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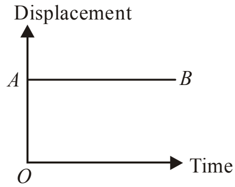
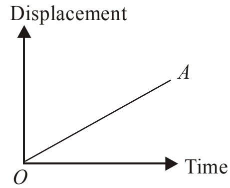
CHAPTER

Kinematics and Vectors

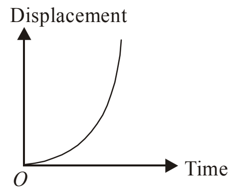
- Mechanics can be divided under two branches- (i) **statics** which deals with the study of stationary objects and (ii) **dynamics** which deals with the study of moving objects.
- An object is said to be at **rest** if it does not change its position with time with respect to its surroundings.
- An object is said to be in **motion** if it changes its position with time, with respect to its surroundings.
- **Rest and motion are relative**. It means an object observed in one frame of reference may be at rest, the same object can be in motion in another frame of reference.
- An object can be considered as a point object if during motion in a given time, it covers a distance much greater than its own size.
- The motion of an object is said to be **one dimensional motion** if only one out of the three coordinates specifying the position of the object changes with respect to time. In such a motion, an object moves along a straight line.
- The motion of an object is said to be **two dimensional motion** if two out of the three coordinates specifying the position of the object change with respect to time. In such a motion, the object moves in a plane.
- The motion of an object is said to be **three dimensional motion** if all the three coordinates specifying the position of the object change with respect to time. In such a motion, the object moves in a space.
- The **distance** travelled by an object is defined as the length of the actual path traversed by an object during motion in a given interval of time. Distance is a scalar quantity. Its value can never be zero or negative, during the motion of an object.
- The **displacement** of an object in a given interval of time is defined as the change in the position of the object along a particular direction during that time and is given by the straight line joining the initial position to final position. The displacement of an object can be positive, zero or negative. The displacement of an object between two positions has a unique value, which is the shortest distance between them. The magnitude of the displacement of an object in a given time interval can be equal or less than the actual distance travelled but never greater than the distance travelled.
- For a stationary body, the displacement-time graph is a straight line (AB) parallel to time axis.



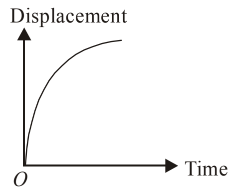
- When a body is moving with a constant velocity, then displacement-time graph will be a straight line.



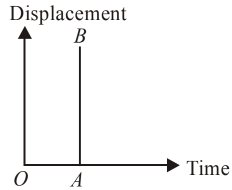
- When a body is moving with a constant acceleration, the displacement-time graph is a parabola.



- When a body is moving with a constant retardation, the displacement-time graph is a curve which bends downwards.



- When a body is moving with infinite velocity, the time-displacement curve is a straight line *AB* parallel to displacement axis. But such motion of a body is never possible.



- The **speed** of an object is defined as the time rate of change of position of the object in any direction,
  i.e. speed = distance travelled/(time taken).
  Speed is a scalar quantity. It can be zero or positive but never negative.

$$v = (s/t) \text{ m/s}.$$

- An object is said to be moving with a **uniform speed**, if it covers equal distances in equal intervals of time, howsoever small these intervals may be.
- An object is said to be moving with a **variable speed** if it covers equal distances in unequal intervals of time or unequal distances in equal intervals of time, howsoever small these intervals may be.
- The **average speed** of an object for the given motion is defined as the ratio of the total distance travelled by the object to the total time taken.

  i.e. $\text{Average speed} = \dfrac{\text{total distance travelled}}{\text{total time taken}}$
- **Average speed-harmonic mean :** When a body travels equal distance with speeds v_1 and v_2, the average speed (v) is the harmonic mean of the two speeds.

$$\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}.$$

- **Average speed and average of speeds:** When a body travels for equal time with speeds v_1 and v_2, the average speed v is the arithmetic mean of the two speeds.

