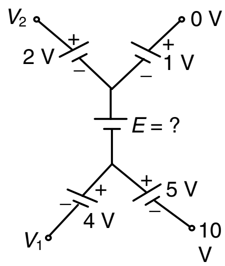
16.



If $V_2 = 1$ V in the abovementioned network, the value of V_1 will be
(A) 2.5 V (B) 4 V (C) 5 V (D) 8 V

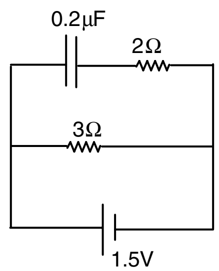
17. If each branch of a Delta circuit has impedance $\sqrt{3}Z$, then each branch of the equivalent Wye circuit has impedance
(A) $\dfrac{Z}{\sqrt{3}}$ (B) $3Z$ (C) $3\sqrt{3}Z$ (D) $\dfrac{Z}{3}$

18. In the given circuit, the value of the voltage source E is



(A) –16 V (B) 4 V (C) –6 V (D) 16 V

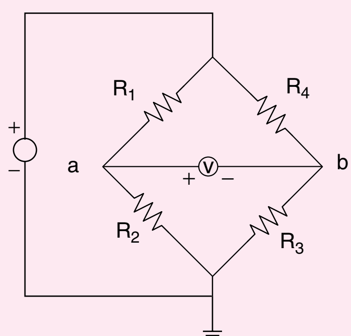
19. What is the current through resistor 2 Ω in the circuit given?



(A) 0.6 V (B) 1.8 (C) 0.9 V (D) 0 V

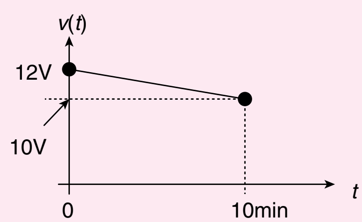
Previous Years' Questions

1. If $R_1 = R_2 = R_4 = R$ and $R_3 = 1.1\ R$ in the bridge circuit shown in figure, then the reading in the ideal voltmeter connected between a and b is **[2005]**



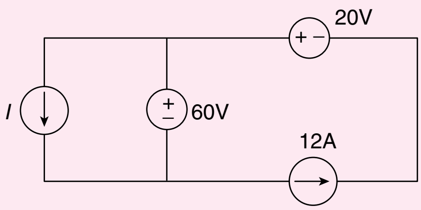
(A) 0.238 V (B) 0.138 V (C) –0.238 V (D) 1 V

2. A fully charged mobile phone with a 12 V battery is good for a 10–min talk time. Assume that, during the talk time, the battery delivers a constant current of 2 A and its voltage drops linearly from 12 V to 10 V, as shown in the figure. How much energy does the battery deliver during this talk time? **[2009]**



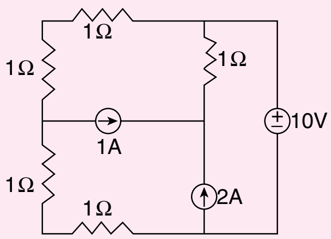
(A) 220 J (B) 12 kJ
(C) 13.2 kJ (D) 14.4 kJ

3. In the interconnection of ideal sources shown in the figure, it is known that the 60 V source is absorbing power



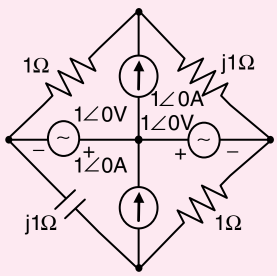
Which of the following can be the value of the current source I? **[2009]**
(A) 10 A (B) 13 A (C) 15 A (D) 18 A

4. In the following circuit, the power supplied by the voltage source is **[2010]**



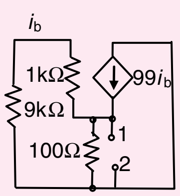
(A) 0 W (B) 5 W
(C) 10 W (D) 100 W

5. In the following circuit, the current through the inductor is **[2012]**



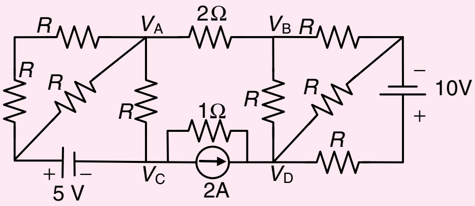
(A) $\dfrac{2}{1+j}$ A (B) $\dfrac{-1}{1+j}$ A (C) $\dfrac{1}{1+j}$ A (D) 0A

