Mathematical Induction

Question1

Product of any r consecutive natural numbers is always divisible by

WB JEE 2009

Options:

A. r!

B. (r + 4)!

C. (r + 1)!

D. (r + 2)!

Answer: A

Solution:

Product of r consecutive natural numbers

= 1 . 2 . 3 . 4 . ... r = r! which is divisible by r! .

Question2

For each n $\in$ N, $2^{3n} - 1$ is divisible by

here N is a set of natural numbers.

WB JEE 2009

Options:

A. 7

B. 8

C. 6

D. 16

Answer: A

Solution:

Let $P(n) = 2^{3n} - 1$

Putting $n = 1$

$P(1) = 2^{3 \cdot 1} - 1 = 7$ which is divisible by 7

Putting $n = 2$

$P(2) = 2^{3 \cdot 2} - 1 = 2^6 - 1 = 63$ which is divisible by 7 and so on

Let $P(k) = 2^{3k} - 1$ is also divisible by 7

$\therefore 2^{3k} - 1 = 7P \Rightarrow 2^{3k} = 7P + 1$

$P(k + 1) = 2^{3(k + 1)} - 1 = 2^{3k} \cdot 2^3 - 1$

$= (7P + 1)8 - 1 = 7 \cdot 8P + 8 - 1 = 7(8P + 1)$

$\therefore$ By process of M.I. P(k) is divisible by 7.

So, $2^{3n} - 1$ is divisible by 7.

Question3

The number $(101)^{100} - 1$ is divisible by

WB JEE 2011

Options:

A. 10^4

B. 10^6

C. 10^8

D. 10^{12}

Answer: A

Solution:

$(1 + 100)^{100} - 1 = (1 + {}^{100}C_1 \cdot 100 + {}^{100}C_2 \cdot 100^2 + ...) - 1$

$= 100 \cdot 100 + {}^{100}C_2 \cdot 100^2 + ...$

$= 10^4(1 + {}^{100}C_2 + ...)$, which is divisible by 10^4.

Question4

The expression $2^{4n} - 15n - 1$, where $n \in \mathbb{N}$ (the set of natural numbers) is divisible by

WB JEE 2025

Options:

A.

125

B.

225

C.

325

D.

425

Answer: B

Solution:

Step 1: Test with n = 1

Let the given expression be $P(n) = 2^{4n} - 15n - 1$.

For $n = 1$:

$P(1) = 2^{4(1)} - 15(1) - 1$

$P(1) = 2^4 - 15 - 1$

$P(1) = 16 - 16 = 0$

Since 0 is divisible by any non-zero number, this result doesn't help us eliminate any options.

Step 2: Test with n = 2

For $n = 2$:

$P(2) = 2^{4(2)} - 15(2) - 1$

$P(2) = 2^8 - 30 - 1$

$P(2) = 256 - 31 = 225$

Step 3: Check the Options

Now, let's see which of the given options divides 225.

- (A) 125: $225 \div 125$ is not an integer.
- (B) 225: $225 \div 225 = 1$. It is divisible.
- (C) 325: $225 \div 325$ is not an integer.
- (D) 425: $225 \div 425$ is not an integer.

Since the expression must be divisible by the same number for all values of n, and for $n = 2$ it's only divisible by 225 among the choices, this must be the correct answer.

⚡ **Pro Tip:** This substitution method is the fastest way to solve such problems in a time-constrained exam. Always start with the smallest natural number ($n = 1$). If it gives 0, move to the next one ($n = 2$).

✍ **Method 2: Detailed Proof (Using Binomial Theorem)**

This method gives a formal proof and explains *why* the expression is always divisible by 225.

Step 1: Rewrite the Expression

The expression is $2^{4n} - 15n - 1$.

We can write 2^{4n} as $(2^4)^n = 16^n$.

So, the expression becomes $16^n - 15n - 1$.

Step 2: Use the Binomial Theorem

The key trick is to write 16 as $(1+15)$.

Now the expression is $(1+15)^n - 15n - 1$.

Let's expand $(1+15)^n$ using the Binomial Theorem:

$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n}b^n$

For $(1+15)^n$:

$= \binom{n}{0}(1)^n(15)^0 + \binom{n}{1}(1)^{n-1}(15)^1 + \binom{n}{2}(1)^{n-2}(15)^2 + \binom{n}{3}(1)^{n-3}(15)^3 + \cdots + \binom{n}{n}(15)^n$

$= 1 + n(15) + \frac{n(n-1)}{2}(15^2) + \frac{n(n-1)(n-2)}{6}(15^3) + \cdots + 15^n$

Step 3: Substitute the Expansion back into the Expression

Now, let's substitute this back into our original expression:

$P(n) = [1 + 15n + \frac{n(n-1)}{2}(15^2) + \ldots] - 15n - 1$

Step 4: Simplify the Expression

The terms cancel out beautifully:

$P(n) = (1-1) + (15n - 15n) + [\frac{n(n-1)}{2}(15^2) + \binom{n}{3}(15^3) + \ldots]$

$P(n) = \frac{n(n-1)}{2}(225) + \binom{n}{3}(15^3) + \cdots + 15^n$

We can factor out $15^2 = 225$ from all the remaining terms:

$P(n) = 225\left[\frac{n(n-1)}{2} + \binom{n}{3}(15) + \ldots\right]$

Since n is a natural number, the term inside the bracket $[\ldots]$ will always be an integer (for $n \geq 2$). For $n = 1$, the value is 0, which is also a multiple of 225.

✅ **Conclusion:** The expression $2^{4n} - 15n - 1$ is always a multiple of 225. Therefore, it is divisible by **225**.

Question5

Let $P(n) = 3^{2n+1} + 2^{n+2}$ where $n \in N$. Then

WB JEE 2023

Options:

A. P(n) is not divisible by any prime integer.

B. there exists prime integer which divides P(n).

C. P(n) is divisible by 5 for all $n \in N$.

D. P(n) is divisible by 3 for all $n \in N$.

Answer: B

Solution:

Given, $P(n) = 3^{2n+1} + 2^{n+2}, n \in N$

At $n = 1, P(1) = 3^3 + 2^3 = 35$

Which is divisible by $(3 + 2) = 5$

i.e. a prime number.

Question6

7^{2n} + 16n -1 (n$\in$ N) is divisible by

WB JEE 2019

Options:

A. 65

B. 63

C. 61

D. 64

Answer: D

Solution:

We have,

$7^{2n} + 16n - 1$

Put n = 1, we get

$7^2 + 16 - 1$

$= 49 + 15$

$= 64$

which is divisible by 64.
