Summary

Gravitation

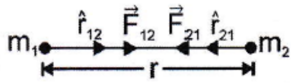
- **Newton's law of gravitation**:

Gravitational attraction force between two point masses

$F_g = \frac{Gm_1m_2}{r^2}$ and its direction will be attractive.

- **Gravitational force on (1) due to (2) in vector form**

$\vec{F}_{12} = \frac{Gm_1m_2}{r^2}\hat{r}_{12}$



- **Gravitational Field**:- Gravitational force acting on unit mass

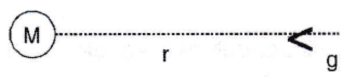
$g = \frac{F}{m}$

- **Gravitational Potential**: - Gravitation potential energy of unit mass

$V_g = \frac{U}{m} \Rightarrow g = -\frac{dV_g}{dr}$ and $V_B - V_A = -\int_A^B \vec{g}.d\vec{r}$

  - For point mass:-

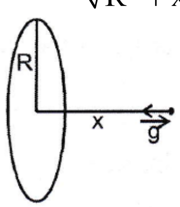
$g = \frac{GM}{r^2}, V = -\frac{GM}{r}$



  - For circular ring

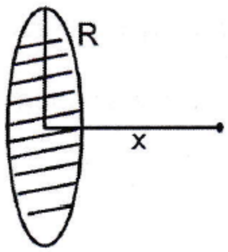
$g = \frac{GMx}{\left(R^2 + x^2\right)^{3/2}}$

$V = -\frac{GM}{\sqrt{R^2 + x^2}}$



  - For thin circular disc:

$g = \frac{2GM}{R^2}\left[1 - \frac{1}{\sqrt{1+\left(\frac{R}{x}\right)^2}}\right]$

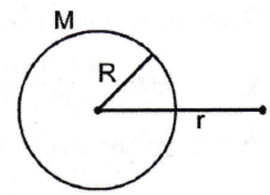


$$V=\frac{-2GM}{R^2}\left(\sqrt{R^2+x^2}-x\right)$$

- Uniform thin spherical shell :-

$$g_{out}=\frac{GM}{r^2}\Rightarrow g_{surface}=\frac{GM}{R^2}$$

$$g_{in}=0$$



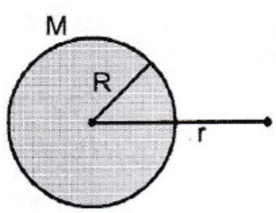
- **Potential :**

$$V_{out}=-\frac{GM}{r}\Rightarrow V_{surface}=-\frac{GM}{R}\Rightarrow V_{in}=-\frac{GM}{R}$$

- Uniform solid sphere:- (Most lmportant)

$$g_{out}=\frac{GM}{r^2}\Rightarrow g_{surface}=\frac{GM}{R^2}$$

$$g_{in}=\frac{GM}{R^3}r\Rightarrow g_{centre}=0$$



- **Potential:**

$$V_{out}=-\frac{GM}{r}\Rightarrow V_{in}=-\frac{GM}{2R^3}\left(3R^2-r^2\right)$$

$$V_{surface}=-\frac{GM}{R}\Rightarrow V_{centre}=-\frac{3}{2}\frac{GM}{R}$$

- **Self energy :**

Self energy of hollow sphere $=U_{self}=-\frac{1}{2}\frac{GM^2}{R}$

Gravitational Self energy of a Uniform Sphere $= U_{self} = -\frac{3}{5}\frac{GM^2}{R}$

Escape speed from earth's surface:

$$V_e = \sqrt{\frac{2GM_e}{R}} = 11.2 km/sec$$

If a satellite is moving around the earth is circular orbit, then its orbital speed is

$$V_o = \sqrt{\frac{GM_e}{r}}$$

where r is distance of satellite from the centre of earth.