6. The impedance looking into nodes 1 and 2 in the given circuit is **[2012]**



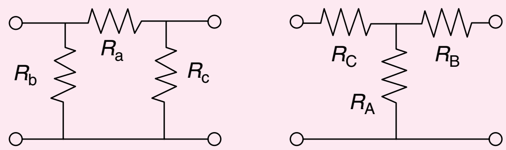
(A) 50 Ω (B) 100 Ω
(C) 5 kΩ (D) 10.1 kΩ

7. If $V_A - V_B = 6$ V, then $V_C - V_D$ is **[2012]**



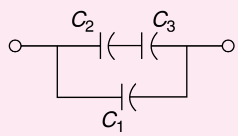
(A) −5 V (B) 2 V (C) 3 V (D) 6 V

8. Consider a delta connection of resistors and its equivalent star connection as shown in the following figure. If all elements of the delta connection are scaled by a factor k, $k > 0$, the elements of the corresponding star equivalent will be scaled by a factor of **[2013]**



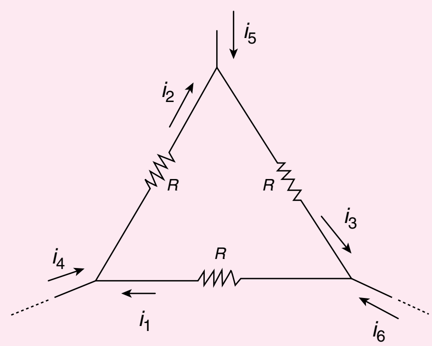
(A) k^2 (B) k (C) $1/k$ (D) $\sqrt{k}$

9. Three capacitors C_1, C_2, and C_3, whose values are 10 μF, 5 μF, and 2 μF, respectively, have breakdown voltages of 10 V, 5 V, and 2 V, respectively. For the interconnection shown in the figure, the maximum safe voltage in Volts that can be applied across the combination and the corresponding total charge in μC stored in the effective capacitance across the terminals are, respectively, **[2013]**



(A) 2.8 and 36 (B) 7 and 119
(C) 2.8 and 32 (D) 7 and 80

10. Consider the configuration shown in the figure, which is a portion of a larger electrical network **[2014]**

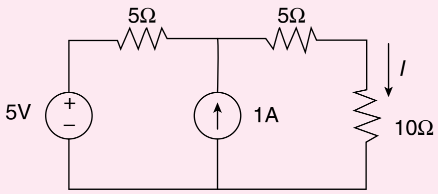


For $R = 1\ \Omega$ and currents $i_1 = 2$ A, $i_4 = -1$ A, and $i_5 = -4$ A, which one of the following is true?

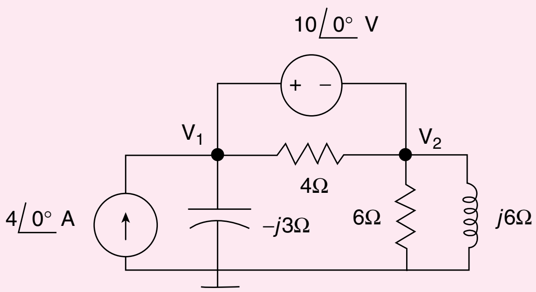
(A) $i_6 = 5$ A
(B) $i_3 = -4$ A
(C) Data is sufficient to conclude that the supposed currents are impossible.
(D) Data is insufficient to identify the currents i_2, i_3, and i_6

11. A Y-network has resistances of 10 Ω each in two of its arms, while the third arm has a resistance of 11 Ω. In the equivalent Δ-network, the lowest value (in Ω) among the three resistances is ________. **[2014]**