$$v = \frac{v_1 + v_2}{2}.$$

  This speed is same as average of speeds.
- The speed of an object at a given instant of time is called its **instantaneous speed**.

$$v = \frac{ds}{dt}$$

- The **velocity** of an object is defined as the time rate of change of displacement of the object.

  i.e. velocity = displacement/time taken.

$$\text{velocity } (\vec{v}) = \frac{\text{displacement}}{\text{time}} = \frac{d\vec{x}}{dt} \qquad \text{or,} \qquad \vec{v} = \frac{\vec{x}_2 - \vec{x}_1}{t_2 - t_1} \text{ m/s}.$$

  where $\vec{x}_1$ and $\vec{x}_2$ are the displacement of an object at instants t_1 and t_2.

  The velocity is a vector quantity. The velocity of an object can be positive, zero or negative.
- If an object undergoes equal displacements in equal intervals of time, it is said to be moving with a **uniform velocity**. If an object undergoes unequal displacements in equal intervals of time or equal displacements in unequal intervals of time, it is said to be moving with a **variable velocity**.
- The average velocity of an object is equal to the ratio of the total displacement, to the total time interval for which the motion takes place.

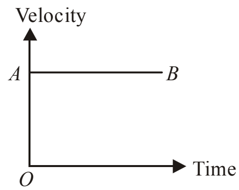
$$\text{Average velocity} = \frac{\text{total displacement}}{\text{total time}}.$$

- The velocity of an object at a given instant of time is called **instantaneous velocity**.

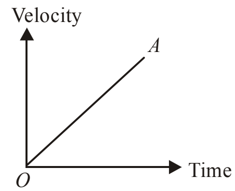
  $\vec{v} = d\vec{x}/dt$

  When a body is moving with a uniform velocity, its instantaneous velocity = average velocity = uniform velocity.

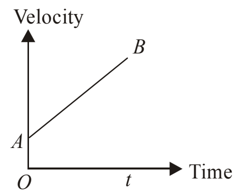
- When a body is moving with a constant velocity, the velocity-time graph is a straight line *AB* parallel to time axis.



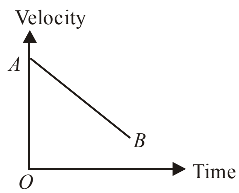
- When a body is moving with a constant acceleration and its initial velocity is zero, the velocity-time graph is an oblique straight line, passing through origin.



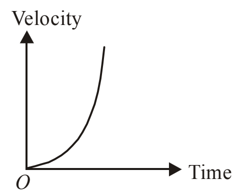
- When a body is moving with a constant acceleration and its initial velocity is not zero, the velocity-time graph is an oblique straight line *AB* not passing through origin.



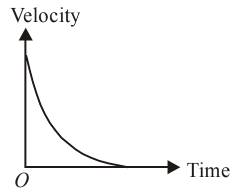
- When a body is moving with a constant retardation and its initial velocity is not zero, the velocity-time graph is an oblique straight line *AB*, not passing through origin.



- When a body is moving with increasing acceleration, the velocity-time graph is a curve which bend upwards.



- When a body is moving with decreasing acceleration, the velocity-time graph is a curve is similar to the one shown here.



- Slope of displacement-time graph gives **average velocity**.
- Slope of velocity-time graph gives **average acceleration**.
- The area of velocity-time graph with time axis gives total distance covered by the body.
- When a body is dropped freely from the top of the tower and another body is projected horizontally from the same point, both will reach the ground at the same time.
- In case of motion under gravity, motion is independent of the mass of the body, *i.e.* if a heavy and light body are dropped from the same height, they reach the ground simultaneously with the same speed because time taken by the body to reach the ground $= \sqrt{2h/g}$.
- **Relative velocity :** The relative velocity of one body with respect to another body is the velocity with which one body moves with respect to another body. If $\vec{v}_A$ and $\vec{v}_B$ are the velocities of two bodies A and B, and θ is the angle between them, then relative velocity of body A with respect to B is given by

  $$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

  where, $$|\vec{v}_{AB}| = \sqrt{|\vec{v}_A|^2 + |\vec{v}_B|^2 + 2|\vec{v}_A||\vec{v}_B|\cos(180° - \theta)}$$