PE . of the satellite $= -\frac{GM_e m}{r}$

KE of the satellite $= \frac{1}{2} m v_o^2 = \frac{GM_e m}{2r}$

TE of the satellite $= -\frac{GM_e m}{2r}$

- **Time period of Geo - stationary satellite = 24 hours**

Kapler's laws: -

- Law of Orbit :- lf a ptanet is revolving around a sun, its path is either elliptical (or circular)
- Law of Area : -

View (i) If a planet is revolving around a sun, the angular momentum of the planet about the sun remains conserved

View (ii) The radius vector from the sum to the planet sweeps area at constant rate

Areal velocity = $= \frac{dA}{dt} = \frac{L}{2m} = \text{constant}$

- For all the planets of a sun

$$T^2 \propto R^3 \Rightarrow T^2 = \left[\frac{4\pi^2}{GM_s}\right] R^3$$

- **Factors Affecting Acceleration Due to Gravity:**
  - **Effect of Altitude**: $g_h = \frac{GM_e}{(R_e + h)^2} = g\left(1 + \frac{h}{R_e}\right)^{-2} \simeq g\left(1 - \frac{2h}{R_e}\right) \text{when } h << R$
  - **Effect or depth:** $g_d = g\left(1 - \frac{d}{R_e}\right)$
  - **Effect of, the surface of Earth**

The equatorial radius is about 21 km longer than its polar radius

We know, $g = \frac{GM_e}{R_e^2}$ Hence $g_{pole} > g_{equator}$

- **Effect of rotation of the Earth**

Consider a particle of mass m at latitude θ. $g' = g - \omega^2 R_e \cos^2\theta$

$\text{At pole } \theta = 90°$

$\Rightarrow g_{pole} = g, \quad \text{At equator } \theta = 0$

$\Rightarrow g_{quator} = g - \omega^2 R_e. \quad \text{Hence } g_{pole} > g_{equator}$

Practical Questions

1. body of mass m is moving in a circular orbit of radius R about a planet of mass M. At some instant, it splits into two equal masses. The first mass moves in a circular orbit of radius $\frac{R}{2}$ and the other mass, in a circular orbit of radius $\frac{3R}{2}$. The difference between the final and initial total energies is :

(2018)

(a) $+\frac{Gm}{6R}$
(b) $-\frac{GMm}{2R}$
(c) $-\frac{GMm}{6R}$
(d) $\frac{GMm}{2R}$

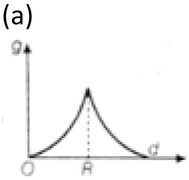
2. Take the mean distance of the moon and the sun from the earth to be 0.4×10^6 km and 150×10^6 km respectively. Their masses are 8×10^{22}kg and 2×10^{30}kg respectively. The radius of the earth is 6400km. Let ΔF_1be the difference in the forces exerted by the moon at the nearest and farthest point on the earth and ΔF_2 be the difference in the force exerted by the sun at the nearest and farthest pointson the earth. Then, the number closest to $\frac{\Delta F_1}{\Delta F_2}$ is : (2018)

(a) 6
(b) 10^{-2}
(c) 2
(d) 0.6

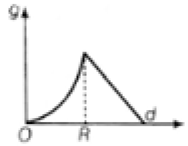
3. Suppose that the angular velocity of rotation of earth is increased. Then as a consequence:
(2018)

(a) Weight of the object everywhere on the earth will decrease
(b) Weight of the object everywhere on the earth will increase
(c) Except at poles weight of the object on the earth will decrease
(d) There will be no change in weight anywhere on the earth

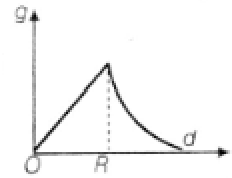
4. The variation of acceleration due to gravity g with distance d from centre of the Earth is best represented by (R = Earth's radius) (2017)
(a)



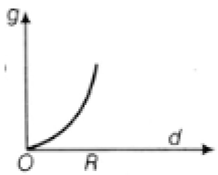
(b)



(c)



(d)



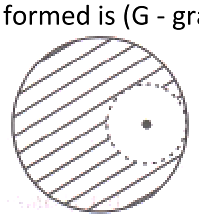
5. If the radius of the earth were to shrink by one per cent, its mass remaining the same, the acceleration due to gravity on the earth's surface would (1981)