12. In the following figure, the value of the current I (in Amperes) is ________ **[2014]**

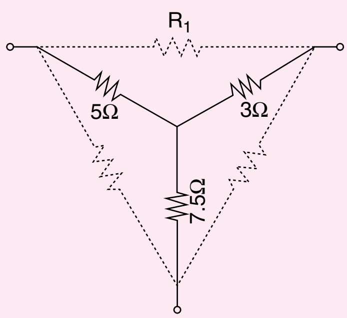


13. In the circuit shown in the figure, the value of node voltage V_2 is **[2014]**

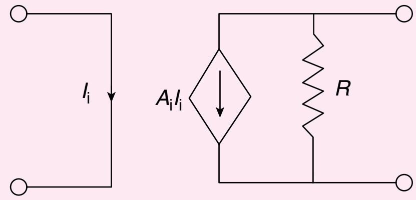


(A) $22 + j\,2$ V
(B) $2 + j\,22$ V
(C) $22 - j\,2$ V
(D) $2 - j\,22$ V

14. For the Y-network shown in the figure, the value of R_1 (in Ω) in the equivalent Δ-network is _____. **[2014]**

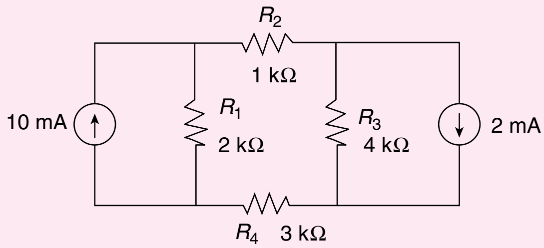


15. The circuit shown in the figure represents a **[2014]**

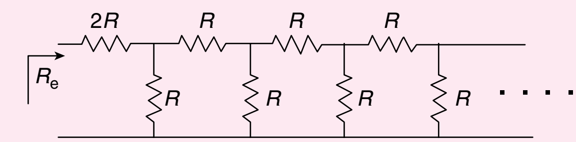


(A) voltage-controlled voltage source
(B) voltage-controlled current source
(C) current-controlled current source
(D) current-controlled voltage source

16. The magnitude of current (in mA) through the resistor R_2 in the following figure is ________ **[2014]**

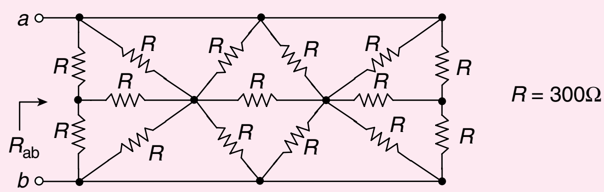


17. The equivalent resistance in the infinite ladder network shown in the figure is R_e. **[2014]**

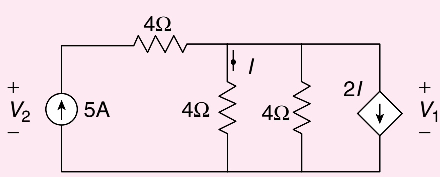


The value of R_e/R is ________

18. In the network shown in the figure, all resistors are identical with $R = 300$ Ω. The resistance R_{ab} (in Ω) of the network is ______. **[2015]**