  $$= \sqrt{|\vec{v}_A|^2 + |\vec{v}_B|^2 - 2|\vec{v}_A||\vec{v}_B|\cos\theta}$$

  and $$\tan\beta = \frac{|\vec{v}_B|\sin(180° - \theta)}{|\vec{v}_A| + |\vec{v}_B|\cos(180° - \theta)} = \frac{|\vec{v}_B|\sin\theta}{|\vec{v}_A| - |\vec{v}_B|\cos\theta}$$

  Here, β is the angle which $\vec{v}_{AB}$ makes with the direction of $\vec{v}_A$.
- If the two bodies A and B are moving in opposite directions with velocities $\vec{u}$ and $\vec{v}$ then, relative velocity of A with respect to B is equal to $(\vec{u} + \vec{v})$.
- If the two bodies A and B are moving with velocities $\vec{u}$ and $\vec{v}$ in the same direction, then relative velocity of A with respect to B is equal to $(\vec{u} - \vec{v})$.
- If rain drops are falling vertically with a velocity $\vec{v}$ and a person is walking horizontally with a velocity $\vec{u}$, then he should hold an umbrella at angle θ with vertical given by $\tan\theta = \frac{|\vec{u}|}{|\vec{v}|}$, to prevent himself from being wet.
- The **acceleration** of an object is defined as the time rate of change of velocity of the object,

  i.e. acceleration = change in velocity/time taken.

  Acceleration is a vector quantity. Acceleration is positive, if the velocity is increasing and is negative if velocity is decreasing. The negative acceleration is called retardation or deceleration.

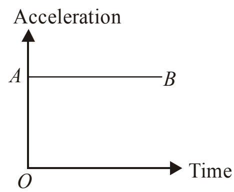
$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

- If the velocity of an object changes by equal amounts in equal intervals of time, it is said to be moving with a **uniform acceleration**. If the velocity of an object changes by unequal amounts in equal intervals of time, it is said to be moving with a **variable acceleration**.
- The **average acceleration** of an object for a given motion is defined as the ratio of the total change in velocity of the object during motion to the total time taken.

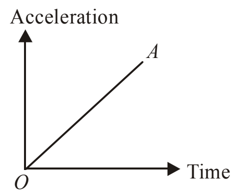
  i.e. average acceleration $= \dfrac{\text{total change in velocity}}{\text{total time taken}}$
- The acceleration of an object at a given instant or at a given point of motion is called its **instantaneous acceleration**. It is defined as the first time derivative of velocity at a given instant or it is also equal to the second time derivative of the position of the object at a given instant.

  i.e. instantaneous acceleration,

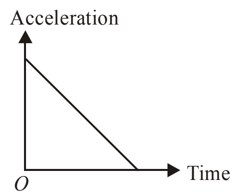
  $$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$$
- Negative acceleration is known as **retardation**. It indicates that velocity of the object is decreasing with respect to time.
- When a body is moving with constant acceleration, the acceleration-time graph is a straight line *AB* parallel to time axis.



- When a body is moving with constant increasing acceleration, the acceleration-time graph is a straight line *OA*.



- When a body is moving with constant decreasing acceleration, the acceleration-time graph is a straight line.



- **Equations of motion** for uniformly accelerated motion along a straight line.

  (i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$

  (iii) $v^2 = u^2 + 2as$ (iv) $s_n = u + \frac{a}{2}(2n-1)$

  where u is initial velocity, v is final velocity, a is uniform acceleration, s is distance travelled in time t, s_n is distance covered in n^{th} second. These equations are not valid if the acceleration is non-uniform.

- **Scalars :** These are those quantities which have only magnitudes but no direction. For example mass, length, time, speed, work, temperature, etc.

- **Vectors :** These are those quantities which have magnitude as well as direction. For example displacement, velocity, acceleration, force, momentum, etc.