(a) decrease
(b) remain unchanged
(c) increase
(d) be zero

6. A satellite is revolving in a circular orbit at a height h from the Earth's surface (radius of earth R, h <<R). The minimum increase in its orbital velocity required, so that the satellite could escape from the Earth's gravitational field, is close to (Neglect the effect of atmosphere) (2016)

(a) $\sqrt{2gR}$
(b) $\sqrt{gR}$
(c) $\sqrt{gR/2}$
(d) $\sqrt{gR}(\sqrt{2}-1)$

7. From a solid sphere of mass M and radius R, a spherical portion of radius $\left(\frac{R}{2}\right)$ is removed as shown in the figure. Taking gravitational potential V = 0 at r= ∞, the potential at the centre of the cavity thus formed is (G - gravitational constant) (2015)



(a) $\frac{-GM}{R}$

(b) $\frac{-GM}{2R}$

(c) $\frac{-2GM}{3R}$

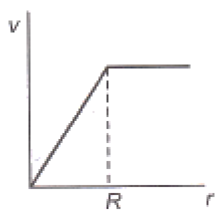
(d) $\frac{-2GM}{R}$

8. A spherically symmetric gravitational system of particles has a mass density $\rho = \begin{cases} \rho_0 & \text{for r} \le \text{R} \\ 0 & \text{for r} > \text{R} \end{cases}$,

where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance r from the centre of the system is represented by

(2008)

(a)



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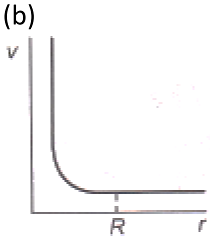

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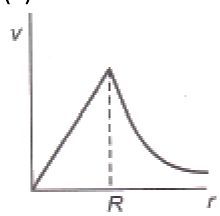
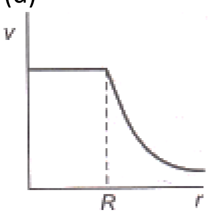


(d)



9. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the Earth, is (1983)

(a) $\frac{1}{2}mgR$

(b) $2mgR$

(c) mgR

(d) $\frac{1}{4}mgR$

10. What is the minimum energy required to launch a satellite of mass in from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R? (2013)

(a) $\frac{5GmM}{6R}$

(b) $\frac{2GmM}{3R}$

(c) $\frac{GmM}{2R}$

(d) $\frac{GmM}{3R}$

11. A satellite is moving with a constant speed v in a circular orbit about the earth. An object of mass m is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection. the kinetic energy of the object is (2011)

(a) $\frac{1}{2}mv^2$

(b) mv^2

(c) $\frac{3}{2}mv^2$

(d) $2mv^2$

12. A geostationary satellite orbits around the earth in a circular orbit of radius 36,000 km. Then, the time period of a spy satellite orbiting a few hundred km above the earth's surface (R_e= 6400km) will approximately be (2002)

(a) 1/2 h

(b) 1 h

(c) 2 h

(d) 4 h

13. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth (1998)

(a) the acceleration of S is always directed towards the centre of the earth

(b) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant
(c) the total mechanical energy of S varies periodically with time
(d) the linear momentum of S remains constant in magnitude

14. A double star system consists of two stars A and B which have time periods T_A and T_B. Radius R_A and R_B and mass M_A and M_B. Choose the correct option. (2006)
(a) If $T_A > T_B$ then $R_A > R_B$
(b) $T_A > T_B$ then $M_A > M_B$
(c) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{R_A}{R_B}\right)^2$
(d) $T_A = T_B$

15. If the distance between the earth and the sun were half its present value, the number of days in a year would have been (1996)
(a) 64.5
(b) 129
(c) 182.5
(d) 730

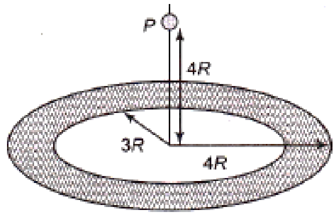
16. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$, then (1989)
(a) T^2 is proportional to R^2
(b) T^2 is proportional to $R^{7/2}$
(c) T^2 is proportional to $R^{3/2}$
(d) T^2 is proportional to $R^{3.75}$

17. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction, the speed of each particle is (2014)
(a) $\sqrt{\frac{GM}{R}}$
(b) $\sqrt{2\sqrt{2}\frac{GM}{R}}$
(c) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$
(d) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

18. A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of T_2/T_1 is
(2001)

(a) 1
(b) $\sqrt{2}$
(c) 4
(d) 2

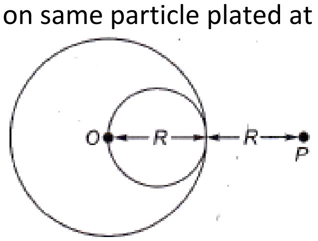
19. A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is (2010)



(a) $\frac{2GM}{7R}(4\sqrt{2}-5)$
(b) $-\frac{2GM}{7R}(4\sqrt{2}-5)$
(c) $\frac{GM}{4R}$
(d) $\frac{2GM}{5R}(\sqrt{2}-1)$

20. Two spheres of radius r and 2r are touching each other. The force of attraction between them is proportional to
(a) r^6
(b) r^4
(c) r^2
(d) r^{-2}

21. A solid sphere of uniform density and radius R applies a gravitational force of attraction equal to F_1 on a particle placed at P, distance 2R from the centre O of the sphere. A spherical cavity of radius R/2 is now made in the sphere as shown in figure. The sphere with cavity now applies an gravitational force F_2 on same particle plated at P. The ratio F_2/F_1 will be



(a) 1/2
(b) 7/9
(c) 3
(d) 7

22. A uniform ring of mass M and radius r is placed directly above a uniform sphere of mass 8 M and of same radius R. The centre of the ring is at a distance of $d = \sqrt{3}R$ from the centre of the sphere. The gravitational attraction between the sphere and the ring is

(a) $\frac{GM^2}{R^2}$

(b) $\frac{3GM^2}{2R^2}$

(c) $\frac{2GM^2}{\sqrt{2}R^2}$

(d) $\frac{\sqrt{3}GM^2}{R^2}$

23. Imagine a light planet revolving around a very massive star in a circular orbit of radius r with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-3/2}$, then T_2 is proportional to

(a) R^3

(b) $R^{5/2}$

(c) $R^{3/2}$

(d) $R^{7/2}$

24. If a planet of given density were made larger its force of attraction for an object on its surface would increase because of planet's greater mass but would decrease because of the greater distance from the object to the centre of the planet. Which effect predominate?

(a) Increase in mass

(b) Increase in radius

(c) Both affect attraction equally

(d) None of the above

25. Both earth and moon are subject to the gravitational force of the sun. As observed from the sun, the orbit of moon

(a)will be elliptical

(b)will not be strictly elliptical because the total gravitational force on it is not central

(c) is not elliptical but will necessarily be a closed curve

(d) deviates considerably horn being elliptical due toinfluence of planets other than earth

26. Different points in earth are at slightly different distances from the sun and hence experience different forms due to gravitation. For a rigid body, we know that if various forces act at various point in it, the resultant motion is as if a net force acts on the CM (centre of mass) causing translation and a net torque at the CM causing rotation around an axis through the CM For the earth-sun system (approximating the earth as a uniform density sphere)

(a) the torque is zero

(b) the torque causes the earth to spin

(c) the rigid body result is not applicable since the earth is not even approximately a rigid body

(d) the torque causes the earth to mow around the sun

27. Two astronauts have deserted their space ships in a region of space far from the gravitational attraction of any other body. Each has a mass of 100 kg and they are 100 m apart. They are initially at rest relative to one another. How long will it be before the gravitational attraction brings them 1 cm closer together?
(a) 2.52 days
(b) 1.41 days
(c) 0.70 days
(d) 0.41 days

28. If three particles each of mass M are placed at the three corners of an equilateral triangle of side a, the forces exerted by this system on another particle of mass M placed (i) at the mid point of a side and (ii) at the centre of the triangle are respectively
(a) $0, 0$
(b) $\frac{4GM^2}{3a^2}, 0$
(c) $0, \frac{4GM^2}{3a^2}$
(d) $\frac{3GM^2}{a^2}, \frac{GM^2}{a^2}$