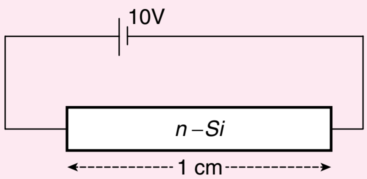


19. In the given circuit, the values of V_1 and V_2, respectively, are **[2015]**

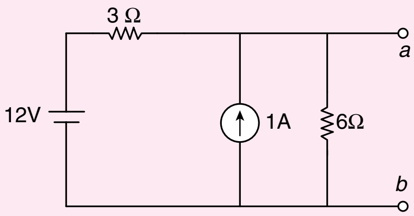


(A) 5 V, 25 V
(B) 10 V, 30 V
(C) 15 V, 35 V
(D) 0 V, 20 V

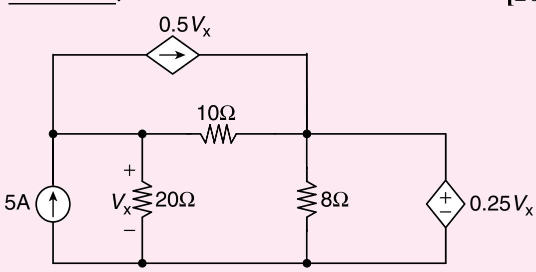
20. A dc voltage of 10 V is applied across an n-type silicon bar having a rectangular cross-section and a length of 1 cm as shown in figure. The donor doping concentration N_D and the mobility of electrons μ_n and 10^{16} cm^{-3} and 1000 $cm^{-2}V^{-1}s^{-1}$, respectively. The average time (in μs) taken by the electrons to move from one end of the bar to other end is ______. **[2015]**



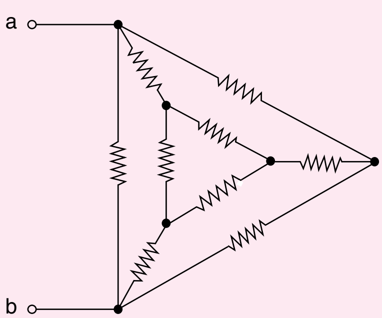
21. For the circuit shown in the figure, the Thevenin's equivalent voltage (in volts) across terminals *a-b* is ______. **[2015]**



22. In the circuit shown, the voltage V_x (in volts) is ______. **[2015]**



23. In the given circuit, each resistor has a value equal to 1 Ω

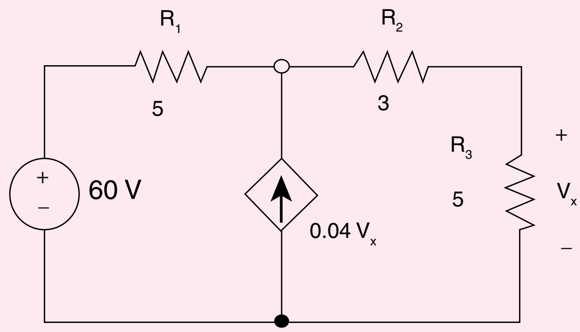


What is the equivalent resistance across the terminals *a* and *b*? **[2016]**

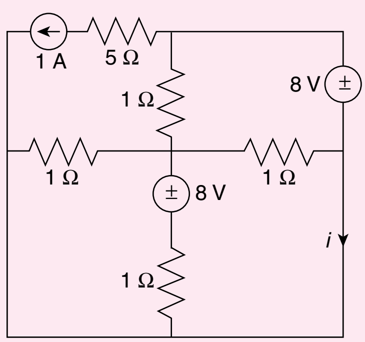
(A) $\frac{1}{6}$ Ω (B) $\frac{1}{3}$ Ω

(C) $\frac{9}{20}$ Ω (D) $\frac{8}{15}$ Ω

24. In the circuit shown in the figure, the magnitude of the current (in amperes) through R_2 is __________. **[2016]**



25. In the figure shown, the current i(in ampere) is _________ . **[2016]**



Answer Keys

Exercises

Practice Problems 1

1. C	**2.** D	**3.** D	**4.** C	**5.** C	**6.** B	**7.** C	**8.** A	**9.** B	**10.** B
11. C	**12.** D	**13.** C	**14.** A	**15.** C	**16.** B	**17.** A	**18.** D	**19.** D	**20.** B
21. B									

Practice Problems 2

1. A	**2.** C	**3.** B	**4.** A	**5.** B	**6.** A	**7.** B	**8.** A	**9.** C	**10.** B
11. D	**12.** C	**13.** C	**14.** C	**15.** C	**16.** D	**17.** A	**18.** A	**19.** D	

Previous Years' Questions

1. C **2.** C **3.** A **4.** A **5.** C **6.** A **7.** A **8.** B **9.** C **10.** A
11. 29.08 to 29.10 **12.** 0.5 **13.** D **14.** 9 to 11 **15.** C **16.** 2.79 to 2.81 **17.** 2.6 to 2.64
18. 99.5 to 100.5 **19.** A **20.** 95 to 105. **21.** 10 **22.** 7.95 to 8.05. **23.** D **24.** 5A
25. −1 Amp