- **Tensor :** A physical quantity which has different values in different directions at the same point is called a tensor. Pressure, stress, modulii of elasticity, moment of inertia, radius of gyration, refractive index, wave velocity, dielectric constant, conductivity, resistivity and density are a few examples of tensor. Magnitude of tensor is not unique.

- **Representation of a vector :**
  - A vector is represented by a straight line.
  - It carries an arrow head at one extremity.
  - The arrow head represents direction of vector.
  - The length of line represents magnitude of vector.
  - Tail is the point from which the vector originates.
  - Head is a point at which the vector ends.

- **Addition of vectors - Properties :**

  (a) Vector addition is commutative, *i.e.* $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.

  (b) Vector addition is associative. It means that

  $\vec{A} + (\vec{B} + \vec{C}) = \vec{B} + (\vec{C} + A) = \vec{C} + (\vec{A} + \vec{B})$.

  (c) Vector addition is distributive. It means that

  $m(\vec{A} + \vec{B}) = m\vec{A} + m\vec{B}$

  $(m + n)\vec{A} = m\vec{A} + n\vec{A}$

  (d) The maximum value of vector addition is $(A + B)$.

  (e) The minimum value of vector addition is $(A - B)$.

- **Rotation of a vector :**

  (a) If the frame of reference is rotated or translated, the given vector does not change. The components of the vector may, however, change.

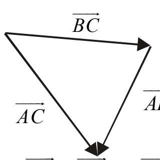
  (b) If a vector is rotated through an angle θ, which is not an integral multiple of 2π, the vector changes.

- **Direction cosines of vector $\vec{A}$:**

  $\cos\alpha = \frac{A_x}{A}$, $\cos\beta = \frac{A_y}{A}$, $\cos\gamma = \frac{A_z}{A}$

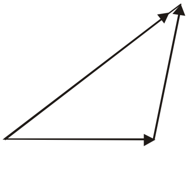
  $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

- Graphically a vector A is represented by a directed segment of a straight line, whose direction is that of the vector it represents and whose length corresponds to the magnitude $|\vec{A}|$ of $\vec{A}$.
- A **unit vector** of a given vector $\vec{A}$ is a vector of unit magnitude and has the same direction as that of the given vector. A unit vector of $\vec{A}$ is written as $\hat{A}$, where $\hat{A} = \vec{A}/|\vec{A}|$. A unit vector is unitless and dimensionless vector and represents direction only.
- The symbol $\hat{i}, \hat{j}, \hat{k}$ represent unit vectors along x, y and z directions of coordinate axes respectively.
- **Null vector** is a vector which has zero magnitude and an arbitrary direction. It is represented by $\vec{0}$ and is also known as **zero vector**. Velocity of a stationary object, acceleration of an object moving with uniform velocity and resultant of two equal and opposite vectors are the examples of null vector.
- **Equal vectors :** Two vectors are said to be equal if they have equal magnitude and same direction.
- A **negative vector** of a given vector is a vector of same magnitude but acting in a direction opposite to that of the given vector. The negative vector of $\vec{A}$ is represented by $-\vec{A}$.
- A vector whose initial point is fixed is called a **localised vector** and whose initial point is not fixed is called **non-localised vector**.
- **Multiplication of a vector by a real number :** When a vector $\vec{A}$ is multiplied by a real number n, it becomes another vector $n\vec{A}$. Its magnitude becomes n times the magnitude of $\vec{A}$. Its direction is same or opposite as that of $\vec{A}$, according as n is positive or negative real number. The unit of $n\vec{A}$ is the same as that of $\vec{A}$.
- **Multiplication of a vector by a scalar :** When a vector $\vec{A}$ is multiplied by a scalar S, it becomes a vector $S\vec{A}$, whose magnitude is S times the magnitude of $\vec{A}$ and it acts along the direction of $\vec{A}$. The unit of $S\vec{A}$ is different from the unit of vector $\vec{A}$.
- For the **addition of two vectors**, represent these two vectors by arrowed lines using the same suitable scale. Displace the second vector such that its tail coincides with the head of the first vector. Then the single vector, drawn from the tail of the first vector to the head of the second vector represents the resultant vector.