29. The gravitational attraction between the two bodies increases when their masses are
(a) reduced and distance is reduced
(b) increased and distance is reduced
(c) reduced and distance is increased
(d)increased and distance is increased

30. A spherical hollow is made in a lead sphere of radius R such that its surface touches the outside surface of the lead sphere and passes through the centre. The mass of the lead sphere before hollowing was M. The force of attraction that this sphere would exert on a particle of mass m which lies at a distance d (> R) from the centre of the lead sphere on the straight line joining the centres of the sphere and the hollow is
(a) $\frac{GMm}{d^2}$
(b) $\frac{GMm}{8d^2}$
(c) $\frac{GMm}{d^2}\left[1+\frac{1}{8\left(1+\frac{R}{2d}\right)}\right]$
(d) $\frac{GMm}{d^2}\left[1-\frac{1}{8\left(1-\frac{R}{2d}\right)^2}\right]$

31. If suppose moon is suddenly stopped and then released (given radius of moon is one-fourth the radius of earth) and the acceleration of moon with respect to earth is 0.0027 ms^{-2}), then the acceleration of the moon just before striking the earth's surface is (Take g = 10 ms^{-2})
(a) 0.0027 ms^{-2}
(b) 5.0 ms^{-2}
(c) 6.4 ms^{-2}
(d) 10 ms^{-2}

32. The acceleration due to gravity on a planet is 1.96 ms^{-2}. If it is safe to jump from a height of 3 m on the earth, the corresponding height on the planet will be
(a) 3 m
(b) 6m
(c) 9 m
(d) 15m

33. The mass of the moon is 1/8 of the earth but the gravitational pull is 1/6 of the earth. It is due to the fact that
(a) moon is the satellite of the earth
(b) the radius of the earth is 8.6 the moon
(c) the radius of the earth is $\sqrt{8/6}$ of the moon
(d) the radius of the moon is 6/8 of the earth

34. If different planets have the same density but different radii, then the acceleration due to gravity on the surface of the planet is related to the radius (R) of the planet as
(a) $g \propto R^2$
(b) $g \propto R$
(c) $g \propto \frac{1}{R^2}$
(d) $g \propto \frac{1}{R}$

35. A thief stole a box full of valuable articles of weight w and while carrying it on his head jumped down from a wall of height It from the ground. Before he reaches the ground, he experienced a load (a) zero
(b) w/2
(c) w
(d) 2w

36. Assuming the earth to be a sphere of uniform mass density, how much would body weigh half way down to the centre of earth if it weighed 260 N on the surface?
(a) 225 N
(b) 325 N
(c) 100 N
(d) 125 N

37. The maximum vertical distance through which a full dressed astronaut can jump on the earth is 0.5 m. Estimate the maximum vertical distance through which he can jump on the moon, which has a mean density 2/3rd that of earth and radius one quarter that of the earth
(a) 1.5 m
(b) 3 m
(c) 6 m
(d) 7.5 m

38. In the above problem, the ratio of the time duration of hisjump on the moon to that of hisjump on the earth is
(a) $1:6$
(b) $6:1$
(c) $\sqrt{6}:1$
(d) $1:\sqrt{6}$

39.Particles of masses 2M, m and M are respectively at 1 points A, B and C with $AB = \frac{1}{2}(BC)$. m is much-much smaller than M and at time t =0, they are all at rest

(a) m will remain at rest
(b) m will move towards M
(c) m will move towards 2M
(d) m will have oscillatory motion

40. The earth is an approximate sphere. If the interior contained matter which is not of the same density everywhere, then on the surface of the earth, the acceleration due to gravity
(a) will be directed towards the centre but not the same everywhere
(b) will have the same value everywhere but not directed towards the centre
(c) will be same everywhere in magnitude directed towards the centre
(d) cannot be zero at any point

41. The masses and radii of the earth and moon are M_1, R_1 and M_2, R_2 respectively. Then centres are distanced d apart. The minimum velocity with which a particle of mass M should be projected from a point midway between their centres so that it escapes to infinity is
(a) $2\sqrt{\frac{G}{d}(M_1 + M_2)}$
(b) $2\sqrt{\frac{2G}{d}(M_1 + M_2)}$
(c) $2\sqrt{\frac{GM}{d}(M_1 + M_2)}$
(d) $2\sqrt{\frac{GM(M_1 + M_2)}{d(R_1 + R_2)}}$