$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

- **Triangle law of vectors :** If two vectors acting simultaneously at a point are represented in magnitude and direction by two sides of a triangle taken in the same order, then their resultant is represented by the third side of the triangle taken in the opposite order.



According to triangle law of vector addition.

$$\vec{P} + \vec{Q} = \vec{R} \quad \text{or} \quad \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

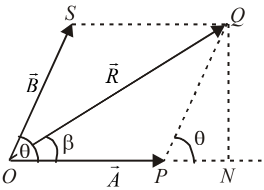
or $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{AC} + \overrightarrow{CA}$

(adding vector $\overrightarrow{CA}$ to both sides of the equation).

or $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{AC} - \overrightarrow{AC}$ $\left[\because \overrightarrow{CA} = -\overrightarrow{AC}\right]$

$= 0.$

- **Parallelogram law of vectors:** It states that if two vectors acting on a particle at the same time be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point.



If $\vec{R}$ is the resultant of $\vec{A}$ and $\vec{B}$, then

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad \text{and} \quad \tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

Note : The magnitude of the resultant vector is maximum if the two vectors are acting in the same direction and is minimum if the two vectors are acting in the opposite directions.

- **Polygon law of vectors :** It states that if number of vectors acting on a particle at a time are represented in magnitude and direction by the various sides of an open polygon taken in same order, their resultant vector is represented in magnitude and direction by the closing side of the polygon taken in opposite order. In fact polygon law of vectors is the outcome of triangle law of vectors.

- **Subtraction of vectors :** Subtraction of a vector $\vec{B}$ from a vector $\vec{A}$ is defined as the addition of vector $-\vec{B}$ (negative of vector $\vec{B}$) to vector $\vec{A}$.

  Thus, $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

- **Rectangular components of a vector in a plane**

  When a vector is splitted into two component vectors at right angles to each other, the component vectors are called rectangular components of a vector. If $\vec{A}$ makes an angle θ with *x*-axis and $\vec{A}_x$ and $\vec{A}_y$ are the rectangular components of $\vec{A}$ along *x*-axis and *y*-axis respectively, then

  $\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{i} + A_y\hat{j}$

  Here, $A_x = A\cos\theta$ and $A_y = A\sin\theta$

  $\therefore \quad A^2(\cos^2\theta + \sin^2\theta) = A_x^2 + A_y^2$

  or, $A = (A_x^2 + A_y^2)^{1/2}$ and $\tan\theta = A_y/A_x$.

- **Dot product of two vectors :** The dot product of two vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A}\cdot\vec{B}$ and is given by $\vec{A}\cdot\vec{B} = AB\cos\theta$, where θ is the smaller angle between $\vec{A}$ and $\vec{B}$. The dot product of two vectors is a scalar.

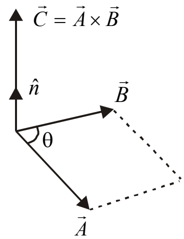
- **Geometrical interpretation of dot product of two vectors :** It is the product of the magnitude of one vector with the magnitude of the component of other vector in the direction of first vector.

- $\vec{A}\cdot\vec{A} = A^2$ or $A = (\vec{A}\cdot\vec{A})^{1/2}$
- $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$ and $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$
- **Dot product in cartesian co-ordinates**

  $\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

  $= A_xB_x + A_yB_y + A_zB_z.$

  $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$.



- **Cross product of two vectors :** The cross product of two vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A}\times\vec{B}$. It is a vector whose magnitude is equal to the product of the magnitudes of the two vectors and sine of the smaller angle between them. If θ is smaller angle between $\vec{A}$ and $\vec{B}$, then $\vec{A}\times\vec{B} = \vec{C} = AB\sin\theta\,\hat{n}$,

  where $\hat{n}$ is a unit vector in the direction of $\vec{C}$.
- **Geometrical interpretation of vector product of two vectors:** The magnitude of vector product of two vectors is equal

  (i) to the area of the parallelogram whose two sides are represented by two vectors.

  (ii) to twice the area of a triangle whose two sides are represented by the two vectors.
- **Properties of cross product**

  (i) Cross product of two parallel vectors is zero. So, $\vec{A}\times\vec{A} = 0$.

  (ii) $\hat{i}\times\hat{i} = 0 = \hat{j}\times\hat{j} = \hat{k}\times\hat{k}$

  (iii) $\hat{i}\times\hat{j} = \hat{k},\ \hat{j}\times\hat{k} = \hat{i},\ \hat{k}\times\hat{i} = \hat{j}$

  (iv) $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$
- **Cross product in cartesian co-ordinates:**

  $\vec{A}\times\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\times(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

  $$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

  $= (A_yB_z - A_zB_y)\hat{i} - (A_xB_z - B_xA_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$
- **Direction of vector cross product :**

  (a) When $\vec{C} = \vec{A}\times\vec{B}$, the direction of $\vec{C}$ is at right angles to the plane containing the vectors $\vec{A}$ and $\vec{B}$. The **direction** is determined by the right hand screw rule and the right hand thumb rule.

  (b) **Right hand screw rule :** Rotate a right handed screw from first vector $(\vec{A})$ towards second vector $(\vec{B})$. The direction in which the right handed screw moves gives the direction of vector $\vec{C}$.

  (c) **Right hand thumb rule :** Curl the fingers of your right hand from $\vec{A}$ to $\vec{B}$. Then the direction of the erect thumb will point in the direction of $\vec{A}\times\vec{B}$.

Angular variables

- Angular displacement is the angle which the position vector sweeps out in a given interval of time. It is represented by θ. In SI, the unit of angular displacement is radian (rad). It is a dimensionless quantity.
- **Angular velocity :** The rate of change of angular displacement is called the angular velocity. It is represented as ω. In SI, the unit of angular velocity is radian per second (rad sec^{-1}), its dimensional formula is $[M^0L^0T^{-1}]$.

  Average angular velocity = $\frac{\Delta\theta}{\Delta t}$.

  Instantaneous angular velocity $= \lim_{\Delta t \to \infty} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \omega$.

  Relation between angular and linear velocity $v = r\omega$.
- **Time period :** The time taken to complete one revolution is called the time period. It is denoted by T.
- **Angular acceleration :** The rate of change of angular velocity is called angular acceleration. It is denoted as α and its unit is radian sec^{-2} (rad sec^{-2}). Dimensions are $[T^{-2}]$.

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2}.$$

- Relation between linear acceleration and angular acceleration

$$a = r\alpha.$$

- **Frequency :** The frequency is defined as the number of revolutions completed per second. It is denoted by υ.

$$\upsilon = \frac{1}{T}. \text{ Its unit is } s^{-1}.$$

Kinematical equations in circular motion

- (i) The angular velocity of the object after time t is given by $\omega = \omega_0 + \alpha t$.

  (ii) The angular displacement of the object after time t is given by

  $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$.

  (iii) If the object attains angular velocity ω, while it covers angular displacement θ, then

  $\omega^2 - \omega_0^2 = 2\alpha\theta$.

CIRCULAR MOTION

- In physics, circular motion is movement of an object with constant speed around in a circle in a circular path or a circular orbit.
- Circular motion involves acceleration of the moving object by a centripetal force which pulls the moving object towards the centre of the circular orbit. Without this acceleration, the object would move inertially in a straight line, according to Newton's first law of motion. Circular motion is accelerated even though the speed is constant, because the velocity of the moving object is constantly changing.