42. At a given place where, acceleration due to gravity is g ms^{-2}, a sphere of lead of density d kgm^{-3} t is gently released in a column of liquid of density ρ kgm^{-3}. If d> p, the sphere will

(a) fall vertically with an acceleration of g ms^{-2}

(b) fall vertically with no acceleration

(c) fall vertically with an acceleration $g\left(\frac{d-\rho}{d}\right)$

(d) fall vertically with an acceleration ρ/d

43. What is the height, the weight of body will be the same as at the same depth from the surface of the earth? Radius of earth is R?

(a) $\frac{R}{2}$

(b) $\sqrt{5}R-R$

(c) $\frac{\sqrt{5}R-R}{2}$

(d) $\frac{\sqrt{3}R-R}{2}$

44. There is a mine of depth about 2.0 km. In this mine the conditions as compared to those at the surface are

(a) lower air pressure, higher acceleration due to gravity

(b) higher air pressure, lower acceleration due to gravity

(c) higher air pressure, higher acceleration due to gravity

(d) lower air pressure, lower acceleration due to gravity

45. A clock S is based on oscillation of a spring and a clock Pis based on pendulum motion. Both clock run at the same rate on earth. On a planet having the same density as earth but twice the radius,

(a) S will run faster than P

(b) P will run faster than S

(c) both will run at the same rate as on the earth

(d) both will run at the same rate which will be different from that on the earth

46. If the radius of the earth were to shrink by 1% its mass remaining same, the acceleration due to gravity on the earth's surface would

(a) decrease by 2%

(b) remain unchanged

(c) increase by 2%

(d) become zero

47. Two spherical planets A and B have same mass but densities in the ratio 8: 1. For these planets, the ratio of acceleration due to gravity at the surface of A to its value at the surface of B is

(a) 1 : 4

(b) 1 : 2

(c) 4 :1

(d) 8:1

48. The height at which the acceleration due to gravity decreases by 36% of its value on the surface of the earth. (The radius of the earth is R).

(a) $\frac{R}{6}$

(b) $\frac{R}{4}$

(c) $\frac{R}{2}$

(d) $\frac{2}{3}R$

49. If the value of g acceleration due to gravity at earth surface is 10 ms^{-2}, its value in ms^{-2} at the centre of the earth, which is assumed to be a sphere of radius R metre and uniform mass density is (a) 5

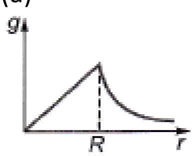
(b) 10/R

(c) 10/2R

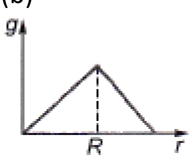
(d) zero

50. When of the following graphs correctly represents the variation of g on earth?

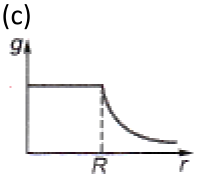
(a)



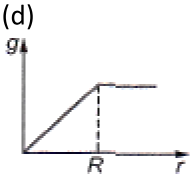
(b)



(c)



(d)



ANSWER KEYS

1. (c) 2. (c) 3. (c) 4. (c) 5. (c) 6. (d) 7. (a) 8. (c) 9. (a) 10. (a) 11. (b) 12. (c) 13. (a) 14. (d) 15. (b) 16. (b) 17. (d) 18. (d) 19. (a) 20. (d) 21. (b) 22. (d) 23. (c) 24. (a) 25. (b) 26. (a) 27. (b) 28. (b) 29. (b) 30. (d) 31. (c) 32. (d) 33. (c) 34. (b) 35. (a) 36. (d) 37. (b) 38. (b) 39. (c) 40. (d) 41. (a) 42. (c) 43. (c) 44. (b) 45. (b) 46. (c) 47. (c) 48. (b) 49. (d) 50. (a)

"Detail solutions are mentioned in the content library"