- Examples of circular motion are: an artificial satellite orbiting the earth in geosynchronous orbit, a stone which is tied to a rope and is being swung in circles (cf. hammer throw), a racecar turning through a curve in a racetrack, an electron moving perpendicular to a uniform magnetic field, a gear turning inside a mechanism.
- A special kind of circular motion is when an object rotates around itself. This can be called **spinning motion**.
- Circular motion is characterized by an orbital radius r, a speed v, the mass m of the object which moves in a circle, and the magnitude F of the centripetal force. These quantities are all related to each other through the equations for circular motion.
- The centripetal force can be tension of the string, gravitational force, electrostatic force or Lorentzian. But the centrifugal force $= m\omega^2 r$ or mv^2/r in stable rotation, is equal to the centripetal force in magnitude and acts outwards.
- A centripetal force of magnitude $\frac{mv^2}{r}$ is needed to keep the particle in uniform circular motion.
- Centrifugal force is the force acting away from the centre and is equal in magnitude to the centripetal force.
- For a safe turn the co-efficient of friction between the road and the tyre should be,

  $\mu_s \geq \frac{v^2}{rg}$

  where v is the velocity of the vehicle

  r is the radius of the circular path
- Angle of banking, $\tan\theta = \frac{v^2}{rg}$ this θ depends on the speed, v and radius of the turn r.
- A cyclist provides himself the necessary centripetal force by leaning inward on a horizontal track.
- The maximum permissible speed for the vehicle is much greater than the optimum value of the speed on a banked road. It is because, friction between road and the tyre of the vehicle also contributes to the required centripetal force.
- Roads are usually banked for the average speed of vehicle passing over them. If μ is the coefficient of friction between the tyres and the road the safe value limit is $v = \sqrt{\frac{rg\tan\theta + \mu}{1 - \mu\tan\theta}}$.
- In case of vertical circle the minimum velocity v, the body should possess at the top so that the string does not slack, is $\sqrt{gr}$.
- The magnitude of velocity at the lowest point with which body can safely go round the vertical circle of radius r is $\sqrt{5gr}$.
- Tension in the string at lowest point $T = 6Mg$.
- The tangent at every point of the circular motion gives the direction of motion in circular motion at that point.

- A car some times overturns while taking a turn. When it overturns it is the inner wheel, which leaves the ground first.
- A car when passes a convex bridge exerts a force on it which is equal to $Mg - \frac{Mv^2}{r}$.
- The driver of a car should brake suddenly rather than taking sharp turn to avoid accident, when he suddenly sees a broad wall in front of him.

Uniform horizontal circular motion

- The instantaneous velocity and displacement act along tangent to the circle at a point.
- The centripetal acceleration and the centripetal force act along radius towards the centre of circle.
- The centripetal force and displacement are at right angles to each other. Hence the work done by the centripetal force is zero.
- Kinetic energy of a particle performing uniform circular motion, in horizontal plane, remains constant.
- The instantaneous velocity of particle and the centripetal acceleration are at right angles to each other. Hence the magnitude of velocity does not change but the direction of velocity changes continuously. It is thus a case of **uniformly accelerated motion**.
- Centripetal acceleration is also called radial acceleration as it acts along radius of circle.
- Momentum of the particle changes continuously along with the velocity.
- The centripetal force does not increase the kinetic energy and angular momentum of the particle moving in a circular path.

Non-uniform horizontal circular motion

- If the magnitude of the velocity of the particle in horizontal circular motion changes with respect to time, the motion is known as non-uniform circular motion.
- The acceleration of particle is called tangential acceleration. It acts along the tangent to the circle at a point. It changes the magnitude of linear velocity of the particle.
- Tangential acceleration $\vec{f}_T$ and angular acceleration $\vec{\alpha}$ are related as $\vec{f}_T = \vec{r} \times \vec{\alpha}$ where $\vec{r}$ denotes radius vector .
- Centripetal acceleration f_C and tangential acceleration f_T act at right angles to each other.

  $\therefore \; f^2 = f_C^2 + f_T^2 = \left(\frac{v^2}{r}\right)^2 + f_T^2.$
- $\tan \phi = \frac{f_T}{f_C} = \frac{r\alpha}{v^2/r} = \frac{r^2\alpha}{v^2}.$

PROJECTILE MOTION

- Anything thrown in space and then allowed to move under the effect of gravity alone is called **projectile**.

- The trajectory of the projectile is a parabola if the particle is projected at an angle θ with the horizontal. θ is between 0 and 90° *i.e.* 0° < θ < 90°.
- The initial velocity u of the projectile can be resolved into two components. (i) $u\cos\theta$ along horizontal direction (ii) $u\sin\theta$ along vertical direction.
- There is no acceleration in the horizontal direction, thus horizontal component of velocity remains the same throughout the motion.
- The vertical component of velocity at the highest point of the path is zero. Only the horizontal component $u\cos\theta$ remains at the highest point of path.
- Horizontal range is same whether the particle is projected at an angle θ or (90° – θ), *i.e.* the range is same for 30° as well as 60° projection angle but the maximum height more for bigger angles.
- **Equation of path** is $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$
- **Time of flight** is defined as the total time for which the projectile remains in air.

$$T = \frac{2u\sin\theta}{g}$$

- **Maximum height** is defined as the maximum vertical distance covered by projectile.

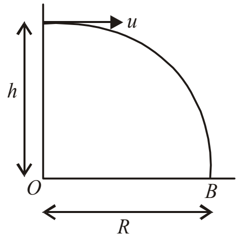
$$H = \frac{u^2\sin^2\theta}{2g}$$

- **Horizontal range** is defined as the maximum distance covered in horizontal distance.

$$R = \frac{u^2\sin 2\theta}{g}$$

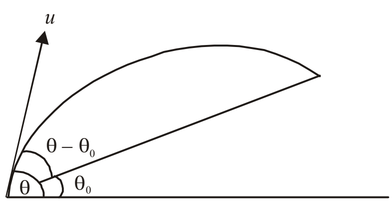
- If the particle is projected from the top of the tower of height h in horizontal direction, then the height of tower, range and time of flight are related as

$$h = \frac{1}{2}gt^2 \quad \Rightarrow \quad R = ut$$



- **Projectile thrown on an inclined plane**

When a projectile is thrown with a velocity u, making an angle θ with the horizontal direction, up the inclined plane while the inclination of the plane with the horizontal direction is θ_0, then



(i) Range of projectile along the inclined plane = R,

$$R = \frac{u^2}{g\cos^2\theta}\ [\sin(2\theta - \theta_0) - \sin\theta_0).$$

(ii) Time of flight on the inclined plane = T.

$$T = \frac{2u\sin(\theta - \theta_0)}{g}$$

(iii) Maximum range on inclined plane = R_{max},

$$R_{max} = \frac{u^2}{g(1+\sin\theta_0)}$$

(iv) The angle at which the horizontal range on the inclined plane becomes maximum

= θ, $\theta = \frac{\pi}{4} + \frac{\theta_0}{2}$.

(v) $T_{max} = \frac{2u}{g}\sin\left(\frac{\pi}{4} - \frac{\theta_0}{2}\right)$

Some salient points about angular projection of projectiles

- Horizontal range of projectile is maximum (= u^2/g) for a given velocity u if its angle of projection is 45°.
- **Horizontal range** (R) will be same if angle of projection is (a) θ or $(90° - \theta)$

  (b) $(45° + \theta)$ or $(45° - \theta)$.
- **Velocity** of projectile is minimum at the highest point of projection (= $u\cos\theta$) and is maximum at the point of projection (= u) or at the point of striking the ground (= u).
- **Linear momentum** at highest point = $mu\cos\theta$.
- Linear momentum at lowest point = mu.
- **Kinetic energy** at highest point = $\frac{1}{2}m(u\cos\theta)^2$.
- Kinetic energy at lowest point = $\frac{1}{2}mu^2$.
- **Acceleration of** projectile is constant throughout the motion and it acts vertically downwards being equal to g.
- **Angular momentum** of projectile = $(mu\cos\theta) \times h$, where h denotes the height.
- In case of angular projection, the angle between velocity and acceleration varies from $0° < \theta < 180°$,
- The maximum height occurs when the projectile covers a horizontal distance equal to half of the horizontal range, *i.e.* $R/2$.
- When the maximum range of projectile is R, then its maximum height is $R/4$.
- Velocity and acceleration are at right angles to each other at the highest point of journey.